# Entry, Spending and Firm Size in a Stochastic R&D Race

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ABSTRACT. A model is developed to explain participation and spending on R&D as a function of firm size. The R&D process is represented as an n-participant race with a Poisson incidence of success, where the winner takes all during some "protection period". Four effects of scale are taken into account: a sunk fixed threshold cost of entry; a flow cost of expenditure for the duration of the race, which affects both the profitability of winning and the speed of development (the Poisson parameter), both with diminishing returns; allowance for an effect of firm size on the effectiveness (profit/cost) of development. The operational decision concerning the level and intensity of commitment in case of participation is modelled in a traditional fashion as the maximization of expected returns. The strategic decision whether or not to participate (at an optimal level and intensity) is modelled as a stochastic process of deliberation between different makers and influencers of decisions in the firm. The latter is to be seen as an introduction of the "political" and "resource dependence" views of organisations. The resulting model of R&D participation as a function of firm size is tested empirically on data from an R&D survey in the Netherlands.

## 1. Introduction

The purpose of the present study is positive: how can one explain observed patterns of entry. This is in contrast with many studies in the Industrial Organisation literature, which are aimed more at performance and its normative implications for market structure. The study needs to take market structure into account, and the results can be used for its further analysis, but that is not the purpose of this article. The study focuses on conduct, on the micro level of individual firms. It was evoked by an attempt to evaluate a state innovation stimulation scheme in the Netherlands (INSTIR), which was aimed in particular at small and medium sized enterprise (SME). In a debate on the effective reach of the scheme (Nooteboom, 1988) the need arose, in particular, for an explanation of the observed relation between firm size and participation in R&D.

In earlier empirical studies of innovation and firm size (for surveys see Kamien and Schwartz (1982), Stoneman (1983), Baldwin and Scott (1987)) there has been a tendency to take a sample of firms of different sizes and to relate some aggregate measure of R&D inputs relative to size (e.g., R&D personnel as a percentage of total employment, or R&D spending as a percentage of sales) or of R&D output relative to size (e.g., patents or a count of successful innovations divided by total sales) to a measure of firm size. This has been done for a linear, quadratic or cubic effect of firm size. For a recent study of the last type, see Acs and Audretsch (1991), who extend earlier results from Scherer (1965, 1984) and Soete (1979).

In such studies two questions are conflated: what percentage of firms, in a given class of firm size, conduct R&D (participation rate), and if a firm conducts R&D, how much does it spend per unit of firm size (input rate), and how much R&D output does it yield relative to firm size (output rate) or relative to R&D input (which we call the rate of effectiveness, which is by definition equal to the ratio between output and input rates). This issue is important if the relation between firm size and participation rate has a different shape from the relation between firm size and input viz. output rate.

In the present study a model is developed which implies that the participation rate increases uniformly with firm size: rising sharply at low firm sizes and then levelling off (see Figure 1). If one

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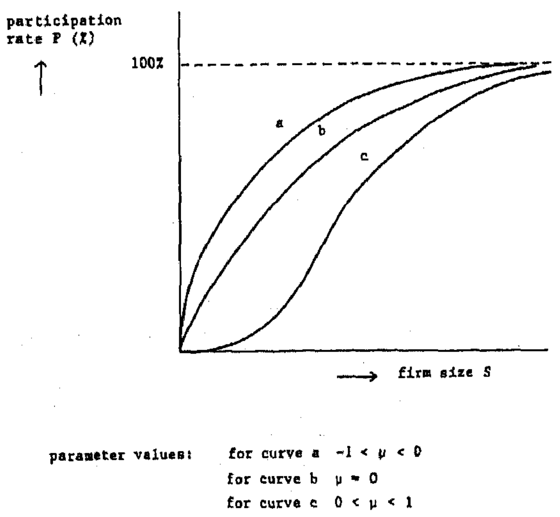


Fig. 1. Participation rate as a function of firm size. Parameter values: for curve $a$, $-1 < \mu < 0$; for curve $b$, $\mu = 0$; for curve $c$, $0 < \mu < 1$.

assumes that R&D effectiveness decreases with firm size, the model implies an input rate which decreases uniformly with firm size (see Figure 2). When the participation and input rates are multiplied to yield the effect of firm size on "relative R&D effort", defined as aggregate R&D inputs relative to aggregate sales, say, including firms which conduct R&D and those who do not, then any of the following outcomes can occur: R&D effort declines uniformly with firm size (at a decreasing rate); it increases uniformly (at a decreasing rate); it first increases and then declines with firm size (see Figure 3). The outcome depends on the exact value of the (negative) elasticity of R&D effectiveness with respect to firm size. In all cases, however, the participation rate maintains its uniform increase, and the input rate its uniform decline.

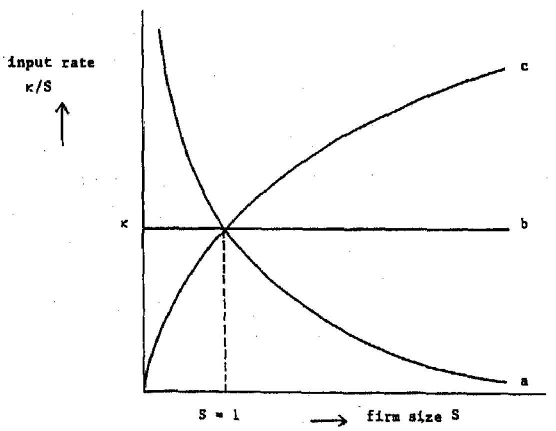


Fig. 2. Input rate as a function of a firm size. Parameter values: for curve $a$, $-1 < \mu < 0$; for curve $b$, $\mu = 0$; for curve $c$, $0 < \mu < 1$.

Hence it is a good idea to systematically separate out the participation rate, the input rate and the rate of effectiveness, as functions of firm size.[1]

Another introductory point is that in studies of innovation one must control for industry effects. Theoretically, the participation rate, input rate and rate of effectiveness all depend on characteristics of the industry, the market or the wider environment, such as technological opportunity (Scherer, 1965), stage in the life cycle or technological trajectory (Dosi, 1982, 1988), scale effects in R&D, appropriability of returns, number of competitors. One should be very surprised to find the same conditions in different industries. The need to control for industry effects also arose sharply from empirical studies. As reported in Baldwin and

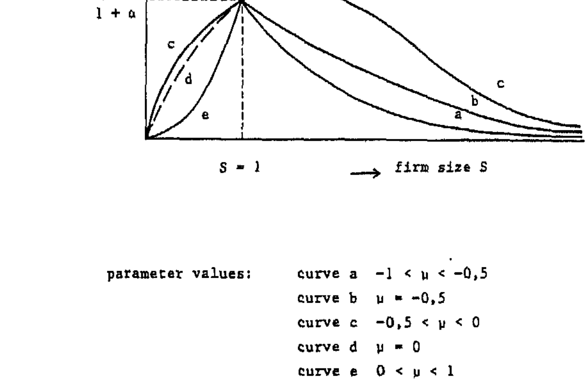


Fig. 3. Aggregate relative R&D effort as a function of firm size. Parameter values: curve $a$, $-1 < \mu < -0.5$; curve $b$, $\mu = -0.5$; curve $c$, $-0.5 < \mu < 0$; curve $d$, $\mu = 0$; curve $e$, $0 < \mu < 1$.

Scott (1987, p. 70), Scherer (1965) concluded that "interindustry effects, most notably "technological opportunity" accounted for about as much of (the) differences in patenting activity as did interfirm sales differences within industries".

Nelson, Peck and Kalachek (1967), as reported by Baldwin and Scott (1987, p. 73), noted that "Across the economy as a whole, larger firms spent a higher percentage of sales on R&D than did smaller ones. But this was in large part because certain industries — aircraft, electronics, and chemicals in particular — were characterized by both higher than average R&D intensity and greater than average size of the firm". Baldwin and Scott (1987, p. 109) report that in an extension of Scherer's findings Levin, Cohen and Mowery (1985) used 8 variables to measure technological opportunity and appropriability, and found "a dramatic reduction in the observed impact of Schumpeterian size and market power variables".

Another point concerns the possible requirement, and the possibility, to endogenize market structure in models of R&D behaviour. This might be straightforward in static conditions of equilibrium. But the core of Schumpeterian economics is that innovation disrupts such structural features in a "gale of creative destriction". From the point of view of "evolutionary theory" (Nelson and Winter (1982), Dosi *et al.* (1988)) a given market structure evokes R&D which then "changes processes, products and indeed the very process of doing R&D, and thereby shapes the evolution of market structure" (Baldwin and Scott (1987, p. 52). Equilibrium is nowhere in sight. It may be possible to endogenize market structure, in a generalized sense including technological opportunity, but in a multi-period evolutionary approach of structural change, rather than in terms of equilibrium.

Another point concerns the way in which organizational decision making is to be treated. There are two issues here which are important. One is that if one allows for uncertainty one should, in view of the finance literature, consider not only expected net returns but also level of risk: for a higher level of risk one will require a higher level of expected net returns. A second issue is how the trade-off between returns and risk is to be treated. I prefer (i.e., consider more plausible) a "behavioral" approach (Simon *et al.*) and an "evolutionary" perspective to a traditional approach in terms of the maximization of expected profit or utility. It would go much too far here to discuss the methodological force of considerations of plausibility[2] and the substantive arguments for a behavioural approach. In the present treatment of decision making an attempt is made to establish a link with the "political paradigm" in the literature on Organizational Behaviour. In short, decision making on a strategic level will be approached in terms of rules of thumb and in terms of conflicting perceptions, interests and aims of different stakeholders or coalitions involved in the process of decision making. On a more operational level, however, optimization will still be employed: e.g., in determining the optimal level of R&D effort, and its optimal time pattern, given a decision to engage in R&D. This hybrid approach reflects observed practice: "rational", calculative approaches (management science, operational research) often prevail in operational decisions (such as scheduling, determination of lot sizes, etc.), while strategic decisions are more in the nature of discourse.

Uncertainty and risk will be incorporated where they appear to be crucial, but perfect knowledge will be assumed concerning the returns on R&D effort, if succesful; the number of competitors and their R&D competence; and R&D technology (notably effects of scale). This is primarily done to keep the complexity of the model within bounds; further lack of information may be incorporated in future elaborations.

A major data problem in studies of R&D input emerges from a survey conducted by Kleinknecht (1987a, 1987b). The problem is that a measure of R&D input in terms of formal R&D departments, specialized staff and separate budgets tends to underestimate R&D in small firms, which is often not formally separated out from production activities. R&D activities are performed in between production activities or in off hours. This poses a threat to the results and interpretations of previous studies of firm size effects, to the extent that they included many firms on the lower end of firm size, with formal measures of R&D inputs.

Concerning R&D effectiveness, a study by Wyatt (1985), based on the Science Policy Research Unit (SPRU, Brighton, UK) innovation data base indicates that it is very much higher in small firms. But, as pointed out in Rothwell (1985), this is no doubt due to some extent to the problem of underestimation of R&D inputs in small firms, yielding an upward bias in the output ratio. However, the apparent excess of effectiveness in small firms is so large that it cannot be fully explained by that, so that the indication remains that R&D effectiveness is higher in smaller firms (Rothwell (1986), p. 127). This is one dimension of effects of scale that should be taken into account, where we leave it up to empirical results to decide whether smaller firms or larger firms are the more effective.

A second dimension, apart from effects of firm size, is that the technology/market combination may yield decreasing returns to scale: going for a more elaborate outcome, at a higher level of development costs, may yield higher rewards, but probably at a decreasing rate. A greater intensity of development, at a higher cost, is likely to speed up development, but here also decreasing returns are likely to occur.

Because it appears to be of increasing importance we also include "threshold costs" involved in investments in knowledge, skills and facilities before R&D activities (or other entry ventures) can effectively start and be maintained. This is a fourth dimension of effects of scale. Besides the obvious resources such as qualified staff, hard- and software of equipment, organizational and managerial support, the threshold may also include patents, licences or access to outside sources of technical or commercial information. To the extent that such a threshold exists and rises, this presents a significant problem to small firms. It may be this aspect, above all, that underlies the hypothesis of the later Schumpeter and of Galbraith that R&D is increasingly an affair of large corporations. The height of the threshold depends on the technology and the type of product or process involved. The development of a new aircraft or superchip, for example, involves such a high threshold that it is feasible only in large firms or joint ventures between them. Due to the increasing complexity and specialization of technology an increase of thresholds in the longer run is likely. However, there are many activities with an emphasis on the "D" side of R&D, particularly in the commercial application of available knowledge and technology, where thresholds may be low and stable, thus yielding opportunities for small, independent firms. In the context of entry decisions, an important question is whether costs, and in particular threshold costs are sunk, if they occur.[3]

## 2. State model

The R&D process is modelled as a race between n participants which lasts until one contestant achieves success. It is assumed that the winner can appropriate incremental returns ("winner takes all") during a "protection period", and after that period incremental net returns (entrepreneurial rents) are zero. If patents are used and provide full closure, the protection period equals the patent period. There is evidence, however, that patents are not used as much as thought previously.[4] In general, therefore, the protection period is determined by the most lasting of several conditions to provide appropriability: secrecy, obstacles to "reverse engineering", blocked resources or other entry barriers, delays in imitation, patents. A Cournot assumption is further made that each decisionmaker acts on an expected number of contestants ($n$), which is equal for all participants, and is not affected by one's decision to enter. With respect to opportunities to speed up development by an intensification of effort, decision makers take into account that competitors will also make optimal use of it (conjectural variations in a Stackelberg setting).

Following Kamien and Schwarz (1976), Loury (1979), Dasgupta and Stiglitz (1980b) and Lee and Wilde (1980), it is assumed that the occurrence of success in development is a Poisson process: for an individual contestant the probability of achieving success in the time interval $(t, t + \Delta t)$ is $\lambda \Delta t$, where $\lambda$ is independent of $t$. This has the debatable implicaton that the process has no memory. But it is this property that enables one to treat new and past entrants in the same way and to disregard the composition of a set of competitors according to their past experience with the current type of development considered. The absence of such information in data is the main reason to make the assumption; the other reason is to keep analytic complexity within bounds. The Poisson process implies:

$$f(t) = \lambda e^{-\lambda t}, \qquad (1)$$

where $f(t)$ is the probability density over development times $t$. According to (1) expected development time is $1/\lambda$.

Still following the preceding literature, it is assumed that, given the selection of a development project, the parameter $\lambda$ can be increased (thus reducing expected development time) by intensifying the effort, i.e., by concentrating development expenditure in time, and that this yields first increasing and then decreasing returns:

$$\frac{d\lambda}{d\gamma} > 0; \frac{d^2\lambda}{d\gamma^2} \gtrless 0 \text{ for } \gamma \lesseqgtr \bar{\gamma}, \qquad (2)$$

where $\gamma$ is a measure of the intensity of expenditure per unit of time during development. In studies before Lee and Wilde (1980), $\lambda$ was assumed to be such a function of expenditure in the form of some fixed, contractual sum of money which is independent of development time.

Following Lee and Wilde, the more reasonable and more flexible assumption is made that there are two types of cost: a fixed sum ($a$), regardless of the time spent on development, and a flow cost per unit of time which stops as soon as one of the contestants achieves success, at which point the race stops. Lee and Wilde's model is extended by adding two features:

Firstly, it is no longer assumed that the flow cost and the reward for success (discounted to the moment that success is achieved) are given, but that the reward ($b$) varies according to the "level" of commitment: in the area of technology considered, one can aim for more or for less sophisticated versions of the object of development, with corresponding higher or lower levels of flow cost ($c$) and associated higher or lower levels of returns ($b$), under the assumption of diminishing returns (from $c$ to $b$). Together with the previous assumption concerning the dependence of the Poisson parameter $\lambda$ on the "intensity" of commitment, the total flow cost now is the product of intensity and level ($\gamma \cdot c$), where $\lambda$ depends on intensity ($\gamma$) and $b$ depends on level ($c$). In other words, the assumption is that the two effects of flow cost are separable: one can raise intensity to speed up development without affecting the level of profitability and one can raise the level of profitability without affecting the speed of development. Once they are selected at the beginning of the development period, both level and intensity of commitment remain fixed during development.

The second extension to the Lee and Wilde model is that allowance is made for a firm size effect on the effectiveness of development, by which is meant the relation between the level of returns ($b$) and the level of commitment ($c$). From the literature there are indications for both theses: that small firms are more effective and the opposite thesis that large firms are more effective (for empirical support of the latter thesis, see Wyatt (1985)). Theoretically, there are arguments that the answer depends on the type and/or stage of development: large firms are likely to be more effective in the development of new basic technologies (requiring highly specialized staff and sophisticated laboratories) and the large-scale penetration of markets by new products or processes (requiring a broad production and marketing base), while small firms are likely to be more effective in the early unorthodox application of new basic technologies and their linking to potential demand (requiring flexibility and a lack of bureaucratic obstacles to unorthodoxy) and innovative product-service adjustments after the declining phase of the product's life cycle has set in (which requires a careful tailoring to the demands of a "die-hard" residual market niche).[5] Thus different firm size effects may be found for different industries or for different periods for a given industry.

The fixed set-up or entry cost (entry ticket to the development gamble), however, may be independent of firm size (but dependent on the industry). This reflects the important issue of a scale effect in the form of a threshold cost. This cost enters the calculation only in so far as it is sunk: no salvage value after development is stopped.

The fixed entry cost ($a$) does not affect profitability ($b$) or speed of development ($\lambda$). In formulae:

$$b = \beta_0 S^{\beta_2} c^{\beta_1}; \quad \beta_1 < 1, \qquad (3)$$

where $c$ is the level of flow cost per unit of firm size, during development, $b$ is reward per unit of firm size, at the time of success, $\beta_1 < 1$ reflects diminishing returns in the level of commitment, $\beta_2$

allows for a firm size effect: $\beta_2 < 0$ if small firms are the more effective, and $S$ is firm size.

Total costs in present value, for any given participant, now are as follows:

$$C = a + \int_0^T \gamma c S e^{-it}\, dt = a + + \frac{\gamma c S}{i}(1 - e^{-iT}), \qquad (4)$$

where $a$ is the fixed entry cost, $\gamma$ is the intensity of the flow cost, $c$ is the level of the flow cost, $i$ is the discount rate, and $T$ is the time that the development stops, i.e., when any contestant achieves success.

On the assumption that the stochastic process of development (1) applies independently to all participants, the density over $T$ is:

$$F(T) = Le^{-LT}, \qquad (5)$$

where $L = \sum_{j=1}^{n} \lambda j,$

$n$ = number of contestants;

$j$ = index for contestants.

Thus we have:

$$E(C) = a + \frac{\gamma c S}{i} \int_0^\infty (1 - e^{-iT}) Le^{-LT} dT = a + \frac{\gamma c S}{L + i}. \qquad (6)$$

A contestant achieves a reward $bS$ at $T$ if it is the first to achieve success at $T$, and nil otherwise. The probabilities involved are as follows: probability of achieving success at $T$:

$$\lambda e^{-\lambda T}, \qquad (7)$$

where $\lambda$ applies to the contestant considered.

Probability of being first to do so = probability that no other contestant achieves success before $T$:

$$e^{-(L-\lambda)T}. \qquad (8)$$

Thus we obtain

$$E(R) = bS \int_0^\infty e^{-iT} e^{-(L-\lambda)T} \lambda e^{-\lambda T}\, dT = \frac{\lambda b S}{L + i}, \qquad (9)$$

where $E(R)$ = expected present value of rewards.

Thus expected net present value is:

$$E = E(R) - E(C) = -a + \frac{\lambda b - \gamma c}{L + i} S. \qquad (10)$$

### 3. Selection model

A traditional maximizing approach is adopted for the selection of an optimal project (level of $c$ with its resulting outcome for $b$) at optimal intensity (value of $\gamma$ with its resulting outcome for $\lambda$), to be considered as the best possible option in the subsequent decision whether or not to enter.

The entry decision will later be modeled in a more behavioral fashion. Regarding selection, (10) is to be maximized with respect to both $c$ and $\gamma$.

$$\frac{\delta E}{\delta c} = \frac{1}{L + i}\left(\frac{\lambda \beta_1 b}{c} - \gamma\right). \qquad (11)$$

In the optimum we have:

$$\hat{b} = \frac{\gamma \hat{c}}{\lambda \beta_1}. \qquad (12)$$

According to (3) we also have:

$$\hat{b} = \beta_0 S^{\beta_2} \hat{c}^{\beta_1}.$$

Which yields:

$$\hat{c} = \left(\frac{\lambda}{\gamma} \beta_1 \beta_0 S^{\beta_2}\right)^{1/1-\beta_1}. \qquad (13)$$

Next we have:

$$\frac{\delta E}{\delta \gamma} = \frac{1}{L+i}\left(b\lambda^1 - c - \frac{(\lambda b - \gamma c)}{L+i}\frac{\delta L}{\delta \gamma}\right), \tag{14}$$

where $\lambda^1 = \dfrac{d\lambda}{d\gamma}$.

Concerning $\delta L/\delta\gamma$, assume the following Stackelberg conjectural variations:

$$\frac{\delta \lambda_j}{\delta \lambda} = 1, \tag{15}$$

where $j$ indexes the $n-1$ competitors.

This is tantamount to a symmetric Nash-Cournot equilibrium: all contestants select the optimal $\gamma$, and take into account that all others do so as well, so that

$$\frac{\delta L}{\delta \gamma} = \frac{d\lambda}{d\gamma}\left(1 + \sum_j \frac{\delta \lambda_j}{\delta \lambda}\right) = n\lambda^1.$$

This yields:

$$\frac{\delta E}{\delta \gamma} = \frac{1}{(L+i)^2}\{(ib + \gamma cn)\lambda^1 - c(L+i)\}. \tag{16}$$

In the optimum we have:

$$\hat{\lambda}^1 = \frac{L+i}{n\hat{\gamma} + \dfrac{ib}{c}}. \tag{17}$$

For an optimum with respect to both $c$ and $\gamma$, conditions (12) and (17) must both apply, so that in the joint optimum:

$$\hat{\lambda}^1 = \frac{\hat{\lambda}}{\hat{\gamma}}\,\frac{\hat{L}+i}{\hat{L}+i/\beta_1} > \frac{\hat{\lambda}}{\hat{\gamma}} \quad \text{for all} \quad n < \infty \text{ (since } \beta_1 < 1). \tag{18}$$

Given the conditions (2), the most efficient point of $\gamma$ is given by:

$$\lambda^1 = \frac{\lambda}{\gamma}. \tag{19}$$

(18) Shows that contestants spend more than this most efficient level. This result is the same as found by Lee and Wilde (in the relevant specifications the present model is the same as theirs). The most efficient point is reached for $n \to \infty$.

All the elements are now available for a systematic analysis of the effects of market structure (in the sense of number of contestants $n$) on spending ($\hat{c}$ and $\hat{\gamma}\hat{c}$), rewards ($\hat{b}$) and expected net present value ($\hat{E}$). Since the results are the same as given by Lee and Wilde, the implications are given without proof: An increase of $n$ yields:

- an increase of both $\hat{c}$ and $\hat{\gamma}\hat{c}$;
- an increase of $\hat{b}$;
- a decrease of $\hat{E}$.

According to (10) and (12) we have:

$$\hat{E} = -a + \frac{\hat{\lambda}b - \hat{\gamma}\hat{c}}{\hat{L}+i}\mathrm{S} = -\mathrm{a} + \frac{\left(\dfrac{1}{\beta_1} - 1\right)\hat{\gamma}\hat{c}}{\hat{L}+i}\mathrm{S}, \tag{20}$$

where $\hat{\lambda}$ and $\hat{\gamma}$ are determined by (18), and $\hat{c}$ by (13). We now proceed to the use of the results for the model of entry decisions in relation to firm size.

## 4. Decision model

For the decision whether or not to participate, a model is chosen in which the entry decision itself is stochastic: for each potential entrant there is probability of entry, which is a function of expected returns and risk. The underlying idea is that decision making is a process in which a decision arises from a collision of arguments, perceptions and opinions. Arguments in favour focus on what one can expect to gain and arguments against focus on the risk of failure. As Nelson and Winter (1977, p. 47) already indicated: "innovation involves uncertainty in an essential way . . . different people, and different organizations, will disagree as to where to place their R&D chips, and when to make their bets." Technical people from R&D or marketing people may focus on what one stands to gain, whereas operations and finance people may focus on what one might loose.

The stochastic decision model, in its most general form, is as follows:

$$p = \frac{(E-m)^{\varepsilon}}{(E-m)^{\varepsilon} + v + \rho\, r};$$

$$\varepsilon, m, v, \rho > 0, \qquad (21)$$

where $p$ = probability of "go", $E$ = expected net present value of returns, $m$ = cut-off value below which net returns (in present value terms) are not considered worthwhile, $v$ serves to model the possibility that entry is not certain ($p < 1$) even if risk is zero and $E > m$, $r$ = risk of failure, $\rho$ is a parameter to indicate the degree of risk aversity, and $\varepsilon$ is a parameter to allow for a non-linear effect of expected returns on entry.

Risk of failure $r$ is defined as the probability that expected net present value of returns will be negative.

A link exists between this model and traditional $\mu - \sigma$ analysis;[6] if we establish a connection between utility levels and probabilities of entry. Solving (21) for $E$, we obtain

$$E = m + \left\{ \frac{p}{1-p} (v + \rho\, r) \right\}^{1/\varepsilon}. \qquad (22)$$

For a given value of $p$, and hence of $p/1 - p$, associated with a level of utility, (22) can be interpreted as an indifference curve in the $E - r$ plane, with curvature and intercept depending on utility level ($p/1 - p$), as illustrated in Figure 4.

The implication of the stochastic decision model is that if there is a single opportunity on a highest indifference curve, it is not certain that the opportunity will be taken: probability of action merely increases with utility. It also implies that two or more opportunities at different levels of utility may all be taken: opportunities with higher utility are merely more probable. There are two ways in which this can be justified:

- A hedging of bets, by undertaking several actions simultaneously, may be both rational and realistic.
- There may be no such thing as a set of known strategic alternatives from which decision makers select the best. Not knowing all options well enough, or being unable to evaluate them jointly organizations may well pick out and discuss opportunities one by one.

If $v = 0$, all indifference curves have the same intercept $m$. If $\varepsilon = 1$, the indifference curves are linear. If $v = m = 0$ all indifference curves go through the origin (no intercepts).

This is not as strange at it may look at first glance, since the opportunity cost of capital has already been incorporated in the discount rate used to calculate net present value of returns $E$.

The simplest meaningful version of (21) arises for $m = v = 0$ and $\varepsilon = 1$:[7]

$$p = \frac{E}{E + \rho\, r}. \qquad (23)$$

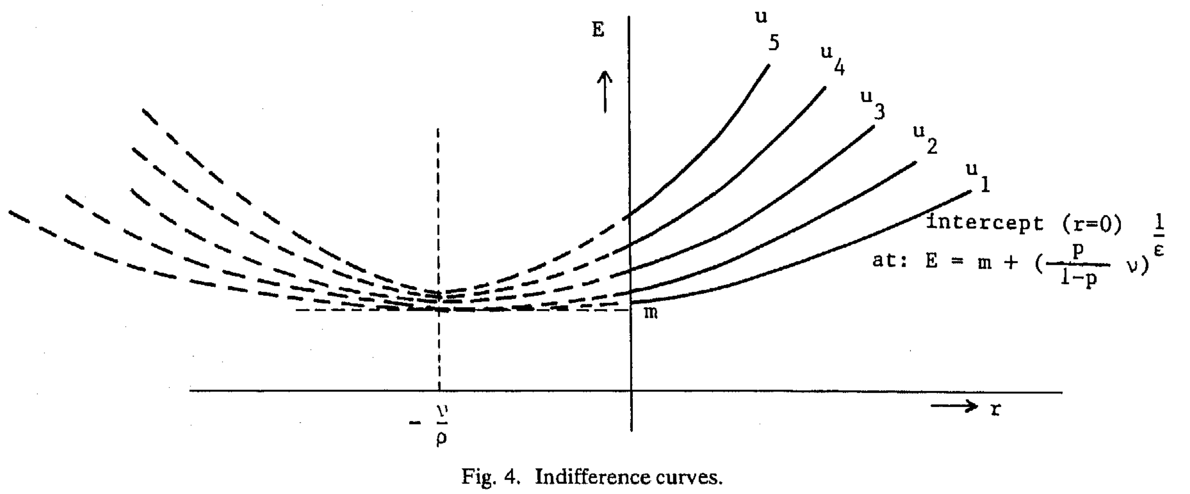


Fig. 4. Indifference curves.

To proceed, we need specifications of $E$ and $r$, based on earlier assumptions and the specification of the state model. The further assumption is made that decision makers proceed as follows:

First one considers what optimal expected returns (in net present value) would be in case of participation, in view of the state model as specified before. This optimum yields a corresponding level of risk $r$. The resulting values of $E$ and $r$ are considered in the evaluation process represented by (21). For the optimal project at optimal intensity, expected net present value is as specified in (20) (from now on we drop the hats (^) that indicate optimal selection).

Substituting (13), we obtain:

$$E = -a + h\, S^{(1+\beta_2/1-\beta_1)}, \tag{24}$$

where $h = \left(\frac{1}{\beta_1} - 1\right) \gamma^{-\beta_1/1-\beta_1}$

$(\lambda\beta_1\beta_0)^{1/1-\beta_1}/(L+i)$.

If there are sunk threshold costs ($a > 0$), $E > 0$ only beyond some critical firm size:

$$S > \left(\frac{a}{h}\right)^{(1-\beta_1)/(1-\beta_1+\beta_2)} \tag{25}$$

The risk of failure ($r$) is defined as the probability that net present value of returns will be negative. If one wins the race, net returns will still be negative if development time $T$ exceeds a critical value $T^*$.

In case of winning at $T$, net present value is (see (4)):

$$R = -a - \frac{\gamma c S}{i}(1 - e^{-iT}) + bSe^{-iT}.$$

Substituting the optimal project (see (12)) we find:

$$R = -a - \frac{\gamma c S}{i} + \left(\frac{1}{i} + \frac{1}{\lambda\beta_1}\right) \gamma c S e^{-iT}. \tag{26}$$

$R$ is negative if:

$$T > T^* = -\frac{1}{i}\log \times$$

$$\times \left\{ \left(1 + \frac{ia}{\gamma c S}\right) \frac{\lambda\beta_1}{\lambda\beta_1 + i} \right\}. \tag{27}$$

If there are no sunk threshold costs ($a = 0$), $T^*$ is independent of $S$. For example, if $\lambda = 0.25$ (expected development period of 4 years) $\beta_1 = 0.5$ and $i = 0.15$, $T^*$ then is 5.25 years. For $a > 0$ $T^*$ is lower for smaller firms. To recover sunk entry costs small firms must be more efficient (i.e., achieve success faster) than larger firms.

The probability that net returns are positive, under the condition that the race is won, thus is, according to (1):

$$p(T < T^*) = \int_0^{T^*} \lambda e^{-\lambda t}\, dt = 1 - e^{-\lambda T^*}. \tag{28}$$

The unconditional probability of positive net returns is as follows: the probability that the development period is $T$ multiplied by the probability that for all competitors the development period exceeds $T$, integrated over all $T < T^*$. This is as follows:

$$\int_0^{T^*} \lambda e^{-\lambda T} e^{-(L-\lambda)T}\, dT = \lambda \int e^{-LT} dT =$$

$$= \frac{\lambda}{L}(1 - e^{-LT^*}).$$

If there are no competitors, $L = \lambda$, and this reduces to (28).

Thus the probability of failure ($r$) is as follows:

$$r = 1 - \frac{\lambda}{L}(1 - e^{-LT^*}). \tag{29}$$

For example, if $\lambda = 0.25$, $T^* = 5.25$, $n = 3$, $r$ is about 2/3.

If there are no sunk threshold costs $r$ does not depend on firm size because $T^*$ does not. If there are sunk threshold costs risk decreases with increasing sales size (since $T^*$ increases with increasing sales size). Without proof it is noted that $r$ rapidly increases with the number of competitors. This illustrates the importance of taking risk into account, next to expected net present value, in studies of the effect of market structure.

## 5. Implications

The claims made in the introduction concerning participation, spending and aggregate R&D effort will now be proved. Without any restrictions on the parameters, the model yields a wide scope of different possible implications. To keep complexity within bounds, the analysis will be conducted on a restricted range of parameters:

- threshold costs are zero ($a = 0$);
- we take the simplest version of the decision model (23);
- the parameter $\mu$, which we define as $\mu = \beta_2/(1 - \beta_1)$ is restricted to the range $-1 \leqslant \mu \leqslant 1$. As a necessary, but not sufficient condition this implies that the elasticity of R&D effectiveness with respect to firm size is restricted to the range $-1 \leqslant \beta_2 \leqslant 1$.

The implications of the model then are as follows: According to (29) risk ($r$) then is independent of size. Substituting $a = 0$ in (24) and the result in (23), the implication for the participation rate is as follows:

$$p = \frac{S^{1+\mu}}{S^{1+\mu} + \alpha}; \quad \text{where} \quad \alpha = \frac{\rho r}{h}. \tag{30}$$

Substituting different values for $\mu$, in the assumed range, we find different functions for the participation rate $p$ as a function of firm size $S$, as illustrated in Figure 1. The proofs are given in Appendix A.

In all cases, the participation rate rises sharply at low firm sizes and then levels off. In case of a positive elasticity of R&D effectiveness with respect to firm size (curve $c$) there is a point of inflection: participation does not rise sharply until some level of firm size.

In the innovation literature several attempts have been made to estimate the relation between firm size and R&D expenditure. The results are not uniform. This section considers some implications of our model for the relation between R&D expenditure and firm size, and considers its capability to reproduce empirical results reported in the literature. One type of study considers only firms that conduct R&D, and looks at their input rate. As discussed, it is another matter if one includes not only firms participating in R&D but all firms, for each size class. Here, relative R&D effort is defined as total R&D spending divided by total assets, employment or turnover of all firms in the size class. This measure is equal to the product of participation rate and the input rate of firms conducting R&D.

It is not always clear which of the two measures is being studied. It appears that most studies are dealing with the latter measure of "R&D effort". In their survey, Kamien and Schwarz (1982) report varying empirical results concerning the relation between R&D intensity and firm size: sometimes it increases with firm size, sometimes it declines, and sometimes it first increases and then declines. Overall, they conclude (p. 103) that "the bulk of the empirical findings indicate that ... R&D activity, measured by either input intensity or output intensity, appears to increase with firm size up to a point and then to level off or decline". A similar assessment is found in the more recent survey of Baldwin and Scott (1987, p. 82): "Studies ... using data from other industrialized countries largely confirmed the findings emerging from U.S. data, notably the lack of evidence of a clearcut causal relationship beyond a size threshold running from firm size to innovative activity".

According to the present model, for firms participating in development, spending depends on firm size as follows (see (13)):

$$K = \frac{a}{\tau} + \kappa S^{1+\mu};$$

$$\kappa = \gamma \left( \frac{\lambda}{\gamma} \beta_0 \beta_1 \right)^{1/1-\beta_1}, \tag{31}$$

where: $K$ = spending per unit of time, and threshold cost $a$ is spread over some period $\tau$.

The input rate, defined as spending per unit of firm size then is:

$$\frac{K}{S} = \frac{a}{\tau S} + \kappa S^{\mu}. \tag{32}$$

If there are threshold costs ($a > 0$) and/or smaller firms have higher R&D effectiveness ($\beta_2 < 0$), the input rate $K/S$ decreases with increasing firm size. If there are threshold costs ($a > 0$) and large firms are more R&D effective ($\beta_2 > 0$), the input rate first decreases and then from some point increases with increasing firm size.

If we assume zero threshold costs ($a = 0$), the alternative outcomes for $-1 < \mu < 1$ (where $\mu =$

$\beta_2/(1 - \beta_1)$) are as illustrated in Figure 2. The results are immediately obvious from (32). The input rate rises or declines uniformly with firm size according to whether the elasticity of effectiveness with respect to firm size is positive or negative. If there is no firm size effect on effectiveness, the input rate is constant (under the assumption that threshold costs are zero).

Now we turn to aggregate R&D effort (firms that do not conduct R&D are included), which equals the product of participation rate and input rate. From (30) on (32) we find (if $a = 0$):

$$I = p\frac{K}{S} = \frac{\kappa S^{1+2\mu}}{S^{1+\mu} + \alpha'} \tag{33}$$

where: $I$ = (aggregate) relative R&D effort. The alternative outcomes are illustrated in Figure 4. Proofs are supplied in Appendix A.

If we limit our attention to the cases with a negative elasticity of R&D effectiveness with respect to firm size, we find: the participation rate increases with firm size everywhere, the input rate declines everywhere, while relative R&D effort may decrease everywhere (curves $a$ and $b$), or may increase everywhere (curve $d$) or may first increase and then decrease (curve $c$), depending on the exact value of the (negative) elasticity of effectiveness with respect to firm size.

Curves $c$ and $d$ appear to be most in line with the empirical results reported in the literature. In the context of our model and further assumptions (notably that threshold costs are zero and the simple version of the decision model applies) this would suggest that the parameter $\mu$ lies in the range $-0.5 < \mu \leqslant 0$. Given that $\mu = \beta_2/(1 - \beta_1)$, this implies that if $\beta_1$ (elasticity of returns with respect to level of R&D effort) ranges between 0.5 and 0.9, $\beta_2$ (elasticity of R&D effectiveness with respect to firm size) would range between 0 and −0.25.

Summing up: it should not come as a surprise when different studies, with samples containing different compositions of small and large firms, containing firms who do and who do not conduct R&D, and containing different industries, yield different results. Note that the present discussion does not exhaust all possible implications. If one allows for threshold costs, for example, yet other implications are obtained.

## 6. An empirical study of the participation rate

In the present section the model of the participation rate is tested and estimated on data concerning R&D participation, for different classes of firm size, in Dutch manufacturing.

The simplest version of such a model is found by taking the simplest form of the decision model (23), and substituting the result for $E$ (24) and $r$ (29) with the assumption that there are no sunk threshold costs ($a = 0$) and there is no effect of firm size on effectiveness of development ($\beta_2 = 0$).

This yields:

$$p = \frac{S}{S + \alpha}; \quad \alpha = \frac{\rho r}{h}, \tag{34}$$

where $h$ is as specified in (24). Under the assumption that $a = 0$, $r$ is independent of firm size. The hypothesis (34) is subjected to two empirical tests. The first involves an estimate of the parameter $\nu$ in the generalized model:

$$p = \frac{S^\nu}{S^\nu + \alpha'}. \tag{35}$$

To interpret $\nu$, substitute $a = 0$, $m = 0$, $v = 0$ and (24) in (21), to find (35) with:

$$\nu = \varepsilon\left(1 + \frac{\beta_2}{1 + \beta_1}\right); \quad \alpha' = \frac{\rho r}{h^\varepsilon}. \tag{36}$$

If the estimate of $\nu$ differs significantly from unity, this is an indication of any of the following possibilities:

- the effect of $E$ on $p$ is non-linear ($\varepsilon \neq 1$);[8]
- development effectiveness depends on firm size ($\beta 2 \neq 0$);
- the hypothesis fails for other reasons.

The parameters $\nu$ and $\alpha$ can be estimated with an ordinary linear least squares procedure on the basis of the following transformation of (35):

$$\log\frac{p}{1-p} = -\log \alpha' + \nu \log S. \tag{37}$$

In this test dummies for industries will be attached to the intercept, thus controlling for possible differences between industries regarding $\alpha$, i.e., level of risk ($r$), degree of risk aversity ($\rho$), profitability of R&D ($h$), and implicitly in $h$ and $r$ the number of competitors ($n$), the expected duration of development ($1/\lambda$), and the discount rate ($i$). Due to lack of degrees of freedom in the available data, no control could be added for differences in the parameter $\nu$, i.e., non-linear effects of expected returns on participation ($\varepsilon$), elasticity of returns with respect to R&D expenditure ($\beta_1$) and the elasticity of R&D effectiveness with respect to firm size ($\beta_2$).

The second test involves an estimate of the parameter $k$ in the following model:

$$p = \frac{S-k}{S-k+\alpha}. \tag{38}$$

To interpret $k$, substitute $\varepsilon = 1$, $v = 0$, $\beta_2 = 0$ and (24) in (21), to find (38), where:

$$k = \frac{a+m}{h}; \quad k/\alpha = \frac{a+m}{\rho r}. \tag{39}$$

It should be noted that if there are any sunk entry costs ($a > 0$) $r$, and hence $\alpha$, depends on firm size, If the estimate of $k$ (or $k/\alpha$) differs significantly from zero, this indicates any of the following:

- There is a non-zero cut-off point of return for the appraisal of opportunities ($m > 0$).
- There are threshold costs of development which are at least partly sunk ($a > 0$).
- The hypothesis fails for other reasons.

The parameters $k$ and $\alpha$ can be estimated with an OLS procedure on the basis of the following transformation of (38):

$$\frac{p}{1-p} = -\frac{k}{\alpha} + \frac{1}{\alpha} S. \tag{40}$$

In this test dummies for industries will be attached to the slope, thus again controlling for industry effects on $\alpha$. Thus the results of the two tests can be compared.

The data for the tests were kindly supplied by Kleinknecht (1987a, 1987b) from his R&D survey conducted in the Netherlands in 1984. The survey was conducted in the manufacturing sector, by mail, on a sample of 3,000 firms with more than 10 persons employed. The overall response rate was 63%. The sample was drawn at random from the database of addresses at the allied chambers of commerce (where firms have to register).

Data on individual respondents are confidential, and hence not available, and had to be grouped so that respondents could not be recognized. The lowest level of aggregation on which the data could be obtained was on the second digit SIC. There was access only to the dichotomous variable whether or not a firm conducted internal R&D, external R&D or both.

The variable to be explained here is the percentage of firms, in a given size class, that conducts R&D (internal, external or both). The data allowed for a grouping into five size classes according to numbers of people employed. As a measure of size, in the model, the midpoint of the size class is taken. Some of the second digit level groups had to be merged.

The resulting numbers of observations per cell are given in Table I. No authorization has been given to also supply the percentages of firms conducting R&D.

Since there are only five observations (five size classes) for each SIC group, while we need to estimate 2 parameters, the data are pooled across SIC groups and we postulate:

- if the parameter $\nu$ in (37) differs from unity, it is the same for all SIC groups;
- if the parameter $k/\alpha$ in (40) differs from zero, it is the same for all SIC groups.

The first assumption implies that if development effectiveness depends on firm size ($\beta_2 = 0$) or expected returns have a non-linear effect on probability of entry ($\varepsilon = 1$), they do so in the same way for different SIC groups. The second assumption implies that if there are sunk threshold costs or a cut-off rate in investment selection, they are proportional to $\alpha$ and hence to the ratio between risk and returns per unit of firm size (see (39)).

For the parameter $\alpha$ we allow different values for different SIC groups, by appending dummy variables. This implies that SIC's are allowed to vary in the ratio between risk and expected returns per unit of firm size. It is only in this limited fashion that we can control for industry effects.

This procedure yields an estimate of 14 param-

TABLE I
Numbers of observations by size class and industry

| SIC group | SIC code | Description | Size classes (persons employed) | | | | |
|---|---|---|---|---|---|---|---|
| | | | 10—20 | 20—50 | 50—100 | 100—200 | 200—500 |
| 1 | 20/21 | Foods, beverages, tobacco | 14 | 56 | 57 | 47 | 36 |
| 2 | 22/23/24 | Textiles, clothing & leather | 13 | 41 | 41 | 27 | 14 |
| 3 | 25 | Wood & furniture | 14 | 42 | 27 | 8 | 4 |
| 4 | 26/27 | Paper & printing | 22 | 79 | 48 | 35 | 35 |
| 5 | 28/29/30 | Oil, chemicals, fibres | 4 | 22 | 29 | 32 | 22 |
| 6 | 31 | Rubber & plastics | 11 | 17 | 22 | 13 | 9 |
| 7 | 32 | Building mat., earthenware, glass | 12 | 32 | 26 | 18 | 6 |
| 8 | 33/34 | Metals & metal prods | 46 | 97 | 84 | 46 | 31 |
| 9 | 35 | Machinery | 23 | 79 | 72 | 50 | 30 |
| 10 | 36 | Electrical goods | 8 | 22 | 16 | 14 | 11 |
| 11 | 37 | Means of transport | 7 | 33 | 12 | 22 | 7 |
| 12 | 38/39 | Instruments, optical goods, remaining | 9 | 30 | 17 | 14 | 8 |
| | | Total | 183 | 550 | 451 | 326 | 213 |
| | | Grand total | 1,723 | | | | |

eters on 60 observations. (2 model parameters and 14 dummies for industry effects.)

The OLS regressions are weighted, to take into account the different numbers of observations in different cells (see Table I). In other words: the variance of the disturbance term in the regressions is assumed to be inversely proportional to the number of observations per cell.

The results are given in Tables II and III. The results from the loglinear regression (Table IIa) show that the hypothesis of a unit slope ($\nu = 1$; see (37)) cannot be rejected. The common intercept of −4.0 yields an estimate of $\alpha$ of 55. Significant deviations from this are found only for SIC groups 5, 6, 9 and 10, and perhaps 4. Leaving the dummies for 4 and 6 as they are, adding up the dummies for 5, 9 and 10 and leaving the other dummies out, we find almost exactly the same results, as indicated in Table IIb.

The results from the linear regression according to (40), supplied in Table III, give some indication of a negative intercept, and hence of a cut-off return in investment selection or sunk threshold costs, but it is not statistically significant. The industry dummies on the slope indicate the same industry differences in the parameter $\alpha$ as found from the intercept dummies in the first test.

Neither of the two empirical tests indicates a need to reject the simplest version of the model (34).

On the basis of this simple model, the estimates of $\alpha$ are given in Table IV.

For a smaller $\alpha$ the participation rate in R&D is higher, at any level of firm size. We see that participation is highest in rubber and plastics (SIC 31), followed fairly closely by the (petro) chemical industry, machinery and electrical goods (SIC 28—30, 35—36). R&D participation is very low in paper and printing (SIC 26—27).

The method used enables us to compare research participation between industries without a bias from different firm sizes. This is important in view of the fact that there often is a correlation between R&D opportunities and average firm size, as reported in the literature. Without a separation of industry effects and firm size effects, one should then expect the results to be biased.

Figure 5 gives a confrontation between obser-

TABLE II
Results of log linear regression. In addition to a constant term, dummy variables have been added to allow for deviations, for SIC groups 1 to 11. $T$-values are indicated between brackets.

TABLE IIa: First estimates

| Intercept | Slope | Intercept dummies for SIC groups: | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| −4.0<br>(−7.5) | 1.10<br>(12.1) | 0.1<br>(0.2) | 0.08<br>(0.2) | 0.4<br>(0.75) | −0.5<br>(−1.2) | 1.0<br>(2.0) | 1.5<br>(2.7) | 0.4<br>(0.7) | 0.04<br>(0.08) | 1.0<br>(2.3) | 1.0<br>(1.9) | −0.3<br>(−0.6) |

TABLE IIb: Second estimates

| Intercept | Slope | Intercept dummies for SIC groups: | | |
|---|---|---|---|---|
| | | 4 | 6 | 5, 9 & 10 |
| −3.9<br>(−10.5) | 1.09<br>(12.9) | −0.6<br>(−2.5) | 1.4<br>(3.6) | 0.9<br>(5.0) |

TABLE III
Results of linear regression. For selected SIC groups (same selection as in Table IIb) dummies have been appended to the slope.

| Intercept | Slope | SIC dummies on the slope for groups: | | |
|---|---|---|---|---|
| | | 4 | 6 | 5, 9 & 10 |
| −2.2<br>(−0.7) | 0.08<br>(3.1) | −0.06<br>(−1.4) | 0.27<br>(3.5) | 0.07<br>(2.1) |

ved participation rates and estimates based on the simplest version of the model, for the classification of SIC codes and corresponding estimates of $\alpha$ given in Table IV. For each of the four classes the corresponding data are pooled. In evaluating the fit, one should take into account the differences between the classes in the total number of firms observed per class: the more observations there are, the closer the fit. Thus for classes I and IV, with 998 and 434 observations respectively, the fit is fairly close, while for classes II and III, with 219 and 72 observations, the fit is less close. For I and II the estimates are lower than observed participation rates at lower firm sizes, and higher than observed at higher firm sizes. For III and IV the converse applies.

## 7. Discussion

According to the model, smaller firms participate less in R&D because expected returns increase with size, while risk of failure is independent of size, or decreases with increasing firm size, so that relative to expected returns risk is higher for smaller firms. The pattern that the model predicts

TABLE IV
Estimates of $\alpha$

| SIC class | SIC groups | SIC codes | Estimate of $\alpha$ | Number of firms observed |
|---|---|---|---|---|
| I | 1, 2, 3, 7, 8, 11, 12 | 20—25, 32—24, 37—39 | 35 | 998 |
| II | 4 | 26—27 | 63 | 219 |
| III | 6 | 31 | 10 | 72 |
| IV | 5, 9, 10 | 28—30, 35—36 | 14 | 434 |

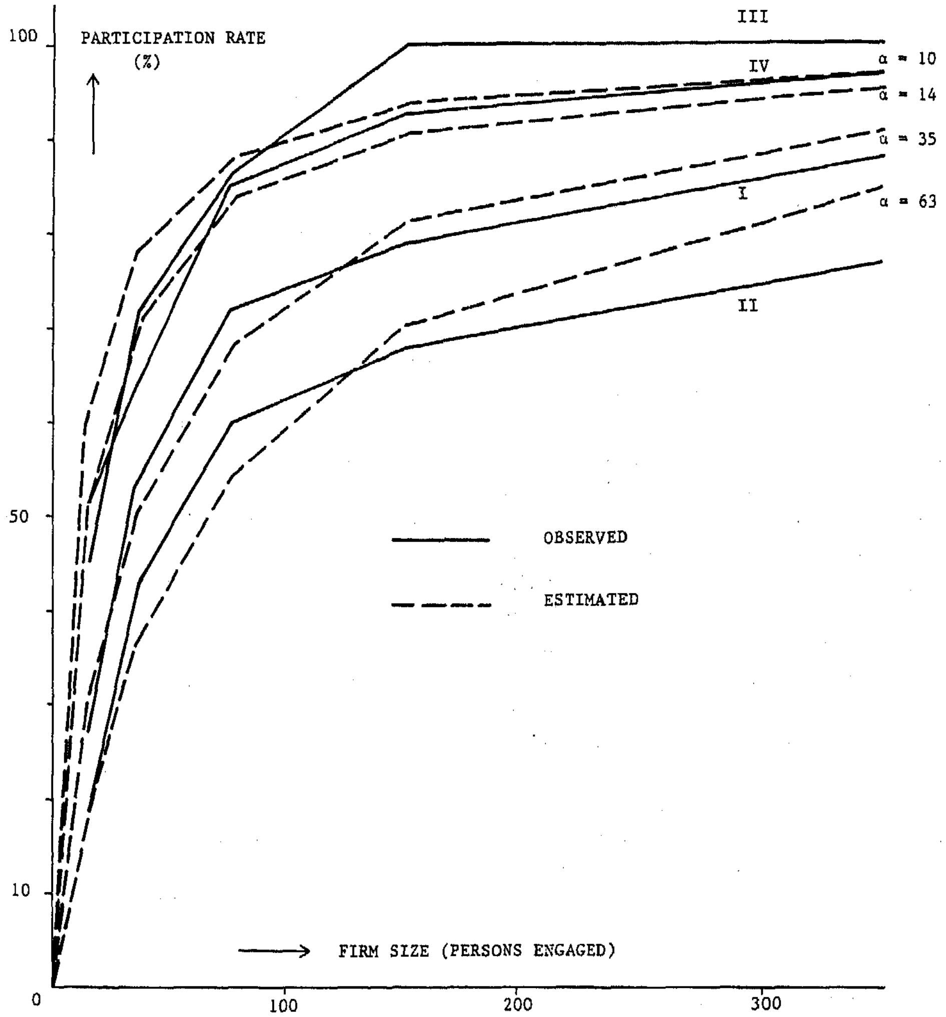


Fig. 5. Participation rates.

for the dependence of participation on size is in accordance with the data. (Controlling for industry effects on some of the key parameters.) The fact that the simplest form of the model is adequate, with the present data, is an indication, within the framework of the model, that there are no sunk threshold costs and that research effectiveness does not depend on firm size. However, this latter conclusion is based on an indirect test, contingent upon model assumptions. Furthermore, due to an insufficient number of size classes per SIC-group or -code, different SIC-groups had to be pooled for the estimation of some coefficients. If all coefficients could be estimated separately for different SIC-codes, sunk threshold costs or size effects in effectiveness might emerge for separate industries. The model was used to extrapolate R&D participation beyond the size classes for which data were available, in order to assess the effective reach of a state innovation stimulation scheme (INSTIR) in the Netherlands.

In so far as the assumptions behind the model are indeed correct, some of the implications are as follows:

— In order to compete, in an R&D technology of a given type, smaller firms have to be more efficient than larger competitors, due to the heavier burden of risk relative to returns. Small firms should seek R&D projects that are unique, in the sense that there are few, and in particular few big competitors, or projects with a shorter development period (emphasis on the D of R&D). That may be exactly what small firms are in fact doing.

- The model also implies that aggregate R&D effort, defined as total R&D expenditure divided by total resources (measured as total assets, employment or turnover), including firms which do not conduct R&D, may decline with firm size, or may increase, or may first increase and then decline, depending on the parameters of the model. This may explain the mixed results of empirical studies of the relation between R&D effort and firm size. The study demonstrates that it is conceptually and empirically useful to separate the participation rate and the input rate of R&D conducting firms, in studies of firm size effects on R&D effort. It confirms the importance of industry effects.
- A priority for further empirical research is to obtain data which allow for a control of industry effects on all parameters (including threshold costs and their sunkenness, and elasticities of returns with respect to R&D effort and firm size).
- A second priority for empirical research is to obtain data which allow for tests of hypotheses concerning the input rate as a function of firm size and a direct test of hypotheses concerning elasticities of returns with respect to R&D effort and firm size.
- One priority for more theoretical analyses is to allow for cumulative effects of R&D learning, whereby the probability of success depends on past R&D (allowing for decay due to forgetting, discontinuity in personnel and obsolescence of experience).
- A second priority would be an effort to endogenize number of competitors and firm size (as a function of R&D participation and success), in a multiperiod model.
- A third priority would be to relax the assumption of "winner takes all", allowing for product differentiation or spill-over between firms.

**Appendix A: Implications for participation rate and aggregate R&D intensity**

In (30) we find for the participation rate:

$$p = \frac{S^{1+\mu}}{S^{1+\mu} + \alpha}; \quad \alpha = \frac{\rho r}{h}. \tag{A.1}$$

For all $-1 < \mu < 1$ we then have $p = 0$ for $S = 0$, and $\lim_{S \to \infty} p = 1$.

From (A.1) we find:

$$\frac{\delta p}{\delta S} = \frac{\alpha(1+\mu)^{\mu} S}{(S^{1+\mu} + \alpha)^2}. \tag{A.2}$$

For all $-1 < \mu < 1$ we have $\lim_{S \to \infty} \frac{\delta p}{\delta S} = 0.$

$$\text{For } -1 < \mu < 0: \frac{\delta p}{\delta S} = \infty \quad \text{for} \quad S = 0.$$

$$\text{For } \mu = 0: \frac{\delta p}{\delta S} = \alpha \quad \text{for} \quad S = 0. \tag{A.3}$$

$$\text{For } 0 < \mu < 1: \frac{\delta p}{\delta S} = 0 \quad \text{for} \quad S = 0.$$

From (A.2) we find:

$$\frac{\delta^2 p}{\delta S^2} = \frac{\alpha(1+\mu)S^{\mu-1}}{(S^{1+\mu} + \alpha)^3} \times$$

$$\times \{\mu\alpha - (\mu + 2)\, S^{\mu+1}\}. \tag{A.4}$$

For $-1 < \mu \leqslant 0$ we have $\frac{\delta^2 p}{\delta S^2} < 0$

for all $S$.

For $0 < \mu < 1$ we find a point of inflection

$$\frac{\delta^2 p}{\delta S^2} > 0 \quad \text{for} \quad S < \text{some } S^*,$$

$$\frac{\delta^2 p}{\delta S^2} = 0 \quad \text{for} \quad S = S^*,$$

$$\frac{\delta^2 p}{\delta S^2} < 0 \quad \text{for} \quad S > S^*.$$

With the same assumptions we find for aggregate relative R&D effort see (33):

$$I = \frac{\kappa S^{1+2\mu}}{S^{1+\mu} + \alpha}. \tag{A.5}$$

This yields:

for all $-1 < \mu < 1: I = \frac{\kappa}{1+\alpha}$ for $S = 1$;

$$\frac{\delta I}{\delta S} = \frac{\kappa \alpha S^{2\mu}}{(S^{1+\mu} + \alpha)^2} \times$$

$$\times \{1 + 2\mu + \mu S^{1+\mu}\}. \tag{A.6}$$

For $-1 < \mu < -0.5$ we find:

$$\operatorname*{Lim}_{S \to 0} I = \infty; \quad \operatorname*{Lim}_{S \to \infty} I = 0; \quad \frac{\delta I}{\delta S} \leqslant 0 \tag{A.7}$$

for all $S$.

For $\mu = -0.5$:

$$I = \frac{\kappa}{S^{0.5} + \alpha}; \quad \frac{\delta I}{\delta S} =$$

$$= \frac{-0.5\kappa\alpha}{S^{0.5}(S^{0.5} + \alpha)^2}, \tag{A.8}$$

$$I = \frac{\kappa}{\alpha} \quad \text{for} \quad S = 0; \quad \operatorname*{Lim}_{S \to \infty} I = 0;$$

$$\frac{\delta I}{\delta S} \leqslant 0 \quad \text{for all } S,$$

$$\operatorname*{Lim}_{S \to 0} \frac{\delta I}{\delta S} = -\infty.$$

For $-0.5 < \mu < 0$:

$$I = 0 \quad \text{for} \quad S = 0; \quad \operatorname*{Lim}_{S \to \infty} I = 0, \tag{A.9}$$

$$\operatorname*{Lim}_{S \to 0} \frac{\delta I}{\delta S} = \infty; \quad \operatorname*{Lim}_{S \to \infty} \frac{\delta I}{\delta S} = 0,$$

$$\frac{\delta I}{\delta S} \gtrless 0 \quad \text{for} \quad S \gtrless S^* =$$

$$= \left\{ \frac{-(1 + 2\mu)}{\mu} \right\}^{1/1+\mu}, \quad \text{where} \quad S^* > 0.$$

For $\mu = 0$:

$$I = \frac{\kappa S}{S + \alpha}; \quad \frac{\delta I}{\delta S} = \frac{\kappa\alpha(1 + S)}{(S + \alpha)^2}, \tag{A.10}$$

whereby:

$$I = 0 \quad \text{for} \quad S = 0; \quad \operatorname*{Lim}_{S \to \infty} I = \kappa;$$

$$\frac{\delta I}{\delta S} = \frac{\kappa}{\alpha} \quad \text{for} \quad S = 0; \quad \operatorname*{Lim}_{S \to \infty} \frac{\delta I}{\delta S} = 0.$$

For $0 < \mu < 1$:

$$I = 0 \quad \text{for} \quad S = 0; \quad \operatorname*{Lim}_{S \to \infty} I = \infty, \tag{A.11}$$

$$\frac{\delta I}{\delta S} = 0 \quad \text{for} \quad S = 0; \quad \operatorname*{Lim}_{S \to \infty} \frac{\delta I}{\delta S} = 0;$$

$$\frac{\delta I}{\delta S} > 0 \quad \text{for all } S \leqslant \infty.$$

## Notes

[1] I do not claim complete novelty on this point. Baldwin and Scott (1987, p. 86) discuss a study by Bound, Cummins, Griliches, Hall and Jaffe (1984), which indicates how empirical results may differ, depending on the inclusion of smaller or larger firms in the sample: "What the previous researchers criticized by Bound et al. have observed is that relatively few smaller firms perform R&D, and not that those small firms that do engage in R&D spend less relative to size than their larger competitors".

[2] For a discussion on this, see Nooteboom (1986). I refrain from giving references to the relevant literature on finance, behavioral theory, evolutionary theory, and organizational behaviour.

[3] For the significance of sunk costs in the context of entry, see the literature on "contestability of markets". For the concept of threshold costs in service industries, see Nooteboom (1982, 1987).

[4] Cf. a recent study by Harabi (1989) which indicates that in fact patents are perceived by business to be at the bottom of the list of measures to provide appropriability.

[5] This reflects the thesis of a "dynamic complementarity" between small and large business; see Rothwell and Zegveld (1982), Nooteboom (1984) and Rothwell (1985).

[6] Cf. Markowitz (1952, 1959); Sinn (1983).

[7] This model was also applied in a study of the adoption of computers in relation to firm size in Nooteboom (1989).

[8] In a $\mu - \sigma$ framework: the corresponding indifference curve is non linear.

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