

## Private Equity and Regulatory Capital

Bongaerts, D.; Charlier, E.

*Publication date:*  
2008

[Link to publication](#)

*Citation for published version (APA):*

Bongaerts, D., & Charlier, E. (2008). *Private Equity and Regulatory Capital*. (CentER Discussion Paper; Vol. 2008-52). *Econometrics*.

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

### Take down policy

If you believe that this document breaches copyright, please contact us providing details, and we will remove access to the work immediately and investigate your claim.

No. 2008–52

**PRIVATE EQUITY AND REGULATORY CAPITAL**

By Dion Bongaerts, Erwin Charlier

May 2008

ISSN 0924-7815

# Private Equity and Regulatory Capital\*

Dion Bongaerts<sup>†</sup>      Erwin Charlier<sup>‡</sup>

May 30, 2008

## Abstract

Regulatory Capital requirements for European banks have been put forward in the Basel II Capital Framework and subsequently in the Capital Requirements Directive (CRD) of the EU. We provide a detailed discussion of the capital requirements for private equity investments under the simple risk weight approach, the PD/LGD approach and the internal model approach. For the latter we present a structural model for which we calibrate the parameters from a proprietary dataset. We modify the standard Merton structural model to make it applicable in practice and to capture stylized facts of these investments. We also show how to implement the early default features of our model in a simulation algorithm with very low computational costs. Our results support capital requirements lower than in Basel II, but not as low as in CRD. A sensitivity analysis shows that this finding is robust to parameter uncertainty and stress scenarios. This is likely to give adverse incentives to banks for using advanced risk models.

JEL codes: G21, G28, G32

Keywords: Private Equity, Regulatory Capital, Risk Management

---

\*We would like to thank our colleagues at ABN AMRO Capital, Magnus Blix, Joost Driessen, Jasper van den Eshof, Jeroen Groothuis, Pieter Klaassen, Franz Palm, Bas Werker, Bas Westerman, Wouter Wijngaards and Stanislav Zakovic and participants at the ECB-CFS conference in Dublin and the Erasmus Finance Day in Rotterdam for helpful comments.

<sup>†</sup>University of Amsterdam, Faculty of Economics and Econometrics, Finance Group, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands. E-mail: d.g.j.bongaerts@uva.nl.

<sup>‡</sup>ABN-AMRO Bank N.V., Credit Portfolio Modelling, Gustav Mahlerlaan 10, 1082 PP Amsterdam and Tilburg University, Department of Econometrics and Operations Research, P.O. Box 90153, 5000 LE, Tilburg, The Netherlands. E-mail: e.charlier@uvt.nl. The views expressed in this paper do not necessarily express the views of ABN-AMRO.

# 1 Introduction

Banking regulation has lately been topic of numerous discussions. With the introduction of the Basel II Capital Framework (Basel Committee on Banking Supervision 2006) and the translation of it into European law in the EU Capital Requirements Directive (CRD) (European Union 2006), some important loopholes and inconsistencies in the 1988 Basel Accord are fixed. In the new regulation, the road of coarse simple risk weights<sup>1</sup> for regulatory capital is largely abandoned for all major banks and only serves in a refined way as a last resort for the smaller ones. This is all meant to stimulate more sophisticated risk measurement and resulting risk management. Meanwhile, it should not impede competition too severely. As a result, with the introduction of the Basel II Capital Framework for regulatory capital, sophisticated quantitative risk management has undergone a boost in activity.

Private Equity (and Buy-Outs (BOs) in particular, its largest component by far) is an asset class that is still not very well understood. Like financial risk management, Private Equity has received much attention lately. A claim often heard from the industry is that it increases economic efficiency by disciplining managers. On the other hand, it has also been referred to as a plague bringing down healthy companies with economic effects that extend far beyond the loss in shareholder value<sup>2</sup>. In view of this debate, banking regulation on PE holdings is important, since it will affect the amount of capital banks allocate to this asset class. Over-allocation is likely to lead to investments destroying economic value, whereas under-allocation might lead to foregoing on investment opportunities that increase economic value. A press release by the EVCA (EVCA 2003) showed that 25% of all the PE funding in Europe originated from banks, making this an economically important issue.

This study makes three main contributions to the existing literature. First, it evaluates whether the regulatory capital requirements for Private Equity (PE) investments set out in the new regulations are consistent with the stated goals: increased stability through more sophisticated management without impeding competition too much. We find that the capital requirements as proposed under the simple risk weight approach of Basel II are higher than the average capital requirements that result from an internal model and could possibly lead to unfair competition. The capital requirements proposed in the CRD legislation of the EU, however are

---

<sup>1</sup>In this paper we will use the terms risk weight and capital requirement as in the Basel II and CRD documents. Thus, the capital requirement equals 8% times the risk weight and the risk weight equals 12.5 times the capital requirement.

<sup>2</sup>The German deputy chancellor Franz Müntefering referred to PE as a "swarm of locusts."

lower and are likely to give banks adverse incentives for using sophisticated risk measurement and management techniques. To our knowledge, there is only one paper that assesses the influence of Basel II on banking book equity investments, which is the paper by Suarez, Dhaene, Henrard and Vanduffel (2006). However, for their empirical test, they use a public equity index, which will not necessarily capture stylized facts of buy-out portfolios correctly.

Second, this study puts forward a structural and comprehensive model to assess the risk on a portfolio of individual BO investments. Thus, we have a much more detailed focus than a substantial part of the academic literature that mainly focuses on private equity funds (for example Kaserer and Diller (2004) and Ljungqvist and Richardson (2003)). Our model captures stylized facts of BO investments that are crucial in determining their risk, like high initial leverage,<sup>3</sup> default before maturity due to covenants and holdings in preferred shares and shareholder loans. To this end, we not only use detailed financial statement data, but also exploit the fact that we have expert opinions of investment managers available. Therefore, we can allow for an even greater level of detail than studies that use financial statement data provided by limited partners of PE funds, like Ljungqvist, Richardson and Wolfenzon (2007). When aggregating to portfolio level, we do not only capture correlations in our portfolio, but also allow for the effects of (imperfect) diversification of size, region, industry and stage of the investment lifecycle. This way, we get a very realistic loss distribution.

Third, we make a contribution with regards to efficient implementation. We allow our investments to go into default before the maturity of the debt that is used to finance them. Moreover, since we are looking at the risk of a portfolio, we want to impose a certain dependence structure. To this end, traditional simulation models of this nature would simulate discretized diffusion paths over the simulation horizon. These discretized paths ignore the possibility of a default between two consecutive points on the path that are both above the default threshold and hence put a negative bias on default probabilities. This can be solved by having a very fine time grid but this comes at the cost of an increase in computational time. Instead, we only simulate asset returns over the whole risk horizon and subsequently, conditional on this realization, we derive an expression for the probability of default over the risk horizon. In the simulation we then only have to do one additional uniform random draw to determine whether the firm has gone into early default. This allows fast

---

<sup>3</sup>For the remainder of the paper, we use the definition of leverage given by the value of the debt over the value of the assets.

simulation with unbiased default probabilities.

For our analysis, we use a proprietary dataset supplied by a large international commercial bank. This way, we can avoid using databases that have been reported to be subject to different kinds of biases. Moreover, in addition to the individual investment financial data, we also have access to expert level data that investment managers are required to file for their investment committees.<sup>4</sup>

The model we use in our analysis is based on the class of structural models as first introduced by Merton (1974). BOs are initially typically highly levered, but on average show a sharp decline in leverage due to asset growth over their lifetime. Since leverage has a large effect on equity volatility and default probabilities, this is an important factor to consider in a risk model for BOs. Because structural models incorporate capital structures explicitly, this class of models is very well suited for our analysis. Not only the leverage effect on volatility, but also the early default effect of initially high default probabilities and the use of covenants is modeled by modeling the equity as a down-and-out call option. Using more option pricing theory, we also can capture the fact that banks tend to not only hold common stock of those investments, but also preferred stock and shareholder loans. Finally, we also acknowledge that liquidity effects play an important role in the valuation of BOs and try to capture those as well.

A sensitivity analysis shows that our main findings are robust to parameter uncertainty and worsening economic conditions. Finally, we explore details in the CRD accord and assumptions in our setup that could change our findings. These strengthen our statement that the CRD capital requirements are too low.

The rest of this paper is organized as follows. Section 2 gives the background on banking regulation and discusses the content of the Basel II capital framework and the CRD legislation regarding private equity. Section 3 gives a short overview of the existing literature and highlights the contribution of this paper. Sections 4 and 5 describe the model and calibration for each individual investment. In Section 6, we link the individual investments, i.e. we describe the dependence structure and simulation procedure. Section 7 then describes the data we will use in our analysis. Section 8 gives the results of our model on different portfolios and Section 9 elaborates further on the possible consequences and problems of the current CRD implementation and our dependence structure. Section 10 concludes.

---

<sup>4</sup>This data is likely to be informative, since it is not uncommon for an investment manager of the bank to sit on a supervisory board of the BO.

## 2 Capital accords on banking regulation

This section starts with describing the general history of banking regulation, the aim of the new Basel II capital Framework and some concerns. Next, we explore the specific proposals for PE investments under both Basel II and CRD.

The call for banking regulation originates from the notion that banks are very important players in local and world-wide economies, but are also commercial firms. As commercial firms, they have to make investment and financing decision and thus make a decision about their capital structure. However, the external costs and benefits of this decision on the economy are so substantial that regulation substantially improves social welfare. This is well illustrated with the trigger for the development of banking regulation: the disastrous overnight failure of Bank Herstatt (Frankfurt, Germany) in 1974, leading to massive overnight losses in the US. The Basel committee of the Bank for International Settlements (BIS) was appointed to design the first Basel capital accord that was introduced in 1988. Only few years later, in 1997, the severity of the Asia crisis underlined the need for supervision of financial institutions.

The first Basel Accord (Basel Committee on Banking Supervision 1988) aimed to

strengthen the soundness and stability of the international banking system; and, secondly, that the framework should be fair and have a high degree of consistency in its application to banks in different countries with a view to diminishing an existing source of competitive inequality among international banks.

To do so, it defines guidelines for minimum capital requirements for investments with different risk profiles. The way to achieve this is to allocate a risk weight of either 0%, 10%, 20%, 50% or 100% to every investment based on its riskiness. The capital requirement is then set to 8% of this risk weight. The role of local governments (initially only the G-10, but later on many others) was crucial in this process. Implementing such a framework on a relatively large scale improves stability and thereby is likely to reduce funding costs for the participants. However, implementation is costly for every participant, which induces a possible freerider problem. By collectively adopting these rules, the local governments prevented the participants from free-riding and ensured that complying by these rules would not weaken the competitive position of the participants, but even strengthen those compared to non-participants.

The positive effects of Basel I are however not undisputed. The coarse risk sensitivity combined with the abundance of credit derivatives seems to stimulate risk taking and regulatory capital arbitrage rather than reducing it (Ambrose, LaCour-Little and Sanders 2004). Sometimes the risk weights fail to properly reflect risk. Duffie and Singleton (2003) give an example where a Turkish government bond gets a risk weight of 0% whereas a AAA corporate bond gets a 100% risk weight.

The Basel II capital framework tries to improve on the deficiencies mentioned above. To do so, it allows banks to use their own internal estimates of the default probability (PD), loss-given-default (LGD) and exposure-at-default (EAD) estimates (after approval by the regulator). Second, the risk mitigating effect of (partially) offsetting exposures is now rewarded. Moreover the capital requirements are also refined for banks that are unable or unwilling to build systems themselves (simple risk weighted assets approach).

What is important for this more advanced and more accurate system to really work is that there are proper incentives to use the more risk sensitive measures that are now allowed (otherwise everyone would still try to opt for the simple risk weighted assets approach) as is also acknowledged by the Basel Committee:

The Committee believes it is important to reiterate its objectives regarding the overall level of minimum capital requirements. These are to broadly maintain the aggregate level of such requirements, while also providing incentives to adopt the more advanced risk-sensitive approaches of the revised Framework.

Crucial in this process is that it is possible to achieve higher risk-adjusted returns when better risk measures are used. Moreover, these additional gains should not be excessively high, because otherwise small banks will be driven out of business, reducing competition and driving up prices in the end. This point is also specifically noted by the Basel Committee:

The fundamental objective of the Committees work to revise the 1988 Accord has been to develop a framework that would further strengthen the soundness and stability of the international banking system while maintaining sufficient consistency that capital adequacy regulation will not be a significant source of competitive inequality among internationally active banks.

However, the implementation of the new capital framework also raises some concerns. Danielsson, Embrechts, Goodhart, Keating, Muennich, Renault and Shin

(2001) for example acknowledge the main improvements, but also point out some severe problems. These include the use of a non-coherent risk measure (which VaR is; see Artzner, Delbaen, Eber and Heath (1999)) and the dependence on elliptic (mainly normal) distributions for the main risk drivers in the model. Like many others, they also address the issue of procyclicality. When internal models become more risk-sensitive, capital requirements are likely to go up when the economy takes a hit. Too little capital supply due to high regulatory capital requirements can then severely hamper the recovery from or even lead to a financial crisis.<sup>5</sup>

## 2.1 PE in the Basel II Capital Framework

In general, the Basel II Capital Framework distinguishes between the Standardized Approach (SA) and the Internal Rating Based (IRB) approach. Under the SA, the capital requirement equals 12% for private equity and 8% for public equity.

Each bank has to decide which of the two approaches it will apply. However, once this choice is made, it will have to use that approach for all of their assets. Because many large international banks apply for the IRB approach on their loans, we will focus on the alternatives for private equity that are available under the IRB approach. The following section will investigate the rules set out in Basel II to calculate capital requirements for PE.<sup>6</sup> The Basel Committee on Banking Supervision (2006)<sup>7</sup> allows for three approaches for calculating capital requirements for PE.

First, the simple risk weight approach prescribes to use a capital requirement of 32% of the economic value<sup>8</sup> of an investment as capital.

Next, we discuss the PD/LGD approach. One year PDs are determined by the banks themselves in a way similar to the one used for corporate bonds. The Capital Framework prescribes a 90% LGD and a maturity  $M$  of 5 years. These parameters are used as arguments in the standard PD/LGD risk weight formula resulting in the

---

<sup>5</sup>This issue can partially be resolved by using through the cycle PDs and LGDs at the cost of some risk-sensitivity.

<sup>6</sup>For investments in PE *funds* other rules apply.

<sup>7</sup>See Section III.E, paragraphs 339 through 361.

<sup>8</sup>This is equal to the value presented in financial statements, i.e. book value for consolidated investments and market value for non-consolidated investments, see Basel Committee on Banking Supervision (2006) Section III.E, paragraph 359.

capital requirement K:

$$K_{B2}(PD, LGD) = LGD \left( N \left( \frac{N^{-1}(PD) + \sqrt{R}N^{-1}(0.999)}{\sqrt{1-R}} \right) - PD \right) \frac{1 + (M - 2.5)b}{1 - 1.5b} \quad (1)$$

where  $N^{-1}$  is the inverse of the standard normal distribution function,

$$R = 0.12 \frac{1 - \exp(-50PD)}{1 - \exp(-50)} + 0.24 \left( 1 - \frac{1 - \exp(-50PD)}{1 - \exp(-50)} \right) \quad (2)$$

$$b = (0.11852 - 0.05478 \ln(PD))^2. \quad (3)$$

Note that (1) is increasing with the PD up to 28% and ranges from 1% to 43%. A capital requirement of 32% (i.e. equal to the simple approach) results for a PD of about 7.2%.

Finally, one can opt for an internal model approach that prescribes to compute the Value at Risk (VaR) for the PE portfolio over a 3-month horizon, a confidence level of 99% and a benchmark return equal to a "properly chosen risk free rate". More details can be found in Basel Committee on Banking Supervision (2006) III.H.11, paragraphs 525 through 536. Compared to the PD/LGD approach, this looks very advantageous because of the shorter horizon of 3-months instead of one-year and the lower confidence level of 99% versus 99.9%.

For all PE investments, there is a minimum capital requirement of 24%, see Basel Committee on Banking Supervision (2006) III.E paragraph 347. Note that in the PD/LGD approach this is attained with a PD of about 2.2%.

For comparative purposes, we also present here the capital requirements applicable to strategic (banking book) public equity holdings. These exposures follow the same rules as private equity investments, but since they are typically perceived to be less risky, they qualify for a 24% capital requirements under the simple risk weighted assets approach and a minimum capital requirement of 16% under all other approaches.

With respect to the different approaches, the (local) supervisor prescribes which approaches are eligible for every bank under its supervision. In general, these approaches should match the internally used best practices.

## 2.2 The CRD legislation of the EU

The CRD legislation of the European Union (2006) used the Basel II Capital Framework as a starting point. It also distinguishes between a Standardised Approach (SA)<sup>9</sup> and an Internal Ratings Based (IRB) approach<sup>10</sup>. In the SA, the capital requirements are identical to those in the Basel II Capital Framework, i.e. 8% for public equity and 12% for private equity.

However, for the IRB approach the CRD differs, as will be explained below. One of the results is that PE investments in the CRD have a *lower* capital requirement than public equity investments, whereas in the Basel II Capital Framework the reverse holds.

Within the IRB approach, the most dramatic effect we see for the simple risk weight approach. We conclude that the capital requirement for PE investments<sup>11</sup> is reduced from 32% to merely 15.2%, whereas for public equity, capital requirements have remained at 24%. In order to qualify for the 15.2% capital requirement, these PE investments should be part of a "well-diversified" portfolio, otherwise 29.6% will apply. However, no guidelines are given for when a portfolio is well diversified. This is left up to the bank to demonstrate. In addition to the capital requirement, the unprovisioned part of the expected loss needs to be deducted from capital.<sup>12</sup> Expected loss equals 0.8% and 2.4% of the economic value, respectively. As provisions are often zero, this results in an effective capital requirement of 16% and 32%.

For the PD/LGD approach the CRD capital requirements also prescribe to use  $M = 5$  but the Basel II capital requirements (1) are multiplied with 1.06, i.e.

$$\begin{aligned}
 & K_{CRD}(PD, LGD) \\
 &= LGD \left( N \left( \frac{N^{-1}(PD) + \sqrt{R}N^{-1}(0.999)}{\sqrt{1-R}} \right) - PD \right) \frac{1 + (M - 2.5)b}{1 - 1.5b} \times 1.06 \quad (4) \\
 &= K_{B2}(PD, LGD) \times 1.06
 \end{aligned}$$

More importantly, exclusively for private equity, the LGD has been reduced from 90% to 65% (for a "well-diversified" portfolio), whereas the LGD of public equity has been preserved at 90%. Like for Basel II, (4) is increasing with PD up to a PD of about 28% and decreasing thereafter. Furthermore, when compared to the simple

---

<sup>9</sup>Described in Annex VI.

<sup>10</sup>Described in Annex VII.

<sup>11</sup>See Annex VII, Section 1.3.1 item 19.

<sup>12</sup>See Annex VII, Section 4 item 36.

risk weight approach, a PD of 0.45% results in a capital requirement of 16% and a PD of about 6% results in a 32% capital requirement.

The internal model approach has not changed compared to Basel II except for the minimum capital requirement for each investment. Rather than 16% and 24% respectively for public and private equity investments, they are now set to  $K_{CRD}(0.09\%, 90\%) = 7.7\%$  and  $K_{CRD}(0.09\%, 65\%) = 5.6\%$ .<sup>13</sup>

In general, the upper bound on the capital requirement is 100%. Only for the PD/LGD method the capital requirement is maximized at 100% minus PD times LGD, See Annex VII, Part I, Section 1.3.2 item 23.

## 2.3 Overview of capital requirements

To summarize the previous sections, we provide Table 1.

Method	Private Equity		Public Equity	
	Basel II	CRD	Basel II	CRD
Simple risk weight	32%	16% or 32%	24%	24%
Min. Risk-Weight for PD/LGD and Internal Model Approach	24%	5.6%	16%	7.7%
PD/LGD	$K_{B2}(PD, 90\%)$	$K_{CRD}(PD, 65\%)$	$K_{B2}(PD, 90\%)$	$K_{CRD}(PD, 90\%)$
Internal Model Approach	3-month horizon 99% confidence level benchmark return equal to risk free rate			

Table 1: Overview of Capital Requirements.

## 3 Internal model approach: existing literature

In this paper, we focus on risk measurement of a portfolio of individual PE investments, for which we have data with a high level of detail, including expert opinion data.

The existing literature on PE investments mainly focusses on the investments in Private Equity funds and tries to explain their performance measured by IRR or related return measures (see for example Kaplan and Schoar (2003), Ljungqvist and Richardson (2003) and Kaserer and Diller (2004)). However, these investment vehicles are not so well suited for our purposes since they are less diversified, closed-end and only have limited information on fund level rather than detailed information

<sup>13</sup>See Annex VII, Part I, Section 1.3.3 item 25.

on deal level. Moreover, the data employed by these studies are often very problematic. Most studies use commercial databases like for example Thomson VentureXpert. Problems with these databases are that reporting is done on a voluntary basis and that there are no strict common guidelines or rules on how to calculate the value of an investment. This leads to all kinds of biases like return smoothing (no reporting and valuation guidelines), selection bias (voluntary reporting; only good funds report), backfill bias (voluntary reporting; only good funds do backfilling), problematic performance measures (no reporting and valuation guidelines; the often used IRR is dependent on reinvestment assumptions), etc. Slowly, more articles appear that manage to obtain proprietary datasets from Limited Partners (LPs) that contain individual firm and cashflow data; see Ljungqvist et al. (2007) for example. However, these still do not contain the expert data that we use in our analysis. Finally, from a risk measurement point of view, these articles do not offer sufficient guidelines for how, for example, losses on individual investments should be modelled and diversification benefits can be estimated. The credit risk literature however, offers a wide array of models and tools that are particularly useful for these kinds of exercises.

Merton (1974) laid the basis for the modern credit risk literature by considering debt and equity as contingent claims on the assets of a firm. These models are better known as structural models and different extensions of the Merton model have been put forward like Black and Cox (1976), Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001). An advantage of these models is that they link debt and equity in a firm in a very intuitive way. This is a feature that is very valuable for investments in highly levered companies (BOs at initiation) and portfolios with a large degree of cross-sectional variety in leverage ratios (BOs in different stages of their lifetime). This type of model is also the basis of a popular credit risk measurement package by Moody's-KMV. As for any model class, there are some limits to this model. For example, a structural model cannot generate a default arbitrarily close to day zero, whereas these 'unexpected' defaults do happen in practice (see for example the Enron case). Duffie and Lando (2000) have a framework with noisy accounting data that generates the unexpected defaults within a structural model framework. Some other papers generate these jumps in a less structural way by adding poisson jumps to the asset process. The class of reduced form models, models defaults as pure jump processes altogether, but sometimes lets the firm characteristics that go as input into the structural models influence the default intensity process.

Regarding banking regulation, there is an abundance of papers, but curiously enough, hardly any on Basel II for PE. To our knowledge there is only one other paper that investigates the different methods that Basel II proposes for banking book equity exposures by Suarez et al. (2006). They adopt a more reduced form approach and argue that extreme value theory should be used for these kind of models, yielding higher capital requirements than in the standardized approach on the DAX and Dow Jones portfolio.

This paper uses a unique and proprietary dataset to calculate Basel II and CRD capital requirements under the internal models approach. Where Suarez et al. (2006) criticize the implementation for Private Equity on more fundamental points that are in fact applicable to the whole accord and are in line with Danielsson et al. (2001), we take the methodological implementation as given and focus on the final regulatory capital requirements these models produce, since that is what is ultimately experienced by the market. We show that the Basel II simple risk weight capital requirements are on the high side, but that the CRD risk weights are likely to be too low. This counteracts the goal set by BIS to create incentives for banks to invest in a more sophisticated risk-management system. We also discuss several externalities of this regulation and the implications of our results. In a sensitivity analysis, we show that the capital requirements resulting from an internal model are relatively stable.

## **4 The firm value model for individual investments**

The Private Equity sub-class considered in this paper is the Buy-Outs (BOs). According to Thomson Financial and PricewaterhouseCoopers (2006), in 2005 BOs took up 80% of the European PE market. A buy-out is a (part of) an existing firm that is acquired by its management and a Private Equity investor. In our model we will try to model the following stylized facts of buy-outs.

1. At initiation, buy-outs are highly levered, resulting in very volatile returns.
2. Due to the initially high leverage and the use of covenants, the probability of default, even before maturity of the debt can be quite substantial.
3. The private equity investor takes a part of both the common and the preferred shares, whereas the management typically only takes common shares.

4. PE investments are typically very illiquid and illiquidity discounts are often deemed appropriate in valuations.

The acquisition of a buy-out firm is most often financed with 65%-70% of (bank)debt. This will initially lead to a high volatility of equity returns as well as a higher probability of default. Most firms have debt with covenants, which will enable debtholders to call a default before maturity if certain financial ratios fall below predefined thresholds. Due to these covenants, a default before the maturity of the debt is not an unrealistic scenario. Next, to create the right incentives, the management in a buy-out only gets common stock, whereas the private equity investor in many cases also gets a substantial amount of the less risky preferred stock or shareholder loans.<sup>14</sup> This increases the risk and possible upside for the position of the management and reduces risk and possible upside for the private equity investor. Finally, private equity investments tend to be long lived and very illiquid. Due to this, an illiquidity discount is often applied to the accounting values of these investments, which is released upon exit.

We also model some portfolio features that do occur in practice, but are sometimes ignored in "hypothetical portfolios."<sup>15</sup> A large PE portfolio is likely to contain investments that are to be exited soon as well as investments that will last for much longer. These differ substantially in leverage and liquidity. Moreover, for the investments close to exit, calculating the risk over the whole risk horizon would over-estimate the risk. On top of that, there is substantial heterogeneity in the size of the investments and allocation to different industries and countries, leading to concentration effects and some non-diversified idiosyncratic risk.

The aim of a buy-out is typically to introduce efficiency improvements and that these relatively small improvements are levered up to obtain high return on the investment. The downside of this structure however, is that the leverage reduces the distance to default. In other words, its credit quality deteriorates significantly and gradually improves again as the value of the equity grows (if successful) and the leverage again decreases. It is widely acknowledged that the main risk from buy-out investments comes from this increased leverage. This motivates the use of a structural model for the firm's asset evolution. The debt and equity are modelled as contingent claims on the assets. Note that in the industry there are other proprietary

---

<sup>14</sup>We consider losses on investments as a whole, so including the preferred shares and shareholder loans. These instruments are so subordinated that they are closer to equity than to debt and are also regarded as (a special form of) equity by the business.

<sup>15</sup>With this we mean for example using the DowJones portfolio as a proxy for a PE portfolio.

models in place that aim at modelling the value of the firm precisely, taking care of many special features. However, these models are not suitable for risk assessment because they lack transparency and require more parameters than can be estimated from the data. In the calibration stage, the structural model presented below will be linked to the more detailed models (Section 5).

## 4.1 The asset process

Our model assumes that the value of the assets follows some stochastic process. By the asset value, we mean the market value of the assets: all the properly discounted future expected cash flows resulting from the assets, rather than the accounting value of the assets. This measures the ability of a company to generate cash rather than the acquisition costs of its possessions minus depreciation.

Let us define a probability space  $(\Omega, \mathcal{F}_t, \mathbb{P})$  where  $\mathbb{P}$  is the real-world measure. Let us for simplicity now assume that the asset process  $A_t$  is a geometric Brownian motion:

$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dW_t^A, \quad (5)$$

where  $W_t^A$  is a standard Brownian motion under  $\mathbb{P}$  and adapted to  $\mathcal{F}_t$ .

## 4.2 A first passage time model

The bank loans contracts that are provided to BOs generally have protective covenants. This can result in the default of a firm even before its debt matures. Inspired by this behavior we assume default occurs as soon as the asset value drops below a certain barrier. Using the reflection principle for Brownian motion (Karatzas and Shreve 1991) one can derive the probability of having an asset value larger than the debt at all times from  $t$  till time  $T$  given an asset value  $A_t$  and survival up to time  $t$ . We can also use the reflection principle to value down-and-out barrier options on the asset value, which will be our approach for valuing the equity part of the BO firm.

## 4.3 The value of the equity

To start with, let us first consider a firm that is financed purely by senior debt with facevalue  $D$  and common stock. Moreover, let us assume that the exit time in case of a success is known to be  $T$ . Let us furthermore assume that early default is

triggered when  $A_t < H$  for some  $t \in [0, T]$ , where  $H$  is conceptually determined by the covenants of the investment (and typically expressed in terms of  $k$  as defined above). To determine the market value of the debt and equity we distinguish the case where  $H > D$  from  $H \leq D$ .

#### 4.3.1 Equity value if $D > H$

To value the common stock consistently within the first passage time model, we can look at the common equity as a European down-and-out call option on the underlying assets. A down-and-out call option is a barrier option that ceases to exist once a barrier  $H$  has been reached by the underlying asset. Hull (2003) gives expressions for the price of a down-and-out call option with an underlying asset following a geometric Brownian motion. Its value however, depends on the time to maturity, while equity typically does not really have a maturity, but the debt has. For public firms with a stable capital structure, the maturities of the different debt issues differ. For BOs however, maturities are generally concentrated on one point. Note that the exercise price of this option will be the debt level of the company, whereas the barrier will not necessarily be the same. The value of the down-and-out call option is given by

$$c_{do} = A_0 N(d_1) - De^{-rT} N(d_2) - (A_0(H/A_0)^{2\gamma} N(y) - De^{-rT}(H/A_0)^{2\gamma-2} N(y - \sigma_A \sqrt{T})), \quad (6)$$

$$\gamma = \frac{r + 0.5\sigma_A^2}{\sigma_A^2}, \quad (7)$$

$$y = \frac{\ln(H^2/(A_0 D))}{\sigma_A \sqrt{T}} + \gamma \sigma_A \sqrt{T}, \quad (8)$$

$$d_1 = \frac{\ln(A_0/D)}{\sigma_A \sqrt{T}} + \gamma \sigma_A \sqrt{T}, \quad (9)$$

$$d_2 = d_1 - \sigma_A \sqrt{T}. \quad (10)$$

#### 4.3.2 Debt value if $D > H$

We can now derive the value of the debt by using Put-Call parity:

$$A_0 + p(D) = D \exp(-rT) + c(D) \quad (11)$$

where  $c(D)$  and  $p(D)$  are plain vanilla put and call options with exercise price  $D$  and  $D$  is the face value of debt. Now realize that

$$p(D) = p_{di}(D, H) + p_{do}(D, H) \quad (12)$$

$$c(D) = c_{di}(D, H) + c_{do}(D, H) \quad (13)$$

and substitute this back into (11):

$$A_0 + p_{di}(D, H) + p_{do}(D, H) = D \exp(-rT) + c_{di}(D, H) + c_{do}(D, H) \quad (14)$$

Now realize that the accounting identity gives us

$$A_0 = DM_0 + E_0 \quad (15)$$

where  $A_0$ ,  $DM_0$  and  $E_0$  are the total market value of assets, the total market values of debt and equity at time zero respectively. Realizing that  $E_0 = c_{do}(D, H)$  and combining (15) and (14) gives us

$$DM_0 = A_0 - c_{do}(D, H) \quad (16)$$

$$= D \exp(-rT) - p(D) + c_{di}(D, H) \quad (17)$$

#### 4.3.3 Debt value if $D \leq H$

Under this assumption, debt is risk free in our model, since the full amount will always be recovered.<sup>16</sup> Thus  $DM_0 = D \exp(-rT)$

#### 4.3.4 Equity value if $D \leq H$

Plug in the expression for the market value of the debt into (11) and combining it with (15) gives

$$E_0 = A_0 - DM_0 \quad (18)$$

$$= c(D) - p(D) \quad (19)$$

---

<sup>16</sup>We also added an idiosyncratic random jump to default component to the asset process to make our debt risky as well (motivated by uncertainty about true asset values; see also Duffie and Lando (2000)), but had problems calibrating the intensities since these vary with rating, but rating information for BOs is typically unavailable (they have mainly bankloans).

### 4.3.5 Extension with preferred stock and shareholder loans

To protect itself against extreme leverage effects and to provide the management with a very strong incentive to perform, a significant share (often 80%-90% of the equity value) of the capital provided by the buy-out investor consists of preferred stock or shareholder loans. Although their tax treatment is in general different, we consider them to be identical for the purpose of risk measurement. Since these instruments are subordinated to debt, they will leave the valuation of the debt intact. However, they are senior to common stock and therefore change the valuation of the stock. Of course, we need to price these instruments themselves too.

From the previous section we know that

$$DM_0 = \begin{cases} D \exp(-rT) - p(D) + c_{di}(D, H) & \text{if } D > H \\ D \exp(-rT) & \text{if } D \leq H \end{cases} \quad (20)$$

where the symbols in brackets indicate the exercise prices. If we substitute part of the equity for something senior to equity and subordinated to debt, say shareholder loans with exit value  $SHL$  (notional amount plus rolled up interest/preferred dividend), the debt is unaffected but the equity value changes to:

$$E_0 = \begin{cases} c_{do}(D + SHL, H) & \text{if } D + SHL > H \\ c(D + SHL) - p(D + SHL) & \text{if } D + SHL \leq H, \end{cases} \quad (21)$$

since for equities only the size of the superordinated claims is important, not its composition. The accounting identity has changed to:

$$A_0 = DM_0 + E_0 + MSHL_0 \quad (22)$$

This leaves us with an expression for the market value of the shareholder loans at time zero  $MSHL_0$ :

$$MSHL_0 = A_0 - E_0 - DM_0 \quad (23)$$

## 5 Calibrating the model for individual investments

This section describes how we sequentially calibrate the different parameters for our individual investments. The advantage of the method described below is that it

allows a stepwise estimation procedure for parameters and is thus much easier than solving equations simultaneously.

## 5.1 Exogenously fixed parameters

Some of the parameters we fix exogenously:

- The short rate is set to the realistic value of 3.5%.
- For the time to maturity we use the investment manager's forecasted expected time to exit and we thus ignore the exit time risk.
- With respect to the barrier level, the contractual covenant barrier is most often larger than the debt, i.e.  $H > D$ . However, in practice waivers are often given for covenant breaches, possibly combined with closer monitoring. Discussions with practitioners indicated that these covenants are typically exercised when asset values hit debt values, i.e. when  $A_t = D$ . Thus, we set  $H = D$ .<sup>17</sup>

## 5.2 Asset volatility

To estimate asset volatilities, we extrapolate asset volatilities from the public equity market to the private equity market. We think this is a reasonable thing to do, since we only consider buy-out investments which are quite sizable and often were part of larger listed companies before they went private. We do however make a fundamental distinction between BOs and public equity. For public equity, we consider the capital structure constant at a long term optimal capital structure, whereas BOs are initially highly levered and leverage decreases over the lifetime of the investment (if it is successful).

We apply the following procedure:

1. Map every BO  $k$  to an industry/region index  $j$
2. For every public equity constituent  $i$  of industry/region index  $j$ , we obtain time series data of its market capitalization ( $E_{i,t}$ ), short-term liabilities ( $S_{i,t}$ ) and long term debt ( $L_{i,t}$ )
3. Make a guesstimate  $\hat{\sigma}_{i,init}$  of  $\sigma_i$

---

<sup>17</sup>In practice it can be the case that a default is triggered when the value of the assets hits a level below  $D$ , i.e.  $H < D$ . The formulas presented would then still apply but we will not investigate further.

4. Following Kealhofer (2003b) we model the equity price as a down-and-out call option as in (10) with  $T = \infty$ ,  $D_{i,t} = S_{i,t} + L_{i,t}$ ,  $H_{i,t} = S_{i,t} + 1/2L_{i,t}$ <sup>18</sup>
5. Calibrate to  $E_{i,t}$  to obtain the time series of  $A_{i,t}$
6. Calculate  $\hat{\sigma}_i$  as the standard deviations of the returns of  $A_{i,t}$
7. Plug in  $\hat{\sigma}_i$  into 3 and iterate until we get convergence
8. Take as an estimate of industry asset volatility the average asset volatility of all constituents in that industry:  $\hat{\sigma}_j = 1/N_j \sum_{i \in j} \hat{\sigma}_i$
9. Use for investment PE investment  $k$  in industry  $j$  the average asset volatility of industry  $j$ :  $\hat{\sigma}_k = \hat{\sigma}_j$

The asset volatilities that we obtain lie between 20% and 30% per annum.

### 5.3 Asset value

As mentioned before, the market values of the assets are typically unavailable since there is no public market. However, Private Equity investors in general have practitioner oriented models in place to calculate the fair market value ( $FMV_t$ ) of their equity investment at a certain time  $t$ . The problem with these practitioner methods is, however, that it is hard to identify the main value drivers let alone impose some reasonable structure in the probability distribution of the value drivers.

Therefore, what we do is the following. We start with imposing our structural model which results in theoretical market values for the PE investments. However, this assumes a completely liquid market in which trading is costless. In general, a substantial illiquidity discount is used when determining the  $FMV_t$  of the PE investments. Due to IFRS, in practice illiquidity discounts are used ranging from 0% up to 30% of the market value. We apply the illiquidity discount that is used to determine the  $FMV_t$  of the private equity investments then also to our theoretical market values. Given the exogenously fixed parameters, the value of the assets  $A_t$  is then determined for each investment by making the theoretical market value (corrected for illiquidity) equal to the internally estimated  $FMV_t$ .

---

<sup>18</sup>Here we assume that listed firms have discovered their optimal capital structure and stick to that.

## 5.4 The expected growth rate $\mu_A$

To obtain expected growth rates of the assets, some additional work needs to be done.

We again take our structural model and calibrate the model expected exit proceeds to the expected exit proceeds on the investment forecasted by the investment managers to obtain the expected asset growth rate. That is, we define

$$h(A_t, \mu_A) = E(w_E E_T + w_{MSHL} MSHL_T | A_t) \quad (24)$$

where  $w_E$  and  $w_{MSHL}$  denote the share of the equity and the shareholder loans that have been invested in. Then we use an iterative procedure to solve

$$h(A_t, \mu_A) = \text{Expected exit proceeds} \quad (25)$$

## 6 Methodology to compute the portfolio risk

In this section we combine the individual investments into a portfolio and we determine the risk at the portfolio level. This requires a dependence structure between the asset processes that underly the investments. Furthermore, we discuss the notion of loss, we describe the steps in the simulation, together with some technicalities.

### 6.1 Setting up a correlation structure

To model the asset dependence structure we use an index based correlation structure. From basic statistical calculus, we know how we can correlate  $A_t$  from (5) to an industry index  $M_t$  that also follows a geometric Brownian motion. Suppose that  $W_t^M$  is the process that drives the index returns and suppose that the correlation between  $M_t$  and  $A_t$  is  $\rho_{M,A}$ . We assume that this correlation is constant over time. The process driving the assets of the individual firm can then be decomposed into two independent Brownian motions

$$W_t^A = \rho_{M,A} W_t^M + \sqrt{1 - \rho_{M,A}^2} W_t^{idio}. \quad (26)$$

Substituting (26) into (5), we obtain

$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A (\rho_{M,A} dW_t^M + \sqrt{1 - \rho_{M,A}^2} dW_t^{idio}). \quad (27)$$

Thus we can simulate independent realizations of Brownian increments to construct correlated asset realizations.

### 6.1.1 Correlations and their calibration

To estimate the correlation coefficients used in the procedure above, we make use of the following structure. We take the Dow Jones (DJ) industry classification, which contains 52 industries in 11 global regions. For all these country/industry combinations and their constituents, we use daily data on monthly total returns from 2002 through 2005.

For each pair  $(I(i), I(j))$  of industry/country indices, we have estimated a correlation coefficient  $\rho_{I(i), I(j)}$ . As we will not differentiate between constituents within an industry/country index, we average the correlations over all pairs of constituents to obtain  $\alpha_j$ . The correlation between two indices is obtained directly from the historical data and is denoted by  $\rho_{I(i), I(j)}$ . The correlation between companies  $i$  and  $j$  is then the product  $\rho_{I(i), I(j)}\alpha_i\alpha_j$ .

Every buy-out investment is allocated to exactly one industry/region index just as for the volatility estimation. Hence, all investments are assumed to inherit the correlation structure and coefficient value resulting from their respective index.

## 6.2 The definition of a loss

The definition of a loss in the context of tail analysis for buy-and-hold portfolios can be subject to some discussion. On the one hand, buy-and-hold portfolios typically consist of loans, bonds and derivatives on those. Models on these investments typically focus on loss of principal outstanding rather than on market losses (although market losses due to credit events are often implicitly captured). Basel II however, explicitly prescribes to define a private equity loss as the return on a properly chosen risk-free investment minus the return on an investment. Since our simulation horizon is only three months, we use as our benchmark the same short rate we also use in the evaluation of the option values, see Section 5.1.

## 6.3 Simulation

This section describes the simulation steps that should be taken. To simplify the exposition, we will introduce the relevant horizon, which is equal to the minimum

of the risk horizon of 3 months and the exit horizon. The steps are illustrated graphically in Figure 1.

1. Simulate  $N(0, 1)$  correlated standardized asset returns by using (27).
2. Transform standardized asset returns to absolute asset returns. If the risk horizon is equal to the relevant horizon, we multiply the standardized asset return with half of the estimated annual volatility ( $\sigma_A$ ) and by adding a quarter of the annual growth rate ( $\mu_A$ ). If the relevant horizon is the exit horizon, we do the same, but scale the volatility and growth rate accordingly and no illiquidity discount is applied anymore.
3. Given the current value of the assets and their value at the relevant horizon, we determine whether the company has gone out of business before the relevant horizon. More details can be found in Section 6.3.1 and Appendix B.
4. In case of the default, the debt is paid and the equity and shareholder loans end up worthless. If no default has taken place, we compute the value of the debt, equity and preferred shares/shareholder loans by using the formulas in Section 4 and correcting for illiquidity, if necessary. Together with the holding shares for every instrument, this determines the value of the investment.
5. Determine losses for each scenario as the value of the portfolio minus the benchmark value of the portfolio, build up the simulated probability distribution and establish the quantile.

### 6.3.1 Simulation of early defaults

To determine whether a company has gone out of business over the risk horizon, we could generate detailed asset value paths. However, this would be time consuming. Instead, we do the following. Conditional on the current value of the assets and their value at the relevant horizon, we determine the probability of default over the relevant horizon by using the expressions in Appendix B. In the simulation we then only have to do one additional uniform random draw to determine whether the firm has gone into early default.<sup>19</sup> This allows fast simulation with unbiased default probabilities.

---

<sup>19</sup>These uniform draws are independent for different companies. However, given that the asset value paths for different companies are correlated, the conditional default probabilities are correlated.

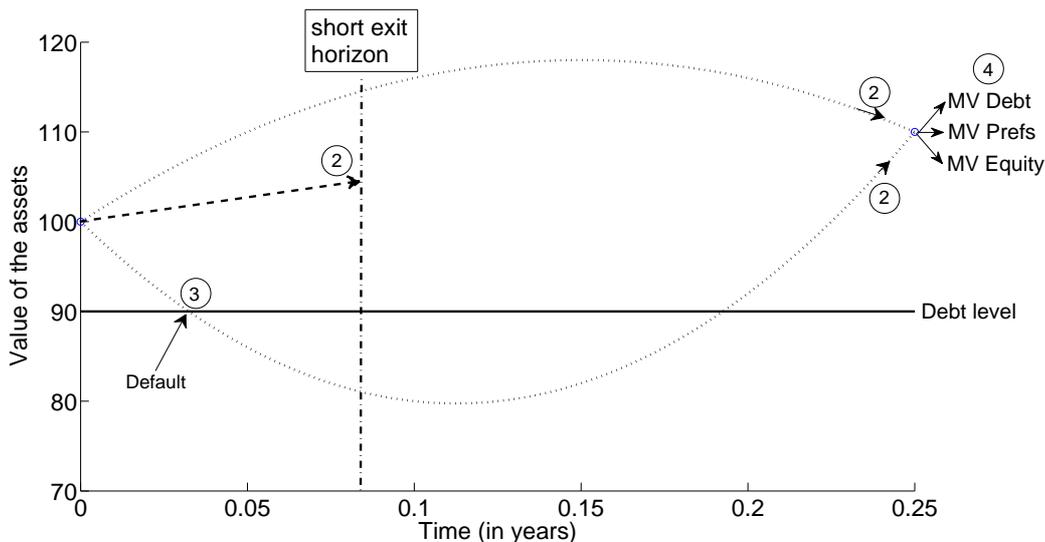


Figure 1: Two potential simulation paths underlying a scenario in which we start from an asset value of 100 and end with an asset value of 110 at the risk horizon of 3 months (dotted arrows). If the exit horizon is shorter than the risk horizon the value of the assets is scaled back to the exit horizon at the dash-dotted vertical line. The third path (dashed arrow) provides an example. The circled numbers indicate the corresponding steps in the simulation process.

## 7 Data

### 7.1 Public equity data

The public equity data that is used to calibrate asset volatilities and correlations is sourced from Datastream. The industry/country indices that are used are the Dow-Jones indices. We use all constituents of these indices that have all required datafields available. Since we are only after volatility in percentages, we used data in local currency. A data analysis revealed that for some currencies, market caps and accounting figures were reported in different orders of magnitude (e.g. thousands vs millions). We algorithmically identify and correct these instances by searching for unrealistically high ( $> 99\%$ ) or low ( $< 1\%$ ) leverage ratios. Moreover, we excluded extreme volatilities (larger than 200%), since they were most often attributable to the decile of smallest companies (and thus substantially smaller than most buy-outs in our portfolio).

## 7.2 Buy-out data

The buy-out data that we used was obtained from a large international commercial bank. The data available on these companies consists of accounting data, valuation model outputs and expert opinion data from the investment manager of each investment. The portfolio has the following characteristics:

- Consists of 82 non-consolidated buy-outs (hence the economic value equals the FMV) with an aggregated FMV of 1.8 bln euros.
- The FMV of the largest individual investment was over 200 times as large as the FMV of the smallest.
- The investments are located in the following countries: The Netherlands, Italy, Spain, France, UK and Australia.
- The investments are in 23 ICB Sectors. When aggregating these into less granular ICB Industries, the main ones are consumer goods and services, industrials and technology.
- Leverage values span the whole range of zero (typically successful deals close to exit) to high values around 70% (typically deals that just started) to even extremely high values of more than 90% (typically deals in financial distress).
- The share of the common equity held is about 40% on average. The share of preferred shares/shareholder loans is often nonzero. For those cases the average share of preferred shares/shareholder is 16%-points more than the equity share.
- The illiquidity discount ranges from 0-30%. The average is 10% so it is somewhat tilted towards the smaller values.
- None of the investments has an exit horizon below 3 months.

## 8 Results

### 8.1 Base Case

Table 2 presents our capital requirements results.<sup>20</sup> We think that the portfolio we consider is quite representative for the European market. It is mainly focussed on buy-outs in a substantial part of the European market and is spread out over several sectors. In the remainder we assume that the portfolio is sufficiently diversified in the regulatory capital computations. It results in a regulatory capital figure for the portfolio of 20.5%. Applying the required CRD floors hardly makes a difference, whereas incorporating the Basel II floors has a substantial impact since there the floor exceeds the average computed capital requirements.

Naturally, this is merely a snapshot of the required capital for this portfolio at the time the snapshot is taken. In absence of enough historical data to do backtesting, we perform a sensitivity analysis in the next subsection in which we show that our results are still valid in more pessimistic conditions.

### 8.2 Sensitivity analysis on the different portfolio results

In this section, we try to see the impact of changes in the granularity of the portfolio or the different parameters to justify that our earlier findings on the snapshot portfolio hold more in general.

One of the reasons for capital requirements exceeding those in the CRD could be due to concentration effects in the portfolio. To investigate this, we split each investment into 5 separate but identical investments. Effectively, this reduces the single-name concentration, but the distribution over regions and industries stays the same. The resulting capital requirement is lowered by 1.5 percentage-point, which hardly affects the conclusions.

To see the effect of accounting or valuation noise on our results, we lower the current fair market value of our investments by 5% and recompute our average capital requirement. Interestingly enough, the result barely changes. The reason for this is that the capital requirement is defined as a percentage of the economic value and both go down with approximately the same percentage.

Next, we consider the case where exit proceeds have been projected too optimisti-

---

<sup>20</sup>Note that for the shareholder loans the loan treatment can be used instead of the internal model approach. Given the close resemblance between shareholder loans and preferred stock, we stick to the internal model approach. See also our discussion in the next section.

cally. The expected exit proceeds are reduced by 5%. This will increase the default probabilities and lower the average gains on any investment that can be used to compensate possible future losses on other investments. The impact of this change is a bit more substantial, but stays within reasonable bounds since the horizon is merely 3 months.<sup>21</sup>

Combining the two adjustments above hardly changed the capital requirement. Therefore, we repeated the analysis with decreases of 10% rather than 5%, while leaving the amount of debt intact. This leads to an increase in leverage but its impact is small.

Two key risk parameters in the model are the volatilities and the correlations. A sudden increase in volatility, proxied by multiplying the volatilities with 1.1 leads to an increase in capital requirement of 2.4% percentage points. For correlations, we stressed both the intra industry correlations ( $\rho_{M,AS}$ ) as well as the inter industry correlations by multiplying each by 1.05. The effects are very similar<sup>22</sup> and also here, the change is only moderate.

Now consider a burst of a bubble, a sudden hit to the economy or some other bad scenario. In these cases, asset values and exit proceeds typically go down, whereas volatility goes up. We proxy for this by increasing equity volatilities by 50%. To establish that we decrease the asset values and expected exit proceeds by 20% while multiplying asset volatilities by 1.33 (this combined with the increased leverage raises the equity volatility by approximately 50%). This results in a disaster scenario capital requirement of 31.6%, which is slightly below the 32% as suggested by Basel II. Note that also here the fact that a horizon of 3 months rather than one year helps a great deal in preventing capital requirements to increase too much in absolute terms.<sup>23</sup>

We also investigated the impact of a change in the risk-free rate where we doubled the risk-free rate, but this seemed to be of minor importance.

Based on the sensitivity analysis above, we do find room to lower the simple risk weighted assets capital requirements for private equity investments, since in all but the disaster scenario, the simple Basel II capital requirement is substantially higher

---

<sup>21</sup>A similar analysis with a one year horizon showed larger and much less stable figures.

<sup>22</sup>The effect of the intra industry correlations looks larger, but the  $\rho_{M,AS}$  are squared when calculating correlations.

<sup>23</sup>Changes in the growth rate and leverage cause the profit and loss distribution at the risk horizon to shift relative to the fixed benchmark of the risk free rate; since this shift is linear in the length of the risk horizon, the effect is much less prevalent for the three months horizon than for the one year horizon.

than the internal model result. However, in view of our results and the fact that prospects were not so bad at the portfolio snapshot date as well as the considerations pointed out in the next section, we think that cutting these capital requirements into half as is done in the CRD is overly optimistic and gives adverse incentives to banks to spend effort on developing sophisticated risk models.<sup>24</sup>

Case	Capital Requirement
Base	20.5%
With CRD floor	20.5%
With Basel II floor	27.0%
Granularity	19.0%
FMV - 5%	20.6%
ExpExit-5%	19.7%
High Leverage	21.8%
Volatility +10%	22.9%
$\alpha$ s +5%	22.6%
Inter Industry Correlations +5%	21.4%
Disaster	31.6%
$r$ double	19.1%

Table 2: Base case and sensitivity analysis of the capital requirement for a portfolio of buy-out investments

## 9 Discussion

We do realize that the model that we present in this paper is not a perfect model, but merely as much as can be sustained by the data at our disposal. A common critique to our model is that a dependence structure stemming from the normal copula might not be the appropriate one and should be replaced by a copula with fatter tails. This is certainly a point for future research. However, with respect to our conclusion regarding the overly optimistic CRD, this argument would only strengthen our conclusion, since fatter tails would increase the risk and hence the capital requirements. It is not clear whether the increase would increase the capital requirements up to the Basel II level. On top of that we would like to add that the selection of a copula and the associated parameters is not trivial. To our knowledge, there is no clear evidence on which copula to use for underlying asset distributions

---

<sup>24</sup>Especially in view of the fact that the portfolio of a bank that is likely to use internal models is likely to be more diversified than a typical simple risk weight approach bank.

in structural models. Just choosing a fat tailed copula would just be as arbitrary as the normal one.

There are two other issues that we would like to highlight here as well. The first one is about ambiguities in the accords. Ambiguities and possible RC arbitrage opportunities in the accords can compromise the goals of the accords severely, since they might lead to deals that are not necessarily structured in an optimal way, but merely structured in such a way that RC is minimized. For example, part of the common equity in a deal might be replaced by shareholder loans if there is ambiguity about how shareholder loans should be qualified. In practice, they are subordinated so that they are much closer to equity than to debt (especially when they are only held by common stock holders as is usually the case), but since they are technically debt, they might qualify for a much lower capital requirement. Another problem is the definition of exposure, which is in our view inconsistent. If an investment is consolidated, the economic value to compute regulatory capital is the book value of the investment whereas for non-consolidated investments, it is the FMV. The internal model approach described in this paper results in a regulatory capital amount that is not affected by the distinction between consolidated (economic value equal to book value) and non-consolidated (economic value equal to FMV). However, when translating the regulatory capital amount into a capital requirement, the result will be that the capital requirements for consolidated investments will be even higher than reported here because they are typically booked at a value lower than FMV. Hence the effective capital charge under the internal model approach for consolidated investments will be even higher, giving further incentives to banks not to develop sophisticated risk models.

## 10 Conclusion

In this paper we have investigated whether the regulatory capital regulations for Private Equity investments under both Basel II and the CRD are consistent with the goals that these regulations try to achieve. To this end, we compared the capital requirements for PE under Basel II and CRD to those of a structural model on a proprietary and representative dataset. We found capital requirements that are substantially lower than those proposed by the Basel II committee and thus may impede competition across banks. On the other hand however, these are still substantially higher than those implemented by the European Union, which is likely to lead to banks not investing in sophisticated risk models. Therefore, we cannot

justify the rigorous changes made in the CRD legislation.

To obtain our results, we have introduced a structural model to take care of several stylized facts of individual PE investments. This way we make sure that these stylized facts and correlation structures are properly reflected in portfolio results. A sensitivity analysis showed that our results are robust to parameter uncertainty and the sudden advent of adverse market conditions.

# A First passage time expressions for Time varying Brownian Motions

In this appendix an analytical expressions will be derived for a first passage time pdf of a generalized geometric Brownian motion. This result will be necessary to derive the cdf of the minimum of a Brownian motion given its terminal realization. This cdf will be used in the valuation formula as well as in the simulation procedure. A structure and arguments similar to Shreve (1997) will be used.

## A.1 Without a drift

Define

$$U(T) = \min_{0 \leq t \leq T} \sqrt{\sigma_t^2} B(t), \quad (28)$$

where  $B(t)$  is a standard Brownian motion. Then we have by applying the reflection principle (scaling the Brownian motion does not change this principle)

$$\begin{aligned} P(U(T) < u, \sqrt{\sigma_T^2} B(T) > b) &= P(\sqrt{\sigma_T^2} B(T) < 2u - b) \\ &= \frac{1}{\sqrt{2\pi T \sigma_T^2}} \int_{-\infty}^{2u-b} \exp\left\{-\frac{x^2}{2T \sigma_T^2}\right\} dx. \end{aligned} \quad (29)$$

We can take the derivative w.r.t.  $u$  and  $b$  to obtain the joint pdf:

$$\begin{aligned} f(U(T), \sqrt{\sigma_T^2} B(T)) &= -\frac{\partial}{\partial u \partial b} \left( \frac{1}{\sqrt{2\pi T \sigma_T^2}} \int_{-\infty}^{2u-b} \exp\left\{-\frac{x^2}{2T \sigma_T^2}\right\} dx \right) \\ &= -\frac{\partial}{\partial u} \left( \frac{1}{\sqrt{2\pi T \sigma_T^2}} \exp\left\{-\frac{(2u-b)^2}{2T \sigma_T^2}\right\} \right) \\ &= \frac{2(b-2u)}{T \sigma_T^2 \sqrt{2\pi T \sigma_T^2}} \exp\left\{-\frac{(2u-b)^2}{2T \sigma_T^2}\right\}. \end{aligned} \quad (30)$$

## A.2 With drift

Now we want to derive a result like this for a drifted (with time varying drift) scaled Brownian motion as well. This is done by a change of measure and applying the result from above.

Let

$$\sigma_t d\tilde{B}_t = \mu_t dt + \sigma_t dB_t = k_t \sigma_t^2 dt + \sigma_t dB_t, \quad (31)$$

where  $k_t = \frac{\mu_t}{\sigma_t^2}$ . Then

$$\begin{aligned}
\sqrt{\bar{\sigma}_T^2} \tilde{B}(T) &= \int_0^T \sigma_t d\tilde{B}_t \\
&= \int_0^T \mu_t dt + \sqrt{\bar{\sigma}_T^2} B(T) \\
&= \int_0^T \sigma_t^2 k_t dt + \int_0^T \sigma_t dB(t),
\end{aligned} \tag{32}$$

where  $\bar{\sigma}_T^2$  is the average value of  $\sigma_t^2$  up till time  $T$ . The Radon-Nikodym derivative is then given by:

$$\begin{aligned}
\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}} &= \exp\left(-\int_0^T k_t \sigma_t dB_t - 0.5 \int_0^T k_t^2 \sigma_t^2 dt\right) \\
&= \exp\left(-\int_0^T k_t \sigma_t d\tilde{B}_t + \int_0^T k_t^2 \sigma_t^2 dt - 0.5 \int_0^T k_t^2 \sigma_t^2 dt\right).
\end{aligned} \tag{33}$$

To see that this is correct, consider the following.

$$\begin{aligned}
E_{\mathbb{P}}\left(\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}} \exp(\beta \sqrt{\bar{\sigma}_T^2} \tilde{B}_T)\right) &= E_{\mathbb{P}}\left(\exp(\beta \sqrt{\bar{\sigma}_T^2} \tilde{B}_T - \int_0^T k_t \sigma_t dB_t - 0.5 \int_0^T k_t^2 \sigma_t^2 dt)\right) \\
&= E_{\mathbb{P}}\left(\exp\left(\int_0^T (\beta - k_t) \sigma_t dB_t - 0.5 \int_0^T k_t^2 \sigma_t^2 dt + \int_0^T \sigma_t^2 k_t dt\right)\right) \\
&= \exp\left(\int_0^T 0.5(\beta - k_t)^2 \sigma_t^2 dt - 0.5 \int_0^T k_t^2 \sigma_t^2 dt + \int_0^T \sigma_t^2 k_t dt\right) \\
&= \exp(0.5\beta^2 \bar{\sigma}_T^2 T) \\
&= E_{\tilde{\mathbb{P}}}(\exp(\beta \sqrt{\bar{\sigma}_T^2} \tilde{B}_T)).
\end{aligned} \tag{34}$$

Using the analysis above and an argument similar to Shreve (1997), we have that

$$\begin{aligned}
f_{\mathbb{P}}(U(T) = u, B(T) = b) &= \\
&= \frac{2(b - 2u)}{T \bar{\sigma}_T^2 \sqrt{2\pi T \bar{\sigma}_T^2}} \exp\left\{-\frac{(2u - b)^2}{2T \bar{\sigma}_T^2}\right\} \cdot \exp\left(\sqrt{\frac{\int_0^T k_t^2 \sigma_t^2 dt}{\int_0^T \sigma_t^2 dt}} b - 0.5 \int_0^T k_t^2 \sigma_t^2 dt\right).
\end{aligned} \tag{35}$$

## B Conditional default pdf

This appendix section gives the derivation of the pdf  $f(U(T) = u | \sqrt{\sigma_T^2} \tilde{B}(T) = b)$ . To start with, note that

$$f(\sqrt{\sigma_T^2} \tilde{B}(T) = b) = \frac{1}{\sqrt{2\pi\sigma_T^2 T}} \exp \left\{ -\frac{(b - \int_0^T m_t dt)^2}{\sigma^2 T} \right\}. \quad (36)$$

Applying Bayes rule and using the results from Appendix A, we have:

$$\begin{aligned} f(U(T) = u | \sqrt{\sigma_T^2} \tilde{B}(T) = b) &= \frac{f(U(T) = u, \sqrt{\sigma_T^2} \tilde{B}(T) = b)}{f(\sqrt{\sigma_T^2} \tilde{B}(T) = b)} = \\ &= \frac{2(b-2u)}{T\sigma_T^2} \exp \left\{ -\frac{4u^2 - 2(2u - \int_0^T m_t dt)b - \left(\int_0^T m_t dt\right)^2}{2T\sigma_T^2} \right\} \\ &\quad \cdot \exp \left\{ \sqrt{\frac{\int_0^T k_t^2 \sigma_t^2 dt}{\int_0^T \sigma_t^2 dt}} b - 0.5 \int_0^T k_t^2 \sigma_t^2 dt \right\}. \quad (37) \end{aligned}$$

When the drift and volatility are constant, this reduces to:

$$\begin{aligned} f(U(T) = u | \tilde{B}(T) = b) &= \frac{f(U(T) = u, \tilde{B}(T) = b)}{f(\tilde{B}(T) = b)} \\ &= \frac{\frac{2(b-2u)}{T\sqrt{2\pi T}} \exp \left\{ -\frac{(2u-b)^2}{2T} \right\} \cdot \exp(mb - 0.5m^2 T)}{\frac{1}{\sqrt{2\pi T}} \exp \left\{ \frac{-(b-mT)^2}{2T} \right\}} \\ &= \frac{2(b-2u)}{T} \exp \left\{ -\frac{4u(u-b)}{2T} \right\}. \quad (38) \end{aligned}$$

Integrating over  $u$  then gives the CDF:

$$F(U(T) \leq u | \tilde{B}(T) = b) = \exp \left\{ -\frac{4u(u-b)}{2T} \right\}. \quad (39)$$

## References

Ambrose, B., LaCour-Little, M. and Sanders, A.: 2004, Does regulatory capital arbitrage or asymmetric information drive securitization? Working Paper.

- Artzner, P., Delbaen, F., Eber, J. and Heath, D.: 1999, Coherent measures of risk, *Mathematical Finance* **9**, 203–228.
- Basel Committee on Banking Supervision: 1988, International convergence of capital measurement and capital standards, *Technical report*, Bank for International Settlements.
- Basel Committee on Banking Supervision: 2006, International convergence of capital measurement and capital standards, *Technical report*, Bank for International Settlements. Comprehensive Version, <http://www.bis.org/publ/bcbs128.pdf>.
- Black, F. and Cox, J.: 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* **31**(2), 351–367.
- Collin-Dufresne, P. and Goldstein, R.: 2001, Do credit spreads reflect stationary leverage ratios?, *Journal of Finance* **56**(5), 1929–1957.
- Danielsson, J., Embrechts, P., Goodhart, C., Keating, C., Muennich, F., Renault, O. and Shin, H.: 2001, An academic response to basel ii. LSE Financial Markets Group ESRC Research Center Special Paper Series no 130.
- Duffie, D. and Lando, D.: 2000, Term structures of credit spreads with incomplete accounting information, *Econometrica* **69**(3), 633–664.
- Duffie, D. and Singleton, K.: 2003, *Credit Risk: Pricing, Measurement and Management*, Princeton Series in Finance, Princeton University Press, Princeton.
- European Union: 2006, Directive 2006/48/ec of the european parliament and of the council, *Official Journal of the European Union* **L 177**, 1–200. [http://eur-lex.europa.eu/LexUriServ/site/en/oj/2006/l\\_177/l\\_17720060630en00010200.pdf](http://eur-lex.europa.eu/LexUriServ/site/en/oj/2006/l_177/l_17720060630en00010200.pdf).
- EVCA: 2003, Position statement basel ii, Press release. <http://www.evca.com>.
- Hull, J.: 2003, *Options, Futures and Other Derivatives*, 5th edn, Prentice Hall, Upper Saddle River.
- Kaplan, S. and Schoar, A.: 2003, Private equity performance: Returns, persistence and capital flows. MIT Sloan Working Paper No. 4446-03.

- Karatzas, I. and Shreve, S.: 1991, *Brownian Motion and Stochastic Calculus*, Graduate Texts in Mathematics, Springer-Verlag, New York.
- Kaserer, C. and Diller, C.: 2004, What drives cash flow based european private equity returns? - fund inflows, skilled gps and/or risks? CEFS Working Paper.
- Kealhofer, S.: 2003b, Quantifying credit risk ii: Debt valuation, *Financial Analysts Journal* pp. 78–92. obtained from the world wide web: [www.defaultrisk.com](http://www.defaultrisk.com).
- Ljungqvist, A. and Richardson, M.: 2003, The cash flow, return and risk characteristics of private equity. Working Paper No. S-FI-03-01.
- Ljungqvist, A., Richardson, M. P. and Wolfenzon, D.: 2007, The Investment Behavior of Buyout Funds: Theory and Evidence, *SSRN eLibrary* .
- Longstaff, F. and Schwartz, E.: 1995, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance* **50**(3), 789–819.
- Merton, R.: 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* **29**(2), 449–470.
- Shreve, S.: 1997, Stochastic calculus and finance, Lecture Slides.
- Suarez, F., Dhaene, J., Henrard, L. and Vanduffel, S.: 2006, Basel ii: Capital requirements for equity investment portfolios. FETEW Research Report, K.U.Leuven.
- Thomson Financial and PricewaterhouseCoopers: 2006, Evca yearbook 2006, *Technical report*, EVCA.