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AN ALTERNATIVE TREATMENT OF SECONDARY PRODUCTS IN INPUT-OUTPUT ANALYSIS

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Abstract—The United Nations System of National Accounts includes an input or "use" table $U = (u_{ij})$ of commodities i consumed by industries j and an output or "make" table $V = (v_{ij})$ of industries producing commodities j. This paper is on the construction of an input-output or "requirements" table $A = (a_{ij})$ of commodities i for commodities j. The established constructs are criticised. The current favourite, the industry technology model, is rejected on the ground that the choice of base year prices affects the results in more than a scaling fashion. The paper presents an alternative to the existing constructs which cancels out the shortcomings and amounts to a rich representation of technology.

THE United Nations (1967) System of National Accounts includes an input or "use" table $U = (u_{ij})$ of commodities i consumed by industries j and an output or "make" table $V = (v_{ij})$ of industries i producing commodities j. This paper is on the construction of an input-output or "requirements" table $A = (a_{ij})$ of commodities i for commodities j. (Industry tables and mixed tables are not considered.) Section I reviews the established constructs. Section II evaluates them. Special attention is given to the so-called industry technology model which is now used by the United States (1980). Section III derives a new construction of a requirements table. Section IV applies the analysis. For convenience we have chosen the well organized tables of Canada (1981) for our experiment. Section V discusses the results. Section VI concludes the paper.

I. The Established Constructs

The established constructs are the commodity technology model, the by-product technology model, the industry technology model, and the mixed technology model of Gigantes (1970). Some notation facilitates the presentation of these models. e denotes the unit column vector. $'$ denotes transposition. $\hat{}$ denotes diagonalization either by suppression of the off-diagonal elements of a square matrix or by placement of the elements of a vector. $\sim$ denotes off-diagonalization by suppression of the diagonal elements of a square matrix. (Thus for a square matrix, $A = \hat{A} + \tilde{A}$.)

The *commodity technology* model (C) rests on the assumption that each commodity has its own input structure. Industries are independent combinations of outputs j with their input structures (a_{ij}^C), $i = 1, \ldots, n$. Thus, industry j needs for the production of v_{jk} units of output k an amount $a_{ik}^C v_{jk}$ of input i. Summing over outputs k yields industry j's total demand for input i: $u_{ij} = \Sigma_k a_{ik}^C v_{jk}$. Hence $U = A_C V'$. Thus the commodity technology requirements table is given by $A_C = UV'^{-1}$. Note that existence may be guaranteed only if the number of commodities equals the number of industries.

The *by-product technology* model (B) rests on the *by-product assumption* that each industry produces outputs in a fixed proportion. All secondary products are by-products and therefore can be treated as negative inputs, yielding net input structures (a_{ij}^B), $i = 1, \ldots, n$ for the primary outputs j. Thus, industry j needs for the production of v_{jj} units of its primary output a net amount $u_{ij} - \tilde{v}_{ji} = a_{ij}^B v_{jj}$ of commodity i. Hence $U - \tilde{V}' = A_B \hat{V}$. Thus the by-product technology requirements table is given by $A_B = (U - \tilde{V}')\hat{V}^{-1}$. Note that again existence may be guaranteed only if the number of commodities equals the number of industries.

The *industry technology* model (I) rests on two assumptions. One is the *industry technology assumption* that each industry j has the same input requirements for any unit of output. Here output is measured in value. The other assumption is that of fixed commodity market shares of industries. Thus, industry k needs $u_{ik}/\Sigma_l v_{kl}$ of input i per unit of output—in particular for commodity j—and its market share $v_{kj}/\Sigma_l v_{lj}$ is fixed. Taking the (market share) weighted average over industries k yields the amount of input i required for

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one unit of output j:

$$a_{ij}^I = \Sigma_k \left(v_{kj}/\Sigma_l v_{lj} \right) u_{ik}/\Sigma_l v_{kl}.$$

Thus the industry technology requirements table is given by

$$A_I = U\,\widehat{Ve}^{-1}\,V\,\widehat{V'e}^{-1}.$$

The *mixed technology* (CI) model as developed by Gigantes (1970) is a combination of the commodity and industry technology models (C) and (I). He splits the make table V into a table V_1 of primary products and "ordinary secondary products" and a table V_2 of by-products. The model rests on the assumptions that primary products and ordinary secondary products fulfill the commodity technology assumption and that "it seems reasonable that [such] by-products have the same input structures as the industries producing them," i.e., that by-products fulfill the industry technology assumption. (See Gigantes (1970, pp. 284–288), also for his mixed technology requirements table A_{CI}.)

II. Critique

The described constructs will now be evaluated. While methodologically sound, the commodity technology model is not found appropriate because it often produces senseless negatives and also because of its hypothesis that no industry's outputs are technologically related. Similarly, the by-product technology model is criticized because of its rigid output proportionality assumption. The industry technology model seems to be found more realistic and more flexible, at least by statisticians, which may explain the United States (1980) subscription even though economists resent its fixed market shares assumption since it violates the dictum of cost minimization.

The industry technology assumption itself has drawn less criticism so far. This paper will attack it, however, on methodological grounds. It will be shown that the industry technology assumption implies that the choice of the base year prices is not only a matter of scaling but becomes an essential determinant of the representation of technology. Consequently, base year prices must be chosen in some rational manner. But no base year prices objectively underlie the representation of technology. For this reason we reject the industry technology model, its implementation in the United States, as well as Gigantes' mixed technology model. Whereas base year prices are usually just a scaling device, they now bear an essential imprint on the very technological relationships.

It remains to show the base year price dependence of the industry technology requirements table. For this purpose, consider

$$U = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$$

and

$$V = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}.$$

(Note that value added equals 1/2 in each industry.) Then

$$\begin{aligned} A_I &= U\,\widehat{Ve}^{-1}\,V\,\widehat{V'e}^{-1} \\ &= \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} \\ &\quad \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} 1/6 & 1/3 \\ 1/3 & 1/6 \end{pmatrix}. \end{aligned}$$

Now suppose that due to an accidental event alternate base year prices were chosen, say twice as big for commodity 1 and the same for commodity 2. Then U and V would have been

$$\begin{pmatrix} 0 & 1 \\ 1/2 & 0 \end{pmatrix}$$

and

$$\begin{pmatrix} 4/3 & 1/3 \\ 2/3 & 2/3 \end{pmatrix},$$

respectively (leaving 7/6 and 1/3 for value added in the respective industries) and A_I would essentially be the same if it were

$$\begin{pmatrix} 1/6 & 2/3 \\ 1/6 & 1/6 \end{pmatrix}.$$

However, substitution of the alternate U and V in the A_I formula yields

$$\begin{aligned} &\begin{pmatrix} 0 & 1 \\ 1/2 & 0 \end{pmatrix} \begin{pmatrix} 5/3 & 0 \\ 0 & 4/3 \end{pmatrix}^{-1} \begin{pmatrix} 4/3 & 1/3 \\ 2/3 & 2/3 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \\ &\quad = \begin{pmatrix} 0 & 3/4 \\ 3/10 & 0 \end{pmatrix} \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} \\ &\quad = \begin{pmatrix} 1/4 & 1/2 \\ 1/5 & 1/10 \end{pmatrix}. \end{aligned}$$

This shows that A_I depends in an essential way on base year prices (or, at a deeper level, on the industrial distribution of value added).

III. An Alternative

In this section we present an alternative to the existing constructs. The alternative amounts to a rich representation of technology in which industry's outputs can be interrelated, but not necessarily to the extent of rigid proportionality. Secondary products may range from totally independent activities to by-products of primary activities. The richness is obtained by using *new empirical data* to fill the information voids on the nature of secondary production. Output time series are employed to classify the secondary products. On this basis a sophisticated construction of an input-output requirements table becomes feasible. The construct will be described now.

Basically, our model cancels out the shortcomings of the commodity technology model and the by-product technology model against each other. This is in the spirit of Gigantes' mixed technology model while avoiding the industry technology assumption trap. In fact, all we do is to amend Gigantes' mixed technology model by assuming that by-products fulfill the by-product assumption instead of the industry technology assumption. Thus our model (CB) rests on the assumptions that primary products and ordinary secondary products fulfill the commodity technology assumption and that by-products fulfill the by-product assumption.

It should be mentioned that whereas Gigantes (1970) deals with industrial by-products, we deal with commodity by-products. Industrial by-products are thought to be proportional to total industry output. The latter notion, however, also depends in an essential way on base year prices. In fact, measurement of total industry output is impossible without an aggregation bias. This bias lies at the heart of our critique of the industry technology assumption and now the notion of industrial by-products. This complication is avoided by focusing on by-products that are proportional to *primary* rather than total industry output. This subtle redefinition from Gigantes' industrial by-products to our commodity by-products eliminates the dependence on the choice of base year prices.

We now spell out our construction. The make table V is split into a table $V_1 = \left(v_{ij}^{(1)}\right)$ of primary products and ordinary secondary products and a table $V_2 = \left(v_{ij}^{(2)}\right)$ of by-products. By-products are treated as negative inputs, yielding net input structures (a_{ij}^{CB}), $i = 1,\dots, n$ for the primary or ordinary secondary outputs j. Thus, industry j needs for the production of $v_{jk}^{(1)}$ units of its primary or ordinary secondary outputs k a net amount $u_{ij} - v_{ji}^{(2)} = \Sigma_k a_{ik}^{CB} v_{jk}^{(1)}$ of commodity i. Hence $U - V_2' = A_{CB} V_1'$. Thus our requirements table is given by $A_{CB} = (U - V_2')V_1'^{-1}$.

Note that as before existence may be guaranteed only if the number of commodities equals the number of industries. Note also that if all secondary products are ordinary, then $V_1 = V$ and $V_2 = 0$ so that then $A_{CB} = UV'^{-1} = A_C$. While if all secondary products are by-products then $V_1 = \hat{V}$ and $V_2 = \tilde{V}$ so that then $A_{CB} = (U - \tilde{V}')\hat{V}^{-1} = A_B$.

IV. Application

We wish to find the just derived requirements table for the Canadian economy. The data bank consists of the 1971–77 use and make tables in constant 1971 prices of Canada (1981) with 43 sectors.

First we have to classify the secondary products into ordinary ones and by-products. Ideally one would work with highly disaggregated tables and use the judgment of industry experts to determine if technical relationships govern output proportions. In this study we have to do it through inference though.

Consider the primary product p and a secondary product v of some industry. (There are $n(n - 1)$ of these cases where n is the number of sectors.) If v can fluctuate independently of p, then it is an ordinary secondary product. If, however, v is bound to be proportional to p, then it is a by-product. Formally, let $v = \beta p + u$, with β a coefficient and u a random variable. The question is if β is highly significant. If so, then v is a by-product. To settle this, we must assume some stochastic structure on u. Originally we thought of assuming that the values of u in the various years are independently and identically normally distributed. Then β can be estimated by ordinary regression and its significance can be tested as usual. The assumption of identical distributions or "homoscedasticity" implies, however, that the random product component, u, becomes small relative to primary output,

p, when the economy grows. But this counters the lesson of input-output analysis which tells us that proportions rather than absolute magnitudes have stable distributions. Therefore, theory suggests that we should assume that the variance of u is proportional to p, i.e., *heteroscedasticity of the first degree*. Formally, we assume that the values of u/p are independently and identically normally distributed. Then the best linear unbiased estimator of β is simply the average of the observed v/p ratios. And the relevant t-statistic is $\sqrt{T}b/s$, where T is the number of observations, b the estimate of β and s^2 is the residual sum of squares of the ratios divided through by $T-1$. These results are obtained by dividing through the equation by p and then applying classical statistical inference. (It should be mentioned that the differences with the ordinary regression results turn out to be minor: the degree of heteroscedasticity is not so important.) If our t-statistic is very high, then the product is classified as a by-product. Since this procedure makes sense only for positively high t-values, we may take, as a first step, a one sided confidence interval about $\beta = 0$. For seven observation years and a significance level of 0.001 the confidence interval is $(-\infty, 5.208)$, according to PTT (1960, p. 22): Higher t-values indicate secondary products which cannot be purely random. However, to accept these as by-products would go too far. To be a by-product a good must not merely have a significant deterministic component, βp, but the random component, u, should be negligible in addition. Ultimately it is a matter of sharp proportionality, i.e., a high coefficient of determination or, equivalently, a large t-value. How high should the value be to indicate a by-product? According to Theil (1971, pp. 164, 181) this is a matter of comparing with similar regressions. The ultimate choice of the cut-off point is made by judging the nature of the secondary products with t-values, just under or over it. We have selected $t = 40$, which agrees with a coefficient of determination of 0.996. Still higher t-values indicate by-products.

We have chosen a moderately sized time period, namely 1971–77. A longer period may seem desirable from the viewpoint of estimation and inference, and, it should be said, is also possible as far as data availability is concerned. But we have resisted this temptation in the knowledge that technical coefficients, including by-product coefficients, are roughly constant in the shorter or intermediate run, but not in the long run due to technical change. This condition prompts the use of a relatively short time span for the determination of by-products.

By selecting a high t-value, we may seem severe in the classification of by-products. Our motivation is two-fold. First, the whole methodology of inference is biased towards by-products. For imagine that the economy is in balanced growth. Then all output proportions are constant, be they governed by technical relations or not. By the described method all secondary products would be classified as by-products, which is clearly false. Note, however, that no bias will persist in forecasts provided that conditions of balanced growth remain. The same considerations hold in our study. There is a bias towards by-products, but this will not poison forecasts based on the resulting input-output matrix, provided that final demand trends remain essentially as in the years which underlie the construction of the matrix. The second reason for being tough in accepting by-products is due to Kishori Lal of Statistics Canada who pointed out to us that aggregation blurs the by-products. While at a high level of disaggregation it is possible to pinpoint the true by-products, at our level of aggregation they disappear among the other secondary products. This argument is in line with Theil (1971, p. 181). We neutralize the two dangers somewhat by employing a high critical t-value. But, to repeat, it would have been better to use highly disaggregated tables and a priori judgment of industry experts.

The estimates of the β-coefficients of the various secondary products in the industries and their t-values are reported in table 1 which is presented in the appendix. Secondary products with sharp β's in the sense described above ($t > 40$) are underlined; these are the *by-products* in the further derivation.

Before we proceed with the construction of the requirements table, theoretical purists may add the following critical note to our just completed classification scheme. Imagine that i is a by-product of j which, on its turn, is an ordinary secondary product of k. Then industry k, engaging itself in the production of j, will generate some i as a by-product of its secondary production. Such further technical relations have been neglected in our scheme. We should have tested for system wide linear output restrictions rather than just within sectors producing the primary outputs considered. But, apart from information obstacles, we expect that little would be gained by such a refined procedure, for the tertiary and further production effects are, in view of the output ratio figures reported in table 1, of negligible magnitudes.

Using table 1 for the splitting of any year's make table V into a primary and ordinary secondary table V_1 and a by-product table V_2, and substituting these and the same year's use table U in the formula for A_{CB} given in the last section, we obtain the requirements table for the year considered. We have actually carried this out for the year 1977. The use and make tables U and V are reprinted in tables 2 and 3, respectively, from Statistics Canada (1981). Some commodities have been aggregated in order to have a system wide level of aggregation (the medium one, M). Tables 4 and 5, respectively, present out requirements table A_{CB} and its inverse $(I - A_{CB})^{-1}$. The percentage deviations of the latter from the industry technology inverse is given in table 6 for the purpose of comparison. Tables 2–6 have been deleted from the appendix to shorten the article, but are available from the authors on request.

V. Discussion

Our requirements table (table 4) depicts several negative coefficients as one expects in the presence of by-products. But even the inverse or total requirements table (table 5) contains some negative coefficients, as well as some diagonal entries which are less than unity. The sizeable negatives (greater than 0.03 in absolute value) and the less than one diagonal entries are the following:

1. the total own requirements of mineral fuels (0.98);
2. the total chemicals requirements of non-metal mines (−0.06);
3. the total transport equipment requirements of rubber & plastic (−0.08) and of textile (−0.13);
4. the total transport equipment requirements of clothing (−0.05);
5. the total knitting requirements of clothing (−0.15);
6. the total machinery requirements of fabricated metals (−0.08);
7. the total electrical products requirements of machinery (−0.05);
8. the total wholesale trade requirements of chemicals (−0.03);
9. the total personal services requirements of wholesale trade (−0.08) and of retail trade (−0.16).

Although the total number of entries is 43^2 or 1,849, the 9 complications warrant scrupulous discussion. The last complication (number 9) is the only one which appeared on the surface when we did preliminary calculations at a more aggregated level. In essence, the personal services by-product of trade is so sizeable that it persists when the indirect requirements are taken into account. *If* final demand for trade were predominant, *then* the supply of personal services would exceed total demand and they would be a free good. In reality, however, final demand for goods other than trade is significant and their total requirements exhaust the supply of services. Moreover, one may argue that trade is a typical intermediate activity which with little loss of information can be incorporated into the other sectors as in Leontief (1967). This procedure would eliminate the negative total requirement coefficients. Or one may simply lump trade and personal services together, introducing a small aggregation bias. This would also eliminate those negative entries. But we prefer to present our results in full detail, including the negative coefficients. The presence of by-products, yielding negative entries in the requirements table and, when persistent, in the inverse as well, should not astound us.

The next to last complications (numbers 6, 7, and 8) are similar. All the observations we made on the persistent by-product (of trade) apply. The present negatives were not detected at the more aggregated level since they were overwhelmed by other total requirements which, at that level, were not reported separately.

The other complications (numbers 1 to 5) have nothing to do with the presence of by-products, but concern the well-known problem of negatives associated with the commodity technology model. The problem shows up here too as our model is a mixture of the commodity and the by-product technology models. Many countries construct their input-output requirements tables according to the commodity technology model and thus face the problem of negatives. They usually handle it by a device of Almon (1970, pp. 110–112). He writes beforehand the factor to be inverted as $V = [I - (I - V\widehat{eV}^{-1})]\widehat{eV}$ and then iterates truncated Neumann series in which matrix multiplications are carried out only to a limited extent to avoid negatives. This arithmetic manipulation goes without justification, is arbitrary and depends on the choice of V-decomposition as well as the iteration scheme. We rather report the pure results and spot the trouble shooters in order to suggest lines for future investigations of this problem which, however important yet underexposed, is not our main concern.

Take the very first complication (number 1). The mineral fuels sector has a secondary activity (petroleum & coal products) with a sizeable mineral fuels input component. In the commodity technology model, secondary activities are subtracted, yielding a negative net mineral fuels input flow, in view of the small recorded amount of mineral fuels used by the mineral fuels sector. In essence, the mineral fuels sector is to some extent vertically integrated into its downstream market. If one would disentangle the vertically integrated processes into elementary activities, then one would report as distinct inputs the intermediate mineral fuels which are further processed in the sector itself. This procedure would eliminate the negative own requirements and the less than one own total requirements. Thus, the reporting of own inputs, even when they are merely throughputs within the sectors, is critical for commodity technology models in a broad sense and our construction in particular. It should be mentioned, however, that own inputs are often within plants of firms, so that the data will be hard to get at.

Somewhat the reverse problem arises in the next two complications (numbers 2 and 3). Here we have secondary activities with large own input components. Upon subtraction, these goods become negative inputs in the sectors at hand. Again, the own inputs (on the diagonal of the use table) are critical. The further complication (number 4) is an immediate consequence. Here, in the clothing sector, textile is the main input. But textile has negative transport equipment requirements by the last consideration. It follows that this is also true for clothing itself. The remaining complication (number 5) is just like the ones of secondary activities with large own input requirements. An anonymous referee suggested to set these own requirements equal to zero, for it can be proved that such a procedure has no effect on the value of off-diagonal inverse coefficients. This procedure would eliminate complications 2–4, but, unfortunately, in the presence of secondary products there is an effect on the inverse, including the off-diagonal part.

Summing up, some secondary products are sizeable and yet classified as by-products, yielding negative requirements (numbers 6 to 9); one sector has little own input and yet downstream secondary products, yielding negative own requirements (number 1); some secondary products have much own input, yielding negative contributions to primary output requirements of the sectors at hand (numbers 2 to 5).

The last two observations concern the problem of negatives in the pure commodity technology model and draw attention to the import of own inputs, i.e., the diagonal of the use table, thus shedding some fresh light on this problem. The first observations is on the classification problem of by-products and also draws attention to the data which are needed for our approach.

The necessary data are a use table, a make table, and a by-products list. The use and make tables are also required for the established constructs. These tables are compiled by a growing number of statistical agencies which subscribe to the UN system of Standard National Accounts. But the by-products list is a new requirement. Such a list is not included in the Standard National Accounts. We have gone about this by setting up a by-products list ourselves. We have done it through statistical inference. It should be repeated, though, that the classification of secondary products is not a statistical matter but a technical question. Whether or not a secondary product is a by-product is most appropriately determined when data are collected. We therefore propose that the questionnaires which are used for the compilation of make tables will include a question pertaining to the nature of secondary products: if they are automatic consequences of the main process or if they result from side activities in which firms engage themselves. Then the Standard National Accounts can be expanded in that the make table entries will be qualified as by-products or other secondary products. This would facilitate direct application of our method for the construction of an input-output coefficients matrix. Also here a caveat is at hand, as the referee pointed out. The black-white qualification will not be easily applicable to gray products such as cogenerated electricity, which could be viewed either as a by-product or a side activity. Some arbitrary judgment is unavoidable. The point is, however, that this is best done at the level of data collection.

The suggested improvements are most relevant, for alternative treatments of secondary products yield very different results as the table of percentage differences between our inverse and the industry technology inverse (table 6) reveals.

VI. Conclusion

The industry technology model, suggested by the United Nations and used by the United States for the construction of requirements tables, is rejected on the ground that it depends in an essential way on the choice of base year prices. An alternative model is derived in this paper. The method is workable as is illustrated by an application to the Canadian economy. More detailed knowledge about secondary production, in particular the relationship to either primary output or its own input structure, is called for to free the analysis from statistical devices and to improve the results. Such improvements are relevant since alternative treatments of secondary products yield greatly varying results.

APPENDIX

Table 1.—Secondary products and their t-ratios.
Table 2.—Use table U (commodities by industries).
Table 3.—Make table V (industries by commodities).
Table 4.—Our requirements table A_{CB} (commodities by commodities).
Table 5.—Inverse $(I - A_{CB})^{-1}$ (commodities by commodities).
Table 6.—Deviations from inverse $(I - A_I)^{-1}$ (percentages).

Table 1 follows. Tables 2–6 have been deleted to shorten the article, but are available from the authors on request.

TABLE 1.—SECONDARY PRODUCTS AND THEIR t-RATIOS

1. Agriculture		
2. .0084 (33.29)	8. .0160 (22.83)	35. .0014 (8.39)
2. Forestry		
1. .0089 (6.50)	15. .0078 (6.50)	21. .0001 (3.03)
29. .0019 (15.01)	31. .0001 (3.05)	32. .0031 (10.23)
35. .0009 (3.21)	39. .0179 (20.98)	40. .0039 (8.28)
3. Fishing		
8. .0095 (4.48)	40. .0147 (10.53)	
4. Metal Mines		
6. .0008 (6.20)	7. .0299 (10.18)	19. .0004 (8.06)
21. .0025 (2.47)	31. .0006 (21.07)	32. .0018 (2.47)
35. .0033 (9.15)	39. .0032 (12.26)	40. .0028 (5.55)
5. Mineral Fuels		
6. .0211 (18.79)	7. .0007 (1.00)	21. .0037 (3.77)
25. .0399 (13.11)	32. .0000 (1.00)	35. .0260 (5.16)
38. .0001 (1.54)	39. .0001 (1.00)	40. .0002 (3.43)
6. Non-Metal Mines		
7. .0131 (8.03)	21. .0093 (4.22)	24. .0147 (18.61)
26. .3542 (18.86)	32. .0053 (7.93)	35. .0079 (3.74)
39. .0012 (14.01)	40. .0075 (5.06)	
7. Mining Services		
21. .0121 (10.99)	35. .0052 (5.79)	40. .0071 (15.56)
8. Food & Beverages		
1. .0010 (11.33)	6. .0001 (2.57)	17. .0003 (12.20)
21. .0004 (22.82)	26. .0125 (22.89)	27. .0001 (3.84)
29. .0001 (4.70)	31. .0000 (3.86)	32. .0247 (42.12)
35. .0006 (11.19)	39. .0004 (19.16)	40. .0002 (6.37)
9. Tobacco		
8. .0018 (3.50)	21. .0003 (3.74)	32. .0043 (11.04)
35. .0001 (2.46)	39. .0002 (50.52)	
10. Rubber & Plastic		
12. .0030 (5.19)	14. .0167 (17.69)	15. .0001 (1.97)
16. .0119 (6.44)	17. .0275 (15.19)	19. .0003 (3.70)
20. .0287 (5.30)	21. .0122 (4.49)	22. .1262 (29.84)
23. .0141 (7.66)	24. .0089 (7.04)	26. .0453 (14.17)
27. .0344 (20.23)	32. .0477 (13.48)	35. .0007 (4.10)
38. .0004 (3.48)	39. .0006 (9.17)	40. .0013 (8.28)

TABLE 1.—(*Continued*)

11. Leather		
6. .0000 (1.00)	8. .0007 (3.25)	10. .0602 (9.42)
12. .0013 (6.05)	14. .0301 (15.77)	16. .0009 (2.12)
26. .0003 (2.70)	27. .0095 (6.30)	32. .0217 (16.11)
35. .0006 (3.55)	38. .0001 (1.00)	39. .0000 (0.15)
40. .0002 (1.49)		
12. Textile		
10. .0098 (8.51)	11. .0001 (2.78)	13. .0008 (2.87)
14. .0282 (22.78)	16. .0001 (11.36)	17. .0142 (36.98)
20. .0023 (6.01)	21. .0012 (7.86)	22. .1135 (18.75)
27. .0084 (9.57)	31. .0001 (4.84)	32. .0171 (19.39)
35. .0001 (2.82)	38. .0001 (2.77)	39. .0004 (9.00)
40. .0001 (3.66)		
13. Knitting		
12. .6978 (48.24)	14. .0869 (9.57)	21. .0003 (1.55)
32. .0053 (8.52)	35. .0015 (6.38)	39. .0003 (3.86)
40. .0000 (1.00)		
14. Clothing		
11. .0007 (9.15)	12. .0036 (10.37)	13. .1534 (25.47)
18. .0000 (2.12)	21. .0000 (1.00)	27. .0003 (2.61)
32. .0170 (8.04)	35. .0013 (7.05)	39. .0002 (7.21)
40. .0002 (10.02)		
15. Wood		
2. .0124 (5.84)	16. .0091 (13.10)	17. .0006 (5.73)
20. .0022 (11.75)	21. .0037 (7.50)	27. .0003 (4.38)
29. .0024 (14.56)	31. .0001 (5.58)	32. .0076 (14.84)
35. .0018 (8.74)	39. .0004 (12.02)	40. .0005 (7.45)
16. Furniture & Fixtures		
12. .0039 (3.29)	14. .0001 (2.12)	15. .0404 (19.17)
20. .0068 (6.06)	23. .0370 (13.89)	27. .0074 (6.60)
32. .0136 (12.39)	35. .0023 (6.36)	39. .0009 (7.73)
40. .0118 (5.07)		
17. Paper		
10. .0090 (9.27)	12. .0005 (6.16)	15. .0008 (18.47)
18. .0053 (17.89)	19. .0019 (5.83)	21. .0067 (10.36)
24. .0014 (10.36)	25. .0009 (5.58)	26. .0025 (24.76)
27. .0030 (8.39)	31. .0019 (10.85)	32. .0084 (10.06)
35. .0014 (6.71)	38. .0002 (15.59)	39. .0002 (12.39)
40. .0005 (9.90)		
18. Printing & Publishing		
10. .0002 (4.13)	15. .0001 (2.11)	17. .0247 (28.48)
21. .0007 (5.59)	27. .0015 (17.64)	32. .0138 (22.48)
35. .0021 (13.56)	38. .0004 (2.20)	39. .0010 (15.76)
40. .0008 (2.88)		
19. Primary Metals		
4. .0191 (8.80)	6. .0002 (2.66)	20. .0563 (30.22)
21. .0144 (9.50)	23. .0019 (14.35)	25. .0029 (15.21)
26. .0065 (17.48)	29. .0003 (2.62)	31. .0032 (18.55)
32. .0028 (8.48)	35. .0004 (6.06)	39. .0001 (21.48)
40. .0001 (9.50)		
20. Fabricated Metals		
10. .0052 (9.59)	14. .0004 (4.58)	15. .0019 (8.49)
16. .0126 (27.37)	17. .0040 (15.93)	18. .0001 (3.25)
19. .0609 (37.72)	21. .0746 (44.15)	22. .0356 (26.64)
23. .0196 (11.47)	24. .0044 (5.77)	26. .0013 (7.02)
27. .0182 (15.88)	31. .0000 (1.00)	32. .0260 (31.54)
35. .0017 (9.11)	38. .0000 (1.00)	39. .0007 (18.84)
40. .0124 (24.61)		
21. Machinery		
10. .0002 (1.84)	11. .0012 (5.02)	16. .0031 (9.92)
18. .0006 (4.10)	19. .0102 (10.26)	20. .0505 (27.43)
22. .0374 (13.63)	23. .0921 (49.77)	27. .0070 (4.37)
32. .0583 (28.66)	35. .0014 (9.07)	38. .0230 (8.23)
39. .0007 (19.84)	40. .0283 (16.30)	

TABLE 1.—(*Continued*)

22. Transport Equipment		
10. .0003 (6.02)	15. .0009 (3.96)	19. .0004 (3.06)
20. .0064 (11.53)	21. .0084 (9.05)	23. .0085 (21.82)
24. .0001 (2.90)	31. .0000 (1.54)	32. .0535 (12.53)
35. .0005 (4.05)	38. .0007 (5.32)	39. .0003 (15.52)
40. .0057 (10.99)		
23. Electrical Products		
10. .0008 (2.78)	16. .0015 (4.70)	19. .0047 (8.08)
20. .0206 (8.73)	21. .0167 (26.26)	22. .0011 (3.89)
27. .0080 (8.33)	31. .0001 (5.27)	32. .0498 (26.87)
35. .0005 (12.28)	38. .0016 (2.60)	39. .0008 (13.69)
40. .0127 (13.09)		
24. Non-Metal. Mineral Products		
6. .0107 (37.63)	15. .0006 (9.03)	19. .0023 (5.39)
21. .0019 (4.29)	22. .0059 (14.44)	26. .0062 (14.22)
31. .0000 (1.00)	32. .0209 (44.33)	35. .0007 (9.69)
39. .0016 (19.07)	40. .0016 (5.10)	
25. Petroleum & Coal Products		
5. .0037 (6.80)	6. .0005 (4.78)	10. .0000 (2.12)
21. .0001 (3.99)	26. .0030 (5.93)	27. .0056 (7.06)
32. .0026 (12.02)	35. .0162 (15.23)	38. .0004 (8.21)
39. .0001 (12.97)	40. .0002 (2.15)	
26. Chemicals		
6. .0013 (9.46)	8. .0061 (14.79)	10. .0047 (6.32)
11. .0002 (5.76)	15. .0008 (7.36)	17. .0019 (15.00)
18. .0002 (5.20)	19. .0036 (39.09)	21. .0014 (6.75)
24. .0005 (4.30)	25. .0100 (20.73)	27. .0128 (8.95)
31. .0000 (3.86)	32. .0515 (69.27)	35. .0010 (20.69)
38. .0010 (8.67)	39. .0009 (10.94)	40. .0002 (4.70)
27. Miscellaneous Manufacturing		
4. .0132 (23.56)	10. .0250 (15.30)	11. .0075 (5.12)
12. .0114 (7.01)	13. .0004 (2.99)	14. .0093 (14.08)
15. .0017 (6.57)	16. .0033 (8.88)	17. .0660 (25.43)
18. .0008 (6.28)	19. .0177 (17.89)	20. .0122 (16.61)
21. .0193 (10.08)	22. .0100 (24.35)	23. .0384 (6.84)
24. .0037 (3.32)	26. .0118 (24.40)	32. .0692 (33.56)
35. .0017 (8.44)	38. .0004 (1.28)	39. .0015 (16.31)
40. .0426 (39.94)		
28. Construction		
35. .0060 (29.89)	38. .0000 (2.06)	40. .0054 (15.76)
29. Transportation & Storage		
31. .0001 (1.93)	22. .0095 (13.18)	25. .0001 (5.65)
35. .0039 (27.72)	32. .0005 (2.83)	33. .0015 (21.79)
40. .0085 (25.95)	38. .0019 (20.68)	39. .0021 (20.83)
30. Communication		
23. .0417 (39.57)	35. .0005 (13.35)	37. .0032 (15.08)
38. .0092 (8.46)	40. .0021 (11.75)	
31. Utilities		
2. .0005 (14.47)	25. .0001 (3.81)	29. .0048 (15.71)
33. .0098 (20.06)	35. .0022 (25.72)	38. .0006 (3.47)
39. .0017 (14.23)	40. .0130 (31.05)	
32. Wholesale Trade		
8. .0020 (26.09)	9. .0001 (5.22)	11. .0009 (39.95)
12. .0017 (48.95)	13. .0000 (28.54)	14. .0001 (37.65)
15. .0002 (42.23)	16. .0007 (62.40)	17. .0003 (42.80)
18. .0004 (88.84)	20. .0015 (25.09)	22. .0006 (27.24)
23. .0006 (37.09)	24. .0002 (9.62)	26. .0001 (29.21)
27. .0061 (62.00)	35. .0070 (13.42)	38. .0043 (3.38)
40. .0984 (55.71)		
33. Retail Trade		
1. .0004 (11.00)	8. .0278 (34.41)	12. .0003 (21.62)
14. .0001 (9.93)	29. .0010 (15.26)	35. .0070 (11.99)
38. .0002 (4.31)	39. .0181 (40.42)	40. .1767 (101.62)

TABLE 1.—(*Continued*)

34. Owner Occupied Dwellings—No secondary products		
35. Other Finance		
29. .0000 (3.37)	31. .0000 (2.12)	38. .0026 (4.51)
39. .0000 (9.24)	40. .0210 (8.41)	
36. Education & Health		
35. .0008 (5.75)		
37. Amusement & Recreation		
27. .0251 (2.08)	<u>33</u>. .0140 (43.76)	35. .0111 (24.27)
38. .0036 (2.38)	39. .0417 (5.56)	
38. Business Services		
32. .0184 (11.77)	33. .0033 (35.59)	35. .0075 (13.80)
37. .0013 (10.04)	39. .0000 (2.30)	40. .1662 (18.02)
39. Accommodation & Food Services		
33. .0067 (14.44)	35. .0098 (30.74)	37. .0001 (6.43)
40. .0057 (27.75)		
40. Personal Services		
31. .0064 (22.49)	<u>33</u>. .0058 (46.98)	35. .0042 (23.21)
39. .0001 (3.97)		
41. Transportation Margins—No secondary products		
42. Operating Office, Lab & Food—No secondary products		
43. Promotion & Advertising—No secondary products		

Note: The industries are numbered. For each industry, the secondary products are listed by the indices which correspond to their sectors. The figures denote the output ratios of secondary-primary production. t-statistics are in parentheses. Underlinings indicate by-products.

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