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AN ALTERNATIVE TREATMENT OF SECONDARY PRODUCTS IN INPUT-OUTPUT ANALYSIS: FRUSTRATION

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Abstract—ten Raa, Chakraborty and Small (1984) rule out industry technology based input-output coefficients in favor of a construct based on the commodity technology model. The latter, however, produces negative coefficients. This note shows that the negatives cannot be ascribed to errors of measurement. The very framework of deriving unique technical coefficients matrices from the black-box of a single pair of input and output flows must be abandoned.

I. Introduction

ten Raa, Chakraborty and Small (1984) derived and implemented a mixed technology model to deal with secondary products in the construction of input-output coefficients matrices. Secondary products were classified as by-products or independent secondary outputs and the former were treated as negative inputs while the latter were subjected to the so called commodity technology model. Some of the input-output coefficients turned out negative. In case this was due to the by-product modeling, the negatives were to be expected and admitted a natural interpretation. Many negatives, however, were a consequence of the commodity technology model.

This problem is assessed statistically in the present note. The mixing of technology is ignored in the interest of clarity and brevity. Our main finding is negative. The negative coefficients are tiny in magnitude and it is common to sweep them under the carpet, but the underlying adjustments of the data which generate nonnegativity of the coefficients are surprisingly large and, in fact, unlikely. Hence the commodity technology model and, therefore, the mixed technology model of ten Raa, Chakraborty and Small (1984) are rejected.

II. The Model and Its Reestimation

The set-up is as in ten Raa, Chakraborty and Small (1984). $U = (u_{ij})$ is the "use" table of commodities i consumed by industries j. $V = (v_{jk})$ is the "make" table of industries j producing commodities k. The commodity technology model postulates technical coefficients a_{ik} no matter the sector of fabrication, j. ten Raa, Chakraborty and Small (1984) derive that the matrix of coefficients is $A = UV^{-T}$ where ^{T} denotes transposition and $^{-1}$ inversion. (Since the latter two

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TABLE 1.—AGGREGATED U, ACCURACIES, AND REESTIMATES

	agricult	mining	fd dr tb	mingaspr	metals	heav man	lght man	construc	services
agricult	1420.20	0.00	2947.20	36.00	2.40	0.20	310.30	4.10	174.40
	5.0%	—	4.5%	5.0%	3.7%	3.5%	2.6%	8.7%	15.7%
	1420.00	0.00	6753.00	37.08	2.10	0.24	269.90	4.15	0.11
mining	3.60	59.30	13.90	3369.30	494.00	17.10	152.30	294.40	1319.10
	6.9%	4.5%	4.9%	4.6%	3.8%	3.3%	4.9%	12.2%	4.2%
	3.60	61.20	14.65	3292.00	494.00	17.10	152.30	292.70	1288.00
fd dr tb	936.70	0.00	2794.60	136.30	0.10	1.70	28.30	2.90	919.70
	4.9%	—	4.4%	4.8%	5.0%	4.4%	2.3%	8.7%	12.3%
	936.70	0.00	2884.00	10.55	0.10	1.77	28.24	2.88	601.30
mingaspr	392.80	128.80	461.20	3775.70	1007.00	392.30	1709.60	340.70	2385.40
	4.5%	4.5%	2.9%	3.4%	2.1%	2.1%	2.0%	6.5%	5.9%
	392.80	128.80	461.20	3776.00	1007.00	392.30	1710.00	340.70	2385.00
metals	45.40	231.60	111.30	142.90	6239.10	3023.50	336.30	1478.00	893.00
	4.6%	2.7%	2.6%	2.8%	1.7%	2.0%	1.5%	5.0%	4.5%
	45.40	231.60	111.30	142.90	6239.00	3023.00	336.30	1478.00	893.00
heav man	55.40	34.20	311.80	237.60	1266.40	2973.70	401.70	408.20	1210.90
	2.7%	2.9%	3.5%	4.2%	2.6%	2.2%	2.1%	7.9%	7.0%
	55.40	34.20	311.80	237.60	1266.00	2974.00	401.70	408.20	1211.00
lght man	97.10	51.80	771.60	378.40	813.10	715.80	6314.40	2328.00	2629.50
	2.2%	2.5%	2.2%	2.5%	1.4%	2.1%	1.7%	5.6%	4.4%
	97.10	51.80	771.60	378.40	813.10	715.80	6314.00	2328.00	2630.00
construc	129.60	106.00	18.00	8.40	128.10	28.30	46.30	2836.30	707.20
	8.7%	7.3%	6.0%	7.8%	7.3%	4.4%	3.4%	15.0%	14.5%
	129.60	106.00	18.00	8.76	128.00	28.30	46.29	2421.00	683.20
services	592.40	593.70	2148.30	2025.70	3397.00	1615.40	3059.60	1151.80	14390.30
	7.5%	5.2%	6.2%	5.2%	3.6%	3.6%	2.6%	13.0%	12.2%
	592.40	593.70	2148.00	2026.00	3397.00	1615.00	3060.00	1152.00	14390.00

Note: agricult = Agriculture etc.
mining = Mining & Gas
fd dr tb = Food, Drink & Tobacco
mingaspr = Mining & Gas Products
metals = Metals
heav man = Heavy Manufacturing
lght man = Light Manufacturing
construc = Construction
services = Services.

operations commute, their composition may be denoted $^{-T}$ without confusion.)

u_{ij} and v_{jk} are now considered true values. Attached are error terms δ_{ij} and ϵ_{jk}. Summing we get the observed data, u_{ij}^0 and v_{jk}^0:

$$u_{ij}^0 = u_{ij} + \delta_{ij}$$

$$v_{jk}^0 = v_{jk} + \epsilon_{jk}.$$

Following ten Raa and van der Ploeg (1988), δ_{ij} and ϵ_{jk} are independent normally distributed with zero means and standard deviations σ_{ij} and τ_{jk}, based on subjective information regarding 1975 U.K. statistics and reported in tables 1 and 2. Minus twice the log-likelihood of real values (U,V) is

$$f(U,V) = \sum_{i,j} \sigma_{ij}^{-2}\left(u_{ij} - u_{ij}^0\right)^2 + \sum_{j,k} \tau_{jk}^{-2}\left(v_{jk} - v_{jk}^0\right)^2.$$

Maximum likelihood reestimation is equivalent to

$$\text{minimize } f(U,V) \text{ subject to } UV^{-T} \geq 0.$$

Data $(U_0,V_0) = ((u_{ij}^0), (v_{jk}^0))$ and reestimates (U,V) are also reported in tables 1 and 2, where the unit is million pounds. The input-output coefficients matrix $U_0V_0^{-T}$ and the adjusted $A = UV^{-T}$ are presented in table 3, multiplied by a factor of 100, so that the unit is pennies per pound.

Table 2 shows that to render input-output coefficients nonnegative some secondary outputs are set to zero. These adjustment steps involve many standard deviations and are very unlikely. One way of obtaining insight into this question is the use of the likelihood-ratio test. Since the variances are assumed to be known, $f(U,V)$ is the test statistic. It is distributed as a $\chi^2(r)$ variate, where r is the number of binding nonnegativity constraints. In our case $r = 9$ and the test statistic is

TABLE 2.—AGGREGATED V, ACCURACIES, AND REESTIMATES

	agricult	mining	fd dr tb	mingaspr	metals	heav man	lght man	construc	services
agricult	5616.90	0.00	0.00	0.00	0.00	0.00	0.00	30.20	0.20
	5.0%	—	—	—	—	—	—	8.7%	17.3%
	5617.00	0.00	0.00	0.00	0.00	0.00	0.00	30.20	0.20
mining	0.00	2622.40	0.00	0.80	0.00	0.00	36.00	12.10	69.60
	—	4.0%	—	5.0%	—	—	7.1%	8.4%	14.8%
	0.00	2622.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
fd dr tb	0.00	0.00	11844.30	24.80	0.00	0.00	5.60	19.10	731.00
	—	—	4.1%	3.7%	—	—	4.0%	5.9%	8.3%
	0.00	0.00	11540.00	24.43	0.00	0.00	5.60	19.07	657.80
mingaspr	0.00	0.00	21.80	12413.20	31.20	3.60	48.90	56.10	277.90
	—	—	5.0%	3.5%	3.2%	5.0%	3.2%	8.2%	10.7%
	0.00	0.00	22.58	12560.00	31.20	3.60	48.89	53.87	288.40
metals	0.00	0.00	0.00	272.20	20450.00	401.50	21.40	55.50	554.80
	—	—	—	4.6%	2.6%	1.7%	2.5%	5.6%	6.3%
	0.00	0.00	0.00	273.80	20780.00	401.50	21.42	55.35	0.00
heav man	0.00	0.00	0.00	1.10	457.10	12582.00	47.20	32.50	363.40
	—	—	—	5.0%	2.0%	2.7%	3.6%	5.4%	8.0%
	0.00	0.00	0.00	1.09	452.60	12580.00	15.73	31.95	307.50
lght man	0.00	33.90	0.10	53.60	23.80	40.40	19482.90	32.90	657.70
	—	7.1%	5.0%	4.4%	3.5%	3.9%	1.9%	4.3%	4.3%
	0.00	33.90	0.10	53.61	23.80	40.40	20650.00	32.90	658.40
construc	0.00	0.00	0.00	0.00	0.00	0.00	0.00	15669.70	21.90
	—	—	—	—	—	—	—	15.0%	27.4%
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	19230.00	21.98
services	0.00	0.00	0.00	0.00	36.20	0.00	0.00	507.40	61567.30
	—	—	—	—	5.0%	—	—	6.2%	20.6%
	0.00	0.00	0.00	0.00	36.20	0.00	0.00	507.90	106100.00

Note: See table 1.

1914.2. Since the critical value of $\chi^2(9)$ at the 5% significance level is 16.92, the nonnegativity constraints are violated at the 5% level. This leaves no room other than for an empirical rejection of the commodity technology model.

III. Discussion

At least in principle, two considerations may muffle the rejection of the commodity technology based input-output coefficients construct. Firstly, the above minimization problem presents one approach to incorporating the nonnegativity constraint—it uses subjective information on data uncertainties to "reallocate" excessive errors to other flows and checks to see if that *particular* reallocation scheme is consistent with all the *subjective* error estimates. Perhaps there are other reallocation schemes or less stringent evaluation criteria. Clearly, the subjective error estimates could themselves be wrong. Secondly, the requirement of nonnegativity of input-output coefficients may be an *inadequate* ground for the rejection of an input-output model. Strict application of this criterion may return the industry technology model to the limelight. The errors of that model are forced to reside under the camouflage of positive coefficients, but are probably larger. I shall take up the two points in turns.

As regards alternative reallocation schemes, it is instructive to view the reestimation procedure as an analogon to the balancing of flows. Basically, the reestimation procedure is obtained by substituting the nonnegativity constraints for the balance constraints in the Stone, Champernowne and Meade (1942) balancing mechanism. Clearly, alternative reallocation schemes are obtained by departing from other balancing mechanisms, such as linear or quadratic programming (Matuszewski, Pitts and Sawyer, 1964, and Harrigan and Buchanan, 1984, respectively) and RAS or entropy methods (Bacharach, 1970, and Theil, 1967, respectively). The main reason that I do not adapt any of these methods is that they do not admit a statistical interpretation, let alone testing. In other words, it is impossible to assess the acceptability of adjustments. A further reason is that the error estimates will not be affected significantly. As is long known to input-output practitioners, a negative coefficient typically emerges when some secondary output has an own input structure of which a component is not used by the sector

TABLE 3.—TECHNICAL COEFFICIENTS AND THEIR REESTIMATES

	agricult	mining	fd dr tb	mingaspr	metals	heav man	lght man	construc	services
agricult	25.28	−0.03	24.86	0.23	−0.00	−0.01	1.58	0.03	0.28
	25.28	0.00	58.50	0.18	0.01	0.00	1.31	0.02	0.00
mining	0.05	2.18	−0.07	27.08	1.99	−0.01	0.63	1.88	2.13
	0.06	2.33	0.00	26.17	2.03	0.03	0.62	1.52	1.21
fd dr tb	16.68	−0.04	23.50	1.02	−0.05	−0.03	0.09	0.02	1.49
	16.68	0.00	24.94	0.03	0.00	0.00	0.12	0.01	0.57
mingaspr	6.98	4.67	3.59	30.27	4.35	2.81	8.54	2.17	3.85
	6.98	4.91	3.80	29.96	4.38	2.89	8.11	1.77	2.24
metals	0.76	8.73	0.84	0.99	29.98	22.87	1.56	9.43	1.36
	0.77	8.83	0.90	1.00	29.55	22.93	1.49	7.68	0.79
heav man	0.97	1.21	2.50	1.83	5.65	23.36	1.93	2.60	1.94
	0.97	1.30	2.63	1.83	5.61	23.40	1.85	2.12	1.13
lght man	1.65	1.35	6.21	2.74	3.65	5.28	32.22	14.85	4.15
	1.66	1.98	6.51	2.77	3.71	5.43	30.46	12.10	2.42
construc	2.21	3.93	0.06	−0.04	0.55	0.13	0.17	18.10	1.00
	2.24	4.04	0.10	0.00	0.58	0.16	0.18	12.59	0.58
services	10.51	21.78	16.65	15.63	15.51	11.53	14.78	7.32	23.30
	10.51	22.64	17.79	15.67	15.88	11.90	14.26	5.97	13.53

Note: See table 1.

under consideration. To render such a coefficient nonnegative, the secondary output or the associated input component *must* be set to zero, irrespective the objective function. This observation is the key to my rejection of the commodity technology model and it is robust with respect to the reallocation scheme or evaluation criterion.

Turning to the adequacy issue, I can be brief. The requirement of nonnegativity yields a sufficient likelihood criterion for the rejection of an input-output model, but not a necessary one. Indeed, the industry technology model passes the test, but this does *not* imply that it is better. Other, theoretical, considerations must be taken into account (see ten Raa, Chakraborty and Small (1984)). In fact, I reject both models, for different reasons though. That is why I am frustrated.

IV. Conclusion

Theoretical considerations of ten Raa, Chakraborty and Small (1984) rule out industry technology based input-output coefficients matrices in favor of a construct based on the commodity technology model. The latter, however, produces negative coefficients. Either the underlying model is wrong or errors in the data produce the negatives. This note renders the latter hypothesis unlikely and, therefore, rejects the commodity technology model and the derived construct of ten Raa, Chakraborty and Small (1984). In conclusion, we must abandon the very framework of deriving unique technical unit coefficients (A) from the black-box of a single pair of input and output flows (U,V). We must accept that technical coefficients vary within and across industries and need more data to model them.

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