JOURNAL OF APPLIED ECONOMETRICS, VOL. 9, 31–45 (1994)

A UNIVARIATE ANALYSIS OF EMS EXCHANGE RATES USING A TARGET ZONE MODEL

F. DE JONG
Department of Econometrics, Centre for Economic Research, Tilburg University, PO Box 90153, 5000 LE, Tilburg, The Netherlands

SUMMARY

The models in the literature on exchange-rate target zones imply a non-linear time series model for the exchange rate. We show how the parameters of such models can be estimated and develop Maximum Likelihood and Method of Simulated Moments estimators for the target zone model of Krugman (1991). The Maximum Likelihood estimator is based on a computationally attractive approximation to the exact predictive density of the continuous time model. Monte Carlo experiments are used to assess the properties of this estimator. In the empirical part we estimate the model with data on recent EMS exchange rates. We find that the Krugman (1991) target zone model is not able to explain the full observed kurtosis and conditional heteroscedasticity of the exchange-rate returns.

1. INTRODUCTION

In recent years, target zone arrangements for exchange rates have become very important in Europe. The EMS has extended to cover nearly all EC countries, and was stable until the crisis of September 1992. The Nordic countries had self-imposed target zones for their currencies, which were also rather successful until 1992. In such target zones, monetary authorities promise to keep the exchange rate within a prespecified band. Following the the seminal model of Krugman (1991), a large theoretical literature of exchange-rate determination in target zones has developed. The crucial observation in these models is that the target zone influences the expectations of future spot prices because the central banks will intervene if the exchange rate deviates too much from its central parity. Hence, in a forward-looking model of exchange-rate determination, the presence of a target zone has an impact on the exchange rate itself, even if there are currently no interventions. One of the most striking implications of the Krugman model, and nearly all other theoretical models, is the non-linear, S-shaped relation between fundamentals and exchange rates.

In this paper, we develop several methods for estimating the Krugman (1991) target zone model. For this model the likelihood function is known, though quite complicated, so efficient estimation and testing based on the method of Maximum Likelihood is possible. However, for almost any extension of the model that is present in the literature, the likelihood function is not known, so other estimators are needed. An alternative class of estimators is given by the Generalized Method of Moments. A recent development in GMM estimation is the Method of Simulated Moments, where the model is simulated to compute the moments of the theoretical distribution. Although the computational burden of simulating artificial data and computing their moments is high, simulated moments estimators are well suited for applications in target

CCC 0883–7252/94/010031–15

Received October 1991
Revised June 1993

zone models, because in these models the stochastic process generating fundamentals and exchange rates is explicitly specified, and simulation of the model is conceptually straightforward.

In the empirical part of this paper the Krugman target zone model is estimated and tested on EMS exchange rate data over a relatively stable period, January 1987 to October 1990. The results indicate that there are significant non-linearities in the exchange rate processes of the Belgian franc, the French franc, and the Danish krone, possibly caused by the impact of the band on expectations of future variables. For the three other currencies, however, there seem to be no non-linearities at all, which result may be explained by the presence of an implicit band that is narrower than the official EMS band. The specification of the model is tested by comparing some moments of the theoretical distribution with the sample moments. These tests reveal that the Krugman target zone model is misspecified: the model is not capable of explaining the full magnitude of the observed conditional heteroscedasticity and leptokurtosis in exchange-rate returns.

The organization of the paper is as follows. Section 2 reviews the Krugman target zone model briefly, and Section 3 gives an overview of the empirical literature on testing for non-linearities in exchange rates and on estimating target zone models. Section 4 discusses Maximum Likelihood estimation, and Section 5 Method of Simulated Moments estimation of the Krugman target zone model. Section 6 presents the empirical results and Section 7 concludes the paper. An appendix gives details on the simulation of the model.

2. A SIMPLE MODEL OF EXCHANGE-RATE DETERMINATION IN A TARGET ZONE

In this section we describe the basic target zone model of Krugman (1991). In this model, the logarithm of the exchange rate, $e(t)$, is a function of the economic fundamental and its own expected rate of change:

$$e(t) = f(t) + \alpha \cdot E_t(\mathrm{d}e(t)/\mathrm{d}t), \qquad \alpha > 0 \tag{1}$$

Excluding 'bubble' solutions, this implies that $e(t)$ is the present discounted value of all expected future fundamental values:

$$e(t) = \alpha^{-1} \int_{\tau=0}^{\infty} \exp(-\tau/\alpha) E_t[f(t+\tau)] \, \mathrm{d}\tau \tag{2}$$

It is assumed that, except for occasional interventions, the fundamental follows a Brownian motion with constant drift, μ, and variance, σ^2:

$$\mathrm{d}f(t) = \mu \mathrm{d}t + \sigma \, \mathrm{d}W(t) \tag{3}$$

where $W(t)$ is the Wiener process with standard normal increments over the unit time interval. If the relevant exchange-rate regime is a free float without interventions, the solution of the model is linear:

$$e(t) = G(f(t)) = f(t) + \alpha\mu \tag{4}$$

To obtain a solution for the model under a target zone regime, first note that the general solution for the exchange rate is

$$e(t) = G(f(t)) = f(t) + \alpha\mu + A_1 \exp(\lambda_1 f(t)) + A_2 \exp(\lambda_2 f(t)) \tag{5}$$

where λ_1 and λ_2 are the roots of the characteristic equation

$$\tfrac{1}{2}\alpha\sigma^2\lambda^2 + \alpha\mu\cdot\lambda = 1$$

and A_1 and A_2 are constants that are to be determined from the relevant boundary conditions provided by the target zone model. In the model of Krugman (1991) there is a fully credible target zone $[\mathbf{e}, \bar{e}]$ with infinitesimal interventions at the margin only. There are no interventions within the band, which is clearly an unrealistic assumption. With infinitesimal marginal interventions the boundary conditions are the value-matching conditions $G(\bar{f}) = \bar{e}$ and $G(\mathbf{f}) = \mathbf{e}$ and the 'smooth-pasting' conditions, which state that the derivative of $G(f(t))$ is zero at the margins of the target zone. The solution for the function $G(f(t))$ is non-linear and has an S-shaped form. Note that in solving the model the value-matching and smooth-pasting conditions have to be solved simultaneously in the four unknown parameters $\mathbf{f}$, $\bar{f}$, A_1 and A_2.

In the Krugman model the fundamental follows a Brownian motion within the implicit band $[\mathbf{f}, \bar{f}]$, and is kept within the band by infinitesimal interventions at the margin. This stochastic process is called a regulated Brownian motion and exhibits a strong Markov property (Harrison, 1985, p. 81) which implies that all relevant information of the model is captured in the current fundamental $f(t)$. Therefore, the exchange rate is a function of the current fundamental only. This indicates another restrictive feature of the model.

More realistic models of exchange rate determination within a target zone have appeared in the literature recently.[1] Miller and Weller (1991) use a model with slow price adjustment and obtain a target zone solution that depends on more than one state variable. The model of Bertola and Svensson (1993) introduces a second state variable that represents the risk of a realignment. Lindberg and Söderlind (1992) allow for mean reversion in the fundamental, which is thought to represent intra-marginal interventions by central banks. These extensions add realism to the very restrictive Krugman model.

3. EMPIRICAL WORK ON TARGET ZONE MODELS

Recently, considerable attention has been paid in the literature to non-linear exchange-rate models. There are different approaches, some papers use very general non-parametric methods, whereas others estimate strictly specified target zone models. In this section we briefly review this literature.

Diebold and Nason (1990) try to improve upon the forecastability of a linear autoregressive time-series model for exchange rates by using a non-parametric Locally Weighted Least Squares predictor. For a sample of weekly US dollar exchange rates from 1973 to 1987, the non-parametric technique produces better within-sample predictions, but the out-of-sample performance is not better than that of the linear models. Their conclusion is that non-linearities have not been important for the exchange rates under consideration.

The work of Meese and Rose (1990, 1991) is similar in spirit, but their models include a set of 'fundamental' determinants of the exchange rate, such as interest rates, money supplies, and output. The aim of their research is twofold: first, to examine whether such structural models predict the exchange rate better than the random walk, and second, to test for non-linearities in the relation between fundamentals and exchange rates. The technique used is a non-parametric Locally Weighted Regression method and a so-called Alternating Conditional Expectations method. Again, using dollar exchange rates, the conclusions are negative: there

[1] The current literature on target zone models is discussed more extensively in Svensson (1992).

are no significant non-linearities and the predictive performance of structural models is not better than that of the random walk.

Flood *et al.* (1990) use the forward-looking exchange rate equation (1) to construct a measure of the fundamental. Assuming uncovered interest rate parity, the expected depreciation in equation (1) is replaced by a very short-term interest rate differential. The only thing that is unknown in this equation is the parameter α, which is assumed to equal 0·1. Inspection of the graphs of the exchange rate versus the constructed fundamental reveals remarkably few non-linearities, and if the relation appears to be non-linear, the shape is not quite the S-shaped type predicted by the Krugman model. A formal test is performed by regressing $e(t)$ on $\hat{f}(t)$ and two exponential terms, similar to those in equation (5). The parameters of the exponential terms are often significant, but the signs of the parameters are not those predicted by the theoretical model.

The two-state model of Bertola and Svensson (1993) with realignment risk is analysed by Rose and Svensson (1991) for the French franc/deutschmark exchange rate. They use a cubic approximation to the Bertola–Svensson model to predict exchange rate changes within the band and find significant mean reversion within the band. Assuming uncovered interest parity holds, the expected rate of realignment can be assessed from the observed interest rate differentials. It appears difficult to predict realignments accurately.

A different approach to target zone modelling is taken by Pesaran and Samiei (1992). They incorporate rational expectations in a discrete-time model with a limited dependent variable, and derive an implicit solution for the expectations variable. Although their model has current instead of future expectations, the model also implies an S-shaped relation between exchange rate and expected fundamental. The relation is, however, not deterministic but stochastic. The results show that the target zone model fits the data of the deutschmark/French franc exchange rate during the EMS period better than a model that does not take the presence of a band on this exchange rate into account.

The models closest in spirit to this paper are Smith and Spencer (1991) and Lindberg and Söderlind (1992). Smith and Spencer use the Method of Simulated Moments to estimate the Krugman target zone model on the deutschmark/Italian lira exchange rate. Estimation of the model appears to be difficult; the paper reports problems in finding an optimum of the criterion function.

Lindberg and Söderlind (1992) estimate their model with a mean reverting fundamental on Swedish data by the Method of Simulated Moments. The mean reversion is significant, indicating that the Krugman model is misspecified for the Swedish exchange rate. Mean reversion in the fundamentals seems *a priori* important for EMS currencies as well because central banks often use intra-marginal interventions. Koedijk *et al.* (1992) present evidence on this point for the Belgian franc. However, given the difficulties of incorporating mean reverting fundamentals into the estimation method proposed, we shall use the Krugman model in the empirical part of this paper, although the absence of mean reversion is restrictive.

4. MAXIMUM LIKELIHOOD ESTIMATION

In this section we show how the Krugman (1991) target zone model can be estimated efficiently by the method of Maximum Likelihood (ML). In order to apply ML, the statistical distribution of a sequence $\{e_1(\vartheta), .., e_T(\vartheta)\}$ generated by the model must be known to derive the likelihood function, defined as the joint density[2] of the observations $D(e_1, .., e_T \mid \mathbf{E}_0; \vartheta)$, where $\mathbf{E}_0$ is the set of initial conditions.

[2] The symbol D is used to denote any (joint) density function.

In the Krugman target zone model the fundamental follows a regulated Brownian motion on the interval $[\mathbf{f}, \bar{f}]$. It is therefore convenient to rewrite the likelihood function in terms of the fundamentals. The mapping $e = G(f; \vartheta)$ is monotone and continuously differentiable for any ϑ so we can apply a change of variables, $f_t \equiv G^{-1}(e_t; \vartheta)$, for all observed values of the exchange rate and rewrite the likelihood function to

$$D(e_1, .., e_T \mid \mathbf{E}_0; \vartheta) = D(f_1, ..., f_T \mid \mathbf{E}_0; \vartheta) \cdot \prod_t G'(f_t; \vartheta)^{-1} \tag{6}$$

where $\mathbf{E}_0$ is the set of initial conditions. The latter part of equation (6) is the Jacobian of the transformation, which is always positive and continuous. Due to the strong Markov property of the regulated Brownian motion, the history $f(t-\tau)$, $\tau > 0$, is irrelevant for the distribution of the future of the process, $f(t+\tau)$, $\tau > 0$, if $f(t)$ is known. This property allows us to condition the distribution of f_t on the previous observation only, and the likelihood function can be rewritten to

$$D(e_1, .., e_T \mid e_0; \vartheta) = \prod_{t=1}^{T} D(f_t \mid f_{t-1}; \vartheta) \cdot \prod_{t=1}^{T} G'(f_t; \vartheta)^{-1} \tag{7}$$

In our empirical work we condition the first observation, e_0 which is transformed to the initial state of the fundamental process through $f_0 \equiv G^{-1}(e_0; \vartheta)$.

Having derived the likelihood of the exchange rate in terms of the fundamentals, what remains to be found is the predictive density function of the regulated Brownian motion. The predictive distribution function of the *one-sided* regulated Brownian motion with regulation at lower bound $\mathbf{f}$ is given in Harrison (1985, p. 49). The expression is

$$P(f \mid f_{t-s}) = \Phi\left(\frac{f - f_{t-s} - \mu s}{\sigma\sqrt{s}}\right) - e^{\tau(f-\mathbf{f})}\Phi\left(\frac{2\mathbf{f} - f - f_{t-s} - \mu s}{\sigma\sqrt{s}}\right), \qquad \tau \equiv 2\mu/\sigma^2 \tag{8}$$

The first part of this function is the usual normal distribution function, whereas the second part represents the probability that $f(t)$ is regulated at the lower bound in the time interval $(t-s, t]$. The marginal distribution is obtained by letting s go to infinity, which yields

$$P(f) = \begin{cases} 1 - e^{\tau(f-\mathbf{f})} & \text{if } \tau < 0 \\ 0 & \text{if } \tau \geqslant 0 \end{cases} \tag{9}$$

If the target zone were one-sided, one could apply these distribution functions directly to obtain the likelihood function. However, most actual target zones are two-sided, so we need the conditional density or distribution function of a *two-sided* regulated Brownian motion. This function is quite complicated[3] and contains an infinite summation, which makes computation very time consuming.

Instead of using this exact density we approximate the conditional distribution of a two-sided regulated Brownian motion by a weighted average of the conditional distributions of two

---

[3] The density of a regulated Brownian motion $f(t)$ with drift μ, variance σ^2, and support $[\mathbf{f}, \bar{f}]$, conditional on $f(0) = f_0$ is

$$p(f \mid f_0) = \frac{\tau e^{\tau(f-\mathbf{f})}}{e^{\tau(\bar{f}-\mathbf{f})} - 1} + \frac{\exp[\tau(f - f_0)/2]}{4(\bar{f} - \mathbf{f})} \sum_{n=1}^{\infty} \frac{y_n(f) \cdot y_n(f_0)}{\lambda_n a^2/\sigma^2} \cdot \exp(-\lambda_n t)$$

$\tau \equiv 2\mu/\sigma^2$

$a \equiv (\bar{f} - \mathbf{f})/\pi$

$y_n(f) \equiv 2n \cdot \cos(n(f - \mathbf{f})/a) + \tau a \cdot \sin(n(f - \mathbf{f})/a)$

$\lambda_n \equiv \sigma^2[n^2/a^2 + \tau^2/4]/2$

These formulas are adapted and corrected from Appendix A3 of the working paper version of Svensson (1991a).

one-sided regulated Brownian motions, regulated at the lower and at the upper bound, respectively. The weights are chosen such as to satisfy two conditions. First, the approximate conditional distribution must converge to the exact marginal distribution as the time between $f(t)$ and the initial value $f(0)$ goes to infinity. Second, the function must converge to the predictive distribution of a one-sided regulated Brownian motion if one of the bounds goes to infinity. The distribution function that satisfies these conditions is

$$P(f\,|\,f_{t-s}) = \Phi\left(\frac{f - f_{t-s} - \mu s}{\sigma\sqrt{s}}\right) - (1 - P(f)\Phi\left(\frac{2\mathbf{f} - f - f_{t-s} - \mu s}{\sigma\sqrt{s}}\right) + P(f)\left[1 - \Phi\left(\frac{2\bar{f} - f - f_{t-s} - \mu s}{\sigma\sqrt{s}}\right)\right] \quad (10)$$

where

$$P(f) = \frac{e^{\tau(f-\mathbf{f})} - 1}{e^{\tau(\bar{f}-\mathbf{f})} - 1} \text{ if } \tau \neq 0, \qquad P(f) = \frac{f - \mathbf{f}}{\bar{f} - \mathbf{f}} \text{ if } \tau = 0$$

is the marginal or 'asymptotic' distribution of $f(t)$ (see Harrison, 1985, p. 90). Numerical experimentation shows that the first derivative of this approximate distribution function gives a very accurate approximation to the exact density function.

In order to test the accuracy and normality of the ML estimator in a reasonably large sample we picked a vector of pseudo-true parameter values and generated a number of artificial time series of exchange rates, using the simulation method of the target zone model described in the Appendix. Each simulated series contains 1000 observations. The pseudo-true parameter values selected are $(\mu, \sigma^2, \alpha) = (0, 4, 0 \cdot 1)$ and the exchange rate is allowed to fluctuate in a band with bounds $-\mathbf{e} = \bar{e} = 2 \cdot 25$; these values imply first and second moments for the exchange rate that are comparable to those of actual EMS exchange rates. For each simulated series the parameter vector was estimated by ML using the approximate likelihood function. In Figures 1–3, the empirical distribution functions of 100 estimates of μ, $\ln(\sigma^2)$ and $\ln(\alpha)$ are plotted

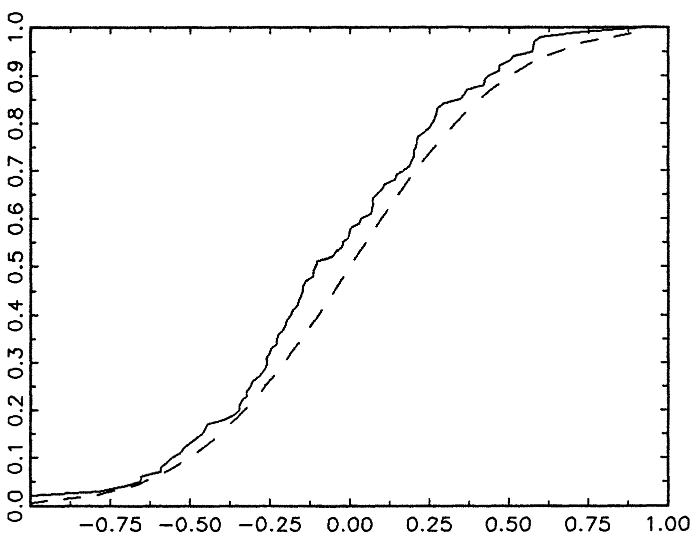

Figure 1. Empirical distribution of $\ln(\hat{\alpha})$. Solid line: empirical distribution function of $\ln(\hat{\alpha})$; dotted line: normal distribution function with mean equal to true pseudo-true parameter value and same variance as the empirical distribution. Average overestimation: $-0 \cdot 0639$ (t-value $1 \cdot 630$)

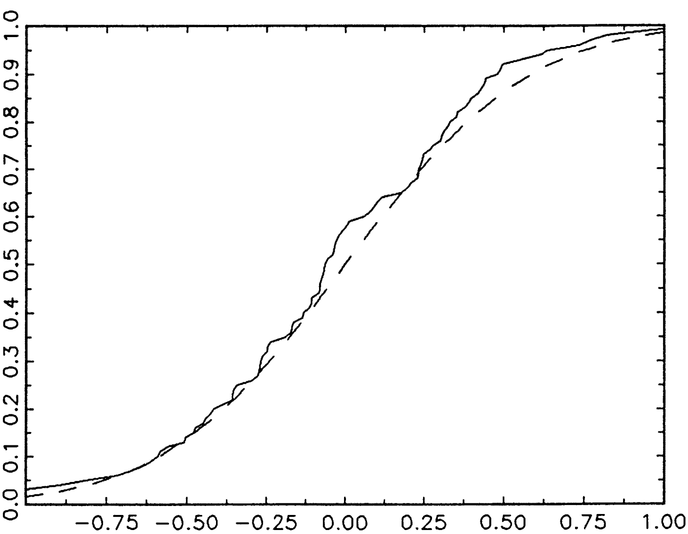

Figure 2. Empirical distribution of $\hat{\mu}$. Solid line: empirical distribution function of $\hat{\mu}$; dotted line: as in Figure 1. Average overestimation: $-0{\cdot}0398$ (t-value $0{\cdot}866$)

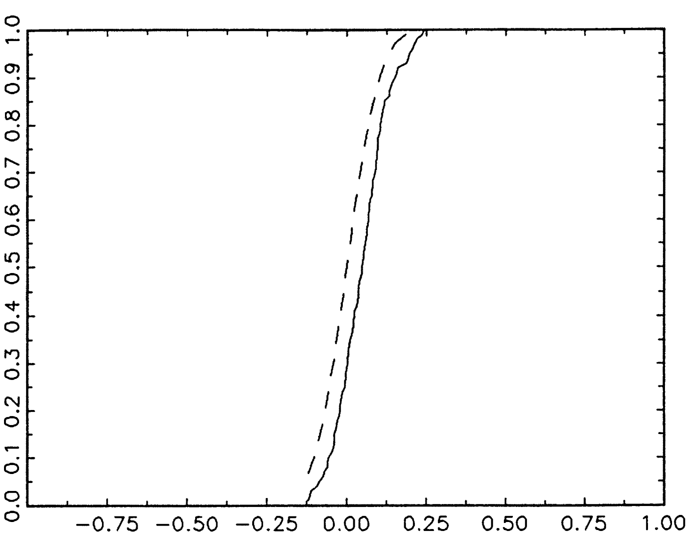

Figure 3. Empirical distribution of $\ln(\hat{\sigma}^2)$. Solid line: empirical distribution function of $\ln(\hat{\sigma}^2)$; dotted line: as in Figure 1. Average overestimation: $0{\cdot}0485$ (t-value $6{\cdot}070$)

against a normal distribution with mean equal to the pseudo-true parameter value and variance equal to the variance of the estimates. The ML estimator appears to be normal with the correct mean, although there is a slight overestimation of the variance of the fundamental. The variance of the estimates of μ and $\ln(\alpha)$ is rather large, indicating that these parameters are not very precisely estimated.

According to Chernoff (1954), the Likelihood Ratio test of H_0: $\alpha = 0$ against H_1: $\alpha > 0$ has a $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$ distribution. This distribution function and the empirical distribution function

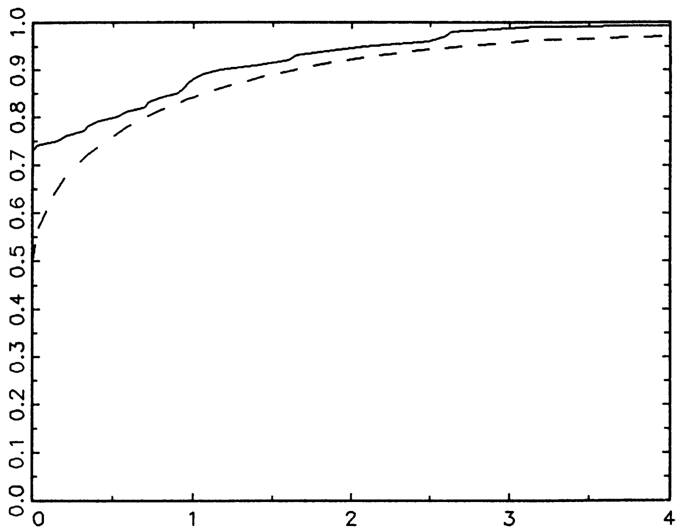

Figure 4. Empirical distribution of LR statistic. Solid line: empirical distribution of Likelihood Ratio statistic for $\alpha = 0$; dotted line: $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$ distribution function

computed from 100 Monte Carlo replications with pseudo-true parameter values $(\mu, \sigma^2, \alpha) = (0, 4, 0)$ is shown in Figure 4. The empirical distribution lies everywhere to the left of the theoretical one, so that using the critical values of Chernoff's distribution gives a conservative test.

The results of this section are not easily extended to a mean reverting process for the fundamental. For example, the fundamental in the Lindberg and Söderlind (1992) model is a two-sided regulated Ornstein–Uhlenbeck process. For this process, the functional form of the predictive density is not known, so Maximum Likelihood estimation is not easily implemented. Therefore, we keep to the Krugman target zone model and leave efficient estimation of other models for further research.

5. METHOD OF SIMULATED MOMENTS ESTIMATION OF TARGET ZONE MODELS

In the Krugman model the marginal and conditional densities of the exchange rate are known so that Maximum Likelihood estimation of the parameters is feasible. For more complicated target zone models the marginal and conditional exchange rate distributions are usually unknown. An estimator for these models is provided by the method of moments, in which the parameters are chosen such as to minimize the distance between a vector of observed (sample) moments and the theoretical (population) moments. However, in the target zone models computation of some useful moments, especially the autocovariances of the first differences (returns) of the exchange rates, requires multidimensional integration which may be problematic. McFadden (1989) and Lee and Ingram (1991) show that this problem can be solved by computing the theoretical moments by simulation. This method is especially attractive for application to target zone models because the stochastic process that drives the exchange rate is usually explicitly specified in these models.

We compute the moments of the exchange rate and its first difference as follows. A number of series $\{\tilde{e}_{it}\}$ of length equal to the sample size are simulated according to the procedure described in the Appendix. For each simulated series its 'sample' moments are computed, and a consistent estimator of the population moments is obtained by averaging the 'sample' moments over all simulated series. Because our sample starts just after a realignment there is no obvious reason why the first observation would come from the marginal distribution. Therefore, the analysis is performed conditional on the first observation, i.e. the simulation of each series is started at the value of the first observation.

The Method of Simulated Moments (MSM) estimator involves minimizing the distance between the sample moments and the simulated population moments, denoted by $d(\vartheta)$, with respect to the parameters of the model. The most efficient MSM estimator is obtained if one uses the inverse of the covariance matrix of the moments, Σ, to define the metric in which the distance is minimized. We estimate by the method of Newey and West (1987) from the simulated values. Lee and Ingram (1991) show that if the simulated exchange rates are independent of the observed series the asymptotic covariance matrix of the distance vector is $[1 + (1/n)]$ times the variance of the moments, where n is the number of simulated exchange rate series. In our applications $n = 50$ so that the variance caused by simulating the moments is relatively small. They also show that the asymptotic distribution of the MSM estimator is

$$\sqrt{T}(\hat{\vartheta} - \vartheta_0) \Rightarrow N\left(0, \left(1 + \frac{1}{n}\right)(D'\Sigma^{-1}D)^{-1}\right), \qquad D \equiv \partial d(\vartheta)/\partial\vartheta \tag{11}$$

In actual applications of the method of moments we want to use moments that are most informative about the parameters to be estimated. Although the exchange rate is a stationary process in the target zone model the moments of the *level* of the exchange rate are not very informative about the parameters, the reason being that the observations on the levels exhibit strong serial correlation and therefore very long time series are needed to obtain precise estimates of the theoretical moments of the level of the exchange rate.[4] Moreover, not all parameters of the target zone model can be estimated from the moments of the marginal distribution of the exchange rate alone, because the function $G(.)$, and the marginal distribution of the fundamental depend only on the ratio μ/σ^2 and the product $\alpha\mu$. This argument also implies that histograms of the marginal distribution of the exchange rate (e.g. used by Bertola and Caballero, 1992) alone do not provide information about all the properties of the model. Information from the conditional distribution of the exchange rate is necessary for estimating all parameters. Therefore, it is necessary to use moments of the first differences of the exchange rate. These moments have the additional advantage that their variance is relatively small because first differences of the exchange rate are not strongly correlated over time.

In our empirical application we use five moments of the first difference of the exchange rate. The first two moments are the expectation and the variance, because these are likely to be sensitive to the drift parameter, μ, and the variance parameter, σ^2, of the fundamental process. In addition, we use the third moment (skewness) and the first and second autocovariances of

[4] This argument may also explain why the estimator in the paper of Smith and Spencer (1991) does not work well. That estimator uses only three moments, the mean and variance of the exchange rate returns and the variance of the level, to estimate three parameters, so that, in principle, a unique estimator could be found. However, the variance of the exchange rate level is not very informative about the parameters. This is probably the cause of the difficulties in finding an optimum for the criterion function and obtaining good estimates for the parameters of the target zone model.

the first difference of the exchange rate. These moments are included to capture the non-linear part of the exchange rate process. In principle, many other moments could be added to this set, but, given computational limitations, we confine ourselves to the five moments mentioned.

6. EMPIRICAL RESULTS

In this section we present the results of an empirical application of the target zone model to exchange rates of six EMS currencies against the deutschmark. The model is estimated and it is tested to see whether the model can explain certain 'stylized' facts of the exchange-rate data.

We use samples of EMS exchange rates from the period after the realignment of January 1987. The sample ends in October 1990, well before the effects of German unification became known and culminated in the crisis in the EMS in September 1992. Although the absence of realignments does not imply complete credibility of the band (see Svensson, 1991b), we think it is worth testing the Krugman model on a sample from a relatively stable period within the EMS. The time series we study are 189 weekly observations from 14 January 1987 until October 1990 of the exchange rates of six major EMS currencies against the deutschmark. The currencies are the Belgian franc, the Dutch guilder, the Danish krone, the French franc, the Irish punt, and the Italian lira. The dependent variable, e_t, is defined as 100 times the relative deviation from the central parity. The upper and lower limit on the exchange rate in the model are put equal to the official bound of the target zone, i.e. a deviation of $+2 \cdot 25$ per cent upward and $-2 \cdot 25$ per cent downward from the central parity is allowed (+6 per cent and −6 per cent for the Italian lira until January 1990). We use weekly data to avoid problems with missing observations due to weekends and holidays. It must be stressed that the estimators developed in Sections 4 and 5 are perfectly suited for dealing with missing observations or any other type of non-equally spaced observations, because the conditional distributions and the simulation schemes can be adjusted for any time interval between two observations.

The Maximum Likelihood parameter estimates and some associated test statistics are reported in Table I. The estimates of α are significant for three currencies, the Belgian franc, the Danish krone, and the French franc. Graphical inspection of the data series shows that these exchange rates come close to the margin, but always revert to the central parity. Also, the volatility of these exchange rates appears to be smaller close to the margin. The target zone model picks up these effects. The large estimates of σ^2 and α for the Danish krone series and their large standard errors are caused by a high correlation between the estimators of these parameters; the estimated asymptotic correlation between $\hat{\sigma}^2$ and $\hat{\alpha}$ is nearly one. The likelihood function is also very flat in these parameters, which is reflected in the large standard errors.

The estimates for the Dutch guilder, the Irish punt, and the Italian lira show that the target zone model is essentially linear for these currencies; the point estimate of α is very close to 0, and the hypothesis that $\alpha = 0$ cannot be rejected by the Likelihood Ratio test at any usual level of significance. The failure of the target zone model to detect any non-linearities in the guilder and lira rates is probably not too surprising, as the guilder is always very close to its central parity in the sample, and also the lira does not get close to the margins of its relatively wide target zone, probably due to intra-marginal interventions.

The Method of Simulated Moments estimates are reported in Table II. The estimates for the guilder and punt are similar to the ML estimates. Note that the estimate of α for these series is close to zero and the model is again essentially linear. In that case, the drift and variance of the fundamental (μ and σ^2) are well estimated by the first two moments of the first difference of the exchange rate. For the other series the MSM estimates are far from the

Table I. Maximum Likelihood estimates for deutschmark exchange rates

	μ	σ^2	α	LR	M-norm	M-arch
Belgian franc	−0·148	1·399	4·089	5·32	0·89	1·19
	(0·387)	(2·124)	(9·099)			
Dutch guilder	0·028	0·170	0·100	0·00	17·01	170·75
	(0·153)	(0·027)	(a)			
Danish krone	−0·028	4972·03	139·37	31·47	12·75	1·96
	(0·017)	(4021·01)	(71·61)			
French franc	1·196	8·256	4·375	7·97	316·14	19·92
	(0·710)	(2·889)	(1·362)			
Irish punt	0·181	1·625	0·008	0·00	8·93	26·47
	(0·710)	(0·282)	(0·002)			
Italian lira	−1·296	5·723	0·240	0·10	105·18	3·71
	(1·286)	(1·515)	(0·443)			

Notes:
(1) Weekly data from 14/01/1987 to 3/10/1990.
(2) Dependent variable $e(t)$ is 100·ln (exchange rate/central parity).
(3) Units of parameters μ and σ^2 is 1/year, unit of α is one year.
(4) Asymptotic standard errors in parentheses.
(5) (a) not identified for this series.
(6) LR: likelihood ratio test for $\alpha = 0 \simeq \chi^2(1)$.
(7) M-norm: M-test on third and fourth moment of $\Delta e \simeq \chi^2(2)$.
(8) M-arch: M-test on first and second autocorrelation of $\Delta e^2 \simeq \chi^2(2)$.

Table II. Simulated moments estimates for deutschmark exchange rates

	μ	σ^2	α	dist	M-norm	M-arch
Belgian franc	−0·432	1·505	5·109	2·05	11·99	0·18
	(11·57)	(77·39)	(383·66)			
Dutch guilder	0·541	0·172	0·000	12·67	15·28	16·23
	(0·248)	(0·023)	(0·031)			
Danish krone	1·600	9·507	0·867	4·32	4·68	0·19
	(31·40)	(385·68)	(44·00)			
French franc	1·094	6·853	0·737	47·44	29·30	0·54
	(14·58)	(182·84)	(30·75)			
Irish punt	0·859	1·719	0·000	1·40	52·73	20·99
	(0·744)	(0·180)	(0·003)			
Italian lira	1·933	6·278	0·001	84·46	155·76	1·27
	(5·371)	(5·286)	(0·082)			

Notes: see Table I.
(1) 'dist' is the minimum of the distance function $\simeq \chi^2(2)$.
(2) Moments used for estimation: mean, variance, skewness, first and second autocorrelation of the first difference of the exchange rate.

Maximum Likelihood estimates. The estimates of the parameters are imprecise, which is reflected in the large standard errors, but a minimum of the distance function is always found. Thus, our choice of moments improves upon the methods of Smith and Spencer (1991) although precise parameter estimates cannot be obtained. A general specification test of the model is provided by the minimum of the criterion function. If the model is correctly specified, the minimum has a χ^2 distribution with degrees of freedom equal to the number of moments minus the number of parameters. The results in Table II show that this test rejects the model for the guilder, French franc, and lira. Lindberg and Söderlind (1992) argue that this may be due to the misspecification of the Krugman model which does not allow for interventions within the band.

We also perform two other specification tests, which test whether the model can explain 'stylized' facts about exchange-rate returns, in particular, ARCH effects and non-normality of the first differences. The test procedure employed is a variant of the M-test developed by Newey (1985). The principle of the test is to check whether some moments of the exchange rate distribution generated by the theoretical model are significantly different from their sample counterparts. The moments used for testing are computed by simulation.[5] Full details on the computation of the test statistics are in an appendix that is available from the author on request.

The first test, M-norm, is based on the unconditional third and fourth moment of the exchange rate returns. The distributional assumption made in deriving the Krugman model, as in nearly all other target zone models, is that the innovations in the fundamental are normally distributed. The S-shaped transformation from fundamental to exchange rate causes the exchange rate distribution to be non-normal and have even thinner tails than the normal. The M-norm statistic tests whether the observed first differences show a higher degree of departure from normality than is implied by the target zone model.

The second test, M-arch, is based on the autocovariances of the squared exchange rate returns, $(\Delta e_t)^2$. In the target zone model we expect a positive correlation in the second moment of the returns, because due to the S-shaped mapping from fundamental to exchange rate the conditional variance is relatively large when the exchange rate is in the middle of the band, but relatively small close to the bounds. Therefore, if the variance of the fundamentals process is small, successive observations on the exchange rate have a similar position within the band. This implies that the conditional variances of successive returns have similar values, so that there is an endogenous ARCH effect in the target zone model. The question is whether this endogenous ARCH effect in the model is strong enough to explain the observed serial correlation in the second moments of the returns.

The tests in Table I reveal that for all series except the Belgian franc the model is misspecified. The M-arch test indicates that the ARCH effect in the data on Dutch guilder, French franc, and Irish punt rates is not fully explained by the model. However, for the other currencies there is no significant difference in observed and predicted ARCH. The M-norm test on the skewness and kurtosis of the exchange rate returns is highly significant for all series but the Belgian franc. Clearly, the target zone model is not capable of explaining one of the most prominent stylized facts of exchange rates, namely, the fat-tailed distribution of the returns.

7. CONCLUSIONS

In this paper we have developed Maximum Likelihood and Method of Simulated Moments estimators for the target zone model of Krugman (1991). It is shown that estimation of this model is feasible. The empirical results indicate that the Krugman model is misspecified for actual EMS exchange rate data. The Maximum Likelihood and Method of Simulated Moments

[5] The theoretical moments, of course, depend on the value of the model's parameters. The M-test requires the moments to be evaluated in a consistent estimate of the parameter vector. This estimate can be obtained by any consistent estimator of the parameter vector, such as the Maximum Likelihood estimator or a method of moments estimator.

parameter estimates differ substantially. Moreover, the model is not capable of explaining two well-known stylized facts of exchange rates: autoregressive conditional heteroscedasticity and a non-normal distribution for the returns. The main deficiencies of the Krugman model seem to be the restrictive dynamics, where the scalar fundamental follows a first-order Markov process, the normal distribution of this process, and the assumption that there are no interventions within the band. Future empirical models should take these features of exchange rates within a target zone into account.

APPENDIX: SIMULATING THE TARGET ZONE MODEL

The method of simulated moments estimator and the M-tests discussed in this paper require simulation of the stochastic process of the target zone model. In this appendix we describe how the numerical simulations are performed. The method is based on the work of Duffie and Singleton (1988), who developed a method for computing the moments of a Brownian motion. The difficulty of a discrete time computer simulation of the continuous time regulated Brownian motion is caused by the assumption that there is *infinitesimal* intervention at the margins, so that the process does not jump and has a zero probability of being exactly at the margin. In a discrete time approximation, interventions are strictly positive, and we have to make some assumption on where the process goes after an intervention. The scheme used by Smith and Spencer (1991) and Beetsma (1991) is

$$f^*(t+\Delta t) = f(t) + \mu\,\Delta t + \sigma\sqrt{\Delta t}\cdot\hat{\varepsilon}(t)$$

$$f(t+\Delta t) = \begin{cases} \bar{f} \text{ if } f^*(t+\Delta t) > \bar{f} \\ \mathbf{f} \text{ if } f^*(t+\Delta t) < \mathbf{f} \\ f^*(t+\Delta t) \text{ otherwise} \end{cases}$$

It is clear that for $\Delta t > 0$ there will be a point mass at $\bar{f}$ and $\mathbf{f}$ in the distribution of $f(t+\Delta t)$, whereas the mass at those points in the continuous time model is zero. This point mass can be large if $f(t)$ is close to $\bar{f}$ or $\mathbf{f}$ relative to the magnitude of Δt. A way to improve the accuracy of the simulation is to choose Δt very small if $f(t)$ is close to $\bar{f}$ or $\mathbf{f}$.

A scheme that gives no point mass at the bounds is found by reflecting the stochastic process in the upper or lower bound if $f^*(t+\Delta t)$ exceeds that bound:

$$f^*(t+\Delta t) = f(t) + \mu\Delta t + \sigma\sqrt{\Delta t}\cdot\hat{\varepsilon}(t)$$

$$f(t+\Delta t) = \begin{cases} 2\bar{f} - f^*(t+\Delta t) - f(t) & \text{if } f^*(t+\Delta t) > \bar{f} \\ 2\mathbf{f} - f^*(t+\Delta t) - f(t) & \text{if } f^*(t+\Delta t) < \mathbf{f} \\ f^*(t+\Delta t) \text{ otherwise} \end{cases}$$

We prefer this scheme because it generates no observations exactly on the bounds. Such observations cause problems in computing the likelihood function for simulated data because, as a result of the smooth-pasting conditions, $G'(f)$ for such observations is zero, so that the Jacobian of the transformation from fundamental to exchange rate and hence the likelihood function are infinite.

The conditional distribution function of $f(t+\Delta t)$ generated by this scheme with initial value $f(t) = f_t$ is

$$P(f\,|\,f_t) = \Phi\left(\frac{f - f_t - \mu\,\Delta t}{\sigma\sqrt{\Delta t}}\right) - \Phi\left(\frac{2\mathbf{f} - f - f_t - \mu\,\Delta t}{\sigma\sqrt{\Delta t}}\right) + \left[1 - \Phi\left(\frac{2\bar{f} - f - f_t - \mu\,\Delta t}{\sigma\sqrt{\Delta t}}\right)\right]$$

The first part is the usual normal distribution function, the second part represents the probability that $f(t)$ has been reflected in the lower bound, and the third part is the probability mass reflected in the upper bound. Comparing this distribution with the approximate distribution function of the continuous time model we see that the discrete simulation overestimates the probabilities of reflection somewhat, due to the omission of interventions that possibly take place within the time interval $(t, t+\Delta t]$. Therefore, for a good approximation to the continuous time distribution it is necessary to use a simulation interval Δt that is small compared with the length of the observation interval. In our applications we use ten drawings per observation.

ACKNOWLEDGEMENTS

The author would like to thank Roel Beetsma, Hans Bloemen, Theo Nijman, Rick van der Ploeg, Hossein Samiei, Casper de Vries, Sweder van Wijnbergen and the Editor for helpful comments. Errors are all the author's. The data were kindly provided by M. M. G. Fase of De Nederlandsche Bank. An earlier version of this paper, entitled 'Estimating Target Zone Models of Exchange Rates', was presented at the Econometric Society European Meeting (ESEM91), 2–6 September 1991, Cambridge, UK.

REFERENCES

Beetsma, R. (1991), 'Bands and statistical properties of EMS exchange rates', CentER discussion paper 9105, Tilburg University.

Bertola, G. and R. Caballero (1992), 'Target zones and realignments', *American Economic Review*, **82**, 520–536.

Bertola, G. and L. Svensson (1993), 'Stochastic devaluation risk and the empirical fit of target zone models', *Review of Economic Studies*, (forthcoming).

Chernoff, H. (1954), 'On the distribution of the Likelihood Ratio', *Annals of Mathematical Statistics*, **25**, 573–578.

Diebold, F. and J. Nason (1990), 'Nonparametric exchange rate prediction', *Journal of International Economics*, **28**, 315–332.

Duffie, D. and K. Singleton (1988), 'Simulated moments estimation of diffusion models of asset prices', Working paper, Graduate School of Business, Stanford University.

Flood, R., A. Rose and D. Mathieson (1990), 'An empirical exploration of exchange rate target-zones', NBER working paper 3543.

Harrison, J. M. (1985), *Brownian Motion and Stochastic Flow Systems*, Wiley, New York.

Koedijk, K., P. Stork, and C. de Vries (1992), 'Between realignments and intervention: the Belgian franc in the European Monetary System', Working paper, University of Limburg.

Krugman, P. (1991), 'Target zones and exchange rate dynamics', *Quarterly Journal of Economics*, **CVI**, 669–682.

Lee, B.-S. and B. F. Ingram (1991), 'Simulation estimation of time-series models', *Journal of Econometrics*, **47**, 197–205.

Lindberg, H. and P. Söderlind (1992), 'Target zone models and intervention policy: the Swedish case', IIES seminar paper 492.

McFadden, D. (1989), 'A method of simulated moments for estimation of discrete response models without numerical integration', *Econometrica*, **57**, 995–1026.

Meese, R. and A. Rose (1990), 'Nonlinear, nonparametric, nonessential exchange rate estimation', *American Economic Review*, **80**, 192–196.

Meese, R. and A. Rose (1991), 'An empirical assessment of non-linearities in models of exchange rate determination', *Review of Economic Studies*, **58**, 603–619.

Miller, M. and P. Weller (1991), 'Exchange rate bands with price inertia', *Economic Journal*, **101**, 1380–1399.

Newey, W. (1985), 'Maximum Likelihood specification testing and conditional moment tests', *Econometrica*, **53**, 1047–1069.

Newey, W. and K. West (1987), 'A simple, positive definite, heteroskedasticity and autocorrelation consistent covariance matrix', *Econometrica*, **55**, 703–708.

Pesaran, M. H. and H. Samiei (1992), 'An analysis of the determination of the DM/FF exchange rate in a discrete-time target-zone model', *Economic Journal*, **102**, 388–401.

Rose, A. and L. Svensson (1991), 'Expected and predicted realignments: the FF/DM exchange rate during the EMS', NBER working paper 3695 and IIES seminar paper 485.

Smith, G. and M. Spencer (1991), 'Estimation and testing in models of exchange rate target zones and process switching', in P. Krugman and M. Miller (eds), *Exchange Rate Targets and Currency Bands*, Cambridge University Press, Cambridge.

Svensson, L. (1991a), 'The term structure of interest rate differentials in a target zone: theory and Swedish data', *Journal of Monetary Economics*, **28**, 87–116.

Svensson, L. (1991b), 'Target zones and interest rate variability', *Journal of International Economics*, **31**, 27–54.

Svensson, L. (1992), 'An interpretation of recent research on exchange rate target zones', *Journal of Economic Perspectives*, **6**, 119–144.