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A DYNAMIC MODEL OF ENDOGENOUS MERGERS AND TRADE LIBERALIZATION

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Abstract

This paper uses a dynamic dominant-firm model with an endogenous merger process to examine the effects of trade liberalization on industry structure. Domestic and cross-border mergers and demergers are allowed for. When firms are myopic and the dominant firm has a sufficiently high pre-merger capital share in any one country, trade liberalization causes the industry to become significantly more concentrated. When firms are forward-looking, this anti-competitive effect of trade liberalization is mitigated. Tariff reduction from a prohibitive to a non-prohibitive level aligns merger patterns across countries and initiates merger (or demerger) waves simultaneously across countries, provided all firms are equally forward-looking. When the dominant firm is more forward-looking than the fringe, however, this result may be reversed. These results, thus, highlight the importance of taking into consideration existing industry structure and firms’ discount rates whilst formulating competition policy in the face of trade liberalization.

JEL Classification Numbers: L41, F13
Keywords: endogenous market structure, horizontal mergers, trade liberalization

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1 Introduction

Successive rounds of international trade negotiations have reduced trade barriers worldwide consistently over the past few decades, with the average tariff level of the GATT and WTO members, as a percentage of 1930 tariff levels, falling from 21.2 subsequent to the Tokyo round (ended 1979) to below 14.8 subsequent to the start of the Doha round (Bowen, Hollander, and Viaene (1998)). At the same time, free trade agreements have also proliferated at a more regional level amongst different groups of countries. In Europe, for example, the move towards free trade has been a necessary step towards closer economic integration. In North America, tariffs on most manufactured goods have decreased substantially as a result of the Canada-US Free Trade Agreement signed in 1989 and the North American Free Trade Agreement signed in 1994. Simultaneously, one of the most significant ways in which firms have been bringing about changes in industry structure is through their decisions to participate in mergers. The total volume of mergers worldwide has been growing at an annual rate of 42% over the period 1980-1999, according to the UN’s World Development Report. Merger activity has steadily grown since, across industries and countries, to reach unprecedented levels in 2006 with the total value of merger activity worldwide surpassing $3 trillion.

Within this context, a natural question that arises is the following. Has trade liberalization played an active role in encouraging mergers? Besides affecting the number of mergers across the world, is trade liberalization influencing the pattern of mergers? In particular, cross-border mergers (as opposed to domestic ones) have been on the rise, constituting approximately 25-30% of total merger activity, between 1987 and 1999 (Brakman, Garresten and Marrewijk (2005)).

Existing studies that examine merger activity in open economies consist of two broad categories. The first imposes an exogenously given pattern of mergers within the industry (Benchekrour and Ray Chaudhuri (2006), Gaudet and Kanouni (2004), Long and Vousden (1995)), whilst the second endogenizes the merger process (Falvey (2005), Horn and Persson (2001b), Yildiz (2003)).¹ The main limitation of the first category is that these studies are restricted to examining a merger pattern that might not be realized at equilibrium. This is because by exogenously imposing an arbitrarily chosen merger pattern, these studies ignore other patterns which might be more profitable. An endogenous merger process allows for all possible potential merger patterns, thus ensuring that the post-merger scenario yielded by the model represents an industry equilibrium.² This model, therefore, incorporates an endogenous merger process.

¹It is noted that there exists a related stream of literature which focuses on the interplay between trade and competition policies (Collie (2003), De Stefano and Rysman (forthcoming), Head and Ries (1995), Horn and Levinsohn (2001), Qiu and Zhou (2006), Richardson (1999) and Saggi and Yildiz (2006)). These studies focus on the welfare implications of mergers within an open economy context.

setting is considered, where firms are rational and forward-looking. There does exist a literature of dynamic models which predicts the evolution of industry structure through time, and allows for entry into and exit from the industry (Ericson and Pakes (1995), Pakes (2000) and Pakes and McGuire (1994)). These models, however, do not allow for any merger activity. Existing dynamic merger models include Gowrisankaran (1999), Gowrisankaran and Holmes (2004), Judd and Cheong (forthcoming) and Marino and Zabojnik (2006). These papers have established that forward-looking firms make significantly different merger decisions than myopic firms. However, they are restricted to closed economy scenarios. Thus, this paper is a first to apply a dynamic model with an endogenous merger process to an open economy setting.

A partial equilibrium scenario with two countries, Home and Foreign, is considered, where trade liberalization occurs bilaterally. This reflects the tariff cuts that are realized subsequent to rounds of international trade negotiations.

In reality, firms within an industry are heterogeneous and enjoy different degrees of market power. Merger decisions of individual firms are dependent on the market power of each merger participant relative to other participants and non-participants. The model in this paper incorporates this heterogeneity at the firm level. For tractability, I follow Gowrisankaran and Holmes (2004) and use a dominant-firm model, that is, an industry consisting of a single firm with market power and others with no market power. There exists a competitive fringe in each country and a single multi-national dominant firm that begins each period with given capital shares in each of the two countries.

Within the multi-period framework, each period consists of two stages. In the first stage, the firms undertake merger decisions, which re-allocate the stock of industry-specific capital amongst the firms. In this case, a merger (demerger) is said to occur whenever the dominant firm buys (sells) capital from (to) the fringe firms. The choices of all firms regarding the amount of capital to buy/sell and the location at which to buy/sell (Foreign or Home) are endogenized. That is, the firms are allowed to choose which other firms they wish to merge with and how many other firms they wish to merge with. In the second stage, the firms undertake their investment and output decisions, that is, how much to invest in next period’s capital stock and how much of the consumption good to produce and to sell in each country. In the consumption goods markets and in the capital goods markets in both countries, the dominant firm moves first, followed by the fringe firms.

Two key assumptions are made about the technology. The first is that all firms have access to an identical constant returns to scale technology. If there were economies of scale or if the dominant firm had a cost advantage over the fringe firms, then the dominant firm would have an extra incentive to

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3Neary (2003, 2007) studies cross-border mergers in a general equilibrium setting, taking into consideration the effects of economy wide shocks such as changes in legal and regulatory environments and asymmetric business cycles on merger activity. On the other hand, this paper uses a partial equilibrium setting, which focuses on the strategic interaction of firms within an industry.

4This feature is similar to that used by Schleifer and Vishny (1986) in their model.

5The two-stage framework is similar to Perry and Porter (1985).
buy capital. I abstract away from this well-understood effect in order to focus on the other factors affecting merger decisions as described below. The second assumption is that the capital stock required for production is industry-specific. If this were not the case, capital could costlessly flow in from other industries, in which case mergers would be rendered futile.

The capital good is assumed to be physically immobile across borders. However, the consumption goods produced in one of the countries need not be sold within the local market. The producer can choose to export the goods. Resale of the consumption good is not allowed for.

Using the dominant-firm framework, it is possible to focus on the key factors affecting the merger decisions of heterogeneous firms (differing in terms of size and behavior) within an industry. First, firms have an incentive to push the industry towards monopoly, since a monopoly maximizes industry profits: the "market power effect". Second, firms have an incentive to not participate in mergers. By remaining outside the merger, firms can free ride on the efforts made by the merger participants to increase the industry price level (Stigler (1950)): the "free-rider effect". Third, large firms, which internalize the effects of their own actions on industry output and price, have different incentives to invest in the industry's capital stock than do small firms: the "size effect". Apart from these three effects, which would also exist in a closed economy model, a fourth factor that affects merger decisions in the open economy case is that the dominant firm can affect the price of capital in both the countries whenever it buys or sells capital in any one. This is labeled the "cross-border effect".

The above four factors hold in both a single-period scenario (where firms behave myopically) as well as in a multi-period model where firms are forward-looking. An additional factor which comes into play only in the dynamic case is the following. As the firms become more forward-looking, the dominant firm finds it increasingly profitable to sell capital in both countries. The key factor driving the differences in the results yielded by the single and multi-period cases (both of which are analyzed in this paper) is the dominant firm's inability to commit to future behavior in the latter case. Subsequent to a merger, forward-looking fringe firms expect the dominant firm to cut production, thereby raising the price of the consumption good. Also, a merger raises the price of capital and signals that the dominant firm might continue to buy capital from the fringe in the future. This encourages the fringe firms to invest at a higher rate. To discourage the fringe from investing, the dominant firm uses the sale of capital as a signal to indicate that it will not raise the price of the consumption good or buy capital in the future. This is labeled the "dynamic effect".

Having determined the factors affecting merger decisions at any given tariff level, the paper addresses the main issue of interest: the effect of trade liberalization on the incentives to merge. The impact of tariff reductions on merger activity is shown to depend on the pre-merger capital shares of the dominant

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6This is applicable to industries where the capital good consists of heavy machinery which is costly to relocate, or where the capital is location specific. For example, if the capital is knowledge pertaining to a particular country’s market (demand conditions or retail network), it may not be useful in another country.
firm in both countries and on the discount factor. Let us now turn to a discussion of the main results.

First, when discussing the effects of trade liberalization, it is important to distinguish between whether the tariff reduction in question is from a prohibitive to a non-prohibitive level or from one non-prohibitive level to another. Given an identical discount factor for all firms and a prohibitive tariff level, there occur mergers in one country and demergers in the other. However, opening up the economies by reducing the tariff to a non-prohibitive level generates merger (or demerger) waves simultaneously in both countries, thereby aligning merger patterns across the countries. This result is driven by the fact that the "cross-border effect" is muted when the tariff is prohibitive, but is kicked into motion as the tariff is reduced to a non-prohibitive level. Five major merger waves have been observed internationally during the period 1900-2000 (Kleinert and Klotz (2002)). The subsequent merger wave had, by 2006, surpassed over five times the value of the previous wave, reaching unprecedented levels. The existing theories proposed to explain merger waves within the industrial organization literature consist of endogenous merger models restricted to closed economy scenarios.7 This paper identifies an alternative possible explanation for merger waves within an open economy scenario, namely, trade liberalization.

An important policy implication arises. At prohibitive tariff levels, the diverging merger patterns across the countries may engender conflicts of interest between the national anti-trust authorities. Consider a scenario where the multinational dominant firm belongs to a third country. Mergers in Home and demergers in Foreign reduce consumer and producer surplus in Home and increase both in Foreign respectively. The net effect on the welfare of the two countries is ambiguous. This creates a role for an international anti-trust authority. Indeed, such situations have been a rising concern of international organizations. At the 2003 Ministerial Conference of the World Trade Organization (WTO), a session was held to discuss "Sustainable Competition Law", where it was proposed that the WTO develop into an umbrella for international competition disciplines, and build mechanisms into the international competition treaties to ensure that changes in industry structure at a global level are Pareto improving (Gehring (2003)). Given a uniformly myopic industry, this model predicts that trade liberalization, by aligning the post-merger industry structure across the countries, reduces the need for such supra-national level intervention. However, these policy implications may be reversed if the firms have heterogenous discount factors.

Different firms within an industry may face different credit constraints or possess asymmetric information regarding the future. In reality, a dominant firm may have deeper pockets than its followers and, being the market leader, may have more information about its own future strategies than do the followers. This paper, therefore, considers the scenario where, due to some combination of the above reasons, the dominant firm is more forward-looking than the fringe firms. In this case, opening up the

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7Exceptions include Bertrand and Zitouna (2006) and Neary (2007). In these papers mergers are driven by differences in technology across firms where low cost firms buy out high cost foreign rivals as trade is liberalized. In contrast, this paper shows that trade liberalization can trigger merger waves even when all firms within an industry and across countries have access to identical technologies.
economy to trade may not align post-merger capital shares across countries. This is because, if mergers in one country affect the price of capital in the other, the dominant firm and the fringe firms, because they weigh the future differently, perceive the change in the value of capital differently. This causes the "cross-border effect" itself to be modified. As long as the dominant firm’s pre-merger capital share in one country is small, opening up the economy, at best, fails to align the merger patterns. Numerical examples are provided where trade liberalization causes the merger patterns in the two countries to diverge.

Second, of particular concern to antitrust authorities are scenarios where all firms are equally myopic and the dominant firm has a sufficiently high pre-merger capital share in any one country. In such cases, trade liberalization causes the industry to become significantly more concentrated by encouraging mergers in both countries simultaneously. This result is driven by a combination of the "size" and "cross-border" effects. If the dominant firm has a larger pre-merger capital share in Home, then it has a greater incentive to buy capital in Home than in Foreign (the "size effect"). Once it buys some capital at Home, the "cross-border effect" ensures that the merger activity spreads from Home to Foreign and a wave of mergers is generated. The lower the tariff, the easier it is for the "cross-border effect" to be set into motion.

Third, the anti-competitive effect of trade liberalization, given myopic firms, is mitigated when firms are more forward-looking. The more forward-looking the fringe firms, the more costly it is for the dominant firm to buy capital (the "dynamic effect"). This effect dominates the other factors affecting merger decisions as the discount factor rises. This results in demergers, regardless of the tariff level, for a sufficiently high discount factor that is common to all firms.

By using a multi-period analysis, this paper thus highlights the importance of taking into consideration the pre-merger industry structure and the rates at which individual firms discount the future in forecasting merger patterns in the face of trade liberalization and in determining international competition policy.

The paper proceeds as follows. Section 2 presents the model.8 Sections 3 and 4 present the analysis at a given tariff level and the effects of trade liberalization respectively. Section 5 concludes.

2 The Model

There are two countries, Home and Foreign. There is a single dominant firm that can produce and sell in Home and Foreign, a competitive fringe in Home and a competitive fringe in Foreign. All firms produce homogenous products, resale of which is not allowed. The industry is modeled in partial equilibrium with demand schedules in each country that are constant over time. A discrete time model is adopted. Firms maximize their expected discounted value of profits. Each period consists of two stages. First stage: Merger decisions are undertaken. Second stage: Investment and output decisions are undertaken. The preferences and technology are discussed below, followed by a description of the equilibrium of the model.

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8For a list of the key variables used in this model, please refer to the Appendix (Part VII).
Preferences and Technologies

The inverse demand is identical across the two countries. In Home, the inverse demand is given by \( p = P(Q) \), where \( Q \) represents total sales, including imports, in the Home market. Similarly, in Foreign, the inverse demand is given by \( p^* = P(Y) \), where \( Y \) represents total sales in the Foreign market. The tariff level, \( t \), is assumed to be equal in both countries and trade liberalization is assumed to be in the form of bilateral tariff reductions, that is, equal tariff reductions in both countries.

One of the inputs required in the production process is industry-specific capital. The dominant firm is endowed with \( K_h \) units of capital located in Home (Foreign). The Home (Foreign) fringe is endowed with \( K_f \) units of capital located in Home (\( K_f \) units located in Foreign). The fringes in both countries consist of a continuum of firms, each of which possesses an infinitesimally small proportion of the capital stock. Thus each fringe firm is a price taker in both the product and the capital markets.

Each firm has access to the same production process represented by \( F(K, L) \), where \( K \) denotes industry-specific capital, and \( L \) denotes non-industry-specific labor. This production process produces joint outputs: the consumption good, \( X \), and future capital, \( K_{next} \). I assume that the two outputs are produced in fixed proportions, as in Gowrisankaran and Holmes (2004). Let \( X = F(K, L) \) be the production of the consumption good and \( K_{next} = \sigma F(K, L) \) be the production of future capital, given inputs, \( K \) and \( L \), where \( 0 < \sigma < 1 \).\(^9\) Current capital is assumed to completely depreciate during the production process.\(^10\) The assumption of the two outputs being produced in fixed proportions simplifies the computation of the model’s equilibrium significantly by essentially collapsing the investment and output decisions of each firm into one.\(^11\)

The production process, \( F(K, L) \), represents a constant returns to scale production process with \( F(0, L) = F(K, 0) = 0 \). It is assumed that \( F(K, L) \) is strictly concave and strictly increasing in \( K \) and \( L \) for \( K > 0 \) and \( L > 0 \). Also, it is assumed that \( \lim_{L \to -\infty} F_L(K, L) = 0 \) and \( \lim_{L \to 0} F_L(K, L) = \infty \) for any \( K > 0 \).

Let \( C(X, K) \) be the labor cost, corresponding to the given technology, of producing \( X \) units of the consumption good and \( \sigma X \) units of the capital good. That is, \( C(X, K) = \omega L' \) for the \( L' \) that solves \( X = F(K, L') \), given the competitive wage \( \omega \). Given the assumptions on \( F(K, L) \), it follows that \( C(X, K) \) is homogeneous of degree one in \( K \), so that \( KC(X/K, 1) = C(X, K) \). Lower case \( x \) denotes output per unit of capital. Let \( c(x) = C(x, 1) \) denote the labor cost per unit of capital necessary to produce \( x \) units of the consumption good and \( \sigma x \) units of the capital good. The assumptions on \( F(K, L) \) imply that \( c(x) \) is strictly convex and strictly increasing, and that \( c'(0) = 0 \).

\(^9\)As long as the production function is characterized by a sufficiently low elasticity of substitution between the capital and consumption goods, it can be shown that the optimal choice of \( \sigma \) remains fairly constant for different market structures.

\(^10\)Alternatively, the interpretation of \( X \) could be the end-of-period capital. In this case, by defining \( \delta \equiv 1 - \sigma \), we could interpret \( \delta \) to be the rate of depreciation of the capital stock, with \( K_{next} = (1 - \delta)X \) and \( \sigma \) representing the proportion of end-of-period capital that survives into the next period.

\(^11\)Gowrisankaran and Holmes (2004) show that for the closed economy version of this model, their results hold qualitatively when the assumption of fixed proportions is relaxed.
Equilibrium of the Model

The Markov-Perfect equilibria (MPE) of the model, in the sense of Maskin and Tirole (2001) are analyzed. In other words, only those equilibria in which the actions are functions solely of payoff-relevant state variables (capital stocks, in this case) are analyzed. Let \((K_0^h, K_0^f, K_{d0}^h, K_{d0}^f)\) denote the pre-merger capital stocks of the fringe and the dominant firm, and let \((K_h, K_f, K_{d}^h, K_{d}^f)\) denote the capital stocks after the merger stage but before the investment/output stage. (Throughout the paper, the superscript \(^0\) will represent pre-merger values and the absence of the superscript will denote post-merger values.)

The total capital stock remains unchanged subsequent to the merger stage, \(K = K_0^h + K_0^f + K_{d0}^h + K_{d0}^f = K_h + K_f + K_{d}^h + K_{d}^f\). The state in each period of the model is represented by the shares of the total capital stock held by the dominant and fringe firms in each country. Let \(m_0^h = K_{d0}^h / K\) and \(m_0^f = K_{d0}^f / K\) denote the proportions of the total industry capital stock held by the dominant firm before (after) the merger stage in Home and in Foreign respectively.

The discounted value of the future stream of profits of firms is a function of the state variables, namely \(m_h, m_f, K,\) and \(K_f\). Henceforth, in order to simplify the notation, the arguments of the value functions are not shown. Define \(w_d (w_d^*)\) to be the discounted value to the dominant firm from its profits in the Home (Foreign) market by selling goods that it produces in Home. Let \(v_d = \frac{w_d}{m_hK}\) and \(v_d^* = \frac{w_d^*}{m_hK}\). That is, \(v_d (v_d^*)\) represents the discounted value to the dominant firm from its profits in the Home (Foreign) market per unit of capital located in Home. Similarly, \(w_x (w_x^*)\) denotes the discounted value to the dominant firm from its profits in the Home (Foreign) market by selling goods that it produces in Foreign, and \(v_x (v_x^*)\) the corresponding values per unit of capital located in Foreign. Let \(v_h\) and \(v_h^*\) be the discounted values to a Home fringe firm per unit of capital from Home profits (that is, profits made from selling the consumption good in the Home market) and from Foreign profits respectively. Analogously, let \(v_f\) and \(v_f^*\) be the discounted values to a Foreign fringe firm per unit of capital from Home profits and from Foreign profits respectively.

The merger decision

During the merger process, the dominant firm with market shares \(m_0^h\) and \(m_0^f\) chooses the post-merger market shares, \(m_h\) and \(m_f\). Given \(m_h\) and \(m_f\), the amount of capital purchased by the dominant firm in Home is given by \(m_hK - m_0^hK\) and in Foreign is given by \(m_fK - m_0^fK\). The equilibrium price of capital in Home is given by:

\[
p_h^b = v_h + v_h^*
\]

At equilibrium, in order to buy a fringe firm, the dominant firm must pay the value that the fringe firm would get post-merger, if the fringe firm decided to remain outside the merger. It is only at this price that the Home fringe firm is indifferent amongst buying, selling and holding onto its capital. Thus, at

\(^{12}\) The notation is kept similar to that used by Gowrisankaran and Holmes (2004) to facilitate the comparison of results.
equilibrium, the price of each unit of Home capital equals the post-merger value of each unit of capital to the Home fringe firm, as shown in (1). Similarly, the equilibrium price of capital in Foreign is given by:

$$p_{K}^{f} = v_{f} + v_{f}^{*}$$

(2)

Thus the dominant firm chooses $m_h$ and $m_f$ to solve

$$\max_{m_h, m_f} \begin{cases} m_h K (v_d + v_d^{*}) + m_f K (v_x + v_x^{*}) \\ -(m_h K - m_h^{0} K)p_{K}^{h} - (m_f K - m_f^{0} K)p_{K}^{f} \end{cases}$$

That is,

$$\max_{m_h, m_f} \begin{cases} m_h K (v_d + v_d^{*}) + m_f K (v_x + v_x^{*}) \\ -(m_h K - m_h^{0} K)(v_h + v_h^{*}) - (m_f K - m_f^{0} K)(v_f + v_f^{*}) \end{cases}$$

(3)

The first two terms are the dominant firm’s return if it enters the investment/production stage with shares of $m_h$ and $m_f$. The third term subtracts the amount spent on the acquisition of capital from the Home fringe. The fourth term subtracts the amount spent on the acquisition of capital from the Foreign fringe. Let the solution be $(\tilde{m}_h(m_h^{0}, m_f^{0}, K, K_f), \tilde{m}_f(m_h^{0}, m_f^{0}, K, K_f))$.

Once the dominant firm chooses $m_h = \tilde{m}_h(m_h^{0}, m_f^{0}, K, K_f)$ and $m_f = \tilde{m}_f(m_h^{0}, m_f^{0}, K, K_f)$, it sets the price of capital in each country according to (1) and (2). It commits to buy (sell) as much capital as the fringe firms wish to sell (buy) at these prices. This ensures that the following conditions are satisfied at equilibrium.

$$v_{h}^{0}(m_h^{0}, m_f^{0}, K, K_f) + v_{h}^{*0}(m_h^{0}, m_f^{0}, K, K_f) = v_{h}(\tilde{m}_h, \tilde{m}_f, K, K_f) + v_{h}^{*}(\tilde{m}_h, \tilde{m}_f, K, K_f)$$

(4)

---

To explain (4), let us consider the case where the dominant firm wishes to buy $(\tilde{m}_h K - m_h^{0} K) > 0$ units of capital from the Home fringe. The dominant firm sets $p_{K}^{h} = v_{h}(\tilde{m}_h, \tilde{m}_f, K, K_f) + v_{h}^{*}(\tilde{m}_h, \tilde{m}_f, K, K_f)$, in accordance with (1). If $p_{K}^{h} < v_{h}^{0}(m_h^{0}, m_f^{0}, K, K_f) + v_{h}^{*0}(m_h^{0}, m_f^{0}, K, K_f)$, however, then the Home fringe firms lose per unit of capital sold to the dominant firm. As a result, at this price, the dominant firm cannot induce any of the fringe firms to sell their capital. The Home fringe firms will instead have an incentive to buy capital from the dominant firm. On the other hand, if $p_{K}^{h} > v_{h}^{0}(m_h^{0}, m_f^{0}, K, K_f) + v_{h}^{*0}(m_h^{0}, m_f^{0}, K, K_f)$, it becomes profitable for the Home fringe firms to sell each unit of capital to the dominant firm. Thus, all fringe firms offer their capital for sale. Given that the dominant firm has precommitted to buying all capital that is supplied at this price, it must buy up all the capital from the fringe instead of buying the optimal amount, $(\tilde{m}_h K - m_h^{0} K)$. Only when (4) is satisfied can the dominant firm induce the Home fringe firms to sell exactly $(\tilde{m}_h K - m_h^{0} K)$ units of capital at the capital market equilibrium. By similar reasoning, for the Foreign capital market to be at equilibrium at $m_f = \tilde{m}_f$, (5) must be satisfied.
\[ v_f^0(m_h^0, m_f^0, K, K_f) + v_f^0(m_h^0, m_f^0, K, K_f) \]
\[ = v_f(m_h, m_f, K, K_f) + v_f^*(m_h, m_f, K, K_f) \]  

(5)

The output/investment decision

Let \( q_d \) and \( q_d^* \) (\( x \) and \( x^* \)) denote the sales, in the Home and Foreign markets respectively, of the dominant firm per unit of capital possessed, which are produced using capital located in Home (Foreign). Let \( q \) and \( q^* \) (\( y \) and \( y^* \)) denote the sales in the Home and in the Foreign market respectively of the Home (Foreign) fringe firms per unit of capital. When the fringe firms make their output decisions, the dominant firm has already made its move. The fringe firms take the dominant firm’s decisions as given when making their own output decisions. Let \( \tilde{q}(q_d, x, q_d^*, x^*, m_h, m_f, K, K_f) \) and \( \tilde{q}^*(q_d, x, q_d^*, x^*, m_h, m_f, K, K_f) \) denote the equilibrium level of sales of each Home fringe firm in Home and Foreign markets respectively, given the dominant firm’s choices and the state of the model. Analogously, \( \tilde{y}(q_d, x, q_d^*, x^*, m_h, m_f, K, K_f) \) and \( \tilde{y}^*(q_d, x, q_d^*, x^*, m_h, m_f, K, K_f) \) denote the equilibrium level of sales of each Foreign fringe firm in Home and Foreign markets respectively. Henceforth, in order to simplify the notation, the arguments of the equilibrium output functions are not shown.

In each market, since each fringe firm is infinitesimally small, it takes the current prices, \( p \) and \( p^* \), and the value of capital next period as given. The Home fringe firm’s choices of the level of sales in each market solve the following.

\[
\arg\max_{q,q^*} \begin{cases} 
 pq + (p^* - t) q^* - c(q + q^*) \\
 + \beta \sigma(q + q^*) \left( v_{h,\text{next}}^0 + v_{h,\text{next}}^* \right) 
\end{cases} 
\]  

(6)

In (6), \( \beta \) denotes the discount factor. Unless otherwise mentioned, for the rest of the paper it is assumed that all firms have the same discount factor, \( \beta \). A choice of the level of sales in the Home market, \( q \), yields current revenues of \( pq \). Similarly, a choice of the level of sales in the Foreign market, \( q^* \), yields current revenues of \( (p^* - t) q^* \). The current cost of producing \( (q + q^*) \) units is given by \( c(q + q^*) \). Together, the choices \( q \) and \( q^* \) also yield \( \sigma(q + q^*) \) units of capital next period by the fixed proportions technology assumption, each unit of which will be worth \( (v_{h,\text{next}}^0 + v_{h,\text{next}}^*) \).

Similarly, the foreign fringe firm’s choices solve the following.

\[
\arg\max_{y,y^*} \begin{cases} 
 (p - t) y + p^* y^* - c(y + y^*) \\
 + \beta \sigma(y + y^*) \left( v_{f,\text{next}}^0 + v_{f,\text{next}}^* \right) 
\end{cases} 
\]  

(7)
In (6) and (7) the following hold:

\[
\begin{align*}
    p &= P(Q) \\
    p^* &= P(Y) \\
    Q &= m_h K q_d + m_f K x^* + ((1 - m_h - m_f) K - K_f) q + K_f y \\
    Y &= m_f K q_d^* + m_f K x^* + ((1 - m_h - m_f) K - K_f) q^* + K_f y^* \\
    v^0_{h, next} &= v^0_h(m^0_{h, next}, m^0_{f, next}, K_{next}, K_{f, next}) \\
    v^*_{h, next} &= v^*_h(m^0_{h, next}, m^0_{f, next}, K_{next}, K_{f, next}) \\
    v^0_{f, next} &= v^0_f(m^0_{h, next}, m^0_{f, next}, K_{next}, K_{f, next}) \\
    v^*_{f, next} &= v^*_f(m^0_{h, next}, m^0_{f, next}, K_{next}, K_{f, next}) \\
    m^0_{h, next} &= \frac{m_h K (q_d + q_d^*)}{Q + Y} \\
    m^0_{f, next} &= \frac{m_f K (x + x^*)}{Q + Y} \\
    K_{next} &= \sigma(Q + Y) \\
    K_{f, next} &= \sigma m_f K (x + x^*) + \sigma K_f y^* + \sigma K_f y \\
\end{align*}
\]

(8)

Now let us turn to the dominant firm’s output/investment decision. Unlike the fringe firms, the dominant firm recognizes that its choice of output affects the current and future prices, and also the fringe supply.

Given the fringe firms’ reactions, \( \tilde{q}, \tilde{q}^*, \tilde{y}\) and \( \tilde{y}^*\), the dominant firm’s choices of the output levels to sell in both countries solve the following.

\[
\begin{align*}
    \max_{q_d, q_d^*, x, x^*} & \quad p (q_d, q_d^*, x, x^*) q_d + (p^* (q_d, q_d^*, x, x^*) - t) q_d^* - c(q_d + q_d^*) \\
    & + (p (q_d, q_d^*, x, x^*) - t) x + p^* (q_d, q_d^*, x, x^*) x^* - c(x + x^*) \\
    & + \frac{\beta}{m_h K} w^0_d \left( m^0_{h, next}, m^0_{f, next}, K_{next}, K_{f, next} \right) \\
    & + \frac{\beta}{m_h K} w^0_d \left( m^0_{h, next}, m^0_{f, next}, K_{next}, K_{f, next} \right) \\
    & + \frac{\beta}{m_f K} w^{*0}_d \left( m^0_{h, next}, m^0_{f, next}, K_{next}, K_{f, next} \right) \\
    & + \frac{\beta}{m_f K} w^{*0}_x \left( m^0_{h, next}, m^0_{f, next}, K_{next}, K_{f, next} \right)
\end{align*}
\]

subject to the same conditions as set out in (8) except that prices are now functions of the quantities chosen by the dominant firm.
In this setting, a Markov Perfect equilibrium is a set of functions
\[ (v_h^0, v_h^0, v_f^0, v_f^0, v_h, v_f, v_d, v_d^*, v_x, v_x^*, w_d^0, w_x^0, q_d^0, q_d^*, \tilde{q}, \tilde{q}_d, \tilde{x}, \tilde{x}^*, m_h, m_f) \]
such that the following conditions hold.

1. The per-unit-of-capital values, \( v_h^0, v_h^0, v_f^0, v_f^0 \) solve (4) and (5).

2. The per-unit-of-capital values \( v_h, v_h^*, v_f, v_f^* \) are the values that solve (6) and (7) for \( q_d, q_d^*, x, x^* \) evaluated at \( q_d = \tilde{q}_d(m_h, m_f, K, K_f), q_d^* = \tilde{q}_d^*(m_h, m_f, K, K_f), x = \tilde{x}(m_h, m_f, K, K_f) \) and \( x^* = \tilde{x}^*(m_h, m_f, K, K_f) \).

3. The per-unit-of-capital values \( v_d, v_d^*, v_x, v_x^* \) solve (9).

4. The total values \( w_d^0, w_d^0, w_x^0, w_x^0 \) solve (3).

5. The policy functions \( \tilde{q}(q_d, x, q_d^*, x^*, m_h, m_f, K, K_f), \tilde{q}^*(q_d, x, q_d^*, x^*, m_h, m_f, K, K_f), \tilde{y}(q_d, x, q_d^*, x^*, m_h, m_f, K, K_f) \) solve (6) and (7).

6. The policy functions \( \tilde{q}_d(m_h, m_f, K, K_f), \tilde{q}_d^*(m_h, m_f, K, K_f), \tilde{x}(m_h, m_f, K, K_f), \tilde{x}^*(m_h, m_f, K, K_f) \) solve (9).

7. The policy functions \( \tilde{m}_h(m_h^0, m_f^0, K, K_f), \tilde{m}_f(m_h^0, m_f^0, K, K_f) \) solve (3).

3 Merger Activity at a given tariff level

This section discusses the factors driving merger decisions at any given tariff level. Having determined these, I will examine the effects of trade liberalization in the following section.

The Single Period Case

Let us begin by characterizing the version of this model which consists of a single-period. The single-period version is equivalent to a static framework. It is also equivalent to the case where all firms are completely myopic, that is, where the discount factor is equal to zero. This version is analytically more tractable than the multi-period version, and the analysis presented in this section allows me to illustrate the key forces driving merger decisions in this model.

When analyzing the single period case, \( K \) is normalized to 1 for notational convenience. It is noted that in the single-period version of the model, \( K \) does not change. With \( K = 1 \), \( m_h \) and \( m_f \) represent the dominant firm’s capital levels as well as shares.

Let us begin by noting that it is not possible to simultaneously have \( q_d^* > 0 \) and \( x > 0 \) at the equilibrium when \( m_h = m_f \). The fact that, in any country, by selling goods that have been locally
produced (denoted by $q_d$ and $x^*$), the dominant firm can avoid paying the tariff, $t$, which it must pay per unit exported (denoted by $q_d^*$ and $x$) drives this result.

I then proceed to show that a perfectly competitive industry structure is an absorbing state, as stated in Proposition 1, the proof of which follows from Lemma 1.\footnote{This result is similar to that obtained by Gowrisankaran and Holmes (2004) for the closed economy case.}

**Lemma 1:** A dominant firm with either $m_h > 0$ or $m_f > 0$, or both, has $MR_d < p$, $MR_x < p$, $MR_d^* < p^*$, and $MR_x^* < p^*$ and hence, $q_d < q$, $x < y$, $q_d^* < q^*$ and $x^* < y^*$.

**Proof:** See Appendix (Part I).

Lemma 1 shows that, given the industry price, the fringe firms always sell more consumption goods per unit of capital than the dominant firm in each market. This follows from the fact that the fringe firms set price to marginal cost in each market whereas the dominant firm sets marginal revenue to marginal cost.\footnote{A more detailed interpretation of Lemma 1 is included in the Appendix (Part 1).} A direct implication of Lemma 1 is that the average value per unit of capital is higher for the fringe firms than for the dominant firm. That is,

$$v_d < v_h, \quad v_d^* < v_h^*, \quad v_x < v_f, \quad v_x^* < v_f^* \quad (10)$$

**Proposition 1:** The equilibrium merger policy functions $\tilde{m}_h(m_h^0, m_f^0, K, K_f)$ and $\tilde{m}_f(m_h^0, m_f^0, K, K_f)$ satisfy $\tilde{m}_h(0, 0, K, K_f) = \tilde{m}_f(0, 0, K, K_f) = 0$.

**Proof:** For the case of $m_h^0 = m_f^0 = 0$, the dominant firm’s problem, (3) is given by:

$$\max_{m_h, m_f} m_h(v_d + v_d^*) + m_f(v_x + v_f^*) - m_h(v_h + v_h^*) - m_f(v_f + v_f^*)$$

From (10) it follows that $\tilde{m}_h(0, 0, K, K_f) = \tilde{m}_f(0, 0, K, K_f) = 0$ is the unique solution to the dominant firm’s problem. \footnote{In this model, industry concentration is influenced by internal investment decisions of firms. This feature is similar to that of Kydland (1979).}

Proposition 1 can be explained as follows. The proportion of the industry-specific capital stock that is owned by the dominant firm is affected by the following two factors: the merger decisions made in the first stage within each period and the investment decisions made in the second stage.\footnote{In this model, industry concentration is influenced by internal investment decisions of firms. This feature is similar to that of Kydland (1979).} In the merger stage, in order to buy a fringe firm, the dominant firm must pay the value that the fringe firm would get post-merger, if the fringe firm decided to remain outside the merger. Since the dominant firm earns less per unit of capital than each fringe firm, as shown by (10), it has to pay the fringe firm a higher price than it can earn per unit of capital. A large dominant firm might still find it profitable to buy capital
since, by undertaking the merger, it can raise the price of the consumption good and thereby increase
the value of all the units of capital owned by itself before the merger. However, for a dominant firm with
very small pre-merger capital shares in both countries buying capital necessarily engenders a loss. This
is the "free rider" effect at work and explains why no merger is realized when the industry is perfectly
competitive. Also, by Lemma 1, we have that the dominant firm invests at a lower rate than the fringe
firm. Thus, the capital shares of the dominant firm in both countries are necessarily lower subsequent to
the investment stage than they were subsequent to the merger stage. This, together with the fact that
no mergers occur when the industry is perfectly competitive to begin with, implies that once the capital
shares of the dominant firm in both countries approach zero, they remain at this level over time.

The paper then proceeds to examine what happens when the dominant firm has a strictly positive
pre-merger capital share in at least one of the countries. Given a single closed economy, small mergers
are always profitable in the single period model.\footnote{Refer to Proposition 2 of Gowrisankaran and Holmes (2004).} In the open economy case, however, the dominant firm does not always have the incentive to undertake positive mergers in any given country. In order to understand the dominant firm’s incentive to merge, the marginal benefit from buying a unit of Home capital is computed as follows. The Home-merger-Marginal-Benefit is defined to be the slope of the dominant firm’s merger choice problem, (3), with respect to its Home capital share, $m_h$.

\[
\text{Home-merger Marginal Benefit} \\

\equiv A_h - (m_h - m_h^0) \frac{d(v_h + v_h^*)}{dm_h} - (m_f - m_f^0) \frac{d(v_f + v_f^*)}{dm_h}
\]

(11)

where

\[
A_h \equiv v_d + v_d^* - v_h - v_h^* + m_h \frac{d(v_d + v_d^*)}{dm_h} + m_f \frac{d(v_x + v_x^*)}{dm_h}
\]

Let us examine the components of $A_h$. If the dominant firm buys one more unit of Home capital, it earns
a profit of $v_d + v_d^*$ on this new unit, since this new unit of capital can be used to produce and sell goods in
both Home and Foreign markets. Buying the capital from the Home fringe firm costs $v_h + v_h^*$. Together,
the term $(v_d + v_d^* - v_h - v_h^*)$ represents the "free-rider effect". That is, this term shows the incentive of the
dominant firm to not buy capital from the fringe firms since, as shown by (10), $(v_d + v_d^* - v_h - v_h^*) < 0$.
Acquiring this new unit of capital drives up the profit per unit of capital that the dominant firm takes
into the output/investment stage in each market: $(\frac{d}{dm_h} (v_d + v_d^*))$ and $(\frac{d}{dm_h} (v_x + v_x^*))$. This is due to the
market power gained by the dominant firm, which allows it to cut production and drive up the industry
price. This is labeled the "market power effect" inducing the dominant firm to buy capital from the fringe
firms. Even though, it follows from Lemma 1 that the average value per unit of capital is higher for the
fringe firm than for the dominant firm, it can be shown that the marginal benefit of transferring one unit
of capital from the fringe to the dominant firm is higher than the fringe value of this capital, that is, \( A_h > 0 \). The last two terms in (11) illustrate how buying a unit of Home capital allows the dominant firm to influence the price of capital at Home and in Foreign. This last term, \( \left( m_f - m_f^0 \frac{d(v_f + v_f^*)}{dm_h} \right) \), is labeled the "cross-border effect".

Similarly, the Foreign-merger-Marginal-Benefit is defined to be the slope of the dominant firm’s merger choice problem, (3), with respect to its Foreign capital share, \( m_f \).

\[
\text{Foreign-merger Marginal Benefit} = A_f - (m_h - m_h^0) \frac{d(v_h + v_h^*)}{dm_f} - (m_f - m_f^0) \frac{d(v_f + v_f^*)}{dm_f} \tag{12}
\]

where

\[
A_f \equiv v_x + v_x^* - v_f - v_f^* + m_h \frac{d(v_d + v_d^*)}{dm_f} + m_f \frac{d(v_x + v_x^*)}{dm_f}
\]

By similar reasoning as to why \( A_h > 0 \), it follows that \( A_f > 0 \).

The dominant firm simultaneously decides how much capital to buy/sell in both countries. By comparing (11) and (12), it follows that when \( m_h^0 > m_f^0 \left( m_h^0 < m_f^0 \right) \), the effect of \( \frac{d(v_h + v_h^*)}{dm_h} \left( \frac{d(v_f + v_f^*)}{dm_f} \right) \) outweighs that of \( \frac{d(v_f + v_f^*)}{dm_f} \left( \frac{d(v_d + v_d^*)}{dm_h} \right) \) on the dominant firm’s merger decisions in the two countries. This is labeled the "size effect", alluding to the fact that the dominant firm’s future merger decisions are dependent on the relative sizes of its existing capital shares across the two countries. Further discussion about the "free-rider", "market power", "size" and "cross-border" effects are postponed until the following section, where numerical examples are used to illustrate the mechanisms by which they affect merger outcomes in this model.

In the following analysis, it is shown that, unlike for the closed economy, it might be optimal for the dominant firm to sell capital, in the single period case, in any one of the countries or simultaneously in both countries depending on the signs and values of \( \frac{dp}{dm_h}, \frac{dp}{dm_h}, \frac{dp}{dm_f} \) and \( \frac{dp}{dm_f} \). This section is restricted to illustrating the different possibilities in terms of equilibrium merger patterns that might emerge in this model. The intuitions behind these results are further developed in the following section with the help of numerical examples.

Before proceeding, for notational convenience, let us define the following.

\[
B_h \equiv \frac{d}{dm_h} (v_h + v_h^*), \quad B_f \equiv \frac{d}{dm_f} (v_h + v_h^*)
\]

\[
C_h \equiv \frac{d}{dm_h} (v_f + v_f^*), \quad C_f \equiv \frac{d}{dm_f} (v_f + v_f^*)
\]

\( See\ Lemma\ 2\ in\ the\ Appendix\ (Part\ II)\ for\ a\ formal\ proof.\)
By the envelope theorem, it can be shown that
\[ \frac{dv}{dm} = q \frac{dp}{dm} \]
and
\[ \frac{dv'}{dm'} = y \frac{dp'}{dm'} \].

It follows that \( \text{sign} (B_h) = \text{sign} (C_h) \) and \( \text{sign} (B_f) = \text{sign} (C_f) \). Depending on the values of \( \frac{dp}{dm} \), \( \frac{dp'}{dm'} \), \( \frac{dp}{dm} \), and \( \frac{dp'}{dm'} \), there arise four possible cases.

Table 1

<table>
<thead>
<tr>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_h)</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>(B_f)</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>

There exist functional forms of demand and supply for which all of the above cases are possible. In the following section, a numerical example is presented using constant elasticity demand and supply where there exist ranges of \( m_h \) and \( m_f \) for which each of the cases A-D occur.

**Proposition 2:** The dominant firm sells capital in both countries iff Case D occurs and the following holds:

\[ (m^0_h - m_h) > \max \{0, \max \{-D_h, -D_f\}\} \]  \hspace{1cm} (13)

**Proof:** See Appendix (Part III).

Proposition 2 determines the conditions under which it is optimal for the dominant firm to sell capital in both countries simultaneously. This occurs when, by selling capital in one country, the dominant firm raises the price of capital in the other country. This is only possible given Case D. Moreover, Proposition 2 shows that it is only when the dominant firm sells a sufficiently large amount of capital in one of the countries that it is able to sufficiently raise the price of capital in the other, such that it becomes lucrative for the dominant firm to start selling capital in the other country.

It is not possible to obtain Case D if \( \frac{dp}{dm_h} > 0 \), \( \frac{dp'}{dm_h} > 0 \), \( \frac{dp}{dm_f} > 0 \) and \( \frac{dp'}{dm_f} > 0 \). In this case, the last two terms of (11) and of (12) illustrate the monopsony power of the dominant firm: that is, as the dominant firm buys capital in any country, it drives up the price of capital in both countries. If zero mergers are realized in both countries, the monopsony effect is zero. Thus, a small merger in Home and/or in Foreign is always better than zero mergers in both countries. Also, it is never optimal for the dominant firm to sell off capital in Home (Foreign) if there is a sell-off in Foreign (Home), since then the monopsony power effect would be non-negative.

**Proposition 3:** The dominant firm buys capital in both countries iff
(i) Given Case A, the following hold:

\[(m_h - m^0_h) \in (0, \min \{D_h, D_f\})\]  \hspace{1cm} (14)

and

\[(m_f - m^0_f) \in \left(0, \min \left\{\frac{A_h}{C_h}, \frac{A_f}{C_f}\right\}\right)\]  \hspace{1cm} (15)

(ii) Given Case B, the following hold:

\[(m_h - m^0_h) \in (0, D_h)\]  \hspace{1cm} (16)

and

\[(m_f - m^0_f) \in \left(0, \frac{A_h}{C_h}\right)\]  \hspace{1cm} (17)

(iii) Given Case C, the following hold:

\[(m_h - m^0_h) \in (0, D_f)\]  \hspace{1cm} (18)

and

\[(m_f - m^0_f) \in \left(0, \frac{A_f}{C_f}\right)\]  \hspace{1cm} (19)

(iv) Given Case D, the following hold:

\[(m_h - m^0_h) > \max \{0, \max \{D_h, D_f\}\}\]  \hspace{1cm} (20)

**PROOF:** See Appendix (Part IV).

Unlike the scenario where the dominant firm sells capital in both countries, as presented by Proposition 2, it might be optimal for the dominant firm to buy capital in both countries given any of Cases A - D. For \(\frac{dp}{dm_h} > 0, \frac{dp^*}{dm_h} > 0, \frac{dp}{dm_f} > 0\) and \(\frac{dp^*}{dm_f} > 0\), Proposition 3(i) implies that if the dominant firm is buying capital in both countries, the mergers in each country must be sufficiently small. This is because, by buying capital in one country, the dominant firm increases the price of capital in both. Thus, as capital is bought in any one country, it becomes more expensive to buy more capital in both countries. On the hand, for \(\frac{dp}{dm_h} < 0, \frac{dp^*}{dm_h} < 0, \frac{dp}{dm_f} < 0\) and \(\frac{dp^*}{dm_f} < 0\), Case D is obtained. For Case D, Proposition 3(iv) implies that if the dominant firm is buying capital in both countries, the mergers in each country must be sufficiently large. This is because, by buying enough capital in any one country, the dominant firm is able to decrease the price of capital in the other.

Having determined the conditions under which the dominant firm either buys or sells capital simultaneously in both countries, I ask whether the dominant firm ever finds it optimal to buy capital in any
one country whilst selling capital in the other. Without loss of generality, I focus on the case where the 
dominant firm buys capital in Home and sells capital in Foreign. By symmetry, the pattern of mergers 
and demergers is reversed if the conditions given in Proposition 4 corresponding to Home are applied to 
Foreign.

**Proposition 4:** The dominant firm buys capital in Home and sells in Foreign iff

(i) Given Case A, the following hold:

\[(m_h - m_h^0) \in (\max \{0, D_f\}, D_h)\]  \hspace{1cm} (21)

and

\[
\begin{cases} 
  \text{either } (m_f^0 - m_f) > \max \{0, \frac{B_f A_h - B_h A_f}{B_f C_h - B_h C_f}\} \quad \text{and} \quad \left(\frac{C_h}{B_h} - \frac{C_f}{B_f}\right) > 0 \\
  \text{or } (m_f^0 - m_f) \in \left(0, \frac{B_f A_h - B_h A_f}{B_f C_h - B_h C_f}\right), \quad \left(\frac{C_h}{B_h} - \frac{C_f}{B_f}\right) < 0 \quad \text{and} \quad \left(\frac{A_h}{B_h} - \frac{A_f}{B_f}\right) < 0
\end{cases}\]  \hspace{1cm} (22)

(ii) Given Case B, (14) is satisfied and the following holds:

\[
(m_f^0 - m_f) > -\frac{A_f}{C_f}\]  \hspace{1cm} (23)

(iii) Given Case C, (20) is satisfied.

(iv) Given Case D, the following hold:

\[(m_h - m_h^0) \in (\max \{0, D_f\}, D_h)\]  \hspace{1cm} (24)

and

\[
\begin{cases} 
  \text{either } (m_f^0 - m_f) > \max \{0, \frac{B_f A_h - B_h A_f}{B_f C_h - B_h C_f}\} \quad \text{and} \quad \left(\frac{C_f}{B_f} - \frac{C_h}{B_h}\right) > 0 \\
  \text{or } (m_f^0 - m_f) \in \left(0, \frac{B_f A_h - B_h A_f}{B_f C_h - B_h C_f}\right), \quad \left(\frac{C_f}{B_f} - \frac{C_h}{B_h}\right) < 0 \quad \text{and} \quad \left(\frac{A_h}{B_h} - \frac{A_f}{B_f}\right) < 0
\end{cases}\]  \hspace{1cm} (25)

**Proof:** See Appendix (Part V).

Proposition 4 determines the conditions under which the dominant firm buys capital in one country and 
sells capital in the other. For \(\frac{\partial p}{\partial m_h} > 0, \frac{\partial p^*}{\partial m_h} > 0, \frac{\partial p}{\partial m_f} > 0\) and \(\frac{\partial p^*}{\partial m_f} > 0\), Case A is obtained. Proposition 
4(i) can be explained as follows. As the dominant firm buys Home capital, it raises the price of both 
Home and Foreign capital. The rising price of Home capital limits the dominant firm from buying all the 
capital in Home. The upper limit of the amount of Home capital bought by the dominant firm is given 
by \(D_h\) in (21). The rising price of Foreign capital makes it more profitable for the dominant firm to sell 
Foreign capital. The amount of Foreign capital sold depends on the relative magnitudes of \(A_h, A_f, B_h,\)
$B_f$, $C_h$ and $C_f$, as shown by (22). On the other hand, for $\frac{dp}{dm_h} < 0$, $\frac{dp^*}{dm_h} < 0$, $\frac{dp}{dm_f} < 0$ and $\frac{dp^*}{dm_f} < 0$, Case D is obtained. Intuitively, Proposition 4(iv) can be explained as follows. As the dominant firm buys Home capital, it decreases the price of both Home and Foreign capital. The dominant firm wishes to take advantage of this low price of Home capital. It requires funds in order to purchase more Home capital. The amount of Foreign capital sold depends on the relative magnitudes of $A_h$, $A_f$, $B_h$, $B_f$, $C_h$ and $C_f$, as shown by (25).

It is noted that the slopes of the Home – merger Marginal Benefit with respect to $m_h^0$ and $m_f^0$ are given by $\left[\frac{du_h}{dm_h} + \frac{dv_h}{dm_h}\right]$ and $\left[\frac{du_f}{dm_h} + \frac{dv_f}{dm_h}\right]$ and thus, are determined by the values of $\frac{dp}{dm_h}$ and $\frac{dp^*}{dm_h}$. Similarly, the slopes of the Foreign – merger Marginal Benefit with respect to $m_h^0$ and $m_f^0$ are given by $\left[\frac{du_h}{dm_f} + \frac{dv_h}{dm_h}\right]$ and $\left[\frac{du_f}{dm_f} + \frac{dv_f}{dm_f}\right]$, which depend on the values of $\frac{dp}{dm_f}$ and $\frac{dp^*}{dm_f}$.

**The Multi-Period Case**

An infinite horizon scenario is considered.

As in the single period case, an industry that begins at perfect competition remains perfectly competitive over time. The proof is very similar to that of Proposition 1, and therefore, is not included.

The main point of departure from the single period case is that in the multi-period case, the dominant firm has a significantly greater incentive to sell capital. This has been shown for the single closed economy model (see Proposition 5(ii), Gowrisankaran and Holmes (2004)). For the single closed economy case, as $\beta \to 0$, the dominant firm always buys capital. However, Proposition 5(ii) of Gowrisankaran and Holmes (2004) shows that there exists a threshold value of $\beta$, less than 1, above which the dominant firm sells capital. This can be explained as follows. A merger in this model has two effects: (i) The dominant firm gains market power by raising the price of the consumption good. (ii) Subsequent to a merger, therefore, each unit of capital earns more, which has the effect of raising the price of capital in the next period. If fringe firms are myopic, they only take into consideration the first effect whilst making their current output and investment decisions. If the fringe firms are forward-looking, they also take into account the second effect and therefore, have an incentive to produce and invest at a higher rate. If the industry is close to being monopolized, so that the fringes in each country are negligibly small, or if the discount rate is very low, indicating that fringe firms do not attach a high weight to the future, then the dominant firm does not need to worry about this aspect of fringe behavior. Otherwise, to discourage such behavior of the fringe, the dominant firm has an additional incentive to sell capital as compared to the single period scenario. This is the "dynamic effect".

There is no reason for the above intuition to be modified in the open economy scenario. By this reasoning, I expect the dominant firm to have a greater incentive to sell capital in both countries than in the single period case. In line with my expectations, the simulation results, presented in the following
show that, at any given tariff level and for any given combination of pre-merger capital shares $m_0^h$ and $m_0^f$, the post-merger capital shares, $m_h$ and $m_f$, are lower at higher levels of $\beta$. This can be seen by comparing Figure 1(a) to 2 (a) and Figures 1(b) to 2 (b) respectively.

Having examined some of the characteristics of the equilibrium of this model, let us proceed to investigate the effects of trade liberalization on merger activity. Ideally, the effect of marginal tariff reductions on the marginal benefits of merging in Home and in Foreign should be computed by partially differentiating (11) and (12) with respect to the tariff level. However, in order to proceed with the analysis, assumptions need to be made on the signs of \( \frac{\partial^2 p}{\partial m_h \partial t} \) and \( \frac{\partial^2 p}{\partial m_f \partial t} \). Since these are not intuitively obvious, I follow a different approach by computing the equilibrium of the model at different tariff levels.\(^{19}\)

### 4 The Effect of Trade Liberalization

For the numerical analysis,\(^{20}\) constant elasticity demand and cost functions are used. This facilitates comparison between the closed and open economy results.

\[
P(Q) = Q^{-\frac{1}{\varepsilon}}
\]
\[
P(Y) = Y^{-\frac{1}{\varepsilon}}
\]
\[
c(z) = \frac{z^{\theta+1}}{\theta + 1}
\]

The results yielded by the numerical analysis support Propositions 1-4. I present below the results for the case where $K = \frac{1}{2}(K_{mon} + K_{comp})$, where $K_{mon}$ refers to the amount of capital that would be produced if the dominant firm owned all the capital stock and $K_{comp}$ refers to the amount of capital that would be produced if the industry were perfectly competitive, $\varepsilon = 4$, $\theta = 1$ and $\sigma = 0.8.\(^{21}\)$

In this section, attention is restricted to the case where the two countries are identical in all but one respect. Although the demand, production technology and capital stock are equal across countries, the dominant firm is allowed to hold different pre-merger capital shares in the two countries. Since the two countries possess equal capital stocks and given that capital is immobile across the border, neither $m_h$ nor $m_f$ can rise above 0.5. I find that merger decisions are exactly symmetric in the following sense: $\tilde{m}_f(m_0^f, m_0^h, K_f) = \tilde{m}_h(m_0^h, m_0^f, K, K_f)$. This is not surprising since the two countries have identical demand and capital stocks. The results are categorized into the following scenarios: (i) $\beta \to 0$, (ii) $\beta \in [\bar{\beta}, 1)$, (iii)

\(^{19}\)I follow the method used by Gowrisankaran and Holmes (2004) to compute the equilibrium, the details of which are discussed in Appendix (Part VI).

\(^{20}\)In order to proceed with the numerical analysis the following restriction is imposed, which greatly increases the speed of convergence of the algorithm: the fringe firms are not allowed to export. Often there exist high fixed costs to exporting, such as surveying the foreign market and interacting with retail outlets in the importing country. This restriction thus reflects this feature of reality. Under this restriction, (that is, setting $v_h^*, v_f$, $q^*$ and $y$ to zero), it is straightforward to show that Lemmas 1 and 2 and Proposition 1-4 hold.

\(^{21}\)A sensitivity analysis is presented where these parameter values are varied.
Forward-looking dominant firm and myopic fringe.

**Scenario 1: \( \beta \to 0 \)**

The effect of trade liberalization on mergers when the discount factor is sufficiently close to zero is illustrated by Figures 1(a)-1(b) and Tables 2-4. Figures 1(a) and 1(b) show the effect of tariff reductions on \( m_f \) and \( m_h \) respectively for different values of \( m_f^0 \), holding \( m_h^0 \) constant at 0.26.\(^{22}\) Tables 2-4 provide a more holistic picture by varying both \( m_f^0 \) and \( m_h^0 \) simultaneously. Let us begin by analyzing Figures 1(a) and 1(b).

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**Result 1:** Tariff reduction from a prohibitive to a non-prohibitive level aligns post-merger capital shares across countries.

In Figures 1(a) and 1(b), \( t = 0.15 \) represents the lowest tariff level (upto three decimal places) at which all exports fall to zero, given the parameter values. At this tariff level, exports of the consumption good fall to zero so that Home and Foreign become two closed economies. Although there is no trade between the two countries at prohibitive tariff levels, merger activity in each country is dependent on merger activity in the other, unlike in the case of a single closed economy as studied by Gowrisankaran and Holmes (2004). This is because, in the two-country case, the dominant firm is able to finance its purchases of capital in one country by selling capital in the other.

In this model, given a prohibitive tariff, the "size" and "market power" effects become the driving forces behind the merger decisions of the dominant firm. When the dominant firm has a large pre-merger capital share in Foreign (that is, \( m_f^0 \) is high) then it has a greater incentive to buy more capital in Foreign than in Home. This is because the gain in market power as a result of buying each additional unit of Foreign capital increases the value of all the units of capital owned pre-merger by the dominant firm in Foreign. This explains why, in Figure 1(a), the dominant firm buys capital in Foreign for \( 0.24 < m_f^0 < 0.45 \). In this range of \( m_f^0 \), the dominant firm sells capital in Home to finance its purchase of capital in Foreign. It is more profitable for the dominant firm to consolidate its capital in one country, rather than

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\(^{22}\)Recall that \( m_h^0 \) can range from 0 to 0.5. We choose \( m_h^0 = 0.26 \) in Figures 1(a)-1(b) since it approaches the mid-point of this range. For merger patterns that result from \( m_h^0 \) significantly different from 0.26, please refer to Tables 2-3.
having moderate capital shares in both countries and sharing both markets with the fringe firms. For 
\( m_f^0 < 0.24 \) and given \( m_h^0 = 0.26 \), by the same reasoning, it becomes optimal for the dominant firm to 
sell capital in Foreign in order to buy capital in Home. Thus, at the prohibitive tariff, for \( m_f^0 \in (0, 0.45) \), 
when the dominant firm buys capital in one country, it sells capital in the other, as shown by Figures 
1(a) and 1(b).

An exception occurs for \( m_f^0 \in (0.45, 0.5) \). As the pre-merger Foreign capital share of the dominant 
firm approaches 0.5, the size effect is offset by the free rider effect, which causes the dominant firm to 
start selling capital in Foreign. As \( m_f^0 \to 0.5 \), the pre-merger price of the consumption good in both 
markets rises. Thus, at levels of \( m_f^0 \) close to 0.5, the price of capital is high. The dominant firm finds it 
optimal to sacrifice its market power in the consumption goods market in order to gain from the capital 
market, by selling capital in both countries at this high price.

Unlike the case of prohibitive tariffs, at non-prohibitive tariff levels the merger patterns are almost 
identical across both countries. Given the numerical example, this holds for all \( t \in [0, 0.15] \). Figures 1(a) 
and 1(b) illustrate this for \( t = 0, 0.03, 0.05 \) and 0.1, given \( m_f^0 \in (0, 0.5) \) and \( m_h^0 = 0.26 \). This is because 
total sales in the Home (Foreign) market is monotonically increasing in \( m_f \) (\( m_h \)). In Figure 1(a), there 
is a merger in Foreign at \( m_h^0 = 0.26 \) and \( m_f^0 = 0.4 \) at all non-prohibitive tariff levels. At \( t = 0.1 \), for 
example, this merger is followed by an increase in total Home sales. Similar increases in Home sales 
are noted at all non-prohibitive tariff levels. Consequently, the price of the consumption good in the 
Home market falls. This causes the value of the Home fringe firms to fall, thereby causing the price of 
Home capital to fall. This is the "cross-border effect" at work. Consequently it becomes cheaper for the 
dominant firm to buy the Home fringe firms. Thus, a merger in Foreign leads to mergers in Home. This 
has a feedback effect, by similar reasoning, on mergers in Foreign and so on. Exports, thus, act as a 
mechanism for transferring merger activity from one country to another, thereby aligning merger patterns 
across the countries. This leads to Result 2.

**Result 2:** Tariff reduction from a prohibitive to a non-prohibitive level initiates merger (or demerger) 
waves.

In Figures 1(a) and 1(b), at the prohibitive tariff, the dominant firm is content to hold moderate levels 
of post-merger capital share in any one of the countries for any given \( m_f^0 \). However, at non-prohibitive 
tariff levels, the dominant firm chooses to hold capital shares in both countries that either approach zero 
or the maximum value of 0.5. At any given \( m_f^0 \), the "cross-border effect" sets into motion a cycle or 
wave of mergers or demergers through the feedback mechanism driven by exports. For a given set of 
parameter values, when faced with two closed economies, there are no merger waves. However, opening 
up the economies generates merger (or demerger) waves simultaneously in both countries.

**Result 3:** For moderate values of \( m_h^0 \) and \( m_f^0 \), the effect of tariff reduction on post-merger capital shares

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is non-monotonic in the tariff level.

The non-monotonicity mentioned in Result 3 occurs when the tariff level jumps from a prohibitive to a non-prohibitive level. This is illustrated by Figures 1(a) and 1(b) for $m^0_f \in (0.2, 0.4)$. For this range of $m^0_f$, Figures 1(a) and 1(b) show that a reduction from a prohibitive tariff to $t = 0.1$ causes $m_f$ to fall to zero whereas a jump to free trade causes $m_f$ to rise to 0.5.

So far I have focused on the results for a given value of $m^0_h$. Now I turn to a more holistic analysis by varying both $m^0_h$ and $m^0_f$ simultaneously. Results 4-6 are illustrated by Tables 2-3, which summarize the merger decisions of the dominant firm at $t = 0$ and $t = 0.1$ respectively for different combinations of pre-merger shares in the two countries. Table 2 shows the optimal capital shares for $\beta = 0.01$ and $t = 0$.

Table 2: Optimal Capital Shares at $t = 0$

<table>
<thead>
<tr>
<th>$m^0_h$</th>
<th>$m^0_f$</th>
<th>$m^0_h$</th>
<th>$m^0_f$</th>
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<tbody>
<tr>
<td>0.04</td>
<td>0.04</td>
<td>0.003</td>
<td>0.38,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>0.46</td>
<td>0.05,</td>
<td>0.38</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.38</td>
<td></td>
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</tbody>
</table>

Table 3 shows the optimal capital shares for $\beta = 0.01$ and $t = 0.1$.

Table 3: Optimal Capital Shares at $t = 0.1$

<table>
<thead>
<tr>
<th>$m^0_h$</th>
<th>$m^0_f$</th>
<th>$m^0_h$</th>
<th>$m^0_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>0.04</td>
<td>0.009</td>
<td>0.0</td>
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<tr>
<td></td>
<td></td>
<td>0.009</td>
<td>0.5</td>
</tr>
<tr>
<td>0.46</td>
<td>0.0</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>0.009</td>
<td></td>
</tr>
</tbody>
</table>

**Result 4:** As $m^0_h \to 0$ and $m^0_f \to 0$, tariff reduction reduces the dominant firm’s incentive to undertake mergers.

Tables 2 and 3 together show that for $m^0_h = m^0_f = 0.04$, trade liberalization does not encourage the dominant firm to undertake mergers in either country. Going from $t = 0.1$ to free trade, the post-merger capital shares of the dominant firm in both countries falls from 0.01 to 0.003. As $m^0_h \to 0$ and $m^0_f \to 0$, at any given tariff level, the "size effect" is negligibly small and outweighed by the "free-rider effect". Starting from $m^0_h = m^0_f = 0.04$, the dominant firm does not wish to buy capital in either one of the countries or in both countries simultaneously. This is because, as it buys capital, the dominant firm raises the price of capital. As $m^0_h \to 0$ and $m^0_f \to 0$, this example illustrates that, at non-prohibitive tariff
levels, total sales in the Home (Foreign) market is decreasing in \( m_f (m_h) \). At \( t = 0.1 \), for example, going from \((m_f = 0, m_h = 0)\) to \((m_f = 0.04, m_h = 0)\), total Home sales, \( Q \), fall by 6\%. Consequently, the values per unit of capital of Home and Foreign fringe firms are increasing in \( m_f \) and \( m_h \) respectively as \( m_h \rightarrow 0 \) and \( m_f \rightarrow 0 \). Recall that the post-merger values of the fringe firms represent the price that the dominant firm must pay to acquire each unit of capital. Starting from \( m_h^0 = m_f^0 = 0.04 \), buying capital in both countries simultaneously, thus, proves to be infeasible for the dominant firm. If either \( m_h^0 \) or \( m_f^0 \) were large, then the gain in market power through the "size effect" could offset the increasing cost of buying capital. This is illustrated by Result 5 below. The gain in market power (through a higher post-merger price of the consumption good) is applicable only to the pre-merger stock of capital belonging to the dominant firm. In the case of \( m_h^0 \rightarrow 0 \) and \( m_f^0 \rightarrow 0 \), this gain is too small to outweigh the increasing cost of buying capital. The only way that the rising price of capital does not deter the dominant firm is if it can finance its purchases of capital in one country by selling capital in the other. For the case where \( m_h^0 \rightarrow 0 \) and \( m_f^0 \rightarrow 0 \), this possibility is also ruled out for the dominant firm.

As tariff is reduced and the volume of exports grows, the capital owned by the dominant firm which is located in Home essentially competes against that located in Foreign. If the dominant firm buys more Home (Foreign) capital, then it hurts its Foreign (Home) capital holdings by taking away the sales of the Foreign (Home) capital through exports. At \( t = 0.1 \), for example, moving from \( m_h^0 = m_f^0 = 0.04 \) to \( m_h^0 = 0.24 \) and \( m_f^0 = 0.04 \left( \text{or } m_h^0 = 0.04 \text{ and } m_f^0 = 0.24 \right) \) leaves exports unchanged at zero. In contrast, at \( t = 0 \), moving from \( m_h^0 = m_f^0 = 0.04 \) to \( m_h = 0.24 \) and \( m_f = 0.04 \left( \text{or } m_h = 0.04 \text{ and } m_f = 0.24 \right) \) increases exports from Home (Foreign) to Foreign (Home) from zero to 0.635, which causes per unit value of the Foreign (Home) capital held by the dominant firm to fall by 69.23\%. This loss outweighs any gain in market power resulting from the merger. Thus, at lower tariffs, there is an additional factor reducing the incentive of the dominant firm to undertake mergers.

In fact, in this example, at any given tariff level, the revenue from selling capital is greater than the gain in market power due to merger. As implied by the above discussion, the merger-induced gain is smaller at lower tariffs. Thus, the dominant firm sells more capital at \( t = 0 \) than at \( t = 0.1 \), so that post-merger capital share in both countries fall by 64.943\% as a result of trade liberalization. However, this pro-competitive effect of trade liberalization only occurs for a small range of \( m_h^0 \) and \( m_f^0 \) close to zero and the magnitude of this effect is very small compared to the anti-competitive effect of trade liberalization that occurs for other combinations of \( m_h^0 \) and \( m_f^0 \), as shown below.

**Result 5:** As either \((m_h^0 \rightarrow 0 \text{ and } m_f^0 \rightarrow 0.5)\) or as \((m_h^0 \rightarrow 0.5 \text{ and } m_f^0 \rightarrow 0)\), at non-prohibitive tariff levels, tariff reduction increases the incentive to merge simultaneously in both countries.

The anti-competitive effect, as mentioned in Result 5, is driven by a combination of the "size " and "cross-border" effects. Consider the case where \( m_h^0 \rightarrow 0 \) and \( m_f^0 \rightarrow 0.5 \). The dominant firm has a greater incentive to buy capital in Foreign than in Home, due to the "size effect". This example il-
illustrates, that the per unit value of Home (Foreign) capital is monotonically decreasing (increasing) in $m_f^0$. At $t = 0$, a purchase of Foreign capital as represented by a move from $\left( m_h^0 = 0.04, \; m_f^0 = 0.46 \right)$ to $\left( m_h = 0.04, \; m_f = 0.5 \right)$ results in $(v_d + v_d^*)$ falling by 0.003 and $(v_x + v_x^*)$ rising by 0.009 causing $(v_d + v_d^* + v_x + v_x^*)$ to rise by approximately 2%. That is, the gain in market power by buying Foreign capital applicable to all units of Foreign capital owned pre-merger by the dominant firm (the "size effect") outweighs the harm caused by this merger to the value of Home capital as a result of increased post-merger exports from Foreign to Home. Thus, the "size effect" induces the dominant firm to buy Foreign capital.

As illustrated by Result 2, once the dominant firm starts buying capital in Foreign, the "cross-border effect" ensures that a wave of mergers follows simultaneously in both countries. The result is that the dominant firm buys all the fringe firms and becomes an international monopoly. The greater the volume of exports, the greater the gain from buying a unit of Foreign capital. Besides gaining market power in Foreign, the dominant firm also gains market share in Home by increasing exports post-merger. That is, the increase in $(v_x + v_x^*)$ as a result of buying Foreign capital is greater the lower the tariff level. At $t = 0$, the rise in $(v_x + v_x^*)$ due to a move from $\left( m_h^0 = 0.04, \; m_f^0 = 0.46 \right)$ to $\left( m_h = 0.04, \; m_f = 0.5 \right)$ is given by 0.002 (as opposed to 0.009 for $t = 0$). This makes it easier for the merger wave to be initiated at lower tariff levels. This is illustrated in Figures 1(a) and 1(b) where the merger wave is initiated at successively lower levels of pre-merger capital shares as the tariff level is reduced. This also explains the merger pattern illustrated in Tables 2 and 3. Starting from $m_h^0 = 0.04$ and $m_f^0 = 0.46 \left( m_h^0 = 0.46 \text{ and } m_f^0 = 0.04 \right)$ trade liberalization from $t = 0.1$ to $t = 0$ causes post-merger capital shares of the dominant firm to rise simultaneously in both countries with $\tilde{m}_h \left( \hat{m}_f \right)$ rising from 0.009 to 0.38 and $\hat{m}_f \left( \tilde{m}_h \right)$ rising from 0 to 0.5. Thus, trade liberalization has a strong anti-competitive effect if the pre-merger capital shares of the dominant firm in the two countries differ significantly from each other.

**Result 6:** As $m_h^0 \rightarrow 0.5$ and $m_f^0 \rightarrow 0.5$, at non-prohibitive tariff levels, the effect of tariff reduction is negligible.

As $m_h^0 \rightarrow 0.5$ and $m_f^0 \rightarrow 0.5$, the "size effect" dominates. This ensures that the dominant firm becomes an international monopoly, regardless of the tariff level, provided that the tariff level is non-prohibitive. This is illustrated in Tables 2 and 3, where starting at $\left( m_h^0 = 0.46, \; m_f^0 = 0.46 \right)$ the dominant firm buys all the fringe capital in both countries to reach $(\tilde{m}_h = 0.5, \; \hat{m}_f = 0.5)$ regardless of whether $t = 0.1$ or $t = 0$.

Table 4 summarizes the effect of tariff reductions for different combinations of pre-merger shares in the two countries.

**Table 4: Optimal Capital Shares in response to Trade Liberalization**
Given this set of parameters, Results 1-6 hold qualitatively for all $\beta \in [0, 0.1)$. Scenario 2 discusses the effect of raising $\beta$ above 0.1 in order to examine the scenario where all firms in the industry attach a greater weight to future profits than in Scenario 1.

Scenario 2: $\beta \in [\tilde{\beta}, 1)$

As discussed in the previous section, the dominant firm has less incentive to undertake mergers (and more incentive to undertake demergers) when firms are more forward-looking. At any given non-prohibitive $t$ and $m^0_h \left(m^0_f \right)$, as $\beta$ is increased, the threshold values of $m^0_f$ and $m^0_h$ at which the merger wave occurs becomes larger. This leads to Result 7.

**RESULT 7:** There exists $\tilde{\beta} \in (0, 1]$ such that at non-prohibitive tariff levels, for $\beta > \tilde{\beta}$, it holds that $m_h \to 0$ and $m_f \to 0$ for all $m^0_h \in [0, 0.5]$ and $m^0_f \in [0, 0.5]$.

Given my parameter values, $\tilde{\beta} = 0.1$. As $\beta$ is increased above $\tilde{\beta}$, the threshold value of $m^0_f \left(m^0_h \right)$ at which the merger wave occurs tends to 0.5, so that at any given non-prohibitive tariff level there occur demerger waves simultaneously in both countries. This is illustrated by Figures 2(a) and 2(b) for $\beta = 0.1$. The demerger patterns are very similar for all $\beta \in [0.1, 1)$.

For a single closed economy, Gowrisankaran and Holmes (2004) show that the dominant firm has less incentive to undertake mergers (and more incentive to undertake demergers) when firms are more forward-looking, as in the two-country model. However, in contrast to my results for non-prohibitive tariffs, in the single closed economy case, the dominant firm’s post-merger capital share remains strictly
positive for strictly positive pre-merger capital shares as $\beta$ is increased. In this two-country framework, as illustrated by Figures 2(a) and 2(b), the dominant firm's post-merger capital shares in both countries tend to zero regardless of the pre-merger capital shares for all $\beta \in [0, 1)$.

The higher the value of $\beta$, the more likely that the dominant firm will sell capital. For all $\beta \in [0, 1)$ and $m_f (m_h) > 0$, the price of the consumption good in Home (Foreign) is monotonically decreasing in $m_f (m_h)$. Thus, the value per unit of capital of the Home (Foreign) fringe firms is monotonically decreasing in $m_f (m_h)$ for all $m_f (m_h) > 0$. As the dominant firm sells capital in one country, it increases the price of capital in the other country. This causes the dominant firm to sell capital in the other country. Subsequent to the initial sale of capital by the dominant firm in either country, there, thus, occurs a wave of demergers simultaneously in both countries, driven by the "cross-border effect".

It is noted that $\beta$, as defined in Result 7, is decreasing in the elasticity of demand, $\varepsilon$. This holds for all $\varepsilon \in (1.5, 1000)$. For instance, as $\varepsilon$ falls from 4 to 3, $\beta$ rises from 0.1 to 0.35.\(^{23}\) Thus, in this model, the incentive to merge is increased as the elasticity of demand is reduced. This is in contrast to the result obtained by Gowrisankaran and Holmes (2004) for the single closed economy case. In their model, the post-merger price of the consumption good is increasing in the elasticity of demand. Thus, the post-merger quantity produced by the non-participating fringe firms is decreasing in the elasticity of demand. This factor induces the dominant firm to buy less capital from the fringe at lower elasticity of demand. On the other hand, in this two-country framework, a merger in Home leads to greater exports to Foreign. The lower the elasticity of demand, the greater the volume of merger-induced exports. Assuming identical $\varepsilon$ across countries, a lower elasticity of demand increases the post-merger profits from sales in Foreign at the expense of sales in Home. Given these parameter values, the former effect outweighs the latter, explaining why $\bar{\beta}$ is decreasing in $\varepsilon$.

It is also noted that $\bar{\beta}$ is increasing in the elasticity of supply, $\theta$. For instance, to achieve the same increase in $\bar{\beta}$ as above (from 0.1 to 0.35), $\theta$ must fall from 1 to 0.5.

Finally, I note that $\bar{\beta}$ is decreasing in the parameter characterizing the fixed proportions technology, $\sigma$. In order to achieve the same increase in $\bar{\beta}$ as above (from 0.1 to 0.35), $\sigma$ must fall from 0.8 to 0.7. These findings are similar to those of Gowrisankaran and Holmes (2004) for the single closed economy case.

### Scenario 3: Forward-looking dominant firm and myopic fringe

So far I have assumed that all firms within the industry are equally forward-looking with a common discount factor, $\beta$. In Scenario 3, this assumption is relaxed. In reality, a dominant firm often has deeper pockets than its followers and, being the market leader, has more information about its own future strategies than do the followers. In such cases, due to the differential of availability of information and/or

\(^{23}\)In this paper, I restrict my attention to values of $\varepsilon$ strictly greater than 1. This is because, given inelastic demand functions, the value of fringe firms in the neighbourhood of an international monopoly approach infinity.
credit across firms, the dominant firm might be in a position to attach a greater weight to its future
profits than do the competitive fringe firms. Therefore, I examine the scenario where the dominant firm
is more forward-looking than the fringe firms. The discount factor of the dominant firm is given by \( \beta_d \)
and that of the fringe firms is given by \( \beta_f \).

**Figure 3(a):** Effect of trade liberalization on \( m_f \) for \( \beta_d = 0.1, \beta_f = 0.01 \) and \( m_h^0 = 0.26 \)

**Figure 3(b):** Effect of trade liberalization on \( m_h \) for \( \beta_d = 0.1, \beta_f = 0.01 \) and \( m_h^0 = 0.26 \)

**Result 8:** There exists \( \tilde{m}_f^0 \in (0, \frac{1}{2}) \), such that for \( m_f^0 < \tilde{m}_f^0 \) tariff reduction from a prohibitive to a non-prohibitive level causes merger patterns across the two countries to diverge.

As long as all firms have the same discount factor, a tariff reduction from a prohibitive to a non-prohibitive level aligns merger patterns across both countries, as pointed out by Result 2. This result does not hold when firms differ in the degree to which they are forward-looking. Result 8 shows that when the pre-merger capital share is sufficiently small in any one country, a move from a prohibitive to a non-prohibitive tariff fails to align merger patterns across Home and Foreign. This is illustrated by Figures 3(a) and 3(b) for \( m_f^0 < \tilde{m}_f^0 = 0.2 \).

Recall that, when all firms are equally forward-looking, the main source of discrepancy between the merger decisions taken by a myopic and forward looking industry is the behaviour of the fringe firms. This would suggest that Scenario 3 (where the fringe firms are myopic) should be similar to Scenario 1 (where all firms are myopic). This explains why Figure 3(b) closely resembles Figure 1(b). The merger decisions in Foreign, however, do not match in Scenarios 1 and 3, as illustrated by Figures 3(a) and 1(a). The demergers that occur in Home for \( m_f^0 < \tilde{m}_f^0 = 0.2 \) cause exports to fall. This increases the value of the Foreign fringe firms. The increase in the value of each unit of capital is higher to the dominant firm than to the fringe firms, since the latter are myopic. Thus, the minimum price that fringe firms require to agree to sell their capital is lower than what the dominant firm is willing to pay for each unit of capital. For this reason, the benefit from buying each unit of capital is greater than the cost to the dominant firm. This results in mergers in Foreign for \( m_f^0 < \tilde{m}_f^0 = 0.2 \).

It is noted that \( \tilde{m}_f^0 \) is decreasing in \( (\beta_d - \beta_f) \). This holds for all \( \beta_d \in (0.01, 1) \), given \( \beta_f = 0.01 \). For instance, for an increase in \( \beta_d \) from 0.1 to 0.3, \( \tilde{m}_f^0 \) falls from 0.2 to 0.16. On the other hand, for a decrease in \( \beta_d \) from 0.1 to 0.05, \( \tilde{m}_f^0 \) rises from 0.2 to 0.22. Also, \( \tilde{m}_f^0 \) is decreasing in \( \epsilon \). For a decrease in \( \epsilon \) from 0.4 to 0.3, for instance, \( \tilde{m}_f^0 \) rises from 0.2 to 0.26. On the other hand, for an increase in \( \epsilon \) from
0.4 to 0.5, \( m_{ij}^0 \) falls from 0.2 to 0.14. Also, \( m_{ij}^0 \) is decreasing in \( \theta \), the elasticity of supply. For a decrease in \( \theta \) from 1 to 0.5, for instance, \( m_{ij}^0 \) rises from 0.2 to 0.5. For an increase in \( \theta \) from 1 to 1.5, \( m_{ij}^0 \) falls from 0.2 to 0.12. According to the simulations, \( m_{ij}^0 \) is decreasing in \( \sigma \). For a decrease in \( \sigma \) from 0.8 to 0.7, \( m_{ij}^0 \) rises from 0.2 to 0.24. For an increase in \( \sigma \) from 0.8 to 0.9, \( m_{ij}^0 \) falls from 0.2 to 0.14.

5 Conclusion

A dynamic dominant-firm model with an endogenous merger process, constant returns to scale production and a two-country framework was used to examine the effects of trade liberalization on industry structure. The factors driving the results, given this setting, were as follows. Merging allows the merger participants to gain by raising prices. The free rider effect hinders the realization of mergers. The dominant firm and the fringes have different incentives to invest in the industry-specific capital stock. By buying (or selling) capital in any one of the countries, the dominant firm can manipulate the price of capital in both countries.

Moreover, the impact that trade liberalization has on merger activity depends crucially on the extent to which firms within the industry in question are forward-looking. It is well understood that economies of scale and barriers to entry must be absent for an industry to tend toward a competitive structure. This paper showed that another important factor affecting industry structure is the discount factor. For an industry spread across several countries, it was shown that a competitive structure is more likely to evolve internationally the more forward-looking the firms. This is because forward-looking fringe firms expect the dominant firm to cut production subsequent to a merger. Also, a merger raises the price of capital and signals that the dominant firm might continue to buy capital from the fringe in the future. This encourages the fringe firms to invest at a higher rate. Capital sell-offs prove to be a substitute for commitment to the dominant firm. To discourage the fringe from investing, the dominant firm uses the sale of capital as a signal to indicate that it will not raise the price of the consumption good or buy capital in the future. Once the dominant firm starts selling capital in one country, it is possible that it raises the price of capital in the other, thus making it profitable to start selling capital in the other country simultaneously. Such behavior causes the merger decisions of firms at high discount factors to induce a competitive structure, provided that the tariff level is non-prohibitive, so that the effect of trade liberalization turns out to be negligible.

At lower values of the discount factor, however, the impact of trade liberalization on industry structure was shown to be significant and dependent on the pre-merger capital shares of the dominant firm in both countries. Of particular concern to antitrust authorities are scenarios with very low discount factors where a jump to free trade pushes the industry to an international monopoly. This was shown to be possible if the pre-merger capital share is sufficiently high in any one of the countries. When the pre-merger capital share is strictly positive but significantly lower than its maximum possible value in one country and close
to its maximum in the other, the anti-competitive effect of trade liberalization is strongest. The dominant firm significantly increases its capital shares in both countries in the face of trade liberalization, even if the new tariff level remains strictly positive. On the other hand, when the pre-merger capital shares in both countries is close to zero, tariff reductions have a pro-competitive effect, that is, post-merger capital shares become lower as the tariff level falls. However, this effect is negligibly small unless the tariff jumps from a prohibitive to a non-prohibitive level. Even when it is significant, this effect only occurs for a small range of pre-merger capital shares very close to zero.

This study was not restricted to non-prohibitive tariff levels. A natural question that arises is the following: What happens when the tariff reduction is from a prohibitive to a non-prohibitive level? At prohibitive tariff levels, the dominant firm buys capital in one country whilst selling in the other. This can lead to changes in the welfare of the two countries in opposite directions. Thus, it might be desirable to create supra-national antitrust authorities to ensure that changes in industry structure are Pareto improving for both countries. It was shown that trade liberalization, in such situations, has the effect of aligning the merger patterns across both countries, provided that all firms are equally forward-looking. Thus, trade liberalization reduces the need for supra-national intervention. This result may, however, be reversed when the dominant firm is more forward-looking than the fringe firms.

The main conclusion from this study is the following. To accurately forecast merger behaviour and formulate international competition policy in the face of trade liberalization, it is essential to take into consideration the existing industry structure and the discount factors of individual firms.

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Appendix

I. Proof of Lemma 1

Proof.

With a single period, the first order condition of the Home fringe firms’ problem (6) in the output/investment stage in the Home market is reduced to

\[ P(Q) - c'(q + q^*) = 0 \] (26)
i.e. price equals marginal cost. The first order condition of the dominant firms' problem (9) in the output/investment stage in the Home market is given by:

\[ m_h P(Q) + (m_h q_d + m_f x) P'(Q) \frac{\partial Q}{\partial q_d} - m_h c'(q_d + q_d^*) = 0 \]

Equivalently,

\[ MR_d - c'(q_d + q_d^*) = 0 \]

where

\[ MR_d \equiv P(Q) + \frac{1}{m_h} (m_h q_d + m_f x) \]

\[ P'(Q) \left( m_h + (1 - m_h - m_f - K_f) \frac{\partial q}{\partial q_d} + K_f \frac{\partial y}{\partial q_d} \right) \]

with \( \left( m_h + (1 - m_h - m_f - K_f) \frac{\partial q}{\partial q_d} + K_f \frac{\partial y}{\partial q_d} \right) > 0 \).

A dominant firm with either \( 0 < m_h < 1 \) or \( 0 < m_f < 1 \) or both, has \( MR_d < p \), and hence \( q_d < q \). The method for showing \( x < y, q_d^* < q^* \) and \( x^* < y^* \), as stated in Lemma 1, is very similar to that presented above. \( \blacksquare \)

To interpret Lemma 1, consider the expression for \( MR_d \), where \( MR_d \) denotes the increase in revenue to the dominant firm from selling an additional unit of \( q_d \).

\[ MR_d = P(Q) + \frac{1}{m_h} (m_h q_d + m_f x) \]

\[ P'(Q) \left( m_h + (1 - m_h - m_f - K_f) \frac{\partial q}{\partial q_d} + K_f \frac{\partial y}{\partial q_d} \right) \]

When the dominant firm increases its sales in Home, \( q_d \), by one unit, it gets price, \( P(Q) \), for the extra unit sold. But the extra supply lowers the price on the \( (m_h q_d + m_f x) \) units it was already selling. If the firm were a monopoly and there did not exist any fringe firms selling in the Home market, then \( (m_h + (1 - m_h - m_f - K_f) \frac{\partial q}{\partial q_d} + K_f \frac{\partial y}{\partial q_d}) \) in (28) would be unity and the change in price would be \( \frac{1}{m_h} (m_h q_d + m_f x) P'(Q) \). This is the direct effect of supplying one more unit in the Home market. If there does exist a competitive fringe in either country then any increase in the dominant firm’s supply is mitigated by the fringe firms’ supply. This is the strategic part of the effect and is of the opposite sign to the direct effect. Thus \( (m_h + (1 - m_h - m_f - K_f) \frac{\partial q}{\partial q_d} + K_f \frac{\partial y}{\partial q_d}) < 1 \) for the case of a dominant firm with a competitive fringe.

**II. Lemma 2**

\[
\begin{align*}
\frac{d}{dm_h} \left( m_h (v_d + v_d^*) + m_f (v_x + v_x^*) \right) &> v_h + v_h^* \\
\frac{d}{dm_f} \left( m_h (v_d + v_d^*) + m_f (v_x + v_x^*) \right) &> v_f + v_f^* \\
\end{align*}
\]

(i) for either \( m_h > 0, m_f > 0 \), or both

(ii) for either \( m_h > 0, m_f > 0 \), or both

\[ 24 \text{Recall that } \left( m_h + (1 - m_h - m_f - K_f) \frac{\partial q}{\partial q_d} + K_f \frac{\partial y}{\partial q_d} \right) = \frac{\partial q}{\partial q_d}. \text{ Assume contradictorily that } \left( m_h + (1 - m_h - m_f - K_f) \frac{\partial q}{\partial q_d} + K_f \frac{\partial y}{\partial q_d} \right) \text{ is non-positive. This implies that as the dominant firm increases output, total output in the Home market does not increase. Thus price does not fall. Therefore, the fringe firms do not contract their outputs. However, together with the fact that the dominant firm is increasing output this implies that total output is increasing, which yields a contradiction.} \]

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Proof.

(i) Consider the fringe policy functions, $\bar{q}(q_d, x, m_h, m_f, K, K_f)$, implicitly defined by (26). Expanding (26) using (6), $K = 1$, and differentiating yields

$$\frac{\partial \bar{q}}{\partial m_h} = \frac{\partial \bar{y}}{\partial m_h} \tag{29}$$

and

$$\frac{\partial \bar{q}}{\partial q_d} = \frac{\partial \bar{y}}{\partial q_d} \tag{30}$$

For the single period case, the dominant firm value of Home production is

$$v_d + v_d^* = P(Q)q_d + P(Y)q_d^* - c(q_d + q_d^*) - tq_d^*$$

evaluated at the optimal $q_d, q_d^*$. Using the envelope theorem,

$$\frac{\partial (v_d + v_d^*)}{\partial m_h} = P'\left(q_d - q + (1 - m_h - m_f - K_f) \frac{\partial \bar{q}}{\partial m_h} + K_f \frac{\partial \bar{y}}{\partial m_h}\right)q_d$$

$$+ P'\left(q_d - q^* + (1 - m_h - m_f - K_f) \frac{\partial \bar{q}^*}{\partial m_h} + K_f \frac{\partial \bar{y}^*}{\partial m_h}\right)q_d^*$$

$$= P'(q_d + q_d^*) \left(\frac{(q - q_d + q^* - q_d^*) c''}{P'(1 - m_h - m_f - 2c'')}\right)$$
Rearranging terms and substituting for $MR_d$ and $MR^*_d$, and using (29) and (30),

\[
\frac{d}{dh} (m_h(v_d + v_d^*) + m_f(v_x + v_x^*)) = v_d + v_d^* + m_h \frac{d}{dh} (v_d + v_d^*) + m_f \frac{d}{dh} (v_x + v_x^*)
\]

\[
= pq_d + p^* q_d^* - c(q_d + q_d^*) - t q_d^* + m_h \frac{d}{dh} (v_d + v_d^*) + m_f \frac{d}{dh} (v_x + v_x^*) + v_h + v_h^* - pq - p^* q^* + t q^* + c(q + q^*)
\]

\[
= v_h + v_h^* + [MR_d q_d + MR^*_d q_d^* - c(q_d + q_d^*) - t q_d^*] - [MR_d q + MR^*_d q^* - c(q + q^*) - t q^*]
\]

The last two terms in the last line sum to a strictly positive number, since $z = q_d$ and $z^* = q_d^*$ are the unique maximizers of $[MR_d z + MR^*_d z^* - c(z + z^*) - t z^*]$, and $q \neq q_d$, $q^* \neq q_d^*$ for either $m_h > 0$, $m_f > 0$, or both. Therefore, I have the following:

\[
v_d(m_h, m_f, K_f) + v_d^*(m_h, m_f, K_f)
\]

\[
+ m_h \frac{d}{dh} (v_d(m_h, m_f, K_f) + v_d^*(m_h, m_f, K_f)) > v_h(m_h, m_f, K_f) + v_h^*(m_h, m_f, K_f)
\]

(31)

(ii) Similar to proof of (i).

III. Proof of Proposition 2

Proof.

It holds that $\tilde{m}_h(m^0_h, m^0_f, K, K_f) < m^0_h$ and $\tilde{m}_f(m^0_h, m^0_f, K, K_f) < m^0_f$ iff (11) and (12) are both negative. Thus the following two inequalities must be satisfied:

\[
(m_h - m^0_h) B_h > A_h - (m_f - m^0_f) C_h
\]

(32)

and

\[
A_f - (m_h - m^0_h) B_f < (m_f - m^0_f) C_f
\]

(33)

For $m_h - m^0_h < 0$ and $m_f - m^0_f < 0$, it is straightforward to show the following. Given Case D, (32) and (33) are satisfied iff (13) holds. It is also straightforward to show that (32) and (33) cannot be satisfied given Cases A-C.

IV. Proof of Proposition 3

Proof.

It holds that $\tilde{m}_h(m^0_h, m^0_f, K, K_f) > m^0_h$ and $\tilde{m}_f(m^0_h, m^0_f, K, K_f) > m^0_f$ iff (11) and (12) are both positive. Thus the following two inequalities must be satisfied:

\[
(m_h - m^0_h) B_h < A_h - (m_f - m^0_f) C_h
\]

(34)
and

\[ A_f - (m_h - m_h^0) B_f > (m_f - m_f^0) C_f \quad (35) \]

For \((m_h - m_h^0) > 0 \) and \((m_f - m_f^0) > 0 \), it is straightforward to show the following. Given Case A, \((34)\) and \((35)\) are satisfied iff \((14)\) and \((15)\) hold. This proves Proposition 3 (i). Given Case B, \((34)\) and \((35)\) are satisfied iff \((16)\) and \((17)\) hold. This proves Proposition 3 (ii). Given Case C, \((34)\) and \((35)\) are satisfied iff \((18)\) and \((19)\) hold. This proves Proposition 3 (iii). Given Case D, \((34)\) and \((35)\) are satisfied iff \((20)\) holds. This proves Proposition 3 (iv).

V. Proof of Proposition 4

Proof.

It holds that \(\tilde{m}_h(m_h^0, m_f^0, K, K_f) > m_h^0\) and \(\tilde{m}_f(m_h^0, m_f^0, K, K_f) < m_f^0\) iff \((11)\) is positive and \((12)\) is negative. Thus, for the dominant firm to be buying in Home and selling in Foreign, the following two inequalities must be satisfied:

\[ (m_h - m_h^0) B_h < A_h - (m_f - m_f^0) C_h \quad (36) \]

and

\[ A_f - (m_h - m_h^0) B_f < (m_f - m_f^0) C_f \quad (37) \]

For \((m_h - m_h^0) > 0 \) and \((m_f - m_f^0) < 0 \), it is straightforward to show the following. Given Case A, \((36)\) and \((37)\) are satisfied iff \((21)\) and \((22)\) hold. This proves Proposition 4 (i). Given Case B, \((36)\) and \((37)\) are satisfied iff \((14)\) and \((23)\) hold. This proves Proposition 4 (ii). Given Case C, \((36)\) and \((37)\) are satisfied iff \((20)\) holds. This proves Proposition 4 (iii). Given Case D, \((36)\) and \((37)\) are satisfied iff \((24)\) and \((25)\) hold. This proves Proposition 4 (iv).

VI. Details of computational algorithm

In Section 4, to simulate the model, the finite grid approximation method is used.\(^{25}\) To compute the equilibrium reached in a given period, I fix \(K\) and \(K_f\). I discretize the state space \((m_h, m_f)\) into a finite rectangular grid and iterate on the fringe and dominant firm value and policy functions until reaching a fixed point. I use a 50x50 grid for the state space and evaluate all state variables over the ranges \(m_h^0 \in [0, 0.5]\) and \(m_f^0 \in [0, 0.5]\).

Using the finite grid approximation, the value functions are not smooth. This leads to convergence problems. To overcome this, I add a tiny logistic smoothing error to the payoffs at each grid point. The policy and value functions are then evaluated by setting the payoffs to the ones yielded by the model plus the smoothing error. The larger the grid, the smaller the error required to make the value functions smooth up to computer precision. The smoothing error I impose is smaller than \(10^{-3}\) times the standard logistic error. I verify using even smaller errors that the impact of this error on the equilibrium is negligible. With the smoothing error, the merger and investment decisions effectively become weighted averages of the values at grid points near the true optimum, where the weights are larger the closer the value at the grid point to the true optimum. The weights are computed using the standard multinomial logit formulas. A randomly selected sample of the simulations have also been run using a 200x200 grid to check that the results do not change significantly when using a finer grid. Further details are available upon request.

\(^{25}\) We essentially adopt the method used by Gowrisankaran and Holmes (2004) and modify it to suit the open economy model.
VII. List of key variables

$p$: Equilibrium market price of consumption good in Home
$p^*$: Equilibrium market price of consumption good in Foreign
$Q$: Total sales of consumption good, including imports, in the Home market
$Y$: Total sales of consumption good, including imports, in the Foreign market
$t$: Tariff level
$K$: Industry-specific capital
$K_h$: Units of capital owned by Home fringe firms jointly
$K_f$: Units of capital owned by Foreign fringe firms jointly
$K^d_h$: Units of capital owned by the dominant firm and located in Home
$K^f_f$: Units of capital owned by the dominant firm and located in Foreign
$L$: Non-industry-specific labor
$F(K, L)$: Production technology
$\sigma$: Units of capital produced per unit of consumption good produced
$c(\cdot)$: Labor cost per unit of capital
$m_h$: Proportion of the total industry capital stock held by the dominant firm in Home post-merger
$m_f$: Proportion of the total industry capital stock held by the dominant firm in Foreign post-merger
$m^0_h$: Proportion of the total industry capital stock held by the dominant firm in Home pre-merger
$m^0_f$: Proportion of the total industry capital stock held by the dominant firm in Foreign pre-merger
$v_h$: Discounted value to a Home fringe firm per unit of capital from Home profits
$v^*_h$: Discounted value to a Home fringe firm per unit of capital from Foreign profits
$v_f$: Discounted value to a Foreign fringe firm per unit of capital from Home profits
$v^*_f$: Discounted value to a Foreign fringe firm per unit of capital from Foreign profits
$v_d$: Discounted value to the dominant firm per unit of capital from Home profits
$v^*_d$: Discounted value to the dominant firm per unit of capital from Foreign profits
$v_x$: Discounted value to the dominant firm per unit of capital from Home profits earned by capital located in Foreign
$v^*_x$: Discounted value to the dominant firm per unit of capital from Foreign profits earned by capital located in Foreign
$q_d$: Sales in Home of the dominant firm per unit of capital located in Home
$q^*_d$: Sales in Foreign of the dominant firm per unit of capital located in Home
$q_x$: Sales in Home of the dominant firm per unit of capital located in Foreign
$q^*_x$: Sales in Foreign of the dominant firm per unit of capital located in Foreign
$q$: Sales per unit of capital in Home of the Home fringe firms
$q^*$: Sales per unit of capital in Foreign of the Home fringe firms
$y$: Sales per unit of capital in Home of the Foreign fringe firms
$y^*$: Sales per unit of capital in Foreign of the Foreign fringe firms
$\beta$: Discount factor

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