The international spillover effects of ageing and pensions

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ter verkrijging van de graad van doctor aan de Universiteit van Tilburg, op gezag van de rector magnificus, prof. dr. F.A. van der Duyn Schouten, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op vrijdag 30 mei 2008 om 14.15 uur door

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CHAPTER 1

INTRODUCTION

The baby boom after the Second World War, the subsequent decline in fertility rates in the 1970s and 1980s and the ongoing increase in life expectancy cause the ageing of populations in many developed countries. Although population ageing is the result of positive developments like the emancipation of women and better provision of health care, the unprecedented rise of the relative number of elderly will put facilities like pension- and health care schemes under pressure. In the EU15, the number of inactive elderly compared to the labour force, the so-called old-age dependency ratio[1], rose from 13.9 percent in 1950 to 30.5 percent in 2005. The United Nations projects that this figure will more than double to 65.4 percent in 2050 (UNCDB, 2007). This enormous increase of the relative number of pensioners forces governments to think about the way their pensions are organised.

Most public pension schemes are financed on a pay-as-you-go (PAYG) basis, which means that the pension benefits of the retirees in a certain period are financed by contributions levied on the working people in the same period. These pension schemes were introduced to provide people with a minimum standard of living during retirement. In the United States a social security scheme was introduced after the stock market crash in the 1930s where many retired people lost all their savings and did not have any resources anymore

[1]The old-age dependency ratio is defined as the number of people aged 65 or older in proportion to the number of people aged 15 to 65 (the labour force).

to finance their consumption during old age. Many European countries introduced a public pension system after the Second World War to improve the living standard of the elderly. At the time of the introduction of these public pension schemes life expectancy was much lower than current life expectancies. Moreover, women gave birth to a larger number of children than nowadays, so that the tax base to finance the pension benefits and the implicit rate of return of the PAYG scheme was larger[2]. In the 1970s the size of public PAYG pension systems increased to a large extent, even though it was known that the demographic situation would change in the future. The fact, however, that more people receive a pension benefit for a longer time period, combined with the fall of the relative number of working people leads to the question whether the public pension schemes currently in place will be sustainable in the coming decades.

Several countries have already taken measures to lower the pressure on their public PAYG pension systems. Some countries, e.g., Germany, will gradually increase the retirement age. Countries like Sweden and Italy moved to a pension scheme where pension benefits change when the demographic composition changes, that is, pension rights will fall when average life expectancy at retirement rises. In the Netherlands the government primarily focuses on increased labour force participation. One of the most suggested solutions by economists is to switch to a more funded pension system where people save for their own pension and realise a higher expected rate of return on their contributions. This type of reform is proposed by the Bush administration. Funded pension schemes, however, will not be immune to ageing either. Because population ageing is a worldwide phenomenon it will have macroeconomic consequences and thus affect the rates of return that pension funds earn on their investments. Moreover it is difficult to implement such a switch to more funding as it will probably hurt some generations during the transition phase of the reform. And finally, although it is generally believed that the rate of return on contributions to a pension fund is higher than the implicit rate of return on PAYG contributions, the higher rate of return of a funded pension scheme goes together with more financial market risk and inflation risk. In order to diversify the various types of risks it is often proposed that the provision of retirement income should be a combination of PAYG-financed

[2]The implicit return on PAYG contributions is equal to the growth rate of the economy, which is the sum of the population growth rate and the rate of technological progress. Higher fertility rates imply a higher rate of population growth and thus a higher implicit rate of return of PAYG pension schemes.

pensions and funded pensions.

The public debate about the sustainability of pension schemes primarily focuses on how the problems that result from an ageing population can be solved within the country itself. However, given the fact that ageing is a global phenomenon and capital markets become increasingly integrated, citizens of one country will also be affected by ageing and the policy measures taken in response to ageing in other countries. Open economies will therefore have to cope with *international spillover effects* of ageing via capital markets. For example, although most OECD countries will have an ageing population in the coming decades, the ageing pattern differs between countries. Population ageing implies a fall of the relative number of working people and therefore makes capital relatively abundant compared to labour. This will reduce the rate of return on capital. In case countries differ in the timing and the extent of ageing the change of the rate of return varies between countries and this will lead to international capital flows. These capital flows will affect the well-being of people by changing the factor rewards on capital and labour. The utility effects of these capital flows are referred to as international spillover effects. This thesis analyses the international spillover effects of ageing via capital markets. We do not focus on differences in ageing patterns, but on the fact that countries differ in the degree of funding of their pension systems. This is especially relevant for the European Economic and Monetary Union (EMU) where the members have one capital market and one monetary policy. Within the EMU, the timing and intensity of the ageing process will be largely the same, but there are large differences in the way pensions are financed. There is a large group of countries with extensive public PAYG pension schemes but almost no funded pensions. In these countries PAYG pensions finance more than 90 percent of the income of retirees (Boeri et al., 2006). On the other hand, there is a smaller group of countries that have large funded pension schemes. In a country like for example the Netherlands total investments of pension funds amount to 124.9 percent of GDP (OECD Global Pension Statistics, 2006). These differences in pension schemes will lead to capital flows between countries, even if the pattern of ageing is identical across countries. These capital flows will have welfare effects, i.e., international spillover effects.

There are various channels why differences in pension schemes will induce international spillover effects when the population is ageing. As described above population ageing will affect PAYG pension systems and funded pensions differently. Pension arrangements financed on a PAYG basis are directly affected by the rise in the old-age dependency ratio, while funded pension schemes will be affected indirectly by changes in the rates of return on investments. Therefore savings reactions in response to ageing will not be the same when countries differ in the retirement scheme they use. Savings will also change if a country reforms its PAYG pension system and moves to a more funded pension system to improve the sustainability of its pension system when the population is ageing. In a common capital market differences in savings between countries will lead to capital flows, which causes international spillover effects. Countries with large PAYG pension schemes may also use government debt to finance their pension burden. High levels of government debt may lead to inflationary pressures or make government bonds more risky. In a monetary union like the EMU this will necessarily affect other members that accumulated large pension funds. This thesis gives a theoretical analysis and economic interpretation of these issues. In particular, the aim of this thesis is to gain insight how people that live in countries that rely to a large extent on funded pensions are affected by the fact that other countries have large PAYG pension schemes when the population is ageing, and vice versa. These international spillover effects will vary for different generations. The analysis in this thesis is positive, i.e., we only analyse how population ageing and certain policy measures in response to ageing affect the welfare of individuals. We do not consider the political decision process of certain policy changes and we assume that governments in funded countries do not redistribute the international spillover effects between generations.

The rest of this introductory chapter is organised as follows. In Section 1.1 we will give some stylised facts about population ageing and the differences in pension schemes currently in place in Europe. Section 1.2 will give a brief overview of the literature and explain some important standard concepts. We end the chapter by giving an overview and summary of the thesis in Section 1.3.

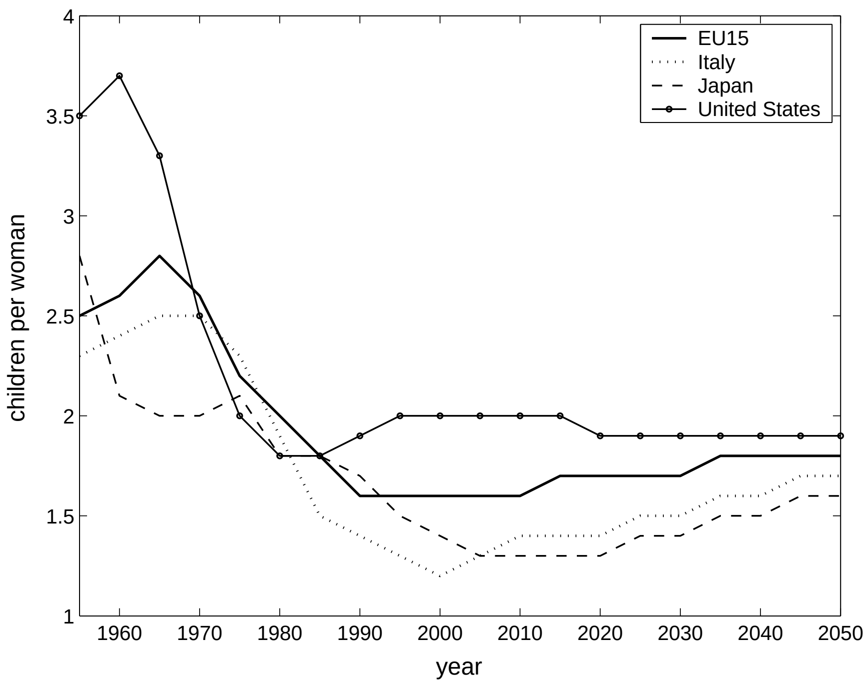
1.1 Some facts

In this section we will give some facts and figures that underly the questions we try to answer in this thesis. First it will be explained why the western populations are ageing in the coming decades. Secondly, we describe how European countries differ in the way they organised their retirement schemes.

1.1.1 The ageing problem

Population ageing arises as a result of changes in two determinants of demographic structure; the fall in fertility rates and a rise in longevity. In Figure 1.1 we show the change in the fertility rate, i.e., the average number of children per woman, from the 1950s until 2050 for a number of developed countries and the EU15. After the second world war there was a baby boom which continued until the mid 1960s. In 1965 the average number of children per woman was 2.8 in the EU15, while the fertility rate in the US was even 3.7 in 1960. From the 1970s onwards, however, there has been a large drop in fertility rates, caused by the introduction of contraceptives and the increased labour force participation of women. In 2005 the average number of children women gave birth to was 2 in the US, 1.6 in the EU15 and 1.3 in Japan. Currently, the fertility rates are particularly low in Italy and Japan. For the coming decades it is expected that fertility rates will improve somewhat, but will stay below the replacement level of 2.1 children per woman.

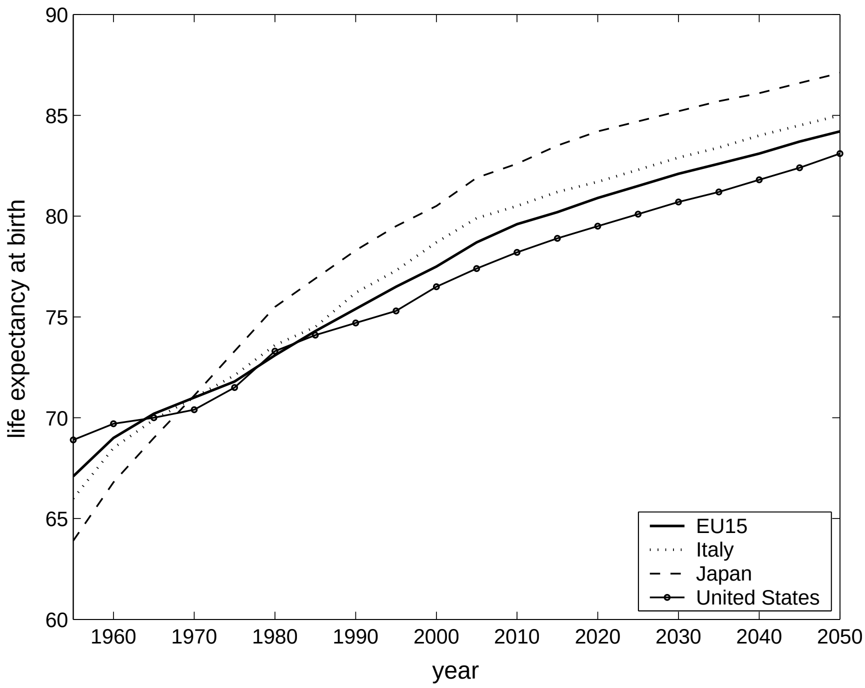
Figure 1.1: Fertility rate



Source: United Nations Common Database (UNCDB), 2007 and own calculations. The figures for 2010 and later are based on the medium variant projection of the UN population division.

The second source of demographic change is the ongoing increase in longevity. The average lifetime of people is increasing because of better nutrition and better provision of health care. Figure 1.2 shows the development of life expectancy at birth between 1955 and 2050. In the EU15, life expectancy was 67.1 in 1955, this increased to 78.7 in 2005 and in 2050 life expectancy at birth is forecasted to be 84.2. The rise in life expectancy is most pronounced in Japan. In the first half of the twentieth century life expectancy mainly increased as a result of reduced rates in infant mortality. The current decline in mortality mainly occurs at old ages, however, which causes the ageing of populations.

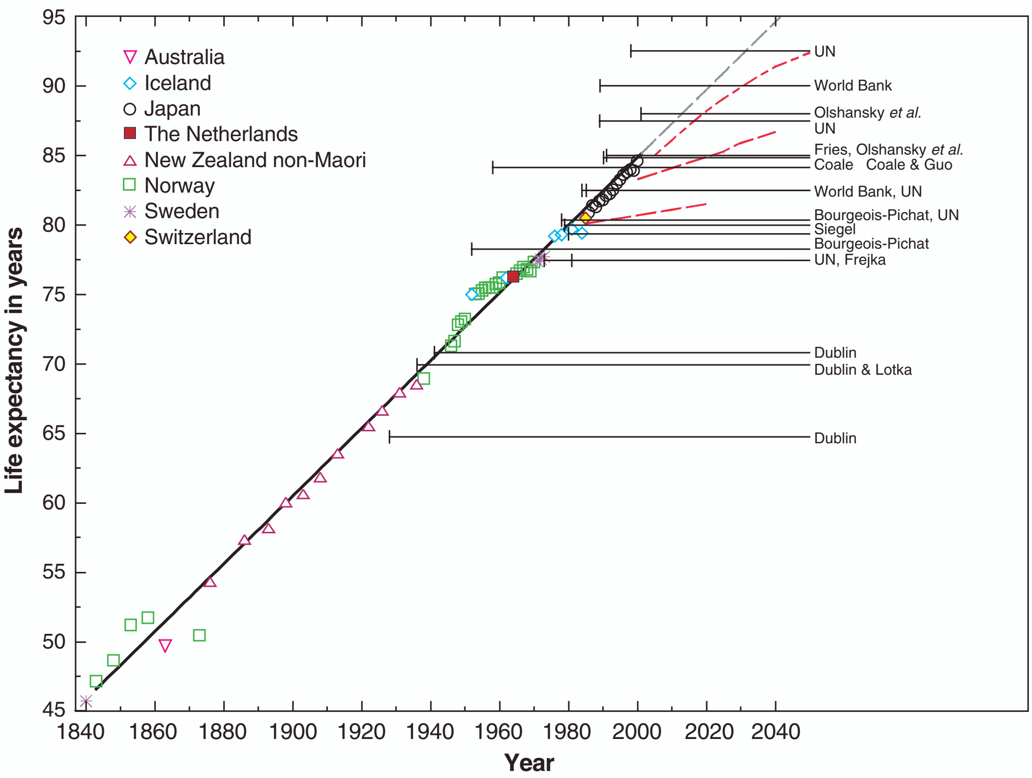
Figure 1.2: Longevity



Source: United Nations Common Database (UNCDB), 2007 and own calculations. The figures for 2010 and later are based on the medium variant projection of the UN population division.

The rising trend in life expectancy has been consistently underestimated in the past. This can be seen in Figure 1.3. The horizontal lines show the asserted limits to life expectancy by various studies, where the short vertical line indicates the year of publication. Dublin (1928) estimated the ultimate figure for life expectancy at 64.75, while female life expectancy in New Zealand was already 65.93 at the time. Olshansky et al. (1990) estimated the upper limit of life expectancy at 85 years, in 1996, however, life expectancy of Japanese women already exceeded this figure. The ceilings on life expectancy of some recent studies have not been reached yet, but this will happen in the coming decades in case the depicted upward trend will continue. The extrapolated trend lies above the projections of female life expectancy in Japan made in 1986, 1999 and 2001 by the United Nations (UN), which are depicted by the dashed lines. This implies that if the rise in life expectancy will continue in this linear fashion, the predictions by the UN are again too low and the population will age even more.

Figure 1.3: Female life expectancy



Source: Oeppen and Vaupel (2002). Reprinted with permission from AAAS. The bold line depicts the linear regression trend and the dashed grey line is the extrapolated trend. The horizontal lines show asserted ceilings on life expectancy, with a short vertical line indicating the year of publication. The dashed lines denote projections of female life expectancy in Japan published by the United Nations in 1986, 1999 and 2001.

The underestimation of the improvement in life expectancy is also shown in Table 1.1, which compares the realised gains in life expectancy at birth with past UN projections. Each number shows the difference between actual life expectancy in 2003 and projected life expectancy for the average of period

2000-2005 made by the UN in 1999. A positive number indicates that actual life expectancy exceeds projected life expectancy. These mistakes in life expectancy forecasts may cause that people make the wrong savings decisions, as on average people always live longer than what they could reasonably have expected at the time they made their savings decisions. In this sense the rise in longevity can be seen as unexpected and therefore we model population ageing as an unexpected shock in this thesis. The assumption that people do not expect ageing only affects the short-run effects of ageing, the long-run effects do not change.

Table 1.1: Comparing actual and projected life expectancy at birth

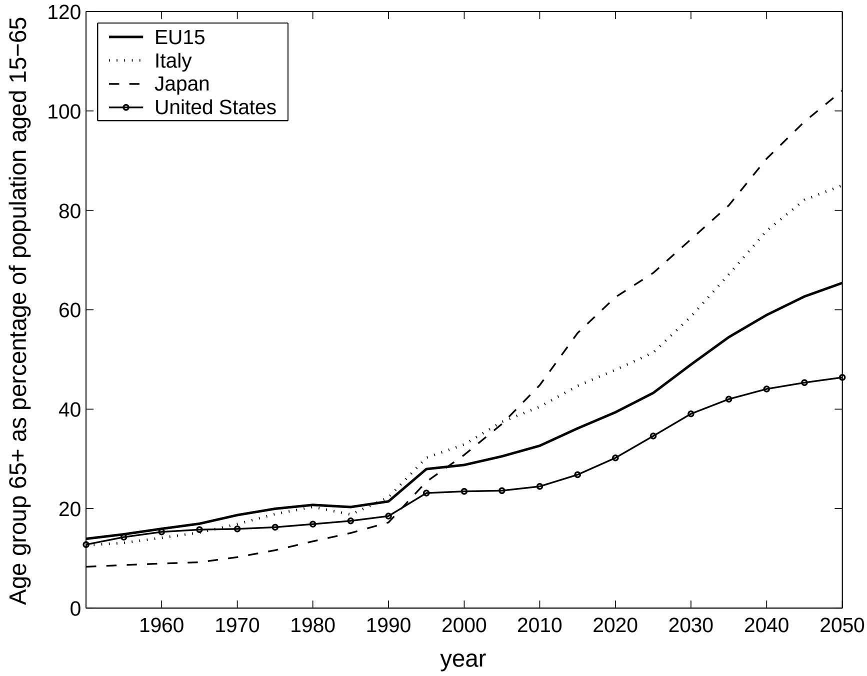
France	0.6	EU15	0.7
Germany	0.6	Japan	1.5
Italy	1.1	US	-0.2
UK	0.5		

Source: Antolin (2007). The numbers reflect the difference between actual life expectancy in 2003 and the 1999 UN forecasted life expectancy for the average of 2000-2005.

The combination of lower fertility rates and higher longevity results in a rise of the relative number of elderly people, i.e., an increase in the so-called old-age dependency ratio. From Figure 1.4 we can infer that from 2000 onwards the number of inactive elderly compared to the labour force will rise to a large extent in all developed regions. In Japan the expectation is that the number of retirees will be even larger than the working-age population in 2050, i.e., the old-age dependency ratio will exceed 100 percent.

The forecasts in Figure 1.1 suggest that fertility rates will not continue to decrease in the coming decades, but stabilise around the current level or will even rise to some extent. Life expectancy, however, will continue to increase. This implies that the fall in mortality rates among the elderly will be the main source of population ageing in the coming decades. The fact that we live longer is in the first place a blessing of course. On the other hand, a rise in the old-age dependency ratio implies that a smaller group of working people have to take care of a larger number of retirees when retirement ages do not change. This puts the pension systems in different countries under pressure. In the next subsection we will discuss the different pension arrangements currently in place. We focus on the European Union as this region is also the main focus for the rest of this thesis.

Figure 1.4: Old-age dependency ratio



Source: United Nations Common Database (UNCDB), 2007 and own calculations. The figures for 2010 and later are based on the medium variant projection of the UN population division.

1.1.2 Pension schemes

A useful way to distinguish between the different components within retirement systems is the three-pillar system (or the Cappuccino model) proposed by the World Bank (Worldbank, 1994). The first pillar (the coffee) is the basic public pension with the main goal to alleviate poverty among the elderly. It is financed on the basis of pay-as-you-go (PAYG), implying that the government collects taxes from the working young to finance the pension benefits of the retired old. The second pillar (the milk) is a supplementary and obligatory pension to ensure that people do not experience a too large fall in income when they retire. It is supplied by the private sector and can be organised both at the occupational or personal level. These pension schemes are mostly funded so that each individual saves for its own pension. The third pillar (the cocoa) refers to voluntary (individual or occupational) saving and insurance plans and provides additional protection to people who like a better income and security during retirement. This classification of the World Bank shows that ideally each pillar has its own goal. In reality, however, we see that in many countries public PAYG pensions are not only used to provide a minimum standard of living for the elderly, but also insure people against large income losses. This means that the goal of the second pillar is shifted to the first pillar and that a relatively large part of pension income is financed by first pillar PAYG pensions. This can be seen in Table 1.2 where we show the relative size of the three pillars in some European countries. In most countries, first pillar public pensions finance more than 90 percent of the income of retirees. The second funded pillar is only significant in Denmark, Sweden and the Netherlands.

Table 1.2: Composition of retirement incomes in 2003 (%)

	First pillar	Second pillar	Third pillar
Greece	98	1.5	0.5
France	97	1.5	1.5
Spain	97	1.5	1.5
Italy	96	2	2
Austria	95	3	2
Germany	94	4	2
Denmark	76	20	4
Sweden	75	20	5
Netherlands	56	37	7

Source: Boeri et al. (2006)

In Table 1.3 we show total expenditure on pensions by the government relative to GDP and Table 1.4 presents the size of pension funds as a percentage of GDP. These two tables confirm the differences in financing methods of pensions sketched in Table 1.2. There is a large group of countries with extensive public PAYG schemes but almost no funded pensions. In large countries like Italy, France and Germany public pension expenditure is already quite large, while the funded component is fairly small. On the other hand, there is a smaller group of countries that have sizeable pension funds, while public expenditure on pensions is smaller (e.g., the Netherlands and the United Kingdom).

Table 1.3: Public expenditure on pensions as % of GDP (2004)

Italy	14.2	Belgium	10.4
Austria	13.4	Luxembourg	10.0
France	12.8	Denmark	9.5
Germany	11.4	Spain	8.6
Portugal	11.1	Netherlands	7.7
Finland	10.7	UK	6.6
Sweden	10.6	Ireland	4.7

Source: European Commission (2006)

Table 1.4: Total investments of pension funds as % of GDP (2005)

Luxembourg	0.4	Portugal	12.9
Italy	2.8	Sweden	14.5
Germany	3.9	Denmark	33.6
Belgium	4.2	Ireland	52.8
Austria	4.7	Finland	66.1
France	5.8	UK	70.1
Spain	9.1	Netherlands	124.9

Source: OECD Global Pension Statistics, 2006

The fact that countries with large public PAYG schemes also use these first-pillar pensions to provide high earnings-related pension benefits, so that the drop in income during retirement is not too large, is shown in Table 1.5. This table presents the division between the so-called Beveridgian- and Bismarckian factor within public pension schemes. A pension system is called Bismarckian in case the pension benefits are linked to previous contributions to the system. In case of a complete Bismarckian system, like in Italy, there is no redistribution between rich and poor people of the same generation, i.e., there is no intragenerational redistribution. The advantage of such a system is that tax distortions are absent. The disadvantage, however, is that the amount of poverty reduction is minimal. A Beveridgian pension scheme is characterised by the fact that there is no link between the PAYG contributions people pay during their working life and the public pension benefit people receive during retirement. In the Netherlands, for example, PAYG contributions are income-dependent, while the pension benefit is flat. This implies that there is a lot of intragenerational redistribution in case the public pension scheme has a large

Beveridgian factor. The Bismarckian factor is large in countries with large public PAYG schemes, which suggests that in these countries first-pillar pensions are not only used to alleviate poverty among pensioners, but also serve to maintain a certain standard of living during retirement. Countries where a relatively large part of retirement income is financed by funded pensions, on the other hand, have a lot of redistribution within their public PAYG schemes. In these countries pension arrangements are organised along the lines suggested by the World Bank. Second-pillar funded pensions ensure that retirees do not endure too large income losses, while public PAYG pensions provide a minimum living standard for the elderly.

Table 1.5: Redistribution within public pension systems (%)

	Beveridgian factor	Bismarckian factor
Italy	-	100
Spain	0.5	99.5
Austria	1	99
Finland	1.6	98.4
Germany	1.9	98.1
Portugal	4.2	95.8
France	6.5	93.5
Belgium	11.1	88.9
Luxembourg	13.8	86.2
Sweden	14.90	85.10
UK	89.5	10.5
Denmark	90	10
Ireland	100	-
Netherlands	100	-

Source: OECD (2005).

In this thesis we will not distinguish between Bismarckian or Beveridgian pension schemes, but we focus on the distinction between PAYG and funded pension systems. Population ageing will affect countries in different ways depending on the way pensions are financed. In countries with extensive public PAYG pension systems (like Italy, France and Germany), the rise in the old-age dependency ratio will put these pension schemes under pressure as the number of contributors falls and the number of recipients rises. This implies that either PAYG contributions have to increase or pension benefits have to fall. Another option would be to use government debt to cope with the costs of ageing. Given the fact that public pension expenditure is already quite large (see Table 1.3), an often discussed solution is to reform the PAYG scheme and switch to a more funded scheme, so that a larger part of pension income is financed by the second pillar. This thesis will analyse the international spillover effects of all these policy scenarios.

Within funded pension systems a rise in the old-age dependency ratio will not cause the type of financing problems that occur in PAYG schemes as people save for their own pension[3]. Funded pensions, however, are vulnerable to changes in rates of return on investments. This implies that in case ageing changes the rates on return this will affect funded pension systems. As population ageing is a world-wide phenomenon we can expect that rates of return will change when the number of working people falls compared to the number of inactive elderly. So funded pensions will not be immune to ageing either. Moreover, countries with a more funded pension scheme will be affected by countries with relatively large PAYG schemes in case there is free movement of capital.

1.2 An overview of the related literature

This section gives an overview of the main literature related to what is done in this thesis. We also explain some important concepts and mechanisms that will used in the thesis.

1.2.1 The basic model

The basic model used in this thesis is the two-period overlapping-generations model first introduced by Samuelson (1958) and later extended by Diamond (1965). In this model individuals live for two periods, people work in the first period and are retired in the second period of life. This implies that in a given period both a young and an old generation are alive. This demographic structure allows for transfers between generations and therefore this model is

[3]A funded pension system can be characterised by defined benefits (DB), however, which implies that there is a PAYG element in the scheme. In a DB funded pension scheme the young participants bear the residual risk of an unexpected shock. This thesis only considers funded pension systems with defined contributions (DC), so that the retirees are the residual risk bearers. Assuming a DB funded scheme instead means that the short-run utility effects of ageing are shifted from the old to young, the spillover effects do not qualitatively change, however.

a very useful tool for modelling PAYG pensions, where the working young finance the pension benefits of the elderly.

Steady state per capita consumption is maximised at the point where the real rate of interest (r) is equal to the growth rate of the economy, which equals the population growth rate (n) when there is no technological progress. This situation is often referred to as the *Golden Rule* point. Diamond (1965) showed that it is possible that consumers save too much, this is a *dynamic inefficient* situation. There is too much capital accumulation in a dynamically inefficient economy and $r < n$. In that case it is possible to increase welfare of all generations by lowering the capital stock, that is, by substituting private savings for consumption. Aaron (1966) showed that this can be done by the introduction of a PAYG pension scheme. In case $r > n$, however, the capital stock is smaller than the Golden Rule level of capital and the economy is *dynamically efficient* as it is not possible to increase long-run welfare without hurting some current or future generations. In that case the introduction of a PAYG pension scheme benefits short-run welfare, but reduces long-run welfare, as PAYG pensions will lead to an even smaller capital stock. Most studies assume that economies are dynamically efficient nowadays, this will also be the assumption in this thesis.

Buiter (1981) and Persson (1985) were the first to extend the overlapping-generations model of Diamond and Samuelson to an open economy with two countries. Buiter (1981) studied the effects of integration of two economies that differ in their rate of time preference. Consumers with a high time preference consume relatively more when they are young and save relatively less, so that capital-labour ratio in the high-time-preference country is relatively low under autarky. The opposite holds for the country where individuals have a low rate of time preference. In case these two countries integrate their capital and commodity markets, international trade and international borrowing and lending make sure that the capital-labour ratio is the same in the two countries. The integration results in a higher capital-labour ratio for the high-time-preference country, while the capital-labour is lower in the low-time-preference country. This change in the capital-labour ratio has welfare effects in both countries because wages and interest rates are affected. The long-run welfare effects of integration crucially depend on the fact whether there is underaccumulation of capital or not, that is, whether the economies are dynamically efficient or not. Moreover, Buiter (1981) shows that it is not necessarily the case that both economies gain from the integration. Consider for example the case where both economies are dynamically efficient under autarky and stay dynamically efficient under openness. The long-run welfare effects of integration will be unambiguously positive in the high-time-preference country in that case. First, because the capital-labour ratio is higher under openness, so that capital underaccumulation is less and the economy is closer to the Golden Rule point. Second, as the high-time-preference country will be a net borrower after integration it gains from the lower interest rate that results for the higher capital-labour ratio. The long-run utility effects of integration for the country with the low rate of time preference can be either positive or negative, however. On the one hand, as the country is a net lender under openness it gains from the lower capital-labour ratio because this leads to a higher interest rate. On the other hand, a lower capital-labour ratio has negative utility effects under dynamic efficiency. This shows that in this model it is not necessarily the case that the opening up of countries improves welfare. The reason for this result is that international trade and financial mobility do not expand the consumption possibility set of consumers and there are no potential gains from specialisation, as only one good is produced. Openness only means that the two countries get one capital market.

The analysis of Persson (1985) focuses on the intergenerational welfare redistribution of an increase in public debt in open economies. It is well-known that in a closed economy with overlapping generations government debt reduces steady state welfare in case of dynamic efficiency, because capital is crowded out and taxes have to increase to service the higher government debt. Persson (1985) shows that the negative welfare effects are smaller in a small open economy, because the capital-labour ratio is exogenous so that future generations only bear the higher tax burden of the higher public debt. In case of two large countries that affect world factor prices with their debt policy, the long-run welfare effects of public debt are not clear anymore, however, as there is an extra terms-of-trade effect besides the two negative effects of increased taxes and a lower capital-labour ratio. If the country is a net borrower on the international capital market it is affected negatively by the higher interest rate resulting from the fall of the capital-labour ratio. In this case the extra terms-of-trade effect reinforces the two other negative effects and an increase in government debt unambiguously decreases steady state welfare. If the country is a net lender, on the other hand, it gains from the higher interest rate and the two negative effects are alleviated. When this positive terms-of-trade effect is large enough, government debt may even increase welfare in the long run.

The models of Buiter (1981) and Persson (1985) serve as a basis for the models used in this thesis. Both models do not include pensions, however. Breyer and Wildasin (1993) examine the steady state welfare effects of unfunded pension schemes in a large open economy. They assume that there is only one country that is large enough to affect the world interest rate, all other countries take the interest rate as given. Their findings are very similar to those of Persson (1985), as an increase in the size of the PAYG scheme is comparable to a rise of government debt. Higher PAYG pensions reduce savings and crowd out the capital stock (just like government debt) and the world interest rate will rise. Breyer and Wildasin (1993) show that a large open economy that finds itself on the Golden Rule path and is a net lender on the international capital market can increase steady state welfare by increasing the size of its PAYG pension scheme. The reason for this result is that a net lender benefits from the rise in the world interest rate. For a country that is a net borrower exactly the opposite result holds true; steady state welfare is increased if it reduces PAYG contributions below the Golden Rule level. In contrast to Breyer and Wildasin (1993), this thesis considers an open-economy overlapping-generations model consisting of two large countries.

Casarico (2001) analyses the effects of the integration of capital markets of two countries that differ in the degree of funding of their pension systems. One country relies on a PAYG pension scheme, while the other country has a fully-funded pension scheme. This difference in pension systems implies that the capital-labour ratios in the two countries differ under autarky. The PAYG country will have a lower per capita capital stock than the funded country, therefore capital will flow from the funded country to the PAYG country in case of integration. Although the source of capital flows between the two countries (differences in pension schemes) differs from the one in Buiter (1981) (different rates of time preferences), the mechanisms behind the results are more or less the same. The PAYG country can be compared to the high-time-preference country and therefore people in this country are better off in the open-economy equilibrium in case of dynamic efficiency. As was the case for the low-time-preference country, long-run welfare can either increase or decrease in the funded country. We follow Casarico (2001) and use a two-country two-period overlapping-generations model where one country uses a PAYG scheme and in the other country pensions are fully funded. In contrast to Casarico (2001) we study the international spillover effects of population ageing and pension reform given that the countries already have a common capital market, but differ in the pension arrangements they use.

1.2.2 Ageing

There is an extensive literature that considers the open economy aspects of population ageing focusing on the fact that differences in the extent and the timing of ageing will lead to capital flows between countries. Cutler et al. (1990) were one of the first to point out that demographic changes in other countries should be taken into account, given the fact that international capital markets are getting more and more integrated. They first explain that the consumption possibilities in an economy are affected in two ways when the population ages. First, the rise in the old-age dependency ratio (see Figure 1.4) implies that the number of working people that can produce falls compared to the number of inactive elderly, so that output per person falls. This *dependency-ratio effect* lowers consumption per capita. Second, the fall in fertility rates lowers the population growth rate and leads to a decreasing labour force. In a standard optimal growth model a decline in the growth rate of the labour force implies that less investment is needed to equip each worker with a certain capital stock. This so-called *capital-thickening effect* reduces the need for saving and increases consumption per capita. Using a Ramsey growth model, Cutler et al. (1990) find that for the United States the positive capital-thickening effect dominates the negative dependency-ratio effect. The optimal response to the upcoming ageing of the population would therefore be to decrease savings and increase consumption. Moreover, the fact that the US ages more slowly than other OECD nations, implies that the capital-labour ratio increases even more in other OECD countries and capital will flow into the US. These capital inflows reinforce the initial positive capital-thickening effect and further increase optimal consumption.

The main drawback of the analysis of Cutler et al. (1990) is that they use a Ramsey growth model where individuals are assumed to live infinitely long and there is no distinction between different generations. This implies that the effects of ageing on intergenerational redistribution and the subsequent effects on savings cannot be taken into account. The fact that agents have an infinite life also means that a Ramsey model is not suitable for studying the effects of a rise in life expectancy, which is the other main determinant of population ageing. Moreover, it is not possible to explicitly model an intergenerational transfer scheme, like a pay-as-you-go pension system and analyse the savings effects of such a pension scheme. To capture these effects it is necessary to use an overlapping-generations (OLG) model. There is a large number of papers, e.g., Attanasio et al. (2006), Börsch-Supan et al. (2006), Brooks (2003), Domeij and Flodén (2006), Fehr et al. (2005) presenting simulation experiments with large multi-country overlapping-generations (OLG) models to study the (international) effects of ageing. The rise in the old-age dependency ratio will have an extra effect on savings when a PAYG pension system is in place. A defined-benefit (DB) PAYG pension scheme implies higher PAYG contributions, which will decrease saving rates. The effect will be opposite in case of a defined-contribution (DC) PAYG system. Fehr et al. (2005) argue that the increase in taxes needed to finance the retirement benefits is so large that the capital-thickening effect is dominated by the negative effect of savings and the capital-labour ratio actually falls when the population is ageing. Therefore they forecast that capital will flow from slowly ageing countries like the US to rapidly ageing regions like the EU and Japan. Most other studies that use multi-country general equilibrium OLG models, however, find that ageing increases the capital stock per worker, see for example Attanasio et al. (2006), Börsch-Supan et al. (2006), Brooks (2003), Domeij and Flodén (2006). It appears that the results of Fehr et al. (2005) are mainly driven by the fact that the effective labour supply is rising due to labour-augmenting technical progress, which more than offsets the relative reduction in the labour force when the population is ageing. But it is not obvious at all whether technical progress will be able to offset the negative effects on the supply of labour. Börsch-Supan et al. (2006) and Brooks (2003) point out that the international capital flows induced by differential ageing processes across countries will change over time. Initially, rapidly ageing regions will export capital, while economies less affected by ageing will be capital importers. This pattern is reversed when the baby boom generation dissave during retirement and countries most affected by ageing will be capital importers.

Less attention, however, has been paid to a second possible reason for international spillover effects in case of ageing, i.e., differences in pension systems between countries. This thesis will focus on this channel. Moreover, an important drawback of these large-scale general equilibrium models is that the underlying mechanisms of the results are not clear because analytical solutions are not feasible. By keeping our models relatively simple, we are able to derive an analytical solution of the transition path after a shock has occurred. This allows us to gain insight into the underlying mechanisms of the results.

1.2.3 Pension reform

In many developed countries ageing has led to a debate on reforming unfunded pay-as-you-go (PAYG) social-security systems. One of the most discussed reform proposals is to switch to a more funded system where people save for their own pensions, and realise a higher expected rate of return on their contributions[4]. This switch to funding increases savings and will increase welfare in the long run in a dynamically efficient economy. The problem is, however, that there will be one transition generation that paid into the PAYG system, but will not receive a PAYG pension benefit anymore. The question is whether it is possible to compensate this transition generation in such a way that no generation is hurt by the pension reform and at least one generation gains. In other words, can a pension reform where (part of) the PAYG system is abolished be Pareto improving? To answer this question a distinction has to be made between the case where collecting PAYG contributions does not involve any distortions and the case where the PAYG scheme leads to distortions in the economy. In the first case the PAYG system is Pareto efficient, as proved by Verbon (1989) for the small open economy and Breyer (1989) for the closed economy. This means that it is not possible that some generations gain from the conversion policy without hurting other generations. The best thing a government in a reforming country can do is to implement the pension reform in such a way that no generation gains or loses.

In case a PAYG scheme is distortionary, that is, when the PAYG tax implies an *excess burden*, a Pareto-improving pension reform is possible, however. This is shown by Homburg (1990) for a small open economy and Breyer and Straub (1993) for the closed economy. The idea is that if the contribution rate is decreased net welfare gains result after the compensation of the older generations alive at the time of the abolishment of the PAYG system. In that way future generations gain from the switch to more funded pensions, which offer a higher rate of return and do not negatively affect savings, while the transition generation is compensated.

In Chapter 3, where we consider the international spillover effects of pension reform, we distinguish between the case where a reform potentially leads to a Pareto improvement in the reforming country, and where this is not possible.

[4]This is the case when the economy is dynamically efficient. Dynamic efficiency implies that the implicit return on PAYG contributions (the growth rate of the economy) is lower than the return on savings (the real rate of interest).

1.2.4 Pensions and macroeconomic risk

Based on the discussion in the previous section one would argue that abolishing a PAYG pension system is a good idea when PAYG contributions induce an excess burden. There is a strand of literature, however, that focuses on another aspect of PAYG pension systems that makes them useful; pension schemes that redistribute among generations can be used to enhance the sharing of aggregate risks between generations. The idea is that financial markets are incomplete because there cannot be trade with unborn generations and human capital is not traded. As a result of these missing markets the young are too much exposed to wage risk and the old bear too much financial market risk. In case financial market returns are imperfectly correlated with wages, this results in suboptimal diversification. By linking PAYG pension benefits to wages, retired households obtain a claim to human capital which is not traded on financial markets. In this way PAYG pension schemes can contribute to better *intergenerational risk sharing* and diversification. So although a PAYG system has a lower return and may decrease savings, it provides opportunities for intergenerational risk sharing and diversification.

There are several studies that investigate what the optimal size of a PAYG pension system would be, given that there is a trade-off between the provision of risk sharing and the reduction of saving incentives. The first strand of literature uses a partial equilibrium approach, where the rates of return are exogenous and constant, see for example Thøgersen (1998), Dutta et al. (2000), Wagener (2003), Matsen and Thøgersen (2004) and Miles and Černý (2006). All these partial equilibrium studies find that the risk sharing benefits outweigh the negative effects of reduced saving incentives, so that it is optimal to finance the pensions of the elderly at least partly on a PAYG basis. In partial equilibrium, however, one does not take into account the capital crowding-out effect of private savings. A PAYG system leads to lower savings and in general equilibrium, this leads to a smaller capital stock, which reduces steady state welfare in case of dynamic efficiency. Most papers that analyse the risk-sharing aspects of a PAYG system in a general equilibrium framework, like Storesletten et al. (1999), Sánchez-Marcos and Sánchez-Martin (2006), and Krueger and Kubler (2006), find that for standard parameter values the crowding-out effect of savings is larger than the risk-sharing benefits, which implies that it is optimal to abolish the PAYG system[5].

[5]These papers do not consider the transition effects of pension reform, however. As ex-

In contrast to this literature, this thesis does not consider optimal PAYG schemes, but takes the PAYG system as given. The model used in Chapters 4 and 5, however, is comparable to the models used in these papers. In these two chapters we develop a stochastic overlapping-generations general equilibrium model with safe PAYG pensions. The above-mentioned papers that study the intergenerational risk-sharing properties of PAYG pensions in stochastic general equilibrium OLG models use computable models. The major advantage of the stochastic general equilibrium model in Chapters 4 and 5 is that it is fairly simple so that analytical solutions are still feasible. There are a few papers that develop an anaytical stochastic general equilibrium OLG model. Bohn (1998, 2001, 2003) studies intergenerational risk sharing of demographic and macroeconomic risk in a stochastic OLG model using government debt or social security and derives a quasi-analytical condition for ex ante efficient intergenerational risk sharing. Beetsma and Bovenberg (2007) investigate intergenerational risk sharing in a two-pillar system with a pay-as-you-go pillar and a funded pillar. By keeping the model relatively simple they obtain analytical expressions for the optimal pension scheme. These papers do not, however, derive the optimal conditions for savings- and portfolio decisions of consumers as we do in our model.

We use the approach of Campbell and Viceira (2002) to obtain an explicit expression for the optimal portfolio share. In this approach it is assumed that asset returns are lognormally distributed, i.e., the logarithms of gross asset returns are normal. By taking logs of the Euler condition for optimal consumption choice one can derive an equation that can be used to explain the risk premia of assets or consumers' portfolio choice. The finance literature keeps the stochastic discount rate (i.e, the marginal value of financial wealth in the next period) exogenous in case they want to study asset pricing, see Campbell (2003) for an overview. For the explanation of portfolio choice, on the other hand, asset returns are taken as exogenous, see for example Campbell and Viceira (1999). The main advantage of our model is that it is a general equilibrium model where we can derive optimal portfolio choice and the equity premium at the same time. The difficulty with the lognormal consumption-based model is that the intertemporal budget constraint and the portfolio return are not loglinear. Campbell and Viceira solve this problem by taking Taylor approximations of the nonlinear functions, this is also the approach we take in

plained in Section 1.2.3, abolishing a PAYG system can be advantageous for future generations without hurting some current generations when PAYG contributions imply an excess burden.

the model of Chapters 4 and 5.

1.3 Outline of thesis

As explained at the beginning of this chapter the main aim of this thesis is to analyse the international spillover effects of ageing when countries rely to a different extent on PAYG pension schemes. The models used in this thesis differ along two dimensions, this is shown in Figure 1.5.

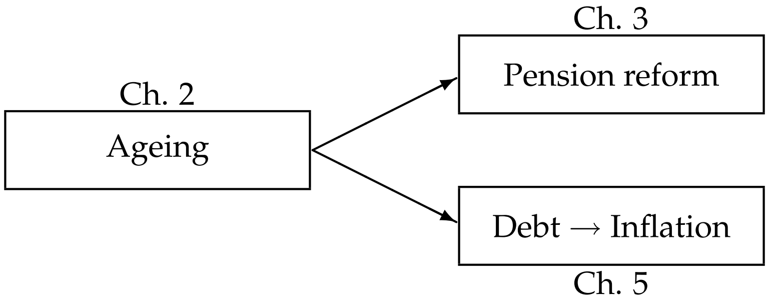
Figure 1.5: Model characteristics

	Certainty	Uncertainty
Open	Ch. 2, 3	Ch. 5
Closed		Ch. 4

The models in Chapters 2 and 3 are completely deterministic, that is, consumers do not face any uncertainty about the return on their investments. Moreover, the model contains two countries where one country relies on PAYG pensions and the other country has a fully-funded pension system. This model is used in Chapter 2 to analyse the international spillover effects of ageing, where the PAYG country keeps its pension system balanced when the population is ageing. This means that PAYG contributions or pension benefits adjust accordingly. Chapter 3 examines the spillover effects of pension reforms in PAYG countries where part of the PAYG scheme is abolished. This is an often proposed solution to improve the sustainability of public pension schemes. In Chapter 4 we develop a closed-economy stochastic overlapping-generations model with safe PAYG pensions. In this model the rates of return on investments are risky. In that case PAYG pensions do not only provide income during retirement, but also offer the possibility to transfer income from the working period to retirement at a non-stochastic rate of return. Chapter 5 combines the open-economy model of Chapter 2 with the stochastic model of Chapter 4. In this chapter we investigate the international spillover effects of ageing when the PAYG country uses government debt to cope with the costs of ageing, which may lead to unexpected inflation or more inflation risk.

The link between chapters 2, 3 and 5 is depicted in Figure 1.6. The remainder of this section gives an overview of the contents and the main conclusions of each chapter.

Figure 1.6: Structure of the thesis



Chapter 2 addresses the question how, in case of population ageing, a country with an extensive funded pension scheme (e.g., the Netherlands) is affected by the fact that other EMU countries (like Italy) rely to a relatively large extent on unfunded pensions, and vice versa. Differences in pension schemes cause savings in the various countries to react differently to the ageing of the population. In a common capital market, these differences in savings lead to capital flows between the countries and in that way induce international spillover effects.

One of the aims of this chapter is to gain insight into the underlying mechanisms of the results and therefore we keep the model as simple as possible. We use a two-country two-period overlapping-generations model where one country uses a PAYG pension system and the other country has a fully-funded retirement scheme. The countries are identical in all other respects. There is perfect capital mobility between the two regions, but labour is immobile. In the initial steady state, the funded country is a net lender on the international capital market as savings are higher than in the PAYG country.

The countries are hit by a symmetric ageing shock, that is, the size and the timing of the ageing shock is the same in both regions. We distinguish between two causes of population ageing, namely an increase in longevity and a decrease in fertility. A rise in longevity is modelled as an increase in the probability that people live throughout old age, while a fall in fertility is reflected by a decrease in the population growth rate. Both a higher longevity and lower fertility rates result in an increase in the relative number of elderly people, i.e., a rise in the so-called old-age dependency ratio. This *dependency-ratio effect* will put a PAYG pension scheme under pressure as the number of contributors falls and the number of recipients rises. In this chapter it is assumed that the PAYG country keeps its PAYG system balanced. This means that either contributions to the PAYG scheme or pension benefits have to adjust in response to ageing. The first case is referred to as a defined-benefit scheme and in that case the working generation bears the burden of an ageing population by paying higher taxes. In the latter case the burden of ageing lies with the elderly and the PAYG scheme is characterised by defined contributions. Both variants are considered in this chapter. The international spillover effects are derived by comparing the utility effects of ageing in case each country adopts a different pension scheme (one PAYG, one funded) to the utility effects in case both countries have the same type of pension system (both PAYG or both funded), which implies that there are no international capital flows. In that way we can deduce how a funded country is affected by the fact that the other country uses a PAYG pension scheme instead of a fully-funded pension system, and vice versa.

The utility effect of ageing in the different cases can be derived as soon as we know the change in the capital-labour ratio, the central variable in the model. The capital-labour ratio is determined by savings in the previous period and the number of working people. A rise in longevity implies that the probability of reaching the second period of life is higher and the expected lifespan of people is longer. This induces people to save more, which increases the capital-labour ratio. A fall in the fertility rate lowers the population growth rate and leads to a shrinking labour force. This implies that less investment is needed to equip each worker with a certain capital stock. This so-called *capital-thickening effect* implies that for a given amount of per capita savings a higher capital-labour ratio can be maintained. So both types of ageing affect the capital-labour ratio positively, however, due to the capital-thickening effect, the rise in the capital-labour ratio is much larger after a fall in fertility compared to an increase in longevity.

The rise in the old-age dependency ratio has an extra effect on savings when a PAYG pension system is in place, however. In case of a defined-benefit PAYG pension system contributions have to go up to keep the pension system balanced, which will decrease savings and weakens the direct positive effect on the capital-labour ratio. The effect will be opposite in case of a fixed-contribution PAYG scheme; pension benefits are reduced due to a higher dependency-ratio, which increases savings and the capital-labour ratio.

We show that for realistic parameter values the capital-labour ratio increases in all considered cases. Moreover, savings in the funded country rise more than in the PAYG country, which leads to capital flows from the funded country to the PAYG country. This results in long-run positive spillover effects for the PAYG country as it benefits both from the higher capital stock and the resulting lower interest rate on its borrowings from abroad. The funded country, on the other hand, is negatively affected by the lower capital stock but gains from the resulting higher interest rate as it is a lender on the international capital market. We find that for realistic parameter values the first effect dominates and therefore a country with funded pensions is in the long run adversely affected by the existence of a PAYG scheme in the other country. The short-run spillovers, however, are opposite to the spillovers in the long run. This means that the initial generations in the funded country gain from the fact that the other country uses a PAYG scheme, whereas these initial generations in the PAYG country are negatively affected by the funded system in the other country.

We show that these results are quite robust for modifications to the basic model. This chapter therefore shows that in the long run countries with funded retirement schemes are negatively affected by the fact that other countries have PAYG pensions, even when the government of the PAYG country keeps its pension system balanced.

Population ageing puts pension schemes financed on a PAYG basis under pressure as the rise in the old-age dependency ratio implies that either contributions to the scheme have to rise to high levels or pension benefits have to fall to a large extent. One of the most discussed proposals to increase the sustainability of the pension system is to reform the PAYG scheme and switch to a more funded scheme. In a multicountry world with integrated capital markets, such a switch to funding will engender spillover effects to other countries. The aim of **Chapter 3** is to look at these international spillover effects of pension reform. More specifically, this chapter analyses how countries with funded pension systems are affected when countries with PAYG pension schemes reform their pension system and finance a larger part of the future pension benefits by accumulated funds.

The model used is comparable to the model in the previous chapter, but some additional features are added. The most important adjustment is that we allow for the fact that the PAYG scheme leads to distortions in the economy, which implies that the PAYG tax induces an excess burden. The existence of an excess burden offers the scope for introducing a Pareto-improving pension reform.

We first investigate the spillover effects of a pension reform in the PAYG country under the assumption that taxes are non-distortionary. This implies that the PAYG system is Pareto efficient and there is a trade-off between the utility of different generations in implementing the reform. That means that, if some generations are allowed to gain from the conversion policy, it is unavoidable that other generations incur a loss. So, in the reforming country a choice has to be made which generations are allowed to gain from the reform and which not. It is shown that in a common capital market this choice spills over to the funded country. In particular, if the reforming country decides that future generations will gain from the reform, future generations in the funded country will gain as well. But these long-run welfare gains come at the cost of lower utility in the short run and these initial losses will also be transmitted to the funded country via the capital market. If the government in the PAYG country, on the other hand, decides that the reform should be welfare-improving for a majority of its current population (the young and old generation alive at the time of the reform), the initial generations in the funded country also gain from the pension reform in the PAYG country. Under this reform policy the future generations in the PAYG country will bear the costs of the reform and the future generations in the funded country will lose as well.

We then proceed by assuming excess burdens in tax collection, which enables the PAYG country to implement a Pareto-improving pension reform. When such a policy is implemented in our two-country world, the long-run gains in the PAYG country are again transferred to the funded country. However, although in the PAYG country the policy is carefully shaped such that no generation loses in both the short run and the long run, in the funded country some generations might lose in the short run. In other words, a reform policy that appears to be Pareto improving for the PAYG country considered separately, does not have to be Pareto improving after taking into account the international spillover effects.

The central conclusion that emerges from this chapter is that in a common capital market the effects of pension reform in PAYG countries spills over to countries with a fully-funded pension system. In the EU, for example, there is a large group of countries that have extensive PAYG-financed parts in their pension system, while a much smaller group of countries have sizeable funded pensions. The message of this chapter is that the latter countries cannot insulate themselves from the effects of reform measures in the former countries. The policy conclusion from this chapter is, therefore, that in a common market like the EU, decisions on pension reform in the countries with a large PAYG scheme should not be taken without considering the effects for non-reforming (funded) countries. In particular, dependent on the type and size of reform in the PAYG countries, compensation for the non-reforming countries might be necessary.

In Chapters 2 and 3 consumers do not face any uncertainty about the return on their savings, i.e., the models used are completely deterministic. In Chapters 4 and 5 we take a different approach and develop a stochastic two-period overlapping-generations general equilibrium model with government debt and safe PAYG pensions. People can invest in two types of assets, stocks and government bonds, where the total amount of bonds is determined by the exogenous level of government debt. Both assets are risky, productivity risk makes the return on stocks uncertain and there is inflation risk on government bonds. The fact that stocks and bonds differ in their riskiness makes that they earn a different rate of return, which allows us to analyse the equity premium and portfolio choice. The function of safe PAYG pensions in our stochastic economy is twofold. First, it provides retirees with a certain minimum amount of income. Second, as people cannot invest in any risk-free asset, safe PAYG pensions will partly fill the gap of this missing market. Chapter 4 analyses a closed-economy version of this model, while in Chapter 5 the model is extended to a two-country model where one country uses a PAYG scheme and the other country relies on funded pensions. This chapter analyses the international spillover effects of ageing in case the PAYG country uses government debt to cope with the costs of ageing, where high levels of government debt may go along with high inflation (risk).

Chapter 4 links the last chapter of this thesis with Chapters 2 and 3 and the main purpose of this chapter is to gain insight into the workings of the stochastic model. We study three shocks that all affect the portfolio decision of households directly; more risk aversion, the introduction of a PAYG pension scheme and higher inflation risk. We examine the effects of the shocks on portfolio choice, savings and the equity premium.

To contrast the results in a general equilibrium model with the partial equilibrium approach typically taken in the finance literature, we first consider the effects on portfolio- and savings decisions in case the rates of return are taken as exogenous. First, we address the effects of more risk aversion. The main reason why this shock is considered is to contrast the results with the effects of introducing safe PAYG pensions, which is the second analysed shock. Safe PAYG pensions reduce the variance of old-age consumption and will therefore affect the investment decisions of consumers. Comparing the effects of PAYG pensions to the effects of higher risk aversion shows that safe PAYG pensions make individuals act like less risk-averse agents. This implies that they save less and have a more risky investment portfolio, i.e., invest a larger part of their savings in stocks. Finally, we also study the effects of an increase in the riskiness of government bonds, i.e., a rise in inflation risk, to address the impact of uncertainty on consumer behaviour. Higher inflation risk makes government bonds less attractive to hold and consumers reoptimise their investment portfolio and invest a larger part of their savings in stocks.

In partial equilibrium, only the effects on individual decisions are considered. The macroeconomic effects all these individual choices together may have, are not incorporated. We show that it matters whether these macroeconomic effects are taken into account or not. In particular, it turns out that in general equilibrium portfolio choice is affected in exactly the opposite way as in partial equilibrium. Individually, consumers would like to adjust their portfolio in a certain direction after a shock. However, as all people in the economy have the same incentive, this would mean that the capital market does not clear anymore. In general equilibrium the rates of return adjust in such a way that both the government debt and the capital stock are financed, and portfolio choice moves in the same direction as savings. In partial equilibrium financing government debt or the capital stock is not a problem as bonds and stocks can be supplied on the international capital market for given rates of return.

Consider for example the effects of introducing safe PAYG pensions. In partial equilibrium individuals increase the riskiness of their investment portfolio when they receive a safe PAYG pension benefit. In general equilibrium, however, this cannot occur because the lower investment in government debt would lead to an oversupply of government bonds. Actually, the fall in savings induced by PAYG pensions implies that the part of savings invested in government bonds has to rise, i.e., people have to move to the less risky asset, to make sure that the government debt is financed. To persuade consumers to invest more in government bonds, the return on government bonds has to rise to a larger extent than the return on stocks, i.e., the risk premium of stocks over bonds has to fall. The effect on the equity premium is opposite in case people become more risk averse, the rise in savings causes the risk premium of stocks over bonds to rise to clear the capital market.

For a rise in inflation risk we have the same result, that is, the effect on portfolio choice in general equilibrium is opposite to the effect in partial equilibrium. Higher inflation risk makes government bonds less attractive to hold and consumers prefer to switch to stocks. In general equilibrium, however, this cannot occur as government debt will not be completely financed in that case. In order to induce people to invest in the given amount of government bonds, the return on bonds has to rise relative to the return on stocks. The higher interest rate on government debt increases the so-called debt tax, i.e., the tax to pay the interest obligations on the government debt in order to keep the debt dynamics stable, and savings will fall. Lower savings imply that a larger part of savings has to be invested in government bonds to have a clearing capital market and the equity premium will fall to make this happen.

These opposing results in partial- and general equilibrium imply that it matters whether one studies portfolio choice in a small open economy or in a large (closed) economy, like the US or the EU. In the first case a partial equilibrium approach would suffice, while a general equilibrium framework is more appropriate in the latter case. Moreover, this result is also relevant in case one would like to analyse the effects of 'global' shocks, like population ageing, where the change of rates of return have to be taken into account.

Chapter 5 investigates the international spillover effects of ageing and pensions when the PAYG country uses government debt to cope with the costs of ageing. This scenario may occur when governments in PAYG countries do not reform their pension system (like in Chapter 3) and do not adjust PAYG contributions or -benefits immediately (like in Chapter 2) to spare the initial generations that suffer most from ageing. This will benefit these initial generations, but future generations will lose as government debt crowds out capital. The utility effects spill over to the funded country that does not use government debt. The initial generations gain from the higher rates of return, while later generations are affected negatively because wage levels fall. This implies that the use of government debt by the PAYG country increases both the positive short-run spillovers and the negative long-run spillovers of ageing for the funded country.

High levels of government debt may give governments an incentive to lobby for surprise inflation at the central bank as this will reduce the fiscal burden of debt service. The question is of course whether the central bank will give in and create unexpected inflation. We assume that the decision-making process about monetary policy is not completely transparent. In that case the monetary authority might not be completely independent and may have an incentive to inflate away nominal debt. We show that in a closed-economy setting unexpected inflation is a zero-sum game. On the one hand, the government gains from unexpected inflation as it decreases the real value of government debt and the real return on government bonds. On the other hand, the people that invested in government bonds lose from the unexpected fall of the real return on government bonds. In a closed economy the gain for the government is exactly high enough to compensate the people who lose from the unexpected inflation shock. In a two-country model where one country relies on PAYG pensions and the other country has a fully-funded pension scheme, however, the PAYG country gains from unexpected inflation at the cost of the funded country. This means that there may be a conflict of interest on monetary policy when countries with different pension schemes form a monetary union. The reason for this result is that residents of the funded country hold a relatively large share of the total amount of government bonds because they save more and have a more conservative portfolio than people in the PAYG country. This implies that the PAYG country can export part of the inflationary tax on debt holders to the funded country, while it still receives the full gain of a lower debt burden and a net gain results. So when a PAYG country forms a monetary union with a funded country it has an incentive to put pressure on the central bank to increase inflation unexpectedly. This gain of unexpected inflation for a PAYG country rises with the amount of nominal government debt. In the coming decades the ageing of the population will put the public finances more under pressure. In case PAYG countries finance their increased pension obligations by issuing more debt, the incentive of PAYG governments to lobby for surprise inflation will rise. On the other hand will unexpected inflation harm funded countries more if PAYG countries issue large amounts of nominal government debt. The conflicts of interest on monetary policy between PAYG- and funded countries may therefore be reinforced if population ageing raises government debt in PAYG countries.

If the decision-making process of the central bank is not completely transparent, it is not clear how the central bank will react to these conflicting interests about the creation of inflation between PAYG- and funded countries. As a consequence, there will be more uncertainty about what the final outcome for inflation will be and inflation risk may rise with the level of government debt in PAYG countries. Higher inflation risk makes government bonds less attractive to hold and the rate of return on bonds will rise relative to the return on equity (i.e., the equity premium falls) to induce people to buy the existing stock of government bonds. This increases the debt tax and lowers savings and the capital-labour ratio, and both countries experience negative utility effects from the rise in inflation risk. Actually, the PAYG country experiences negative spillover effects from the fact that they form a monetary union with a funded country in case inflation risk increases. This result arises because the funded country owns a relatively large part the government bonds and people in the funded country are more risk averse as they do not receive a safe PAYG pension benefit. This implies that these people need to be compensated more in order to hold the more risky government bonds, that is, the rate of return on government bonds has to rise to a larger extent when a funded pension scheme is in place. The rise in the debt tax and its negative consequences will therefore be larger in the PAYG country when it forms a monetary union with a country that uses a fully-funded pension scheme instead of a PAYG pension system.

All the scenarios analysed in this chapter show that in the long run funded countries are affected even more negatively by the fact that other countries in the monetary union rely on PAYG-financed pensions when these countries use government debt to cope with the ageing costs. The use of government debt itself has negative long-run spillover effects for funded countries. Moreover, a high debt burden may lead to unexpected inflation or a rise in inflation risk, which both increase the negative spillover effects even more. In the coming decades it will therefore be important for funded countries that the rules of the Stability and Growth Pact are met, so that debt levels of PAYG countries do not rise to too high levels. The use of government debt by PAYG countries to cope with the costs of ageing benefits the initial generations in these countries that suffer most from ageing. Future generations, however, are harmed by the larger debt burden. In a monetary union part of this burden can be shifted to the funded country and the debt burden can be reduced by an unexpected rise in inflation when the PAYG country successfully lobbies at the central bank. If this lobbying only raises perceived inflation risk, the negative effects of a rise in the riskiness of government bonds in response to a higher level of public debt cannot be shared with the funded country, however. It may therefore also be in the interest of PAYG countries to obey the fiscal constraints stated in the Stability and Growth Pact to prevent large increases in government debt, which may raise inflation risk. Moreover, it is important for all countries that a central bank like the ECB is independent, credible and transparent, so that it is clear how monetary policy is determined and the risk of inflation does not rise when debt levels are high.

CHAPTER 2

INTERNATIONAL SPILLOVERS OF PENSIONS UNDER AGEING[1]

2.1 Introduction

In the coming decades, differences in the extent and the timing of ageing in the developed world will lead to international spillover effects through the capital market. This has been established in an extensive literature, starting with a seminal paper by Cutler et al. (1990), followed by a number of papers presenting simulation experiments with large multi-country overlapping-generations models (see for example Attanasio et al., 2006, Börsch-Supan et al., 2006, Brooks, 2003, Domeij and Flodén, 2006, Fehr et al., 2005).

Less attention, however, has been paid to a second possible reason for international spillover effects in case of ageing, i.e., differences in pension systems between countries. Within the EMU, for example, there are large differences in pension arrangements. Some countries, such as the Netherlands, have large funded schemes, while other countries, e.g., Italy and Germany, rely almost completely on pay-as-you-go (PAYG) systems. Together with differences in the institutional set-up of PAYG schemes this causes savings in the various countries to react differently to the ageing of the population, even if the pattern of ageing is identical across countries. In a common capital market, these

[1]This chapter is based on Adema et al. (2008*a*).

differences in savings will spill over to other countries.

The aim of this chapter is to explore the international spillover effects of a symmetric ageing shock when countries in a common capital market have different pension systems. In other words, we want to shed light on the question how, in case of population ageing, a country with an extensive funded pension scheme (e.g., the Netherlands) is affected by the fact that other EMU countries (such as Italy) rely to a relatively large extent on unfunded pensions, and vice versa.

Some studies (for example Casarico, 2001, van Groezen, 2003, Jousten and Legros, 2002) investigate what happens with capital flows when two countries that differ in the degree of funding of their pension systems integrate their capital markets. Casarico (2001) finds that the young and the future generations in the PAYG country are better off in the open economy equilibrium. In the funded country, however, welfare can either increase or decrease for the young and future generations. So it is not necessarily true, as one might expect, that both countries gain from the integration of capital markets. In contrast to the above-mentioned papers, we study the consequences of ageing given that countries have an integrated capital market but different pension arrangements. To our knowledge this chapter is one of the first studies taking this approach. One exception is the paper by Börsch-Supan et al. (2006) who also allow for differences in the generosity of public PAYG pension schemes. In contrast to that paper, we develop a simple model that enables us to derive an analytical solution of the transition path, in order to gain insight into the underlying mechanisms of the results.

We use a two-country two-period overlapping-generations model where one country has a PAYG pension system and the other country has a fully-funded retirement scheme. The countries are identical in all other respects and are hit by an identical demographic shock. Two typical variants of the PAYG scheme are distinguished: a defined-benefit scheme where the working generation bears the burden of an ageing population by paying higher taxes, and a defined-contribution scheme where the burden of ageing lies with the elderly. Moreover, we consider two forms of population ageing, namely an increase in longevity and a decrease in fertility.

We find that, in general, a country with funded pensions is in the long run adversely affected by the existence of a PAYG scheme in the other country.

This means that a country using a funded system is more vulnerable to an ageing shock when it has a common capital market with a country relying on PAYG pensions. In the short run, however, the spillover effects may be opposite: generations born in the country with the funded pension scheme around the time of the demographic shock may gain from the fact that the other country has a PAYG scheme. The reason for this is that the rise in the capital-labour ratio, resulting from the ageing shock, is smaller in case the funded country has integrated capital markets with a country that uses a PAYG pension scheme instead of a funded system. As a result the fall in the interest rate, which especially harms these initial generations, is less.

The rest of the chapter is organised as follows. Section 3.2 presents the benchmark model. In Section 2.3 we discuss the spillover effects when ageing is modelled as an increase in longevity, first in case of a defined-benefit PAYG scheme and then shortly for the case of defined contributions. Section 2.4 deals with modifications of the model. We analyse the spillover effects with a decline of fertility instead of an increase in longevity (2.4.1) and with a more general type of utility function (2.4.2). Moreover, we discuss how the results would change in case of some other modifications, like proportional taxation and endogenous labour supply (2.4.3). The final section concludes.

2.2 The model

Following Buiter (1981), we use a two-period overlapping-generations model of an open economy. The world consists of two countries, country P and country F, and the only difference between the two countries is the way the pensions are financed. Country P uses a PAYG system and country F has a fully-funded retirement scheme.

2.2.1 Demographics

Both economies are populated with non-altruistic, identical individuals who live for at most two periods. So in each period both a young and old generation are alive. We assume that an agent born at t lives throughout old age with probability ε_{t+1}. ε_t can be interpreted as average longevity; when ε_t rises, the expected lifespan of people is longer.

The active population, L_t^i, is supposed to increase at the same exogenous rate

n_t[2]. The number of active (young) people at time t is then:

$$L_t^i = L_0^i \cdot \prod_{j=0}^{t}(1+n_j) \tag{2.1}$$

where L_0^i is the initial population size in country i, $i = P,\ F$. A decrease in n_t can be interpreted as a fall in the fertility rate. As the countries may differ in their initial population size, the model allows for scale differences between the two countries. Define $\frac{L_0^F}{L_0^P} = \nu$, so if L_0^P is normalised to 1, then ν tells us the relative size of L_0^F.

2.2.2 Production

All variables in the model are expressed as the amount per young individual in the country. For example, k_t^i stands for the amount of capital per young individual in period t in country i. Production per young individual is described by a standard neoclassical constant-returns-to-scale production function, $f(k_t^i)$. Perfect competition among producers results in the usual equilibrium conditions, $r_t^i = f'(k_t^i) - \delta$ and $w_t^i = f(k_t^i) - k_t^i f'(k_t^i)$, where r_t^i is the interest rate, w_t^i denotes the real wage, and δ is the depreciation rate of capital. There is perfect capital mobility between the two countries, but labour is immobile. Since capital can freely move across countries, the interest rates will be equalised, i.e., $r_t^P = r_t^F = r_t,\ \forall t$. As both countries are endowed with the same production technology, we have $k_t^P = k_t^F = k_t$, and consequently $w_t^P = w_t^F = w_t$.

2.2.3 Government

The government in country P runs a PAYG pension system, that is, the pension benefits of the elderly (z_t^P) are covered by lump-sum taxes of the young (τ_t^P). Since, at time t, there are $\varepsilon_t L_{t-1}^P$ old agents and L_t^P young agents, a PAYG system satisfies:

$$z_t^P = \frac{1+n_t}{\varepsilon_t}\,\tau_t^P \tag{2.2}$$

In country F, the government invests the contributions of the young (τ_t^F) and returns them with interest in the next period in the form of transfers to

[2]Throughout this chapter, both economies are assumed to be dynamically efficient, i.e., $r_t > n_t,\ \forall t$.

the then old agents (z^F_{t+1}). As only a fraction of ε_{t+1} of young people born at t survives to period $t+1$, the contributions of those who deceased will fall to surviving contemporaries. In this case, we have:

$$z^F_{t+1} = \frac{1+r_{t+1}}{\varepsilon_{t+1}}\,\tau^F_t \tag{2.3}$$

2.2.4 Households

Expected lifetime utility of a representative individual born at t is given by the following utility function:

$$E_t U\left(c^{y,i}_t, c^{o,i}_{t+1}\right) = \log\left(c^{y,i}_t\right) + \frac{1}{1+\rho}\,\varepsilon_{t+1}\log\left(c^{o,i}_{t+1}\right) \tag{2.4}$$

where $\rho > 0$ stands for the (constant) pure rate of time preference of an individual, $c^{y,i}_t$ is consumption when young, and $c^{o,i}_{t+1}$ is consumption in the second period of life.

Young agents inelastically supply one unit of labour. We assume perfect annuity markets, which implies that the assets of those who deceased are distributed among the people who survived. The total return on savings is therefore $\frac{1+r_{t+1}}{\varepsilon_{t+1}}$. The consolidated lifetime budget constraint is:

$$c^{y,i}_t + \frac{\varepsilon_{t+1}\,c^{o,i}_{t+1}}{1+r_{t+1}} = w_t - \tau^i_t + \frac{\varepsilon_{t+1}\,z^i_{t+1}}{1+r_{t+1}} \tag{2.5}$$

Maximising lifetime utility with respect to the lifetime budget constraint gives the following expressions for individual optimal savings in both countries:

$$s^P_t = \frac{\varepsilon_{t+1}}{1+\rho+\varepsilon_{t+1}}\left[w_t - \tau^P_t\right] - \frac{1+\rho}{1+\rho+\varepsilon_{t+1}}\,\frac{\varepsilon_{t+1}\,z^P_{t+1}}{1+r_{t+1}} \tag{2.6}$$

$$s^F_t + \tau^F_t = \frac{\varepsilon_{t+1}}{1+\rho+\varepsilon_{t+1}}\,w_t \tag{2.7}$$

Note that optimal savings in country F do not depend on the interest rate. The reason is that, given a logarithmic utility function, the intertemporal substitution elasticity is equal to one. For the same reason, optimal savings in country P only react to the interest rate because it changes the net present value of the pension benefit.

2.2.5 Equilibrium international capital market

Individuals invest their savings either in the home country or abroad. Their portfolios will be composed such that interest rates are equalised. Equilibrium in the international capital market is given by:

$$s_t^P + \nu\left(s_t^F + \tau_t^F\right) = (1+\nu)(1+n_{t+1})k_{t+1} \tag{2.8}$$

From equations (2.6) and (2.7), it can be seen that country F has higher savings than country P, implying that country F exports capital abroad.

2.3 Increase in longevity

One of the causes of population ageing is that people live longer. Therefore, in this section, we analyse the international spillover effects when longevity increases permanently at $t = 0$. The demographic shock is *unexpected*, i.e., people do not adjust their behaviour in the period before the shock. Moreover, the size and the timing of the ageing shock is the same in both countries. As a result, ageing will induce capital flows between the two countries only because the pension systems differ.

We employ the method of comparative dynamics, adopted from Judd (1982), to calculate the effect of the longevity shock analytically. The probability of reaching the second period of life at time t is given by:

$$\varepsilon_t = \varepsilon + \pi h_t \tag{2.9}$$

where ε denotes the initial steady-state value[3], h_t describes the time pattern of a perturbation of this steady-state value and π reflects the magnitude of this perturbation. Ageing is reflected by a positive value of h_t[4]. To focus solely on the effects of an increase in longevity we assume that there is no population growth, i.e., $n_t \equiv 0\ \forall t$. The effects of an ageing shock can be traced by linearising the capital-accumulation equation (2.8) with respect to π around the initial steady state. The resulting first-order difference equation for k_t describes the capital-labour ratio changes over time and the determining factors. Given the change in the capital-labour ratio we can derive the changes in all other variables.

First, we explain the effects of an increase in longevity when the PAYG system is characterised by defined benefits. After that we shortly describe what

[3]Throughout the chapter, we omit time subscripts to denote the (initial) steady-state value of the respective variable.

[4]Because we assume that longevity increases *permanently*, this means that $h_0 = h_1 = \ldots = h_\infty > 0$. However, as people do not anticipate the rise in longevity we have that $E_{t=-1}(h_0) = 0$.

changes when the PAYG system is characterised by defined contributions instead.

2.3.1 Defined benefits

This section analyses the international spillover effects of population ageing in case the PAYG scheme is characterised by defined benefits. In terms of the model this means that the pension benefit in country P is fixed at z^P ($z_t^P = z^P \,\forall t$). This implies that in response to a longer expected lifespan taxes have to increase to keep the PAYG scheme balanced. So the burden of ageing is entirely borne by the working population.

The change in the capital-labour ratio

Using the method described above we obtain the following first-order difference equation for the evolution of the capital-labour ratio[5]:

$$\begin{aligned}\frac{\partial k_{t+1}^{FP}}{\partial \pi} = & -\frac{\varepsilon f''(k)k}{\Psi}\frac{\partial k_t^{FP}}{\partial \pi} - \overbrace{\frac{z^P \varepsilon}{(1+\nu)\Psi}}^{3} h_t \\ & + \left[\overbrace{\frac{(1+\rho)(w - \frac{\tau^P}{1+\nu})}{\Psi(1+\rho+\varepsilon)}}^{1} - \overbrace{\frac{(1+\rho)^2 z^P}{(1+\nu)\Psi(1+\rho+\varepsilon)(1+r)}}^{2} \right] h_{t+1}\end{aligned} \tag{2.10}$$

where $\Psi \equiv (1+\rho+\varepsilon) - \frac{(1+\rho)\varepsilon z^P f''(k)}{(1+\nu)(1+r)^2} > 0$.

Equation (2.10) shows the change in the capital-labour ratio after an increase in longevity when the two economies have different pension schemes. To analyse the international spillover effects we derive the same kind of equations for the situation where the two economies use the same pension system. The first-order difference equation for the case where both countries have a PAYG scheme is given by:

$$\begin{aligned}\frac{\partial k_{t+1}^{PP}}{\partial \pi} = & -\frac{\varepsilon f''(k)k}{\Psi^P}\frac{\partial k_t^{PP}}{\partial \pi} - \overbrace{\frac{z^P \varepsilon}{\Psi^P}}^{3} h_t \\ & + \left[\overbrace{\frac{(1+\rho)(w - \tau^P)}{\Psi^P(1+\rho+\varepsilon)}}^{1} - \overbrace{\frac{(1+\rho)^2 z^P}{\Psi^P(1+\rho+\varepsilon)(1+r)}}^{2} \right] h_{t+1}\end{aligned} \tag{2.11}$$

[5] In Appendix 2.A we show the derivation of this expression.

while the expression for the change in k_{t+1} in case both economies use a funded system is given by:

$$\frac{\partial k_{t+1}^{FF}}{\partial \pi} = -\frac{\varepsilon f''(k)k}{\Psi^F}\frac{\partial k_t^{FF}}{\partial \pi} + \overbrace{\frac{(1+\rho)w}{\Psi^F(1+\rho+\varepsilon)}}^{1} h_{t+1} \tag{2.12}$$

with $\Psi^P \equiv (1+\rho+\varepsilon) - \frac{(1+\rho)\varepsilon z^P f''(k)}{(1+r)^2} > 0$ and $\Psi^F \equiv (1+\rho+\varepsilon) > 0$.

By comparing the capital-labour ratio changes in case the two countries have the same pension system (equations (2.11) and (2.12)) with the change in the capital-labour ratio when the two countries have different pension schemes (equation (2.10)), we derive the pure spillover effects of pensions and ageing in a common capital market[6,7].

As the increase in longevity at $t = 0$ is not anticipated, savings at $t = -1$ do not adjust and, as a result, the capital stock per worker does not change at the time of the shock, that is, $\frac{\partial k_0}{\partial \pi} = 0$. At $t = 0$, there are several effects that influence optimal savings in both countries in different directions. First of all, people have more incentives to save because the chance of reaching the second period of life is higher (this effect is indicated by a 1 in the first-order difference equations). In country P, however, the existence of PAYG pension benefits has a depressing effect on savings when longevity goes up. The reason for this effect (indicated by a 2) is that the present value of the pension benefit rises, which raises lifetime income and thus consumption in both periods of life. For a given net wage income, the increase in consumption when young implies lower savings. Moreover, with a defined-benefit PAYG pension scheme, savings decrease as contributions have to go up to keep the PAYG system balanced (this is the so-called *dependency-ratio effect*, indicated by a 3). So the effect on total savings, and the capital-labour ratio, is ambiguous. From equation (2.10) it follows that, given that $h_t = h_{t+1}$, the capital-labour ratio rises after a permanent increase in longevity if:

$$\frac{z^P}{1+\nu} < \frac{(1+\rho)(1+r)\,w}{\varepsilon(1+\rho)(1+r) + (1+\rho)^2 + \varepsilon(1+\rho+\varepsilon)(1+r)} \tag{2.13}$$

[6] To exclude the effects of integration it is assumed that the initial steady state is the same in all cases.

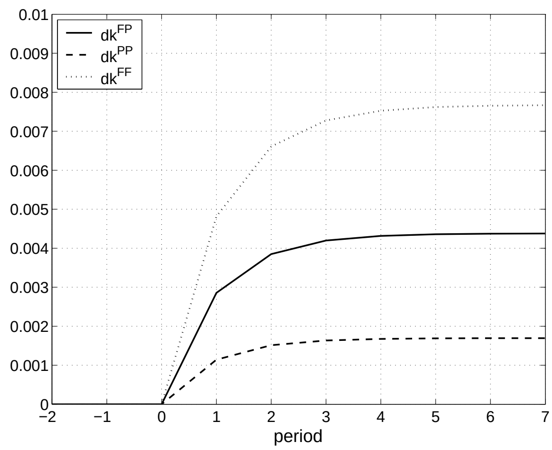
[7] Assuming that both countries have the same type of pension system and comparing this to the situation where each adopts a different scheme is the same as comparing the open equilibrium to the closed-economy outcome: when both countries have the same pension design and when they are subject to the same ageing shock, there are no international capital flows and therefore it is as if the countries were closed.

As shown in Appendix 2.B.1 this condition holds in the benchmark case (see footnote 9) as long as $\tau^P < 0.6w$. For other parameter values the PAYG system can be even larger. Therefore, we can say that for realistic parameter values, the capital-labour ratio increases after a rise in longevity when one country has a PAYG scheme and the other country uses a funded system[8].

The increase in the common capital-labour ratio at $t = 1$ leads to higher wages, which engenders higher savings in both countries (this is the first term in the three equations). Due to these higher savings the capital-labour ratio continues to rise.

Savings unambiguously rise more in the funded country than in the PAYG country, as effects 2 and 3 do not appear in equation (2.12). When the funded country has a common capital market with a PAYG country, part of its extra savings flow to country P. To illustrate the mechanics of the model we also show some numerical simulation experiments[9]. The change in the capital-labour ratio for the three different cases can be seen in Figure 2.1. In the steady

Figure 2.1: Change in k_t



[8]The appendix also shows, however, that in case both countries have a PAYG scheme the contribution rate does not have to be unrealistically high for the capital-labour ratio to *decrease* after a longevity shock. This would not change the international spillover effects qualitatively, however.

[9]The graphs are based on simulations with $f(k_t) = k_t^{0.3}$, $\nu = 1$, $\frac{\tau^P}{w} = 0.2$, $\varepsilon = 0.94$ and $h_t = 0.05$. Capital depreciates at 5% per year and assuming that one period is 30 years this means that $\delta = 1 - (0.95)^{30} = 0.7854$. Agents are relatively patient with a time preference rate of 1% per year, so that $\rho = (1.01)^{30} - 1 = 0.3478$. We also derived numerically the non-linear transition path and compared the numerical results with those found with the method of comparative dynamics. The relative error of the linearised path was one percent at most.

state $k = 0.12$, which implies that the capital-labour ratio rises by about 3.6% in the benchmark case where one country uses a PAYG scheme and the other country relies on fully-funded pensions ($\mathrm{d}k^{PF}$).

The change in utility

To infer whether a country gains or loses from being in an integrated capital market with a country that has another pension system, we compare the utility effects of ageing in the different cases. In Appendix 2.C we derive the long-run change of utility in both countries, for the PAYG country we get:

$$\frac{\partial U^P_\infty}{\partial \varepsilon_\infty} = \frac{f''(k)}{c^{y,P}(1+r)} \left[s^P - k(1+r) \right] \frac{\partial k_\infty}{\partial \varepsilon_\infty} - \frac{1}{c^{y,P}} \left(z^P + \frac{s^P}{\varepsilon} \right) \tag{2.14}$$

The last term indicates that for a given production capacity, an increase in longevity leads to a lower utility as individuals have to share total life-time consumption with more people[10]. As explained in Appendix 2.C.1 the whole term in front of $\frac{\partial k_\infty}{\partial \varepsilon_\infty}$ is positive, which implies that a rise in the capital-labour ratio, resulting from an increase in longevity, has positive utility effects. There are two reasons for this result. First, a higher capital-labour ratio leads to a lower interest rate, which is good for the country that borrows money on the international capital market (the PAYG country). Second, a lower interest rate means that the economy is closer to the Golden Rule point ($r = n$). This means that the rise in the capital-labour ratio after an increase in longevity reduces the direct negative utility effects. However, this indirect positive effect on utility is a second-order effect and, therefore, not large enough to compensate for the negative utility effects, as can also be seen in Figure 2.2.

The rise in the capital-labour ratio is larger in case the PAYG country is in a common capital market with a funded country instead of with a PAYG country. As a result, in the long run the PAYG country gains from having a common capital market with a country that has a fully-funded pension scheme. Notice, however, that the spillovers are exactly opposite for the generation born at the time of the ageing shock ($t = 0$). The reason for this is that the fall in the interest rate resulting from the rise in the capital-labour ratio, which especially harms this initial generation, is less in case both countries use a PAYG system.

[10] In order to compare lifetime utility before and after ageing properly, we take the value of ε_t constant at unity. So we do not take into account that a higher life expectancy is actually something that is nice for people. If this is taken into account the overall utility effects of an increase in longevity are still negative, however. An increase in longevity only results in a higher utility if people can work longer. We leave this for future research.

As these initial generations do not enjoy the gains that result from a larger increase in wages when the economies have different pension schemes, the negative utility effects of the lower interest rate are smaller in case the PAYG country does *not* have a common capital market with a country using a funded pension system.

Figure 2.2: Change in U_t^P

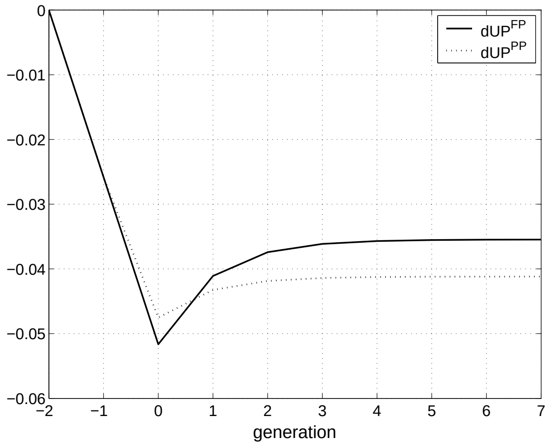
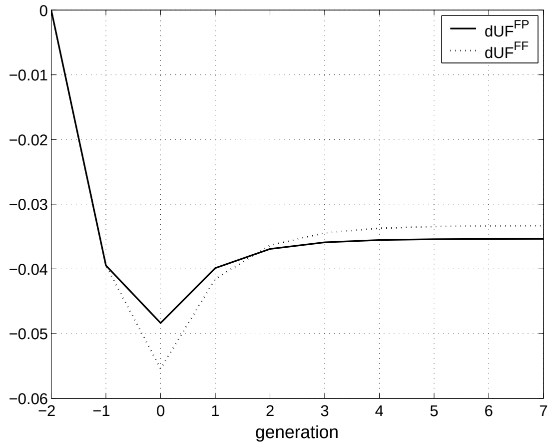


Figure 2.3: Change in U_t^F



The change in long-run utility in the funded country is given by:

$$\frac{\partial U_\infty^F}{\partial \varepsilon_\infty} = \frac{f''(k)}{c^{y,F}(1+r)}\left[(s^F+\tau^F) - k(1+r)\right]\frac{\partial k_\infty}{\partial \varepsilon_\infty} - \frac{1}{c^{y,F}}\frac{(s^F+\tau^F)}{\varepsilon} \tag{2.15}$$

Again there is the direct negative effect on utility because consumption has to be shared with more people. For the funded country, however, it is not necessarily true that a higher capital-labour ratio results in a higher utility. As explained in Appendix 2.C.2, there are two opposing mechanisms that can make the whole expression in front of $\frac{\partial k_\infty}{\partial \varepsilon_\infty}$ either negative or positive. First, we have the positive Golden Rule effect we also had for the PAYG country: a higher capital-labour ratio is good for the funded country because a lower interest rate causes the economy to be closer to its Golden Rule point. On the other hand, a lower interest rate is bad for the funded country as it is a lender on the international capital market. In Appendix 2.C.2 we show that for realistic values of the various parameters the positive Golden Rule effect dominates the negative interest rate effect, implying that a rise in the capital-labour ratio has positive utility effects in the funded country. Because the capital-labour ratio increases less in case the funded country has a common capital market with a PAYG country rather than with a funded-pension country, the fall in utility in country F is larger in the former case. So the funded country experiences negative spillover effects from the PAYG country in the long run, as shown in Figure 2.3.

As was the case for the PAYG country, the short-run spillovers are opposite to the spillovers in the long run. The reason is that the initial generations only incur the losses that result from a falling interest rate, they do not have the gains from the higher wages. As the fall in the interest rate is larger in case both countries use a fully-funded system these generations experience a larger utility loss. Notice that the effect of interest rate decreases have more impact in the funded country than in the PAYG country, as households save more in the funded country. As a result, it takes more generations to invert the spillover effects.

In these simulation graphs we assumed that the two economies have the same size, that is, $\nu = 1$. However, as most EMU countries mainly use PAYG pension schemes, a country like the Netherlands, relying to a relatively large extent on funded pensions, can be considered as relatively small. In case the funded country is smaller than the PAYG country, that is, when $\nu < 1$, the negative spillover effects for the funded country become larger.

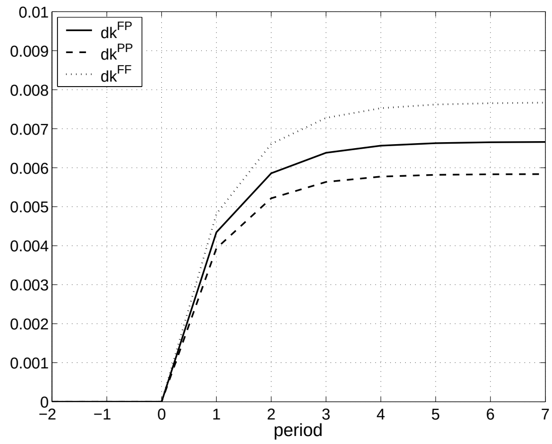
2.3.2 Defined contributions

If the PAYG pension system is characterised by defined contributions, the change in the capital-labour ratio in case of different pension schemes, is described by:

$$\frac{\partial k_{t+1}^{FP}}{\partial \pi} = -\frac{\varepsilon f''(k)k}{\Psi}\frac{\partial k_t^{FP}}{\partial \pi} + \left[\overbrace{\frac{(1+\rho)(w-\frac{\tau^P}{1+\nu})}{\Psi(1+\rho+\varepsilon)}}^{1} - \overbrace{\frac{(1+\rho)^2 z^P}{(1+\nu)\Psi(1+\rho+\varepsilon)(1+r)}}^{2} + \overbrace{\frac{(1+\rho)\,\tau^P}{(1+\nu)\Psi(1+r)\varepsilon}}^{3}\right] h_{t+1} \tag{2.16}$$

The only difference with equation (2.10) is the dependency-ratio effect (indicated by a 3 in both equations). In case of defined benefits, the higher dependency ratio led to less savings because contributions had to rise to keep the PAYG system balanced. With a fixed PAYG tax, however, pension benefits are reduced due to a higher dependency-ratio, which increases savings. This implies that the capital-labour ratio increases more in case the PAYG scheme is characterised by defined contributions, resulting in a smaller decrease in utility after a longevity shock. However, as shown formally in Appendix 2.D, savings in country F still increase more than in country P, so that the rise in the capital-labour ratio is smaller when country F has integrated capital markets with a country that uses a PAYG pension system. This can also be seen in Figure 2.4[11]. In the long run, this results in positive spillover effects for the

Figure 2.4: Change in k_t



PAYG country and negative spillovers for the funded country. It should be noticed, however, that the spillovers are smaller than in the defined-benefits case.

Our main findings can be summarised as follows:

Result 2.1 *In case ageing is characterised by an unexpected increase in longevity:*

1. *Savings in the funded country unambiguously rise more than in the PAYG country, which leads to capital flows from the funded country to the PAYG country.*
2. *In the long run, the funded country experiences negative spillover effects from the PAYG scheme in the other country.*
3. *The short-run spillovers are opposite to the spillovers in the long run.*
4. *The spillovers are smaller in case the PAYG pension system is characterised by defined contributions compared to a defined-benefit PAYG scheme.*

[11]The capital-labour ratio now rises by about 5.4% due to the larger increase of savings in the PAYG country.

2.4 Modifications

In this section the analysis presented above is modified in several ways to investigate the robustness of our results. First, we study the international spillover effects in case population ageing is caused by a lower fertility rate. Second, we consider a more general type of utility function. More precisely, we analyse whether the spillover effects change when preferences are represented by a CES utility function instead of a logarithmic utility function. Finally, we briefly discuss some other modifications.

2.4.1 Decrease in fertility

The ageing of the population in the industrialised world is not only caused by the fact that people live longer. Another important reason is that women give birth to a smaller number of children, that is, the fertility rate is declining. In this section, we analyse the spillover effects of pensions and population ageing in case of an unanticipated permanent decrease in fertility at $t = 0$. As before, we employ the method of comparative dynamics. Let the fertility rate at time t be given by $n_t = n + \pi g_t$. Ageing is reflected by a negative value of g_t[12]. To isolate the effects of a fertility decline, it is assumed that agents live throughout their old-age with certainty, i.e., $\varepsilon_t \equiv 1 \, \forall t$. And again we distinguish between a defined-benefit and a defined-contribution PAYG scheme.

The main difference with a longevity shock is that a decline in the rate of population growth leads to a shrinking labour force, so that less investment is needed to keep the capital-labour ratio constant. This so-called *capital-thickening effect* has a direct positive effect on the capital-labour ratio. As a result of this capital-thickening effect, a fall in fertility can actually have positive utility effects for individuals.

[12]A *permanent* fall in fertility implies that $g_0 = g_1 = \ldots = g_\infty < 0$, but as the decline in fertility is not anticipated $E_{t=-1}(g_0) = 0$.

Defined benefits

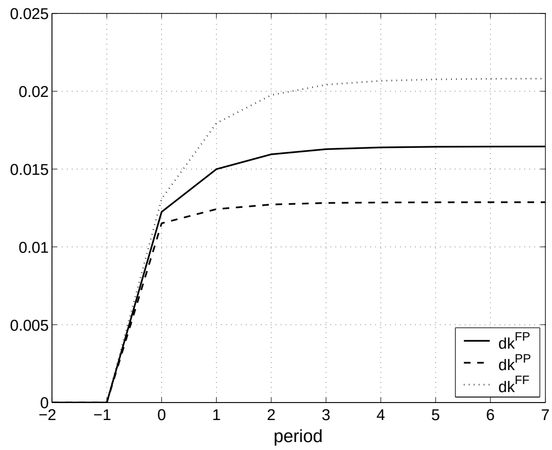
The first-order difference equation for the change in the capital-labour ratio is:

$$\frac{\partial k_{t+1}^{FP}}{\partial \pi} = -\frac{f''(k)\,k}{\Delta}\frac{\partial k_t^{FP}}{\partial \pi} + \overbrace{\frac{z^P}{(1+\nu)\Delta}}^{3} g_t - \overbrace{\frac{(2+\rho)k}{\Delta}}^{4} g_{t+1} \tag{2.17}$$

with $\Delta \equiv (2+\rho) - \frac{(1+\rho)z^P f''(k)}{(1+\nu)(1+r)^2} > 0$.

The just described capital-thickening effect is indicated by a 4 in equation (2.17). As before the fall in fertility has a *dependency-ratio effect* in the PAYG country (effect 3), which affects savings negatively in case of a defined-benefit system[13]. Because the country with the funded pension system does not have the negative dependency-ratio effect, the capital-labour ratio increases more when this country is not integrated with country P. Figure 2.5 confirms these results[14]. Note that, due to the capital-thickening effect, the rise in the capital-labour ratio is much larger after a fall in fertility compared to an increase in longevity. In particular, in the steady state $k = 0.13$, which implies that the capital-labour ratio rises by about 12.6% in the benchmark case where one country uses a PAYG scheme and the other country relies on fully-funded pension (dk^{PF}).

Figure 2.5: Change in k_t



[13]As in section 2.3.1 we can derive a condition that has to hold for the capital-labour ratio to rise after an ageing shock. From equation (2.17) it follows that, given $g_t = g_{t+1}$, k_t rises after a permanent decline in the fertility rate if $\frac{z^P}{1+\nu} < (2+\rho)k$. As shown in Appendix 2.B.2 this condition holds for a large range of parameter values.

[14]In these simulation graphs it is assumed that the population size is constant in the initial steady state ($n = 0$). Assuming a positive population growth rate in the original steady state does not qualitatively change our results. We take $g_t = -0.1$.

Figures 2.6 and 2.7 illustrate the effects of the decrease in fertility on utility in the PAYG country and the funded country respectively. Notice that lifetime utility is higher in the funded country in the long run as compared to the initial steady state, which is due to the capital-thickening effect. In the PAYG country the long-run utility effects are negative however, as in our benchmark simulation the dependency-ratio effect dominates the capital-thickening effect. In case the PAYG system is not too large, that is, when the dependency-ratio effect is not too large, people in country P also experience positive utility effects of a fall in fertility in the long run. From simulations it follows that this is the case if $\tau^P < 0.13w$.

Figure 2.6: Change in U_t^P

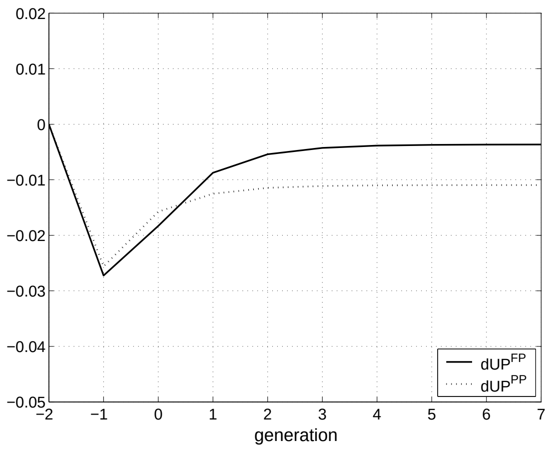
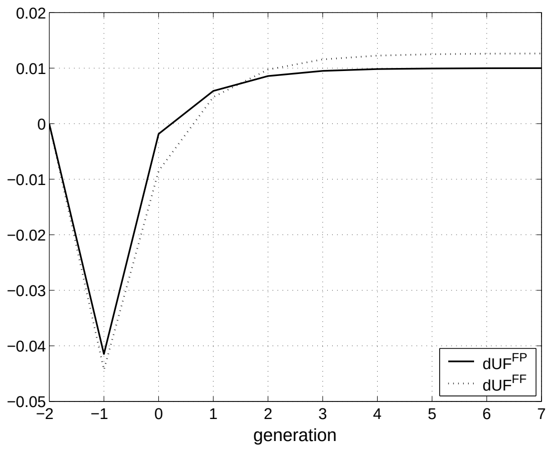


Figure 2.7: Change in U_t^F



The international spillover effects of a fertility shock can also be seen from these graphs. Figure 2.7 shows that in the long run, individuals in country F experience negative spillover effects from the PAYG scheme in the other country. That is, utility rises more in the long run if country F has a common capital market with a country that also has funded pensions. People in the PAYG country, however, gain in the long run from having integrated capital markets with a country that uses a funded pension system (Figure 2.6). In the short run the spillovers are opposite to the effects in the long run as was the case in Section 2.3.1. So the international spillover effects do not change qualitatively compared to the effects of the longevity shock.

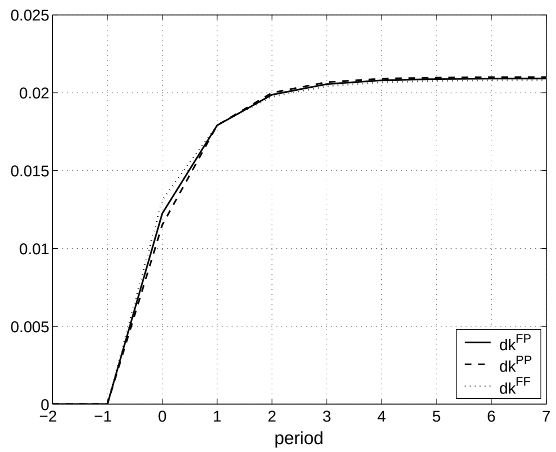
Defined contributions

The first-order difference equation for $\frac{\partial k_{t+1}}{\partial \pi}$ in case of defined contributions is:

$$\frac{\partial k_{t+1}^{FP}}{\partial \pi} = -\frac{f''(k)\,k}{\Delta}\frac{\partial k_t^{FP}}{\partial \pi} - \left[\overbrace{\frac{(2+\rho)k}{\Delta}}^{4} + \overbrace{\frac{(1+\rho)\,\tau^P}{(1+\nu)\Delta\,(1+r)}}^{3}\right] g_{t+1} \tag{2.18}$$

As in Section 2.3.2, the dependency-ratio effect (effect 3) works in the opposite direction as in the defined-benefit case, i.e., leads to higher savings as pension benefits have to decrease after a fall in fertility[15]. As a result, this effect intensifies the capital-thickening effect, so that individuals in the PAYG country now also experience positive long-run utility effects of a fertility decline. Moreover, because the funded country has no dependency-ratio effect, the capital-labour ratio actually rises less in this country. This means that the spillover effects turn around, this holds both for the short run and the long run, so that in the long run citizens in country P suffer and those living in country F gain from having an integrated capital market with a country that uses another pension scheme. Figure 2.8, however, shows that the difference between the change in k_t in the two countries is very small, especially in the long run. This implies that these opposite spillovers are also very small. Moreover, the next section shows that these spillovers are not robust to changes in the intertemporal elasticity of substitution.

Figure 2.8: Change in k_t



[15]This implies that the rise in the capital-labour ratio is larger, i.e., the long-run rise of k_t^{FP} is about 16%.

2.4.2 CES utility function

The results presented until now are based on a logarithmic utility function, which means that the intertemporal elasticity of substitution (σ) is equal to one. In the literature, values taken for σ range from $\frac{1}{4}$ to $\frac{1}{2}$ [16], which implies that our assumption of $\sigma = 1$ is probably not very realistic. Therefore, we also simulated the model for smaller values of σ[17]. If $\sigma < 1$ the income effect is larger than the substitution effect, and a falling interest rate leads to higher savings. This positive effect on savings is largest in the funded country, because it has the highest savings in the initial steady state. Consequently, the negative spillover effects for the funded country in case $\sigma = 1$ will only be reinforced if $\sigma < 1$. This gives the following result:

Result 2.2 *The findings stated in Result 1 do not change if the intertemporal elasticity of substitution is smaller than one.*

In the previous subsection it was shown that in case of a fertility shock *and* a defined-contributions PAYG scheme, savings in country P increased more than in country F when $\sigma = 1$, resulting in small *positive* spillover effects for the funded country. For a small enough value of σ, however, the interest-rate effect of $\sigma < 1$ dominates the dependency-ratio effect, so that savings in country F increase more than in country P. This is illustrated below, where the change in k_t for $\sigma = \frac{1}{4}$ is shown[18]. This means that the funded country experiences negative spillover effects of the PAYG scheme as in the other cases. So we can conclude that for smaller and probably more realistic values for the intertemporal elasticity of substitution the long-run spillovers for the funded country are negative in all considered cases.

The main result of a fertility decline is:

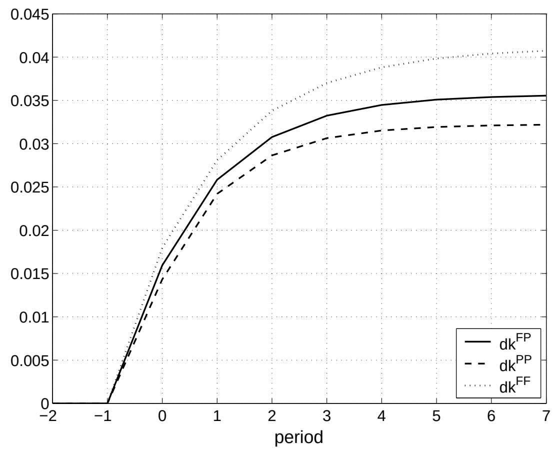
Result 2.3 *In case ageing is characterised by an unexpected decrease in fertility, for realistic values of the intertemporal elasticity of substitution the PAYG scheme causes negative spillovers to the funded country in the long run, while in the short run the spillovers are positive. So the spillovers do not differ qualitatively from those in case of an increase in longevity.*

[16] Auerbach and Kotlikoff (1987) for example assume a value of $\frac{1}{4}$.

[17] For expositional clarity, we do not show the analytical expressions for the case of a CES utility function.

[18] Note that rise in k_t is much larger due to the fact that savings rise in response to lower interest rates in case $\sigma < 1$. In particular, the rise of k_t^{PF} is about 30% in the long run ($k = 0.12$).

Figure 2.9: Change in k_t



2.4.3 Possible other modifications

In this subsection we discuss the effects of some other possible modifications to our model. We contend that, in general, these modifications do not qualitatively affect our results.

Anticipated and asymmetric shocks

So far we have only analysed unexpected demographic shocks. It is often argued, however, that the demographic changes that characterise population ageing are largely anticipated. Therefore, we also studied the results of an increase in longevity and a decrease in fertility that is anticipated one period before the shock actually takes place. Only the short-run effects of the shocks are slightly different in this case, the long-run results are identical to the ones presented in the previous sections. Differences are largest in case of an anticipated increase in longevity and a defined-benefits PAYG scheme. In that case savings already adjust in the period before the shock, which leads to a rise in the capital-labour ratio at $t = 0$ as a net result of effects 1 and 2 in equations (2.10)-(2.12). Just as with an unexpected shock, from $t = 0$ on there is a dependency-ratio effect which negatively affects the capital-labour ratio in the next period. As a result of this, in case both countries use a PAYG system, the increase in k_t at $t = 1$ is smaller than at $t = 0$. The spillover effects are not qualitatively different, however.

A further robustness check is to assume asymmetric ageing, i.e., longevity or fertility may change more in one country than in the other. The model can easily be extended to allow for this. Let us consider two extreme cases: ageing only in country *F* and ageing only in country *P*. In the former case, savings will only increase in the funded country which obviously leads to a large capital flow from country *F* to country *P* and thus reinforces our results. In the latter case, savings will only increase in the PAYG country. This clearly reverses the spillover effect: capital now flows from the ageing unfunded country to the funded country. Due to the PAYG pensions, the capital flow in this case is smaller than in the opposite one. Therefore, when both extremes cases are combined as we did in our model, a net capital flow from country *F* to country *P* results.

Proportional taxes

Our basic model assumes lump-sum taxes, but of course, it would be more realistic to assume proportional taxes. Given the assumption of inelastic labour supply, it can easily be shown, however, that proportional taxes would only complicate the analytical expressions but do not qualitatively change our results. In particular, assuming proportional taxes would result in a smaller dependency-ratio effect because the rise in taxes can be less as a result of the increase in wages. This, in turn, would imply smaller spillover effects.

Endogenous labour supply

At first sight, one might presume that allowing for proportional taxes in combination with endogenous labour supply could qualitatively change the spillover effects. We argue, however, that endogenous labour supply may reduce but most likely not reverse the international spillover effects derived in our basic model.

It should first be noted that, given the assumption of perfect capital mobility, gross wages are always equal in both countries. Thus, as labour supply reacts to changes in net wages, there can only be differences in the spillovers via the capital market (compared to the situation with fixed labour supply) when PAYG contributions change. Hence, endogenous labour supply can only affect our results in case the pension scheme in the PAYG country is characterised by defined benefits. In case of defined contributions, the results of our basic model remain in full force also when labour supply is elastic.

Now let us look at the case of defined benefits and assume that labour supply in a country rises when net wages rise. In that case, after an increase in longevity, labour supply rises in the funded country relative to the PAYG country, the reason being that gross wages are equal and PAYG contributions rise in country *P*. The relative increase in labour supply in the funded country has two effects. Firstly, as especially in the funded country a large part of wage income is saved, the relative increase in labour supply in country *F* will increase savings in this country even more than with fixed labour supply, thus reinforcing the capital flow from country *F* to country *P*. On the other hand, the increase in relative labour supply tends to increase the return on capital in this country which will reduce the capital flow to the PAYG country as compared to the situation with fixed labour supply. Given these two opposing effects, it is unlikely that the change in relative labour supply will offset the primary effect on the capital flow due to the differences in savings in both countries. This can only happen if the elasticity of labour supply is high and changes in relative labour supply lead to large changes in the return on capital, i.e., capital and labour are weak substitutes. This seems not very likely. Empirical studies find that the labour supply elasticity is fairly small for the principle earner of a household, see Blundell and MaCurdy (1999) for an overview. Moreover, papers simulating large general equilibrium models often assume a Cobb-Douglas production function, i.e., a substitution elasticity between capital and labour of one, see for example Altig et al. (2001).

One could also allow for the fact that a rise in longevity induces people to work longer, i.e., increases the retirement age. The rise in savings after an increase in longevity will be smaller in both countries in that case. Moreover, the adjustment of PAYG contributions or PAYG pension benefits can be less because the increase in the dependency ratio will be smaller. The spillover effects will not change qualitatively, however, as savings in the funded country still rise more than in the PAYG country.

Imperfect annuity markets

Our basic model assumes perfect annuity markets. Although all savings in our model are pension savings that in principle could be organised in pension funds, this may be considered a strong assumption. However, relaxing this assumption would probably only reinforce our results. In the absence of perfect annuity markets, households will have a stronger incentive to save for old age. This especially holds true in the funded country where there is no other source of old-age income than private savings. It is therefore likely that with imperfect annuity markets the capital flows from the funded to the unfunded country will be larger.

Bequests

In case of bequests, it is important to distinguish between intentional and unintentional bequests. Unintentional bequests are the result of the individual's uncertainty about his/her longevity. In our model, the individual is uncertain whether (s)he will die at the end of period one or will live on in the second period. It is assumed that this "risk" can be insured via perfect annuity markets, so that there are no unintentional bequests. As argued above, unintentional bequests, i.e., the absence of perfect annuity markets will reinforce our results.

Intentional bequests are deliberate transfers from one generation to the next one at the end of life. Our basic model does not allow for such bequests. Introducing them will not qualitatively change our conclusions, but may reinforce the spillover effects from the funded to the unfunded country. The latter will happen when bequests and thus savings are smaller in the PAYG country because households in this country have a lower lifetime income as (assuming dynamic efficiency) the return on the PAYG contribution is lower than that on private savings.

2.5 Concluding remarks

In the coming decades, most European countries will have an ageing population. These countries differ in the degree of funding of their pension systems. On the one hand there are countries like the UK and the Netherlands, that have an extensive funded pension system. Most other countries, on the other hand, mainly rely on unfunded pensions. These differences in pension schemes engender different saving responses when the population is ageing, which leads to capital flows. This chapter focuses on the question how, in case of population ageing, countries are affected by the fact that other countries use other pension systems. We find that in the long run a PAYG country gains from having a common capital market with a country relying on funded pensions, the main reason being that in the funded country savings increase more in response to ageing. A country using a funded retirement scheme, on the other hand, experiences long-run negative spillover effects from the PAYG system in the other country. The short-run spillovers, however, are opposite to the spillovers in the long run. In other words, the initial generations in the funded country gain from the fact that the other country uses a PAYG scheme, whereas these initial generations in the PAYG country are negatively affected by the funded system in the other country.

We have shown that our results are quite robust for modifications to the basic model used in this chapter. Extensions of the analysis that we leave for future research would, for example, be to allow for endogenous labour supply and the fact that an increase in longevity can also lengthen the working period of people. Moreover, this chapter considers *balanced* PAYG pension systems, that is, in response to population ageing contributions or pension benefits adjust in such a way so that the PAYG scheme does not run a deficit. Governments in funded countries, however, are mainly concerned about the negative effects of *unbalanced* PAYG schemes. In other words, they worry how their country will be affected when countries with PAYG schemes use government debt to cope with the costs of ageing. In Chapter 5 we extend the model presented in this chapter to allow for government debt. Moreover, by including inflation, it is also possible to analyse the spillover effects in case governments put pressure on the central bank to accommodate, so that their debt burden is reduced. This chapter, however, already shows that countries with funded pensions are mostly negatively affected by the fact that other countries have PAYG pension schemes, even when the government of the PAYG country keeps its pension system balanced.

Appendix to Chapter 2

2.A Derivation first-order difference equation k_{t+1}

In this appendix we derive the first-order difference equation for the evolution of the capital-labour ratio given in equation (2.10). Linearising the capital-accumulation equation (2.8) with respect to π around the initial steady state gives:

$$\frac{\partial s_t^P}{\partial \pi} + \nu \frac{\partial (s_t^F + \tau_t^F)}{\partial \pi} = (1+\nu)\frac{\partial k_{t+1}}{\partial \pi} \tag{2.19}$$

Then we derive expressions for $\frac{\partial s_t^P}{\partial \pi}$ and $\frac{\partial (s_t^F + \tau_t^F)}{\partial \pi}$, using equations (2.6) and (2.7):

$$\begin{aligned} \frac{\partial s_t^P}{\partial \pi} &= \frac{(1+\rho)(w-\tau^P)}{(1+\rho+\varepsilon)^2}\frac{\partial \varepsilon_{t+1}}{\partial \pi} + \frac{\varepsilon}{1+\rho+\varepsilon}\left[\frac{\partial w_t}{\partial \pi} - \frac{\partial \tau_t^P}{\partial \pi}\right] \\ &\quad + \frac{(1+\rho)\varepsilon z^P}{(1+\rho+\varepsilon)^2(1+r)}\frac{\partial \varepsilon_{t+1}}{\partial \pi} - \frac{(1+\rho)z^P}{(1+\rho+\varepsilon)(1+r)}\frac{\partial \varepsilon_{t+1}}{\partial \pi} \\ &\quad + \frac{(1+\rho)\varepsilon z^P}{(1+\rho+\varepsilon)(1+r)^2}\frac{\partial r_{t+1}}{\partial \pi} \end{aligned} \tag{2.20}$$

$$\frac{\partial (s_t^F + \tau_t^F)}{\partial \pi} = \frac{(1+\rho)w}{(1+\rho+\varepsilon)^2}\frac{\partial \varepsilon_{t+1}}{\partial \pi} + \frac{\varepsilon}{1+\rho+\varepsilon}\frac{\partial w_t}{\partial \pi} \tag{2.21}$$

$\frac{\partial w_t}{\partial \pi}$ and $\frac{\partial r_{t+1}}{\partial \pi}$ are given by:

$$\frac{\partial w_t}{\partial \pi} = -kf''(k)\frac{\partial k_t}{\partial \pi} \tag{2.22}$$

$$\frac{\partial r_{t+1}}{\partial \pi} = f''(k)\frac{\partial k_{t+1}}{\partial \pi} \tag{2.23}$$

Combining equations (2.19) - (2.23) and simplifying gives:

$$\begin{aligned}\frac{\partial k_{t+1}}{\partial \pi} =& -\frac{\varepsilon f''(k)k}{\Psi}\frac{\partial k_t}{\partial \pi} - \frac{\varepsilon}{(1+\nu)\Psi}\frac{\partial \tau_t^P}{\partial \pi} \\ &+ \left[\frac{(1+\rho)(w-\frac{\tau^P}{1+\nu})}{\Psi(1+\rho+\varepsilon)} - \frac{(1+\rho)^2 z^P}{(1+\nu)\Psi(1+\rho+\varepsilon)(1+r)}\right]\frac{\partial \varepsilon_{t+1}}{\partial \pi}\end{aligned} \tag{2.24}$$

with $\Psi \equiv (1+\rho+\varepsilon) - \frac{(1+\rho)\varepsilon z^P f''(k)}{(1+\nu)(1+r)^2}$.

Using equations (2.2) and (2.9) we know that:

$$\frac{\partial \tau_t^P}{\partial \pi} = z^P \frac{\partial \varepsilon_t}{\partial \pi} \tag{2.25}$$

$$\frac{\partial \varepsilon_{t+1}}{\partial \pi} = h_{t+1} \tag{2.26}$$

Substituting these last two expressions into equation (2.24) we obtain equation (2.10).

2.B Conditions for a long-run increase in k

2.B.1 Longevity shock

As explained in Section 2.3.1 the capital-labour ratio rises after a permanent increase in longevity as long as:

$$\frac{z^P}{1+\nu} < \frac{(1+\rho)(1+r)\,w}{\varepsilon(1+\rho)(1+r) + (1+\rho)^2 + \varepsilon(1+\rho+\varepsilon)(1+r)} \tag{2.27}$$

In the same way we can obtain a condition for the case where the PAYG country does not have a common capital market with a country that uses a funded system:

$$z^P < \frac{(1+\rho)(1+r)\,w}{\varepsilon(1+\rho)(1+r) + (1+\rho)^2 + \varepsilon(1+\rho+\varepsilon)(1+r)} \tag{2.28}$$

Figures 2.10 and 2.11 show how the sign of these two conditions changes for different parameter values[19]. For positive values on the vertical axis the above conditions hold, and an increase in longevity leads to an increase in the capital-labour ratio. The horizontal axis shows the relative size of the lump-sum PAYG tax compared to the wage. As can be seen from Figure 2.10, con-

[19]The standard case is characterised by $\alpha = 0.3$, $\varepsilon = 0.94$, $\nu = 1$, $\delta = 0.7854$, and $\rho = 0.3478$. See also footnote 9 in the main text.

Figure 2.10: Condition (2.27)

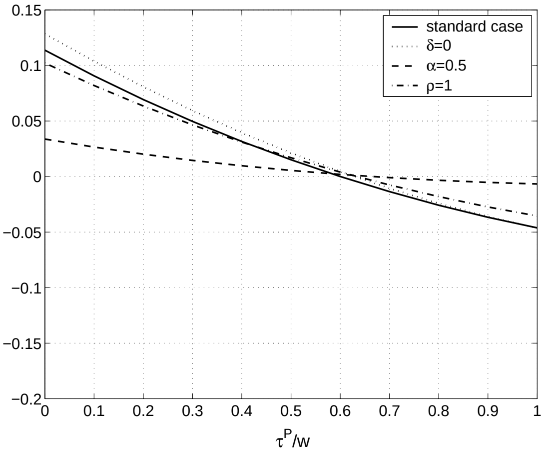
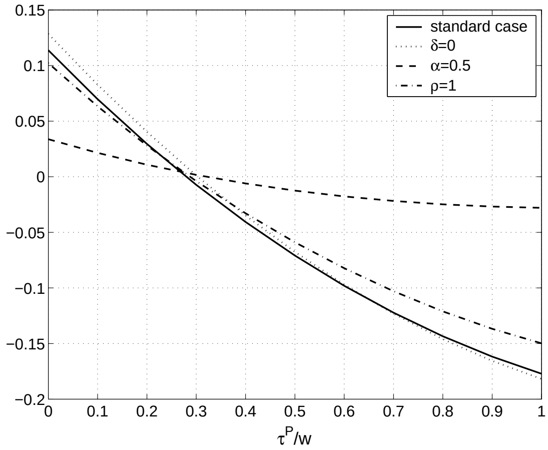


Figure 2.11: Condition (2.28)



dition (2.27) holds in the standard case as long as $\tau^P < 0.6w$. For other parameter values the PAYG system can be even larger. Therefore we can say that for realistic parameter values the capital-labour ratio increases after a rise in longevity when one country has a PAYG scheme and the other country uses a funded system.

In case both countries use a PAYG system (see Figure 2.11), the capital-labour ratio only rises after a permanent increase in longevity as long as $\tau^P < 0.28w$. This is not an unrealistically high value for the contribution rate of the PAYG system, especially when expenditures on medical care are included, implying that the capital-labour ratio may actually fall after a rise in longevity when both countries use a PAYG scheme. This is the point Fehr et al. (2005) make. They argue that the increase in taxes needed to finance the benefits is so large that the capital-labour ratio falls after an ageing shock. Most other studies that use multi-country general equilibrium models, however, find that ageing increases the capital stock per worker, see Attanasio et al. (2006), Börsch-Supan et al. (2006), Brooks (2003), Domeij and Flodén (2006), McMorrow and Röger (2004), Miles (2001). It appears, however, that the results of Fehr et al. (2005) are mainly driven by the fact that the effective labour supply is rising due to labour-augmenting technical progress, which more than offsets the reduction in the labour force when the economy is ageing. But it is not obvious at all whether technical progress will be able to offset the negative effects on the supply of labour. Therefore, we follow the other studies and assume that an increase in longevity results in a higher capital-labour ratio if both countries rely on PAYG pensions when we present the simulation graphs in the main text[20]. The spillovers do not change qualitatively when k_t^{PP} falls instead, the reason being that the spillovers depend on the relative change of k_t^{FF} to k_t^{PP}. As the dependency-ratio effect is absent when both countries use funded pensions, the capital-labour ratio unambiguously increases more in that case, that is, we always have that $\mathrm{d}k_t^{FF} > \mathrm{d}k_t^{PP}$.

2.B.2 Fertility shock

As explained in footnote 13 the capital-labour ratio rises after a permanent decline in fertility if:

$$\frac{z^P}{1+\nu} < (2+\rho)k \tag{2.29}$$

while this condition in case both countries have a PAYG pension scheme is equal to:

$$z^P < (2+\rho)k \tag{2.30}$$

As before we produce graphs that show how the sign of these two conditions changes for different parameter values. Figure 2.12 shows that condition (2.29)

Figure 2.12: Condition (2.29)

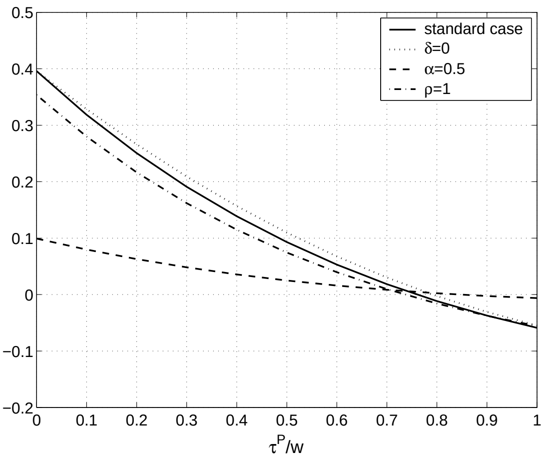
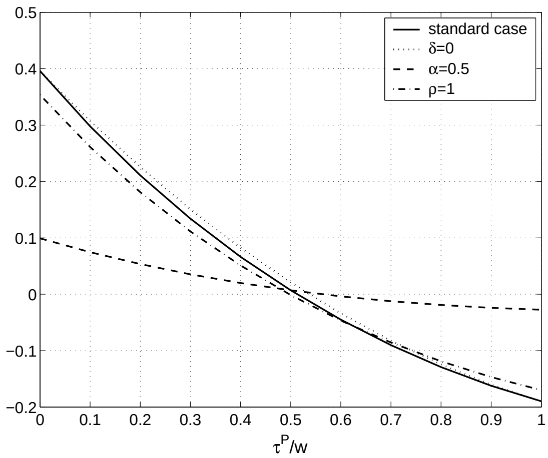


Figure 2.13: Condition (2.30)



holds in the standard case as long as $\tau^P < 0.8w$, while we can see from Figure 2.13 that condition (2.30) holds if $\tau^P < 0.55w$. This means that in case ageing is characterised by a fall in the fertility rate, the capital-labour ratio only falls when the PAYG system is unrealistically large.

[20]The simulation graphs are based on the assumption that $\tau^P = 0.2w$.

2.C Effects on utility

In this appendix we derive the expressions for the long-run change of utility in both countries. From these equations we can infer the relationship between the change in the capital-labour ratio and utility.

2.C.1 PAYG country

First we derive the change of long-run utility in country P. Therefore we first need to know what happens with consumption in both periods of life in the long run:

$$\frac{\partial c_\infty^{y,P}}{\partial \pi} = \frac{\partial w_\infty}{\partial \pi} - \frac{\partial \tau_\infty^P}{\partial \pi} - \frac{\partial s_\infty^P}{\partial \pi} \tag{2.31}$$

$$\frac{\partial c_\infty^{o,P}}{\partial \pi} = \frac{(1+r)}{\varepsilon}\frac{\partial s_\infty^P}{\partial \pi} + \frac{s^P}{\varepsilon}\frac{\partial r_\infty}{\partial \pi} - \frac{(1+r)s^P}{\varepsilon^2}\frac{\partial \varepsilon_\infty}{\partial \pi} + \frac{\partial z_\infty^P}{\partial \pi} \tag{2.32}$$

The long-run change in utility is[21]:

$$\frac{\partial U_\infty^P}{\partial \pi} = \frac{1}{c^{y,P}}\frac{\partial c_\infty^{y,P}}{\partial \pi} + \frac{\varepsilon}{1+\rho}\frac{1}{c^{o,P}}\frac{\partial c_\infty^{o,P}}{\partial \pi} \tag{2.33}$$

Using the Euler condition ($\frac{c^{y,P}}{c^{o,P}} = \frac{1+\rho}{1+r}$), equation (2.33) can be written as:

$$\frac{\partial U_\infty^P}{\partial \pi} = \frac{1}{c^{y,P}}\left(\frac{\partial c_\infty^{y,P}}{\partial \pi} + \frac{\varepsilon}{1+r}\frac{\partial c_\infty^{o,P}}{\partial \pi}\right) \tag{2.34}$$

Substituting equations (2.31) and (2.32) into equation (2.34) gives:

$$\begin{aligned}\frac{\partial U_\infty^P}{\partial \pi} &= \frac{1}{c^{y,P}}\left[\frac{\partial w_\infty}{\partial \pi} - \frac{\partial \tau_\infty^P}{\partial \pi} - \frac{\partial s_\infty^P}{\partial \pi}\right] \\ &+ \frac{1}{c^{y,P}}\left[\frac{\varepsilon}{1+r}\left(\frac{(1+r)}{\varepsilon}\frac{\partial s_\infty^P}{\partial \pi} + \frac{s^P}{\varepsilon}\frac{\partial r_\infty}{\partial \pi} - \frac{(1+r)s^P}{\varepsilon^2}\frac{\partial \varepsilon_\infty}{\partial \pi} + \frac{\partial z_\infty^P}{\partial \pi}\right)\right]\end{aligned} \tag{2.35}$$

Using that $\frac{\partial w_\infty}{\partial \pi} = -kf''(k)\frac{\partial k_\infty}{\partial \pi}$ and $\frac{\partial r_\infty}{\partial \pi} = f''(k)\frac{\partial k_\infty}{\partial \pi}$, we can write:

$$\frac{\partial U_\infty^P}{\partial \pi} = \frac{1}{c^{y,P}}\left[-kf''(k)\frac{\partial k_\infty}{\partial \pi} - \frac{\partial \tau_\infty^P}{\partial \pi} + \frac{f''(k)s^P}{(1+r)}\frac{\partial k_\infty}{\partial \pi} - \frac{s^P}{\varepsilon}\frac{\partial \varepsilon_\infty}{\partial \pi} + \frac{\varepsilon}{1+r}\frac{\partial z_\infty^P}{\partial \pi}\right] \tag{2.36}$$

[21]This equation shows the change in utility for a person living for two periods with certainty, i.e., $\varepsilon = 1$.

With defined benefits it holds that $\frac{\partial \tau_\infty^P}{\partial \pi} = z^P \frac{\partial \varepsilon_\infty}{\partial \pi}$ and $\frac{\partial z_\infty^P}{\partial \pi} = 0$, so that this equation can be rearranged to:

$$\frac{\partial U_\infty^P}{\partial \pi} = \frac{f''(k)}{c^{y,P}(1+r)} \left[s^P - k(1+r) \right] \frac{\partial k_\infty}{\partial \pi} - \frac{1}{c^{y,P}} \left(z^P + \frac{s^P}{\varepsilon} \right) \frac{\partial \varepsilon_\infty}{\partial \pi} \tag{2.37}$$

Dividing this expression by $\frac{\partial \varepsilon_\infty}{\partial \pi}$, we get an expression for the change in U^P in the long run when longevity changes (this is equation (2.14) in the text):

$$\frac{\partial U_\infty^P}{\partial \varepsilon_\infty} = \frac{f''(k)}{c^{y,P}(1+r)} \left[s^P - k(1+r) \right] \frac{\partial k_\infty}{\partial \varepsilon_\infty} - \frac{1}{c^{y,P}} \left(z^P + \frac{s^P}{\varepsilon} \right) \tag{2.38}$$

The following things can be said about the terms in front of $\frac{\partial k_\infty}{\partial \varepsilon_\infty}$:

1. $\frac{f''(k)}{c^{y,P}(1+r)} < 0$, because $f''(k) < 0$.
2. $s^P - k(1+r) < 0$, because we know that $s^P < k(1+n)$: country P is a capital importer in the initial steady state. And because we assumed dynamic efficiency ($r > n$), we know that $k(1+n) < k(1+r)$. So that $s^P < k(1+r)$.

These two points imply that an increase in the capital-labour ratio, after an increase in longevity, has positive utility effects. There are two reasons for this result:

1. A higher capital-labour ratio leads to a lower interest rate, which is good for the country that borrows money (the PAYG country).
2. Moreover, a lower interest rate means that the economy is closer to the Golden Rule point ($r = n$).

As the rise in the capital-labour ratio is larger in case the PAYG country has integrated capital markets with a country that uses a funded system instead of PAYG system, utility is also higher in that case.

2.C.2 Funded country

Unfortunately it is not possible to draw such a clear conclusion for the country that uses a funded pension system. The main intuition for this is as follows. Again we have that a higher capital-labour ratio is good for the funded country, because a lower interest rate causes the economy to be closer to its Golden

Rule point. On the other hand, a lower interest rate is bad for the funded country because it is a lender of money. This can be shown more formally as follows (using the same kind of technique used above for the PAYG country). The expressions for the long-run change of consumption and utility are given by:

$$\frac{\partial c_{\infty}^{y,F}}{\partial \pi} = \frac{\partial w_{\infty}}{\partial \pi} - \frac{\partial (s_{\infty}^{F} + \tau_{\infty}^{F})}{\partial \pi} \tag{2.39}$$

$$\frac{\partial c_{\infty}^{o,F}}{\partial \pi} = \frac{(1+r)}{\varepsilon} \frac{\partial (s_{\infty}^{F} + \tau_{\infty}^{F})}{\partial \pi} + \frac{(s^{F} + \tau^{F})}{\varepsilon} \frac{\partial r_{\infty}}{\partial \pi} - \frac{(1+r)(s^{F} + \tau^{F})}{\varepsilon^{2}} \frac{\partial \varepsilon_{\infty}}{\partial \pi} \tag{2.40}$$

$$\frac{\partial U_{\infty}^{F}}{\partial \pi} = \frac{1}{c^{y,F}} \left(\frac{\partial c_{\infty}^{y,F}}{\partial \pi} + \frac{\varepsilon}{1+r} \frac{\partial c_{\infty}^{o,F}}{\partial \pi} \right) \tag{2.41}$$

Substituting the equations for consumption (2.39) and (2.40) into the equation for utility (2.41), and using the same expressions for $\frac{\partial w_{\infty}}{\partial \pi}$ and $\frac{\partial r_{\infty}}{\partial \pi}$ as before, the following expression for the change in utility in country F can be obtained:

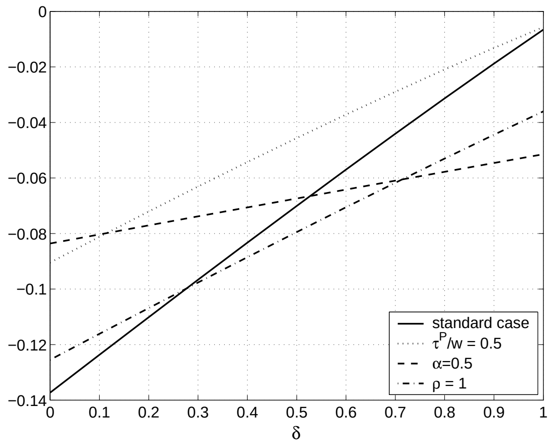
$$\frac{\partial U_{\infty}^{F}}{\partial \pi} = \frac{f''(k)}{c^{y,F}(1+r)} \left[(s^{F} + \tau^{F}) - k(1+r) \right] \frac{\partial k_{\infty}}{\partial \pi} - \frac{1}{c^{y,F}} \frac{(s^{F} + \tau^{F})}{\varepsilon} \frac{\partial \varepsilon_{\infty}}{\partial \pi} \tag{2.42}$$

Dividing by $\frac{\partial \varepsilon_{\infty}}{\partial \pi}$ we get (equation (2.15) in the text):

$$\frac{\partial U_{\infty}^{F}}{\partial \varepsilon_{\infty}} = \frac{f''(k)}{c^{y,F}(1+r)} \left[(s^{F} + \tau^{F}) - k(1+r) \right] \frac{\partial k_{\infty}}{\partial \varepsilon_{\infty}} - \frac{1}{c^{y,F}} \frac{(s^{F} + \tau^{F})}{\varepsilon} \tag{2.43}$$

which is comparable to equation (2.38) for the PAYG country. As before we have that the first term $\left(\frac{f''(k)}{c^{y,F}(1+r)} \right)$ is negative. But we cannot say anything about the sign of the second term $\left((s^{F} + \tau^{F}) - k(1+r) \right)$. Firstly, we know that in the initial steady state country F is a capital exporter, which means that $s^{F} + \tau^{F} > k(1+n)$. Secondly, it holds that $r > n$, because we assume dynamic efficiency, implying that $k(1+r) > k(1+n)$. So there are two opposing mechanisms, meaning that we do not know whether $s^{F} + \tau^{F} - k(1+r)$ will be negative or positive. The first effect reflects the fact that a lower interest rate is bad for a country that is a lender on the international capital market, the second effect is the Golden Rule effect. In case the first effect dominates the second, $s^{F} + \tau^{F} - k(1+r)$ is positive and the whole term in front of $\frac{\partial k_{\infty}}{\partial \varepsilon_{\infty}}$ is negative, implying that an increase in the capital-labour ratio after an increase in longevity actually affects utility negatively. On the other hand, when the second effect dominates the first, we have the same result as for the PAYG country: a higher capital-labour ratio leads to a higher utility in the funded country. This is the same conclusion as in Casarico (2001). Below we present a graph that shows how the sign of $s^F + \tau^F - k(1+r)$ (vertical axis) changes for various parameter values[22]. The horizontal axis shows the depreciation rate (δ). This graph shows that $s^F + \tau^F - k(1+r) < 0$ for different values of the

Figure 2.14: $s^F + \tau^F - k(1+r)$



parameters, which means that the Golden Rule effect dominates the interest rate effect. This in turn implies that for realistic parameter values, a rise in the capital-labour ratio has positive utility effects in the funded country[23]. As the capital-labour ratio increases more in case the funded country has a common capital market with a country that also uses a funded pension scheme, opposed to a PAYG system, utility is higher in that case.

2.D Appendix to Section 2.3.2

In this appendix we prove that savings in the funded country still increase more than in the PAYG country after a rise in longevity when the PAYG scheme is characterised by defined contributions. This implies that the capital-labour ratio in the funded country increases less in case it has a common capital market with a country using a PAYG pension system.

Savings in the funded country increase more than in the PAYG country after

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[22]In the standard case we have the following parameter values: $\rho = 0.3478$, $\frac{\tau^P}{w} = 0.2$, $\nu = 1$, $\alpha = 0.3$, and $\varepsilon = 0.94$.

[23]It is actually possible to have $s^F + \tau^F - k(1+r) > 0$. This is for example the case when $\rho = 0$ *and* $\delta > 0.87$. A time preference rate of zero is not very realistic, however. Therefore we conclude that for realistic parameter values it holds that $s^F + \tau^F - k(1+r) < 0$.

an increase in longevity in case the following condition holds:

$$\frac{(1+\rho)(w-\tau^P)}{\Psi^P(1+\rho+\varepsilon)} - \frac{(1+\rho)^2 z^P}{\Psi^P(1+\rho+\varepsilon)(1+r)} + \frac{(1+\rho)\tau^P}{\Psi^P(1+r)\,\varepsilon} < \frac{(1+\rho)\,w}{\Psi^F(1+\rho+\varepsilon)} \tag{2.44}$$

which can written as:

$$\begin{aligned} &\frac{\varepsilon(1+\rho)(1+r)(w-\tau^P) - \varepsilon(1+\rho)^2 z^P + (1+\rho)(1+\rho+\varepsilon)\,\tau^P}{\Psi^P(1+\rho+\varepsilon)(1+r)\,\varepsilon} \\ &< \frac{(1+\rho)\,w}{\Psi^F(1+\rho+\varepsilon)} \end{aligned} \tag{2.45}$$

where:

$$\begin{aligned} \Psi^P &\equiv (1+\rho+\varepsilon) - \frac{(1+\rho)\,\varepsilon\, z^P f''(k)}{(1+r)^2} > 0 \\ \Psi^F &\equiv (1+\rho+\varepsilon) > 0 \end{aligned}$$

and $\Psi^P > \Psi^F$.

Simplifying and using the fact that $z^P = \frac{\tau^P}{\varepsilon}$ gives:

$$\frac{-\tau^P r}{\Psi^P(1+r)} < \frac{w}{\Psi^F} - \frac{w}{\Psi^P} \tag{2.46}$$

The left-hand side is always negative and because $\Psi^F < \Psi^P$ we know that $\frac{w}{\Psi^F} > \frac{w}{\Psi^P}$, so that the right-hand side is always positive. This implies that equation (2.46) always holds and that the positive dependency-ratio effect in case of defined contributions is not large enough to compensate for the negative effects a PAYG system has in general, so that savings in the funded country increase more than in the PAYG country.

CHAPTER 3

INTERNATIONAL SPILLOVERS OF PENSION REFORM[1]

3.1 Introduction

In many developed countries ageing has led to a debate on reforming unfunded pay-as-you-go (PAYG) social-security systems. One of the most discussed reform proposals is to switch to a more funded pension system where people save for their own pensions, and realise a higher expected rate of return on their contributions (see for example Feldstein (2005) and Bovenberg (2003)). Many countries with PAYG-financed pension schemes actually (are planning to) implement such pension reforms. In a multicountry world with integrated capital markets, such a switch to funding will engender spillover effects to other countries. The aim of this chapter is to look at these international spillover effects of pension reform. More specifically, we will analyse how countries with funded pension systems are affected when countries with PAYG pension schemes reform their pension system.

Some papers (e.g. Fehr et al. (2005), Börsch-Supan et al. (2006) and INGENUE (2001)) that address pension reform issues in an open-economy framework develop large multi-country overlapping-generation models to study the effects of pension reform. These large-scale general equilibrium models are particularly useful in case one wants to obtain reliable forecasts

[1]This chapter is based on Adema et al. (2008*b*).

on several key economic variables. An important drawback of these models is, however, that analytical solutions are not feasible and the underlying mechanisms of the results are not clear. We use a simple two-country two-period overlapping-generations model with an integrated capital market, where one country has a PAYG pension system and the other country has a fully-funded retirement scheme. By keeping the model this simple, we are able to derive an analytical solution for the transition path of the common capital-labour ratio, and to derive the spillover effects of pension reform.

There are some other papers that study the open-economy aspects of pension schemes using smaller models. Pestieau et al. (2006) consider the issue of the optimal PAYG scheme in a setting where the number of symmetric countries changes. They show that a rise in the number of countries implies larger PAYG systems and less capital accumulation. Bräuninger (1999) focuses on the optimal degree of social security funding in a two-country framework, given that the two countries have identical social security systems in the initial steady state. He concludes that the optimal level of social security funding is below the golden rule level. We, however, assume that the two countries use different pension schemes in the initial steady state. Moreover, we do not consider optimality issues, but investigate how pension reform in a PAYG country affects a country with a fully-funded retirement scheme.

Casarico (2001) concentrates on the effects of the integration of capital markets of two countries that differ in the degree of funding of their pension systems. She finds that a country with a PAYG system gains from the switch to perfect capital mobility. In contrast, we assume that the two countries already have a common capital market. Casarico (2001) also shortly touches upon the issue of pension reform in an open economy. We, however, consider different pension reform scenarios, derive the full dynamic path for the capital-labour ratio, show simulation graphs to clarify the mechanisms of the model further, and distinguish between the case where PAYG contributions are distorting and the case where the PAYG taxes do not lead to distortions.

In analysing the effects of a pension reform in the PAYG country, we first assume that collecting contributions in the PAYG country does not involve any distortions. It is well-known that in such a case the PAYG system is Pareto efficient, see Verbon (1989) and Breyer (1989)[2]. That means that, if some gen-

[2] Actually, we already know from the classic papers of Samuelson (1958) and Gale (1973) that if the economy is dynamically efficient no intergenerational redistribution can be Pareto improving.

erations are allowed to gain from the conversion policy, it is unavoidable that other generations incur a loss. So, in the reforming country a choice has to be made which generations are allowed to gain from the reform and which not. In general, future generations in the PAYG country gain if the reform implies an increase in the capital-labour ratio, the reason being that the PAYG country is a borrower on the common capital market, and therefore gains from both the lower interest rate that goes along with an increase in the capital-labour ratio and from the higher wages. As the funded country is a lender on the common capital market, the increasing capital-labour ratio does not necessarily imply welfare gains for future generations in the funded country. However, we can show that under plausible parameter configurations the reforming country's choice spills positively over to the funded country in a common capital market. In particular, if the reforming country decides that future generations will gain from the reform, future generations in the non-reforming country will gain as well. But, just like in the reforming country, this long-run welfare gain may come at the cost of lower utility in the short run as, due to the rising capital-labour ratio, the increase in wages for initial generations is relatively small compared to the decrease in the interest rate.

We then proceed by assuming excess burdens in tax collection. It has been shown that in that case in a closed economy a Pareto-improving pension reform is possible (Homburg (1990)). The idea is that if the contribution rate is decreased net welfare gains result which make it possible to compensate the elderly for the loss of their benefits. When a Pareto-improving policy is implemented in our two-country world, the long-run gains in the PAYG country are again transferred to the funded country. However, although in the PAYG country the policy is carefully shaped such that no generation loses in both the short run and the long run, in the funded country some generations might lose in the short run. The reason for this is that in the open capital market the additional savings that emerge during the conversion policy in the PAYG country will depress the interest rate, which especially harms the generations that are alive in the funded country when the PAYG country starts the conversion policy. As these generations do not, or do not fully, receive the gains that result from increasing wages, they are not compensated for the negative utility effects of the lower interest rate.

In principle, replacing a PAYG-financed public pension scheme by a funded pension scheme implies that pension claims of the current retired are no longer honored. In this respect the switch to funding is comparable to imposing a lump-sum tax on the current old generations in order to decrease the size of the government debt. Exactly the reverse of this policy scenario, i.e., granting a lump-sum tax decrease to older generations financed by increasing the government debt, was already analysed by Diamond (1965) in a one-country model and by Persson (1985) in a two-country model[3]. Based on this similarity, this chapter can be seen as a merger between the pension reform literature, where international spillover effects received relatively little attention, and the literature on the effects of government debt, where the international externalities did not remain unnoticed. Some of our results are closely linked to the conclusions of Persson (1985). For example, the spillover effects of reforming a Pareto-efficient PAYG scheme without compensating the initial generation of elderly are qualitatively the same as those of a decrease in government debt: both lead to a decrease in the interest rate on the international capital market. However, other reform scenarios that we discuss, and especially the Pareto-improving reform, do not fit into Persson's set up and therefore extend his analysis.

Steigum and Raffelhüschen (1994) find in a calibrated two-country model, with North America and the rest of the OECD taken as the two countries of interest, that the welfare effects of debt changes in one country are relatively minor. The reason for this result is that the change in debt in one country is small compared to the total capital stock in the common capital market[4]. We allow for differences in the size of the two countries by introducing a parameter that measures the relative size of the funded country. The analysis presented in this chapter can then be applied to the case where a group of countries shifts to more funded pensions, while a small number of countries do not reform their pension system. An interesting example of this can be observed in the EU where, broadly speaking, two groups of countries can be distinguished: a large group of countries with extensive PAYG schemes but almost no funded pensions, and a much smaller group of countries having sizeable funded pen-

[3]Other papers that analyse public debt in an international setting are, for example Fried and Howitt (1988) and Homburg and Richter (1993). These papers use models based on assumptions that essentially differ from ours. The former abstracts from capital accumulation and examines the effect of government debt on capital gains and losses, the latter analyses the problem of harmonising debt and public pension schemes when labour is perfectly mobile.

[4]Another reason is that Steigum and Raffelhüschen (1994) assume endogenous labour supply. In that case a tax decrease, due to pension reform, stimulates labour supply which dampens the savings, and thereby the general-equilibrium effect of the reform. Empirical estimates of labour-supply elasticities for the principal earner of a household, however, generate relatively low values, see Blundell and MaCurdy (1999) for an overview.

sions (see Table 3.1). Many of the countries in the first group are discussing (or have already started) a transition to more funded pensions. This reform will lead to sizeable changes in the capital stock in the long run, irrespective of whether the reform is gradually implemented or not, and therefore have important consequences for the countries that already have more elaborate funded pension schemes, especially because this group of countries is small compared to the group of reforming countries.

Table 3.1: Composition of retirement incomes in 2003 (%)

	First pillar	Second pillar	Third pillar
Greece	98	1.5	0.5
France	97	1.5	1.5
Spain	97	1.5	1.5
Italy	96	2	2
Austria	95	3	2
Germany	94	4	2
Denmark	76	20	4
Sweden	75	20	5
Switzerland	72	26	2
Netherlands	56	37	7

Source: Boeri et al. (2006)

The rest of the chapter is organised as follows. Section 3.2 presents the benchmark model and shows how the pension reform is modelled. In Section 3.3 we analyse different types of reform scenarios under the assumption that the PAYG scheme is Pareto efficient, while Section 3.4 considers the effects in case the reform is Pareto improving in the PAYG country. In these sections we illustrate the effects of the reform by numerical simulation examples using Cobb-Douglas specifications. Analytical derivations of the results using more general utility and production functions are presented in the Appendix. The chapter winds up with a concluding section.

3.2 The model

We will use a two-period overlapping-generations model of an open economy. Following Buiter (1981) and Persson (1985), the world consists of two countries, country P and country F. Countries differ in the way the pensions are financed. Country P uses a pay-as-you-go (PAYG) system and country F has a fully-funded retirement scheme. We assume a constant population size and dynamic efficiency in both countries. Countries may, however, differ in population size. In this way we allow for scale differences between the two countries. When we define $\frac{L^F}{L^P} = \nu$ and normalise L^P to 1, then ν tells us the relative size of L^F. The countries are identical in all other respects.

3.2.1 Production

Production per young individual is described by a standard neoclassical constant-returns-to-scale production function, $f(k_t^i)$, where k_t^i stands for the amount of capital per young individual in period t in country i, $i = P,\ F$. Perfect competition among producers gives the usual equilibrium conditions, $r_t^i = f'(k_t^i) - \delta$ and $w_t^i = f(k_t^i) - k_t^i f'(k_t^i)$, where r_t^i is the interest rate, w_t^i denotes the real wage, and δ is the depreciation rate of capital. There is perfect capital mobility between the two countries, but labour is immobile. Since capital can freely move across countries, the interest rates will be equalised, i.e., $r_t^P = r_t^F = r_t$, $\forall t$. And because both countries are endowed with the same production technology, we have $k_t^P = k_t^F = k_t$, and consequently $w_t^P = w_t^F = w_t$.

3.2.2 Modelling pensions and reform

Initially the government in country P runs a balanced PAYG pension system, that is, taxes of the young (τ_t^P) are used to finance the pension benefits of the elderly (z_t^P). As explained in the introduction we distinguish between the case where the PAYG system is efficient and the case where the PAYG scheme leads to distortions in the economy. In the latter case the PAYG tax implies an excess burden. This has been modelled in various ways in the literature. Homburg (1990) and Breyer and Straub (1993) assume that taxes distort the labour-leisure decision so that a decrease in the contribution rate for the PAYG system will increase labour supply, and, therefore, restrict the loss in revenue for financing the pension benefits. Pemberton (2000), however, considers the case where a conversion policy goes along with an income tax being replaced by a consumption tax. Then young individuals under the conversion policy have an incentive to save more than under the income tax, which provides the means to compensate the older generations when the PAYG system is abolished. Analogously, Belan et al. (1998) offer the first generation a subsidy on the return on their savings. Yet another way of modelling can be found in

Köthenbürger and Poutvaara (2006) who assume that a decrease of taxation goes along with an increase in the value of a fixed factor. Let us note that we merely conclude that the existence of an excess burden offers the scope for introducing a Pareto-improving reform. However, we do not go into the issue here whether this is a 'good' or 'bad' reason for reform[5].

Instead of explicitly specifying how behaviour is affected when the excess burden of the premium contributions is lifted, we assume that the tax base is constant and independent of the size of the contribution. The existence of an excess burden under the PAYG system is modelled by assuming a so-called 'iceberg' formulation of tax distortions (as in e.g., Casamatta et al. (2000) and Perotti (2001)), that is, for a given tax imposed on young individuals, only $\tau_t^P - (\tau_t^P)^2$ can be redistributed, so $(\tau_t^P)^2$ is 'wasted'. For a tax on elderly individuals, however, such a waste does not occur. This specification of the excess burden of the tax is a short cut for the labour-supply interpretation. Regarding this interpretation, it should be the case, as we assume, that taxing the young leads to tax revenue losses if the young curtail their labour-supply efforts as a result of the tax. But, when the elderly are retired, taxes cannot have an effect on their labour-supply behaviour, and so, there is no excess burden[6].

In general, in the initial steady state the pension benefit of an old individual is equal to[7]:

$$z^P = \tau^P - \xi(\tau^P)^2 \tag{3.1}$$

where $\xi = 1$ implies that the PAYG tax leads to an excess burden, and where $\xi = 0$ refers to the situation where the financing of the PAYG scheme is not

[5]It should be noted here that the existence of excess burdens in taxation cannot be the prime motive for converting the PAYG system into a fully-funded system. The excess burden arises because the individual link between pension benefits and contributions is broken. The reason for this is that the pension system is also used for intragenerational redistribution. As proved by Fenge (1995) and Brunner (1996), in a PAYG system in which such a link exists the system is Pareto efficient, even if contributions are a proportional tax on labour income. Such PAYG systems exist. For instance, as noted by Sinn (2000), Germany has had a PAYG system since 1957 where benefits are proportional to contributions, and so a Pareto improving transition to a funded system is not possible in Germany. Sinn (2000) and Belan and Pestieau (1999) give overviews of the issue. Their conclusion in the words of the last authors is that "reduced distortions can be achieved without privatization".

[6]The tax on the elderly can be seen, for example, as a short-cut for a tax on old-age consumption that is non-distortionary.

[7]By omitting time subscripts we denote the initial steady state value of the respective variable.

distortionary. For the moment we develop the model in the absence of distortions so that $\xi = 0$ and $z^P = \tau^P$ in the initial steady state. In Section 3.4 we consider the case where $\xi = 1$.

At $t = -1$ the government in country P announces that it will reform its pension system in the next period ($t = 0$). Individuals take the economic consequences of the reform into account when they make their optimising decisions in period $t = -1$. A pension reform leads to a lower contribution level and lower benefits. We model this as follows:

$$\tau_t^P = \mu_t \, \tau^P \tag{3.2}$$

$$z_t^P = \lambda_t \, \tau^P \tag{3.3}$$

where $\mu_t < 1$ and $\lambda_t \leq 1$.

One of the crucial issues in pension reform is whether or not the (partial or complete) switch to funding is accompanied by a compensation for the older generations. We consider both possibilities. In the first case there is no compensation: benefits and contributions simultaneously decrease by the same amount. The elderly individuals at the time of the reform lose as a consequence, while the current and future young individuals fully gain from the higher rate of return on their contributions under the funded system.

In the second case the elderly are compensated, implying that $\mu_t < \lambda_t$ will hold during the initial periods of the reform. We assume that public debt, (b_t^P), is used to finance the shortfall in contributions. It is assumed that the government issues one-period debt, which yields the same rate of interest as capital. At a later stage, additional contributions (τ_t^B) are raised to finance the interest obligations on the debt, so as to keep debt per worker constant. With debt, therefore, the budget constraint of the government (public debt dynamics) in per capita terms is:

$$b_{t+1}^P = (1 + r_t) b_t^P + z_t^P - \tau_t^P - \tau_t^B \tag{3.4}$$

If part of the benefits are financed by government debt, we assume that at a certain point in time benefits match contributions again, i.e., the benefits should have decreased as: $\lambda_t = \mu_t < 1$. So the PAYG system is balanced again, but at a permanently lower level. At the moment that contributions and benefits are equal again, the government introduces τ_t^B, such that debt per worker is stabilised from then on. Furthermore, we assume that there is no government debt in the initial steady state ($b^P = 0$), so that τ^B is zero too.

In country F the government invests the contributions of the young and returns them with interest in the next period in the form of transfers to the then old agents. The funded scheme has fixed contributions, which implies that the system is actuarially fair for every individual and contributions to the pension scheme are exactly offset by an equal reduction in private savings. This means that the funded pension system is neutral and the economy behaves in exactly the same way as if there were no pension scheme. Therefore we do not distinguish between contributions to the funded pension scheme and private savings, that is, pension contributions are included in total savings s_t^F.

3.2.3 Households

Lifetime utility of a representative individual born at t is given by the following separable utility function[8]:

$$U\left(c_t^{y,i}, c_{t+1}^{o,i}\right) = u\left(c_t^{y,i}\right) + \frac{1}{1+\rho}\, v\left(c_{t+1}^{o,i}\right) \quad (i = P, F) \tag{3.5}$$

where $\rho > 0$ stands for the (constant) pure rate of time preference of an individual, $c_t^{y,i}$ is consumption when young, and $c_{t+1}^{o,i}$ is consumption in the second period of life.

Young agents inelastically supply one unit of labour. The budget constraints in the PAYG country are as follows:

$$c_t^{y,P} = w_t - \tau_t^P - s_t^P \tag{3.6a}$$

$$c_{t+1}^{o,P} = (1 + r_{t+1})s_t^P + z_{t+1}^P - \tau_{t+1}^B \tag{3.6b}$$

$$c_t^{y,P} = w_t - \tau_t^P - \tau_t^B - s_t^P \tag{3.7a}$$

$$c_{t+1}^{o,P} = (1 + r_{t+1})s_t^P + z_{t+1}^P \tag{3.7b}$$

The government in country P can either levy the additional tax to stabilise government debt (τ_t^B) on the elderly (eq. (3.6b)) or on the working people (eq. (3.7a)). The budget constraints for the funded country are same, except that $\tau_t^P = \tau_t^B = z_{t+1}^P = 0$.

Maximising lifetime utility subject to the budget constraints gives the following expression for individual optimal consumption in both countries:

$$u'\left(c_t^{y,i}\right) = \frac{1 + r_{t+1}}{1+\rho}\, v'\left(c_{t+1}^{o,i}\right) \quad (i = P, F) \tag{3.8}$$

[8] We assume that the felicity functions $u(c_t^{y,i})$ and $v(c_{t+1}^{o,i})$ satisfy the Inada conditions.

where $u'(c_t^{y,i}) = \frac{\mathrm{d}u(c_t^{y,i})}{\mathrm{d}c_t^{y,i}}$ is the marginal utility of young-age consumption, and $v'(c_{t+1}^{o,i}) = \frac{\mathrm{d}v(c_t^{o,i})}{\mathrm{d}c_t^{o,i}}$ is the marginal utility of old-age consumption. For future reference we define the elasticity $\sigma_t^i \equiv -\frac{v'(c_t^{o,i})}{c_t^{o,i} v''(c_t^{o,i})}$, $i = F, P$ with $v''(c_t^{o,i})$ the second-order derivative of felicity when old. The value of these elasticities in the steady state is important for the effects of a reform. In particular, we exclude extremely low values of σ as these imply large increases in savings in reaction to a decrease in the interest rate.

3.2.4 Equilibrium international capital market

Individuals invest their savings either in the home country or abroad. Their portfolios will be composed such that interest rates are equalised. Equilibrium in the international capital market is given by:

$$s_t^P + \nu s_t^F = (1+\nu)k_{t+1} + b_{t+1}^P \tag{3.9}$$

As old-age consumption in country P is partly financed by a transfer from the young, while in country F old-age consumption has to be completely financed by own savings, the latter country has higher savings than country P, implying that country F exports capital abroad.

3.3 Pension reform under Pareto efficiency

This section investigates the spillover effects of a pension reform in the PAYG country, under the assumption that the PAYG system is Pareto efficient, i.e., taxes are non-distortionary. This means that there is a trade-off between the utility of different generations in implementing the reform. We consider three different types of reforms. In the first reform, the government in the PAYG country does not compensate the elderly at the time of the reform (Section 3.3.1). In the second and third reform we analyse the effects in case the government fully compensates the current old (Section 3.3.2). In that case government debt is created and the extra tax needed to pay the interest obligations on the debt can either be levied on the pensioners or on the workers.

For all cases considered we are able to calculate analytically the effect of a pension reform on the common capital stock. We employ the method of comparative dynamics, adopted from Judd (1982). The processes for μ_t and λ_t are given by:

$$\mu_t = 1 + \pi g_t \tag{3.10}$$

$$\lambda_t = 1 + \pi f_t \tag{3.11}$$

where $g_t < 0$ and $f_t \leq 0$ describe the time pattern of a perturbation of μ_t and λ_t from their steady-state values and π reflects the magnitude of this perturbation. The effects of a pension reform can be traced by linearising the capital-accumulation equation (3.9) with respect to π around the initial steady state. The resulting first-order difference equations for k_t describe the capital-labour ratio changes over time and the determining factors. Moreover, we produce numerical simulations in order to illustrate the mechanics of the model. The qualitative results of these simulations are robust for changes in the adopted values of the parameters[9].

3.3.1 No compensation for the current old

At $t = -1$ the government announces that it will decrease both the contributions to the PAYG system (τ_t^P) and the pension benefits (z_t^P) permanently in the next period ($t = 0$)[10]. So the old at $t = 0$ bear all the costs of the reform.

The change in the capital-labour ratio

To analyse the international spillover effects we compare the effects of reform when the two economies have a joint capital market (indicated by superscript PF) to the situation where the two economies are closed (indicated by superscripts P and F, respectively). Obviously, in country F nothing happens when it is a closed economy, as there is no need for reform.

Using the method described above we obtain the following first-order difference equation for the evolution of the capital-labour ratio when the two

[9]We derived numerically the non-linear transition path, and compared the numerical results with those found with the method of comparative dynamics. The accuracy of the linearised path was quite satisfactory with a relative error of one percent at most. This is in line with the findings by Meijdam and Verhoeven (1998). They conclude that using comparative dynamics in a dynamic model is just as accurate as using comparative statics in a static model.

[10]This means that $g_0 = g_1 = \ldots = g_\infty < 0$ and $f_t = g_t < 0$. This could either be a full privatisation ($g_t = f_t = -1$) or a partial privatisation ($-1 < g_t = f_t < 0$).

economies have a joint capital market[11]:

$$\frac{\partial k_{t+1}^{PF}}{\partial \pi} = \frac{\left[\frac{1}{\lambda^P} - 1 + \nu\left(\frac{1}{\lambda^F} - 1\right)\right] k f''(k)}{\Delta} \frac{\partial k_t^{PF}}{\partial \pi} + \frac{\left(\frac{1}{\lambda^P} - 1\right)\tau^P}{\Delta} g_t - \frac{\tau^P}{\lambda^P \Delta (1+r)} f_{t+1} \tag{3.12}$$

with $\Delta \equiv 1 + \nu + f''(k)\left(\frac{\beta^P}{\lambda^P} + \nu \frac{\beta^F}{\lambda^F}\right)$, and $\lambda^i \equiv 1 + \frac{1+\rho}{(1+r)^2}\frac{u''(c^{y,i})}{v''(c^{o,i})} > 1$, $i = F, P$. Note that $\frac{1}{\lambda^i} < 1$ can be interpreted as the propensity to consume out of lifetime income when young (for a given interest rate). The β's indicate the effect of a change in the interest rate on consumption when young (and thus on savings) and are defined as $\beta^F \equiv \frac{(1-\sigma^F)c^{o,F}}{(1+r)^2}$, $\beta^P \equiv \frac{(1-\sigma^P)c^{o,P} - z^P}{(1+r)^2}$. Note that in case of logarithmic utility functions $\lambda^i = \frac{2+\rho}{1+\rho}$, $\beta^F = 0$ and $\beta^P = \frac{-z^P}{(1+r)^2}$, $i = P, F$. The first-order difference equation for country P when it is closed is given by:

$$\frac{\partial k_{t+1}^{P}}{\partial \pi} = \frac{\left[\frac{1}{\lambda^P} - 1\right] k f''(k)}{\Delta^P} \frac{\partial k_t^{P}}{\partial \pi} + \frac{\left(\frac{1}{\lambda^P} - 1\right)\tau^P}{\Delta^P} g_t - \frac{\tau^P}{\lambda^P \Delta^P (1+r)} f_{t+1} \tag{3.13}$$

with $\Delta^P \equiv 1 + f''(k)\frac{\beta^P}{\lambda^P}$. We assume[12] $0 < \Delta^P < \Delta$. By comparing the capital-labour ratio changes in the closed-economies case (eq. (3.13)) with the change in the capital-labour ratio when the two countries have integrated capital markets (eq. (3.12)), we derive the pure spillover effects of pension reform in a common capital market[13].

At the time of the announcement ($t = -1$), young individuals living in country P increase their savings because they know they will receive a lower pension benefit, i.e., $f_0 < 0$. As a result, the reform leads to a positive change in the capital-labour ratio at $t = 0$, as $\frac{\partial k_0^{PF}}{\partial \pi} = \frac{-\tau^P}{\lambda^P \Delta (1+r)} f_0 > 0$ (eq. (3.12)). Note that (given that $\Delta > \Delta^P$) this effect is larger in case of a closed economy (see eq. (3.13)). The increase in the common capital-labour ratio at $t = 0$ leads to higher wages, which engenders higher savings in both countries. Due to these higher savings the capital-labour ratio continues to rise (the first term in eq. (3.12) > 0). Citizens in the PAYG country have an additional incentive to save

[11] In Appendix 3.A.1 we show the derivation of this expression.

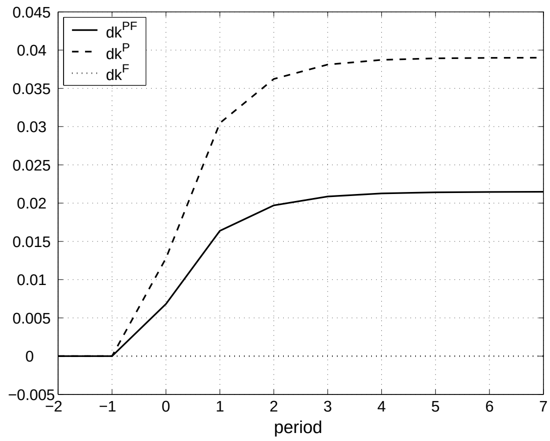
[12] In case of logarithmic utility this condition always holds. For the more general case this condition holds for a wide range of parameter values.

[13] To exclude the effects of integration it is assumed that the initial steady state is the same in all cases.

more, because, as of $t = 0$ contributions to the PAYG system fall (second term in eq. (3.12) > 0), and they will receive lower benefits. So, we have the general result (see also Figure 3.1 where we show the change in the capital-labour ratio for the different cases)[14]:

Result 3.1 *In case the elderly are not compensated at the time of a given pension reform, the capital-labour ratio in country P increases less in case it has a common capital market with a funded country compared to the case where it is closed, as part of the extra savings flow to country F.*

Figure 3.1: Change in k_t



In Figure 1 the two economies have the same size ($\nu = 1$). However, as most EU countries mainly use PAYG pension schemes, the group of countries that has sizeable funded pensions can be considered as relatively small. If the group of funded countries is relatively small (i.e., $\nu < 1$) and a large group of countries with extensive PAYG schemes reform their pension system, the spillover effects for the funded countries will be larger. For the PAYG countries, however, it holds that the larger they are relative to the funded countries, the more the effects of the reform resemble the effects of a closed economy.

[14]The graphs are based on simulations with logarithmic utility functions and countries of equal size ($\nu = 1$). The initial value of the tax rate is 0.2. It is assumed that half of the PAYG system is privatised, that is, from $t = 1$ onwards both the contributions and the benefits fall by 50% permanently. The results do not qualitatively change when the PAYG system is totally privatised, however. Moreover, we used the following production function, $f(k_t) = k_t^{0.3}$. Capital depreciates at 5% per year, and assuming that one period is 30 years, this means that $\delta = 1 - (0.95)^{30} = 0.7854$. Agents are relatively patient with a time preference of 1.2% per year, so that $\rho = (1.012)^{30} - 1 = 0.4303$. In the steady state $k = 0.12$, which implies that the capital-labour ratio rises by about 17% in the open economy case.

The change in consumption and utility

When we know the change in the capital-labour ratio we can derive the changes in all other variables. The analytical derivations are presented in Appendix 3.A.2 for the consumption and utility effects in country F. Here we only show simulation graphs. The change in consumption when young and old in the two countries is displayed in Figures 3.2 and 3.3, respectively. The spillover effects for consumption are summarised in the following results:

Result 3.2 *In case there is no compensation for the current old:*

(a) The reform in country P implies an increasing capital-labour ratio and higher wages, which enhances the consumption possibilities for the young in country F.[15]

(b) The fall in the interest rate, however, lowers the return of the savings of old people living in country F, which has a negative effect on their consumption. Savings in country F do not rise sufficiently to offset the negative effects of the lower interest rate in the long run, so that the consumption of the elderly in the non-reforming country decreases due to the reform.[16]

The changes in lifetime utility are shown in Figure 3.4, lifetime utility rises in the long run. However, this type of pension reform, in which benefits are decreased without compensation, will obviously hurt the elderly in country P at the time of the reform. It is, however, interesting to note that this loss spills over to the elderly in country F as well. In particular, we have[17]:

[15]This is the case if $\beta^F < k$, see Appendix 3.A.2 for details. Simulations show that this condition holds for realistic parameter values. In case of a logarithmic utility function this condition always holds as $\beta^F = 0$. In the benchmark simulation of footnote 14, $c^{y,F} = 0.22$ in the initial steady state. This implies that as a result of the reform in the PAYG country, consumption during young age in the funded country increases by about 5.18% in the long run.

[16]As shown in Appendix 3.A.2, this result holds if $s^F > \frac{\sigma^F(\lambda^F-1)(1+r)(k-\beta^F)}{\lambda^F}$. Simulations show that this condition holds for realistic parameter values. The condition reduces to $s^F > \frac{k(1+r)}{2+\rho}$ in case of logarithmic utility, which holds as long as the production elasticity of capital is not too large (< 0.5). $c^{o,F} = 0.23$ in the benchmark simulation of footnote 14, which implies that the long-run decrease of old-age consumption in the funded country is about 5.21%.

[17]As shown in Appendix 3.A.2 utility in the funded country rises in the long run if $s^F < k(1+r)$. Figure 3.13 in Appendix 3.A.2 shows that this condition holds for a wide range of parameter values. The utility in the funded country decreases initially if the positive wage effect when young is outweighed by the negative effect on the interest rate (which

Result 3.3 *In case the current elderly in country P are not compensated, the generation in country F born at the time of the reform ($t = 0$) may experience a loss, while this is not the case for the same generation in country P. Later generations in country F gain from the pension reform in country P, but the consumption gap between young and old people has increased*[18].

Figure 3.2: Change in $c_t^{y,i}$

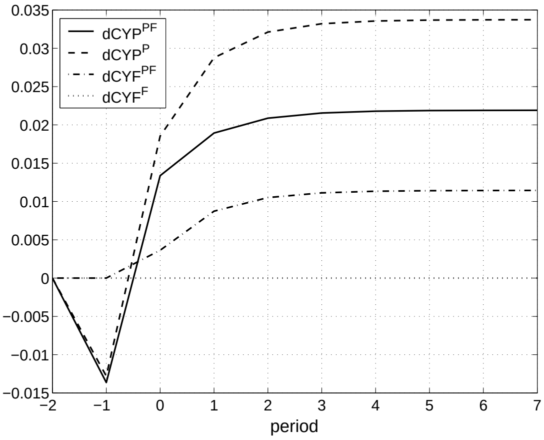


Figure 3.3: Change in $c_t^{o,i}$

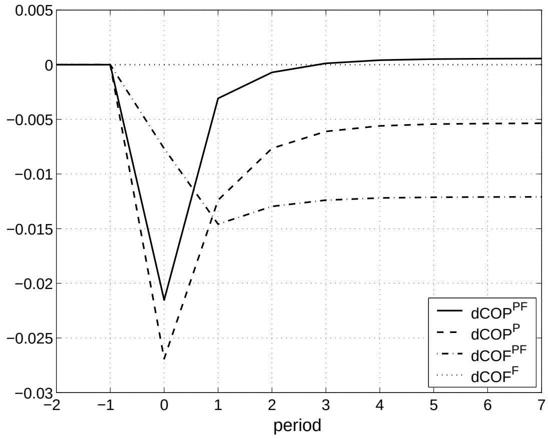
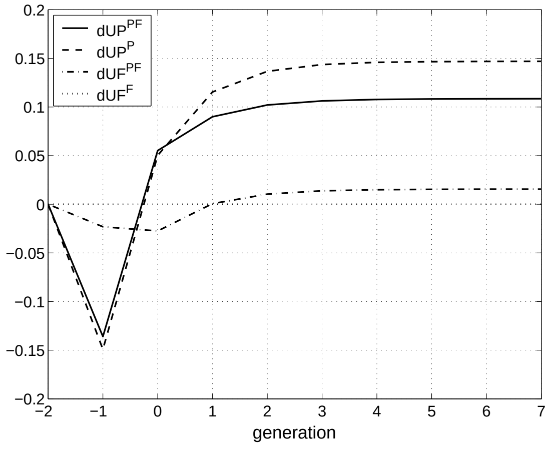


Figure 3.4: Change in U_t^i



may be relatively large because k increases over time) when old. The condition for this is $s^F > \frac{(1+r)k}{\lambda^P(1+r)-r+[\frac{1}{\lambda^P}-1+\nu(\frac{1}{\lambda^F}-1)]kf''(k)/\Delta}$. This condition holds as long as the production elasticity of capital is not too large (≤ 0.5).

[18]If the government in the funded country compensates the first two generations that experience negative spillovers from the reform in the PAYG country, later generations will lose. See Appendix 3.A.2 for details.

The latter result is noteworthy. An important motive for introducing a reform is to get higher rates of return on pension contributions and, as a result, a higher retirement income. In the funded country, however, the result is just the opposite: the pensioners in the funded country will achieve lower incomes than if the PAYG country had not implemented a reform. The reason for this counterintuitive result is that the higher savings in the PAYG country, generated by the reform, lead to a fall in the interest rate and, therefore, lower returns on the pension contributions paid in the funded country.

3.3.2 Compensation

In the pension reform described in the previous section the current elderly bear all the costs of the reform. It is probably more realistic to assume that the government compensates the current old, so that individuals have more time to adjust their behaviour to the smaller PAYG system. Political economy arguments could also play a role for implementing such a reform policy. Cooley and Soares (1999), for example, argue that privatising a PAYG system is only politically feasible in case a transition policy uses debt to finance the benefits during the transition period, shifting at least part of the cost to future generations. Therefore, in this and the next subsection we assume that while contributions to the PAYG scheme fall permanently at $t = 0$, benefits are kept constant in that period. This is again communicated one period before the reform actually takes place (at $t = -1$). The government also announces that at $t = 1$ pension benefits will fall as much as the contributions, so that the PAYG system is balanced again from then on[19]. Since taxes are lower than the benefits during one period ($t = 0$), there will be government debt in country P at $t = 1$. At the moment that contributions and benefits are equal again ($t = 1$), the government introduces an extra tax (τ_t^B) to pay the interest obligations on its debt, such that debt per worker is stabilised from then on. This extra tax can either be levied on the working people or on the elderly.

Tax levied on the future old

When τ_t^B is levied on the elderly, starting at $t = 1$, the pension reform is Pareto neutral, that is, there is no generation that gains or loses from the pension reform. The reason for this result is that savings in the PAYG country increase just as much as government debt increases. All that has happened is that the

[19]This means that $g_0 = g_1 = \ldots = g_\infty < 0$ and $f_0 = 0$, $f_t = g_t < 0$ for $t > 0$.

implicit debt inherent to the PAYG system has been made explicit. This is a standard result in the pension reform literature, see for example Verbon (1989), Breyer (1989) and Homburg (1990). So we have[20]:

Result 3.4 *In case the old at the time of the reform are fully compensated and the tax to finance the debt is levied on the future elderly, the capital-labour ratio remains constant over time.*

A constant capital-labour ratio implies that consumption and utility also do not change. Moreover, there are no international spillover effects for the funded country.

Tax levied on the future young

Instead of imposing τ_t^B on the elderly, the government can also levy the tax on the working people, starting at $t = 1$. So the first young generation under the reform, born at $t = 0$, does not have to pay the debt tax, i.e., $\tau_0^B = 0$, as $b_0^P = 0$. Like the future young generations, they get the lower PAYG tax, but, unlike the future young generations, they do not have to contribute to the compensation the elderly at $t = 0$ receive. The young generation at $t = 0$, therefore, will get a windfall gain[21]. This gives the following result[22]:

Result 3.5 *In case there is full compensation for the elderly at the time of the reform and the debt tax is levied on the future workers, the capital-labour ratio will start decreasing in the period after the reform in country P. This in turn leads to capital flows from country F to country P.*

The intuition for this result is obvious. The first young generation under the reform consumes part of its gain in the first period, and saves part of it. As the gain this generation receives equals the created debt, the increase in savings at $t = 0$ is lower than the created debt. In other words, the public debt crowds out part of the capital stock.

Future working generations get the lower PAYG tax, but also pay a debt tax τ_t^B. Moreover, they inherit a lower capital-labour ratio, which leads to lower

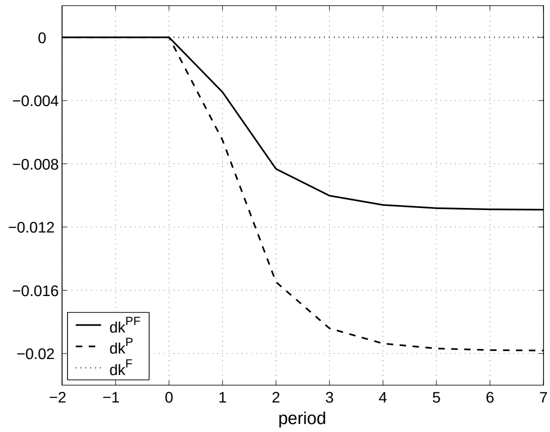
[20]More information and a formal proof of this result can be found in Appendix 3.B.1.

[21]The government in the PAYG country may decide to implement the pension reform in this way for political economy reasons. For example, when it is assumed, as in Cooley and Soares (1999), that a reform is only implementable in case it is welfare-improving for a majority of the current population.

[22]More details and a formal proof of this result are given in Appendix 3.B.2.

wages. Therefore savings are lower and, as a result, the capital-labour ratio continues to decline. Because country P can finance part of its government debt with savings of country F, the capital-labour ratio falls more when country P does not have an integrated capital market with country F. This can be seen in Figure 3.5. Notice that the results of this reform are exactly the opposite of the pension reform described in Section 3.3.1: the capital-labour ratio falls over time instead of rises. This implies that the effect on the other endogenous variables is also reversed. Actually, all simulation graphs are almost the mirror images of those of the pension reform in Section 3.3.1, the peaks are only one period later, because young individuals in the PAYG country do not adjust their behaviour at $t = -1$. The main findings are as follows (see also Figures 3.6 - 3.8)[23]:

Figure 3.5: Change in k_t



Result 3.6 *In case the elderly are fully compensated at the time of the reform and the tax to finance the debt is levied on the future workers:*

(a) The pension reform in country P leads to less consumption possibilities for the young in country F, while the elderly in this country gain.

(b) In the period after the reform both the elderly and the working people in the funded country gain from the pension reform in the PAYG country. In the long run, however, people in the funded country experience negative spillover effects.

[23]The analytical framework of Appendix 3.A.2 still applies, as all the expressions for the change in consumption and utility are expressed as a function of the capital-labour ratio.

Figure 3.6: Change in $c_t^{y,i}$

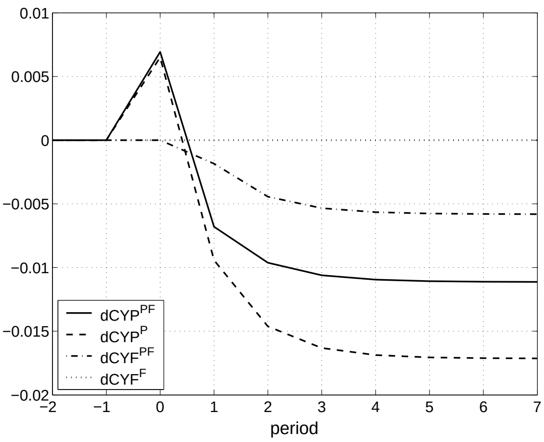


Figure 3.7: Change in $c_t^{o,i}$

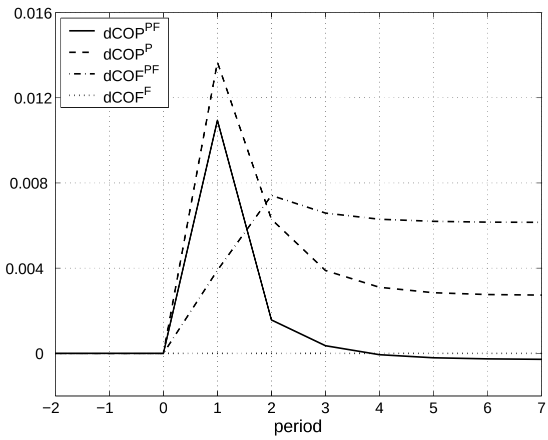
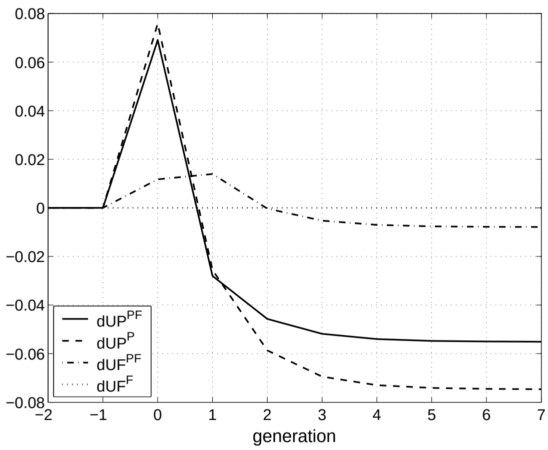


Figure 3.8: Change in U_t^i



So the fact that they have a common capital market with country F protects the generations living in country P to some extent, as part of the burden of this reform policy is transmitted to country F via the capital market[24].

[24]Of course the government in the PAYG country could decide to stabilise the debt and levy τ_t^B in a later period, so that even more generations get a windfall gain. Government debt would crowd out a larger part of the capital stock and the long-term losses would be larger. However, our central result that the funded country shares in the gains and losses of the reform in the PAYG country, would not change.

3.3.3 Concluding remarks

In general we conclude the following: In case the PAYG system is Pareto efficient and the government in the PAYG country implements a reform that leads to losses for at least one generation, these losses will be transmitted to the funded country via the capital market. It should be noted that the pension reforms analysed in this section are extreme, in the sense that no compensation at all or full compensation is granted at the time of the reform. Of course it is possible to have pension reforms where the elderly are *partly* compensated. However, such scenarios will not change the general conclusion from our analysis that a reform policy in one country unavoidably spills over to other countries.

3.4 Pareto-improving pension reform

In this section we analyse the international spillover effects of a pension reform in the PAYG country in case there is scope for a Pareto improvement, due to a distortionary PAYG tax. As explained in Section 3.2.2, we model the excess burden as a (quadratic) loss of tax revenue. This corresponds to $\xi = 1$ in equation (3.1), i.e., $z^P = \tau^P - (\tau^P)^2$, so $(\tau_t^P)^2$ is wasted. We start from the Pareto neutral pension reform scenario of the previous section. So the government compensates the elderly at the time of the reform completely, and then from $t = 1$ onwards imposes an extra tax on the pensioners to pay the interest obligations on the debt in order to keep debt per capita constant. The budget constraint of the government (eq. (3.4)) changes to:

$$b_{t+1}^P = (1 + r_t)b_t^P + z_t^P - (\tau_t^P - (\tau_t^P)^2) - \tau_t^B \tag{3.14}$$

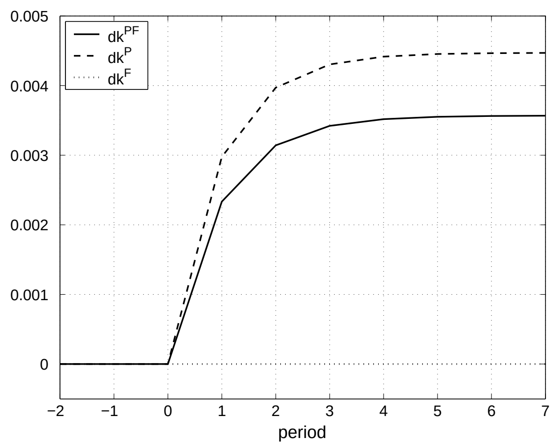
As argued earlier, we assume that the debt tax on the elderly, τ_t^B, does not imply an excess burden, given the labour-supply motivation for the excess burden. Now we can state the following[25]:

Result 3.7 *In case the PAYG tax implies an excess burden, a pension reform in the PAYG country where the elderly are compensated and the debt tax is levied on the pensioners leads to an increase in the capital-labour ratio in the period after the reform. A higher capital-labour ratio in turn leads to higher wages and savings, so that the capital-labour ratio continues to increase.*

[25] In Appendix 3.C this is formally derived.

We also show this in Figure 3.9. The intuition behind this result is that when PAYG taxes induce an excess burden, abolishing (part of) the PAYG system leads to efficiency gains, so that the capital-labour ratio actually rises, instead of staying constant (as was the case in Result 3.4). It is then obvious that this

Figure 3.9: Change in k_t



reform leads to a Pareto improvement in the PAYG country. This can indeed be seen in Figure 3.12, all generations get a higher utility. In the funded country[26], however, only consumption of the working people increases (see Figure 3.10). The elderly in the non-reforming country get less consumption possibilities (see Figure 3.11), mainly because the interest rate decreases after an increase in k_t. For the generation born at the time of the reform consumption when young does not change, while they can consume less at their old-age. This necessarily implies that this generation loses from the pension reform in the other country (see Figure 3.12). All later generations in country F gain from the pension reform in country P. So we have the following main result:

Result 3.8 *A Pareto-improving pension reform in country P does not necessarily lead to a Pareto improvement in the funded country: the generation born at the time of the reform is hurt by the pension reform in the PAYG country.*

[26]Appendix 3.A.2 shows the analytical framework for the change in consumption and utility.

Figure 3.10: Change in $c_t^{y,i}$

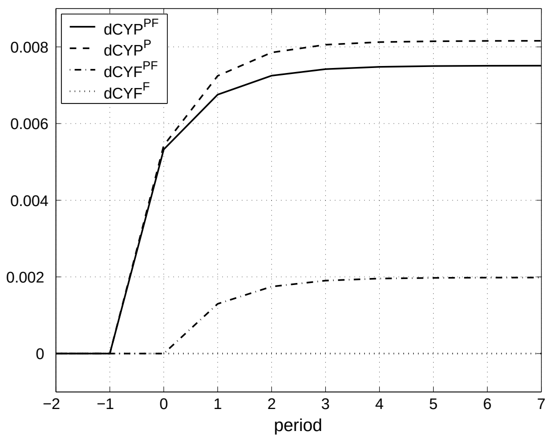


Figure 3.11: Change in $c_t^{o,i}$

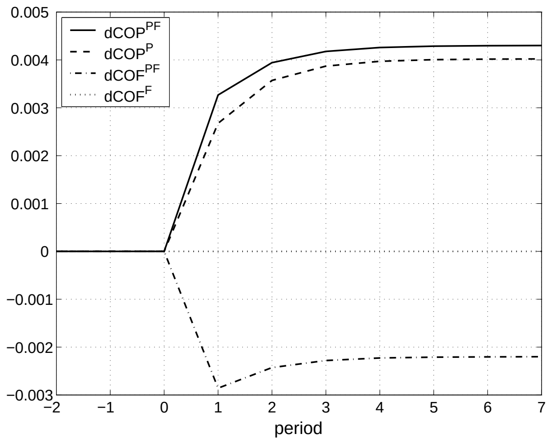
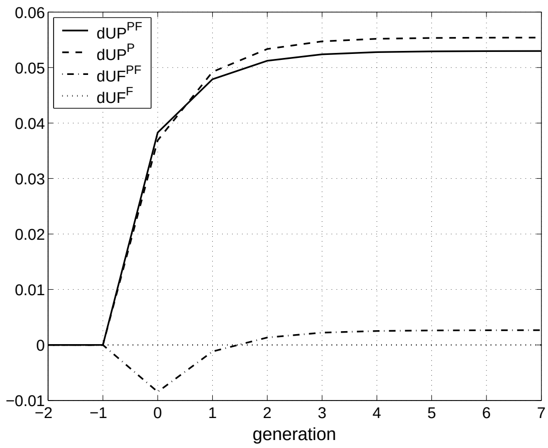


Figure 3.12: Change in U_t^i



3.5 Concluding remarks

Currently, in many countries with an extensive pay-as-you-go (PAYG) financed public pension system reforms are considered to finance a larger part of the future pension benefits by accumulated funds. A central conclusion emerging from this chapter is that in a common capital market the effects of such a pension reform in a PAYG country spills over to countries with a fully-funded pension system. In Europe, e.g., differences in financing methods of pension systems abound. For instance, countries like Germany and Italy have extensive PAYG-financed parts in their pension system, while in the UK and the Netherlands the larger part of the pension benefits are financed out of accumulated assets. The message of this chapter is that the latter countries cannot insulate themselves from the effects of reform measures in the former countries. In the European pension debate these spillover effects of pension reform have not been an issue till now, as far as we know. Yet, the consequences of pension reform in a PAYG country can be rather adverse for a funded country as we have shown in this chapter. Some key results illustrate this.

First, consider the case where the PAYG country compensates the elderly during the transition phase, and the PAYG system is Pareto efficient. The introduction of public debt, necessary to finance the compensation during the transition phase of the reform, will lead to crowding out of the capital stock in both the PAYG and the funded country, as soon as one generation is allowed to gain during the transition. As a result, future generations in both types of countries will lose under this reform policy.

Second, if a PAYG country reforms its public pension system such that its own future generations gain, then, although future generations in the funded country gain as well, the distribution of consumption between young and old individuals at a certain time will change at the expense of old individuals. In the funded country the elderly will even consume less in absolute amounts after the reform. Although the deterioration of old-age consumption is the result of free choice by individuals in the funded country, the resulting consumption allocation between young and old individuals might not be desirable from a societal point of view.

Third, we have shown that if excess burdens in tax collection enable a Pareto-improving pension reform in the PAYG country, during the transition phase some initial generations in the funded country might suffer a loss under the reform nevertheless. In other words, a reform policy that appears to be Pareto improving for the PAYG country considered separately, does not have to be Pareto improving after taking into account the international spillover effects.

Obviously, our model has oversimplified the real world in many ways, and the issue of spillover effects of pension reform merits further study in especially larger and more applied models. Yet, we think that our central result, i.e., that pension reform in PAYG countries can have adverse effects on the welfare of some generations, or some type of individuals in funded countries will remain to stand out in a more general model. The obvious policy conclusion from our model is, therefore, that in a common market like the EU, decisions on pension reform in the countries with a large PAYG scheme cannot be taken without considering the effects for non-reforming (funded) countries. In particular, dependent on the type and size of reform in the PAYG countries, compensation for the non-reforming countries might be necessary.

APPENDIX TO CHAPTER 3

3.A Appendix to Section 3.3.1

3.A.1 Derivation first-order difference equation k_{t+1}

In this appendix we derive the first-order difference equation for the evolution of the capital-labour ratio given in equation (3.12). Linearising the capital-accumulation equation (3.9) with respect to π around the initial steady state gives:

$$\frac{\partial s_t^P}{\partial \pi} + \nu \frac{\partial s_t^F}{\partial \pi} = (1+\nu) \frac{\partial k_{t+1}}{\partial \pi} \tag{3.15}$$

where we used the fact that $\frac{\partial b_{t+1}^P}{\partial \pi} = 0$ in case the current old are not compensated. Then we derive expressions for $\frac{\partial s_t^P}{\partial \pi}$ and $\frac{\partial s_t^F}{\partial \pi}$, using equations (3.6)-(3.8) and the fact that $\frac{\partial \tau_t^B}{\partial \pi} = 0$:

$$\frac{\partial s_t^P}{\partial \pi} = \frac{\lambda^P - 1}{\lambda^P} \left[\frac{\partial w_t}{\partial \pi} - \frac{\partial \tau_t^P}{\partial \pi} \right] - \frac{1}{\lambda^P} \left[\beta^P \frac{\partial r_{t+1}}{\partial \pi} + \frac{1}{1+r} \frac{\partial z_{t+1}^P}{\partial \pi} \right] \tag{3.16}$$

$$\frac{\partial s_t^F}{\partial \pi} = \frac{\lambda^F - 1}{\lambda^F} \frac{\partial w_t}{\partial \pi} - \frac{1}{\lambda^F} \beta^F \frac{\partial r_{t+1}}{\partial \pi} \tag{3.17}$$

where $\lambda^i \equiv 1 + \frac{1+\rho}{(1+r)^2} \frac{u''(c^{y,i})}{v''(c^{o,i})} > 1, i = F, P,$ $\beta^F \equiv \frac{(1-\sigma^F)c^{o,F}}{(1+r)^2}$ and $\beta^P \equiv \frac{(1-\sigma^P)c^{o,P} - z^P}{(1+r)^2}$.

$\frac{\partial w_t}{\partial \pi}$ and $\frac{\partial r_{t+1}}{\partial \pi}$ are given by:

$$\frac{\partial w_t}{\partial \pi} = -kf''(k)\,\frac{\partial k_t}{\partial \pi} \tag{3.18}$$

$$\frac{\partial r_{t+1}}{\partial \pi} = f''(k)\,\frac{\partial k_{t+1}}{\partial \pi} \tag{3.19}$$

Combining equations (3.15) - (3.19) and simplifying gives:

$$\begin{aligned}\frac{\partial k^{PF}_{t+1}}{\partial \pi} = & \frac{\left[\frac{1}{\lambda^P} - 1 + \nu\left(\frac{1}{\lambda^F} - 1\right)\right] kf''(k)}{\Delta}\,\frac{\partial k^{PF}_t}{\partial \pi} + \frac{\left(\frac{1}{\lambda^P} - 1\right)}{\Delta}\,\frac{\partial \tau^P_t}{\partial \pi} \\ & - \frac{1}{\lambda^P \Delta (1+r)}\,\frac{\partial z^P_{t+1}}{\partial \pi}\end{aligned} \tag{3.20}$$

with $\Delta \equiv 1 + \nu + f''(k)\left(\frac{\beta^P}{\lambda^P} + \nu\,\frac{\beta^F}{\lambda^F}\right)$. Using equations (3.2)-(3.3) and (3.10)-(3.11) we know that:

$$\frac{\partial \tau^P_t}{\partial \pi} = \tau^P g_t \tag{3.21}$$

$$\frac{\partial z^P_{t+1}}{\partial \pi} = \tau^P f_{t+1} \tag{3.22}$$

Substituting these two last expressions into equation (3.20) gives equation (3.12).

3.A.2 Derivation Results 3.2 and 3.3

In this appendix we derive analytical expressions for the change in consumption and utility in the funded country. From these equations we can infer the relationship between the change in the capital-labour ratio and consumption and utility. The framework also applies to the pension reform scenarios discussed in Sections 3.3.2 and 3.4.

Result 3.2a: Change in consumption when young

Linearising the first-period budget constraint gives:

$$\frac{\partial c^{y,F}_t}{\partial \pi} = \frac{1}{\lambda^F}\left[\beta^F f''(k)\frac{\partial k_{t+1}}{\partial \pi} - kf''(k)\frac{\partial k_t}{\partial \pi}\right] \tag{3.23}$$

The first term between the brackets shows the effect of the lower interest rate resulting for the higher capital-labour ratio, which affects consumption when young negatively in case $\sigma^F < 1$. The second term represents the positive wage effect. In the long run it holds that $\frac{\partial k_{t+1}}{\partial \pi} = \frac{\partial k_t}{\partial \pi} = \frac{\partial k_\infty}{\partial \pi}$ and we can write:

$$\frac{\partial c_\infty^{y,F}}{\partial \pi} = \frac{1}{\lambda^F}\left[\beta^F f''(k) - kf''(k)\right]\frac{\partial k_\infty}{\partial \pi} \tag{3.24}$$

which is positive as long as $\beta^F < k$, implying that σ^F should not be extremely low so that the positive wage effect dominates the negative interest rate effect in case $\sigma^F < 1$. Simulations show that for realistic parameter values $\beta^F < k$. In case of a logarithmic utility function $\sigma^F = 1$ and $\beta^F = 0$, so that only the wage effect occurs and young-age consumption always rises.

Result 3.2b: Change in old-age consumption

Linearising the second-period budget constraint gives:

$$\frac{\partial c_{t+1}^{o,F}}{\partial \pi} = (\lambda^F - 1)(1+r)\frac{\partial c_t^{y,F}}{\partial \pi} + \frac{c^{o,F} f''(k)}{(1+r)\,\sigma^F}\frac{\partial k_{t+1}}{\partial \pi} \tag{3.25}$$

Substituting this equation and equation (3.23) and noting that $\frac{\partial k_{t+1}}{\partial \pi} = \frac{\partial k_t}{\partial \pi} = \frac{\partial k_\infty}{\partial \pi}$ gives the long-run change of old-age consumption:

$$\frac{\partial c_\infty^{o,F}}{\partial \pi} = \left[\frac{(\lambda^F - 1)(1+r)}{\lambda^F}(\beta^F - k) + \frac{s^F}{\sigma^F}\right] f''(k)\,\frac{\partial k_\infty}{\partial \pi} \tag{3.26}$$

The first term between the brackets shows the positive effect of higher savings, the second term shows the negative effects of the lower interest rate. As $\frac{\partial k_\infty}{\partial \pi} > 0$ and $f''(k) < 0$, $\frac{\partial c_\infty^{o,F}}{\partial \pi} < 0$ if $\frac{(1+r)\sigma^F(\lambda^F - 1)(k-\beta^F)}{\lambda^F} < s^F$. Simulations show that this condition holds for realistic parameter values. Note that this condition reduces to $\frac{k(1+r)}{2+\rho} < s^F$ in case of logarithmic utility. In that case simulations show that the condition does not hold in case the production elasticity of capital is relatively large (≥ 0.5).

Result 3.3: Change in utility

Long-run change in utility

Linearisation of the lifetime utility function and substitution of the first-order condition gives:

$$\frac{\partial U_t^F}{\partial \pi} = u'(c^{y,F}) f''(k)\left[-k\frac{\partial k_t}{\partial \pi} + \frac{s^F}{1+r}\frac{\partial k_{t+1}}{\partial \pi}\right] \tag{3.27}$$

Using that in the long run $\frac{\partial k_{t+1}}{\partial \pi} = \frac{\partial k_t}{\partial \pi} = \frac{\partial k_\infty}{\partial \pi}$ it follows that

$$\frac{\partial U^F_\infty}{\partial \pi} = \frac{u'(c^{y,F})f''(k)}{1+r}\left[s^F - k(1+r)\right]\frac{\partial k_\infty}{\partial \pi} \tag{3.28}$$

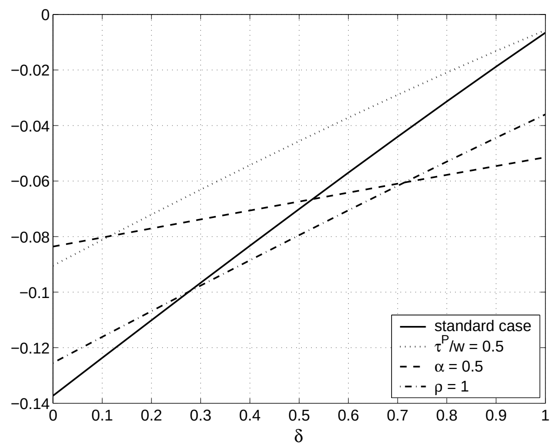
which is positive if $s^F < k(1+r)$. A rise in the capital-labour ratio has two opposing effects on the utility of individuals living in the funded country:

1. Country F is a lender in the international capital market, which means that $s^F > k(1+n)$. This implies that the lower interest rate that results for a higher capital-labour ratio hurts people living in the funded country.

2. A lower interest rate also implies, however, that the economy is closer to the Golden Rule point ($r = n$), which has positive utility effects in case of dynamic efficiency ($r > n$).

If $s^F > k(1+r)$, the first effect dominates the second, implying that an increase in the capital-labour ratio affects utility negatively. On the other hand, when $s^F < k(1+r)$, the second effect dominates the first and a higher capital-labour ratio leads to a higher utility in the funded country.

Figure 3.13: $s^F - k(1+r)$



In Figure 3.13 we present a graph that shows how the sign of $s^F - k(1+r)$ (vertical axis) changes for different parameter values[27]. The horizontal axis shows the depreciation rate (δ). This graph shows that $s^F - k(1+r) < 0$ for

[27]In the standard case, we have the following parameter values: $\rho = 0.4303$, $\frac{\tau^P}{w} = 0.2$, $\nu = 1, \sigma^i = 1, i = P, F$ and $f(k_t) = k_t^\alpha$ where $\alpha = 0.3$.

various values of the parameters, which means that the Golden Rule effect dominates the interest rate effect. This in turn implies that, for realistic parameter values, a rise in the capital-labour ratio has positive utility effects in the funded country[28].

Short-run change in utility

For $t = 0$ we can substitute the difference equation for the capital stock (equation (3.20)) in equation (3.27) and use the fact that $\frac{k_{-1}}{\partial \pi} = 0$ to find:

$$\frac{\partial U_0^F}{\partial \pi} = u'(c^{y,F})f''(k)\left[-k + \frac{s^F}{1+r}\left(1 + (\lambda^P - 1)(1+r) + \frac{\left[\frac{1}{\lambda^P} - 1 + \nu\left(\frac{1}{\lambda^F} - 1\right)\right]kf''(k)}{\Delta}\right)\right]\frac{\partial k_0}{\partial \pi} \quad (3.29)$$

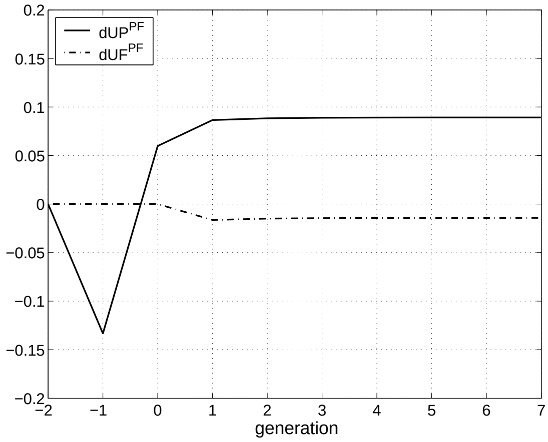
Utility of people living in the funded country born at the time of the reform ($t = 0$) falls as long as $s^F > \frac{(1+r)k}{\lambda^P(1+r) - r + \frac{\left[\frac{1}{\lambda^P} - 1 + \nu\left(\frac{1}{\lambda^F} - 1\right)\right]kf''(k)}{\Delta}}$ $(< (1+r)k)$. This condition holds as long as the production elasticity of capital is not too large (≤ 0.5).

Compensation first two generations

The government in the funded country can also decide to compensate the two generations that are negatively affected by the pension reform in the PAYG country (generations -1 and 0). This means that in those two periods government debt is created. Future generations will lose from this policy because they suffer from the higher debt burden and have to pay an extra tax to pay the interest obligations on the debt to keep the debt dynamics sustainable. This scenario is shown in Figure 3.14. This graph shows that the funded country experiences negative spillovers from a pension reform in the PAYG country in the long run if it compensates the generations that are initially harmed by the reform.

[28] It is actually possible to have $s^F - k(1+r) > 0$. This is the case, for example, when $\rho = 0$ *and* $\delta > 0.87$. A time preference rate of zero is not very realistic however. Therefore, we conclude that, for realistic parameter values, it holds that $s^F - k(1+r) < 0$.

Figure 3.14: Change in U_t^i



3.B Appendix to Section 3.3.2

3.B.1 Tax levied on the future old

With government debt, linearising the capital-accumulation equation (3.9) with respect to π around the initial steady state gives:

$$\frac{\partial s_t^P}{\partial \pi} + \nu \frac{\partial s_t^F}{\partial \pi} = (1+\nu)\,\frac{\partial k_{t+1}}{\partial \pi} + \frac{\partial b_{t+1}}{\partial \pi} \tag{3.30}$$

From this we can derive the following first-order difference equation for the change in the capital-labour ratio in case of different pension schemes:

$$\begin{aligned}\frac{\partial k_{t+1}^{PF}}{\partial \pi} = {} & \frac{\left[\frac{1}{\lambda^P} - 1 + \nu\left(\frac{1}{\lambda^F} - 1\right)\right] k f''(k)}{\Delta}\,\frac{\partial k_t^{PF}}{\partial \pi} + \left(\frac{1}{\lambda^P} - 1\right)\frac{\tau^P g_t}{\Delta} \\ & - \frac{1}{\lambda^P \Delta (1+r)}\left(\tau^P f_{t+1} - \frac{\partial \tau_{t+1}^B}{\partial \pi}\right) - \frac{1}{\Delta}\frac{\partial b_{t+1}^P}{\partial \pi}\end{aligned} \tag{3.31}$$

where Δ has the same definition as in Section 3.3.1. Comparing this equation with equation (3.12) we see that there are two extra terms, $\frac{\partial b_{t+1}^P}{\partial \pi}$ and $\frac{\partial \tau_{t+1}^B}{\partial \pi}$, because of the government debt created to compensate the old at $t = 0$. As can be seen debt has a direct negative impact on the change in the capital-labour ratio. However, because τ_t^B is levied on the pensioners, they will increase their savings when young which has a positive effect on the capital-labour ratio. Below we show formally that savings increase exactly by the same amount as the government debt, so that the capital-labour stays constant with this pension reform scenario:

Proof Result 3.4 At $t = 0$ we have that $\frac{\partial \tau_0^P}{\partial \pi} = \tau^P g_0$, $\frac{\partial z_0^P}{\partial \pi} = 0$. Then, using equation (3.4), we know that $\frac{\partial b_1^P}{\partial \pi} = -\frac{\partial \tau_0^P}{\partial \pi} = -\tau^P g_0$ and $\frac{\partial \tau_1^B}{\partial \pi} = r\frac{\partial b_1^P}{\partial \pi}$. So that we can write $\frac{\partial k_1^{PF}}{\partial \pi} = (\frac{1}{\lambda^P} - 1))\frac{\tau^P}{\Delta}\, g_0 - \frac{1}{\lambda^P \Delta(1+r)}(\tau^P f_1 + r\tau^P g_0) + \frac{\tau^P\, g_0}{\Delta}$ Noting that $f_1 = g_0$, it is easy to verify that $\frac{\partial k_1^{PF}}{\partial \pi} = 0$. And when this is the case for $t = 1$, this holds for all following periods. ∎

3.B.2 Tax levied on the future young

In this case the first-order difference equation for k_t is:

$$\begin{aligned} \frac{\partial k_{t+1}^{PF}}{\partial \pi} = & \frac{\left[\frac{1}{\lambda^P} - 1 + \nu(\frac{1}{\lambda^F} - 1)\right] kf''(k)}{\Delta} \frac{\partial k_t^{PF}}{\partial \pi} + \left(\frac{1}{\lambda^P} - 1\right) \frac{\tau^P g_t + \frac{\partial \tau_t^B}{\partial \pi}}{\Delta} \\ & - \frac{1}{\lambda^P \Delta(1+r)}\, \tau^P f_{t+1} - \frac{1}{\Delta}\frac{\partial b_{t+1}^P}{\partial \pi} \end{aligned} \tag{3.32}$$

As can be seen this equation looks almost the same as equation (3.31), the only difference is that now τ_t^B has a negative impact on the change in the capital-labour ratio.

Proof Result 3.5 The change in the capital-labour ratio at $t = 1$ is $\frac{\partial k_1^{PF}}{\partial \pi} = \frac{(\frac{1}{\lambda^P} - 1)\tau^P}{\Delta} g_0 - \frac{\tau^P}{\lambda^P \Delta(1+r)}\, f_1 + \frac{1}{\Delta}\, \tau^P g_0$. This equation can be simplified to $\frac{\partial k_1^{PF}}{\partial \pi} = \frac{\tau^P r}{\Delta \lambda^P (1+r)}\, g_0 < 0$. So we know that as soon τ_t^B is levied on the future young, the capital-labour ratio decreases at $t = 1$. ∎

3.C Appendix to Section 3.4

Proof Result 3.7 Using equation (3.14), we know that the change in government debt at $t = 1$ is equal to $\frac{\partial b_1^P}{\partial \pi} = -(1 - 2\tau^P)\frac{\partial \tau_0^P}{\partial \pi} = -(1 - 2\,\tau^P)\,\tau^P g_0$. Moreover, from $t = 1$ onwards we have that $\frac{\partial z_t^P}{\partial \pi} = (1 - 2\tau^P)\frac{\partial \tau_t^P}{\partial \pi}$, implying that $f_1 = (1 - 2\tau^P)g_0$, where we used the fact that $g_1 = g_0$. Then, noting that the capital-accumulation equation looks the same as equation (3.31) gives $\frac{\partial k_1^{PF}}{\partial \pi} = \left(\frac{1}{\lambda^P} - 1\right) \frac{\tau^P}{\Delta}\, g_0 - \frac{1}{\lambda^P \Delta(1+r)} \left[(1 - 2\tau^P)\,\tau^P g_0 + r(1 - 2\tau^P)\,\tau^P g_0\right] + \frac{1}{\Delta}(1 - 2\tau^P)\,\tau^P g_0$, which can be written as $\frac{\partial k_1^{PF}}{\partial \pi} = \frac{2(\tau^P)^2(1-\lambda^P)}{\lambda^P \Delta}\, g_0 > 0$. ∎

CHAPTER 4

PENSIONS, INFLATION RISK AND PORTFOLIO CHOICE

4.1 Introduction

This chapter connects the last chapter of this thesis with the previous two chapters. In the models used in Chapters 2 and 3 consumers did not face any uncertainty about the return on their savings, the models were completely deterministic. In this chapter we develop a stochastic two-period overlapping-generations general equilibrium model with safe pay-as-you-go (PAYG) pensions. People can invest in two types of assets, stocks and government bonds. Both assets are risky, productivity risk makes the return on stocks uncertain and there is inflation risk on government bonds. The fact that stocks and bonds differ in their riskiness makes that they earn a different rate of return, so that this model can be used to analyse the equity premium and portfolio choice. We can infer, for example, what happens with the equity premium in case the population ages and how people are affected when inflation risk goes up after large increases of government debt. This will be done in Chapter 5 where we extend the closed-economy model of this chapter to an open-economy version to analyse the spillover effects of population ageing in case the PAYG country uses government debt to cope with the costs of ageing, which may increase inflation (risk). The main purpose of this chapter is to gain insight into the workings of the model. We consider three shocks that all affect portfolio choice directly; more risk aversion, the introduction of a PAYG pension scheme and higher inflation risk. We concentrate on the effects on portfolio choice, savings and the equity premium.

The function of safe PAYG pensions in our stochastic model is twofold. First, it provides retirees with a certain minimum amount of income. Second, as people cannot invest in any risk-free asset, safe PAYG pensions partly remove this market incompleteness, as the system offers the possibility to transfer income from the working period to retirement at a non-stochastic rate of return. In that respect the PAYG pension scheme has a comparable role as in the literature that focuses on the intergenerational risk-sharing properties of pension schemes. The idea of this literature is that financial markets are incomplete because there cannot be trade with unborn generations and human capital is not traded. As a result of these missing markets the young are too much exposed to wage risk and the old bear too much financial market risk. In case financial market returns are imperfectly correlated with wages, this results in suboptimal diversification. By linking PAYG pension benefits to wages, retired households obtain a claim to human capital which is not traded on financial markets. In this way PAYG pension schemes can contribute to better intergenerational risk sharing and diversification. There are several studies that investigate how the optimal pension scheme should look like using stochastic OLG models. In contrast to this literature we do not consider optimal pension schemes, but we take the pension scheme as given. Moreover, we do not link pension benefits to wages because in our model wages are perfectly correlated with stock market returns and therefore wage-linked pension benefits will not improve diversification opportunities. PAYG pension benefits are modelled as safe lump-sum benefits and in this way pension benefits are imperfectly correlated with financial market returns and therefore contribute to better diversification. Our model, however, is comparable to the models used in these papers and therefore we discuss the main differences below.

Matsen and Thøgersen (2004) develop a partial equilibrium model where they treat the PAYG system as a 'quasi-asset' and derive the optimal share invested in this PAYG asset. They use the approach of Campbell and Viceira (2002) to obtain explicit expressions for the optimal portfolio shares. This approach will also be used in this chapter. They assume that people only consume in the second period of life, so that the complete net labour income received in the first period of life is saved. In contrast to Matsen and Thøgersen (2004) we model the savings decisions of individuals and more importantly, we develop a general equilibrium model where the effects on the rates of return are taken into account.

Most papers that analyse the risk-sharing aspects of a PAYG system in a general equilibrium framework, like Storesletten et al. (1999), Sánchez-Marcos and Sánchez-Martin (2006), and Krueger and Kubler (2006), use computable models. The major advantage of our general equilibrium model is that it is fairly simple so that analytical solutions are still feasible. This allows us to interpret the various shocks in a very intuitive way as we are able to derive analytical expressions for the time path of the various variables after a shock has occurred.

There are a few papers that develop an analytical stochastic general equilibrium OLG model. Bohn (1998, 2001, 2003) studies intergenerational risk sharing of demographic and macroeconomic risk in a stochastic OLG model using government debt or social security. He log-linearises the budget constraints and first-order conditions and in that way he derives a quasi-analytical condition for ex ante efficient intergenerational risk sharing. He does not, however, obtain explicit solutions for portfolio choice, savings and the risk premium as we do in our model. Beetsma and Bovenberg (2007) investigate intergenerational risk sharing in a two-pillar system with a pay-as-you-go pillar and a funded pillar. By keeping the model relatively simple they obtain analytical expressions for the optimal pension scheme. Their model only contains two periods with two generations that overlap during one period. In the first period the generation is born that is old in the second period, while in the second period the young generation is born. Our model, however, has an infinite horizon and can be used to derive the whole transition path of the predetermined variables to study the effects of various shocks. Moreover, Beetsma and Bovenberg (2007) also do not derive the optimal conditions for savings- and portfolio decisions of consumers.

In contrast to the above-mentioned papers, a recent paper by Campbell and Nosbusch (2007) does not consider the welfare implications of risk-sharing social security systems, but concentrates on the effects of these schemes on equilibrium asset prices. They use a Blanchard model where agents face a constant death probability in each period, while we use a two-period OLG model. They assume a fixed supply of physical capital which implies that the effects of a PAYG pension scheme on asset prices are quite extreme as the effects on capital accumulation are not taken into account. We *do* model production and capital accumulation so that the capital crowding-out effects of PAYG pensions are accounted for. Moreover, in our model individuals can invest in government bonds besides stocks, while in Campbell and Nosbusch (2007) physical capital is the only asset in positive net supply so that agents have to invest all their savings in the risky asset and there is no portfolio choice. Finally, they have to rely on numerical simulations to infer the effect of social security on the risk premium, whereas our model gives unambiguous results.

We analyse three shocks that affect the portfolio decision of households directly and examine the effects of these shocks on portfolio choice, savings and the risk premium of stocks over bonds. First, we address the effects of more risk aversion. The main reason why this shock is considered is to being able to contrast the results with the effects of introducing safe PAYG pensions, which is the second analysed shock. As explained, the fact that there is no risk-free asset in our economy implies that markets are incomplete. Safe PAYG pensions will partly fill the gap of this missing market and will therefore affect the investment decisions of consumers. Comparing the effects of introducing PAYG pensions to the effects of higher risk aversion shows that consumers behave like less risk averse individuals when they receive a safe PAYG pension benefit. Finally, we also consider the effects of an increase in inflation risk to address the impact of uncertainty on consumer behaviour and asset returns.

To contrast the results in a general equilibrium model with the partial equilibrium approach typically taken in the finance literature, we first consider the effects on portfolio- and savings decisions in case the rates of return are taken as exogenous. It turns out that the effects on portfolio choice in general equilibrium are opposite to the effects in partial equilibrium. The intuition for this result is that in general equilibrium portfolio choice has to be such that the capital market clears. The rate of return on government bonds adjusts in such a way that enough savings are invested in government bonds to make sure that the government debt is financed. In partial equilibrium, financing the government debt is not a problem as the government can supply the bonds on the international capital market, without effect on the rate of return that has to be granted to those who invest in the bonds. This result implies that it matters whether you study portfolio choice in a small open economy or in a (large) closed economy, like the US or the EU. Moreover, this result is also relevant in case one would like to analyse the effects of 'global' shocks, like population ageing.

In the model of this chapter government debt and the PAYG pension system differ in their degree of risk. Government debt is denominated in nominal terms and therefore contains inflation risk, while the PAYG pension scheme is totally risk-free. This assumption may look strange as a PAYG system is often seen as implicit debt and it is not obvious a priori why PAYG pension benefits will not be guaranteed in nominal terms and therefore also contain inflation risk. We make this assumption, however, to incorporate the fact that a PAYG pension scheme also serves as a risk-sharing and diversification device. Government debt, on the other hand, provides an extra investment opportunity for people besides the investment in physical capital. We also discuss whether the results would change if we relax this assumption about the difference in riskiness between government bonds and PAYG pension benefits and contend that this will not qualitatively change our main results.

The chapter is organised as follows. First, we present a stochastic version of the standard two-period overlapping-generations model with government debt and PAYG pensions. We use the method of Campbell and Viceira (2002) to derive an explicit solution for portfolio choice. This model is used to analyse the effects of risk aversion, PAYG pensions, and inflation risk on portfolio choice and savings, both in partial equilibrium (Section 4.3) and general equilibrium (Section 4.4). In general equilibrium we also obtain the effects on the excess return. Section 4.5 discusses whether the results would change if we relax the assumption that government debt and PAYG pensions differ in their riskiness. The main findings of the chapter are summarised in Section 4.6.

4.2 Model

In order to analyse the effects of PAYG pensions and inflation risk on portfolio choice and the excess return we develop a two-period overlapping-generations (OLG) model of a closed economy with a PAYG pension scheme and government debt. We use the framework of Campbell and Viceira (2002) to derive an explicit solution for the portfolio choice of consumers. All variables in the model are expressed as the amount per young individual in the country and small letters refer to the logarithm of the respective variable.

4.2.1 Risk factors

There are two risk factors, there is productivity risk on stocks and inflation risk on government bonds.

Production

Production per worker is described by a standard neoclassical constant-returns-to-scale Cobb-Douglas production function:

$$F(A_t, K_t) = A_t K_t^{\alpha} \tag{4.1}$$

where A_t denotes productivity at time t, α the production elasticity, and K_t the amount of capital per young individual at time t. Profit maximisation and perfect competition among producers results in the usual equilibrium conditions:

$$W_t = (1-\alpha) A_t K_t^{\alpha} \tag{4.2}$$

$$R_{k,t} + \delta = \alpha A_t K_t^{\alpha-1} \tag{4.3}$$

where W_t is the real wage, $R_{k,t}$ the return to capital and δ the depreciation rate of capital. We adopt the Campbell and Viceira (2002) approach and assume that the gross return on capital $(1 + R_{k,t})$ is lognormal distributed[1]. To achieve this we have to assume that A_t is lognormal distributed and that there is 100% depreciation, i.e., $\delta = 1$. This implies that both W_t and $R_{k,t}$ are stochastic. People do not have to form expectations about W_t, however, as A_t is already known before W_t is paid (see Figure 4.1). People base their saving- and portfolio decisions on the future return of capital, so they *do* have to form expectations about this variable. In Appendix 4.A.1 we derive that the variance of the log of the gross return on capital is equal to the variance of the log of productivity, i.e.,

$$\mathrm{Var}_t[\log(1 + R_{k,t+1})] = \mathrm{Var}_t[\log A_{t+1}] \tag{4.4}$$

$$\sigma_{k,t}^2 = \sigma_{a,t}^2 \tag{4.5}$$

[1] In case a variable X is lognormally distributed, this means that:

$$\log X \sim N\left(E(\log X), \sigma_x^2\right)$$

The expectation and variance of X are equal to:

$$E(X) = \exp[E(\log X) + \frac{1}{2}\sigma_x^2]$$

$$\sigma_X^2 = \exp[2E(\log X) + \sigma_x^2][\exp(\sigma_x^2) - 1]$$

Inflation

Government debt is denominated in nominal terms and therefore there is inflation risk on the return on government bonds. The real return on government bonds is equal to:

$$1 + R_{b,t} = \frac{1 + R_{b,t}^{N}}{1 + \pi_t} \tag{4.6}$$

where $R_{b,t}^{N}$ is the nominal return on government bonds and π_t the inflation rate between $t-1$ and t. The nominal return $R_{b,t}^{N}$ is a predetermined variable and we assume that the inflation factor $(\frac{1}{1+\pi_t})$ is lognormally distributed. In Appendix 4.A.2 we derive that:

$$\mathrm{Var}_t\left[\log(1 + R_{b,t+1})\right] = \mathrm{Var}_t\left[\log\left(\frac{1}{1+\pi_{t+1}}\right)\right] \tag{4.7}$$

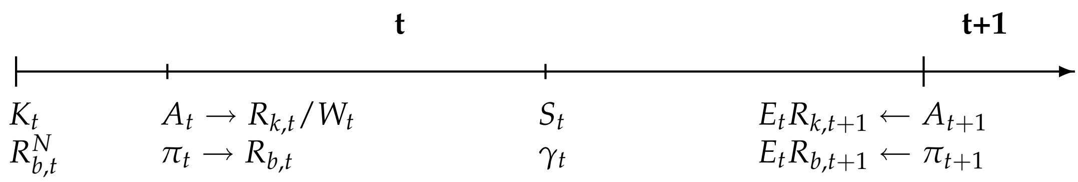
$$\sigma_{b,t}^2 = \sigma_{\pi,t}^2 \tag{4.8}$$

The risk on government bonds could also be interpreted as default risk. There are no risk-free indexed bonds, which implies that there is a missing market. Safe income in the second period of life, like PAYG pensions or defined-benefit funded pensions, would (partly) fill this gap.

4.2.2 Timing

The sequence of events is shown in Figure 4.1. At the beginning of period t, the capital stock K_t and the nominal interest rate on government bonds $R_{b,t}^{N}$ are inherited from the last period, as they are determined by the savings and portfolio decisions made in the previous period. Then, productivity and inflation are revealed. With this knowledge firms choose factor employment and the real return on government bonds is determined. Subsequently, households make their portfolio choice γ_t and saving decisions S_t (and thereby consumption decisions), which are also based on the expected future asset returns. This implies that consumers only face uncertainty about the return on their savings.

Figure 4.1: Timing of events



4.2.3 Consumers

Expected lifetime utility of a representative individual born at t is given by the following constant-relative-risk-aversion (CRRA) utility function:

$$E_t U(C_t^Y, C_{t+1}^O) = \frac{\left(C_t^Y\right)^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\,\varepsilon_{t+1} E_t \left[\frac{\left(C_{t+1}^O\right)^{1-\theta} - 1}{1-\theta} \right] \tag{4.9}$$

where C_t^Y is consumption when young at time t, C_{t+1}^O is consumption in the second period of life, θ is the coefficient of relative risk aversion, $\rho > 0$ stands for the rate of time preference and ε_{t+1} is the probability that an agent born at t lives throughout old age. Only consumption in the second period of life is uncertain because the rates of return depend on the realisations of A_{t+1} and π_{t+1}, which are unknown at t when decisions about savings S_t are made (see Figure 4.1).

People can either invest in firm stocks which yield the stochastic return $R_{k,t+1}$ or in government bonds with the stochastic return $R_{b,t+1}$[2]. The part of savings that is invested in equities is denoted by γ_t, so that the return on the portfolio can be defined as:

$$R_{p,t+1} \equiv \gamma_t R_{k,t+1} + (1-\gamma_t) R_{b,t+1} \tag{4.10}$$

Young agents inelastically supply one unit of labour. We assume perfect annuity markets, which implies that the assets of those who deceased are distributed among the people who survived. The total return on savings is therefore $\frac{1+R_{p,t+1}}{\varepsilon_{t+1}}$. The consolidated lifetime budget constraint is:

$$W_t - T_t^P - T_t^B - C_t^Y = \frac{\varepsilon_{t+1}}{1+R_{p,t+1}} \left(C_{t+1}^O - Z_{t+1}^P \right) \tag{4.11}$$

where T_t^P are lump-sum PAYG pension contributions and T_t^B are taxes to finance the interest obligations on the government debt, so as to keep debt per capita constant. Individuals receive a pension benefit Z_{t+1}^P when old[3]. Maximising lifetime utility with respect to the lifetime budget constraint gives the

[2]We assume that individuals or firms do not issue bonds. In a two-period OLG model individuals can only issue bonds if agents are heterogenous. Moreover, if firms can issue bonds besides stocks the decision-making process about the financing structure of firms has to be modelled. Both extensions will complicate the analysis to a large extent and therefore we leave these for future research.

[3]Now we assumed a lump-sum PAYG scheme, but as soon as the PAYG pension benefits are linked to (past or current) wages, Z_{t+1}^P becomes stochastic. This is important to consider in

Euler condition:

$$1 = \frac{1}{1+\rho} \left(C_t^Y\right)^\theta E_t \left[\left(C_{t+1}^O\right)^{-\theta} (1 + R_{j,t+1}) \right] \tag{4.12}$$

where $j = p, k, b$. To derive an explicit solution for the portfolio choice γ_t, we follow the approach of Hansen and Singleton (1983) and Campbell and Viceira (2002) and assume that the joint distribution of consumption and returns is lognormal. In Appendix 4.B.1 we show the derivation of the optimal solution for portfolio choice, which is given by:

$$\gamma_t = \frac{\log E_t(1 + R_{k,t+1}) - \log E_t(1 + R_{b,t+1})}{\theta(1 - z_t)\sigma_{k-b,t}^2} - \frac{\sigma_{k-b,bt}}{\sigma_{k-b,t}^2} \tag{4.13}$$

where $\sigma_{k-b,t}^2$ is the variance of the excess return of stocks over bonds, $\sigma_{k-b,bt}$ is the covariance between the excess return and the return on bonds, and z_t is the part of expected old-age consumption financed by PAYG pensions:

$$z_t = \frac{Z_{t+1}^P}{\frac{E_t(1+R_{p,t+1}) \exp(-\frac{1}{2}\sigma_{pt}^2)}{\varepsilon_{t+1}} S_t^P + Z_{t+1}^P} \tag{4.14}$$

Appendix 4.B.2 derives the optimal conditions for consumption when young and savings in the general case. For later reference we show here the optimal conditions for savings in three special cases. First, consider the case where there is no PAYG scheme, i.e., $z = T^P = Z^P = 0$, we refer to this as the funded case. This means that we assume that people treat funded pension savings the same as their private savings and the pension scheme does not affect the equilibrium. In that case the optimal solution for savings is:

$$S_t^F = \frac{\exp[\frac{1}{2}(\theta - 1)\sigma_{pt}^2] \left[E_t(1 + R_{p,t+1})\right]^{\frac{1}{\theta}-1} \varepsilon_{t+1}}{\exp[\frac{1}{2}(\theta - 1)\sigma_{pt}^2] \left[E_t(1 + R_{p,t+1})\right]^{\frac{1}{\theta}-1} \varepsilon_{t+1} + (1+\rho)^{\frac{1}{\theta}}} (W_t - T_t^{B,F}) \tag{4.15}$$

It is also useful to show the optimal conditions in case $\theta = 1$, that is, when the

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case one wants to analyse the risk-sharing properties of PAYG pension schemes. Wage-linked PAYG pension benefits will only result in better diversification when wages are imperfectly correlated with stock market returns. This is not the case in our model, however, as productivity risk affects both wages and the rate of return on capital. To allow for imperfect correlation between labour and capital income an often-used trick is to assume that there is depreciation risk besides productivity risk. We leave this for future research as this would complicate the analysis to a large extent.

utility function is logarithmic:

$$\begin{aligned} S_t^P\Big|_{\theta=1} &= \frac{\exp\left(\frac{1}{2}z_t^2\sigma_{pt}^2\right)\varepsilon_{t+1}}{\exp\left(\frac{1}{2}z_t^2\sigma_{pt}^2\right)\varepsilon_{t+1}+1+\rho}\left[W_t - T_t^P - T_t^B\right] - \\ & \quad \frac{1+\rho}{\exp\left(\frac{1}{2}z_t^2\sigma_{pt}^2\right)\varepsilon_{t+1}+1+\rho}\,\frac{\varepsilon_{t+1}Z_{t+1}^P}{E_t(1+R_{p,t+1})\exp(-\frac{1}{2}\sigma_{pt}^2)} \qquad (4.16) \\ S_t^F\Big|_{\theta=1} &= \frac{\varepsilon_{t+1}}{1+\rho+\varepsilon_{t+1}}\left(W_t - T_t^{B,F}\right) \qquad (4.17) \end{aligned}$$

Notice that as soon people have a logarithmic utility function and there are no PAYG pensions, they do not react to changes in uncertainty or rates of return (see equation (4.17)).

4.2.4 Government

The government budget constraint is given by:

$$(1+n_{t+1})B_{t+1} = (1+R_{b,t})B_t + \frac{\varepsilon_t}{1+n_t}Z_t^P - T_t^P - T_t^B \qquad (4.18)$$

where public debt per young individual at time t is denoted by B_t and n_t is the population growth rate between $t-1$ and t. The government can accumulate debt for a certain period of time, but on a given moment it has to raise additional contributions so as to keep debt per worker constant. These taxes are denoted by T_t^B and in case of a balanced PAYG system, i.e., $Z_t^P = \frac{1+n_t}{\varepsilon_t}T_t^P$, they have to be equal to:

$$T_t^B = (R_{b,t} - n_{t+1})\,B \qquad (4.19)$$

Note that $R_{b,t}$ is known at time t as both $R_{b,t}^N$ and π_t are already known at the beginning of period t (see Figure 4.1).

4.2.5 Capital market equilibrium

The capital market is in equilibrium when savings at time t finance the capital stock and the government debt in the next period:

$$S_t = (1+n_{t+1})\,(B_{t+1}+K_{t+1}) \qquad (4.20)$$

Moreover, portfolio choice has to be such that the right part of savings goes to the capital stock and the government debt:

$$\begin{aligned} \gamma_t S_t &= (1+n_{t+1})K_{t+1} \qquad (4.21) \\ (1-\gamma_t)S_t &= (1+n_{t+1})B_{t+1} \qquad (4.22) \end{aligned}$$

As soon equation (4.20) and equation (4.21) (equation (4.22)) hold, equation (4.22) (equation (4.21)) also holds. This implies that there are two equilibrium conditions and K_{t+1} and $R^N_{b,t+1}$ adjust to make sure that these equilibrium conditions hold. In Appendix 4.C we discuss the stability of the model.

4.3 Partial equilibrium

We consider three shocks that affect the portfolio decision of households directly; more risk aversion, the introduction of a PAYG pension scheme and higher inflation risk. Our model can be used to analyse these shocks in a general equilibrium framework. First, however, we consider what happens if we take a partial equilibrium approach, the approach normally taken in the finance literature. This is done in order to gain insight into the differences between the two approaches. In partial equilibrium one only considers the effects on individual choices and does not take into account that these individual choices together may have macroeconomic consequences and change the rates of return. In other words, in partial equilibrium the capital stock and the rates of return are exogenous and fixed.

We use the method of comparative dynamics, adopted from Judd (1982), to calculate the effects of various shocks analytically. The process of the variable that changes (x_t) is given by:

$$x_t = x + \zeta h^x_t \qquad (4.23)$$

where x is the steady state value of x_t, h^x_t is the time pattern of a perturbation of this steady-state value and ζ is the magnitude of this perturbation. By taking the derivative with respect to ζ, we derive the changes of the various variables. A rise in variable x_t in reflected by $h^x_t > 0$.

4.3.1 Risk aversion

In this subsection we address the effects of higher risk aversion by analysing the move from a utility function where $\theta = 1$ to a utility function where $\theta > 1$. We mainly do this to compare it to the effects of the second shock, the effects of introducing safe PAYG pensions, which shows that PAYG pensions make people less risk averse. Moreover, the effect of higher risk-aversion on savings are not very straightforward. We obtain a simple expression for the change in savings where the various effects can easily be distinguished. To focus solely on the effects of higher risk aversion, we assume that there is no PAYG scheme in place, that is, $z = T^P = Z^P = 0$. The time pattern of this shock is:

$$\theta_t = 1 + \zeta h_t^\theta \tag{4.24}$$

where $h_t^\theta > 0$.

Taking the derivative of equation (4.13) with respect to ζ gives:

$$\frac{\partial \gamma_t}{\partial \zeta} = \frac{-[\log E(1+R_k) - \log E(1+R_b)]}{\sigma_{k-b}^2} h_t^\theta < 0 \tag{4.25}$$

which shows that people invest a smaller part of their savings in stocks when they become more risk averse. The fall in γ_t results in a lower variance and expected return on the portfolio:

$$\frac{\partial \sigma_{pt}^2}{\partial \zeta} = (2\gamma\sigma_{k-b}^2 + 2\sigma_{k-b,b}) \frac{\partial \gamma_t}{\partial \zeta} \tag{4.26}$$

$$\frac{E_t R_{p,t+1}}{\partial \zeta} = [E(R_k) - E(R_b)] \frac{\partial \gamma_t}{\partial \zeta} \tag{4.27}$$

where the term between the brackets in equation (4.26) is positive as long as the hedge term ($\sigma_{k-b,b}$) is not too negative.

Since θ represents both the coefficient of relative risk aversion and the inverse of the intertemporal elasticity of substitution, the effect of a higher θ on savings is ambiguous:

$$\frac{\partial S_t}{\partial \zeta} = \frac{\varepsilon(1+\rho)\left[\frac{1}{2}\sigma_p^2 + \log\left(\frac{1+\rho}{E(1+R_p)}\right)\right]}{(1+\rho+\varepsilon)^2} (W - T^B)\, h_t^\theta \tag{4.28}$$

Savings are affected in two ways when there is uncertainty (see also Sandmo (1970) and Rothschild and Stiglitz (1971)). First, higher riskiness makes it necessary to save more in order to protect oneself against very low levels of future consumption, this is the income effect. Secondly, there is a substitution effect; an increase in the degree of risk makes the consumer less inclined to expose his/her resources to the possibility of loss, so that savings fall. Or in the words of Rothschild and Stiglitz (1971): "Increased uncertainty in the return on savings will lower savings because 'a bird in the hand is worth two in the bush'" (p. 69). In case $\theta > 1$, the income effect dominates the substitution effect, so that uncertainty leads to higher savings. This is the precautionary savings motive and is represented by the first term between the brackets: for a given level of uncertainty σ_p^2 a rise of the coefficient of relative risk aversion from the point $\theta = 1$ increases savings.

The second term between the brackets shows the consumption smoothing motive. A higher θ also implies a lower intertemporal substitution elasticity, which means that individuals find it more important to smooth consumption. The sign of this effect depends on the relative size of the rate of time preference ρ compared to the expected portfolio return $E(R_p)$. In case $\rho > E(R_p)$, the rate of time preference is relatively high and consumption when young will be relatively high. But when θ increases this means that an individual would like to smooth consumption more, which implies that the consumer increases his/her savings, so that consumption when young and old-age consumption will be closer to each other. In that case the consumption smoothing effect of a higher θ reinforces the precautionary savings effect, so that savings unambiguously rise when θ rises. In case $\rho < E(R_p)$, however, the consumption smoothing effect is opposite to the precautionary savings effect, the reason being that consumption when young will be relatively low as the rate of time preference is relatively low, which implies that savings fall when θ rises. We assume that savings rise when a consumer becomes more risk averse, which is the case when $\rho > E(R_p)$[4] or when the precautionary savings effect dominates the consumption smoothing effect in case $\rho < E(R_p)$.

4.3.2 PAYG pensions

In this subsection we analyse the effects of introducing safe PAYG pensions and take $\theta = 1$. It turns out that in that case consumers act in the exact opposite way as a consumer that becomes more risk averse. Safe PAYG pensions make people less risk averse and they act as someone with $0 < \theta < 1$ who does not get a PAYG pension. We assume that there is no PAYG system in the initial steady state, i.e., $T^P = Z^P = z = 0$, and there is a marginal increase in the PAYG pension benefit:

$$Z_{t+1}^P = \zeta h_{t+1}^z \tag{4.29}$$

where $h_{t+1}^z > 0$.

[4] This assumption is supported by the fact that the estimates for the rate of time preference are normally quite high, see Frederick et al. (2002) for an overview of various studies.

This increases the part of old-age consumption financed by PAYG pensions:

$$\frac{\partial z_t}{\partial \zeta} = \frac{E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)S/\varepsilon}{\left[E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)S/\varepsilon + Z^P\right]^2} h_{t+1}^z > 0 \tag{4.30}$$

A rise of z_t induces people to invest a larger part of their savings in stocks. This can be seen by taking the derivative of equation (4.13) with respect to ζ:

$$\frac{\partial \gamma_t}{\partial \zeta} = \frac{\log E(1+R_k) - \log E(1+R_b)}{\sigma_{k-b}^2} \frac{\partial z_t}{\partial \zeta} > 0 \tag{4.31}$$

The variance of the log portfolio return σ_{pt}^2 and the expected return on the portfolio $E_t R_{p,t+1}$ rise because γ_t goes up (see equations (4.26) and (4.27) in the previous subsection). This result shows that for a marginal increase of the PAYG pension benefit, where there was no PAYG system in the initial steady state, people will take more risk and increase their relative holdings of stocks. We cannot prove this for the general case when there already is a PAYG scheme in place, the reason being that in that case we have feedback effects from σ_{pt}^2, $E_t R_{p,t+1}$, and S_t to z_t, see equation (4.14). Simulations show[5], however, that the direct effect via Z_{t+1}^P dominates, and that the other effects are of a second-order nature. Moreover, only the effect via $E_t R_{p,t+1}$ weakens the direct effect of higher PAYG pension benefit. A higher σ_{pt}^2 and lower savings (see below) reinforce the initial effect on z_t.

Actually what happens is that a safe PAYG pension benefit gives retired people a source of income that is non-stochastic, so that the variance of old-age consumption is reduced. This can be seen very clearly from equation (4.77) in Appendix 4.B.1. This induces people to reoptimise their portfolio when PAYG benefits increase.

The PAYG pension scheme lowers savings, that is, when people know they will receive a safe PAYG pension benefit when they are retired and have to pay PAYG contributions when they are young, they save less:

$$\frac{\partial S_t}{\partial \zeta} = \frac{-\varepsilon}{1+\rho+\varepsilon}\left[\frac{\varepsilon}{1+n} h_t^z + \frac{1+\rho}{E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)} h_{t+1}^z\right] < 0 \tag{4.32}$$

4.3.3 Inflation risk

The last shock we analyse that directly affects the portfolio decision of consumers is an increase in the riskiness of government bonds, i.e, inflation risk

[5]See Section 4.3.4 for more details.

goes up. This shows how people react to more uncertainty. A rise in inflation risk can be modelled as follows:

$$\sigma_{\pi t}^2 = \sigma_\pi^2 + \zeta h_t^{\sigma_\pi} \tag{4.33}$$

where $h_t^{\sigma_\pi} > 0$.

In case inflation risk rises this implies that the risk on bonds increases and people rebalance their portfolio and invest more in stocks:

$$\frac{\partial \gamma_t}{\partial \zeta} = \frac{1-\gamma}{\sigma_{k-b}^2} h_t^{\sigma_\pi} > 0 \tag{4.34}$$

Taking the derivative of equation (4.69) in Appendix 4.B.1 with respect to ζ and substituting equation (4.34) gives:

$$\frac{\partial \sigma_{pt}^2}{\partial \zeta} = \Bigg[\underbrace{1-\gamma^2}_{>0} + \underbrace{\frac{2\sigma_{k-b,b}(1-\gamma)}{\sigma_{k-b}^2}}_{<0 \text{ when } \sigma_{k-b,b}<0}\Bigg] h_t^{\sigma_\pi} \tag{4.35}$$

A rise in inflation risk affects portfolio risk in three ways. First, a rise in inflation risk increases the risk on the portfolio directly. Second, as people move to the more risky asset the risk on the portfolio increases even more. Third, when inflation risk rises, the hedge term ($\sigma_{k-b,bt} = \sigma_{kb,t} - \sigma_{bt}^2$) falls, which decreases portfolio risk. As long as the third effect is not too strong, the riskiness of the portfolio σ_{pt}^2 will rise after a rise in the variance of the log benchmark return, not only because this increases portfolio risk directly, but also because people invest a larger part of their savings in stocks. The rise in γ_t also increases the expected return on the portfolio (see equation (4.27)).

For the effect on savings we distinguish three cases. In the first case we analyse the effects when there is no PAYG scheme (the funded case) and the coefficient of relative risk aversion is equal to one. In the second case there is still no PAYG scheme, but $\theta > 1$ and the third case analyses the effects when people have PAYG pension benefits, but $\theta = 1$.

Equation (4.17) shows that people do not react to changes in risk or return when there is no PAYG scheme and $\theta = 1$, so that:

$$\left.\frac{\partial S_t^F}{\partial \zeta}\right|_{\theta=1} = 0 \tag{4.36}$$

In case there are no PAYG pensions, but $\theta > 1$, we have to take the derivative of equation (4.15):

$$\frac{\partial S_t^F}{\partial \zeta} = \frac{\frac{1}{2}(\theta-1)(W-T^{B,F})(1+\rho)^{\frac{1}{\theta}}\exp[\frac{1}{2}(\theta-1)\sigma_p^2][E(1+R_p)]^{\frac{1}{\theta}-1}\varepsilon}{\left[\exp[\frac{1}{2}(\theta-1)\sigma_p^2][E(1+R_p)]^{\frac{1}{\theta}-1}\varepsilon+(1+\rho)^{\frac{1}{\theta}}\right]^2}\frac{\partial \sigma_{pt}^2}{\partial \zeta} - \frac{(1-\frac{1}{\theta})(W-T^{B,F})(1+\rho)^{\frac{1}{\theta}}\exp[\frac{1}{2}(\theta-1)\sigma_p^2][E(1+R_p)]^{\frac{1}{\theta}-2}\varepsilon}{\left[\exp[\frac{1}{2}(\theta-1)\sigma_p^2][E(1+R_p)]^{\frac{1}{\theta}-1}\varepsilon+(1+\rho)^{\frac{1}{\theta}}\right]^2}\frac{\partial E_t R_{p,t+1}}{\partial \zeta} \tag{4.37}$$

Savings are affected in two ways when inflation risk rises. On the one hand there is an upward pressure on savings because portfolio risk rises (see equation (4.35)). People with $\theta > 1$ will have more precautionary savings when the risk on the portfolio increases. On the other hand, savings fall because the expected return on the portfolio rises. To know which effect dominates we rewrite equation (4.15) to:

$$S_t^F = \frac{\exp[\frac{1}{2}(1-\theta)(\gamma_t^2\sigma_{k-b,t}^2-\sigma_{bt}^2)][E_t(1+R_{b,t+1})]^{\frac{1}{\theta}-1}\varepsilon_{t+1}}{\exp[\frac{1}{2}(1-\theta)(\gamma_t^2\sigma_{k-b,t}^2-\sigma_{bt}^2)][E_t(1+R_{b,t+1})]^{\frac{1}{\theta}-1}\varepsilon_{t+1}+(1+\rho)^{\frac{1}{\theta}}}(W_t-T_t^{B,F}) \tag{4.38}$$

Taking the derivative of this equation with respect to ζ gives:

$$\frac{\partial S_t^F}{\partial \zeta} = \frac{\frac{1}{2}(\theta-1)(1-\gamma)^2(W-T^{B,F})(1+\rho)^{\frac{1}{\theta}}\varepsilon}{\left[\exp[\frac{1}{2}(1-\theta)(\gamma^2\sigma_{k-b}^2-\sigma_b^2)][E(1+R_b)]^{\frac{1}{\theta}-1}\quad \varepsilon+(1+\rho)^{\frac{1}{\theta}}\right]^2}\cdot \frac{\exp[\frac{1}{2}(1-\theta)(\gamma^2\sigma_{k-b}^2-\sigma_b^2)][E(1+R_b)]^{\frac{1}{\theta}-1}}{\left[\exp[\frac{1}{2}(1-\theta)(\gamma^2\sigma_{k-b}^2-\sigma_b^2)][E(1+R_b)]^{\frac{1}{\theta}-1}\varepsilon+(1+\rho)^{\frac{1}{\theta}}\right]^2} h_t^{\sigma\pi} \tag{4.39}$$

This expression is positive in case $\theta > 1$, so that the first effect $\left(\frac{\partial \sigma_{pt}^2}{\partial \zeta}\right)$ in equation (4.37) dominates and savings rise when inflation risk increases.

To analyse the effect of a rise in inflation risk in case of a PAYG pension system, where $\theta = 1$, we take the derivative of equation (4.16) with respect to ζ:

$$\left.\frac{\partial S_t^P}{\partial \zeta}\right|_{\theta=1} = \frac{\frac{1}{2}z^2(1+\rho)\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon}{\left[\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon + 1 + \rho\right]^2} \cdot \left[W - T^P - T^{B,P} + \frac{\varepsilon Z^P}{E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)}\right] \underbrace{\frac{\partial \sigma_{pt}^2}{\partial \zeta}}_{3} + \frac{(1+\rho)\varepsilon Z^P \exp(-\frac{1}{2}\sigma_p^2)}{(\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon + 1 + \rho)[E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)]^2} \left[\underbrace{\frac{\partial E_t R_{p,t+1}}{\partial \zeta}}_{1} \underbrace{- \frac{1}{2}E(1+R_p)\frac{\partial \sigma_{pt}^2}{\partial \zeta}}_{2}\right] \tag{4.40}$$

The third line shows that the net present value of the PAYG pension benefit, i.e., $\frac{\varepsilon_{t+1} Z_{t+1}^P}{E_t(1+R_{p,t+1})\exp(-\frac{1}{2}\sigma_{pt}^2)}$, is affected in two ways. First, the rise in the expected portfolio return decreases the net present value of the pension benefit, which affects savings positively (effect 1). Second, the increase of the variance of the log portfolio return affects savings negatively (effect 2). This second effect is in line with the reasoning of Section 4.3.2 where we noticed that people with a PAYG pension and $\theta = 1$ act as consumers with $0 < \theta < 1$ without a PAYG pension. In that case the substitution effect of uncertainty dominates the income effect (see Section 4.3.1), so that savings fall when the risk on the portfolio rises. Moreover, on the basis of equation (4.39) we expect that the effect of the increase in the riskiness of the portfolio (σ_{pt}^2) dominates the effect of the higher expected portfolio return ($E_t R_{p,t+1}$) and savings fall when inflation risk rises. There is, however, an extra effect of PAYG pensions, which is shown in the second line of equation (4.40) and is denoted by a 3. This effect shows that when part of old-age consumption is safe, that is, when people receive a safe PAYG pension benefit (i.e., $z > 0$), a rise in the level of uncertainty (σ_{pt}^2 rises) induces people to save more. The intuition is that a (higher) PAYG pension benefit increases safe income in the second period of life, which increase savings as the substitution effect of uncertainty dominates. This effect will become more important when the level of uncertainty increases. The ultimate effect of inflation risk on individual savings is therefore ambiguous. In the baseline simulation of Section 4.3.4 savings fall after a rise in inflation risk, which implies that effect 2 is dominant. A consumer with a PAYG pension benefit and $\theta = 1$ indeed acts as someone with $0 < \theta < 1$.

4.3.4 Summary partial equilibrium effects

To make the comparison with the effects in general equilibrium relatively easy we summarise the main findings of this section in Table 4.1. The first column shows the effects of a marginal increase of the coefficient of relative risk aversion from $\theta = 1$ to $\theta > 1$ as discussed in Section 4.3.1. The second column summarises the effects of safe PAYG pensions as described in Section 4.3.2. Comparing these two columns shows that safe PAYG pensions have exactly the opposite effects as higher risk aversion, which demonstrates that safe PAYG pensions make individuals act like less risk averse agents. This implies that they save less and invest a larger part of their savings in stocks. The last three columns show the effects of a rise in the riskiness of the benchmark asset, i.e., more inflation risk (Section 4.3.3). This induces people to reoptimise their investment portfolio and invest a larger part of their savings in stocks. The saving effects of an increase in the overall amount of risk in the economy depend on whether a PAYG pension scheme is in place and the value of the coefficient of relative risk aversion. In case retirees do not receive a safe PAYG pension benefit and $\theta = 1$ (funded case, third column) savings does not react to changes in risk. Savings rise in case $\theta > 1$ (and no PAYG pension scheme is in place, fourth column) as precautionary savings will be higher when the level of uncertainty rises. Safe PAYG pensions in combination with a coefficient of relative risk aversion of one make people behave like less risk averse individuals, so that savings fall after a rise in inflation risk.

Table 4.1: Partial equilibrium effects

			σ_π^2		
	$\theta > 1$	PAYG	Funded	$\theta > 1$	PAYG
γ	-	+	+	+	+
σ_p^2	-	+	+	+	+
$E(R_p)$	-	+	+	+	+
S	+	-	0	+	-

Table 4.1 shows the qualitative effects of the various experiments analysed in this section. The analytical derivations in this section are based on specific experiments and serve to disentangle the various mechanisms behind the results. To infer whether the results also hold in more general cases we have to resort to numerical simulations. For the baseline simulation we took

$F(A_t, K_t) = K_t^{0.3}$ so we take $E(A) = 1$, $\varepsilon = 0.94$, $n = 0$, $\delta = 1$ and $E(\pi) = 0$. Agents are relatively patient with a time preference rate of 1% per year and we assume that one period is 30 years so that $\rho = (1.01)^{30} - 1 = 0.3478$. There is no PAYG pension scheme in place and $\theta = 1$. Moreover, the level of government debt is equal to 0.025, which implies that there is still an equilibrium (see also Appendix 4.C). We take $\sigma_a^2 = 0.2$ and $\sigma_\pi^2 = 0.01$, which roughly corresponds to an annual standard deviation of 8.2% for stock returns and 1.8% for bond returns, assuming that the returns are serially uncorrelated[6]. In this baseline scenario $\gamma = 0.84$. To test the sensitivity of the results we ran many simulation experiments with various parameter values. This did not change our qualitative findings, however.

To get some feeling for the size of the effects, Table 4.2 shows the effect on portfolio choice with some other parameter configurations. A rise of the coefficient of relative risk aversion to 5 results in a more realistic value of 0.21 for γ (see Guiso et al., 2001). The introduction of a (safe) PAYG pension scheme where $T^P = 0.2W$ increases γ to 1.49 in case $\theta = 1$ and to 0.32 when $\theta = 5$, implying that safe PAYG pension benefits have a significant effect on portfolio choice. A rise in inflation risk to $\sigma_\pi^2 = 0.05$, on the other hand, increases γ from 0.84 to 0.87 when $\theta = 1$ and from 0.21 to 0.33 when $\theta = 5$, showing that more risk averse people respond much more strongly to a rise in uncertainty.

Table 4.2: Portfolio choice (γ)

	$\theta = 1$	$\theta = 5$
Funded ($T^P = 0$)	0.84	0.21
PAYG ($T^P = 0.2W$)	1.49	0.32
$\sigma_\pi^2 = 0.05$	0.87	0.33

4.4 General equilibrium

In general equilibrium the capital stock and the rates of return are endogenously determined, which implies that we can also analyse the effects of the various shocks on the excess return. It turns out that in general equilibrium the portfolio effects of the shocks analysed in the previous section are oppo-

[6]Here we follow Campbell and Viceira (2005) who show that returns on stocks are significantly less volatile when the investment horizon is long and inflation risk on nominal bonds increases with the investment horizon.

site to the partial equilibrium effects. In Appendix 4.D we derive a general framework to analyse the effects of the different shocks.

4.4.1 Risk aversion

We analyse the same shock as in partial equilibrium, that is, we analyse the effect of a marginal increase of θ from the point $\theta = 1$ and the time pattern of this shock is given in equation (4.24). We use the framework in Appendix 4.D to derive the first-order conditions for K_{t+1} and $R^N_{b,t+1}$. Notice that in case of a rise of θ, the PAYG contributions and benefits do not change, this also holds for the (co)variances: $\frac{\partial T^P_t}{\partial \zeta} = \frac{\partial Z^P_{t+1}}{\partial \zeta} = \frac{\partial z_t}{\partial \zeta} = \frac{\partial \sigma^2_{k-b,t}}{\partial \zeta} = \frac{\partial \sigma_{k-b,bt}}{\partial \zeta} = 0$. Using equations (4.100) and (4.115) from Appendix 4.D.1, we can write:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \underbrace{\frac{\varepsilon \alpha W}{\Psi K}\frac{\partial K_t}{\partial \zeta}}_{4} \underbrace{- \frac{\varepsilon B}{\Psi(1+\pi)}\frac{\partial R^N_{b,t}}{\partial \zeta}}_{3} + \underbrace{\frac{\varepsilon(1+\rho)\left[\frac{1}{2}\sigma^2_p + \log\left(\frac{1+\rho}{E(1+R_p)}\right)\right]}{\Psi(1+\rho+\varepsilon)}(W - T^B)h^\theta_t}_{2} \tag{4.41}$$

$$\frac{\partial R^N_{b,t+1}}{\partial \zeta} = \underbrace{\Phi}_{<0}\frac{\partial K_{t+1}}{\partial \zeta} \underbrace{-(1+R^N_b)\left[\log E(1+R_k) - \log E(1+R_b)\right] h^\theta_t}_{1} \tag{4.42}$$

These two equations can be used to analyse the effects of a higher θ over time. Risk aversion rises permanently at time t, i.e., $h^\theta_t = h^\theta_{t+1} = \ldots = h^\theta_\infty > 0$. There are two partial equilibrium effects. First, as people become more risk averse they would like to have a more conservative portfolio and move from stocks to government bonds. In that case, however, too much savings go to the government debt and because the level of government debt is given, $R^N_{b,t+1}$ will fall to make sure that γ does not fall. This effect is denoted by a 1 in equation (4.42). Second, we have the direct effect on savings, see equation (4.28). As explained in Section 4.3.1 the effect of a higher θ depends on the sign of $\rho - E(R_p)$, but we assume that a higher θ results in higher savings[7]. For a given level of government debt, higher savings result in a higher capital-labour ratio, this is effect 2. Moreover, higher savings combined with

[7] In case $\rho < E(R_p)$ and this consumption smoothing effect is large enough to dominate the precautionary savings effect, so that savings actually fall after a rise of θ, the effects described

a constant level of government debt have to result in a higher γ_t, to make this happen $R^N_{b,t+1}$ will fall. This effect is captured by the first term in equation (4.42); when K_{t+1} increases, the nominal interest rate on government debt will fall. Effects 1 and 2 are the initial effects of a rise of θ on K_{t+1} and $R^N_{b,t+1}$.

What happens over time? The fall of $R^N_{b,t+1}$ results in a lower debt tax, which implies that people can spend more and increase their savings. As explained above, higher savings result in a higher capital-labour ratio (K_{t+2} goes up, effect 3) and a lower interest rate on the government debt, i.e., $R^N_{b,t+2}$ falls. The rise in the capital-labour ratio at time $t+1$ results in higher wages, so that savings at $t+1$ increase even more. This reinforces the effect via $R^N_{b,t+1}$, i.e., K_{t+2} increases (effect 4) and $R^N_{b,t+2}$ decreases. This process continues until the economy reaches a new steady state.

From equation (4.103) in Appendix 4.D we know that:

$$\frac{\partial \gamma_t}{\partial \zeta} = \frac{(1+n)(1-\gamma)}{S} \frac{\partial K_{t+1}}{\partial \zeta} > 0 \tag{4.43}$$

This equation shows very clearly that, with a constant level of government debt, a larger capital stock has to lead to a higher γ. This is even more clear when you look at equation (4.22), which we repeat here for convenience:

$$(1-\gamma_t)S_t = (1+n_{t+1})B_{t+1} \tag{4.44}$$

As savings rise and the right-hand side does not change, γ has to increase to make sure that the capital market is in equilibrium. The rise of γ also leads to a higher variance of the log portfolio return. The rate of return on stocks falls as a result of the higher capital-labour ratio and the rate of return on bonds falls as well, so that the expected return on the portfolio would decrease. A higher γ, on the other hand, results in an upward pressure on the portfolio return. From equation (4.118) in Appendix 4.D.2 we deduce that it is most likely that the expected return on the portfolio falls after a rise of θ, which means that the effects of the lower rates of return dominate the effect of a higher γ. A general equilibrium framework also allows us to analyse the effect on the excess return of stocks over bonds. From equation (4.120) in Appendix 5.D we can infer that

in the main text will be opposite. That is, in the long run, K will fall, R^N_b rises and γ falls. This is the case, for example, when the PAYG system is large enough. The reason for this is that a PAYG system leads to crowding out of capital, which result in higher rates of return. In the baseline simulation of Section 4.3.4, $E(R_p)$ is larger than ρ when $T^P = 0.15W$.

the log excess return of stocks over bonds rises. More risk averse investors will demand a higher risk premium.

There are two points of interest when $\theta > 1$ in a general equilibrium model. First, higher values for the coefficient of relative risk aversion do not lead to more realistic values for γ_t. In a general equilibrium framework portfolio choice has to be such that the capital market is in equilibrium. In case θ rises there is a downward pressure on γ_t. The number of government bonds is given, however, so that the nominal return on government bonds adjusts to make sure that γ_t does not fall. A fall in $R^N_{b,t+1}$ means that the debt tax falls and savings rise. Higher savings with a constant level of government debt imply a higher γ_t. So in general equilibrium higher risk aversion leads to a higher γ_t instead of a lower γ_t. This implies that assuming a high value for θ will not give more realistic values for γ_t in general equilibrium. In general equilibrium portfolio choice is determined by the relative size of the capital stock compared to the government debt, which implies that γ_t is always relatively high[8].

Second, savings rise with the level of θ and the economy may become dynamically inefficient when $\theta > 1$. The problem is, however, that it is difficult to determine whether the economy is dynamically inefficient or not in a stochastic OLG model. As shown by Demange (2002) a stochastic economy with overlapping-generations is Pareto efficient in case there does not exist a Pareto improving allocation *in the market*. This has to be the case for all states of nature, which makes the verification quite difficult. A solution for this problem could be to use Epstein-Zin preferences (Epstein and Zin (1989, 1991)), where the intertemporal elasticity of substitution is not necessarily the inverse of the coefficient of relative risk aversion. This implies that portfolio choice and saving decisions can be separated from each other. Risk aversion is the main determinant of portfolio choice, while the intertemporal substitution elasticity is more important for savings decisions. In that case one could analyse the effects of higher risk aversion without changing the elasticity of intertemporal substitution, so that savings are hardly affected and the economy stays dynamically efficient. With Epstein-Zin preferences the model would lose its analytical tractability, however, and we leave this possibility for future research.

[8]This result may change in case individuals and firms also issue bonds. This would complicate the analysis to a large extent, however, see footnote 2.

4.4.2 PAYG pensions

The time pattern of the shock is given in equation (4.29), so we analyse the effects of a marginal increase of the PAYG pension benefit in case there is no PAYG scheme in the initial steady state. We use the framework in Appendix 4.D.1 to derive the first-order conditions for K_{t+1} and $R^N_{b,t+1}$. Notice that $\frac{\partial \theta}{\partial \zeta} = \frac{\partial \sigma^2_{k-b,t}}{\partial \zeta} = \frac{\partial \sigma_{k-b,bt}}{\partial \zeta} = 0$ in case the PAYG benefit rises. Moreover, it holds that:

$$\frac{\partial T^P_t}{\partial \zeta} = \frac{\varepsilon}{1+n} h^z_t \tag{4.45}$$

$$\frac{\partial Z^P_{t+1}}{\partial \zeta} = h^z_{t+1} \tag{4.46}$$

Using equation (4.100) from Appendix 4.D.1, we can then write:

$$\begin{aligned}\frac{\partial K_{t+1}}{\partial \zeta} = & \underbrace{\frac{\varepsilon \alpha W}{\Psi K} \frac{\partial K_t}{\partial \zeta}}_{5} \underbrace{- \frac{\varepsilon B}{\Psi(1+\pi)} \frac{\partial R^N_{b,t}}{\partial \zeta}}_{4} \underbrace{- \frac{\varepsilon^2}{\Psi(1+n)} h^z_t}_{3} \\ & \underbrace{- \frac{\varepsilon(1+\rho)}{\Psi E(1+R_p)\exp(-\frac{1}{2}\sigma^2_p)} h^z_{t+1}}_{2}\end{aligned} \tag{4.47}$$

Substituting equation (4.30) from Section 4.3.2 into equation (4.115) from Appendix 4.D.1, the first-order condition for $R^N_{b,t+1}$ can be written as:

$$\begin{aligned}\frac{\partial R^N_{b,t+1}}{\partial \zeta} = & \underbrace{\Phi}_{<0} \frac{\partial K_{t+1}}{\partial \zeta} + \\ & \underbrace{\frac{(1+R^N_b)[\log E(1+R_k) - \log E(1+R_b)]E(1+R_p)\exp(-\frac{1}{2}\sigma^2_p)S/\varepsilon}{\left[E(1+R_p)\exp(-\frac{1}{2}\sigma^2_p)S/\varepsilon + Z^P\right]^2} h^z_{t+1}}_{1}\end{aligned} \tag{4.48}$$

We use equations (4.47) and (4.48) to analyse what happens over time when the government announces at time t that it will introduce a PAYG pension scheme in the next period, that is, at time $t+1$[9]. People that are young at time t know they will receive a safe PAYG pension benefit when they are retired, which makes them less risk averse. This implies that young people would like to take more risk with the investment of their savings and increase their

[9]This means that $h^z_t = 0$ and $h^z_{t+1} = h^z_{t+2} = \ldots = h^z_\infty > 0$.

holdings of stocks. In that case, however, the given stock of government debt cannot be financed, so that $R^N_{b,t+1}$ will rise to make sure that γ_t does not rise. This effect is denoted by a 1 in equation (4.48). Young people at time t will also decrease their savings as they know they will receive a safe pension benefit when they are old. Lower savings imply that less money is available for the capital stock, so that the capital-labour ratio falls, this is effect 2 in equation (4.47). Moreover, lower savings with a given level of government debt imply that γ_t has to fall, to make sure that enough savings go to the government debt. To attract a larger part of the savings the nominal rate of return on government debt ($R^N_{b,t+1}$) will rise. This effect reinforces effect 1.

At the time of the implementation, i.e., at $t+1$, we have the same effects as at time t, but the working people at $t+1$ also have to pay PAYG contributions. Therefore savings of young people at $t+1$ decrease even more, which means that K_{t+2} falls even more (effect 3) and $R^N_{b,t+2}$ rises even more. Moreover, as savings already fell in t, we had that $R^N_{b,t+1}$ went up and K_{t+1} went down, resulting in a higher debt tax and lower wages. This implies that people have less to spend and savings continue to decrease, so that K continues to decrease (effects 4 and 5) and R^N_b continues to rise. This process continues until the economy reaches a new steady state.

Note that effects 2 and 3 are the direct negative savings effects of PAYG pensions that were also present in partial equilibrium, see equation (4.32). These affect the capital-labour ratio negatively. Using equations (4.43) and (4.44) from the previous subsection we know that γ has to decrease to make sure that the capital market is in equilibrium. The fall of γ also leads to a smaller variance of the log portfolio return. Equation (4.118) in Appendix 4.D.2 shows that the expected return on the portfolio probably rises when the government introduces a PAYG pension scheme. As noted before, a safe PAYG pension benefit makes consumers less risk averse, this will reduce the risk premium of stocks over bonds. Using equation (4.120) in Appendix 5.D, we can indeed conclude that the log excess return falls after the introduction of a PAYG scheme. This is in line with the findings of Campbell and Nosbusch (2007). They have to rely on numerical simulations, however, and assume that physical capital is in fixed supply.

In partial equilibrium as well as in general equilibrium a PAYG pension system leads to lower savings. In partial equilibrium, moreover, a PAYG system induces individuals to take more risk, i.e., γ will increase. In general equilibrium, however, PAYG pensions cannot result in a higher γ, the reason being that lower investment in government debt would lead to an oversupply of government bonds. Therefore, γ has to fall to make sure that the government debt is financed. The nominal rate of return on government bonds will rise to make this happen. In partial equilibrium, however, rates of return are given as government bonds can be supplied on the international capital market.

4.4.3 Inflation risk

The time pattern of an increase in inflation risk is given in equation (4.33). We distinguish between the funded and PAYG case. We do not consider the case where $\theta > 1$ like we did in partial equilibrium, because (as described in Section 4.4.1) in general equilibrium a $\theta > 1$ may lead to a dynamic inefficient economy and does not help to obtain more realistic values for portfolio choice. In case $\sigma^2_{\pi t}$ rises, it holds that $\frac{\partial T^P_t}{\partial \zeta} = \frac{\partial Z^P_{t+1}}{\partial \zeta} = \frac{\partial z_t}{\partial \zeta} = \frac{\partial \theta}{\partial \zeta} = 0$.

Funded

In case the country has a fully-funded pension system, the process of K_{t+1} and $R^N_{b,t+1}$ is given by:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \underbrace{\frac{\varepsilon \alpha W}{\Psi K}\frac{\partial K_t}{\partial \zeta}}_{3} - \underbrace{\frac{\varepsilon B}{\Psi(1+\pi)}\frac{\partial R^N_{b,t}}{\partial \zeta}}_{2} \tag{4.49}$$

$$\frac{\partial R^N_{b,t+1}}{\partial \zeta} = \underbrace{\Phi}_{<0}\frac{\partial K_{t+1}}{\partial \zeta} + \underbrace{(1+R^N_{b,t})(1-\gamma)h^{\sigma_\pi}_t}_{1} \tag{4.50}$$

If inflation risk increases, people would like to invest more in stocks, that is, there is an upward pressure on γ_t. For a given level of savings, this implies that government debt is not financed. To correct for this, $R^N_{b,t+1}$ will rise, this is effect 1. In case the nominal interest rate on government bonds increases, the debt tax has to go up and people can spend less, which decreases their savings. With a constant level of government debt, a fall in savings imply a lower capital-labour ratio at $t+2$ (effect 2). Moreover, lower savings imply that γ_t has to decrease, to make this happen $R^N_{b,t+2}$ will rise. A lower capital-labour ratio implies lower wages, which decreases savings even more, so that K continues to decrease (effect 3) and R^N_b continues to increase.

As the capital-labour ratio falls, we know that γ will decrease. The change of σ_{pt}^2 is given by:

$$\frac{\partial \sigma_{pt}^2}{\partial \zeta} = (2\gamma\sigma_{k-b}^2 + 2\sigma_{k-b,b})\underbrace{\frac{\partial \gamma_t}{\partial \zeta}}_{<0} + \underbrace{(1-\gamma)h^{\sigma_\pi}}_{>0} \tag{4.51}$$

As long as $\sigma_{k-b,b}$ is not too negative, the term before $\frac{\partial \gamma_t}{\partial \zeta}$ is positive and there are two opposing effects on σ_{pt}^2. First, the last term shows the direct effect of a rise in inflation risk. The fall in γ, however, dampens this direct effect. We assume that the variance of the log portfolio return rises in case σ_π^2 rises, as the effect via γ is second order. The expected return increases as both the return on stocks and the return on bonds rise, see equation (4.118) in Appendix 4.D.2. As government bonds become less attractive to hold, the log excess return on stocks over bonds falls, see Appendix 5.D.

In partial equilibrium savings did not react to changes in uncertainty. In general equilibrium, however, the nominal interest rate on government debt changes to make sure that the capital market is in equilibrium. This results in a higher debt tax, which decreases savings. So in general equilibrium savings *do* react to changes in uncertainty. In general equilibrium lower savings also imply that γ will fall, so again we have the opposite result for the portfolio choice compared to partial equilibrium.

PAYG

The analysis becomes more complicated in case the country uses a PAYG pension scheme, because people react to changes of risk and return. In Appendix 4.E we derive the dynamic equations below. We can still use the method of comparative dynamics to separate the various effects. However, as there are different opposing mechanisms, the ultimate signs of the derivatives cannot be determined. Using equations (4.145) and (4.144) in Appendix 4.E and the fact that $\frac{\partial \sigma_{\pi t}^2}{\partial \zeta} = h_t^{\sigma_\pi}$, we get:

$$\begin{aligned}\frac{\partial K_{t+1}^P}{\partial \zeta} =& \underbrace{\frac{\varepsilon \alpha W}{\Psi^P \Gamma K}\frac{\partial K_t^P}{\partial \zeta}}_{5} \underbrace{- \frac{\varepsilon B^P}{\Psi^P \Gamma(1+\pi)}\frac{\partial R_{b,t}^N}{\partial \zeta}}_{4} \\ &+ \frac{\overbrace{S_{\sigma_p^2}^P \left(1-\gamma^P\right)^2}^{3} \overbrace{+ S_{R_p}^P (1-z)\left(1-\gamma^P\right)^2 E(1+R_b)}^{2}}{\Gamma(1+n)} h_t^{\sigma_\pi}\end{aligned} \tag{4.52}$$

$$\frac{\partial R^N_{b,t+1}}{\partial \zeta} = \underbrace{\Phi^P}_{<0} \frac{\partial K^P_{t+1}}{\partial \zeta} \underbrace{+(1+R^N_b)(1-z)(1-\gamma^P)}_{1} h^{\sigma_\pi}_t \tag{4.53}$$

Effect 1 shows the effect we also had in the funded case; because there is large upward pressure on γ_t and to make sure that the government debt is financed, $R^N_{b,t+1}$ will rise. Effects 2 and 3 are the effects of σ^2_{pt} and $E_t R_{p,t+1}$ on savings, comparable to the partial equilibrium effects described in Section 4.3.3. On the one hand, the return on the portfolio rises, which decreases the net present value of the pension benefit and induces people to increase their savings ($S^P_{R_p} > 0$). This effect causes an upward pressure on the capital-labour ratio (effect 2) and a downward pressure on the nominal interest rate on government bonds. On the other hand, the variance of the log portfolio return will rise and when we assume that the substitution effect of uncertainty dominates when there is a PAYG pension system in place, savings will fall ($S^P_{\sigma^2_p} \lessapprox 0$). Lower savings induce a lower K^P_{t+1} (effect 3) and a higher $R^N_{b,t+1}$. Simulations show that savings fall[10] after an increase of $\sigma^2_{\pi t}$, so effect 3 dominates effect 2 and K^P_{t+1} falls and $R^N_{b,t+1}$ rises. This reinforces the initial effect on savings via the nominal interest rate on government debt and the debt tax, so that K^P_{t+1} and $R^N_{b,t+1}$ move in the same direction as in the funded case. This implies that the effects on the other variables are also qualitatively the same, only quantitatively the effects will be stronger.

4.4.4 Summary general equilibrium effects

Table 4.3 gives an overview of the general equilibrium effects. Comparing these results with the results in Table 4.1, shows that in general equilibrium portfolio choice is affected in exactly the opposite way as in partial equilibrium. Individually, consumers would like to adjust their portfolio in a certain direction after a shock. However, as all people in the economy have the same incentive, this would mean that the capital market does not clear anymore. In general equilibrium the rates of return adjust in such a way that both the government debt and the capital stock are financed, and portfolio choice moves in the same direction as savings. The introduction of a PAYG pension scheme, for example, where the pension benefits are safe (shown in the second column) makes people less risk averse and they prefer to hold a more risky investment portfolio. At the same time, however, PAYG pensions reduce savings, implying that a larger part of savings has to be invested in government bonds (γ has

[10]This was also the case in partial equilibrium.

to fall) to finance the fixed supply of public debt. In order to persuade consumers to invest more in government bonds, the return on government bonds has to rise relative to the return on stocks, i.e., the excess return of stocks over bonds has to fall. In the same way will the fall in savings that results from higher inflation risk (last two columns) result in a lower equity premium so that people invest a larger part of their savings in government bonds and the capital market is in equilibrium. In case people become more risk averse, the risk premium of stocks over bonds will rise to clear the capital market as savings rise.

Table 4.3: General equilibrium effects

			σ^2_π	
	$\theta > 1$	PAYG	Funded	PAYG
γ	+	-	-	-
S	+	-	-	-
K	+	-	-	-
R^N_b	-	+	+	+
$E(R_k)$	-	+	+	+
$E(R_p)$	-	+	+	+
log excess return	+	-	-	-
σ^2_p	+	-	+	+

We also ran some numerical simulation experiments. In the baseline scenario described in Section 4.3.4, individuals invested 84% of their savings in stocks. In contrast to the partial equilibrium analysis it is not possible to obtain more realistic values for portfolio choice with a higher value for the coefficient of relative risk aversion. In general equilibrium portfolio choice has to be such that the capital market is in equilibrium, and $\gamma > 0.8$ in all considered cases. A general equilibrium framework also allows us to derive the equity premium. The baseline simulation produces a simple annual excess return of 0.6 percentage points, which is quite low. Assuming a coefficient of relative risk aversion of 5 results in an equity premium of 3 percentage points, which is more realistic. In that case, however, savings rise to a large extent and the rates of return on both assets become negative, which makes the economy probably dynamically inefficient and the results hard to interpret. The introduction of a PAYG pension scheme where $T^P = 0.2W$ decreases the annual risk premium of stocks over bonds to 0.4 percentage points, while the simple risk premium falls to 0.5 percentage points after a rise of σ^2_π to 0.05.

4.5 Alternative risk characteristics of debt and pensions

In the model developed in this chapter government debt and the PAYG pension scheme differ in riskiness. Government debt is denominated in nominal terms, implying that the return on government bonds contains inflation risk. PAYG pension benefits, on the other hand, are guaranteed in real terms and are therefore completely risk-free. The fact that there is no risk-free asset in our economy means that markets are incomplete and safe PAYG pensions partly fill the gap of this missing market[11]. In this section we will discuss whether the effects of PAYG pensions on portfolio choice and the excess return will change if we depart from the assumption of different degrees of risk between government debt and a PAYG pension system.

First consider the case where investment in government debt does not involve any risk because government debt is denominated in real terms. This implies that there is no market incompleteness anymore as the government issues indexed bonds that are risk-free. In that case safe PAYG pensions do not increase diversification opportunities anymore. The effect of PAYG pensions on portfolio choice and the risk premium will not change qualitatively, however. The introduction of a PAYG pension system or an increase in PAYG pension benefits still results in an adjustment of the optimal investment portfolio of consumers. The fact that part of old-age consumption will consist of a (larger) safe PAYG pension benefit induces individuals to invest less in safe government bonds and more in stocks, that is, they will reallocate their investments more towards stocks in partial equilibrium. In general equilibrium, however, the return on government bonds will rise relative to the return on stocks, i.e., the equity premium will fall, to make sure that people do not invest more in stocks and the government debt is financed. The fall in savings

[11] As explained in the introduction financial markets are also incomplete because there cannot be trade with unborn generations and human capital is not traded. Wage-linked PAYG benefits could partly remove this market incompleteness and diversification opportunities are improved when wages are not perfectly correlated with financial market returns. This is another possibility to allow for imperfect correlation between PAYG pension benefits and the return on savings, but would not qualitatively change our analysis. As explained in footnote 3, linking pension benefits to wages and allowing for imperfect correlation between financial market returns and wages would complicate the analysis to a large extent as the PAYG benefit becomes stochastic in that case and an extra stochastic factor like depreciation risk has to be added. Therefore we leave this possibility for future research.

that results from the PAYG pension system will reinforce the negative effect on the equity premium.

We can also analyse the opposite case where PAYG pension benefits are guaranteed in nominal terms. In this case there is inflation risk on government bonds as well as on PAYG pension benefits, implying that the PAYG pension system does not remove any of the market incompleteness. Just like the above case, the introduction of a PAYG pension scheme or higher PAYG pension benefits means that individuals are in principle "forced" to hold an "asset" that has the same amount of risk as government bonds and therefore they reduce their investments in government bonds in partial equilbrium. Moreover, the PAYG pension scheme still reduces savings even though the pension benefit is risky. The fact that the pension benefit has the same amount of risk as government bonds implies that the PAYG pension scheme does not affect precautionary savings. Life cycle savings still fall, however, as people want to smooth consumption. The combination of lower savings and the preference of consumers to move away from government bonds implies again that the excess return of stocks over bonds will fall in general equilbrium.

The discussion in this section thus shows that the effects of PAYG pensions do not depend on the assumption that PAYG pension benefits are risk-free and the investment in government bonds is risky. This assumption just gives the PAYG pension the extra function of partly removing a missing market, but is not crucial for the analysis of this chapter.

4.6 Conclusion

In this chapter we develop a stochastic version of the standard two-period overlapping-generations model with government debt and PAYG pensions. The risk on stock returns arises from productivity risk, while inflation risk makes investing in government bonds risky. We use the method of Campbell and Viceira (2002) to derive an explicit solution for the optimal portfolio choice of individuals. The major advantage of the model is that it is a general equilibrium model that is fairly simple and therefore allows us to interpret the various shocks in a very intuitive way. We study three shocks that all have a direct effect on the portfolio choice of consumers. First we analyse the effects of these shocks in a partial equilibrium framework, where the rates of return are taken as exogenous. In partial equilibrium, only the effects on individual decisions are considered. The macroeconomic effects all these individual choices together may have, are not incorporated. We show that it really matters whether the macroeconomic effects are taken into account or not. In general equilibrium portfolio choice is determined by the relative size of the capital stock compared to the level of government debt. The rates of return adjust to make sure that portfolio choice is such that the capital market is in equilibrium. In case these general-equilibrium effects are not taken into account, the analysed shocks affect portfolio choice in exactly the opposite way. This implies that it is important to distinguish between analysing portfolio choice in a small open economy or in a large (closed) economy, like the US or the EU. In the first case a partial equilibrium approach would suffice, while a general equilibrium framework is more appropriate in the latter case. Moreover, it is necessary to take a general equilibrium approach in case a shock affects a large number of countries, so that it is not very realistic to assume that rates of return do not change. Population ageing is global phenomenon, for example, where the change of rates of return have to be taken into account. The next chapter will analyse population ageing in an (large) open economy version of the model presented in this chapter.

We also show that in partial equilibrium individuals increase the riskiness of their investment portfolio when they receive a safe PAYG pension benefit, as this reduces the variance of their consumption during old-age. In general equilibrium, however, this cannot occur because government debt will not be completely financed in that case and the capital market does not clear. Actually, the fall in savings induced by PAYG pensions implies that the part of savings invested in government bonds has to rise, i.e., people have to move to the less risky asset. To attract a larger part of savings the return on government bonds has to rise to a larger extent than the return on stocks, so that the excess return on stocks over bonds falls. Safe PAYG pensions make consumers act as if they are less risk averse. In partial equilibrium this means that the investment portfolio is more risky, while in general equilibrium the risk premium of stocks over bonds will be lower. Recent proposals to reform PAYG pension schemes where PAYG pension benefits are reduced and private retirement savings increased have of course the opposite effects. Higher savings will boost capital accumulation and lower the rates of return. A lower PAYG pension benefit also means that a smaller part of old-age consumption is safe, however, which makes people more risk averse. This increases the demand for the relatively safe asset, i.e., government bonds, and the equity premium will rise to restore the equilibrium on the capital market. One could also argue that in the coming decades people will perceive their PAYG pension benefit as less safe because they think that the PAYG system will not be sustainable with the upcoming ageing of the population. This means that the political risk of PAYG pension systems will increase over time. In that case people would like to reduce the riskiness of their investment portfolio and the risk premium of stocks over bonds will rise in general equilibrium.

In order to gain insight into the effects of a higher level of uncertainty in the economy we also studied a rise in the riskiness of the benchmark asset, i.e., a rise in inflation risk. Higher inflation risk makes government bonds less attractive to hold and to induce people to invest in the given amount of government bonds, the return on bonds has to rise relative to the return on stocks. The higher interest rate on government debt increases the debt tax and savings will fall when the coefficient of relative risk aversion equals unity. Lower savings imply that a larger part of savings has to be invested in government bonds to have a clearing capital market, which reinforces the initial effect on the equity premium. In the next chapter we will also analyse the effects of higher inflation risk in a two-country version of the model presented in this chapter. There, inflation risk rises as a result of large increases in government debt to cope with the costs of population ageing.

APPENDIX TO CHAPTER 4

4.A Derivation variances

In this appendix we derive the variances of the log gross returns.

4.A.1 Return on capital/stocks

Take the logarithm of optimality condition (4.3) and recall the assumption that $\delta = 1$:

$$\log(1 + R_{k,t}) = \log(\alpha) + \log(A_t) + (\alpha - 1)\log(K_t) \tag{4.54}$$

Now define $\log(1 + R_{k,t}) \equiv r_{k,t}$ and $\log X_t \equiv x_t$, where X_t can be any variable, with the exception of the returns. Now we can write:

$$r_{k,t+1} = \log(\alpha) + a_{t+1} + (\alpha - 1)k_{t+1} \tag{4.55}$$

The expectation of $r_{k,t+1}$ at time t is:

$$E_t r_{k,t+1} = \log(\alpha) + E_t a_{t+1} + (\alpha - 1)k_{t+1} \tag{4.56}$$

The variance of $r_{k,t+1}$ can be derived using equations (4.55) and (4.56).

4.A.2 Return on bonds

Taking logs of equation (4.6) gives:

$$\log(1 + R_{b,t}) = \log(1 + R^N_{b,t}) + \log\left(\frac{1}{1 + \pi_t}\right) \tag{4.57}$$

which can be used to write:

$$r_{b,t+1} = r^N_{b,t+1} + \log\left(\frac{1}{1+\pi_{t+1}}\right) \tag{4.58}$$

$$E_t r_{b,t+1} = r^N_{b,t+1} + E_t \log\left(\frac{1}{1+\pi_{t+1}}\right) \tag{4.59}$$

These two equations can be used to the derive the variance of $r_{b,t+1}$.

4.B Consumer optimisation

4.B.1 Portfolio choice

Using equation (4.12) we take logs of the portfolio-return Euler condition, i.e., $j = p$:

$$\begin{aligned}\log 1 = &\log\left(\frac{1}{1+\rho}\right) + \theta \log(C^Y_t) + E_t\left[-\theta\log(C^O_{t+1}) + \log(1+R_{p,t+1})\right] \\ &+ \underbrace{\frac{1}{2}\mathrm{Var}_t\left[-\theta\log(C^O_{t+1}) + \log(1+R_{p,t+1})\right]}_{\frac{1}{2}\theta^2\sigma^2_{c^o t} + \frac{1}{2}\sigma^2_{pt} - \theta\mathrm{Cov}_t\left(c^o_{t+1}; r_{p,t+1}\right)}\end{aligned} \tag{4.60}$$

where we used the Jensen's inequality condition for a lognormal random variable X (see also footnote 1):

$$\log E_t X_{t+1} = E_t \log X_{t+1} + \frac{1}{2}\mathrm{Var}_t \log X_{t+1} \tag{4.61}$$

Equation (4.60) can be rewritten to:

$$E_t c^o_{t+1} - c^y_t = \frac{1}{\theta}\log\left(\frac{1}{1+\rho}\right) + \frac{1}{\theta}E_t r_{p,t+1} + \frac{1}{2}\theta\sigma^2_{c^o t} + \frac{1}{2\theta}\sigma^2_{pt} - \mathrm{Cov}_t\left(c^o_{t+1}; r_{p,t+1}\right) \tag{4.62}$$

where

$$\begin{aligned}\sigma^2_{c^o t} &\equiv \mathrm{Var}_t\left[\log(C^O_{t+1})\right] \\ \sigma^2_{pt} &\equiv \mathrm{Var}_t\left[\log(1+R_{p,t+1})\right]\end{aligned}$$

In the same way we can derive the log of the Euler equation of the return on bonds, we call this the benchmark-return Euler condition:

$$E_t c^o_{t+1} - c^y_t = \frac{1}{\theta}\log\left(\frac{1}{1+\rho}\right) + \frac{1}{\theta}E_t r_{b,t+1} + \frac{1}{2}\theta\sigma^2_{c^o t} + \frac{1}{2\theta}\sigma^2_{bt} - \mathrm{Cov}_t\left(c^o_{t+1}; r_{b,t+1}\right) \tag{4.63}$$

Now subtract the benchmark equation (4.63) from portfolio-return equation (4.62):

$$E_t r_{p,t+1} - E_t r_{b,t+1} + \frac{1}{2}(\sigma_{pt}^2 - \sigma_{bt}^2) = \theta \left[\mathrm{Cov}_t(c_{t+1}^o; r_{p,t+1}) - \mathrm{Cov}_t(c_{t+1}^o; r_{b,t+1})\right] \tag{4.64}$$

First we derive the terms on the left-hand side of equation (4.64). As the return on the portfolio is a linear combination of the return on stocks and the return on bonds (see equation (4.10)) and the log of a linear combination is not the same as a linear combination of logs, we follow Campbell and Viceira (2002) and use a Taylor approximation of the nonlinear function relating log individual-asset returns to log portfolio returns[12]. First note that equation (4.10) can be written as:

$$1 + R_{p,t+1} = 1 + R_{b,t+1} + \gamma_t[(1 + R_{k,t+1}) - (1 + R_{b,t+1})]$$

Dividing by $(1 + R_{b,t+1})$ gives:

$$\frac{1 + R_{p,t+1}}{1 + R_{b,t+1}} = 1 + \gamma_t \left[\frac{1 + R_{k,t+1}}{1 + R_{b,t+1}} - 1\right] \tag{4.65}$$

The log of equation (4.65) is:

$$r_{p,t+1} - r_{b,t+1} = \underbrace{\log\left[1 + \gamma_t(\exp(r_{k,t+1} - r_{b,t+1}) - 1)\right]}_{f(r_{k,t+1} - r_{b,t+1})} \tag{4.66}$$

Now we take a second-order Taylor expansion of $f(\cdot)$ around $r_{k,t+1} - r_{b,t+1} = 0$, which gives:

$$r_{p,t+1} \approx r_{b,t+1} + \gamma_t(r_{k,t+1} - r_{b,t+1}) + \frac{1}{2}\gamma_t(1 - \gamma_t)\sigma_{k-b,t}^2 \tag{4.67}$$

where $\sigma_{k-b,t}^2 \equiv \mathrm{Var}_t[\log(1 + R_{k,t+1}) - \log(1 + R_{b,t+1})] = \sigma_{kt}^2 + \sigma_{bt}^2 - 2\sigma_{kb,t}$, is the variance of the excess return of the risky asset over the benchmark return. From equation (4.67) we know that:

$$E_t r_{p,t+1} \approx E_t r_{b,t+1} + \gamma_t(E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma_t(1 - \gamma_t)\sigma_{k-b,t}^2 \tag{4.68}$$

[12]This approximation holds exactly in continuous time and is an accurate approximation over short discrete time periods. One may wonder, however, whether this approximation can still be used in a two-period OLG model with social security, where one period is around 30 years. Viceira (2001), Campbell and Viceira (2002) and in particular the detailed calculations of Barberis (2000) show that the magnitude of the horizon effects are negligible, which implies that the approximation is still satisfactory for longer holding periods.

Using equation (4.67) and (4.68) we can derive the variance of the log gross portfolio return:

$$\sigma_{pt}^2 = \sigma_{bt}^2 + (\gamma_t)^2 \sigma_{k-b,t}^2 + 2\gamma_t \sigma_{k-b,bt} \tag{4.69}$$

where $\sigma_{k-b,bt}$ is the covariance of the excess return with the benchmark return, that is, $\sigma_{k-b,bt} \equiv \mathrm{Cov}_t[\log(1+R_{k,t+1}) - \log(1+R_{b,t+1}); \log(1+R_{b,t+1})] = \sigma_{kb,t} - \sigma_{bt}^2$.

We derive the terms on the right-hand side of equation (4.64) using the lifetime budget constraint of an individual. Equation (4.11) can be rewritten to:

$$C_{t+1}^O = \frac{1+R_{p,t+1}}{\varepsilon_{t+1}} \left[W_t - T_t^P - T_t^B + \frac{\varepsilon_{t+1} Z_{t+1}^P}{1+R_{p,t+1}} - C_t^Y \right] \tag{4.70}$$

Taking logs gives:

$$\log C_{t+1}^O = \log(1+R_{p,t+1}) - \log \varepsilon_{t+1} + \log \left[W_t - T_t^P - T_t^B + \frac{\varepsilon_{t+1} Z_{t+1}^P}{1+R_{p,t+1}} - C_t^Y \right] \tag{4.71}$$

which is equal to:

$$\begin{aligned} c_{t+1}^o &= r_{p,t+1} - \log \varepsilon_{t+1} \\ &+ \log \left[\exp w_t - \exp \tau_t^P - \exp \tau_t^B + \exp(\log \varepsilon_{t+1} + z_{t+1}^P - r_{p,t+1}) - \exp c_t^y \right] \end{aligned} \tag{4.72}$$

We approximate the term between the brackets with a first-order Taylor expansion around $r_{p,t+1} = E_t r_{p,t+1}$[13]:

$$\begin{aligned} c_{t+1}^o &\approx r_{p,t+1} - \log \varepsilon_{t+1} \\ &+ \log \underbrace{\left[\exp w_t - \exp \tau_t^P - \exp \tau_t^B + \exp(\log \varepsilon_{t+1} + z_{t+1}^P - E_t r_{p,t+1}) - \exp c_t^y \right]}_{l_t} \\ &- \underbrace{\frac{\exp(\log \varepsilon_{t+1} + z_{t+1}^P - E_t r_{p,t+1})}{l_t}}_{z_t} (r_{p,t+1} - E_t r_{p,t+1}) \end{aligned} \tag{4.73}$$

The term z_t can be rewritten to:

$$z_t = \frac{Z_{t+1}^P}{\frac{E_t(1+R_{p,t+1}) \exp(-\frac{1}{2}\sigma_{pt}^2)}{\varepsilon_{t+1}} S_t^P + Z_{t+1}^P} \approx \frac{\text{PAYG pension benefit}}{\text{expected old-age consumption}} \tag{4.74}$$

[13] This is comparable to the approximation of a linear budget constraint in Campbell (1993).

so z_t is the part of expected old-age consumption financed by PAYG pensions.

Now define $y_t \equiv \log l_t + z_t E_t r_{p,t+1}$, so that we can write:

$$c^o_{t+1} \approx r_{p,t+1} - \log \varepsilon_{t+1} + y_t - z_t\, r_{p,t+1} \tag{4.75}$$
$$E_t c^o_{t+1} \approx E_t r_{p,t+1} - \log \varepsilon_{t+1} + y_t - z_t\, E_t r_{p,t+1} \tag{4.76}$$

Equation (4.75) and (4.76) can be used to derive the variance of the log of old-age consumption and the covariance between the log of old-age consumption and the log portfolio return:

$$\sigma^2_{c^o t} = (1 - z_t)^2 \sigma^2_{pt} \tag{4.77}$$
$$\text{Cov}_t(c^o_{t+1}; r_{p,t+1}) = (1 - z_t)\sigma^2_{pt} \tag{4.78}$$

and the covariance with the log benchmark return:

$$\text{Cov}_t(c^o_{t+1}; r_{b,t+1}) = (1 - z_t)\sigma^2_{bt} + (1 - z_t)\gamma_t \sigma_{k-b,bt} \tag{4.79}$$

Equation (4.77) shows very clearly that safe PAYG pension benefits lower the variance of old-age consumption.

Substituting equations (4.68), (4.69), (4.78) and (4.79) into equation (4.64) gives:

$$\begin{aligned} \gamma_t(E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma_t \sigma^2_{k-b,t} + \gamma_t \sigma_{k-b,bt} &= \theta(1 - z_t)(\gamma_t)^2 \sigma^2_{k-b,t} \\ &\quad + \theta(1 - z_t)\gamma_t \sigma_{k-b,bt} \end{aligned} \tag{4.80}$$

Dividing by γ_t and rearranging gives:

$$\gamma_t = \frac{E_t r_{k,t+1} - E_t r_{b,t+1} + \frac{1}{2}\sigma^2_{k-b,t}}{\theta(1 - z_t)\sigma^2_{k-b,t}} - \frac{\theta - 1}{\theta(1 - z_t)} \frac{\sigma_{k-b,bt}}{\sigma^2_{k-b,t}} + \frac{z_t}{1 - z_t} \frac{\sigma_{k-b,bt}}{\sigma^2_{k-b,t}} \tag{4.81}$$

Using the fact that:

$$E_t r_{k,t+1} = E_t \log(1 + R_{k,t+1}) = \log E_t(1 + R_{k,t+1}) - \frac{1}{2}\sigma^2_{kt} \tag{4.82}$$
$$E_t r_{b,t+1} = E_t \log(1 + R_{b,t+1}) = \log E_t(1 + R_{b,t+1}) - \frac{1}{2}\sigma^2_{bt} \tag{4.83}$$

gives the expression for γ_t in terms of simple returns (equation (4.13)).

4.B.2 Savings

Using equation (4.76) and the definition of y_t we can write:

$$E_t c^o_{t+1} = E_t r_{p,t+1} - \log \varepsilon_{t+1} + \log l_t \tag{4.84}$$

Substitute this equation into the log portfolio-return Euler condition (4.62):

$$\left(1 - \frac{1}{\theta}\right) E_t r_{p,t+1} - \log \varepsilon_{t+1} + \log l_t - c^y_t = \frac{1}{\theta} \log\left(\frac{1}{1+\rho}\right) + \frac{1}{2}\theta\sigma^2_{c^o t} + \frac{1}{2\theta}\sigma^2_{pt} - \mathrm{Cov}_t\left(c^o_{t+1}; r_{p,t+1}\right) \tag{4.85}$$

Using equations (4.77) and (4.78) this can be written as:

$$\left(1 - \frac{1}{\theta}\right) E_t r_{p,t+1} - \log \varepsilon_{t+1} + \log l_t - c^y_t + \sigma^2_{pt}\left[(1 - z_t) - \frac{1}{2}\theta(1 - z_t)^2 - \frac{1}{2\theta}\right] = \frac{1}{\theta}\log\left(\frac{1}{1+\rho}\right) \tag{4.86}$$

Now use the fact that $E_t r_{p,t+1} = E_t \log(1 + R_{p,t+1}) = \log E_t(1 + R_{p,t+1}) - \frac{1}{2}\sigma^2_{pt}$, so that we can write:

$$\left(1 - \frac{1}{\theta}\right) \log E_t(1 + R_{p,t+1}) - \log \varepsilon_{t+1} + \log l_t - \log C^Y_t - \frac{1}{\theta}\log\left(\frac{1}{1+\rho}\right) = \left[\frac{1}{2}\theta z^2_t - (z_t - \frac{1}{2})(\theta - 1)\right]\sigma^2_{pt} \tag{4.87}$$

This can be rewritten to:

$$\left[E_t(1 + R_{p,t+1})\right]^{1-\frac{1}{\theta}} \frac{1}{\varepsilon_{t+1}} \left[W_t - T^P_t - T^B_t + \frac{\varepsilon_{t+1} Z^P_{t+1}}{E_t(1 + R_{p,t+1})\exp(-\frac{1}{2}\sigma^2_{pt})} - C^Y_t\right] \cdot \frac{1}{C^Y_t}\left[\frac{1}{1+\rho}\right]^{-\frac{1}{\theta}} = \exp\left[\left(\frac{1}{2}\theta z^2_t - (z_t - \frac{1}{2})(\theta - 1)\right)\sigma^2_{pt}\right] \tag{4.88}$$

which can be used to derive the optimal solutions for consumption when young and savings:

$$C^Y_t = \frac{(1+\rho)^{\frac{1}{\theta}}}{\exp\left[\left(\frac{1}{2}\theta z^2_t + (\frac{1}{2} - z_t)(\theta - 1)\right)\sigma^2_{pt}\right]\left[E_t(1 + R_{p,t+1})\right]^{\frac{1}{\theta}-1}\varepsilon_{t+1} + (1+\rho)^{\frac{1}{\theta}}} \cdot \left[W_t - T^P_t - T^B_t + \frac{\varepsilon_{t+1} Z^P_{t+1}}{E_t(1 + R_{p,t+1})\exp(-\frac{1}{2}\sigma^2_{pt})}\right] \tag{4.89}$$

$$S_t = \frac{\exp\left[\left(\frac{1}{2}\,\theta z_t^2 + (\frac{1}{2} - z_t)(\theta - 1)\right)\sigma_{pt}^2\right]\left[E_t(1 + R_{p,t+1})\right]^{\frac{1}{\theta}-1}\varepsilon_{t+1}}{\exp\left[\left(\frac{1}{2}\,\theta z_t^2 + (\frac{1}{2} - z_t)(\theta - 1)\right)\sigma_{pt}^2\right]\left[E_t(1 + R_{p,t+1})\right]^{\frac{1}{\theta}-1}\varepsilon_{t+1} + (1+\rho)^{\frac{1}{\theta}}} \cdot \left[W_t - T_t^P - T_t^B\right] - \frac{(1+\rho)^{\frac{1}{\theta}}}{\exp\left[\left(\frac{1}{2}\,\theta z_t^2 + (\frac{1}{2} - z_t)(\theta - 1)\right)\sigma_{pt}^2\right]\left[E_t(1 + R_{p,t+1})\right]^{\frac{1}{\theta}-1}\varepsilon_{t+1} + (1+\rho)^{\frac{1}{\theta}}} \cdot \frac{\varepsilon_{t+1} Z_{t+1}^P}{E_t(1 + R_{p,t+1})\exp(-\frac{1}{2}\sigma_{pt}^2)} \tag{4.90}$$

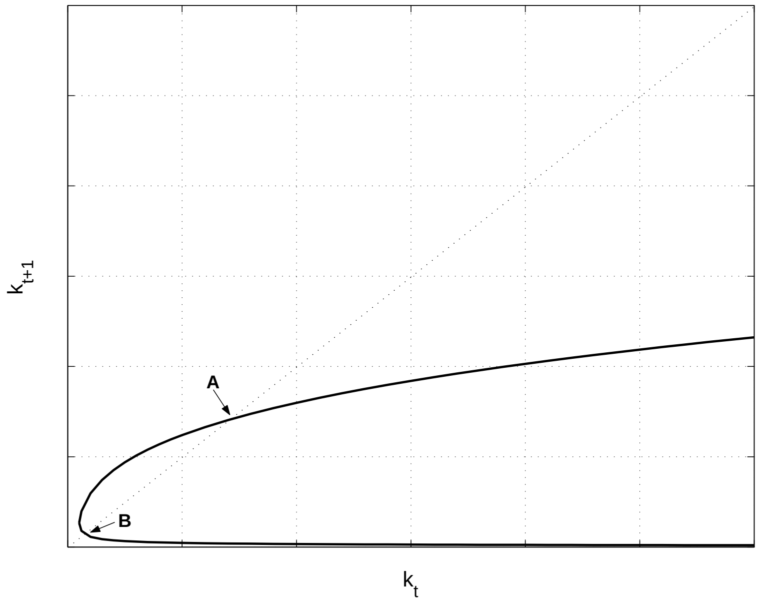
4.C Stability

In case government debt is included in the model, the model can have two equilibria, a stable and an unstable equilibrium. This is shown in Figure 4.2, point A shows the stable equilibrium and point B is the unstable equilibrium[14]. So one has to make sure that the steady state of the economy is at point A. Another thing to keep in mind is that there is maximum amount of government debt, i.e., when government debt is too large there is no equilibrium anymore.

There are two dynamic equations; equations (4.20) and (4.21), and there are two pre-determined variables, the capital stock (K_t) and the nominal interest rate on government bonds ($R_{b,t}^N$). K_t is determined by savings in period $t-1$. $R_{b,t}^N$ is the nominal interest rate that is paid at time t on the government debt that is issued in period $t-1$. $R_{b,t}^N$ makes sure that γ_{t-1} is such that this government debt is financed, so $R_{b,t}^N$ is fixed at the beginning of period t. This implies that for a stable equilibrium the two eigenvalues of the dynamic system given in equations (4.20) and (4.21) have to be within the unit circle. We checked this for various parameter values, and both eigenvalues were always within the unit circle implying that we have stable global equilibrium.

[14]For stability we need that $\frac{\mathrm{d}K}{\mathrm{d}B} < 0$, see Croix and Michel (2002) (p. 229f) and Atkinson and Stiglitz (1980) (p. 252).

Figure 4.2: Phase diagram



4.D Derivation first-order conditions

In this appendix we present a general framework, which we will use to analyse the effects of the different shocks. Again, we use the method of comparative dynamics and equation (4.23) describes the process of the variable that changes. First we derive the first-order difference equations for the evolution of the two pre-determined variables, the capital-labour ratio and the nominal interest rate on government debt. We also derive the first-order difference equations for the expected portfolio return and the excess return of stocks over bonds.

4.D.1 Capital-labour ratio and nominal return on government bonds

First we take the derivative of equation (4.20) and rewrite it to:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{1}{1+n}\frac{\partial S_t}{\partial \zeta} - \frac{\partial B_{t+1}}{\partial \zeta} \tag{4.91}$$

We assume that government debt does not change, i.e., $\frac{\partial B_{t+1}}{\partial \zeta} = 0$. The total derivative of savings is:

$$\frac{\partial S_t}{\partial \zeta} = \frac{\partial S_t}{\partial \theta}\frac{\partial \theta}{\partial \zeta} + \frac{\partial S_t}{\partial W_t}\frac{\partial W_t}{\partial \zeta} + \frac{\partial S_t}{\partial T_t^B}\frac{\partial T_t^B}{\partial \zeta} + \frac{\partial S_t}{\partial T_t^P}\frac{\partial T_t^P}{\partial \zeta} + \frac{\partial S_t}{\partial Z_{t+1}^P}\frac{\partial Z_{t+1}^P}{\partial \zeta} \tag{4.92}$$

We derive every part in this equation around the point where $\theta = 1$ and $Z^P = T^P = z = 0$:

$$\frac{\partial S_t}{\partial \theta} = \left[\frac{(1+\rho)\varepsilon\left\{\frac{1}{2}\sigma_p^2 + \log\left(\frac{1+\rho}{E(1+R_p)}\right)\right\}}{(1+\rho+\varepsilon)^2}\right](W - T^B) \tag{4.93}$$

$$\frac{\partial S_t}{\partial W_t} = \frac{\varepsilon}{1+\rho+\varepsilon} \tag{4.94}$$

$$\frac{\partial S_t}{\partial T_t^B} = \frac{\partial S_t}{\partial T_t^P} = \frac{-\varepsilon}{1+\rho+\varepsilon} \tag{4.95}$$

$$\frac{\partial S_t}{\partial Z_{t+1}^P} = \frac{-(1+\rho)\varepsilon}{(1+\rho+\varepsilon)E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)} \tag{4.96}$$

Using equations (4.2), (4.6) and (4.19) we get:

$$\frac{\partial W_t}{\partial \zeta} = \frac{\alpha W}{K}\frac{\partial K_t}{\partial \zeta} \tag{4.97}$$

$$\frac{\partial R_{b,t}}{\partial \zeta} = \frac{1}{1+\pi}\frac{\partial R_{b,t}^N}{\partial \zeta} \tag{4.98}$$

$$\frac{\partial T_t^B}{\partial \zeta} = \frac{B}{1+\pi}\frac{\partial R_{b,t}^N}{\partial \zeta} \tag{4.99}$$

Substituting equations (4.92)-(4.99) into equation (4.91), we get:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{\varepsilon\alpha W}{\Psi K}\frac{\partial K_t}{\partial \zeta} - \frac{\varepsilon B}{\Psi(1+\pi)}\frac{\partial R_{b,t}^N}{\partial \zeta} - \frac{\varepsilon}{\Psi}\frac{\partial T_t^P}{\partial \zeta} - \frac{\varepsilon(1+\rho)}{\Psi E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)}\frac{\partial Z_{t+1}^P}{\partial \zeta}$$
$$+ \frac{\varepsilon(1+\rho)\left[\frac{1}{2}\sigma_p^2 + \log\left(\frac{1+\rho}{E(1+R_p)}\right)\right]}{\Psi(1+\rho+\varepsilon)}(W - T^B)\frac{\partial \theta}{\partial \zeta} \tag{4.100}$$

where:

$$\Psi \equiv (1+n)(1+\rho+\varepsilon) \tag{4.101}$$

Now we use equation (4.21) to derive:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{\gamma}{1+n}\frac{\partial S_t}{\partial \zeta} + \frac{S}{1+n}\frac{\partial \gamma_t}{\partial \zeta} \tag{4.102}$$

Using equation (4.91) we can rewrite this equation to:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{S}{(1+n)(1-\gamma)} \frac{\partial \gamma_t}{\partial \zeta} \tag{4.103}$$

The total derivative of γ_t is:

$$\begin{aligned} \frac{\partial \gamma_t}{\partial \zeta} = \frac{\partial \gamma_t}{\partial \theta} \frac{\partial \theta}{\partial \zeta} + \frac{\partial \gamma_t}{\partial E_t R_{k,t+1}} \frac{\partial E_t R_{k,t+1}}{\partial \zeta} + \frac{\partial \gamma_t}{\partial E_t R_{b,t+1}} \frac{\partial E_t R_{b,t+1}}{\partial \zeta} \\ + \frac{\partial \gamma_t}{\partial \sigma^2_{k-b,t}} \frac{\partial \sigma^2_{k-b,t}}{\partial \zeta} + \frac{\partial \gamma_t}{\partial \sigma_{k-b,bt}} \frac{\partial \sigma_{k-b,bt}}{\partial \zeta} + \frac{\partial \gamma_t}{\partial z_t} \frac{\partial z_t}{\partial \zeta} \end{aligned} \tag{4.104}$$

We derive every part in this equation around the point where $\theta = 1$ and $Z^P = T^P = z = 0$:

$$\frac{\partial \gamma_t}{\partial \theta} = \frac{-\left[\log E(1+R_k) - \log E(1+R_b)\right]}{\sigma^2_{k-b}} \tag{4.105}$$

$$\frac{\partial \gamma_t}{\partial E_t R_{k,t+1}} = \frac{1}{\sigma^2_{k-b} E(1+R_k)} \tag{4.106}$$

$$\frac{\partial \gamma_t}{\partial E_t R_{b,t+1}} = \frac{-1}{\sigma^2_{k-b} E(1+R_b)} \tag{4.107}$$

$$\frac{\partial \gamma_t}{\partial \sigma^2_{k-b,t}} = \frac{-\gamma}{\sigma^2_{k-b}} \tag{4.108}$$

$$\frac{\partial \gamma_t}{\partial \sigma_{k-b,bt}} = \frac{-1}{\sigma^2_{k-b}} \tag{4.109}$$

$$\frac{\partial \gamma_t}{\partial z_t} = \frac{\left[\log E(1+R_k) - \log E(1+R_b)\right]}{\sigma^2_{k-b}} \tag{4.110}$$

$$\frac{\partial E_t R_{k,t+1}}{\partial \zeta} = \frac{(\alpha-1)E(1+R_k)}{K} \frac{\partial K_{t+1}}{\partial \zeta} \tag{4.111}$$

$$\frac{\partial E_t R_{b,t+1}}{\partial \zeta} = \frac{E(1+R_b)}{1+R_b^N} \frac{\partial R^N_{b,t+1}}{\partial \zeta} \tag{4.112}$$

Substituting equations (4.105)-(4.112) into equation (4.104) we get:

$$\begin{aligned} \frac{\partial \gamma_t}{\partial \zeta} = \frac{1}{\sigma^2_{k-b}} \Bigg\{ \frac{\alpha-1}{K} \frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R_b^N} \frac{\partial R^N_{b,t+1}}{\partial \zeta} \\ + \left[\log E(1+R_k) - \log E(1+R_b)\right] \left(\frac{\partial z_t}{\partial \zeta} - \frac{\partial \theta}{\partial \zeta} \right) + (1-\gamma) \frac{\partial \sigma^2_{\pi t}}{\partial \zeta} \Bigg\} \end{aligned} \tag{4.113}$$

where we used the fact that $\frac{\partial \sigma^2_{k-b,t}}{\partial \zeta} = \frac{\partial \sigma^2_{\pi t}}{\partial \zeta}$ and $\frac{\partial \sigma_{k-b,bt}}{\partial \zeta} = -\frac{\partial \sigma_{\pi t}}{\partial \zeta}$. Substitute this equation into equation (4.103) and simplifying gives:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{1}{\Phi}\left\{\frac{\partial R^N_{b,t+1}}{\partial \zeta} - (1+R^N_b)\left[\log E(1+R_k) - \log E(1+R_b)\right]\left(\frac{\partial z_t}{\partial \zeta} - \frac{\partial \theta}{\partial \zeta}\right) - (1+R^N_b)(1-\gamma)\frac{\partial \sigma^2_{\pi t}}{\partial \zeta}\right\} \tag{4.114}$$

where $\Phi \equiv \frac{\left[K(1+n)(1-\gamma)\sigma^2_{k-b} + (1-\alpha)S\right](1+R^N_b)}{-SK} < 0$. Equation (4.114) can be rewritten to:

$$\frac{\partial R^N_{b,t+1}}{\partial \zeta} = \Phi\frac{\partial K_{t+1}}{\partial \zeta} + (1+R^N_b)\left[\log E(1+R_k) - \log E(1+R_b)\right]\left(\frac{\partial z_t}{\partial \zeta} - \frac{\partial \theta}{\partial \zeta}\right) + (1-\gamma)\frac{\partial \sigma^2_{\pi t}}{\partial \zeta} \tag{4.115}$$

where $\frac{\partial K_{t+1}}{\partial \zeta}$ is given in equation (4.100). Equation (4.100) and (4.115) can be used to analyse the effects of different shocks over time.

4.D.2 Expected portfolio return

The change in the expected return on the portfolio is equal to:

$$\frac{\partial E_t R_{p,t+1}}{\partial \zeta} = \left[E(R_k) - E(R_b)\right]\frac{\partial \gamma_t}{\partial \zeta} + \gamma\frac{\partial E_t R_{k,t+1}}{\partial \zeta} + (1-\gamma)\frac{\partial E_t R_{b,t+1}}{\partial \zeta} \tag{4.116}$$

where the last two terms are given in equations (4.111) and (4.112). Using equation (4.103) we know that the change of γ_t has to be equal to:

$$\frac{\partial \gamma_t}{\partial \zeta} = \frac{(1+n)(1-\gamma)}{S}\frac{\partial K_{t+1}}{\partial \zeta} \tag{4.117}$$

Substituting this expression into equation (4.116) gives:

$$\frac{\partial E_t R_{p,t+1}}{\partial \zeta} = \underbrace{-\frac{\gamma}{K}}_{<0}\Big[\underbrace{E(R_k)(\gamma-\alpha)}_{>0 \text{ as long as } \gamma>\alpha} + \underbrace{E(R_b)(1-\gamma)+1-\alpha}_{>0}\Big]\frac{\partial K_{t+1}}{\partial \zeta} + \underbrace{\frac{(1-\gamma)E(1+R_b)}{1+R^N_b}}_{>0}\frac{\partial R^N_{b,t+1}}{\partial \zeta} \tag{4.118}$$

where we used the fact that $\gamma S = (1+n)K$.

4.D.3 Excess return

As people base their portfolio decisions on the log excess return, $\log E_t(1+R_{k,t+1}) - \log E_t(1+R_{b,t+1})$, see equation (4.13), we first derive the change in the log excess return:

$$\frac{\partial \text{log excess return}}{\partial \zeta} = \frac{1}{E(1+R_k)}\frac{\partial E_t R_{k,t+1}}{\partial \zeta} - \frac{1}{E(1+R_b)}\frac{\partial E_t R_{b,t+1}}{\partial \zeta} \tag{4.119}$$

Substituting equations (4.111), (4.112) and (4.115) and simplifying gives:

$$\begin{aligned}\frac{\partial \text{log excess return}}{\partial \zeta} &= \frac{\gamma(1-\gamma)\sigma_{k-b}^2}{K}\frac{\partial K_{t+1}}{\partial \zeta} \\ &- \left[\log E(1+R_k) - \log E(1+R_b)\right]\left(\frac{\partial z_t}{\partial \zeta} - \frac{\partial \theta}{\partial \zeta}\right) - (1-\gamma)\frac{\partial \sigma_{\pi t}^2}{\partial \zeta}\end{aligned} \tag{4.120}$$

Note that the derivative of a log is the percentage change of the respective variable. This implies that the consumer adjusts his/her portfolio decision on the basis of the percentage change of the gross returns.

The derivative of the simple excess return is:

$$\frac{\partial \text{excess return}}{\partial \zeta} = \frac{\partial E_t R_{k,t+1}}{\partial \zeta} - \frac{\partial E_t R_{b,t+1}}{\partial \zeta} \tag{4.121}$$

Substituting equations (4.111), (4.112) and (4.115) and simplifying gives:

$$\begin{aligned}\frac{\partial \text{excess return}}{\partial \zeta} &= \frac{-(1-\alpha)[E(R_k) - E(R_b)]}{K}\frac{\partial K_{t+1}}{\partial \zeta} \\ &+ E(1+R_b)\frac{\partial \text{log excess return}}{\partial \zeta}\end{aligned} \tag{4.122}$$

In case $E(R_k) \approx E(R_b)$ or $\alpha \to 1$ the term in front of $\frac{\partial K_{t+1}}{\partial \zeta}$ approaches zero and the simple excess return moves in the same direction as the log gross excess return. On the other hand when $E(R_k)$ and $E(R_b)$ are far apart, the simple excess return could move in the opposite direction as the log gross excess return. All shocks analysed in this chapter affect γ directly, and simulations show that in that case the simple excess return moves in the same direction as the log gross excess return. The reason for this is that the direct pressure on γ causes a large response by the nominal return on government bonds.

4.E Effect inflation risk in case of a PAYG scheme

In case there is a PAYG pension scheme in place people *do* react to changes of uncertainty and rates of return. This means that the analysis of Appendix 4.D changes somewhat. Equation (4.92) becomes[15]:

$$\frac{\partial S_t^P}{\partial \zeta} = \frac{\partial S_t^P}{\partial \sigma_{pt}^2}\frac{\partial \sigma_{pt}^2}{\partial \zeta} + \frac{\partial S_t^P}{\partial W_t}\frac{\partial W_t}{\partial \zeta} + \frac{\partial S_t^P}{\partial T_t^{B,P}}\frac{\partial T_t^{B,P}}{\partial \zeta} + \frac{\partial S_t^P}{\partial E_t R_{p,t+1}}\frac{\partial E_t R_{p,t+1}}{\partial \zeta} \tag{4.123}$$

Note that we do not take into account that changes of σ_{pt}^2 also affect z_t, this would complicate the analytical expressions to a large extent. Therefore we consider this effect as second order and simulations show that this is indeed the case. Using equation (4.16) we can derive:

$$\begin{aligned}\frac{\partial S_t^P}{\partial \sigma_{pt}^2} &= \frac{\frac{1}{2}z^2(1+\rho)\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon}{\left[\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon + 1 + \rho\right]^2}\left[W - T^P - T^{B,P} + \frac{\varepsilon Z^P}{E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)}\right] \\ &\quad - \frac{\frac{1}{2}(1+\rho)\varepsilon Z^P}{\left[\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon + 1 + \rho\right]E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)}\end{aligned} \tag{4.124}$$

$$\frac{\partial S_t^P}{\partial W_t} = \frac{\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon}{\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon + 1 + \rho} > 0 \tag{4.125}$$

$$\frac{\partial S_t^P}{\partial T_t^{B,P}} = -\frac{\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon}{\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon + 1 + \rho} < 0 \tag{4.126}$$

$$\frac{\partial S_t^P}{\partial E_t R_{p,t+1}} = \frac{(1+\rho)\varepsilon Z^P \exp(-\frac{1}{2}\sigma_p^2)}{\left[\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon + 1 + \rho\right]\left[E(1+R_p)\exp(-\frac{1}{2}\sigma_p^2)\right]^2} > 0 \tag{4.127}$$

Equation (4.104) becomes:

$$\begin{aligned}\frac{\partial \gamma_t^P}{\partial \zeta} &= \frac{\partial \gamma_t^P}{\partial E_t R_{k,t+1}}\frac{\partial E_t R_{k,t+1}}{\partial \zeta} + \frac{\partial \gamma_t^P}{\partial E_t R_{b,t+1}}\frac{\partial E_t R_{b,t+1}}{\partial \zeta} \\ &\quad + \frac{\partial \gamma_t^P}{\partial \sigma_{k-b,t}^2}\frac{\partial \sigma_{k-b,t}^2}{\partial \zeta} + \frac{\partial \gamma_t^P}{\partial \sigma_{k-b,bt}^2}\frac{\partial \sigma_{k-b,bt}}{\partial \zeta}\end{aligned} \tag{4.128}$$

where:

$$\frac{\partial \gamma_t^P}{\partial E_t R_{k,t+1}} = \frac{1}{(1-z)\sigma_{k-b}^2 E(1+R_k)} \tag{4.129}$$

$$\frac{\partial \gamma_t^P}{\partial E_t R_{b,t+1}} = \frac{-1}{(1-z)\sigma_{k-b}^2 E(1+R_b)} \tag{4.130}$$

And $\frac{\partial E_t R_{k,t+1}}{\partial \zeta}$, $\frac{\partial E_t R_{b,t+1}}{\partial \zeta}$, $\frac{\partial \gamma_t^P}{\partial \sigma_{k-b,t}^2}$ and $\frac{\partial \gamma_t^P}{\partial \sigma_{k-b,bt}}$ are the same as in equations (4.111), (4.112), (4.108) and (4.109). Then we can write:

$$\frac{\partial \gamma_t^P}{\partial \zeta} = \frac{1}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^P}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right] + \frac{1-\gamma^P}{\sigma_{k-b}^2}\frac{\partial \sigma_{\pi t}^2}{\partial \zeta} \tag{4.131}$$

[15]To simplify the analysis we take $\theta = 1$.

Now we can derive the change of σ_{pt}^2, as:

$$\frac{\partial \sigma_{pt}^2}{\partial \zeta} = \frac{\partial \sigma_{pt}^2}{\partial \sigma_{bt}^2}\frac{\partial \sigma_{bt}^2}{\partial \zeta} + \frac{\partial \sigma_{pt}^2}{\partial \gamma_t^P}\frac{\partial \gamma_t^P}{\partial \zeta} \tag{4.132}$$

$$\frac{\partial \sigma_{pt}^2}{\partial \zeta} = \left(1-\gamma^P\right)^2 \frac{\partial \sigma_{\pi t}^2}{\partial \zeta} + \frac{2\gamma^P \sigma_{k-b}^2 + 2\sigma_{k-b,b}}{(1-z)\sigma_{k-b}^2} \cdot \left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^P}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta} + (1-z)(1-\gamma^P)\frac{\partial \sigma_{\pi t}^2}{\partial \zeta}\right] \tag{4.133}$$

And the change of $E_t R_{p,t+1}$ is equal to:

$$\frac{\partial E_t R_{p,t+1}}{\partial \zeta} = \frac{\gamma^P(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}^P}{\partial \zeta} + \frac{(1-\gamma^P)E(1+R_b)}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta} + \frac{E(R_k)-E(R_b)}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^P}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta} + (1-z)(1-\gamma^P)\frac{\partial \sigma_{\pi t}^2}{\partial \zeta}\right] \tag{4.134}$$

The change of the wage and the debt tax are the same as equations (4.97) and (4.99). Now we write equation (4.123) as:

$$\begin{aligned}\frac{\partial S_t^P}{\partial \zeta} &= \frac{\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon}{\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon + 1 + \rho}\left[\frac{\alpha W}{K}\frac{\partial K_t^P}{\partial \zeta} - \frac{B^P}{1+\pi}\frac{\partial R_{b,t}^N}{\partial \zeta}\right] \\ &+ \frac{\partial S_t^P}{\partial \sigma_{pt}^2}\left\{\frac{2\gamma^P\sigma_{k-b}^2 + 2\sigma_{k-b,b}}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^P}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right.\right. \\ &\left.\left.+ (1-z)(1-\gamma^P)\frac{\partial \sigma_{\pi t}^2}{\partial \zeta}\right] + (1-\gamma^P)^2\frac{\partial \sigma_{bt}^2}{\partial \zeta}\right\} \\ &+ \frac{\partial S_t^P}{\partial E_t R_{p,t+1}}\left\{\frac{\gamma^P(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}^P}{\partial \zeta} + \frac{(1-\gamma^P)E(1+R_b)}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta}\right. \\ &\left. + \frac{E(R_k)-E(R_b)}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^P}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^N}{\partial \zeta} + (1-z)(1-\gamma^P)\frac{\partial \sigma_{\pi t}^2}{\partial \zeta}\right]\right\}\end{aligned} \tag{4.135}$$

To simplify the calculations, we define the following:

$$S^P_{\sigma^2_p} \equiv \frac{\partial S^P_t}{\partial \sigma^2_{pt}} \tag{4.136}$$

$$S^P_{R_p} \equiv \frac{\partial S^P_t}{\partial E_t R_{p,t+1}} \tag{4.137}$$

$$\Phi^P \equiv \frac{\left[K(1+n)(1-\gamma^P)(1-z)\sigma^2_{k-b} - (\alpha-1)S^P\right](1+R^N_b)}{-S^P K} < 0 \tag{4.138}$$

$$\Delta_{\sigma^2_p} \equiv \frac{2\gamma^P\sigma^2_{k-b} + 2\sigma_{k-b,b}}{(1-z)\sigma^2_{k-b}} \frac{(\alpha-1)(1+R^N_b) - K\Phi^P}{K(1+R^N_b)} \tag{4.139}$$

$$\begin{aligned}\Delta_{R_p} \equiv\ & \frac{1}{K}\Big\{ -\gamma^P(\gamma^P-\alpha)\left[E(R_k)-E(R_b)\right] + (\alpha-1)E(1+R_b) \\ & -\gamma^P(1-\gamma^P)E(1+R_b)(1-z)\sigma^2_{k-b}\Big\}\end{aligned} \tag{4.140}$$

$$\Gamma \equiv \frac{1+n-S^P_{\sigma^2_p}\Delta_{\sigma^2_p} - S^P_{R_p}\Delta_{R_p}}{1+n} \tag{4.141}$$

$$\Psi^P \equiv \frac{\left[\exp(\frac{1}{2}z^2\sigma^2_p)\varepsilon + 1 + \rho\right](1+n)}{\exp(\frac{1}{2}z^2\sigma^2_p)} \tag{4.142}$$

Then, using equations (4.103), (4.131) and (4.138), we can write:

$$\frac{\partial K^P_{t+1}}{\partial \zeta} = \frac{1}{\Phi^P}\left\{\frac{\partial R^N_{b,t+1}}{\partial \zeta} - (1+R^N_b)(1-z)(1-\gamma^P)\frac{\partial \sigma^2_{\pi t}}{\partial \zeta}\right\} \tag{4.143}$$

$$\frac{\partial R^N_{b,t+1}}{\partial \zeta} = \Phi^P\frac{\partial K^P_{t+1}}{\partial \zeta} + (1+R^N_b)(1-z)(1-\gamma^P)\frac{\partial \sigma^2_{\pi t}}{\partial \zeta} \tag{4.144}$$

Combining equation (4.144) with equations (4.91) and (4.135) and the definitions in equations (4.136)-(4.142) gives:

$$\begin{aligned}\frac{\partial K^P_{t+1}}{\partial \zeta} =\ & \frac{\varepsilon\alpha W}{\Psi^P\Gamma K}\frac{\partial K^P_t}{\partial \zeta} - \frac{\varepsilon B^P}{\Psi^P\Gamma(1+\pi)}\frac{\partial R^N_{b,t}}{\partial \zeta} \\ & + \frac{S^P_{\sigma^2_p}\left(1-\gamma^P\right)^2 + S^P_{R_p}(1-z)\left(1-\gamma^P\right)^2 E(1+R_b)}{\Gamma(1+n)}\frac{\partial \sigma^2_{\pi t}}{\partial \zeta}\end{aligned} \tag{4.145}$$

CHAPTER 5

Ageing, debt and inflation in a monetary union

5.1 Introduction

In Chapter 2 we analysed the international spillover effects of pensions under population ageing in case countries with PAYG pension schemes keep their pension system balanced, that is, either contributions to the scheme or pension benefits adjusted in response to an ageing population. In Chapter 3 we investigated the spillovers of a pension reform where PAYG pension systems are reduced. This is an often-proposed policy solution to cope with the upcoming problems within social security systems induced by an ageing population. Another possibility, however, is that governments in PAYG countries use debt instead to cope with the costs of ageing. High levels of nominal government debt may give governments an incentive to lobby for surprise inflation at the central bank as this will reduce the fiscal burden of debt service. The question is of course whether the central bank will give in and create unexpected inflation. The European Central Bank (ECB), for example, is formally independent, but the decision-making process about monetary policy is not completely transparent. In case it is not completely clear how monetary policy is determined, there is uncertainty about what the final outcome for inflation will be when public debt levels are high. As a consequence of this uncertainty about monetary policy, inflation risk may rise with the level of government debt. This will affect the spillover effects of ageing discussed in Chapter 2. The purpose of this chapter is to explore how these spillovers change. More precisely, we will investigate the international spillover effects of ageing when the PAYG country uses government debt to cope with the ageing costs, which will either lead to unexpected inflation or more inflation risk.

The model in this chapter is a combination of the models presented in Chapters 2 and 4. This means that the model presented in the previous chapter is extended to a two-country model, where one country has fully-funded pensions and the other country relies on PAYG-financed pensions. The two countries form a monetary union, which implies that the rate of inflation is the same. In contrast to Chapter 2 consumers allocate their investment portfolio between stocks and bonds. These assets differ in riskiness, which allows us to analyse the effect of ageing on the excess return of stocks over bonds. In a model where the asset returns are uncertain the difference in pension schemes also implies that people in the two countries have different attitudes towards risk. As shown in Chapter 4, people who receive a safe PAYG pension benefit act like less risk averse individuals and prefer to have a more risky portfolio compared to people that do not receive a safe pension benefit, i.e., have funded pensions. In an open economy, consumers can take advantage of this difference in risk aversion between the two countries, implying that individuals in the PAYG country have a more risky investment portfolio than people in the funded country.

An important channel in this chapter is that countries with high debt levels might be able to put pressure on the central bank to follow an accommodating policy in order to reduce the debt burden. This implies that we assume that the monetary authority might not be completely independent and may have an incentive to inflate away nominal debt. One could argue that this assumption is not very realistic as monetary policy has become more and more independent in the last decades. We argue, however, that in case PAYG countries finance their increased pension obligations by issuing more debt, that, given the scope of projected increases in expenditures (see European Commission (2006)), the accumulation of debt is extensive. These high debt levels substantially increase the pressure on the central bank to give in. In the European Economic and Monetary Union (EMU) it may happen, for example, that a group of large countries that rely to a large extent on PAYG-financed pensions (e.g. Italy, Germany and France) put pressure on its national delegates on the board of the ECB to accommodate. Another argument, put forward for example by Annicchiarico et al. (2006), Leith and Wren-Lewis (2006), Schabert and van Wijnbergen (2006), could be that the combination of high nominal debt and an active inflation targeting policy by the central bank may result in unstable debt dynamics. The reasoning is that a high level of government debt increases inflationary pressures. If the central bank raises the nominal interest rate in response to higher expected inflation, the service costs of debt rise, which further increases the real debt and thus leads to unstable debt dynamics. To avoid an explosive path for debt, the monetary authority has to follow a "passive" policy such that the rise in inflation reduces real interest rates and the debt is stabilised. These studies assume Calvo-type price rigidities and the central bank follows a Taylor-type interest rate rule. In contrast to these papers we do not model monetary policy explicitly, but we introduce a simple ad-hoc specification of the link between government debt and inflation (risk). Moreover, we assume that governments only increase their debt for a while and make sure that the debt dynamics are stable. The nominal rigidity in our model does not originate from the fact that imperfectly competitive firms can only change their prices infrequently, but from the fact that people invest in long-term nominal bonds. This assumption is justified by the fact that we focus on savings that have to finance old-age consumption, which are indeed long-term savings. Moreover, the introduction of market-based valuation by regulators forces pension funds to use the yields of fixed-income instruments as the basis for discounting liabilities. In response to this, pension funds hold a larger fraction of long-term bonds to limit the volatility of the regulatory funding ratios and to reduce the duration gap between assets and liabilities. Long-term bonds are still mostly denominated in nominal terms[1]. Unexpected inflation has real effects because it creates a wedge between the ex post real interest rate and the ex ante rate.

We show that in a closed-economy setting unexpected inflation is a zero-sum game. On the one hand, the government gains from unexpected inflation as it decreases the real value of government debt and the real return on government bonds. On the other hand, those who own government bonds lose from the unexpected fall of the real return on government bonds[2]. In a closed

[1]This is especially relevant for the Netherlands, where a large part of old-age consumption is financed by pension benefits which are taken care of by pension funds. In 2005 the investment portfolio of the largest pension fund in the Netherlands, the ABP, consisted for 40 percent of nominal bonds (see ABP, 2006, Table 5.5).

[2]Expected inflation will not have any real effects because people will ask a higher nominal rate of return on government bonds, so that the real rate of return does not change.

economy the gain for the government is exactly high enough to compensate the people who lose from the unexpected inflation shock. In a two-country model where one country relies on PAYG pensions and the other country has a fully-funded pension scheme, however, the PAYG country gains from unexpected inflation at the cost of the funded country. This means that there may be conflict of interest on monetary policy when countries with different pension schemes form a monetary union. The reason for this result is that residents of the funded country hold a relatively large share of the total amount of government bonds because savings in the funded country are higher and people in the funded country have a more conservative investment portfolio. This implies that the PAYG country can export part of the inflationary tax on debt holders to the funded country, while it still receives the full gain of a lower debt burden and a net gain results. This gain of unexpected inflation for a PAYG country rises with the amount of nominal government debt, but this will also increase the loss for the funded country. In the coming decades the conflict of interest on monetary policy between PAYG and funded countries may therefore be reinforced if population ageing induces governments of PAYG countries to raise debt to "finance" their pension system.

We also analyse the spillover effects of ageing when large debt burdens in PAYG countries cause a rise in inflation risk. The idea is that the conflicting interests between PAYG- and funded countries about the creation of inflation results in more uncertainty about what the final outcome of inflation will be as the decision-making process of the central bank may not be completely transparent. This uncertainty raises inflation risk and thus increases the riskiness of government bonds and the risk on the total portfolio. Higher inflation risk makes government bonds less attractive to hold and the rate of return on bonds will rise relative to the return on equity (i.e, the equity premium falls) to finance the total amount of government debt. This effect is relatively strong if the funded country holds a relatively large part of the government bonds as people in the funded country are more risk averse and therefore need to be compensated more, that is, the rate of return on government bonds has rise to a larger extent. The increase of the rate of return on government bonds implies that the costs of government debt rise, which harms both groups of countries. The analysis in this chapter therefore points out that in a monetary union like the EMU it may be in the interest of both funded- and PAYG countries to obey the fiscal constraints stated in the Stability and Growth Pact to prevent large increases in government debt, which may raise inflation risk. Moreover, it is important for all countries that a central bank like the ECB is independent, credible and transparent to prevent an increase in inflation risk.

A number of other papers have also argued in favour of fiscal restrictions in case of monetary unification. Chari and Kehoe (2007), for example argue that there will be a free-rider problem in fiscal policy in case there is a time-inconsistency problem in monetary policy resulting from an inflation bias by the central bank. This free-rider problem arises because the fiscal authorities do not fully take into account the cost of higher inflation due to a higher stock of public debt, as part of these costs wil be borne by residents of other countries in the monetary union. This results in too much debt accumulation and an inflation rate that is too high. Beetsma and Bovenberg (1999) also argue that monetary unification leads to excessive debt accumulation due to the lack of commitment in monetary policy and myopic governments. In these papers the countries in the monetary union are symmetric and the interaction between monetary policy and fiscal policy is modelled as a game. The objective function of the central bank comprises the inflation rate, output (gap) and government debt. In contrast to these papers we take monetary policy as exogenous, we just note that high debt levels may lead to high inflation (risk). Moreover, the countries in our model are asymmetric and we show that this asymmetry has the effect that one country gains from unexpected inflation at the expense of the other country.

This chapter is organised as follows. Section 5.2 presents the open-economy version of the model presented in Chapter 4. In Section 5.3 we consider the spillover effects in the benchmark case where no government debt is used by the PAYG country to cope with the ageing costs. The international spillover effects are very comparable to the results in Chapter 2. However, as stocks and bonds differ in their riskiness, the returns on these two assets will differ, which allows us to derive the effects of population ageing on portfolio choice and the risk premium. The effects of government debt are discussed in Section 5.4. Government debt crowds out capital, resulting in negative utility effects in the long run in both countries. The international spillover effects (positive or negative) become larger in both countries. Section 5.5 analyses the effects of an unexpected inflation shock where both countries try to compensate the people that lose from this shock; the elderly. As explained above the government in the PAYG country can implement a policy that is Pareto improving in its own country, while the residents in the funded country necessarily lose. This result shows that within the EMU there will be conflicting interests between different groups of countries in setting monetary policy in case debt of capital-importing (PAYG) countries reach high levels. Section 5.6 explores the spillover effects in case these conflicts of interest on monetary policy raise inflation risk. This will increase the riskiness of government bonds and will affect utility negatively in both countries. Section 5.7 concludes the chapter.

5.2 Model

As the model in this chapter builds on the model presented in the previous chapter, we do not repeat the whole model, but only present the model equations that have to be added to make it an open economy model. The country with the fully-funded pension scheme is referred to as country F and the country that relies on a PAYG pension system is denoted as country P. Production and the risk factors are modelled in exactly the same way as in the previous chapter. There is productivity risk on stocks, which are used to invest in the capital stock of firms. The return on government bonds is denominated in nominal terms, which implies that there is inflation risk on government bonds. The timing of events is also the same.

5.2.1 Demographics

Individuals born at time t reach the second period of life with a probability ε_{t+1}, which can be interpreted as average longevity. A rise of ε implies that people expect to live longer. We will vary this parameter to analyse the effects of ageing.

The working population L_t^i increases at the same exogenous rate n_t. The number of working (young) people at time t is then:

$$L_t^i = L_0^i \cdot \prod_{j=0}^{t} (1 + n_j) \tag{5.1}$$

where L_0^i is the initial population size in country i, $i = P, F$. Countries may differ in their initial population size, i.e., $L_0^F \neq L_0^P$, in this way we allow for scale differences between the two countries. We normalise L_0^P to 1 and define $\frac{L_0^F}{L_0^P} = \nu$, so ν can be interpreted as the relative size of L_0^F.

5.2.2 Households

As explained in the previous chapter, as soon as the coefficient of relative risk aversion (θ) is larger than one, savings are so large that the economy may become dynamically inefficient. Moreover, in general equilibrium a large value for θ does not really help to obtain more realistic values for γ. Therefore, and because we want to keep the model as simple as possible we assume that people have a logarithmic utility function:

$$E_t U(C_t^{Y,i}, C_{t+1}^{O,i}) = \log(C_t^{Y,i}) + \frac{1}{1+\rho}\,\varepsilon_{t+1} E_t\left[\log(C_{t+1}^{O,i})\right] \tag{5.2}$$

where ρ is the rate of time preference, $C_t^{Y,i}$ is consumption when young of an individual living in country i and $C_{t+1}^{O,i}$ is old-age consumption. Labour supply is inelastic and annuity markets are perfect.

In the funded country contributions of the young (T_t^F) are invested and returned to them with interest in the next period when they are retired (Z_{t+1}^F), that is, $Z_{t+1}^F = \frac{1+R_{p,t+1}}{\varepsilon_{t+1}} T_t^F$. The funded scheme has fixed contributions, which implies that contributions to the pension scheme are exactly offset by an equal reduction in private savings. This means that the pension fund is neutral and the economy behaves in exactly the same way as if there were no pension scheme. This makes the derivation of the optimal portfolio decision a lot more simple, as we do not have to take a first-order Taylor approximation of the consumer's budget constraint to obtain the covariance between the returns and old-age consumption[3]. The optimal portfolio choices in the funded and PAYG country, γ_t^F and γ_t^P, are given by:

$$\gamma_t^F = \frac{\log E_t(1+R_{k,t+1}) - \log E_t(1+R_{b,t+1})}{\sigma_{k-b,t}^2} - \frac{\sigma_{k-b,bt}}{\sigma_{k-b,t}^2} \tag{5.3}$$

$$\gamma_t^P = \frac{\log E_t(1+R_{k,t+1}) - \log E_t(1+R_{b,t+1})}{(1-z_t)\sigma_{k-b,t}^2} - \frac{\sigma_{k-b,bt}}{\sigma_{k-b,t}^2} \tag{5.4}$$

where $R_{k,t+1}$ and $R_{b,t+1}$ are the return on stocks and bonds respectively. The variance of the excess return of stocks over bonds is denoted by $\sigma_{k-b,t}^2$, $\sigma_{k-b,bt}$ is the covariance between the excess return and the return on bonds, and z_t is the part of expected old-age consumption financed by PAYG pensions (see equation (4.14) in Chapter 4). People in the country with the PAYG pension scheme invest more in the risky asset, i.e., $\gamma_t^P > \gamma_t^F$, as they receive a safe

[3]See Appendix 5.A for the details.

PAYG pension benefit when they are old (this is shown by the $(1 - z_t)$ term in equation (5.4)). In the closed economy case discussed in the previous chapter PAYG pensions resulted in a lower γ in general equilibrium. The reason is that a PAYG scheme decreases savings and the nominal return on government bonds adjusts in such way that the part of savings invested in bonds increases (γ decreases) so that the given level of government debt is financed. In the open economy case where the other country uses a funded pension scheme, however, people in the PAYG country can take advantage of the fact that people in the funded country have a relatively high savings level and are more risk averse because they do not receive a safe pension benefit. This implies that a relatively large part of government debt is in the hands of the funded country and people in the PAYG country can hold a more risky portfolio.

Optimal savings in case of a logarithmic utility function were already derived in Chapter 4, we repeat them here for convenience[4]:

$$S_t^F = \frac{\varepsilon_{t+1}}{1+\rho+\varepsilon_{t+1}} \left(W_t - T_t^{B,F} \right) \tag{5.5}$$

$$\begin{aligned} S_t^P = & \frac{\exp\left(\frac{1}{2} z_t^2 (\sigma_{pt}^2)^P\right) \varepsilon_{t+1}}{\exp\left(\frac{1}{2} z_t^2 (\sigma_{pt}^2)^P\right) \varepsilon_{t+1} + 1 + \rho} \left[W_t - T_t^P - T_t^{B,P} \right] \\ & - \frac{1+\rho}{\exp\left(\frac{1}{2} z_t^2 (\sigma_{pt}^2)^P\right) \varepsilon_{t+1} + 1 + \rho} \frac{\varepsilon_{t+1} Z_{t+1}^P}{E_t(1 + R_{p,t+1}^P) \exp(-\frac{1}{2}(\sigma_{pt}^2)^P)} \end{aligned} \tag{5.6}$$

where W_t is the real wage, $T_t^{B,i}$ is the debt tax to keep debt per capita constant, T_t^P are the lump-sum PAYG pension contributions and Z_{t+1}^P is the PAYG pension benefit. We added the superscript P to the variance of the portfolio return σ_{pt}^2 and the return on the portfolio $R_{p,t+1}$, because these are affected by the portfolio decision and this differs between the funded and PAYG country (compare equations (5.3) and (5.4)).

5.2.3 Government

Pensions in the funded country are organised by pension funds and not by the government, therefore they do not enter the government budget constraint:

$$(1 + n_{t+1}) B_{t+1}^F = (1 + R_{b,t}) B_t^F - T_t^{B,F} \tag{5.7}$$

[4]Savings in the funded country S_t^F include pension contributions T_t^F.

where public debt per young individual is denoted by B_t^F. The government in the funded country keeps its debt constant by raising a debt tax that is equal to:

$$T_t^{B,F} = (R_{b,t} - n_{t+1})B^F \tag{5.8}$$

The public debt dynamics of the PAYG country are already given in the previous chapter. It is assumed that the level of government debt is the same in both countries in the initial steady state, i.e., $B^P = B^F = B$.

5.2.4 Equilibrium international capital market

Individuals invest their savings in both countries either in capital or in government debt. Perfect capital mobility equalises the rates of return between the two countries. This implies that the capital-labour ratio will be same in the two countries, i.e., $K_{t+1}^P = K_{t+1}^F = K_{t+1}^{FP}$, as the same production technology is used. Equilibrium in the international capital market is given by:

$$S_t^P + \nu S_t^F = (1 + n_{t+1}) \left[(1+\nu)K_{t+1}^{FP} + (B_{t+1}^P + \nu B_{t+1}^F) \right] \tag{5.9}$$

$$\gamma_t^P S_t^P + \nu \gamma_t^F S_t^F = (1 + n_{t+1})(1+\nu)K_{t+1}^{FP} \tag{5.10}$$

$$(1 - \gamma_t^P)S_t^P + \nu(1 - \gamma_t^F)S_t^F = (1 + n_{t+1})(B_{t+1}^P + \nu B_{t+1}^F) \tag{5.11}$$

where one of the equations is abundant[5]. In Appendix 5.B we show the equations for the trade balance and the current account.

5.3 Ageing

In this section we analyse the effects of an ageing shock that is comparable to the ageing shock discussed in Section 2.3.1 in Chapter 2. In particular, we analyse the effects of an increase in longevity. This is modelled as an increase in ε_{t+1}, i.e., the probability that people reach old age increases. The time pattern of this shock is given by:

$$\varepsilon_{t+1} = \varepsilon + \zeta h_{t+1} \tag{5.12}$$

As before we assume that the rise in life expectancy is unexpected.

[5] As in Chapter 4 we checked for stability by verifying whether the two eigenvalues of the dynamic system given in equations (5.9) and (5.10) were within the unit circle. This was the case for various parameter values.

The government in the PAYG country does not use government debt to cope with the costs of ageing. We assume that the PAYG scheme is of the defined-benefit type, which implies that the young have to pay higher PAYG contributions when more people reach old age. The change of T_t^P should be equal to:

$$\frac{\partial T_t^P}{\partial \zeta} = \frac{Z^P}{1+n}\, h_t \tag{5.13}$$

The economic effects and the spillovers are very comparable to the effects described in Section 2.3.1 in Chapter 2. In contrast to Chapter 2, however, the current model also gives the effects on portfolio choice and the excess return of stocks over bonds. In Appendix 5.C we derive the following system of first-order difference equations:

$$\begin{aligned}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} =\; & \underbrace{\frac{\varepsilon\alpha W}{\Psi K}\frac{\partial K_t^{FP}}{\partial \zeta}}_{5} \underbrace{-\frac{\Omega\nu\varepsilon}{\Psi^F}\frac{\partial T_t^{B,F}}{\partial \zeta} - \frac{\varepsilon}{\Psi^P}\frac{\partial T_t^{B,P}}{\partial \zeta}}_{4} \underbrace{-\frac{\varepsilon}{\Psi^P}\frac{\partial T_t^P}{\partial \zeta}}_{3} \\ & + \Bigg\{ \underbrace{\frac{(1+\rho)(W-T^P-T^{B,P})}{\Psi^P[\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon+1+\rho]} + \frac{\Omega\nu(1+\rho)(W-T^{B,F})}{\Psi^F(1+\rho+\varepsilon)}}_{1} \\ & \underbrace{- \frac{(1+\rho)^2 Z^P}{\Psi^P[\exp(\frac{1}{2}z^2\sigma_p^2)\varepsilon+1+\rho]E(1+R_p)\exp(\frac{1}{2}\sigma_p^2(z^2-1))}}_{2} \Bigg\} h_{t+1}\end{aligned} \tag{5.14}$$

$$\frac{\partial S_t^F}{\partial \zeta} = \underbrace{\frac{\varepsilon\alpha W}{(1+\rho+\varepsilon)K}\frac{K_t^{FP}}{\partial \zeta}}_{5} \underbrace{-\frac{\varepsilon}{(1+\rho+\varepsilon)}\frac{\partial T_t^{B,F}}{\partial \zeta}}_{4} + \underbrace{\frac{(1+\rho)(W-T^{B,F})}{(1+\rho+\varepsilon)^2}h_{t+1}}_{1} \tag{5.15}$$

$$\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta} = \underbrace{\Phi}_{<0}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} + \underbrace{\frac{\nu(\gamma^F-\gamma^P)\sigma_{k-b}^2(1+R_b^N)}{\nu S^F+\frac{S^P}{1-z}}}_{<0}\frac{\partial S_t^F}{\partial \zeta} \tag{5.16}$$

where Ψ, Ψ^P, Ψ^F and Ω are defined in Appendix 5.C. The debt tax changes because the nominal interest rate on government debt changes:

$$\frac{\partial T_t^{B,F}}{\partial \zeta} = \frac{\partial T_t^{B,P}}{\partial \zeta} = \frac{B}{1+\pi}\frac{\partial R_{b,t}^{N,FP}}{\partial \zeta} \tag{5.17}$$

The spillover effects are derived in the same way as in Chapter 2. We derive the same kind of system of dynamic equations for the case where both countries use a PAYG system and for the case where both economies use a funded system. By comparing the results in case the two countries have the same pension system with the effects when they have different pension schemes, we derive the pure spillover effects of pensions and ageing in a common capital market[6]. We do not show the system of dynamic equations for these two cases, however, as the system above can be used quite easily to explain the spillover effects.

Longevity increases at $t = 1$, but this increase in life expectancy is not anticipated at $t = 0$. This implies that people do not adjust their savings and portfolio decisions in the period before the longevity shock, and that the capital stock per worker and the nominal interest rate on government debt at $t = 1$ do not change. The direct effects of a rise in longevity on savings are exactly the same as in Chapter 2. A rise in life expectancy increase the incentives to save (effect 1). A PAYG pension scheme, however, weakens the direct positive effect on savings when longevity goes up. Not only because it raises the present value of the pension benefit (effect 2), but also because contributions to the PAYG scheme have to rise (the dependency-ratio effect, effect 3). As in Chapter 2 we assume that savings in the PAYG country still rise after an increase in longevity[7]. For a given level of government debt, higher savings imply a higher capital-labour ratio. This also means that a larger part of savings has to go to the capital stock, i.e., γ has to rise. To make this happen the nominal return on government bonds will fall, this can be seen in equation (5.16); $R_b^{N,FP}$ falls in case K^{FP} rises.

The fall of the interest rate on the government debt leads to a lower debt tax, so that people have more money to spend and increase their savings (effect 4). Moreover, a higher capital-labour ratio increases wages, which also engenders higher savings (effect 5). Both these effects imply that K^{FP} continues to rise and $R_b^{N,FP}$ continues to fall.

Savings unambiguously rise more in the funded country than in the PAYG country, as funded pensions do not lead to effects 2 and 3. When the funded country has a common capital market with a PAYG country, part of its extra savings flow to country P. This implies that the rise of the capital-labour ratio

[6]To exclude the effects of integration it is assumed that the initial steady state is the same in all cases.

[7]We can derive the same kind of condition as in Chapter 2. There we showed that the capital-labour ratio rises for realistic parameter values, for details see Appendix 2.B.1 in Chapter 2. For the spillover effects, however, it only matters that savings in the funded country increase more than in the PAYG country.

and the fall in the nominal interest rate on government debt will be larger in case both countries use a funded system. In case both countries use a PAYG scheme exactly the opposite holds, i.e., the rise in K and the fall of R_b^N are smaller. To illustrate the mechanics of the model we show the same kind of simulation graphs as in Chapter 2[8]. Figures 5.1 and 5.2 show the change in the capital-labour ratio and the nominal interest rate on government bonds for the different cases. Notice the resemblance with Figure 2.1 in Chapter 2. In the initial steady state $k = 0.078$ and $R_b^N = 0.58$, which implies that in the long run the capital-labour ratio rises by about 7.4% (to 0.084) and the fall of the nominal interest rate on government bonds is about 14.7% (to 0.49) in the benchmark case where one country uses a PAYG scheme and the other country relies on fully-funded pensions ($\mathrm{d}k^{PF}$).

Figure 5.1: Change in K

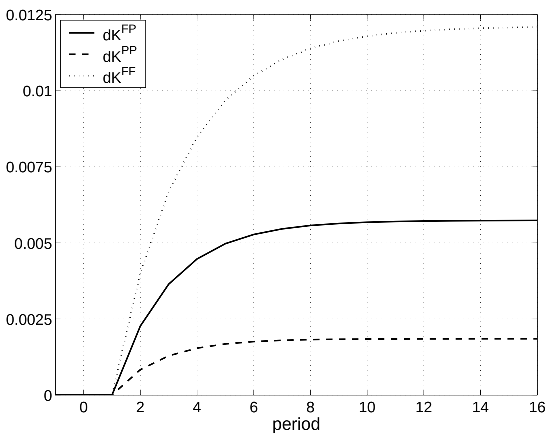
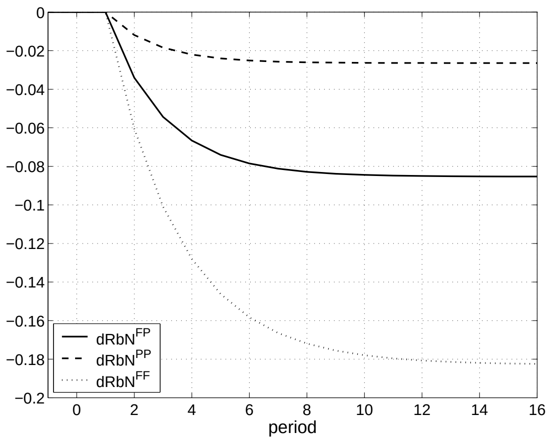


Figure 5.2: Change in R_b^N



Notes: The solid lines refer to the case where one country uses a funded pension scheme and the other country relies on a PAYG pension system. The dotted lines show the changes of the variables in case both economies use a funded system, while the striped lines indicate the changes for the case where both countries use a PAYG scheme.

[8] The graphs are based on simulations with $F(A_t, K_t) = K_t^{0.3}$ so we take $E(A) = 1$, $\varepsilon = 0.94$, $n = 0$, $\delta = 1$, $E(\pi) = 0$, $\nu = 1$, $\frac{T^P}{W} = 0.2$ and $h_t = 0.05$. Agents are relatively patient with a time preference rate of 1% per year and we assume that one period is 30 years so that $\rho = (1.01)^{30} - 1 = 0.3478$. The level of government debt is equal to 0.025, which implies that there is still an equilibrium. We take $\sigma_a^2 = 0.2$ and $\sigma_\pi^2 = 0.01$, which roughly corresponds to an annual standard deviation of 8.2% for stock returns and 1.8% for bond returns, assuming that the returns are serially uncorrelated. Here we follow Campbell and Viceira (2005) who show that returns on stocks are significantly less volatile when the investment horizon is long and inflation risk on nominal bonds increases with the investment horizon. In the initial steady state $\gamma^F = 0.64$, $\gamma^P = 0.95$ and the simple excess return on stocks over bonds is equal to 0.43 percentage points.

The model in this chapter, however, can also be used to analyse the effects on the portfolio choice of consumers and the excess return of stocks over bonds. Figures 5.3 and 5.4 show the change in γ in both countries in the different cases. As explained a larger part of savings has to be invested in the capital stock because savings rise and the debt level of the government is constant, therefore γ rises in all cases. Savings in the funded country increase more in

Figure 5.3: Change in γ^F

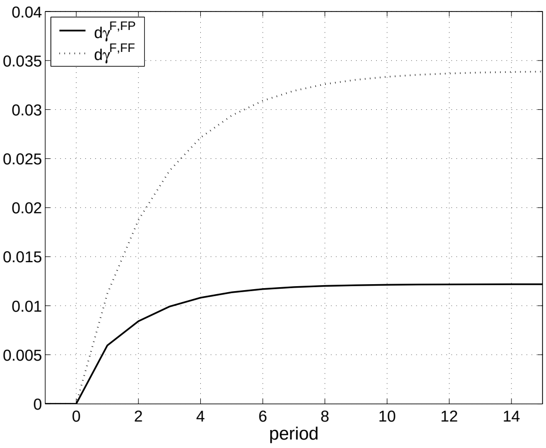
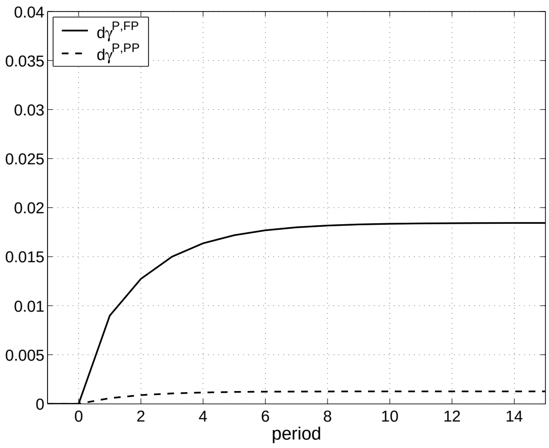


Figure 5.4: Change in γ^P

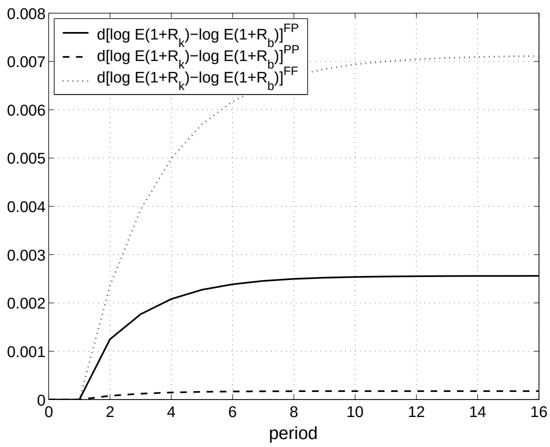


case the other country uses a funded scheme too compared to the case where the other country uses a PAYG scheme (because wages increase more and the fall in the debt tax is larger), therefore the rise in γ will be larger in the former case. The reasoning for the PAYG country is exactly opposite; savings rise less in case the other country also uses a PAYG system instead of a funded scheme, therefore $\mathrm{d}\gamma^{P,PP} < \mathrm{d}\gamma^{P,FP}$. In the benchmark case where the two countries use different pension schemes, however, the rise in γ is larger in the PAYG country than in the funded country, i.e., $\mathrm{d}\gamma^{P,FP} > \mathrm{d}\gamma^{F,FP}$, even though savings in the funded country increase more than in the PAYG country. The intuition for this result is as follows. The change in the rates of return is determined by the total increase in savings in the two countries. The only factor that causes the difference in portfolio choice between the two countries is the safe PAYG pension benefit, which makes people in the PAYG country to behave less risk averse[9]. This implies that for a given rise in total savings, the rise of γ will be larger in the PAYG country.

[9]Actually people in the PAYG country have the same risk aversion parameter as individuals in the funded country, namely $\theta = 1$. However, as shown in Chapter 4 consumers with a safe PAYG pension act as if $0 < \theta < 1$, so they act like a consumer that is less risk averse. People can take more risk when they receive a safe PAYG pension benefit when they are old.

In analysing the effects on the excess return we focus on the log gross excess return, $\log E_t(1 + R_{k,t+1}) - \log E_t(1 + R_{b,t+1})$, as people base their portfolio decisions on the log gross excess return and not on the simple excess return, see equations (5.3) and (5.4)[10]. We know that the log gross excess return has to increase, as γ has to increase to make sure that the capital market clears[11]. Figure 5.5 shows that the relative change in the log gross excess return in the different cases is the same as the relative change in portfolio choice. The log gross excess return is equal to 0.125 in the initial steady state, so in the long run the log gross excess return rises by about 2% (to 0.128) in the benchmark case where the countries use different pension schemes.

Figure 5.5: Change in log excess return



In Figures 5.6 and 5.7 we show the change in utility in the different cases. The graphs are very comparable to the graphs shown in Chapter 2 and the intuition is basically the same. We just repeat the main reasoning for convenience. An increase in longevity has negative utility effects in this model, because individuals have to share total lifetime consumption with more people. Over time the direct negative utility effects are reduced because the capital-labour ratio rises. The relative rise of the capital-labour ratio in the different cases determines the spillover effects. The rise in the capital-labour ratio is smaller in case the funded country is in a common capital market with a PAYG country instead of a funded country, so that the spillovers for the funded country are

[10]Note that the derivative of a log is the percentage change of the respective variable. This implies that the consumer bases his/her portfolio decision on the percentage change of the gross returns.

[11]This does not have to be true for the simple excess return, $E_t R_{k,t+1} - E_t R_{b,t+1}$, see Appendix 5.D for details.

negative in the long run. The short-run spillovers, however, are positive for the funded country. The reason is that these initial generations only incur the losses that result from a falling portfolio return, while they do not enjoy the gains from the higher wages. As the fall in the portfolio return is smaller in case the funded country has a common capital market with a PAYG country these initial generations experience a smaller utility loss. The spillovers for the PAYG country are exactly opposite.

Benchmark

Figure 5.6: Change in U^F

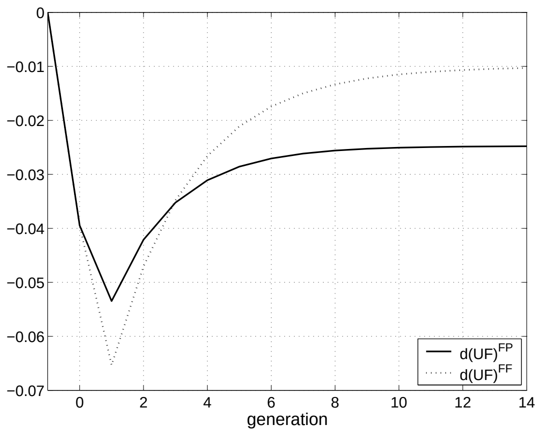
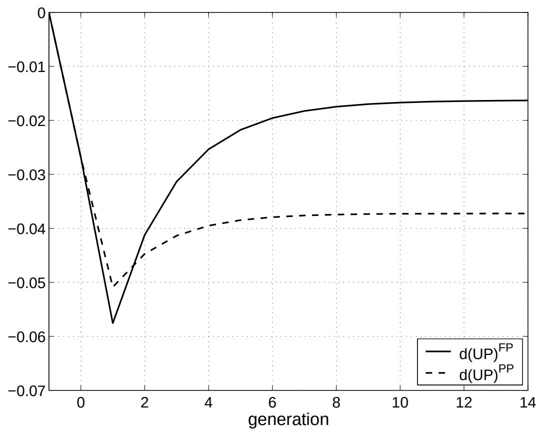


Figure 5.7: Change in U^P



5.4 Government debt

As can be seen from Figures 5.6 and 5.7 especially the initial generations born around the time of the ageing shock experience negative utility effects. The main reason is that these generations only incur the negative effects of the longevity shock (i.e., they share old-age consumption with more people, the return on their savings goes down and they pay higher PAYG contributions) and do not enjoy the positive effect of higher wages. Suppose that the government in the PAYG country wants to compensate these initial generations to some extent by not increasing the PAYG contributions right away. Instead they temporarily use government debt to finance the increased pension obligations after a rise in longevity. This means that the PAYG pension scheme is not balanced for a while, that is, it does *not* hold that $T_t^P = \frac{\varepsilon_t}{1+n_t} Z_t^P$. Assume that the government does not adjust the PAYG contributions T_t^P until period t^*, and that from period t^*+1 onwards the PAYG scheme is balanced again:

$$\frac{\partial T_t^P}{\partial \zeta} = \begin{cases} 0 & \text{for } t = [-1, t^*] \\ \dfrac{Z^P}{1+n} h_t & \text{for } t = [t^*+1, \infty) \end{cases} \tag{5.18}$$

The government can only do this for a certain number of periods, otherwise the government debt dynamics will be unstable. We assume that as soon the government increases pension contributions so that the PAYG scheme is balanced again, the debt tax is increased in such a way that the debt per capita is constant again but at a higher level as in the initial steady state:

$$\frac{\partial T_t^{B,P}}{\partial \zeta} = \begin{cases} \dfrac{B^P}{1+\pi}\dfrac{\partial R_{b,t}^N}{\partial \zeta} & \text{for } t = [-1, t^*] \\ \dfrac{B^P}{1+\pi}\dfrac{\partial R_{b,t}^N}{\partial \zeta} + (R_b - n)\dfrac{\partial B_t^P}{\partial \zeta} & \text{for } t = [t^*+1, \infty) \end{cases} \tag{5.19}$$

The debt dynamics are given by:

$$\frac{\partial B_{t+1}^P}{\partial \zeta} = \frac{1}{1+n}\left[(1+R_b)\frac{\partial B_t^P}{\partial \zeta} + \frac{B^P}{1+\pi}\frac{\partial R_{b,t}^N}{\partial \zeta} + \frac{Z^P}{1+n}h_t - \frac{\partial T_t^P}{\partial \zeta} - \frac{\partial T_t^{B,P}}{\partial \zeta}\right] \tag{5.20}$$

We can follow the calculations in Appendix 5.C, but now $\frac{\partial B_{t+1}^P}{\partial \zeta} > 0$. The dynamic system is the same as in the previous section plus the extra $\frac{\partial B_{t+1}^P}{\partial \zeta}$-terms:

$$\begin{aligned} \frac{\partial K_{t+1}^{FP}}{\partial \zeta} &= \left.\frac{\partial K_{t+1}^{FP}}{\partial \zeta}\right|_{\text{no debt}} \\ &+ \underbrace{\frac{1}{(1+\nu)(1+n) - S_{\sigma_p^2}^P \Delta_{\sigma_p^2} - S_{R_p}^P \Delta_{R_p}}}_{\gtrless 0}\left[\underbrace{\Omega_{B^P}\frac{\partial B_{t+1}^P}{\partial \zeta}}_{8}\underbrace{-(1+n)\frac{\partial B_{t+1}^P}{\partial \zeta}}_{6}\right] \end{aligned} \tag{5.21}$$

$$\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta} = \left.\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta}\right|_{\text{no debt}} + \underbrace{\frac{\gamma^P(1+n)\sigma_{k-b}^2(1+R_b^N)}{\nu S^F + \frac{S^P}{1-z}}\frac{\partial B_{t+1}^P}{\partial \zeta}}_{7} \tag{5.22}$$

where:

$$\begin{aligned} \Omega_{B^P} \equiv \frac{\gamma^P(1+n)}{\nu S^F(1-z) + S^P}\Big\{ & S_{R_p}^P(1-\gamma^P)E(1+R_b)(1-z)\sigma_{k-b}^2 \\ & - S_{R_p}^P[E(R_k) - E(R_b)] - S_{\sigma_p^2}^P(2\gamma^P\sigma_{k-b}^2 + 2\sigma_{k-b,b})\Big\} \gtrless 0 \end{aligned} \tag{5.23}$$

The direct effects of the longevity shock are the same as in the previous section and in the discussion below we concentrate on the extra effects of government debt. As soon as the government uses government debt to finance the higher pension obligations instead of increasing the contributions to the

PAYG scheme, this has a direct crowding-out effect on the capital stock. This crowding-out effect is indicated by a 6 in equation (5.21) and implies that the capital-labour ratio increases less after the ageing shock.

To finance the higher level of government debt, the nominal interest rate on government debt will rise (fall less) to induce people to invest a larger part of their savings in government bonds. This effect is indicated by a 7 in equation (5.22). The higher interest rate on government debt increases the debt tax in both countries. The debt tax in the PAYG country also increases because the level of debt is higher (see equation (5.19)). Both the lower wages that result from the smaller capital-labour ratio and the higher debt tax affect savings in both countries negatively. This implies that in the long run both the rise of the capital-labour ratio and the fall in the nominal interest rate are less after a rise in longevity in case government debt increases.

Consumers have to invest less in stocks in order to finance the higher level of government debt. The fall in γ^P will be even larger in case the PAYG country does not have integrated capital markets with a funded country, as they have to absorb the whole government debt by themselves. To make γ fall, the log excess return on stocks over bonds will fall.

Equation (5.21) shows that also a positive effect of government debt on the capital stock occurs (effect 8). This effect is due to the higher portfolio return and the lower variance on the portfolio, which has fallen because people invest relatively less in stocks. A higher portfolio return decreases the net present value of the pension benefit and therefore affects savings in the PAYG country positively. A lower variance on the portfolio also increases savings in the PAYG country. This implies that these effects via σ_p^2 and $E(R_p)$ dampen the direct negative crowding-out effect of debt on the capital-labour ratio. It is unlikely however, that these indirect effects turn around the effect on the capital-labour ratio as they are of a second-order nature. Simulations show that it is indeed the case that the direct negative crowding-out effect of debt on the capital-labour ratio dominates the indirect positive effects of debt on the capital-labour ratio.

In Figures 5.8 and 5.9 we show the effects on utility in case the PAYG contributions are only adjusted one period after longevity increased. This means that the level of government debt increases at $t = 2$ and stays at this higher level afterwards. The lines with the bullets show the utility effects of this policy scenario. For convenience we also show the lines of the scenario discussed in the previous section where no government debt was used, these are the thin grey lines without bullets.

Debt

Figure 5.8: Change in U^F

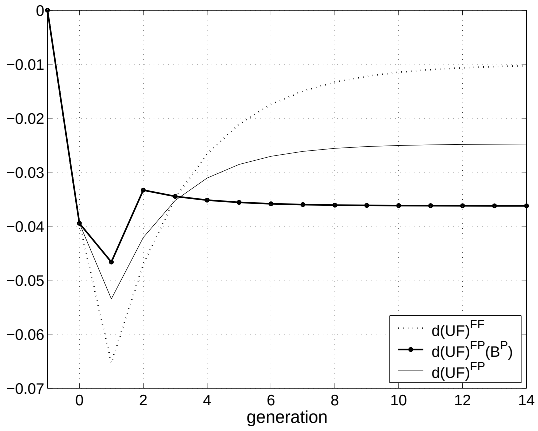
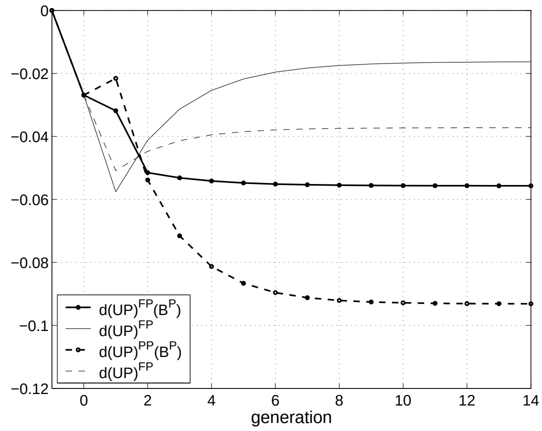


Figure 5.9: Change in U^P



Notes: The thin grey lines indicate the utility effects in the benchmark case where no government debt is used. The dotted line in Figure 5.8 refers to the case where both countries use a funded scheme, while the striped lines in Figure 5.9 denote the case where both countries rely on PAYG pensions. The solid lines demonstrate the utility effects in case one country uses a funded scheme while the other country has PAYG pensions. The lines with the bullets show the effects in case government debt is used by the PAYG country.

The utility effects in case both countries use a funded pension scheme (the dotted line in Figure 5.8) do not change, as government debt is only used by the PAYG country. The distance between the dotted line and the solid line with the bullets shows the spillover effects for the funded country in case the PAYG country uses government debt. Comparing the distance between these two lines with the distance between the dotted and the thin grey solid line (showing the spillover effects in the benchmark case discussed in the previous section) indicates that both the short-run and long-run spillovers are larger for the funded country. The positive spillovers in the short run are larger because the fall in the portfolio return is smaller in case the PAYG country uses government debt that crowds out capital (higher R_k) and makes it more difficult to finance the debt (higher R_b). The negative long-run spillover effects of the PAYG scheme are also larger, however, as the rise in wages is smaller.

Figure 5.9 shows the utility effects for the PAYG country in the different cases. The thin grey solid and -striped lines indicate the effects in the benchmark case, while the solid and striped lines with the bullets point out what happens in case government debt is used. First note that using government debt at the time of the longevity shock ($t = 1$) instead of increasing PAYG contributions makes people born at $t = 1$ better off; the fall in utility is less. Future generations lose, however, as the capital stock is crowded out. This policy can be implemented by governments that put more weight on the welfare of current generations than future generations' welfare.

The long-run utility losses are much smaller in case the PAYG country shares one capital market with a funded country, as the funded country absorbs part of the extra government debt so that the crowding out of capital is less. This implies that it is easier to implement such a policy in case the PAYG country forms a monetary union with a country that uses a funded system instead of a PAYG scheme. Actually, by comparing the distance between the solid and striped lines in case government debt is used (lines with bullets) with the distance between these two lines where no government debt is used (lines without bullets), shows that the positive long-run spillovers of being integrated with a funded country are larger in case government debt is used.

5.5 Unexpected inflation

In case government debt is very large and this debt is denominated in nominal terms, the government has an incentive to put pressure on the central bank to create surprise inflation as this will erode the real value of government debt and lower the real return on government bonds. In this section we will analyse the effects of an unexpected inflation shock. The idea behind this is that the PAYG country has an incentive to lobby for unexpected inflation after it created government debt to cope with the ageing costs. First we consider the situation where the other country also uses a PAYG scheme. This implies that there are no capital flows between the two countries and can therefore be regarded as a closed economy. In that case the gain for the government is exactly high enough to compensate the elderly that experience a loss when inflation rises unexpectedly. So in a closed economy unexpected inflation is only a matter of redistribution between the old and the young (or future generations) and can be implemented in a neutral way (so that no generation loses). In an open economy where the other country uses a funded scheme, however, unexpected inflation affects the PAYG country positively, while the funded country is affected negatively. The reason for this result is that the funded country exports financial capital to the PAYG country and individuals in the funded country have a more conservative portfolio. This implies that residents of the funded country own a relatively large part of the government bonds, so they are affected more negatively by the unexpected inflation shock. The government in the funded country cannot fully compensate its residents for this loss. To isolate the effects of an unexpected inflation shock, we leave out the effects of ageing in the analytical analysis below. In the simulation graphs, however, we combine the ageing and unexpected inflation shock.

5.5.1 Closed economy

First define the inflation factor:

$$\beta_t = \frac{1}{1+\pi_t} \tag{5.24}$$

Suppose that the time pattern of β_t is as follows:

$$\beta_t = \beta + \zeta g_t \tag{5.25}$$

where $g_t < 0$ in case of inflation. By taking the derivative with respect to ζ we analyse the effects of inflation.

In case of unexpected inflation there will be a difference between the expected real return on government bonds and the realised real return:

$$\begin{aligned}
\frac{\partial R_{b,t+1}}{\partial \zeta} &= (1+R_b^N)\frac{\partial \beta_{t+1}}{\partial \zeta} + \beta \frac{\partial R_{b,t+1}^N}{\partial \zeta} && (5.26)\\
\frac{\partial E_t R_{b,t+1}}{\partial \zeta} &= (1+R_b^N)\frac{\partial E_t \beta_{t+1}}{\partial \zeta} + \beta \frac{R_{b,t+1}^N}{\partial \zeta} && (5.27)\\
\frac{\partial R_{b,t+1}}{\partial \zeta} &= \frac{\partial E_t R_{b,t+1}}{\partial \zeta} + (1+R_b^N)\left[\frac{\partial \beta_{t+1}}{\partial \zeta} - \frac{\partial E_t \beta_{t+1}}{\partial \zeta}\right] && (5.28)
\end{aligned}$$

where we assumed that in steady state $\beta = E(\beta)$, so the steady state value of the inflation factor is equal to its expected value. In case people do not expect any inflation, i.e., $\frac{\partial E_t \beta_{t+1}}{\partial \zeta} = 0$ and unexpected inflation is created $\frac{\partial \beta_{t+1}}{\partial \zeta} < 0$, the real return on bonds will fall, while nothing has happened with the expected return.

We assume that consumers have static expectations; their inflation expectation is based on the current inflation rate:

$$\frac{\partial E_t \beta_{t+1}}{\partial \zeta} = \frac{\partial \beta_t}{\partial \zeta} \tag{5.29}$$

This implies that inflation can only be raised unexpectedly during one period, after that people adjust their inflation expectations. Suppose that inflation rises unexpectedly at time τ, then:

$$\frac{\partial \beta_{\tau+1}}{\partial \zeta} = \frac{\partial \beta_{\tau}}{\partial \zeta} = \frac{\partial E_{\tau}\beta_{\tau+1}}{\partial \zeta} \tag{5.30}$$

so the inflation expectations for period $\tau + 1$ are correct again.

The result of this policy is that the real interest rate on government debt is lower, which affects the financial position of the government positively, but will harm people that own government bonds. There are various options for the government how to use this interest gain. Firstly, it can give the benefit to the young people by lowering the debt tax in the same period. Second, it can repay part of its debt so that future generations gain. In both these scenarios however, the elderly alive at the time of the unexpected inflation episode will lose. The third policy option would therefore be to compensate these elderly. We will analyse this last policy option.

The compensation to the elderly is denoted by $Z_t^{B,P}$. In case the elderly receive the whole advantage of a lower real interest rate on government debt the change of $Z_t^{B,P}$ is equal to:

$$\frac{\partial Z_{\tau}^{B,P}}{\partial \zeta} = -\frac{1+n}{\varepsilon}\, B^P(1+R_b^N)\,\frac{\partial \beta_{\tau}}{\partial \zeta} \tag{5.31}$$

where we used the fact that at time τ only a fraction ε of young people at $\tau - 1$ reached period τ and the population grows with n. The change in old-age consumption is:

$$\begin{aligned}
\frac{\partial C_{\tau}^{o,P}}{\partial \zeta} &= \frac{S^P}{\varepsilon}\frac{\partial R_{p,\tau}}{\partial \zeta} + \frac{\partial Z_{\tau}^{B,P}}{\partial \zeta} \\
&= \frac{S^P}{\varepsilon}(1-\gamma^P)(1+R_b^N)\frac{\partial \beta_{\tau}}{\partial \zeta} - \frac{B^P(1+n)(1+R_b^N)}{\varepsilon}\frac{\partial \beta_{\tau}}{\partial \zeta} \\
&= \frac{1+R_b^N}{\varepsilon}\Big[S^P(1-\gamma^P) - B^P(1+n)\Big]\frac{\partial \beta_{\tau}}{\partial \zeta}
\end{aligned} \tag{5.32}$$

In the closed-economy case it has to hold that $S^P(1-\gamma^P) = (1+n)B^P$, so that $\frac{\partial C_{\tau}^{o,P}}{\partial \zeta} = 0$ and the elderly are exactly compensated. Young people at time τ know that inflation is higher and will ask a higher nominal rate of return on government bonds, and the real rate of return will be back at its old value. Combined with the fact that government debt does not change, this implies that an unexpected inflation shock does not have any real effects. So indeed the government can set its policy in such a way that no generation is hurt after an unexpected rise in inflation. The gain of the lower real interest rate on government debt is exactly high enough to compensate the people who lose from the unexpected inflation shock.

5.5.2 Open economy

In this subsection we analyse the effects of unexpected inflation in the open-economy case where one country uses a PAYG pension scheme, while the other country relies on funded pensions. First we consider the scenario discussed in the previous subsection where the whole interest gain is used to compensate the old at the time of the shock. It turns out that in an open economy where the PAYG country imports capital from the funded country, the interest gain for the PAYG country is larger than the loss of the old owners of capital, so that these people gain. In the funded country we have exactly the opposite result, the interest gain is not high enough to fully compensate the elderly, so they lose. In the second scenario we consider the effects of unexpected inflation where the current old are fully compensated and the rest of the gain/loss is transferred to future generations in the form of a lower/higher debt tax.

Scenario 1

In case a PAYG country has an integrated capital market with a funded country, the policy described in the previous subsection will have real effects. Equation (5.32) shows the change in old-age consumption in the PAYG country in case the whole interest gain is transferred to the old at time τ. In the open-economy case equation (5.11) has to hold. Because residents of the funded country save more and invest a larger part of their savings in government bonds than people living in the PAYG country, we know that $S^F(1-\gamma^F) > S^P(1-\gamma^P)$. Defining $S^F(1-\gamma^F) \equiv \alpha S^P(1-\gamma^P)$ where $\alpha > 1$, we can use equation (5.11) to derive[12]:

$$S^P(1-\gamma^P) = \underbrace{\frac{1+\nu}{1+\nu\alpha}}_{<1}(1+n)B \tag{5.33}$$

[12] We assume that the two countries have the same level of government debt in the initial steady state, i.e., $B^F = B^P = B$.

so that $S^P(1-\gamma^P) < (1+n)B$ and the term between brackets in equation (5.32) is smaller than zero. Combining this result with the fact that $\frac{\partial \beta_\tau}{\partial \zeta} < 0$ in case of unexpected inflation, we see that old-age consumption at time τ rises. So there is at least one generation in the PAYG country that gains from the unexpected inflation shock, while no other generation loses.

In the funded country the government also gains from the lower real interest rate on government debt at time τ. Suppose that this government follows the same strategy as the government in the PAYG country, that is, the gain of the lower real interest rate is used to compensate the elderly at the time of the inflation shock:

$$\frac{\partial Z_\tau^{B,F}}{\partial \zeta} = -\frac{1+n}{\varepsilon} B^F(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} \tag{5.34}$$

The change of old-age consumption at τ is:

$$\begin{aligned} \frac{\partial C_\tau^{o,F}}{\partial \zeta} &= \frac{S^F}{\varepsilon}\frac{\partial R_{p,T}}{\partial \zeta} + \frac{\partial Z_\tau^{B,F}}{\partial \zeta} \\ &= \frac{1+R_b^N}{\varepsilon}\Big[S^F(1-\gamma^F) - B^F(1+n)\Big]\frac{\partial \beta_\tau}{\partial \zeta} \end{aligned} \tag{5.35}$$

Using the same method as for the PAYG country above, we can write:

$$S^F(1-\gamma^F) = \underbrace{\frac{1+\nu}{\frac{1}{\alpha}+\nu}}_{>1}(1+n)B \tag{5.36}$$

which implies that $S^F(1-\gamma^F) > (1+n)B$ and the term between brackets in equation (5.35) is positive. This means that the effects of an unexpected inflation shock are still negative for the elderly, that is, $\frac{\partial C_\tau^{o,F}}{\partial \zeta} < 0$. Even though the government in the funded country uses the whole interest gain to compensate the pensioners, they still experience welfare losses.

So in an open economy the PAYG country gains from an unexpected inflation shock at the expense of the funded country. It takes advantage of the fact that the funded country owns a relatively large part of the total supply of government bonds in the two economies. It is actually the case that:

$$\frac{\partial C_\tau^{o,P}}{\partial \zeta} = -\nu\frac{\partial C_\tau^{o,F}}{\partial \zeta} \tag{5.37}$$

so the effects on old-age consumption are opposite to each other. In case the two countries are of equal size, i.e., if $\nu = 1$, the gain of the elderly in the

PAYG country is exactly as large as the loss of the elderly in the funded country. If $\nu > 1$, the funded country is relatively large and there will be more capital flows to the PAYG country. The larger ν, the more the PAYG country can profit from these capital flows and the larger the gain of unexpected inflation is. There is no possibility, however, that the PAYG country compensates the funded country for its loss and the union as a whole experiences a Pareto improvement. It is true that the gain in old-age consumption in the PAYG country is larger than the loss of the old in the funded country in case $\nu > 1$, but this is exactly offset by the fact that the funded country has more inhabitants.

Scenario 2

Instead of transferring the whole gain of the lower interest rate to the elderly alive at the time of the shock, the government in the PAYG country can also decide to compensate those elderly in such a way that they do not experience any welfare gains or losses. This means that these people get full compensation but do not gain as was the case when the whole advantage was transferred to them. The rest of the gain is used to pay off some of the government debt, to decrease the debt burden for future generations. The compensation is such that the consumption of the elderly at τ does not change:

$$\frac{\partial Z_{\tau}^{B,P}}{\partial \zeta} = -\frac{S^P}{\varepsilon}(1-\gamma^P)(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} \tag{5.38}$$

The fall in government debt is then:

$$\frac{\partial B_{\tau+1}^{P}}{\partial \zeta} = \frac{1+R_b^N}{(1+n)^2}\left[B^P(1+n) - S^P(1-\gamma^P)\right]\frac{\partial \beta_\tau}{\partial \zeta} \tag{5.39}$$

Suppose that from period $\tau + 1$ onwards the government debt is kept constant again, but then at this lower level. Future generations indeed profit from the lower debt burden because the interest obligations on the debt and thus the debt tax paid by the young decrease:

$$\frac{\partial T_{\tau+1}^{B,P}}{\partial \zeta} = (R_b - n)\frac{\partial B_{\tau+1}^{P}}{\partial \zeta} \tag{5.40}$$

Suppose that the government in the funded country also fully compensates the elderly at time τ:

$$\frac{\partial Z_{\tau}^{B,F}}{\partial \zeta} = -\frac{S^F}{\varepsilon}(1-\gamma^F)(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} \tag{5.41}$$

As the gain of the lower interest rate was smaller then the loss of the elderly, the funded country has to use government debt to be able to give full compensation. The rise in government debt is equal to:

$$\frac{\partial B^F_{\tau+1}}{\partial \zeta} = \frac{1+R^N_b}{(1+n)^2}\left[B^F(1+n) - S^F(1-\gamma^F)\right]\frac{\partial \beta_\tau}{\partial \zeta} \tag{5.42}$$

so that future generations have to pay a higher debt tax:

$$\frac{\partial T^{B,F}_{\tau+1}}{\partial \zeta} = (R_b - n)\,\frac{\partial B^F_{\tau+1}}{\partial \zeta} \tag{5.43}$$

The total amount of government debt in the two countries does not change:

$$\frac{\partial B^P_{\tau+1}}{\partial \zeta} + \nu\frac{\partial B^F_{\tau+1}}{\partial \zeta} = \frac{1+R^N_b}{(1+n)^2}\left[B^P(1+n) - S^P(1-\gamma^P) + \nu B^F(1+n) - \nu S^F(1-\gamma^F)\right]\frac{\partial \beta_\tau}{\partial \zeta} \tag{5.44}$$

because the term between the brackets equals zero as can be seen from equation (5.11). So the rise of government debt in the funded country and the fall in government debt in the PAYG country exactly cancel. This implies that the capital-labour at $\tau+1$ will not change.

In the next period people adjust their inflation expectations and demand a higher nominal rate of return:

$$\frac{\partial R^N_{b,\tau+1}}{\partial \zeta} = -\frac{1+R^N_b}{\beta}\frac{\partial E_\tau \beta_{\tau+1}}{\partial \zeta} \tag{5.45}$$

so that the real rate of return is back at its old level. The debt tax in the PAYG country is lower at $\tau+1$, which affects savings positively, while the debt tax is higher for inhabitants of the funded country, which decreases their savings. So it is ambiguous what happens to the capital-labour ratio at $\tau+2$. Simulations show, however, that these effects more or less cancel and that the effect on the capital-labour ratio is negligible. This implies that the effect on the debt tax determines the long-run welfare effects of this policy. Future generations in the funded country lose and people in the PAYG country gain.

The utility effects of an ageing shock combined with government debt and the policy described here are shown in Figures 5.10 and 5.11 by the lines with the diamonds. Inflation rises unexpectedly at time $t = 7$. Comparing the

Unexpected inflation

Figure 5.10: Change in U^F

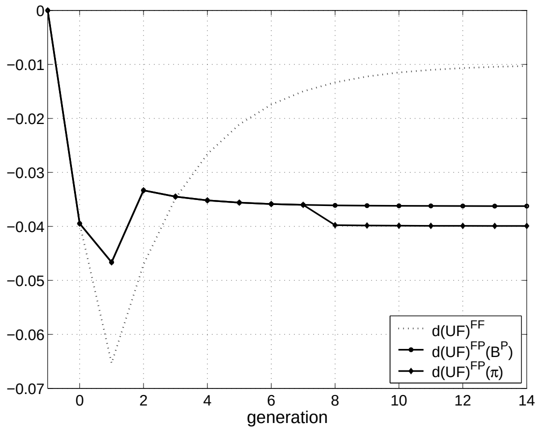
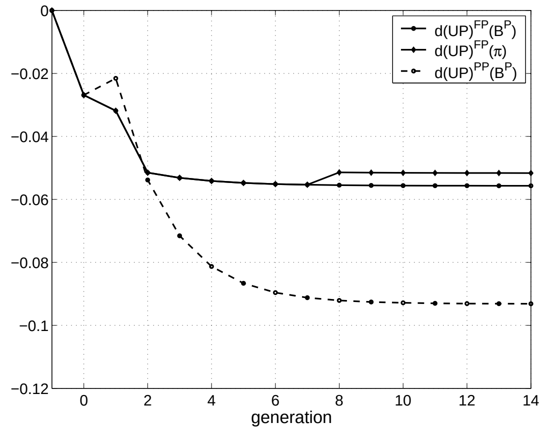


Figure 5.11: Change in U^P



Notes: The dotted line in Figure 5.10 refers to the case where both countries use a funded scheme, where no government debt is used and no surprise inflation is created. The solid lines demonstrate the utility effects in case one country uses a funded scheme while the other country has PAYG pensions. The lines with the bullets show the effects in case government debt is used by the PAYG country, while the lines with the diamonds indicate the effects where not only government debt is used but also unexpected inflation is created. The inflation rate rises to 3.58 percentage point. The elderly at the time of the unexpected inflation shock are fully compensated and the rest of the gain/loss is transferred to future generations in the form of a lower/higher debt tax. The case where both countries rely on PAYG pensions is denoted by the striped line with the bullets in Figure 5.11. As explained in the main text there are no capital flows between the two countries in that case and the gain of surprise inflation is exactly high enough to compensate the elderly. Therefore the line with the diamonds (unexpected inflation) coincides with the line with the bullets (only government debt) in case both countries have a PAYG pension scheme.

utility effects of this scenario with those of the scenario described in the previous section where only government debt was used (the lines with the bullets), shows indeed that the generations after the unexpected inflation shock gain in the PAYG country and lose in the funded country. This implies that the negative long-run spillovers of the PAYG scheme for the funded country become larger.

The two scenarios discussed in this subsection show that the governments in both countries can decide how the total gain or loss of unexpected inflation is shared between generations. The main result is, however, that unexpected inflation is always advantageous for capital-importing PAYG countries, while in funded countries there are always some generations that lose. The gains of unexpected inflation for PAYG countries will be larger the larger the debt burden is. In the coming decades the public finances will be put more under pressure because of the ageing of the population. In case government debt is used to cope with the ageing costs, the incentive of PAYG countries to put pressure on the central bank to accommodate and create surprise inflation will rise. On the other hand will unexpected inflation harm funded countries more if PAYG countries issued large amounts of nominal government debt. The conflict of interest on monetary policy between PAYG- and funded countries will therefore be reinforced if population ageing raises government debt in PAYG countries. If the decision-making process of the central bank is not completely transparent and it is not clear how the central bank will react to these conflicting interests, inflation risk may rise when public debt levels are high. This scenario will be analysed in the next section.

5.6 Inflation risk

In the previous section we showed that PAYG governments have an incentive to lobby for surprise inflation when their debt burden is large. If it is not completely clear how monetary policy is determined inflation risk may rise, that is, inflation risk rises with the level of government debt[13]. In this section we will analyse this scenario. This means that inflation risk is a function of the level of government debt in the PAYG country:

$$\sigma^2_{\pi t}\left(B^P_{t+1}\right) = \lambda B^P_{t+1} \tag{5.46}$$

where λ shows the responsiveness of inflation risk to government debt, so when government debt changes inflation risk also changes:

$$\frac{\partial \sigma^2_{\pi t}}{\partial \zeta} = \lambda \frac{\partial B^P_{t+1}}{\partial \zeta} \tag{5.47}$$

Government debt in the funded country does not affect inflation risk as the funded country does not have an incentive to lobby for unexpected inflation.

Inflation risk has a direct effect on the portfolio choice of consumers and the variance of the portfolio (see equations (4.113), (4.131)-(4.133) in Chapter 4).

[13]The increase in riskiness of government bonds in case government debt rises can also be interpreted as a rise in default risk.

Following the calculations in Appendix 5.C we get the following system of equations:

$$\frac{\partial K^{FP}_{t+1}}{\partial \zeta} = \left.\frac{\partial K^{FP}_{t+1}}{\partial \zeta}\right|_{\text{debt}} + \underbrace{\frac{\Omega_{\sigma_b^2}}{(1+\nu)(1+n) - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p}}\frac{\partial \sigma^2_{bt}}{\partial \zeta}}_{10} \tag{5.48}$$

$$\frac{\partial R^{N,FP}_{b,t+1}}{\partial \zeta} = \left.\frac{\partial R^{N,FP}_{b,t+1}}{\partial \zeta}\right|_{\text{debt}} + \underbrace{\frac{[(1-\gamma^P)S^P + \nu S^F(1-\gamma^F)](1+R^N_b)}{\nu S^F + \frac{S^P}{1-z}}\frac{\partial \sigma^2_{bt}}{\partial \zeta}}_{9} \tag{5.49}$$

where:

$$\begin{aligned}\Omega_{\sigma_b^2} \equiv \frac{(1-\gamma^P)S^P + \nu S^F(1-\gamma^F)}{(\nu S^F(1-z) + S^P)\sigma^2_{k-b}} \Big\{ & S^P_{R_p}(1-\gamma^P)E(1+R_b)(1-z)\sigma^2_{k-b} \\ & - S^P_{R_p}[E(R_k) - E(R_b)] - S^P_{\sigma_p^2}(2\gamma^P\sigma^2_{k-b} + 2\sigma_{k-b,b})\Big\} \\ & + S^P_{R_p}\frac{[E(R_k) - E(R_b)](1-\gamma^P)}{\sigma^2_{k-b}} + S^P_{\sigma_p^2}(1-\gamma^P)\left(1 + \gamma^P + \frac{2\sigma_{k-b,b}}{\sigma^2_{k-b}}\right)\end{aligned} \tag{5.50}$$

and:

$$\frac{\partial \sigma^2_{bt}}{\partial \zeta} = \frac{\partial \sigma^2_{\pi t}}{\partial \zeta} \tag{5.51}$$

For a given rise in the riskiness of government bonds people would like to switch to stocks and this effect will be stronger in the funded country. The intuition for this result is twofold. First, people in the funded country do not receive a safe PAYG pension benefit during retirement, which makes them relatively more risk averse (compare the denominators of equations (5.3) and (5.4)). Second, in the initial steady state inhabitants of the funded country own a relatively large part of the total amount of government debt in the two countries because they save more and invest a larger part of their savings in government bonds than the PAYG country. The upward pressure on γ causes the nominal rate of return on government bonds to rise to make sure that government debt is financed. This is effect 9 in equation (5.49) and the rise of R^N_b has to be larger in case the PAYG country has integrated capital markets with a country that uses a funded pension scheme instead of PAYG pensions.

The rise in R^N_b increases the debt tax and implies lower savings, which affects the capital-labour ratio negatively. A lower capital-labour ratio in turn, affects wages negatively so that savings continue to decrease. As can be seen from equation (5.48) there is an extra effect 10 on the capital-labour ratio. This is the indirect (second-order) effect of changes in $(\sigma^2_{pt})^P$ and $E_t R^P_{p,t+1}$, which affect savings in the PAYG country. See Section 4.4.3 in Chapter 4 for more details.

The higher riskiness of government bonds makes bonds much less attractive to hold and the return on government bonds has to rise to a much larger extent than the return on stocks to make sure that the capital markets clears. This implies that the excess return of stocks over bonds will fall.

The fall in savings implies that γ has to fall in case both countries use a PAYG scheme. When the two countries use different pension schemes the effects on portfolio choice are not that straightforward, however. The equilibrium condition on the bond market, equation (5.11), shows that the part of savings invested in bonds in the two countries together $((1-\gamma^P)+(1-\gamma^F))$ has to rise in case total savings (S^P+S^F) decline. Simulations show that the movement of γ^F is opposite to the movement of γ^P. People in the funded country move to stocks; they move away from the asset that becomes more risky. People in the PAYG country, on the other hand, move to government bonds. Consumers in the funded country invest more in stocks even though the excess return on stocks falls, so the direct effect of the higher risk on bonds dominates for these people. They can do this because people in the PAYG country put less weight on risk in their portfolio decision and decrease γ^P in response to the lower excess return.

In Figures 5.12 and 5.13 we compare the utility effects of a longevity shock in the different cases. The dotted line in Figure 5.12 shows the utility effects for the funded country when it has an integrated capital market with a country that uses a funded scheme too. In that case no government debt is created and inflation risk will not rise. The solid lines show the effect in case one country uses a funded system and the other country relies on PAYG pensions. The bullets refer to the case where only government debt is used, whereas the diamonds denote the case where inflation risk also rises. The spillovers are determined by the relative change of the capital-labour ratio. A rise in inflation risk increases the interest rate on government debt, which implies a higher debt tax and lower savings. This results in a smaller capital-labour ratio and lower wages and utility is negatively affected[14]. This implies that the negative spillover effects in the long run increase for the funded country.

[14]Higher inflation risk also has a direct negative effect on utility because people are risk averse. This direct effect is negligible compared to the negative utility effects of the higher debt tax, however.

Inflation risk

Figure 5.12: Change in U^F

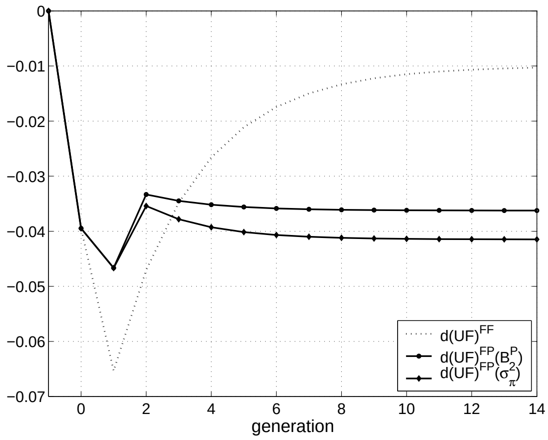
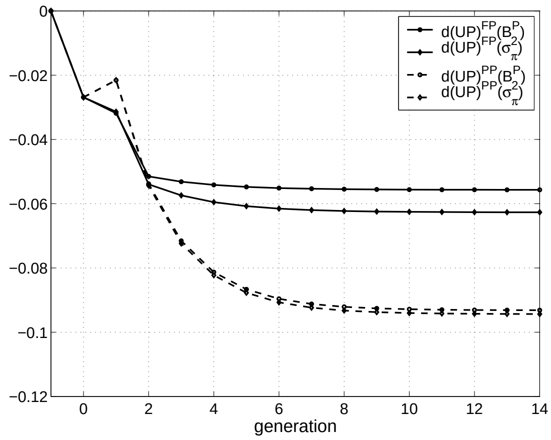


Figure 5.13: Change in U^P



Notes: The dotted line in Figure 5.12 refers to the case where both countries use a funded scheme, where no government debt is used and inflation risk does not rise. The case where both countries rely on PAYG pensions is denoted by the striped lines in Figure 5.13. The solid lines demonstrate the utility effects in case one country uses a funded scheme while the other country has PAYG pensions. The lines with the bullets show the effects in case government debt is used by the PAYG country, while the lines with the diamonds indicate the effects where not only government debt is used but inflation risk also rises. Inflation risk (σ_π^2) rises with 0.08 to 0.09.

Figure 5.13 shows that the change in utility effects in the PAYG country after a rise in inflation risk is much larger in case the PAYG country has a common capital market with a funded country (the solid lines), compared to the case where it shares one capital market with a country that also uses a PAYG scheme (the striped lines). This means that higher inflation risk affects residents in the PAYG country more negatively in case they form a monetary union with a funded country. The reason for this result is that in that case a large part of government bonds is in the hands of citizens of the funded country and because these people are more risk averse, the rise of the nominal interest rate on government bonds has to be much larger compared to the case where both countries rely on PAYG pensions. This implies that the effects on the debt tax and thus the negative effects on savings and the capital-labour ratio are also much larger. This shows that the PAYG country cannot share the adverse effects of higher inflation risk with the funded country, which was the case for example with the use of government debt. The positive long-run spillovers of having a common capital market with a funded country are therefore smaller for people in the PAYG country.

In contrast to unexpected inflation, a rise in inflation risk has adverse effects in both countries of the monetary union. Inflation risk rises in case PAYG countries have a large debt burden, which results in conflicting interests between PAYG- and funded countries about the creation of inflation. If the decision-making process about monetary policy is not completely transparent, it is not clear what the final outcome will be for inflation and this uncertainty raises inflation risk. The fact that both funded- and PAYG countries lose from a rise in inflation risk shows that it may be in the interest of both countries that government debt levels stay at low levels. In case government bonds are treated as more risky when debt levels are high, there is a trade-off for the PAYG country between the short-run benefits of debt and the increase in inflation risk this debt induces. This implies that not only funded countries in the EMU benefit from the rules in the Stability and Growth Pact, but that PAYG countries may benefit from these rules as well. Moreover, the independence, credibility and transparency of the ECB is important for all countries to prevent an increase in inflation risk.

5.7 Conclusion

The ageing of the population will confront governments in developed countries with serious challenges. As a large number of countries rely on pension schemes that are financed on the basis of pay-as-you-go, the rise of the number of retirees relative to the number of working people forces governments to think about the way their pension systems are organised. This thesis analyses how countries that rely to a different extent on PAYG pensions affect each other during an ageing episode, where countries with large PAYG pension schemes react in different ways to population ageing. In Chapter 2 we assume that governments with PAYG schemes will keep their PAYG system balanced and the pension benefits or -contributions are adjusted accordingly. It is also often proposed that countries that only rely on PAYG pensions should downsize their pension schemes and move to a more funded scheme. Chapter 3 investigates how such reforms affect countries that already have fully-funded schemes. In this chapter we analyse the international spillover effects of ageing and pensions in case PAYG governments do not increase the contributions to the pension scheme or do not lower the pension benefits for the initial generations that suffer most from ageing. They neither reform their pension system. These PAYG governments use debt instead to spare the generations alive around the time ageing starts. This will benefit these initial generations, but future generations will lose as government debt crowds out capital. The utility effects spill over to the funded country that does not use government debt. The initial generations gain from the higher rates of return, while later generations are affected negatively because wage levels fall. This implies that the use of government debt by the PAYG country increases both the positive short-run spillovers and the negative long-run spillovers for the funded country.

We also show that in a monetary union where a PAYG country has a joint capital market with a funded country, unexpected inflation will affect the PAYG country positively. In a closed economy, unexpected inflation is only a matter of redistribution from the old to the young- or future generations and can be implemented in such a way that no generation gains or loses. The reason why unexpected inflation creates a net gain in the PAYG country when it forms a monetary union with a funded country is that capital flows from the funded country to the PAYG country, so that part of the government bonds of the PAYG country is in the hands of citizens in the funded country. Therefore, part of the costs of unexpected inflation are transmitted to the funded country, while the gain of the lower debt burden does not change and a net gain results. So when a PAYG country forms a monetary union with a funded country it has an incentive to put pressure on the central bank to increase inflation unexpectedly. The gains from unexpected inflation for the PAYG country will be larger the larger the level of government debt is. In the coming decades the ageing of the population will put the public finances more under pressure and when PAYG countries finance their increased pension obligations by issuing more debt, the incentive of PAYG governments to lobby for surprise inflation will rise. It will lead to negative spillovers for the funded country when the central bank gives in.

These opposing utility effects of surprise inflation in the PAYG country and the funded country imply that there are conflicting interests between the two countries about the direction of monetary policy. Because the monetary policy strategy is not completely transparent, it is not clear how the central bank will react to these conflicting interests. As a consequence, inflation risk may rise with the level of government debt. Higher inflation risk makes government bonds less attractive to hold and the rate of return on bonds will have to increase to induce people to buy the existing stock of government bonds. This increases the debt tax and lowers savings and the capital-labour ratio, and both countries experience negative utility effects from the rise in inflation risk. Actually, the PAYG country experiences negative spillover effects from the fact that they form a monetary union with a funded country in case inflation risk increases. This result arises because the funded country owns a relatively large part of the government bonds and people in the funded country are more risk averse as they do not receive a safe PAYG pension benefit. This implies that these people need to be compensated more in order to be willing to hold the more risky government bonds, that is, the rate of return on government bonds has to increase more in case a funded pension scheme is in place. The rise in the debt tax and its negative consequences will therefore be larger in the PAYG country when it forms a monetary union with a country that uses a fully-funded pension scheme instead of a PAYG pension system.

All the scenarios analysed in this chapter show that in the long run funded countries are affected even more negatively by the fact that other countries in the monetary union rely on PAYG-financed pensions when these countries use government debt to cope with the ageing costs. The use of government debt itself has negative long-run spillover effects for funded countries. Moreover a high debt burden may lead to unexpected inflation or a higher inflation risk, which both increase the negative spillover effects even more. In the coming decades it will therefore be important for funded countries that the rules of the Stability and Growth Pact are met, so that debt levels of PAYG countries do not rise to too high levels. The use of government debt by PAYG countries to cope with the costs of ageing benefits the initial generations in these countries that suffer most from ageing. Future generations, however, are harmed by the larger debt burden. In a monetary union part of this burden can be shifted to the funded country. The debt burden for the PAYG country can be reduced by an unexpected rise in inflation when this country successfully lobbies at the central bank. If this lobbying only raises perceived inflation risk, the negative effects of a rise in the riskiness of government bonds in response to a higher level of public debt cannot be shared with the funded country, however. It may therefore also be in the interest of PAYG countries to obey the fiscal constraints stated in the Stability and Growth Pact to prevent that government debt rises to too high levels. Moreover, it is important for all countries that a central bank like the ECB is independent, credible and transparent about its monetary policy strategy, so that it is clear how monetary policy is determined and the risk of inflation does not rise when debt levels are high.

Appendix to Chapter 5

5.A Portfolio choice funded country

Taking logs of the portfolio-return and benchmark-return Euler condition and subtracting gives (see equation (4.64) in Chapter 4):

$$E_t r^F_{p,t+1} - E_t r_{b,t+1} + \frac{1}{2}((\sigma^2_{pt})^F - \sigma^2_{bt}) = \text{Cov}_t(c^{o,F}_{t+1}; r_{p,t+1}) - \text{Cov}_t(c^{o,F}_{t+1}; r_{b,t+1}) \tag{5.52}$$

We can use the second-order Taylor approximation of $r_{p,t+1}$ from the previous chapter to derive:

$$E_t r^F_{p,t+1} \approx E_t r_{b,t+1} + \gamma^F_t (E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma^F_t (1-\gamma^F_t)\sigma^2_{k-b,t} \tag{5.53}$$

$$(\sigma^2_{pt})^F \approx \sigma^2_{bt} + \left(\gamma^F_t\right)^2 \sigma^2_{k-b,t} + 2\gamma^F_t \sigma_{k-b,bt} \tag{5.54}$$

The budget constraint of an individual in a funded country is:

$$C^{O,F}_{t+1} = \frac{1+R^F_{p,t+1}}{\varepsilon_{t+1}} \left[W_t - T^F_t - T^{B,F}_t + \frac{\varepsilon_{t+1} Z^F_{t+1}}{1+R_{p,t+1}} - C^{Y,F}_t\right] \tag{5.55}$$

Using the fact that $Z^F_{t+1} = \frac{1+R_{p,t+1}}{\varepsilon_{t+1}} T^F_t$, equation (5.55) can be written as:

$$C^{O,F}_{t+1} = \frac{1+R^F_{p,t+1}}{\varepsilon_{t+1}} \left[W_t - T^{B,F}_t - C^{Y,F}_t\right] \tag{5.56}$$

Dividing by $C_t^{Y,F}$ gives:

$$\frac{C_{t+1}^{O,F}}{C_t^{Y,F}} = \frac{1+R_{p,t+1}^F}{\varepsilon_{t+1}} \underbrace{\left[\frac{W_t - T_t^{B,F}}{C_t^{Y,F}} - 1\right]}_{g_t} \tag{5.57}$$

In case of a logarithmic utility function people consume a fixed proportion of their wealth, which implies that the term g_t is constant. Taking logs gives:

$$\log C_{t+1}^{O,F} - \log C_t^{Y,F} = \log(1+R_{p,t+1}^F) - \log \varepsilon_{t+1} + \log g_t \tag{5.58}$$

which gives:

$$\begin{aligned} c_{t+1}^{o,F} &= c_t^{y,F} + r_{p,t+1}^F - \log \varepsilon_{t+1} + \log g_t & (5.59) \\ E_t c_{t+1}^{o,F} &= c_t^{y,F} + E_t r_{p,t+1}^F - \log \varepsilon_{t+1} + \log g_t & (5.60) \end{aligned}$$

Equations (5.59) and (5.60) imply that the variance of the log of old-age consumption equals the variance of the log portfolio return, that is, $(\sigma_{c^o t}^2)^F = (\sigma_{pt}^2)^F$, and that:

$$\begin{aligned} \text{Cov}_t(c_{t+1}^{o,F}, r_{p,t+1}^F) &= (\sigma_{pt}^2)^F & (5.61) \\ \text{Cov}_t(c_{t+1}^{o,F}, r_{b,t+1}) &= \sigma_{bt}^2 + \gamma_t^F \sigma_{k-b,bt} & (5.62) \end{aligned}$$

Substituting equations (5.53), (5.54), (5.61) and (5.62) into equation (5.52) gives:

$$\gamma_t^F(E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma_t^F \sigma_{k-b,t}^2 + \gamma_t^F \sigma_{k-b,bt} = (\gamma_t^F)^2 \sigma_{k-b,t}^2 + \gamma_t^F \sigma_{k-b,bt} \tag{5.63}$$

Dividing by γ_t^F and rearranging gives:

$$\gamma_t^F = \frac{E_t r_{k,t+1} - E_t r_{b,t+1} + \frac{1}{2}\sigma_{k-b,t}^2}{\sigma_{k-b,t}^2} \tag{5.64}$$

Or in simple returns:

$$\gamma_t^F = \frac{\log E_t(1+R_{k,t+1}) - \log E_t(1+R_{b,t+1})}{\sigma_{k-b,t}^2} - \frac{\sigma_{k-b,bt}}{\sigma_{k-b,t}^2} \tag{5.65}$$

5.B Trade balance and current account

The balance of trade surplus is the excess of domestic product over domestic absorption. Domestic absorption is the sum of consumption and domestic capital formation[15]:

$$BT_t^i = F_t^i(\cdot) - C_t^{Y,i} - \frac{\varepsilon_t}{1+n_t} C_t^{O,i} - (1+n_{t+1})K_{t+1}^i \tag{5.66}$$

where we take into account that only a fraction ε_t of young people in $t-1$ survived to period t.

The current account surplus is the excess of national product over domestic absorption. National product equals domestic product plus net foreign investment income. Thus, the current account surplus is defined as the balance of trade surplus plus the returns on foreign investments:

$$CA_t^P = BT_t^P + R_{k,t}\left[\frac{\gamma_{t-1}^P S_{t-1}^P}{1+n_t} - K_t^P\right] + R_{b,t}\left[\frac{(1-\gamma_t^P)S_{t-1}^P}{1+n_t} - B_t^P\right] \tag{5.67}$$

$$CA_t^F = BT_t^F + R_{k,t}\left[\frac{\gamma_{t-1}^F S_{t-1}^F}{1+n_t} - K_t^F\right] + R_{b,t}\left[\frac{(1-\gamma_t^F)S_{t-1}^F}{1+n_t} - B_t^F\right] \tag{5.68}$$

Using the budget constraints of the consumers and the government and the fact that in case of perfect competition the following condition has to hold:

$$F_t^i(\cdot) = W_t + (1+R_{k,t})K_t^i \tag{5.69}$$

we can rewrite equations (5.67) and (5.68) to:

$$CA_t^P = S_t^P - \frac{S_{t-1}^P}{1+n_t} - \left[(1+n_{t+1})B_{t+1}^P - B_t^P\right] - \left[(1+n_{t+1})K_{t+1}^P - K_t^P\right] \tag{5.70}$$

$$CA_t^F = S_t^F - \frac{S_{t-1}^F}{1+n_t} - \left[(1+n_{t+1})B_{t+1}^F - B_t^F\right] - \left[(1+n_{t+1})K_{t+1}^F - K_t^F\right] \tag{5.71}$$

These equations show that, because there is no official settlements balance, the current account surplus of a country should be equal to its capital account deficit. Therefore the current account surplus can be viewed as the net foreign investment by the country. Net foreign investment is the excess of domestic wealth accumulation (savings) over domestic capital formation. Market clearing requires that world savings are equal to world investment. In other words, the current account surpluses of the two countries must sum to zero in equilibrium: $CA_t^P + CA_t^F = 0$. When $n_t > 0$, country F exports capital abroad and therefore runs a current account surplus. Conversely, country P imports capital and will run a current account deficit. Moreover, because we assumed

[15]Note that we have to assume that the capital stock depreciates completely in one period to make sure that the gross return on stocks is lognormally distributed.

dynamic efficiency we know that country F runs a steady state trade balance deficit and country P has a steady state trade balance surplus. So country F uses its interest income from its savings abroad to pay for its net imports of commodities, and country P runs a surplus on its trade account to service its external debt.

5.C Ageing

In this appendix we show the derivation of the first-order difference equations in case longevity increases. The PAYG system is balanced, that is, contributions adjust in response to the larger number of elderly. The government does not use government debt to cope with the costs of ageing.

Linearise equation (5.9) with respect to ζ around the initial steady state:

$$\frac{\partial S_t^P}{\partial \zeta} + \nu \frac{\partial S_t^F}{\partial \zeta} = (1+n)(1+\nu)\frac{\partial K_{t+1}^{FP}}{\partial \zeta} + (1+n)\left[\underbrace{\frac{\partial B_{t+1}^P}{\partial \zeta}}_{=0} + \nu \underbrace{\frac{\partial B_{t+1}^F}{\partial \zeta}}_{=0}\right] \tag{5.72}$$

Simplifying gives:

$$\frac{\partial K_{t+1}^{FP}}{\partial \zeta} = \frac{1}{(1+\nu)(1+n)}\left[\frac{\partial S_t^P}{\partial \zeta} + \nu \frac{\partial S_t^F}{\partial \zeta}\right] \tag{5.73}$$

From equation (5.5) we can derive the change of savings in the funded country:

$$\frac{\partial S_t^F}{\partial \zeta} = \frac{(1+\rho)(W - T^{B,F})}{(1+\rho+\varepsilon)^2}\frac{\partial \varepsilon_{t+1}}{\partial \zeta} + \frac{\varepsilon}{1+\rho+\varepsilon}\left(\frac{\partial W_t}{\partial \zeta} - \frac{\partial T_t^{B,F}}{\partial \zeta}\right) \tag{5.74}$$

The total derivative of savings in the PAYG country is[16]:

$$\begin{aligned}\frac{\partial S_t^P}{\partial \zeta} = {} & \frac{\partial S_t^P}{\partial \varepsilon_{t+1}}\frac{\partial \varepsilon_{t+1}}{\partial \zeta} + \frac{\partial S_t^P}{\partial (\sigma_{pt}^2)^P}\frac{\partial (\sigma_{pt}^2)^P}{\partial \zeta} + \frac{\partial S_t^P}{\partial W_t}\frac{\partial W_t}{\partial \zeta} + \frac{\partial S_t^P}{\partial T_t^{B,P}}\frac{\partial T_t^{B,P}}{\partial \zeta} \\ & + \frac{\partial S_t^P}{\partial T_t^P}\frac{\partial T_t^P}{\partial \zeta} + \frac{\partial S_t^P}{\partial E_t R_{p,t+1}^P}\frac{\partial E_t R_{p,t+1}^P}{\partial \zeta}\end{aligned} \tag{5.75}$$

[16] As in Chapter 4 we do not take into account the second-order effects on z_t, as this complicates the analysis to a large extent.

Using equation (5.6) we can derive:

$$\frac{\partial S_t^P}{\partial \varepsilon_{t+1}} = \frac{(1+\rho)\exp(\frac{1}{2}z^2(\sigma_p^2)^P)(W - T^P - T^{B,P})}{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon + 1 + \rho\right]^2} - \frac{(1+\rho)^2 Z^P}{E(1+R_p^P)\exp(-\frac{1}{2}(\sigma_p^2)^P)\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon + 1 + \rho\right]^2} \gtrless 0 \tag{5.76}$$

$$\frac{\partial S_t^P}{\partial (\sigma_{pt}^2)^P} = -\frac{\frac{1}{2}(1+\rho)\varepsilon Z^P}{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon + 1 + \rho\right] E(1+R_p^P)\exp(-\frac{1}{2}(\sigma_p^2)^P)} + \frac{\frac{1}{2}z^2(1+\rho)\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon}{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon + 1 + \rho\right]^2}\left[W - T^P - T^{B,P} + \frac{\varepsilon Z^P}{E(1+R_p^P)\exp(-\frac{1}{2}(\sigma_p^2)^P)}\right] \lessgtr 0 \tag{5.77}$$

$$\frac{\partial S_t^P}{\partial W_t} = \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon}{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon + 1 + \rho} > 0 \tag{5.78}$$

$$\frac{\partial S_t^P}{\partial T_t^{B,P}} = \frac{\partial S_t^P}{\partial T_t^P} = -\frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon}{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon + 1 + \rho} < 0 \tag{5.79}$$

$$\frac{\partial S_t^P}{\partial E_t R_{p,t+1}^P} = \frac{(1+\rho)\varepsilon Z^P \exp(-\frac{1}{2}(\sigma_p^2)^P)}{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon + 1 + \rho\right]\left[E(1+R_p^P)\exp(-\frac{1}{2}(\sigma_p^2)^P)\right]^2} > 0 \tag{5.80}$$

Equation (5.12) gives:

$$\frac{\partial \varepsilon_{t+1}}{\partial \zeta} = h_{t+1} \tag{5.81}$$

First we need to know how γ_t^P changes in order to derive the change in $(\sigma_{pt}^2)^P$:

$$\frac{\partial \gamma_t^P}{\partial \zeta} = \frac{1}{(1-z)\sigma_{k-b}^2 E(1+R_k)}\frac{\partial E_t R_{k,t+1}}{\partial \zeta} - \frac{1}{(1-z)\sigma_{k-b}^2 E(1+R_b)}\frac{\partial E_t R_{b,t+1}}{\partial \zeta} \tag{5.82}$$

where:

$$\frac{\partial E_t R_{k,t+1}}{\partial \zeta} = \frac{(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} \tag{5.83}$$

$$\frac{\partial E_t R_{b,t+1}}{\partial \zeta} = \frac{E(1+R_b)}{1+R_b^N}\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta} \tag{5.84}$$

Then the changes in γ_t^P and $(\sigma_{pt}^2)^P$ are:

$$\frac{\partial \gamma_t^P}{\partial \zeta} = \frac{1}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta}\right] \tag{5.85}$$

$$\frac{\partial (\sigma_{pt}^2)^P}{\partial \zeta} = \frac{2\gamma^P\sigma_{k-b}^2 + 2\sigma_{k-b,b}}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta}\right] \tag{5.86}$$

The changes of W_t and $E_t R_{p,t+1}^P$ are given by:

$$\frac{\partial W_t}{\partial \zeta} = \frac{\alpha W}{K}\frac{\partial K_t^{FP}}{\partial \zeta} \tag{5.87}$$

$$\begin{aligned}\frac{\partial E_t R_{p,t+1}^P}{\partial \zeta} &= \frac{E(R_k)-E(R_b)}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta}\right] \\ &+ \frac{\gamma^P(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} + \frac{(1-\gamma^P)E(1+R_b)}{1+R_b^N}\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta}\end{aligned} \tag{5.88}$$

Using all these equations we can write equations (5.74) and (5.75) as:

$$\frac{\partial S_t^F}{\partial \zeta} = \frac{\varepsilon\alpha W}{(1+\rho+\varepsilon)K}\frac{\partial K_t^{FP}}{\partial \zeta} - \frac{\varepsilon}{1+\rho+\varepsilon}\frac{\partial T_t^{B,F}}{\partial \zeta} + \frac{(1+\rho)(W-T^{B,F})}{(1+\rho+\varepsilon)^2}h_{t+1} \tag{5.89}$$

$$\begin{aligned}\frac{\partial S_t^P}{\partial \zeta} &= \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon\alpha W}{[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon+1+\rho]K}\frac{K_t^{FP}}{\partial \zeta} - \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon}{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon+1+\rho}\frac{\partial T_t^{B,P}}{\partial \zeta} \\ &- \frac{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon}{\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon+1+\rho}\frac{\partial T_t^P}{\partial \zeta} + \left\{\frac{(1+\rho)\exp(\frac{1}{2}z^2(\sigma_p^2)^P)(W-T^P-T^{B,P})}{\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon+1+\rho\right]^2}\right. \\ &\left. - \frac{(1+\rho)^2 Z^P}{E(1+R_p^P)\exp(-\frac{1}{2}(\sigma_p^2)^P)\left[\exp(\frac{1}{2}z^2(\sigma_p^2)^P)\varepsilon+1+\rho\right]^2}\right\}h_{t+1} \\ &+ S_{\sigma_p^2}^P\frac{2\gamma^P\sigma_{k-b}^2+2\sigma_{k-b,b}}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} - \frac{1}{1+R_b^N}\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta}\right] \\ &+ S_{R_p}^P\left\{\frac{E(R_k)-E(R_b)}{(1-z)\sigma_{k-b}^2}\left[\frac{\alpha-1}{K}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} - \frac{1}{R_b^N}\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta}\right]\right. \\ &\left. + \frac{\gamma^P(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}^{FP}}{\partial \zeta} + \frac{(1-\gamma^P)E(1+R_b)}{1+R_b^N}\frac{\partial R_{b,t+1}^{N,FP}}{\partial \zeta}\right\}\end{aligned} \tag{5.90}$$

where $S^P_{\sigma^2_p} \equiv \frac{\partial S^P_t}{\partial (\sigma^2_{pt})^P}$ and $S^P_{R_p} \equiv \frac{\partial S^P_t}{\partial E_t R^P_{p,t+1}}$, see equations (5.77) and (5.80).

Now use the other dynamic equation (5.10) to derive:

$$\frac{\partial K^{FP}_{t+1}}{\partial \zeta} = \frac{1}{(1+\nu)(1+n)} \left\{ \gamma^P \frac{\partial S^P_t}{\partial \zeta} + S^P \frac{\partial \gamma^P_t}{\partial \zeta} + \nu\gamma^F \frac{\partial S^F_t}{\partial \zeta} + \nu S^F \frac{\gamma^F_t}{\partial \zeta} \right\} \tag{5.91}$$

Using equation (5.73) we can rewrite this equation to:

$$\frac{\partial K^{FP}_{t+1}}{\partial \zeta} = \frac{1}{(1-\gamma^P)(1+\nu)(1+n)} \left\{ \nu(\gamma^F - \gamma^P) \frac{\partial S^F_t}{\partial \zeta} + S^P \frac{\partial \gamma^P_t}{\partial \zeta} + \nu S^F \frac{\partial \gamma^F_t}{\partial \zeta} \right\} \tag{5.92}$$

The changes of S^F_t and γ^P_t are given in equation (5.89) and (5.85). The change of γ^F_t is:

$$\frac{\partial \gamma^F_t}{\partial \zeta} = \frac{1}{\sigma^2_{k-b}} \left[\frac{\alpha - 1}{K} \frac{\partial K^{FP}_{t+1}}{\partial \zeta} - \frac{1}{1+R^N_b} \frac{\partial R^{N,FP}_{b,t+1}}{\partial \zeta} \right] \tag{5.93}$$

Substituting this information into equation (5.92) and simplifying gives:

$$\frac{\partial K^{FP}_{t+1}}{\partial \zeta} = \frac{1}{\Phi} \frac{\partial R^{N,FP}_{b,t+1}}{\partial \zeta} \underbrace{- \frac{\nu(\gamma^F - \gamma^P)\sigma^2_{k-b}(1+R^N_b)}{\Phi(\nu S^F + \frac{S^P}{1-z})}}_{<0} \frac{\partial S^F_t}{\partial \zeta} \tag{5.94}$$

where $\Phi \equiv \frac{[(1+\nu)K(1+n)(1-\gamma^P)\sigma^2_{k-b} + (1-\alpha)(\nu S^F + \frac{S^P}{1-z})](1+R^N_b)}{-(\nu S^F + \frac{S^P}{1-z})K} < 0$, combining this with the fact that $\gamma^F < \gamma^P$ this implies that the term in front of $\frac{\partial S^F_t}{\partial \zeta}$ is negative. Equation (5.94) can be rewritten to:

$$\frac{\partial R^{N,FP}_{b,t+1}}{\partial \zeta} = \Phi \frac{\partial K^{FP}_{t+1}}{\partial \zeta} + \frac{\nu(\gamma^F - \gamma^P)\sigma^2_{k-b}(1+R^N_b)}{\nu S^F + \frac{S^P}{1-z}} \frac{\partial S^F_t}{\partial \zeta} \tag{5.95}$$

Use this equation to substitute for $\frac{\partial R^{N,FP}_{b,t+1}}{\partial\zeta}$ in equation (5.90):

$$\begin{aligned}
\frac{\partial S^P_t}{\partial\zeta} &= \frac{\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon\alpha W}{[\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon+1+\rho]K}\frac{K^{FP}_t}{\partial\zeta} - \frac{\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon}{\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon+1+\rho}\frac{\partial T^{B,P}_t}{\partial\zeta} \\
&- \frac{\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon}{\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon+1+\rho}\frac{\partial T^P_t}{\partial\zeta} + \left\{\frac{(1+\rho)\exp(\frac{1}{2}z^2(\sigma^2_p)^P)(W-T^P-T^{B,P})}{\left[\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon+1+\rho\right]^2}\right. \\
&\left. - \frac{(1+\rho)^2Z^P}{E(1+R^P_p)\exp(-\frac{1}{2}(\sigma^2_p)^P)\left[\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon+1+\rho\right]^2}\right\}h_{t+1} \\
&+ S^P_{\sigma^2_p}\left\{\Delta_{\sigma^2_p}\frac{\partial K^{FP}_{t+1}}{\partial\zeta} - \frac{\nu(\gamma^F-\gamma^P)(1-z)\sigma^2_{k-b}}{\nu S^F(1-z)+S^P}\frac{2\gamma^P\sigma^2_{k-b}+2\sigma_{k-b,b}}{(1-z)\sigma^2_{k-b}}\frac{\partial S^F_t}{\partial\zeta}\right\} \\
&+ S^P_{R_p}\left\{\Delta_{R_p}\frac{\partial K^{PF}_{t+1}}{\partial\zeta} + \frac{\nu(\gamma^F-\gamma^P)(1-z)\sigma^2_{k-b}}{\nu S^F(1-z)+S^P}\cdot\right. \\
&\left.\left[(1-\gamma^P)E(1+R_b) - \frac{E(R_k)-E(R_b)}{(1-z)\sigma^2_{k-b}}\right]\frac{\partial S^F_t}{\partial\zeta}\right\}
\end{aligned} \tag{5.96}$$

where

$$\Delta_{\sigma^2_p} \equiv \frac{2\gamma^P\sigma^2_{k-b}+2\sigma_{k-b,b}}{(1-z)\sigma^2_{k-b}}\frac{(\alpha-1)(1+R^N_b)-\Phi K}{K(1+R^N_b)} > 0 \tag{5.97}$$

$$\begin{aligned}
\Delta_{R_p} \equiv\ & \frac{[E(R_k)-E(R_b)][(\alpha-1)(1+R^N_b)-\Phi K]}{(1-z)\sigma^2_{k-b}K(1+R^N_b)} \\
&+\frac{\gamma^P(\alpha-1)E(1+R_k)}{K} + \frac{(1-\gamma^P)E(1+R_b)\Phi}{1+R^N_b} \lesseqgtr 0
\end{aligned} \tag{5.98}$$

Substituting equations (5.89) and (5.96) into equation (5.73) and simplifying gives:

$$\begin{aligned}
\frac{\partial K^{FP}_{t+1}}{\partial\zeta} &= \frac{\varepsilon\alpha W}{\Psi K}\frac{\partial K^{FP}_t}{\partial\zeta} - \frac{\Omega\nu\varepsilon}{\Psi^F}\frac{\partial T^{B,F}_t}{\partial\zeta} - \frac{\varepsilon}{\Psi^P}\frac{\partial T^{B,P}_t}{\partial\zeta} - \frac{\varepsilon}{\Psi^P}\frac{\partial T^P_t}{\partial\zeta} \\
&+ \left\{\frac{(1+\rho)(W-T^P-T^{B,P})}{\Psi^P[\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon+1+\rho]} + \frac{\Omega\nu(1+\rho)(W-T^{B,F})}{\Psi^F(1+\rho+\varepsilon)}\right. \\
&\left. - \frac{(1+\rho)^2Z^P}{\Psi^P[\exp(\frac{1}{2}z^2(\sigma^2_p)^P)\varepsilon+1+\rho]E(1+R^P_p)\exp(\frac{1}{2}(\sigma^2_p)^P(z^2-1))}\right\}h_{t+1}
\end{aligned} \tag{5.99}$$

where:

$$\Omega \equiv 1 + \frac{\gamma^F - \gamma^P}{\nu S^F(1-z) + S^P} \Big\{ S^P_{R_p}(1-\gamma^P)E(1+R_b)(1-z)\sigma^2_{k-b} - S^P_{R_p}[E(R_k) - E(R_b)] - S^P_{\sigma^2_p}(2\gamma^P\sigma^2_{k-b} + 2\sigma_{k-b,b}) \Big\} \gtrless 0 \tag{5.100}$$

$$\Psi \equiv \frac{\left[\exp(\tfrac{1}{2}z^2(\sigma^2_p)^P)\varepsilon + 1 + \rho\right](1+\rho+\varepsilon)\left[(1+\nu)(1+n) - S^P_{\sigma^2_p}\Delta_{\sigma^2_p} - S^P_{R_p}\Delta_{R_p}\right]}{\exp\left(\tfrac{1}{2}z^2(\sigma^2_p)^P\right)(1+\rho+\varepsilon) + \Omega\nu\left[\exp(\tfrac{1}{2}z^2(\sigma^2_p)^P)\varepsilon + 1 + \rho\right]} \gtrless 0 \tag{5.101}$$

$$\Psi^P \equiv \frac{\left[\exp(\tfrac{1}{2}z^2(\sigma^2_p)^P)\varepsilon + 1 + \rho\right]\left[(1+\nu)(1+n) - S^P_{\sigma^2_p}\Delta_{\sigma^2_p} - S^P_{R_p}\Delta_{R_p}\right]}{\exp\left(\tfrac{1}{2}z^2(\sigma^2_p)^P\right)} \gtrless 0 \tag{5.102}$$

$$\Psi^F \equiv (1+\rho+\varepsilon)\left[(1+\nu)(1+n) - S^P_{\sigma^2_p}\Delta_{\sigma^2_p} - S^P_{R_p}\Delta_{R_p}\right] \gtrless 0 \tag{5.103}$$

Equations (5.99), (5.95) and (5.89) together give the system of equations in Section 5.3.

5.D Excess return

First we derive the change in the log excess return:

$$\frac{\partial \text{log excess return}^{FP}}{\partial \zeta} = \frac{1}{E(1+R_k)}\frac{\partial E_t R_{k,t+1}}{\partial \zeta} - \frac{1}{E(1+R_b)}\frac{\partial E_t R_{b,t+1}}{\partial \zeta} \tag{5.104}$$

Substituting equations (5.83), (5.84) and (5.95) and simplifying gives:

$$\frac{\partial \text{log excess return}^{FP}}{\partial \zeta} = \frac{\sigma^2_{k-b}}{\nu S^F + \frac{S^P}{1-z}}\left[(1+\nu)(1+n)(1-\gamma^P)\frac{\partial K^{FP}_{t+1}}{\partial \zeta} \underbrace{-\nu(\gamma^F - \gamma^P)}_{>0}\frac{\partial S^F_t}{\partial \zeta}\right] \tag{5.105}$$

which indeed shows that the log excess return increases when savings (and thus the capital stock) rise.

The derivative of the simple excess return is:

$$\frac{\partial \text{excess return}^{FP}}{\partial \zeta} = \frac{\partial E_t R_{k,t+1}}{\partial \zeta} - \frac{\partial E_t R_{b,t+1}}{\partial \zeta} \tag{5.106}$$

Substituting equations (5.83), (5.84) and (5.95) and simplifying gives:

$$\frac{\partial \text{excess return}^{FP}}{\partial \zeta} = \underbrace{-\frac{(1-\alpha)[E(R_k)-E(R_b)]}{K}}_{<0}\frac{\partial K^{FP}_{t+1}}{\partial \zeta}$$
$$+\underbrace{\frac{E(1+R_b)\sigma^2_{k-b}}{\nu S^F + \frac{S^P}{1-z}}\left[(1+\nu)(1+n)(1-\gamma^P)\frac{\partial K^{FP}_{t+1}}{\partial \zeta} - \nu(\gamma^F-\gamma^P)\frac{\partial S^F_t}{\partial \zeta}\right]}_{=E(1+R_b)\frac{\partial \log \text{excess return}^{FP}}{\partial \zeta}>0} \quad (5.107)$$

In case $E(R_k) \approx E(R_b)$ or $\alpha \to 1$ the term in front of $\frac{\partial K^{FP}_{t+1}}{\partial \zeta}$ approaches zero and the simple excess return moves in the same direction as the log gross excess return. In case $E(R_k)$ and $E(R_b)$ are far apart, however, the simple excess return could move in the opposite direction as the log gross excess return. In case of an ageing shock that does not affect portfolio choice directly this was indeed the case in our simulations.

BIBLIOGRAPHY

Aaron, H. (1966), 'The social insurance paradox', *Canadian Journal of Economics and Political Science* **32**, 371–374.

ABP (2006), *Annual Report*, ABP.

Adema, Y., Meijdam, L. and Verbon, H.A.A. (2008*a*), 'Beggar thy thrifty neighbour: The international spillover effects of pensions under population ageing', *Journal of Population Economics* **forthcoming**.

Adema, Y., Meijdam, L. and Verbon, H.A.A. (2008*b*), 'The international spillover effects of pension reform', *International Tax and Public Finance* **forthcoming**.

Altig, D., Auerbach, A., Kotlikoff, L., Smetters, K. and Walliser, J. (2001), 'Simulating fundamental tax reform in the United States', *American Economic Review* **91**(3), 574–595.

Annicchiarico, B., Giammarioli, N. and Piergallini, A. (2006), 'Fiscal policy in a monetary economy with capital and finite lifetime', *ECB Working Paper* **no. 661**.

Antolin, P. (2007), 'Longevity risk and private pensions', *OECD Working Papers on Insurance and Private Pensions* **no. 3**.

Atkinson, A.B. and Stiglitz, J.E. (1980), *Lectures on public economics*, McGraw-Hill, United Kingdom.

Attanasio, O., Kitao, S. and Violante, G. (2006), 'Quantifying the effects of the demographic transition in developing economies', *Advances in Macroeconomics* **6**(1).

Auerbach, A. and Kotlikoff, L. (1987), *Dynamic Fiscal Policy*, Cambridge University Press.

Barberis, N. (2000), 'Investing for the long run when returns are predictable', *Journal of Finance* **55**(1), 225–264.

Beetsma, R. and Bovenberg, A. (1999), 'Does monetary unification lead to excessive debt accumulation?', *Journal of Public Economics* **74**, 299–325.

Beetsma, R. and Bovenberg, L. (2007), 'Pension systems, intergenerational risk sharing and inflation', *CEPR Discussion Paper* **no. 6089**.

Belan, P., Michel, P. and Pestieau, P. (1998), 'Pareto-improving social security reform', *Geneva Papers on Risk and Insurance - Theory* **23**(2), 119–125.

Belan, P. and Pestieau, P. (1999), 'Privatizing social security: A critical assessment', *Geneva Papers on Risk and Insurance - Issues and Practice* **24**(1), 114–130.

Blundell, R. and MaCurdy, T. (1999), Labor supply: a review of alternative approaches, *in* O. Ashenfelter and D. Card, eds, 'Handbook of Labor Economics', Vol. 3A, Elsevier, Amsterdam, Chapter 27.

Boeri, T., Bovenberg, L., Coeuré, B. and Roberts, A. (2006), 'Dealing with the new giants: Rethinking the role of pension funds.', *Geneva Reports on the World Economy* **8**.

Bohn, H. (1998), 'Risk sharing in a stochastic overlapping generations economy', *mimeo*.

Bohn, H. (2001), Social security and demographic uncertainty: The risk-sharing properties of alternative policies, *in* J.Y. Campbell and M. Feldstein, eds, 'Risk Aspects of Investment-Based Social Security Reform', University of Chicago Press, Chicago, pp. 203–246.

Bohn, H. (2003), 'Intergenerational risk sharing and fiscal policy', *mimeo*.

Börsch-Supan, A., Ludwig, A. and Winter, J. (2006), 'Ageing, pension reform and capital flows: A multi-country simulation model', *Economica* **73**(292), 625–658.

Bovenberg, A. (2003), 'Financing retirement in the European Union', *International Tax and Public Finance* **10**, 713–734.

Bräuninger, M. (1999), 'Generalised social security finance in a two-country world', *Scottish Journal of Political Economy* **46**(3), 287–302.

Breyer, F. (1989), 'On the intergenerational Pareto efficiency of pay-as-you-go financed pension systems', *Journal of Institutional and Theoretical Economics* **145**, 643–658.

Breyer, F. and Straub, M. (1993), 'Welfare effects of unfunded pension systems when labor supply is endogenous', *Journal of Public Economics* **50**(1), 77–91.

Breyer, F. and Wildasin, D. (1993), 'Steady state welfare effects of social security in a large open economy', *Journal of Economics* **Suppl. 7**, 43–49.

Brooks, R. (2003), 'Population aging and global capital flows in a parallel universe', *IMF Staff Papers* **50**(2).

Brunner, J. (1996), 'Transition from a pay-as-you-go to a fully funded pension system: the case of differing individuals and intragenerational fairness', *Journal of Public Economics* **60**, 131–146.

Buiter, W. (1981), 'Time preference and international lending and borrowing in an overlapping-generations model', *Journal of Political Economy* **89**(August), 769–797.

Campbell, J. (1993), 'Intertemporal asset pricing without consumption data', *American Economic Review* **83**(3), 487–512.

Campbell, J. (2003), Consumption-based asset pricing, *in* G.M. Constantinides, M. Harris and R.M. Stulz, eds, 'Handbook of the Economics of Finance', Vol. 1, part 2, Elsevier, Chapter 13, pp. 803–887.

Campbell, J. and Nosbusch, Y. (2007), 'Intergenerational risksharing and equilibrium asset prices', *Journal of Monetary Economics* **54**(8), 2251–2268.

Campbell, J. and Viceira, L. (1999), 'Consumption and portfolio decisions when expected returns are time varying', *Quarterly Journal of Economics* **114**(2), 433–495.

Campbell, J. and Viceira, L. (2002), *Strategic asset allocation - Portfolio choice for long-term investors*, Oxford University Press, New York.

Campbell, J. and Viceira, L. (2005), 'The term structure of the risk-return trade-off', *Financial Analysts Journal* **61**(1), 34–44.

Casamatta, G., Cremer, H. and Pestieau, P. (2000), 'The political economy of social security', *Scandinavian Journal of Economics* **102**(3), 503–522.

Casarico, A. (2001), 'Pension systems in integrated capital markets', *Topics in Economic Analysis & Policy* **1**(1), 1–17.

Chari, V. and Kehoe, P.J. (2007), 'On the need for fiscal constraints in a monetary union', *Journal of Monetary Economics* **forthcoming**.

Cooley, T.F. and Soares, J. (1999), 'Privatizing social security', *Review of Economic Dynamics* **2**(3), 731–755.

Croix, D. and Michel, P. (2002), *A theory of economic growth: dynamics and policy in overlapping generations*, Cambridge University Press, Cambridge, United Kingdom.

Cutler, D., Poterba, J., Sheiner, L. and Summers, L. (1990), 'An ageing society: Opportunity or challenge?', *Brookings Papers on Economic Activity* **1**, 1–73.

Demange, G. (2002), 'On optimality in intergenerational risk sharing', *Economic Theory* **20**, 1–27.

Diamond, P. (1965), 'National debt in a neoclassical growth model', *American Economic Review* **55**(December), 1126–1150.

Domeij, D. and Flodén, M. (2006), 'Population aging and international capital flows', *International Economic Review* **47**(3), 1013–1032.

Dublin, L. (1928), *Health and Wealth*, Harper, New York.

Dutta, J., Kapur, S. and Orszag, J.M. (2000), 'A portfolio approach to the optimal funding of pensions', *Economics Letters* **69**(2), 201–206.

Epstein, L.G. and Zin, S.E. (1989), 'Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework', *Econometrica* **57**(4), 937–969.

Epstein, L.G. and Zin, S.E. (1991), 'Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis', *The Journal of Political Economy* **99**(2), 263–286.

European Commission (2006), 'The impact of ageing on public expenditure: projections for EU25 member states on pensions, health care, long-term care, education and unemployment transfers (2004-2050)', *European Economy* **Special Report no. 1**.

Fehr, H., Jokisch, S. and Kotlikoff, L. (2005), The developed world's demographic transition - the role of capital flows, immigration, and policy, *in* R. Brooks and A. Razin, eds, 'Social Security Reform', Cambridge University Press, Cambridge, pp. 11–43.

Feldstein, M. (2005), 'Structural reform of social security', *Journal of Economic Perspectives* **19**(2), 22–55.

Fenge, R. (1995), 'Pareto efficiency of the pay-as-you-go pension system with intergenerational fairness', *Finanzarchiv* **52**, 357–363.

Frederick, S., Loewenstein, G. and O'Donoghue, T. (2002), 'Time discounting and time preference: A critical review', *Journal of Economic Literature* **40**, 351–401.

Fried, J. and Howitt, P. (1988), 'Fiscal deficits, international trade and welfare', *Journal of International Economics* **24**.

Gale, D. (1973), 'Pure exchange equilibrium of dynamic economic models', *Journal of Economic Theory* **6**(1), 12–36.

Groezen, B., van (2003), International spillover effects of unfunded pensions and population ageing, *in* 'The wealth of generations: Pensions and welfare from a macroeconomic perspective', CentER, Tilburg.

Guiso, L., Haliassos, M. and Jappelli, T., eds (2001), *Household Portfolios: Theory and Evidence*, MIT Press, Cambridge.

Hansen, L. and Singleton, K. (1983), 'Stochastic consumption, risk aversion, and the temporal behavior of asset returns', *Journal of Political Economy* **91**(2), 249–265.

Homburg, S. (1990), 'The efficiency of unfunded pension schemes', *Journal of Institutional and Theoretical Economics* **146**, 640–647.

Homburg, S. and Richter, W.F. (1993), 'Harmonizing public debt and public pension schemes in the European Community', *Journal of Economics* **Suppl. 7**, 51–63.

INGENUE (2001), 'Macroeconomic consequences of pension reforms in Europe: An investigation with the INGENUE world model', *Working Paper* **no. 2001-16**.

Jousten, A. and Legros, F. (2002), 'Pensions and savings in a monetary union: an analysis of capital flows', *CEPR Discussion Paper* **no. 3509**.

Judd, K. (1982), 'An alternative to steady-state comparisons in perfect foresight models', *Economics Letters* **10**, 55–59.

Köthenbürger, M. and Poutvaara, P. (2006), 'Social security reform and investment in education: Is there scope for a Pareto-improvement?', *Economica* **73**, 299–319.

Krueger, D. and Kubler, F. (2006), 'Pareto-improving social security reform when financial markets are incomplete!?', *American Economic Review* **96**(3), 737–755.

Leith, C. and Wren-Lewis, S. (2006), 'Compatibility between monetary and fiscal policy under EMU', *European Economic Review* **50**(6), 1529–1556.

Matsen, E. and Thøgersen, O. (2004), 'Designing social security - a portfolio choice approach', *European Economic Review* **48**(4), 883–904.

McMorrow, K. and Röger, W. (2004), *The Economic and Financial Market Consequences of Global Ageing*, Springer-Verlag Press, New York.

Meijdam, L. and Verhoeven, M. (1998), 'Comparative dynamics in perfect-foresight models', *Computational Economics* **12**, 115–124.

Miles, D. (2001), Should monetary policy be different in a greyer world, *in* A. Auerbach and H. Herrmann, eds, 'Ageing, Financial Markets, and Monetary Policy', Springer-Verlag, Berlin.

Miles, D. and Černý, A. (2006), 'Risk, return and portfolio allocation under alternative pension systems with incomplete and imperfect financial markets', *Economic Journal* **116**(April), 529–557.

OECD (2005), *Pensions at Glance: Public Policies across OECD countries*, Organization for Economic Cooperation and Development, Paris.

OECD Global Pension Statistics (2006), Organization for Economic Cooperation and Development, Paris.

Oeppen, J. and Vaupel, J.W. (2002), 'Broken limits to life expectancy', *Science* **296**(5570), 1029–1031.

Olshansky, S., Carnes, B. and Cassel, C. (1990), 'In search of Methuselah: Estimating the upper limits to human longevity', *Science* **250**(4981), 634–640.

Pemberton, J. (2000), 'National and international privatisation of pensions', *European Economic Review* **44**(10), 1873–1896.

Perotti, R. (2001), 'Is a uniform social policy better? Fiscal federalism and factor mobility', *American Economic Review* **91**(3), 596–596–610.

Persson, T. (1985), 'Deficits and intergenerational welfare in open economies', *Journal of International Economics* **19**(1-2), 67–84.

Pestieau, P., Piaser, G. and Sato, M. (2006), 'PAYG pension systems with capital mobility', *International Tax and Public Finance* **13**, 587–599.

Rothschild, M. and Stiglitz, J. (1971), 'Increasing risk II: Its economic consequences', *Journal of Economic Theory* **3**, 66–84.

Samuelson, P. (1958), 'An exact consumption loan model of interest with or without the social contrivance of money', *Journal of Political Economy* **66**, 1002–1011.

Sánchez-Marcos, V. and Sánchez-Martin, A. (2006), 'Can social security be welfare improving when there is demographic uncertainty?', *Journal of Economic Dynamics and Control* **30**, 1615–1646.

Sandmo, A. (1970), 'The effect of uncertainty on saving decisions', *Review of Economic Studies* **37**(3), 353–360.

Schabert, A. and van Wijnbergen, S. (2006), 'Debt, deficits and destabilizing monetary policy in open economies', *CEPR Discussion Paper* **no. 5590**.

Sinn, H. (2000), 'Why a funded pension system is useful and why it is not useful', *International Tax and Public Finance* **7**, 389–410.

Steigum, E. and Raffelhüschen, B. (1994), 'Deficit spillovers, labor supply and intergenerational welfare', *SNF-report* **no. 72/1994**.

Storesletten, K., Telmer, C.I. and Yaron, A. (1999), 'The risk-sharing implications of alternative social security arrangements', *Carnegie-Rochester Conference Series on Public Policy* **50**, 213–259.

Thøgersen, O. (1998), 'A note on intergenerational risk sharing and the design of pay-as-you-go pension programs', *Journal of Population Economics* **11**(3), 373–378.

UNCDB (2007), 'United Nations Common Database'.

Verbon, H.A.A. (1989), Conversion policies for public pension plans in a small open economy, *in* B. Gustafsson and N. Klevmarken, eds, 'The Political Economy of Social Security', Elsevier Science, Amsterdam, pp. 83–95.

Viceira, L.M. (2001), 'Optimal portfolio choice for long-horizon investors with nontradable labor income', *Journal of Finance* **56**(2), 433–470.

Wagener, A. (2003), 'Pensions as a portfolio problem: fixed contribution rates vs. fixed replacement rates reconsidered', *Journal of Population Economics* **16**(1), 111–134.

Worldbank (1994), *Averting the Old Age Crisis: Policies to Protect the Old and Promote Growth*, Oxford University Press, New York.

SAMENVATTING (SUMMARY IN DUTCH)

Veel westerse landen zullen de komende decennia te maken krijgen met een vergrijzende bevolking. Vergrijzing wordt met name veroorzaakt door het feit dat we steeds langer leven en minder kinderen krijgen. In 1950 kregen vrouwen in Europa gemiddeld nog 2,5 kinderen per persoon, terwijl dat aantal in 2005 terug is gelopen tot 1,6 kinderen per persoon. De levensverwachting bij geboorte is in Europa tussen 1950 en 2005 met 11,6 jaren toegenomen van 67,1 jaar naar 78,7 jaar. De verwachting is dat het aantal kinderen per vrouw de komende decennia zal stabiliseren of zelfs iets zal toenemen. De toename van de levensverwachting is echter structureel; de Verenigde Naties voorspellen dat de levensverwachting in 2050 zal zijn toegenomen tot 84,2 jaar. Aan de ene kant is vergrijzing het gevolg van positieve ontwikkelingen zoals de emancipatie van vrouwen en een verbeterde gezondheidszorg. Aan de andere kant worden collectief gefinancierde voorzieningen als pensioenen en zorgverzekeringen onder druk gezet door de ongekende stijging van het relatieve aantal ouderen ten opzichte van het aantal werkenden. In Europa is de zogenaamde ouderen-afhankelijkheidsratio[17] tussen 1950 en 2005 gestegen van 13,9 procent naar 30,5 procent. De voorspelling van de Verenigde Naties is dat dit percentage in 2050 verder zal stijgen naar 65,4. Deze enorme stijging van het relatieve aantal gepensioneerden dwingt overheden om na te denken over de manier waarop hun pensioenen georganiseerd zijn. In de meeste landen wordt gebruik gemaakt van een publiek omslagstelsel, waarbij

[17]De ouderen-afhankelijkheidsratio is gedefinieerd als het aantal mensen dat 65 jaar of ouder is ten opzichte van de potentiële beroepsbevolking, oftewel het aantal mensen tussen de 15 en 65 jaar.

de huidige beroepsbevolking de pensioenen van de huidige ouderen betaalt. Deze publieke pensioenstelsels zullen onder druk komen te staan als het aantal pensioengerechtigden stijgt en er relatief minder werkende mensen zijn. Veel economen pleiten er daarom voor dat overheden een gedeelte van hun omslaggefinancierde pensioenen vervangen door een pensioenstelsel dat via kapitaaldekking gefinancierd is. Bij een kapitaaldekkingsstelsel sparen individuen zelf of via een pensioenfonds voor hun oude dag en daarom zou een kapitaalgedekt pensioensysteem beter bestand zijn tegen vergrijzing. Bovendien wordt er over het algemeen vanuit gegaan dat het rendement dat op de kapitaalmarkt behaald kan worden hoger is dan het impliciete rendement van een omslagstelsel[18]. Kapitaaldekkingsstelsels zullen echter ook beïnvloed worden door vergrijzing. Omdat vergrijzing een wereldwijd fenomeen is zal het macro-economische gevolgen hebben en op die manier de rendementen van investeringen beïnvloeden. Daarnaast zal de omschakeling naar meer kapitaaldekking politiek moeilijk haalbaar zijn omdat er een generatie is die zowel de pensioenen van de huidige ouderen moet betalen als moet sparen voor de eigen oude dag. Verder is het zo dat het hogere verwachte rendement van kapitaalgedekte pensioenen samengaat met meer financieel risico en inflatierisico.

Het publieke debat over de houdbaarheid van pensioenstelsels richt zich met name op de vraag hoe de problemen als gevolg van een vergrijzende bevolking binnen het land zelf opgelost dienen te worden. Vergrijzing is echter een wereldwijd fenomeen en internationale kapitaalmarkten raken in toenemende mate geïntegreerd. Dit heeft tot gevolg dat inwoners van een bepaald land beïnvloed zullen worden door vergrijzing en de in reactie daarop genomen beleidsmaatregelen in andere landen. De toenemende integratie van internationale kapitaalmarkten betekent dat financieel kapitaal steeds mobieler wordt en meer reageert op internationale verschillen in rendementen. Vergrijzing kan leiden tot rendementsverschillen tussen landen omdat de besparingsreacties en de kapitaalbehoeften als gevolg van vergrijzing verschillend kunnen zijn. Deze rendementsverschillen zullen leiden tot internationale kapitaalstromen en op die manier de welvaart in andere landen beïnvloeden. De welvaartseffecten van deze kapitaalstromen worden wel aangeduid als internationale spillovereffecten. Dit proefschrift analyseert *de internationale spillovereffecten van vergrijzing via kapitaalmarkten*. De aandacht is gericht op het

[18] Het impliciete rendement van een omslagstelsel is gelijk aan de groeivoet van de economie, d.w.z., de som van de bevolkingsgroei en de technologische vooruitgang.

feit dat landen verschillen in de omvang van de kapitaaldekkingscomponent van hun pensioensysteem. Deze internationale verschillen in pensioenstelsels hebben tot gevolg dat landen verschillend reageren op een vergrijzende bevolking, wat tot kapitaalstromen en dus tot internationale spillovereffecten zal leiden. Dit geval is in het bijzonder relevant voor de Europese Economische en Monetaire Unie (EMU) waar de lidstaten één kapitaalmarkt vormen en één centrale bank hebben. Aan de ene kant is er een grote groep landen met omvangrijke publieke pensioenstelsels die volgens het omslagprincipe gefinancierd worden en waar kapitaaldekkingsstelsels te verwaarlozen zijn. In deze landen financieren omslagpensioenen meer dan 90 procent van het totale inkomen van de ouderen (Boeri et al., 2006). Aan de andere kant is er een kleinere groep landen die grote kapitaaldekkingsstelsels hebben. De investeringen van pensioenfondsen in een land als Nederland zijn bijvoorbeeld gelijk aan 124,9 procent van het BBP (OECD Global Pension Statistics, 2006).

Het doel van dit proefschrift is om inzicht te krijgen in hoe, bij vergrijzing, mensen die in landen wonen met een omvangrijk kapitaaldekkingsstelsel worden beïnvloed door het feit dat andere landen omvangrijke omslagstelsels hebben en vice versa. Er worden verscheidende scenario's bestudeerd waarbij landen met omvangrijke omslagstelsels op verschillende manieren reageren op vergrijzing. Deze landen kunnen hun omslagstelsel in evenwicht houden. In dat geval moeten óf de bijdragen van de jongeren worden verhoogd óf de pensioenuitkeringen van de ouderen worden verlaagd als gevolg van vergrijzing. Een andere mogelijkheid is dat ze ervoor kiezen om hun pensioenstelsel te hervormen en een groter gedeelte van de pensioenen op basis van kapitaaldekking te financieren om zo hun pensioensysteem houdbaarder te maken bij vergrijzing. Deze landen zouden er echter ook voor kunnen kiezen om overheidsschuld te gebruiken om de kosten van vergrijzing op te vangen. Als de overheidsschuld erg hoog is kan dit tot een inflatoire druk leiden of overheidsobligaties risicovoller maken. In een economische en monetaire unie zoals de EMU zullen al deze scenario's effect hebben op andere landen die een grote hoeveelheid pensioenvermogen hebben omgebouwd via hun kapitaaldekkingsstelsel. Dit proefschrift geeft een theoretische uiteenzetting en economische interpretatie van deze scenario's. Het basismodel dat in elk hoofdstuk gebruikt wordt is het model met twee overlappende generaties zoals dat ontwikkeld is door Samuelson (1958) en Diamond (1965). In dit model leven individuen gedurende (maximaal) twee perioden. Mensen werken in de eerste periode en het arbeidsinkomen wordt gebruikt om de pensioenpremies te betalen, te consumeren en te sparen. In de tweede periode zijn mensen gepensioneerd en de pensioenuitkering en de opbrengst van de besparingen worden gebruikt om de oudedagsconsumptie te financieren. Dit model is uitermate geschikt om de vragen in de verschillende hoofdstukken te beantwoorden. In het onderstaande zal een samenvatting per hoofdstuk gegeven worden.

Hoofdstuk 2 gaat in op de vraag hoe landen (zoals Nederland) die een omvangrijk kapitaaldekkingsstelsel hebben, na een vergrijzingsschok beïnvloed worden door het feit dat andere EMU landen (zoals Italië en Duitsland) hun pensioenstelsels voornamelijk op omslagbasis hebben gefinancierd en vice versa. Om deze vraag te analyseren wordt een tweelandenmodel gebruikt, waarbij het ene land de pensioenen volledig via een omslagstelsel financiert en het andere land een volledig kapitaalgedekt pensioensysteem heeft. In het oorspronkelijke evenwicht leent het land met het kapitaaldekkingsstelsel financieel kapitaal uit aan het land met het omslagstelsel omdat de besparingen in het kapitaaldekkende land hoger zijn. We maken onderscheid tussen de twee voornaamste oorzaken van vergrijzing, namelijk een stijging van de levensverwachting en een daling van het geboortecijfer. Zowel het feit dat mensen ouder worden als dat ze minder kinderen krijgen resulteert in een stijging van het relatieve aantal ouderen, oftewel een stijging van de ouderen-afhankelijkheidsratio. Vergrijzing heeft een verschillend effect op omslagstelsels en kapitaaldekkingsstelsels. Omslagstelsels worden direct beïnvloed door de stijging van de ouderen-afhankelijkheidsratio, terwijl kapitaaldekkingsstelsel indirect beïnvloed worden door de verandering van het rendement op investeringen. Daarom zullen de besparingsreacties als gevolg van vergrijzing verschillen tussen de landen als ze verschillende pensioensystemen gebruiken. Als deze landen een gezamenlijke kapitaalmarkt vormen dan zullen deze verschillen in besparingen tot kapitaalstromen tussen de landen leiden en op die manier internationale spillovereffecten veroorzaken.

De stijging van de ouderen-afhankelijkheidsratio zal omslagstelsels onder druk zetten omdat het aantal jongeren dat pensioenpremies betaalt daalt ten opzichte van het aantal ouderen dat pensioenuitkeringen ontvangt. In dit hoofdstuk wordt verondersteld dat de overheid in het land met het omslagstelsel het pensioensysteem in evenwicht houdt. Dit betekent dat ofwel de pensioenpremies van de jongeren moeten stijgen of de pensioenuitkeringen van de ouderen moeten dalen als de bevolking vergrijst. In het eerste geval is er sprake van een systeem van vaste uitkeringen (defined benefits', DB) en draagt de werkende generatie de lasten van vergrijzing door hogere pensioenpremies te betalen. In het laatste geval liggen de vergrijzingslasten bij de gepensioneerden en wordt het omslagstelsel gekenmerkt door vaste bijdragen ('defined contributions', DC). Beide varianten worden in dit hoofdstuk geanalyseerd.

Beide vormen van vergrijzing leiden tot een toename van de kapitaalarbeidsverhouding, de centrale variabele van het model. Op het moment dat mensen verwachten langer te leven zullen ze meer gaan sparen. Hogere besparingen leiden tot meer kapitaalaccumulatie en hebben dus een positief effect op de kapitaalarbeidsverhouding. Een daling van het geboortecijfer leidt tot een lagere bevolkingsgroei en zorgt ervoor dat de beroepsbevolking relatief kleiner wordt. Dit impliceert dat minder investeringen nodig zijn om elke werknemer te voorzien van een bepaalde hoeveelheid kapitaal. Dit zogenaamde 'capital-thickening' effect betekent dat bij een gegeven hoeveelheid besparingen per persoon er een hogere kapitaalarbeidsverhouding gehandhaafd kan worden.

De stijging van de ouderen-afhankelijkheidsratio heeft echter een extra effect op de besparingen wanneer er een omslagstelsel gebruikt wordt om de pensioenen te financieren. In het geval van een DB omslagstelsel moeten de pensioenbijdragen van de jongeren stijgen om het pensioensysteem in evenwicht te houden. Dit zal de besparingen negatief beïnvloeden en zal het directe positieve effect op de kapitaalarbeidsverhouding verzwakken. Op het moment dat er een DC omslagstelsel in gebruik is zal dit effect precies tegengesteld zijn; de pensioenuitkeringen dalen als gevolg van de hogere afhankelijkheidsratio, wat leidt tot hogere besparingen en een hogere kapitaalarbeidsverhouding.

We laten zien dat voor realistische waarden van de verschillende parameters de kapitaalarbeidsverhouding in alle geanalyseerde gevallen per saldo stijgt. Tevens is het zo dat de besparingen in het land met het kapitaaldekkingsstelsel meer stijgen dan in het land met het omslagstelsel, wat tot gevolg heeft dat financieel kapitaal van het land met het kapitaaldekkingsstelsel naar het land met het omslagstelsel stroomt. Dit betekent dat de stijging van de kapitaalarbeidsverhouding in het land met het omslagstelsel kleiner zou zijn als het niet geïntegreerd was met een kapitaaldekkend land. Op de lange termijn resulteert dit in positieve spillovereffecten voor het land met het omslagstelsel, omdat het zowel voordeel ondervindt van de hogere kapitaalgoederenvoorraad als van de resulterende lagere rente die betaald moet worden op de leningen die ze bij het kapitaaldekkende land hebben. In het land met het kapitaaldekkingsstelsel zou de stijging van de kapitaalarbeidsverhouding echter groter zijn als het niet één kapitaalmarkt zou vormen met een land met een omslagstelsel. Enerzijds heeft de lagere kapitaalgoederenvoorraad negatieve effecten voor het kapitaaldekkende land. Anderzijds ondervindt het land voordeel van de resulterende hogere rendementen op de buitenlandse investeringen. Simulaties laten zien dat voor realistische parameterwaarden het eerstgenoemde effect domineert. De centrale conclusie is daarom dat, als de bevolking vergrijst, landen met een groot kapitaaldekkingsstelsel op de lange termijn negatief beïnvloed worden door het feit dat andere landen omvangrijke omslagstelsels in gebruik hebben, ook al houden ze hun omslagstelsels in balans. De spillovers op de korte termijn zijn echter tegengesteld aan de spillovers op de lange termijn. De reden hiervoor is dat de stijging van de kapitaalarbeidsverhouding tot een daling van de rente leidt, wat met name de generaties treft die geboren zijn ten tijde van de vergrijzingsschok. Dit betekent dat deze initiële generaties in het kapitaaldekkende land baat hebben van het feit dat het andere land een omslagstelsel in gebruik heeft omdat de daling van de rente in dat geval minder groot is. De eerste generaties in het land met het omslagstelsel ondervinden op de korte termijn juist negatieve spillovereffecten van het land met het kapitaaldekkingsstelsel.

Vergrijzing zet pensioenstelsels die op omslagbasis gefinancierd worden onder druk omdat de enorme stijging van de ouderen-afhankelijkheidsratio impliceert dat óf de pensioenbijdragen van de werkenden flink moeten stijgen óf de pensioenuitkeringen van de gepensioneerden sterk moeten dalen. Eén van de meest besproken voorstellen om de houdbaarheid van publieke omslagstelsels te verbeteren is om het pensioensysteem te hervormen en naar een meer kapitaalgedekt stelsel over te stappen. Ook deze (gedeeltelijke) overstap naar een kapitaaldekkingsstelsel zal spillovereffecten hebben in een geïntegreerde kapitaalmarkt. Het doel van **hoofdstuk 3** is om deze internationale spillovereffecten van pensioenhervorming te bestuderen. In het bijzonder richten we ons op de vraag hoe landen die al een kapitaaldekkingsstelsel hebben worden beïnvloed wanneer landen met omslagstelsels een groter gedeelte van hun toekomstige pensioenuitkeringen met geaccumuleerd kapitaal gaan financieren.

Het model dat in dit hoofdstuk wordt gebruikt is vergelijkbaar met het model in het voorgaande hoofdstuk. De belangrijkste aanpassing is dat we het mogelijk maken dat het omslagstelsel tot verstoringen in de economie leidt, wat betekent dat de omslagpremie tot welvaartsverliezen of een zogenaamde 'excess burden' leidt. De aanwezigheid van een 'excess burden' geeft de ruimte om een pensioenhervorming in te voeren die tot een Paretoverbetering leidt[19].

We analyseren eerst de spillovereffecten van een pensioenhervorming in het land met het omslagstelsel onder de aanname dat de omslagpremies niet verstorend zijn. In dat geval is het omslagstelsel Pareto-efficiënt en bestaat er bij de implementatie van de hervorming een afruil tussen het nut van verschillende generaties. Dit betekent dat als er een aantal generaties zijn die baat hebben bij de overgang naar meer kapitaaldekking, het onvermijdelijk is dat andere generaties nadeel ondervinden van deze conversie. Het hervormende land moet daarom een keuze maken welke generaties voordeel ondervinden van de hervorming en welke niet. We laten zien dat deze keuze doorwerkt naar het kapitaaldekkende land als de twee landen een gezamenlijke kapitaalmarkt vormen. Als het hervormende land besluit dat toekomstige generaties profijt zullen hebben van de hervorming, dan zullen de toekomstige generaties in het land met het kapitaaldekkingsstelsel er ook op vooruit gaan. Deze welvaartswinsten op de lange termijn gaan echter gepaard met een lager nut voor de initiële generaties en deze verliezen op de korte termijn zullen ook via de kapitaalmarkt worden doorgegeven aan het kapitaaldekkende land. Als de overheid in het omslagland echter besluit dat de hervorming tot een welvaartsverbetering moet leiden voor een meerderheid van de huidige bevolking (de werkende en gepensioneerde generaties die leven op het moment dat de hervorming wordt ingevoerd), dan zullen de initiële generaties in het kapitaaldekkende land ook baat hebben van de hervorming in het andere land. Bij dit hervormingsscenario zullen de toekomstige generaties van het land met het omslagstelsel de kosten van de hervorming dragen en de toekomstige generaties in het land met het kapitaaldekkingsstelsel zullen er ook op achteruit gaan.

Vervolgens veronderstellen we dat het heffen van omslagpremies tot een 'excess burden' leidt, zodat het voor het land met het omslagstelsel mogelijk wordt om een pensioenhervorming in te voeren die leidt tot een Paretoverbetering. Op het moment dat zo'n Paretoverbeterend hervormingsbeleid wordt ingevoerd, deelt het kapitaaldekkende land opnieuw de lange termijn voordelen met het land met het omslagstelsel. Echter, hoewel de hervorming in het land met het omslagstelsel zo vormgegeven is dat geen enkele generatie in dit land verliest, kunnen enkele generaties in het kapitaaldekkende land op korte termijn toch nadeel ondervinden. Met andere woorden, een hervormingsbeleid dat tot een Paretoverbetering leidt als het land met het omslagstelsel

[19]Dit betekent dat de pensioenhervorming zo ingevoerd kan worden dat minstens één generatie erop vooruit gaat zonder dat een andere generatie erop achteruit gaat.

alleen in ogenschouw wordt genomen, hoeft niet Paretoverbeterend te zijn als er rekening wordt gehouden met de internationale spillovereffecten.

De centrale conclusie van dit hoofdstuk is dat, in een gezamenlijke kapitaalmarkt, de effecten van pensioenhervormingen in landen met omvangrijke omslagstelsels doorwerken op landen met grote kapitaaldekkingsstelsels. De laatstgenoemde landen kunnen zich niet afschermen van de effecten van de hervormingsmaatregelen in de eerstgenoemde landen. De beleidsconclusie van dit hoofdstuk is daarom dat, als in een gemeenschappelijke markt zoals de EU bij besluiten over pensioenhervormingen rekening gehouden wordt met de effecten voor niet-hervormende landen, het noodzakelijk kan zijn om deze landen te compenseren.

In hoofdstukken 2 en 3 bestaat er geen onzekerheid over het rendement op besparingen. De modellen in deze hoofdstukken zijn compleet deterministisch. In hoofdstukken 4 en 5 kiezen we een andere aanpak en ontwikkelen we een stochastische versie van het tweeperioden-overlappende-generatiemodel. Individuen kunnen in twee soorten vermogenstitels beleggen, aandelen en overheidsobligaties, waarbij de totale hoeveelheid obligaties bepaald wordt door de gegeven omvang van de overheidsschuld. Beide vermogenstitels zijn risicovol; productiviteitsrisico maakt het rendement op aandelen onzeker en er rust een inflatierisico op overheidsobligaties. Het is dus niet mogelijk om in een risicovrije vermogenstitel te investeren. Er wordt verondersteld dat de pensioenen die op omslagbasis worden gefinancierd gegarandeerd zijn, wat betekent dat er geen onzekerheid bestaat over de hoogte van de pensioenuitkering. Deze gegarandeerde omslagpensioenen hebben een tweeledige functie in onze stochastische economie. Ten eerste voorziet de uitkering pensioengerechtigden van een bepaald minimuminkomen. Ten tweede levert het gegarandeerde omslagpensioen een risicovrije vermogenstitel die de markt niet aanbiedt. Hoofdstuk 4 analyseert een versie van dit model met een gesloten economie, terwijl in hoofdstuk 5 het model wordt uitgebreid naar een tweelandenmodel waar het ene land een omslagstelsel gebruikt en het andere land een kapitaaldekkingsstelsel in gebruik heeft. Dit laatste hoofdstuk analyseert de internationale spillovereffecten wanneer het land met het omslagstelsel overheidsschuld gebruikt om de vergrijzingskosten op te vangen, waarbij een hoge schuld samengaat met meer inflatie(risico).

Hoofdstuk 4 verbindt het laatste hoofdstuk van dit proefschrift met hoofdstukken 2 en 3 en het voornaamste doel van dit hoofdstuk is om inzicht te krijgen in de werking van het stochastische model. We bestuderen drie schokken die alledrie de portefeuillebeslissing van individuen direct beïnvloeden; meer risico-aversie van consumenten, de introductie van een pensioenstelsel dat op omslagbasis gefinancierd wordt en een hoger inflatierisico. We onderzoeken de effecten van deze schokken op de portefeuillekeuze, de besparingen en de risicopremie op aandelen.

De analyse van de portefeuillebeslissing van consumenten wordt in de financieringsliteratuur vaak gedaan onder de veronderstelling dat de rendementen op de vermogentitels exogeen zijn. Dit betekent dat de rendementen buiten het model om worden bepaald en als gegeven worden beschouwd. In dat geval is er sprake van een partieel evenwichtsmodel. Het model dat in dit hoofdstuk wordt ontwikkeld is echter een algemeen evenwichtsmodel waar de rendementen op de vermogenstitels endogeen worden bepaald. Om de resultaten in een algemeen evenwichtsmodel te vergelijken met die in een partieel evenwichtsmodel, bekijken we eerst de effecten op de portefeuille- en besparingsbeslissing onder de veronderstelling dat de rendementen op de vermogenstitels exogeen zijn. Als eerste analyseren we de effecten van meer risico-aversie van individuen. De voornaamste reden waarom deze parameterwijziging in beschouwing wordt genomen is om de resultaten te kunnen vergelijken met de effecten van de tweede te analyseren schok; het introduceren van gegarandeerde omslagpensioenen. Gegarandeerde omslagpensioenen verminderen de variantie van de oudedagsconsumptie en zullen op die manier de investeringsbeslissingen van consumenten beïnvloeden. Als we de effecten van omslagpensioenen vergelijken met de effecten van een hogere risico-aversie dan zien we dat gegarandeerde omslagpensioenen ervoor zorgen dat individuen zich als minder risico-averse agenten gaan gedragen. Dit betekent dat ze minder sparen en een risicovollere investeringsportefeuille aanhouden; ze investeren een groter gedeelte van hun besparingen in aandelen. Om aan te geven wat de invloed van onzekerheid is op het gedrag van consumenten bestuderen we tenslotte de effecten van een stijging van het inflatierisico, wat inhoudt dat het risico op overheidsobligaties stijgt. Een hoger inflatierisico zorgt ervoor dat het minder aantrekkelijk wordt om overheidsobligaties aan te houden en consumenten zullen dus een groter gedeelte van hun besparingen in aandelen investeren.

In partieel evenwicht worden alleen de effecten op individuele beslissingen in ogenschouw genomen. Er wordt geen rekening gehouden met de macro-economische effecten die al deze individuele beslissingen tezamen hebben. We laten zien dat het uitmaakt of deze macro-economische effecten worden meegenomen of niet. De portefeuillebeslissing beweegt in algemeen evenwicht precies in tegengestelde richting als in partieel evenwicht. Consumenten willen individueel hun beleggingsportefeuille in een bepaalde richting aanpassen na een schok. Omdat alle mensen in de economie dezelfde prikkel hebben zal in dat geval de kapitaalmarkt niet meer in evenwicht zijn. In algemeen evenwicht passen de rendementen zich zo aan dat zowel de overheidsschuld als de kapitaalgoederenvoorraad volledig gefinancierd worden. Omdat de hoogte van de overheidsschuld vastligt en daarmee dus ook het aantal obligaties waarin belegd kan worden, zal het deel van de besparingen dat in kapitaal en dus in aandelen wordt belegd in dezelfde richting moeten bewegen als de besparingen. In partieel evenwicht is de financiering van de overheidsschuld of de kapitaalgoederenvoorraad geen enkel probleem omdat de obligaties en aandelen op de internationale kapitaalmarkt kunnen worden aangeboden voor gegeven rendementen.

Neem bijvoorbeeld de effecten van het introduceren van gegarandeerde omslagpensioenen. In partieel evenwicht zullen individuen meer risico nemen met hun investeringsportefeuille als ze een gegarandeerde pensioenuitkering ontvangen. In algemeen evenwicht is dit echter niet mogelijk omdat er dan te weinig geïnvesteerd wordt in de overheidsschuld. Het is zelfs zo dat vanwege het feit dat omslagpensioenen tot een daling van de besparingen leiden, het deel van de besparingen dat in overheidsobligaties wordt belegd moet stijgen. Om consumenten over te halen om meer in overheidsobligaties te investeren moet het rendement op overheidsobligaties stijgen ten opzichte van het rendement op aandelen. Dit impliceert dat de risicopremie op aandelen ten opzichte van obligaties zal dalen. Een gegarandeerde pensioenuitkering zorgt ervoor dat mensen zich minder risico-avers gaan gedragen en in algemeen evenwicht betekent dit dat de risicopremie op aandelen lager kan zijn. Het effect op de risicopremie is tegengesteld op het moment dat mensen risico-averser worden, in dat geval zorgt de stijging van de besparingen ervoor dat de risicopremie van aandelen ten opzichte van obligaties stijgt om evenwicht op de kapitaalmarkt te bewerkstelligen.

Als het inflatierisico stijgt hebben we hetzelfde resultaat: het effect op de portefeuillebeslissing is in algemeen evenwicht tegengesteld aan het effect in partieel evenwicht. Een hoger inflatierisico maakt het minder aantrekkelijk om overheidsobligaties aan te houden en consumenten geven er de voorkeur aan om meer in aandelen te beleggen. In algemeen evenwicht kan dit echter niet gebeuren omdat de overheidsschuld dan niet volledig gefinancierd is. Om ervoor te zorgen dat mensen investeren in de gegeven hoeveelheid overheidsobligaties, moet het rendement op obligaties stijgen. De hogere rente op de overheidsschuld leidt tot een stijging van de zogenaamde schuldbelasting. Dit is de belasting die betaald moet worden om de rentebetalingen op de overheidsschuld te voldoen om zo de schulddynamiek stabiel te houden. De besparingen zullen dalen als gevolg van deze hogere schuldbelasting. Lagere besparingen impliceren dat een groter gedeelte van de besparingen geïnvesteerd moet worden in overheidsobligaties om kapitaalmarktevenwicht te hebben. De risicopremie op aandelen zal dalen om dit bewerkstelligen.

Deze tegengestelde effecten in partieel en algemeen evenwicht betekenen dat het uitmaakt of de portefeuillekeuze van consumenten voor een kleine open economie wordt bestudeerd of voor een grote (gesloten) economie zoals de VS of de EU. In het eerste geval kan men volstaan met de partiële evenwicht aanpak, terwijl het algemene-evenwichtsraamwerk geschikter is in het laatste geval. Daarnaast is het resultaat ook relevant voor de analyse van schokken die wereldomvattend zijn, zoals bijvoorbeeld vergrijzing, waar rendementen niet constant kunnen worden gehouden.

Wanneer landen met omvangrijke omslagsystemen hun pensioenstelsel niet hervormen (zoals in hoofdstuk 3) en niet gelijk de pensioenbijdragen of pensioenuitkeringen aanpassen (zoals in hoofdstuk 2) om de initiële generaties te sparen die het meeste nadeel ondervinden van vergrijzing, zou ook overheidsschuld gebruikt kunnen worden om de kosten van vergrijzing op te vangen. **Hoofdstuk 5** onderzoekt de internationale spillover effecten van vergrijzing voor dit scenario. Het gebruik van overheidsschuld komt ten goede aan de initiële generaties, maar toekomstige generaties zullen er op achteruit gaan omdat overheidsschuld kapitaal verdringt (dit wordt wel het zogenaamde 'crowding out' effect van overheidsschuld genoemd). De nutseffecten zullen doorwerken op het kapitaaldekkende land dat geen overheidsschuld gebruikt. De initiële generaties profiteren van de hogere rendementen die het gevolg zijn van de daling van de kapitaalarbeidsverhouding. Een lagere kapitaalarbeidsverhouding leidt echter ook tot lagere lonen en dit zal latere generaties negatief beïnvloeden. Dit betekent dat het gebruik van overheidsschuld door het omslagland zowel de positieve spillovers van vergrijzing op de korte termijn als de negatieve spillovers op de lange termijn vergroot voor het kapitaaldekkende land.

Overheden kunnen de prikkel krijgen om bij de centrale bank te lobbyen voor onverwachte inflatie als het niveau van de overheidsschuld hoog wordt, omdat dit de begrotingsdruk van de rentebetalingen op de schuld verlaagt. De vraag is natuurlijk of de centrale bank zal toegeven en onverwachte inflatie zal creëren. De veronderstelling in dit hoofdstuk is dat het besluitvormingsproces over monetair beleid niet volledig transparant is. In dat geval kan het zo zijn dat mensen niet overtuigd zijn van de onafhankelijkheid van de centrale bank. De monetaire autoriteit zou dan een prikkel kunnen hebben om nominale schuld weg te infleren. We laten zien dat onverwachte inflatie in een gesloten economie een nul-somspel ('zero-sum game') is. Aan de ene kant ondervindt de overheid voordeel van onverwachte inflatie omdat het de reële waarde van overheidsschuld en het reële rendement op overheidsobligaties verlaagt. Aan de andere kant is het zo dat mensen die in overheidsobligaties hebben belegd nadeel ondervinden van de onverwachte daling van het reële rendement op overheidsobligaties, dit is de zogenaamde inflatiebelasting. In een gesloten economie is het voordeel voor de overheid precies groot genoeg om de mensen die nadeel ondervinden van onverwachte inflatie te compenseren. In een tweelandenmodel waar het ene land een omslagstelsel heeft en het andere land een kapitaaldekkingsstelsel in gebruik heeft, heeft het omslagland echter een netto voordeel van onverwachte inflatie wat ten koste gaat van het kapitaaldekkende land. Dit betekent dat er tegenstrijdige belangen ontstaan over de richting van monetair beleid als landen met verschillende pensioensystemen een monetaire unie vormen. De reden voor dit resultaat is dat inwoners van het land met het kapitaaldekkingsstelsel een relatief groot gedeelte van de totale hoeveelheid overheidsobligaties in handen hebben omdat ze meer sparen en een conservatievere beleggingsportefeuille aanhouden dan mensen die gegarandeerde omslagpensioenen ontvangen. Dit betekent dat het land met het omslagstelsel een deel van de inflatiebelasting naar het land met het kapitaaldekkingsstelsel kan exporteren, terwijl het nog wel het volledige profijt van de lagere schuldenlast heeft en er dus een netto voordeel resulteert. Een land met een omslagstelsel heeft dus een prikkel om de centrale bank onder druk te zetten om de inflatie onverwacht te verhogen als het een monetaire unie vormt met een land met een kapitaaldekkingsstelsel. Het voordeel van onverwachte inflatie voor het land met het omslagstelsel stijgt met de hoogte van de nominale overheidsschuld. In de komende decennia zal vergrijzing de overheidsfinanciën verder onder druk zetten. Wanneer landen met omvangrijke pensioenstelsels hun toegenomen pensioenverplichtingen financieren met overheidsschuld, zal de prikkel van overheden in deze landen om voor onverwachte inflatie te lobbyen stijgen. Onverwachte inflatie zal landen met kapitaaldekkingsstelsels echter meer benadelen als landen met omslagstelsels grote hoeveelheden nominale overheidsschuld uitgeven. De tegenstrijdige belangen bij monetair beleid tussen landen met omslagstelsels en kapitaaldekkingsstelsels zullen dus versterkt worden als vergrijzing de overheidsschuld in omslaglanden verhoogt.

Als het besluitvormingsproces van de centrale bank niet helemaal transparant is, is het niet duidelijk hoe de centrale bank zal reageren op de tegenstrijdige belangen bij de creatie van inflatie van landen met omslagstelsels en kapitaaldekkende landen. Als gevolg daarvan zal er meer onzekerheid bestaan over wat de uiteindelijke uitkomst voor inflatie zal zijn en het inflatierisico zou kunnen stijgen met de hoogte van de overheidsschuld in landen met omslagstelsels. Een hoger inflatierisico betekent dat het minder aantrekkelijk wordt om overheidsobligaties aan te houden en om er voor te zorgen dat mensen de bestaande hoeveelheid overheidsobligaties kopen, zal het rendement op obligaties stijgen ten opzichte van het rendement op aandelen (de risicopremie op aandelen daalt dus). Dit leidt tot een hogere schuldbelasting en een daling van de besparingen en de kapitaalarbeidsverhouding, en beide landen zullen negatieve nutseffecten ondervinden van de stijging van het inflatierisico. Het is zelfs zo dat het land met het omslagstelsel negatieve spillovereffecten ervaart van het feit dat het een monetaire unie vormt met een land met een kapitaaldekkingsstelsel als het inflatierisico stijgt. Dit resultaat doet zich voor omdat het land met het kapitaaldekkingsstelsel een relatief groot gedeelte van de overheidsobligaties in handen heeft en mensen in het kapitaaldekkende land risico-averser zijn omdat ze geen gegarandeerde pensioenuitkering ontvangen. Dit betekent dat deze mensen meer gecompenseerd dienen te worden om bereid te zijn de risicovollere overheidsobligaties in hun beleggingsportefeuille te houden. Het rendement op overheidsobligaties moet dus meer stijgen als individuen kapitaalgedekte pensioenen ontvangen. De stijging van de schuldbelasting zal daarom groter zijn in het land met het omslagstelsel als het een monetaire unie vormt met een land dat een kapitaaldekkingsstelsel gebruikt in plaats van een omslagstelsel.

Alle geanalyseerde scenario's in dit hoofdstuk laten zien dat het kapitaalgedekte land op de lange termijn nog negatiever wordt beïnvloed door het feit dat het andere land in de monetaire unie een omslagstelsel in gebruik heeft als dit land overheidsschuld gebruikt om de vergrijzingskosten op te vangen. Het gebruik van overheidsschuld op zichzelf heeft op de lange termijn negatieve spillovereffecten voor het land met het kapitaaldekkingsstelsel. Daarnaast kan een hoge schuldenlast tot onverwachte inflatie of een hoger inflatierisico leiden, wat beide de negatieve spillovereffecten versterkt. In de komende decennia is het daarom van belang voor landen met grote kapitaaldekkingsstelsels dat er wordt voldaan aan de regels van het Stabiliteits en

Groei Pact, zodat de schuldniveaus in landen met omvangrijke omslagstelsels niet te hoog worden. Het gebruik van overheidsschuld door omslaglanden om de vergrijzingskosten op te vangen bevoordeelt de initiële generaties die het meeste nadeel ondervinden van vergrijzing. Toekomstige generaties zullen echter worden benadeeld door de grotere schuldenlast. In een monetaire unie kan een deel van deze last afgewenteld worden op het land met het kapitaaldekkingsstelsel. De schuldenlast kan tevens verminderd worden door een onverwachte stijging van de inflatie als het land met het omslagstelsel succesvol lobbyt bij de centrale bank. Als dit lobbyen echter alleen maar het gepercipieerde inflatierisico verhoogt, dan kunnen de negatieve effecten van een stijging van het risico op overheidsobligaties als gevolg van de hogere overheidsschuld niet gedeeld worden met het land met het kapitaaldekkingsstelsel. Het kan daarom ook in het belang zijn van landen met omslagstelsels om te voldoen aan de restricties van het Stabiliteits en Groei Pact. Daarnaast is het belangrijk voor alle landen dat een centrale bank zoals de ECB onafhankelijk, geloofwaardig en transparant is, zodat het duidelijk is hoe het monetaire beleid wordt bepaald en het inflatierisico niet stijgt als de schuldniveaus hoog oplopen.