Nonnegativity constraints and intratemporal uncertainty in multi-good life-cycle models

Adang, P.J.M.; Melenberg, B.

Published in:
Journal of Applied Econometrics

Publication date:
1995

JOURNAL OF APPLIED ECONOMETRICS, VOL. 10, 1-15 (1995)

NONNEGATIVITY CONSTRAINTS AND INTRATEMPORAL UNCERTAINTY IN A MULTI-GOOD LIFE-CYCLE MODEL

PIM ADANG AND BERTRAND MELENBERG
Department of Econometrics, Tilburg University, The Netherlands, PO Box 90153, 5000 LE, Tilburg

SUMMARY

In the standard multi-good life-cycle consumption model (with intertemporal additive utility) the intratemporal relations between the marginal utilities of the different goods are deterministic. However, these deterministic identities will not usually be satisfied by the data. To avoid these deterministic relationships, we apply an approach which consists of introducing intratemporal uncertainty, and which is, in particular, interesting when additional nonnegativity constraints are present. We estimate some simple versions of the model with this so-called intratemporal uncertainty. The estimation results are, in general, in accordance with the theory, and most versions of the model are not rejected by Hansen and Singleton's misspecification test.

1. INTRODUCTION

Since Hall (1978) many economists have studied consumer behaviour under uncertainty within the context of the Life-Cycle Hypothesis (LCH) by means of Euler equations. The standard LCH states that a consumer decides in each period on (total) consumption by maximizing an intertemporally additive (von Neumann–Morgenstern) expected utility function subject to a lifetime wealth budget constraint. From the first order conditions of this optimization problem one can derive Euler equations, which have an attractively simple form: The marginal utility of consumption evolves according to a random walk with trend. By using the Euler equations, the model can be estimated by the Generalized Method of Moments (GMM), as proposed by Hansen and Singleton (1982).

If Hall's (1978) life-cycle model is extended to deal with more than one good per period, the first-order conditions that should hold at the optimum not only result in intertemporal Euler equations but also imply deterministic intratemporal relations between the marginal utilities of the different goods. The deterministic nature of these intratemporal relations has serious consequences for empirical applications of this model. The intratemporal relations must hold *exactly* for each observation in the data set used for the particular application. As it is very unlikely, or even impossible, that this requirement will be met, the presence of such deterministic relations indicates some form of misspecification.

In order to overcome this misspecification, the multi-good version of Hall's (1978) model needs to be modified. Standard approaches consist of incorporating random preferences or measurement errors. However, if one considers models extended with inequality constraints, such as nonnegativity constraints, these modifications can have some disadvantagese. In order to obtain empirically applicable moment restrictions, one has to impose assumptions that do not always seem reasonable.

Alternatively, one can modify the life-cycle model by allowing the uncertainty in the model

CCC 0883-7252/95/010001-15 *Received April 1991*
Revised January 1994

to be not only intertemporal of nature but also intratemporal. This modification, initially noticed by Melenberg and Alessie (1989), has the advantage, considered from an econometric point of view, that deterministic intratemporal relationships are converted into moment restrictions or that these can no longer be obtained, whereas intertemporal Euler equations can still be obtained under reasonable assumptions. In Section 2 we discuss the incorporation of this intratemporal uncertainty into the life-cycle model. We argue that this modification of the life-cycle model is also reasonable from an economic point of view, and we make a comparison with the alternative approaches, in particular random preferences.[1] In Section 3, the estimation and testing results of some (relatively simple) two good versions of the life cycle model with intratemporal uncertainty are presented. For this, a Dutch panel containing information on the monthly expenditures on several commodity categories is used. The estimates are, generally, in accordance with consumer theory. The test results imply that all but one of the versions incorporating intratemporal uncertainty are not rejected.

2. THE LIFE-CYCLE MODEL WITH INTRATEMPORAL UNCERTAINTY

2.1. A standard life-cycle model formulation

Our starting-point is the familiar life-cycle model in which consumers are confronted with uncertainty induced by variables such as income, prices, interest rates, and taste shifters. We shall call these input variables. Following the literature, we assume uncertainty in the sense that future realizations of these input variables are unknown, but that the probability distribution generating these variables is known to the consumer and remains unaffected by the consumer's actions.

In the standard approach (cf. Hall, 1978) it is assumed that at the beginning of a particular period t the consumer knows the realizations of the input variables up to and including period t, whereas the variables dated $t+1$ or later are uncertain. In period t the consumer determines period t's consumption and plans consumption for the periods $\tau > t$. The planned consumption of period τ is allowed to depend upon the input variables up to and including period τ.

In the resulting *standard life-cycle model* a consumer solves at the beginning of each period $t = 1, \ldots, L$, with L the consumer's lifetime, the following problems:

$$\begin{aligned} &\operatorname{Max}_{(q_t', \ldots, q_L')'} E_t \sum_{\tau=t}^{L} u_\tau(q_\tau) \\ &\text{s.t.} \sum_{\tau=t}^{L} i_{t\tau} p_\tau' q_\tau \leq (1 + r_{t-1}) A_{t-1} + \sum_{\tau=t}^{L} i_{t\tau} y_\tau . \end{aligned} \tag{1}$$

where q_τ is an M-dimensional vector of quantities of goods in period τ, p_τ is the corresponding M-dimensional price vector, y_τ denotes nominal non-property income in period τ,

$$i_{tt} = 1, \, i_{t\tau} = \prod_{j=t}^{\tau-1} (1 + r_j)^{-1}, \, \tau > t,$$

[1] In the paper by Melenberg and Alessie (1989) actually only the possibility of the modification of the life-cycle model used in the present paper is noticed and worked out from a purely technical point of view. The contribution of the present paper is the motivation from an economic point of view as well as a comparison with random preferences.

r_τ is the nominal interest rate in period τ, and A_{t-1} is non-human wealth at the end of period $t-1$.

Let prices, interest rates, and income be the input variables. Assume y_τ, p_τ, and $r_{\tau-1}$ to be realized at the beginning of period τ. Then, in period t, q_τ, $\tau = t, \ldots, L$ is allowed to depend upon the input variables contained in the set

$$\{y_{t+1}, p_{t+1}, r_t, \ldots, y_\tau, p_\tau, r_{\tau-1}\} \tag{2}$$

The expectation operator, E_t, is conditional upon the variables contained in the information set, I_t, which includes the set $\{y_1, p_1, r_0, \ldots, y_t, p_t, r_{t-1}\}$. Hence we can write, for some function $f(.)$: $E_t[f(.)] = E[f(.)\mid I_t]$.

Hall (1978) considered only total consumption, and obtained the Euler equation by means of a calculus of variations technique. In the multi-good case studied here that same technique can be applied to obtain in addition to the Euler equations the following intratemporal relations between marginal utilities:

$$(\partial u_t(q_t)/\partial q_{kt})/p_{kt} - (\partial u_t(q_t)/\partial q_{lt})/p_{lt} = 0 \tag{3}$$

These relations are deterministic of nature, and will generally not be satisfied in empirical applications, indicating model misspecification. In order to avoid this misspecification, the usual approach is to add randomness to the model, which concerns the econometrician but not the consumer. For instance, in case of the two-stage budgeting approach one uses equation (3) to derive a demand system, which is estimated after tacking on error terms. See, for example, Blundell (1987), Blundell *et al.* (1991), and Alessie *et al.* (1989). Possible motivations for the additional randomness may be that the researcher does not exactly know the functional form of the utility function, i.e. random preferences (cf. MaCurdy, 1983), or that measurement errors are present (cf. Altonji and Siow, 1987).

In order to avoid equation (3), let us take two goods and assume random preferences. Measurement errors can be dealt with analogously. Make u_t dependent upon a random parameter α, with probability distribution P_α. A straightforward way to introduce random preferences is to take[2] $u_t(q_{1t}, q_{2t}) = \bar{u}_t(q_{1t}, q_{2t}) + \alpha_1 q_{1t} + \alpha_2 q_{2t}$, with $\bar{u}_t$ not a function of α. Equation (3) becomes:

$$(\partial \bar{u}_t(q_t)/\partial q_{1t})/p_{1t} - (\partial \bar{u}_t(q_t)/\partial q_{2t})/p_{2t} + \alpha_1/p_{1t} - \alpha_2/p_{2t} = 0 \tag{4}$$

Obviously, the above equation is not a deterministic relationship, due to the occurrence of α. If we make the assumption that $\alpha = (\alpha_1, \alpha_2)$ is independent of the input variables and has expectation (0, 0), then taking expectation in equation (4) with respect to P_α leads to

$$E_\alpha[(\partial \bar{u}_t(q_t)/\partial q_{1t})/p_{1t} - (\partial \bar{u}_t(q_t)/\partial q_{2t})/p_{2t}] = 0 \tag{5}$$

where E_α is still needed, since q_t will depend upon α. For the sample analogue of equation (5) α is not needed, so that (5) is empirically applicable. In addition, the standard Euler equations will remain in force under the same modifications and assumptions regarding α. Thus the life-cycle model (1) can be estimated and tested by combining the usual Euler equations and moment restrictions (5).

Thus, in the case of model (1), it is straightforward to avoid the occurrence of (3) by imposing, for instance, random preferences. Consider next the case where additional nonnegativity constraints are present. We shall concentrate on the case with two goods per

[2] This choice is particularly attractive in the case of a quadratic utility function (cf. Section 3).

period, where the nonnegativity constraint may be binding with respect to the second good. Thus, in addition to equation (1), we require

$$q_{2\tau} \geqslant 0, \tau = t, ..., L \tag{6}$$

The following Euler equations can easily be derived:

$$E_t[(\partial u_{t+1}(q_{t+1})/\partial q_{1,t+1})/(i_{t,t+1}p_{1,t+1}) - (\partial u_t(q_t)/\partial q_{1t})/p_{1t}] = 0 \tag{7a}$$

$$E_t[[(\partial u_{t+1}(q_{t+1})/\partial q_{1,t+1})/(i_{t,t+1}p_{1,t+1})) - (\partial u_t(q_t)/\partial q_{2,t})/p_{2t}]I_{(0,\infty)}(q_{2t})] = 0 \tag{7b}$$

with $I_{(0;\infty)}(q_{2t})$ the usual indicator function. Without additional randomness, equation (3) remains valid for $q_{2t} > 0$, i.e. we have[3]

$$[(\partial u_t(q_t)/\partial q_{1t})/p_{1t} - (\partial u_t(q_{1t})/\partial q_{2t})/p_{2t}]I_{(0;\infty)}(q_{2t}) = 0 \tag{8}$$

Suppose that we want to avoid equation (8) by assuming random preferences of the form also used in the standard case. Instead of equation (8) we then obtain the following equation (cf. the transformation from equation (3) to equation (4)):

$$[(\partial \bar{u}_t(q_t)/\partial q_{1t})/p_{1t} - (\partial \bar{u}_t(q_t)/\partial q_{2t})/p_{2t} + \alpha_1/p_{1t} - \alpha_2/p_{2t}]I_{(0;\infty)}(q_{2t}) = 0 \tag{9}$$

Like equation (4), equation (9) is a nondeterministic one, due to the presence of α. The Euler equations (7a)–(7b) have to be transformed into the following two equations, respectively, where the additional terms are analogous to the one occurring in equation (9):

$$E_t[(\partial \bar{u}_{t+1}(q_{t+1})/\partial q_{1,t+1})/(i_{t,t+1}p_{1,t+1}) - (\partial \bar{u}_t(q_t)/\partial q_{1t})/p_{1t} + \alpha_1/(i_{t,t+1}/p_{1,t+1}) - \alpha_1/p_{1t}] = 0 \tag{10a}$$

$$E_t[[(\partial \bar{u}_{t+1}(q_{t+1})/\partial q_{1,t+1})/(i_{t,t+1}p_{1,t+1}) - (\partial \bar{u}_t(q_t)/\partial q_{2,t})/p_{2t} + \alpha_1/(i_{t,t+1}/p_{1,t+1}) - \alpha_2/p_{2t}]I_{(0;\infty)}(q_{2t})] = 0 \tag{10b}$$

If we now average over α and impose that α is independent of the input variables and has zero expectation (cf. the transformation from equation (4) to equation (5)), it is obvious that in equation (10a) the term in which α is explicitly present will disappear. For equations (9) and (10b), however, we need[4]

$$E_\alpha[[(\alpha_1/p_{1t} - \alpha_2/p_{2t})]I_{(0,\infty)}(q_{2t})] = 0 \tag{11a}$$

$$E_\alpha[[(\alpha_1/(i_{t,t+1}p_{1,t+1}) - \alpha_2/p_{2t})]I_{(0,\infty)}(q_{2t})] = 0 \tag{11b}$$

where equation (11a) corresponds to (9) and (11b) to (10b). In general, both (11a) and (11b) will not follow from α only being independent of the input variables and having zero expectation. The reason is clear: q_{2t} is a (generally unknown) function of α. Therefore, both equations (11a) and (11b) consist of products of the exogenous random terms α_1 and α_2 and the endogenous censored variable $I_{(0;\infty)}(q_{2t})$, which depends upon α. These products will not have zero expectation (unless, of course, either $q_{2t} = 0$ or $q_{2t} > 0$ with probability one). It is not easy, if possible at all, to strengthen the assumptions on α in a reasonable way such that they will imply equations (11a) and (11b).

Thus the situation is as follows. Without random preferences, one will have to reject the model on the basis of equation (8). With random preferences (of the form presented), one can actually only use the Euler equation (7a) or, equivalently, (10a). Using also equations (9) or

[3] Thus, if $q_{2t} > 0$, equation (8) is equivalent to equation (3), and if $q_{2t} = 0$, we have the obvious identity $0 = 0$.
[4] In equation (11b) the expectation with respect to α is conditional upon p_{1t+1} and $i_{t,t+1}$.

(10b) requires the imposition of conditions (11a) or (11b), respectively. These latter conditions, however, are *ad hoc*, and can hardly, if at all, be motivated.

Similar consequences are to be expected if we consider other forms of random preferences (or, alternatively, measurement errors). Therefore, we shall discuss an alternative modification of the life-cycle model formulation, which avoids the occurrence of relationships such as equations (3) and (8), but which nevertheless allows one to obtain both Euler equations (7a) and (7b), under quite plausible assumptions. This will be the topic of the next subsection.

2.2. Intratemporal Uncertainty

Melenberg and Alessie (1989) generalize the standard life-cycle model by no longer assuming that the consumer's uncertainty pertaining to a particular period τ completely resolves at the beginning of that period (the moment the consumer is supposed to decide). Instead, they allow the uncertainty to resolve partly during the period, and also, loosely speaking, differently with respect to different goods.

From a technical point of view, their approach basically consists of using, in the case of period τ, not just one set of input variables upon which all the components $q_{1\tau}, \ldots, q_{M\tau}$ of q_τ are assumed to depend. Instead, they allow for M different sets in each period τ, one for each consumption good $q_{m\tau}$, $m = 1, \ldots, M$. Define for each $\tau \geqslant t$ (assuming that only prices, income, and interest rates induce uncertainty): $\eta_\tau = (y_\tau, p_\tau', r_{\tau-1})'$. Then assume $\eta_\tau = (\boldsymbol{\eta}_\tau', \bar{\eta}_\tau')'$, with the interpretation that the realization of $\boldsymbol{\eta}_\tau$ is known at the beginning of period τ, whereas the realization of $\bar{\eta}_\tau$ is not yet known. Using this notation, the set of input variables corresponding to good m in period τ is no longer given by equation (2), but becomes $\{\bar{\eta}_t, \eta_{t+1}, \ldots, \eta_{\tau-1}, \boldsymbol{\eta}_\tau, \eta_{m\tau}\}$, where $\eta_{m\tau}$ consists of those elements of $\bar{\eta}_\tau$ which the consumer knows when deciding upon $q_{m\tau}$. Notice that the standard modelling corresponds to $\eta_\tau = \boldsymbol{\eta}_\tau$, $\tau = t, \ldots, L$, so that $\eta_{m\tau}$ and $\bar{\eta}_\tau$ are empty for all m and τ.

The present modification implies that the expectation operator E_t becomes conditional upon the variables contained in the original information set I_t, except the variables of period t which realizations are not yet known at the beginning of period t. Thus $\bar{\eta}_t$ is excluded from I_t. Denote the new information set by I_t'. Then we can define $E_t[\mathrm{f}(.)] = E[\mathrm{f}(.) | I_t']$.

Using this way of modelling the consumer's uncertainty, we avoid the restrictive and also arbitrary assumption that the consumer, when deciding or planning at the beginning of a period, already knows the realizations of all input variables concerning that period. Various examples can be devised to motivate the present generalization. A simple one is to assume that the realization of the prices takes place during a period and in some order, for example, given by the numbering of the goods. For instance, think of a consumer going from market to market in each period. Suppose the consumer wants to consume each good as soon as its price is known. This can be modelled by assuming that $\eta_{1\tau}$ does not contain $p_{2\tau}, \ldots, p_{M\tau}$, $\eta_{2\tau}$ does not contain $p_{3\tau}, \ldots, p_{M\tau}$, and so on. Thus $q_{1\tau}$ is not a function of $p_{2\tau}, \ldots, p_{M\tau}$, so that, as soon as $p_{1\tau}$ is known and $p_{2\tau}, \ldots, p_{M\tau}$ are still unknown, the consumer nevertheless knows (how to choose) $q_{1\tau}$. In this construction, $q_{1\tau}$ does not depend upon the price *realizations* of the other goods but it will depend upon them through the probability distribution of the input variables (which includes these prices). In this example one can change, without difficulty, the ordering of the goods. In addition, the ordering of the goods may be different for different consumers.

The price example is just one possible reason for intratemporal uncertainty. As another example, suppose that a consumer's income stream arises from two sources: labour income and a holiday allowance. Some consumers may then wish to make vacation expenditures dependent upon both labour income and the holiday allowance, whereas other expenditures are allowed only to depend upon labour income. Such an arrangement will also introduce intratemporal uncertainty. It can easily be allowed for in the present framework.

In what follows we shall assume intratemporal uncertainty, however, without specifying its origin. All we impose is that $\eta_{m\tau}$ is nonempty, and that $\eta_{m\tau} \neq \eta_{n\tau}$, for $m \neq n$.

2.3. The construction of moment restrictions

In order to derive Euler equations and moment restrictions in the modified life-cycle model, we shall apply a Lagrange multiplier rule as given in, for instance, Neustadt (1976, Chapter 3). We concentrate on a life-cycle model with two goods, extended with inequality constraints concerning the second good. According to the first-order conditions, there should hold for *all* possible functions $(h_t', ..., h_L')'$ of the input variables, where $h_\tau = (h_{1\tau}', h_{2\tau}')'$, $\tau = t, ..., L$, and where $h_{m\tau}$ is allowed to depend upon the same input variables as $q_{m\tau}$, $m = 1,2$, $\tau = t, ..., L$:

$$E_t\left[\sum_{\tau=t}^{L} (\partial u_\tau/\partial q_{1\tau})h_{1\tau} + (\partial u_\tau/\partial q_{2\tau})h_{2\tau} - \lambda_t \sum_{\tau=t}^{L} i_{t\tau} p_\tau' h_\tau - \sum_{\tau=t}^{L} \mu_{2\tau} h_{2\tau}\right] = 0 \tag{12}$$

together with

$$E_t[q_{2\tau} \ \mu_{2\tau}] = 0, \quad \tau = t, .., L \tag{13}$$

Here, λ_t is the Lagrange multiplier corresponding to the lifetime wealth budget constraint, which is a function of *all* input variables, and $\mu_{2\tau}$, $\tau = t, ..., L$, are the Lagrange multipliers corresponding to the nonnegativity constraints, which are allowed to depend upon the same input variables as $q_{2\tau}$, $\tau = t, ..., L$, respectively, and which have to be nonnegative.

By choosing the h-functions in equations (12) and (13) in a suitable way, we may be able to obtain empirically applicable moment restrictions, i.e. moment restrictions in which the unknown Lagrange multipliers do not show up.

Notice first that the deterministic relationship (8) cannot be obtained now. In order to derive (8) we have to choose something like $h_{1t} = (1/p_{1t})I_{(0;\infty)}(q_{2t})$, $h_{2t} = -(1/p_{2t})I_{(0;\infty)}(q_{2t})$, and the other h-functions equal to zero.[5] However, in case of intratemporal uncertainty, q_{1t} depends upon input variables other than q_{2t}. Consequently, the presented choice for h_{1t} is not allowed, since it depends upon q_{2t}, and thus upon the input variables of q_{2t}.

Therefore, in order to obtain empirically applicable moment restrictions, other h-functions have to be chosen. For a first empirically applicable moment restriction, take $h_{1t} = -1/p_{1t}$, $h_{1,t+1} = 1/(i_{t,t+1}p_{1,t+1})$, and the other h-functions equal to zero. This results in the Euler equation (7a) for the first good. Equation (7b) can be derived by taking all h-functions equal to zero except $h_{1t} = (-1/p_{2t})I_{(0,\infty)}(q_{2t})$ and $h_{1,t+1} = (1/(i_{t,t+1}p_{1,t+1})) \times I_{(0;\infty)}(q_{2t})$. Using equation (13) and the nonnegativity of μ_{2t} to ensure that the term $E_t[\mu_{2t}h_{2t}]$ in equation (12) equals zero, equation (7b) will follow.

Thus the null hypothesis that can be tested consists of *both* Euler equations (7a) and (7b). The model cannot be rejected on the basis of equation (8), since this restriction is not allowed. The alternative hypothesis contains as a special case random preferences of the form discussed in Section 2.1, at least if equation (11b) does not hold. Accepting the null hypothesis is, therefore, an indication that this form of random preferences might not be present. Of course,

[5] Formally, we must be able to choose $h_{1t} = A \times [(1/p_{1t})I_{(0;\infty)}(q_{2t})]$ and $h_{2t} = A \times [(-1/p_{2t})I_{(0;\infty)}(q_{2t})]$, with A the left-hand side of equation (8). Substituting these choices into equation (12) gives $E_t[A]^2 = 0$, or, equivalently, equation (8).

other forms of random preferences and measurement errors are also included in the alternative hypothesis.

In order to test the null hypothesis, one can construct unconditional moment restrictions from equations (7a) and (7b) in the usual way. Notice that the instruments, to be used in case of period t, may only depend upon what is known by the consumer at the *beginning* of period t, i.e. I'_t. This is a slight but important difference with the standard approach.

Finally, we briefly consider the life-cycle model without inequality constraints ($\mu_{2\tau}=0$ in equation (12)). If we choose $h_{1t}=1/p_{1t}$ and $h_{2t}=-1/p_{2t}$ and all other h-functions equal to zero, we obtain from equation (12), instead of equation (3):

$$E_t[(\partial u_t(q_t)/\partial q_{1t})/p_{1t}-(\partial u_t)/\partial q_{2t})/_{2t})/p_{2t}]=0 \tag{14}$$

With intratemporal uncertainty, we have to allow for nonempty η_{1t} and η_{2t}, with $\eta_{1t}\neq\eta_{2t}$, so that the components not entering both η_{1t} and η_{2t} have to be averaged out, hence the expectation operator in equation (14). Thus now the deterministic relationship (3) is converted into a moment restriction. The standard Euler equations are obtained by choosing $h_{mt}=-1/p_{mt}$, $h_{m,t+1}=1/(i_{t,t+1}p_{m,t+1})$ ($m=1$ or 2), and the other h-functions equally zero:

$$E_t[(\partial u_{t+1}(q_{t+1})/\partial q_{m,t+1})/(i_{t,t+1})-(\partial u_t(q_t)/\partial q_{mt})/p_{mt}]=0 \tag{15}$$

In the next section we shall test some versions of the life-cycle model with intratemporal uncertainty. By assuming intratemporal uncertainty, we do not claim that random preferences or measurement errors cannot be present. The present approach only avoids the *need* for them: One can test various versions of the life-cycle model in which random preferences or measurement errors do not play a role without having to reject such models immediately on the basis of intratemporal deterministic relationships that are not satisfied.

3. EMPIRICAL APPLICATION

3.1. The data

The objective of this section is to assess the empirical relevance of the life-cycle model with intratemporal uncertainty and nonnegativity constraints with respect to one of the goods. We will do this on the basis of a two-goods version of the life-cycle model. For comparison, we shall also report the results of a two-goods life-cycle model without nonnegativity constraints.

The data come from the so-called 'Intomart consumer expenditure panel'. This panel contains information on monthly expenditures of households on several commodity categories, and a number of demographic characteristics of these households (including social class and household composition) which are registered on an annual basis. As prices we added the national price indices corresponding to the commodity classes as reported by the Netherlands Central Bureau of Statistics. The panel covers the 42 months from April 1984 through September 1987.

There are some characteristics of the data set that need to be reported. First, almost no household participates in the panel for the complete spell April 1984–September 1987. Only 91 of the 2897 households participate in all 42 periods. Second, when constructing sample analogues of the moments that are used in estimation, different moments correspond with different data requirements. The way in which we formulate the moment restrictions (see Section 3.2) implies that all 32,456 observations (households times periods) can be used for constructing sample analogues of the intratemporal moments which have a demographic variable as instrument. For the intratemporal relations which have the one-period lagged expenditure or price as instrument, as well as for the intertemporal ones which have a demographic variable as instrument, we only use those households participating at least two consecutive periods. This requirement is met by 29,732 observations reported by 2566 households. Finally, for the intertemporal restrictions which have the one-period lagged expenditure or price as instrument, we only use those households that participate at least three consecutive periods. This requirement reduces the number of observations that can be used to 27,334, which are reported by 2382 households. We make the assumption that both types of selection (attrition in the original panel and selection resulting from creating sample analogues of the different moment restrictions) are random.

3.2. Moment restrictions

As mentioned above, the application is limited to the two-goods case. Consider as categories vacation and nonvacation. As can be seen from Table I, vacation is a clear example of an infrequently purchased good, which implies that the nonnegativity constraint for this good will be binding for many observations. In the version without nonnegativity constraints, the two goods are food and non-food. Depending on which model is estimated, either food or vacation is the second good.

The following specification is chosen. We assume that the intratemporal utility function depends on τ only through the discounting factor, i.e.

$$u_\tau(.) = \left(\frac{1}{1+\rho}\right)^{\tau-t} u(.)$$

with ρ the time preference rate, assumed to be constant over time as well as over households. Second, as it is not clear which observable interest rate corresponds to the interest rate of the model, the $r_\tau - s$ in equation (1) are taken to be unknown parameters. Since the actual interest rates, which might have been used for r_τ, remained stable over the sample period, we assume

Table I. Percentage of households with zero vacation expenditures

Period	NH	PZ	Period	NH	PZ	Period	NH	PZ
1	921	79.8	15	753	71.2	29	798	69.3
2	966	74.1	16	757	63.0	30	787	80.8
3	884	66.6	17	767	71.0	31	837	83.2
4	922	59.2	18	789	80.0	32	858	90.2
5	855	68.3	19	806	86.0	33	978	89.3
6	757	81.5	20	764	91.4	34	956	84.1
7	889	85.9	21	742	90.2	35	1022	83.5
8	849	91.5	22	676	84.2	36	1018	80.6
9	789	89.2	23	667	83.2	37	981	78.5
10	736	85.7	24	680	82.7	38	1024	71.5
11	693	82.1	25	706	78.1	39	1052	66.5
12	856	82.9	26	676	71.0	40	968	60.6
13	816	77.6	27	776	69.9	41	954	66.4
14	751	71.7	28	818	59.8	42	n898	76.8

NH = number of households participating in the original panel in a certain month.
PZ = percentage of these households that register zero expenditures for vacation in that month.
Period 1 = April 1984.
Period 42 = September 1987.

that r_τ is constant over time. This assumption reduces the number of parameters considerably, but implies that we are not able to estimate the time preference parameter ρ. We can only estimate the quotient $(1+r)/(1+\rho)$.

We consider a quadratic specification of the intratemporal utility function $u(.)$, where the normalization $a.c - b^2 = 1$ is imposed to ensure identification:[6]

$$u(q_\tau) = \tfrac{1}{2}\{a.q_{1\tau}^2 + 2b.q_{1\tau} + c.q_{2\tau}^2\} + d.q_{1\tau} + e.q_{2\tau} \tag{16}$$

where a $(=(1+b^2)/c)$, b, c, d and e are parameters to be estimated.

As a generalization of this basic version we will make the parameters d and e household dependent, thus allowing the bliss point of the quadratic utility function ($bc - cd$ for the first good, and bd-$(1+b^2)/c$ for the second) to be household-specific. The particular form in which we model this is as follows, with fs the household size:

$$d = d_0 + d_1.\log(fs), e = e_0 + e_1.\log(fs) \tag{17}$$

The chosen specification may seem rather restrictive. However, since our main interest is in investigating the empirical applicability of the Life-Cycle Hypothesis with intratemporal uncertainty, we kept the specification of the life-cycle model as simple as possible.[7]

We use the moment restrictions derived in Section 2. For the vacation/nonvacation case one could derive the moments on the basis of equations (7a) and (7b). A disadvantage of equation (7b) is that it only uses those households in period t, which register a positive amount of consumption of vacation in this period. As can be seen from Table I, this implies that for this second moment most observations will be left unused in estimation. Although from a theoretical point of view not using these observations should not affect the outcome, it turned out to lead to some numerical problems in the empirical application.[8] Therefore we replaced moment restriction (7b) by the sum of (7a) and (7b). This results in two times 41 intertemporal moment restrictions. The set of instruments, used to obtain unconditional moment restrictions, consists of the set of demographic variables, described in the Appendix, extended with the one-period lagged holiday expenditure,[9] resulting in 11 instruments.

For the food/nonfood case, a system of moment restrictions follows from equations (14) and (15). In equation (15) we use the first good, i.e. nonfood. Notice that adding to this Euler equation the intratemporal moment restrictions of the corresponding two periods results in the Euler equation of the second good. This latter Euler equation has therefore been dropped. We thus obtain 42 intratemporal and 41 intertemporal conditional moment restrictions. The set of instruments which is used to convert these conditional moment restrictions into unconditional ones consists of a set of demographic variables, described in the Appendix, extended with the one-period lagged food expenditure and price of food. This leads to 12 instruments.

---

[6] This particular normalization is chosen because it implies that all that remains to be checked to ensure the concavity of the utility function, is whether the parameter c is negative.

[7] Notice, that more general specifications, like nonconstant interest rates or time-preference rates, will hardly complicate the theoretical derivation of moment restrictions but are likely to make empirical analysis difficult, since many more parameters have to be estimated.

[8] The computational difficulties arose when trying to determine the inverse of the outer product of the vector of moment restrictions, which is necessary in order to determine the optimal weighting matrix. Although this matrix should be positive semidefinite, it was found not to be so. Subsequent computation of the eigenvalues of this matrix indicated that some of them were very close to zero, but negative. Given the size of the negative eigenvalues, we concluded that this problem was due to rounding errors.

[9] In the case of the inclusion of the one-period lagged price of holidays in the instrument set the iterative procedure used to determine consistent estimates, which are needed for constructing the optimal weighting matrix, did not converge within acceptable time limits.

When constructing sample analogues of the two systems of moment restrictions it is often observed that one should be aware of possible effects of economy-wide shocks. As pointed out by, for instance, Chamberlain (1984), Hayashi (1985) and Hotz *et al.* (1988), if such shocks are present, averaging over time is essential to ensure the consistency of the estimators. Therefore, we first averaged the moment restrictions over time.[10] In the holiday/nonholiday case we thus end up with 22 moment restrictions, and in the food/nonfood case with 24 moment restrictions.

The resulting systems of moment restrictions are estimated by means of the Generalized Method of Moments (GMM), using the efficient weighting matrices, as discussed in, for instance, Hansen and Singleton (1982).

3.3. Estimation results

In Table II the estimation results for the different versions of the two models are given. Comparing the holiday and food cases we can see a clear difference which concerns not so much the estimates but the corresponding standard errors. In particular, the estimates of the parameters corresponding to the linear part of the utility function, i.e. d_0, d_1, e_0, and e_1, have large standard errors in the food cases. A possible explanation for this is that these parameters correspond with terms in the moment restrictions which are mainly determined by prices.

Although all are rather stable during the survey period, the price variation in the food cases is even smaller than the variation in the holiday cases. Therefore the estimates of these parameters are likely to be less precise in the food cases.

Turning next to the estimates themselves, it can be seen from Table II that the estimate of the parameter c is negative (and significant) for all cases, implying a strictly concave utility function, as required.[11]

Another condition that should hold for the models to be consistent with consumer theory is that the bliss point (i.e. the top of the 'utility hill') is located such that all observations are situated on the part of the utility function where it is increasing in both its arguments.[12] For the basic versions of the food case (food1), this requirement is met by all reported food expenditures and by all but 0.9% of the nonfood expenditures. For the basic holiday version (holiday1) the percentage of wrongly situated observations rises to 2.8 for the holiday and 2.1 for the nonholiday goods, respectively.

The dependence of the parameters d and e on the logarithm of the household size for the household-specific versions implies a similar dependence for the bliss point. Hence, the above-mentioned 'bliss point condition' must be checked for each household size separately. As can be seen from Table II, the estimates of the parameters d_1 and e_1 are positive in all versions, implying that the bliss point increases with the household size, as one would expect. Notice that although neither of the estimates of these parameters is significantly different from zero for the food version, the value of the Wald statistic, T2, reported in Table II, nevertheless indicates that they are jointly significant.

---

[10] There is also a practical reason for doing this, since if the moment restrictions are not averaged over time there would be 830 of these restrictions. Obtaining efficient GMM estimates requires a square matrix weighting the moments. In order to determine this matrix of dimension 830×830 a matrix of the same dimension must be inverted (cf. Hansen and Singleton, 1982). However, the mainframe on which the computations for this paper were performed (a VAX 8700) did not allow for matrices of such a dimension.

[11] Although (quasi-)concavity of the utility function is usually required in models of consumer behaviour, it is not always found in empirical work. See, for example, Hansen and Singleton (1984).

[12] Observations not satisfying this requirement are incompatible with the assumed rational behaviour of consumers, as the same expected utility level can be obtained from a lower consumption level.

Table II. Estimation results

Version	food1	food2	holiday1	holiday2
b	−0.187	−0.133	−0.771	−0.523
	(0.145)	(0.131)	(0.163)	(0.182)
c	−1.652	−2.515	−1.856	−1.660
	(0.464)	(0.631)	(0.102)	(0.069)
d_0	86.945	85.801	87.044	88.616
	(69.129)	(240.555)	(24.303)	(28.105)
d_1	–	6.575	–	31.323
		(227.423)		(13.574)
e_0	83.843	85.083	93.592	94.405
	(67.297)	(234.047)	(24.419)	(28.611)
e_1	–	12.849	–	27.616
		(221.239)		(14.036)
$\frac{1+r}{1+\rho}$	1.001	1.001	0.999	1.000
	(0.001)	(3.10^{-4})	(0.009)	(0.003)
T1	15.7	24.3	31.5	18.8
df1	19	17	17	15
p1	0.677	0.112	0.017	0.222
T2	–	7.1	–	37.9
df2	–	2	–	2
p2	–	0.028	–	6.10^{-9}

Consumption measured in hundreds of guilders.
Standard errors in parentheses.
food1, holiday1 = basic versions.
food2, holiday2 = versions with household-specific parameters d and e.
T1 = chi-square value for Hansen and Singleton's misspecification test.
df1 = degrees of freedom of misspecification test.
p1 = significance level of misspecification test.
T2 = value of Wald test on significance of combined household effect.
df2 = degrees of freedom of Wald test.
p2 = significance level of Wald test.

Checking the 'bliss point condition' for the household-specific versions, it follows that for the food version it is met, as far as food expenditures are concerned, by all observations except two. For the nonfood purchases, the percentage of violations varies somewhat with the household size (between 0% and 0.6%), but is around 0.2% for most household sizes. The percentages for the holiday case are somewhat larger, but do not differ significantly. The percentage of rejections for the holiday expenditures varies between 0 and 0.6, whereas this percentage lies between 0.4 and 2.4 for the nonholiday expenditures. In general we consider the number of observations rejecting the 'bliss point condition' to be acceptable.

Furthermore, it can be seen from Table II that for all cases the term $(1+r)/(1+\rho)$ is estimated close to one. The small standard error for the household-specific food case implies that $(1+r)/(1+\rho)$ is significantly larger than one, which means that the time preference parameter ρ is smaller than the nominal interest rate. The corresponding estimates of $(1+r)/(1+\rho)$ indicate that this difference, although significant, is really quite small. Of greater importance is that under the assumption that r is positive, which does not seem too unrealistic since r is the nominal interest rate, these estimates imply for all versions a positive value for the time-preference parameter ρ. This contrasts with the negative estimates of ρ reported in the studies of Alessie *et al.* (1989), Hotz *et al.* (1988), and Eichenbaum *et al.* (1988). Since a negative value of ρ implies the postponement of all consumption until the last period, such an outcome is counterintuitive.

Finally, the results of Hansen and Singleton's (1982) test on overidentifying restrictions, which is a general misspecification test, are presented in Table II. The resulting values for the food cases do not lead to rejection of the models. Furthermore, a comparison of the basic food version and the household-specific food version shows that the household dependency that was introduced does not improve the test results, despite the earlier reported joint significance of the household-effect. In contrast, for the holiday case, incorporating the household—specific components in the utility function does lead to a considerable improvement, as it results in acceptance of the model.[13] Notice that we also do not reject the model in favour of a version with random preferences as discussed in Section 2.1, for which equation (11) does not hold.

4. SUMMARY OF CONCLUSIONS

In this paper we have studied a problem inherent in the often-applied multi-good version of Hall's (1978) life-cycle model, i.e. the fact that the first-order conditions characterizing the optimal consumption path imply not only intertemporal Euler equations but also deterministic intratemporal relations. As these deterministic relations will generally not hold exactly in empirical applications, their presence indicates a form of misspecification. Several ways of modifying the life-cycle model in order to overcome this problem are possible. Because of its general applicability, we have chosen the modification proposed by Melenberg and Alessie (1989), who extend the standard life-cycle model by dropping the assumption that there is no uncertainty within the consumer's decision period. Instead, the consumption plan for each period is allowed to depend on some input variables, which are still uncertain at the beginning of the period but are realized during the period. As a consequence of the presence of this so-called intratemporal uncertainty, the intratemporal relations need no longer hold exactly for each separate consumer but only 'on average', while the intertemporal Euler equations remain essentially unchanged.

In order to assess the empirical relevance of the modification, we estimated and tested some two-good versions of the model, using a panel running for 42 periods during which 2897 households participated, which resulted in a total of about 30,000 observations. The following conclusions can be drawn from the estimation and testing results presented in Section 3.

First, the estimates are generally in accordance with the theory, i.e. the estimated utility functions are concave and increasing in their arguments for almost all observations; the bliss points are increasing with household size; and in all versions the estimates imply a positive time-preference parameter.

Second, the results of Hansen and Singleton's (1982) misspecification test show that, apart from the basic holiday case, all estimated versions are accepted. Given the rather parsimonious specifications we used, this result may be somewhat surprising. It might indicate that the misspecification tests we used lack some power. On the other hand, it might also support the intratemporal uncertainty approach, since the same dataset has also been applied by Alessie *et al.* (1989), who find quite less plausible results, although their specification differed substantially from ours.

The general applicability of the intratemporal uncertainty framework is, in our opinion, an

[13] For completeness, we also checked whether the intratemporal equations held exactly, as they should if the standard life-cycle model were to be correct. Not surprisingly, this was not the case.

important advantage. By making use of this advantage, more complex life-cycle models can also be estimated and tested. For instance, in Adang (1991) a specification is used which is often applied in labour supply studies, i.e. fixed costs related to entering the labour market. Investigating this and other specifications will be the topic of further research.

APPENDIX: THE INSTRUMENTS USED IN THE ESTIMATION AND TESTING

The following variables were included as instruments (cf. Section 3.2):

Constant term
One-period lagged expenditure on food and holiday, respectively
One-period lagged price of food for the basic model
Degree of urbanization
Region
Province
Social class
Number of household members older than 11
Number of children between 0 and 6
Number of children between 7 and 11
Number of children between 12 and 17
Number of children older than 18

Because the demographic variables are reported only once a year, and since the changes of these variables over time are limited, we decided to keep them constant over the complete survey period. That is, the instruments were given the value reported by the household in the first month it participated in the panel.

The following values are possible for the variables degree of urbanization, region, province,and social class:

Degree of urbanization
1 = villages with more than 50% agrarians
2 = villages with between 40% and 50% agrarians
3 = villages with between 30% and 40% agrarians
4 = villages with between 20% and 30% agrarians
5 = industrialized rural villages with less than 5000 inhabitants
6 = industrialized rural villages with between 5000 and 20,000 inhabitants
7 = commuter suburbs
8 = small cities with between 2000 and 10,000 inhabitants
9 = small cities with between 10,000 and 30,000 inhabitants
10 = medium cities with between 30,000 and 50,000 inhabitants
11 = medium cities with between 50,000 and 100,000 inhabitants
12 = large cities with more than 100,000 inhabitants
13 = Amsterdam, Rotterdam, The Hague

Region
1 = the four major cities (Amsterdam, Rotterdam, The Hague and Utrecht)
2 = remainder of western part of the Netherlands (except 1 and 6)
3 = northern part of the Netherlands

4 = eastern part of the Netherlands
5 = southern part of the Netherlands
6 = suburbs of the four major cities

Province
1 = Groningen
2 = Friesland
3 = Drenthe
4 = Overijssel
5 = Gelderland
6 = Utrecht
7 = Noord Holland (except 12)
8 = Zuid Holland (except 12)
9 = Zeeland
10 = Noord Brabant
11 = Limburg
12 = Amsterdam, Rotterdam, The Hague
13 = Flevoland

Social class
5 = upper class
4 = upper middle class
3 = middle class
2 = lower middle class
1 = lower class

Because the differences between the different values of the urbanization variable are minor, we also estimated the models using a less detailed urbanization variable as instrument. The value one of this new variable corresponds to the values one to five of the old one, the value two to the values six to ten, the value three to the values eleven and twelve and the value four to the value thirteen. Moreover, because the variables region and province are correlated (though not perfectly), we also re-estimated the models of Section 3 excluding the province variable from the instrument set. Both these changes did not alter the outcome of the estimation process in any significant way.

ACKNOWLEDGEMENTS

The authors wish to thank Rob Alessie, Arie Kapteyn, Arthur van Soest, and Guglielmo Weber for stimulating discussions and helpful suggestions. They also thank the editor and two anonymous referees for helpful comments. The second author also wishes to thank the Netherlands Organization for Scientific Research (NWO) for financial support.

REFERENCES

Adang, P. J. M. (1991), 'Expenditure versus consumption in the multi-good life cycle model', working paper, Tilburg University.

Alessie, R., A. Kapteyn and B. Melenberg (1989), 'The effects of liquidity constraints on consumption', *European Economic Review*, **33**, 547–55.

Altonji, J. G. and A. Siow (1987), 'Testing the response of consumption to income changes with (noisy) panel data', *Quarterly Journal of Economics*, **CII**, 293–328.
Blundell, R. (1987), 'Econometric approaches to the specification of life-cycle labour supply and commodity demand behaviour', *Econometric Reviews*, **6**, 103–65.
Blundell, R., M. Browning and C. Meghir (1991), 'A microeconometric model of intertemporal substitution and consumer demand', working paper.
Chamberlain, G. (1984), 'Panel data', in Z. Griliches and M. D. Intriligator (eds), *Handbook of Econometrics*, Vol. II, North-Holland, Amsterdam.
Eichenbaum, M. S., L. P. Hansen and K. J. Singleton (1988), 'A time series analysis of representative agent models of consumption and leisure choice under uncertainty', *Quarterly Journal of Economics*, **CIII**, 51–78.
Hall, R. E. (1978), 'Stochastic implications of the life cycle permanent income hypothesis: Theory and evidence', *Journal of Political Economy*, **86**, 971–88.
Hansen, L. P. and K. J. Singleton (1982), 'Generalized instrumental variables estimation of non-linear rational expectations models', *Econometrica*, **50**, 1269–89.
Hansen, L. P. and K. J. Singleton (1984), Errata, *Econometrica*, **52**, 267–8.
Hayashi, F. (1985), 'Tests for liquidity constraints; a critical survey and some new observations', in *Advances in Econometrics Fifth World Congress*, Vol. II, Cambridge University Press, Cambridge.
Hotz, V. J., F. E. Kydland and G. L. Sedlacek (1988), 'Intertemporal preferences and labor supply', *Econometrica*, **56**, 335–60.
MaCurdy, T. (1983), 'A simple scheme for estimating an intertemporal model of labor supply and consumption in the presence of taxes and uncertainty', *International Economic Review*, **24**, 265–89.
Melenberg, B. and R. Alessie (1989), 'A method to construct moments in the multi-good life cycle consumption model', FEW Research Memorandum, 419, Tilburg University.
Neustadt, L. W. (1976), *Optimization: A theory of necessary conditions*, Princeton University Press, Princeton, NJ.

LINKED CITATIONS

You have printed the following article:

Nonnegativity Constraints and Intratemporal Uncertainty in a Multi-Good Life-Cycle Model
Pim Adang; Bertrand Melenberg
Journal of Applied Econometrics, Vol. 10, No. 1. (Jan. - Mar., 1995), pp. 1-15.
Stable URL:
http://links.jstor.org/sici?sici=0883-7252%28199501%2F03%2910%3A1%3C1%3ANCAIUI%3E2.0.CO%3B2-3

This article references the following linked citations. If you are trying to access articles from an off-campus location, you may be required to first logon via your library web site to access JSTOR. Please visit your library's website or contact a librarian to learn about options for remote access to JSTOR.

[Footnotes]

[10] **Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models**
Lars Peter Hansen; Kenneth J. Singleton
Econometrica, Vol. 50, No. 5. (Sep., 1982), pp. 1269-1286.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198209%2950%3A5%3C1269%3AGIVEON%3E2.0.CO%3B2-G

[11] **Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models**
Lars Peter Hansen; Kenneth J. Singleton
Econometrica, Vol. 52, No. 1. (Jan., 1984), pp. 267-268.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198401%2952%3A1%3C267%3AGIVEON%3E2.0.CO%3B2-1

References

Testing the Response of Consumption to Income Changes with (Noisy) Panel Data
Joseph G. Altonji; Aloysius Siow
The Quarterly Journal of Economics, Vol. 102, No. 2. (May, 1987), pp. 293-328.
Stable URL:
http://links.jstor.org/sici?sici=0033-5533%28198705%29102%3A2%3C293%3ATTROCT%3E2.0.CO%3B2-W

NOTE: *The reference numbering from the original has been maintained in this citation list.*

LINKED CITATIONS

A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty
Martin S. Eichenbaum; Lars Peter Hansen; Kenneth J. Singleton
The Quarterly Journal of Economics, Vol. 103, No. 1. (Feb., 1988), pp. 51-78.
Stable URL:
http://links.jstor.org/sici?sici=0033-5533%28198802%29103%3A1%3C51%3AATSAOR%3E2.0.CO%3B2-V

Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence
Robert E. Hall
The Journal of Political Economy, Vol. 86, No. 6. (Dec., 1978), pp. 971-987.
Stable URL:
http://links.jstor.org/sici?sici=0022-3808%28197812%2986%3A6%3C971%3ASIOTLC%3E2.0.CO%3B2-K

Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models
Lars Peter Hansen; Kenneth J. Singleton
Econometrica, Vol. 50, No. 5. (Sep., 1982), pp. 1269-1286.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198209%2950%3A5%3C1269%3AGIVEON%3E2.0.CO%3B2-G

Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models
Lars Peter Hansen; Kenneth J. Singleton
Econometrica, Vol. 52, No. 1. (Jan., 1984), pp. 267-268.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198401%2952%3A1%3C267%3AGIVEON%3E2.0.CO%3B2-1

Intertemporal Preferences and Labor Supply
V. Joseph Hotz; Finn E. Kydland; Guilherme L. Sedlacek
Econometrica, Vol. 56, No. 2. (Mar., 1988), pp. 335-360.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198803%2956%3A2%3C335%3AIPALS%3E2.0.CO%3B2-W

A Simple Scheme for Estimating an Intertemporal Model of Labor Supply and Consumption in the Presence of Taxes and Uncertainty
Thomas E. MaCurdy
International Economic Review, Vol. 24, No. 2. (Jun., 1983), pp. 265-289.
Stable URL:
http://links.jstor.org/sici?sici=0020-6598%28198306%2924%3A2%3C265%3AASSFEA%3E2.0.CO%3B2-S

NOTE: *The reference numbering from the original has been maintained in this citation list.*