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MEASUREMENT SCALES AND RESOLUTION IV DESIGNS: A NOTE

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SYNOPTIC ABSTRACT

Three measurement scales and models are considered, namely standardized, original, and centered. It is argued that the effects at the midpoint (of the experimental area) are relevant. Effects per unit and effects over the whole range are distinguished. Resolution IV designs permit unbiased estimation of these effects, even if quantitative factors are present.

Key Words and Phrases: scaling, standardization, centering,  
midpoint, quantitative factors.

### 1. INTRODUCTION.

This note discusses two issues:

- (i) The use of different scales, namely the original, the standardized, and the centered scales. Each scale gives different values for "the" effects (or regression parameters).
- (ii) Resolution IV designs give unbiased estimators of the standardized and the centered main effects, but not of the original main effects.

Textbooks on experimental design do not pay much attention to scaling effects. Their designs are always presented in standardized form; that is, the original factors (say)  $z_j$  are scaled so that the standardized factors  $x_j$  satisfy  $-1 \leq x_j \leq 1$  (also see equation (4) below). The technical literature shows that the interpretation of standardization remains controversial even today; that is, the mathematics is simple, but its meaning takes us outside the area of mathematical proofs and into the fuzzy area of "semantics"; see Arnold (1986, pp. 96-100, 169-175), Box and Draper (1987, p. 667), Box and Hunter (1957, p. 221), Gunst (1983, pp. 2224, 2237-2241), Hocking (1983, pp. 235-236), Hocking (1984), Mendenhall (1963, pp. 221-229, 251-257), Mihram (1972, pp. 359-360), Snee and Marquardt (1984), etc. The social sciences, especially sociology and psychology, pay much more attention to the issue of choosing a correct scale; see Ghiselhi, Campbell and Zedeck (1981), Krantz, Luce, Suppes, and Tversky (1971), Suppes and Zinnes (1963), etc. We show that different scales give different conclusions. Moreover, if interactions are possible, then "Resolution IV" designs (defined in Section 4) are popular in the literature and in practice. However, Seely and Birkes (1984, pp. 84-85) state: "an aspect of the resolution IV property that oftentimes appears to be neglected in the literature [is] ... there is no guarantee that the

resolution IV property will carry over to a different parametrization". Actually, Kleijnen (1987, p. 308) states that Resolution IV designs do not preserve their acclaimed property in case quantitative factors are present in the experiment; this note, however, withdraws that statement; for a correct statement see Section 4.

## 2. THREE MEASUREMENT SCALES.

The following three measurement scales or models are relevant. The original model is

$$y = \gamma_0 + \sum_{j=1}^k \gamma_j z_j + \sum_{j=1}^{k-1} \sum_{j'=j+1}^k \gamma_{jj'} z_j z_{j'} \quad (1)$$

where  $z_j$  ( $j=1, \dots, k$ ) are the actual values of the factor variables, for which

$$L_j \leq z_j \leq H_j \quad (-\infty < L_j < H_j < \infty). \quad (2)$$

The standardized model is

$$y = \beta_0 + \sum_{j=1}^k \beta_j x_j + \sum_{j=1}^{k-1} \sum_{j'=j+1}^k \beta_{jj'} x_j x_{j'} \quad (3)$$

where

$$x_j = \frac{z_j - b_j}{a_j} \quad \text{with } a_j = \frac{H_j - L_j}{2} \quad \text{and } b_j = \frac{H_j + L_j}{2} \quad (4)$$

and

$$-1 \leq x_j \leq 1 \quad (j=1, \dots, k). \quad (5)$$

The centered model is

$$y = \delta_0 + \sum_{j=1}^k \delta_j (z_j - \bar{z}_j) + \sum_{j=1}^{k-1} \sum_{j'=j+1}^k \delta_{jj'} (z_j - \bar{z}_j)(z_{j'} - \bar{z}_{j'}) \quad (6)$$

where

$$\bar{z}_j = \frac{1}{n} \sum_{i=1}^n z_{ij} \quad (7)$$

and  $z_{ij}$  denotes the value of factor  $j$  in observation  $i$  ( $i=1, \dots, n$ ). Note that we restrict ourselves to balanced experiments where  $\bar{z}_j = (L_j + H_j)/2 = b_j$ ; in practice most designs are balanced.

If we are interested in prediction only, then it does not matter which model we use: it is easy to prove that  $y(\hat{\beta}) = y(\hat{\gamma}) = y(\hat{\delta})$ ; see Kleijnen (1987, p. 345). However, we may also use the model for explanation, for example, to answer questions like "what is the effect of a change in factor 1?". We distinguish between the "effect of factor 1 when the other factors are at their midpoints ( $z_2 = \bar{z}_2, \dots, z_k = \bar{z}_k$ )" and the "effect of factor 1 when the other factors are at the origin ( $z_2 = 0, \dots, z_k = 0$ )". This origin may be outside  $[L_2, H_2] \times \dots \times [L_k, H_k]$ . For example, in a medical experiment  $z_2$  may refer to body temperature and range between  $36^\circ \text{C}$  and  $42^\circ \text{C}$ . Then the effect of  $z_1$  when  $z_2 = 0$  is of no interest. (We return to this issue in Section 3.)

For the "main" or "first order" effects  $\beta_j$ ,  $\gamma_j$  and  $\delta_j$  equations (8), (9), and (10) hold, which can be seen as follows. From (3) we see that  $\beta_j$  is the effect of factor  $j$  at the point where all other factors  $j'$  ( $j' \neq j$ ) are zero in the standardized scale:  $x_{j'} = 0$ . Likewise (1) shows that  $\gamma_j$  is the effect of factor  $j$  if all other factors are zero in the original scale. Finally, (6) shows that  $\delta_j$  is the effect of factor  $j$  if all

other factors are at their midpoints  $\bar{z}_j$ , in the centered scale. Briefly:

$$\beta_j = \left. \frac{dy}{dx_j} \right]_{x_j' = j(x_j,=0)} \quad (8)$$

$$\gamma_j = \left. \frac{dy}{dz_j} \right]_{z_j' = j(z_j,=0)} \quad (9)$$

$$\delta_j = \left. \frac{dy}{dz_j} \right]_{z_j' = j(z_j,=\bar{z}_j)} \quad (10)$$

Upon substitution of (5) into (6) we obtain

$$y = \delta_0 + \sum_{j=1}^k \delta_j a_j x_j + \sum_{j=1}^{k-1} \sum_{j'=j+1}^k \delta_{jj'} a_j a_{j'} x_j x_{j'}$$

which, combined with (3), implies

$$\beta_j = \delta_j a_j \quad (11)$$

Equation (11) gives a relationship between the standardized model (3) and the centered model (6). The relationship between the original model (1) and the centered model (6) follows from computing  $dy/dz_j$  at the point  $z_j,=0$  for model (6) and at  $z_j, = \bar{z}_j$ , for model (1) respectively; this yields

$$\gamma_j = \delta_j - \sum_{j' \neq j} \delta_{jj'} b_{j'} \quad (12)$$

and

$$\delta_j = \gamma_j + \sum_{j' \neq j} \gamma_{jj'} b_{j'} \quad (13)$$

### 3. INTERPRETATION.

Before we give an example in Section 4, we interpret the mathematical results of Section 2. First, note that the experimenter is interested in the effect of (say) a temperature change of one degree Centigrade, not in a change of one "standard" degree; only the original and the centered model use Centigrade (see (1) and (6)).

Second, the experimenter is interested in the effect of such a unit change at the midpoint of the experimental area ( $\bar{z}_j$ ), not at the origin ( $\bar{z}_j = 0$ ), in general. For example, if in a medical experiment  $z_j$  ranges between  $36^\circ$  and  $42^\circ$ , then the effect of any other variable at  $0^\circ$  is irrelevant. A (rare) counter-example is an experiment in which it is technically infeasible to make observations at zero degrees Kelvin (approximately  $-273^\circ\text{C}$ ); then we may restrict the experimental area to (say)  $-250^\circ \leq z_1 \leq -100^\circ$  and from the experimental results we extrapolate the effect at the point  $z_1 = -273^\circ$ .

Third,  $\delta_j$  (and also  $\gamma_j$ ) measures the change per unit change of the original variable  $z_j$ . The total change as  $z_j$  varies over the experimental area  $[L_j, H_j]$ , equals the product of  $\delta_j$  and  $H_j - L_j$ . Consequently, if factor  $j$  has a wider range  $R_j = H_j - L_j$  than factor  $j'$  has, then the total effect of  $z_j$  may be higher than the total effect of  $z_{j'}$ , even if  $\delta_j < \delta_{j'}$ ; also see Kleijnen (1987, pp. 141-142). Now (5) and (11) yield

$$\delta_j(H_j - L_j) = \delta_j (2 a_j) = 2 \beta_j. \quad (14)$$

So the unit effect is best measured by  $\delta_j$ ; the relative "importance" (total effect) of a factor is best measured by the relative  $\beta$ 's and the "original" effects  $\gamma_j$  are misleading.

Note that the ranges may be unknown at the beginning of an experiment. For example, in Response Surface Methodology (RSM) the quantitative factors are to be optimized. Therefore local experiments are combined with the steepest ascent technique applied to first-order models (where  $\beta_{jj'} = \gamma_{jj'} = \delta_{jj'} = 0$ ). Kleijnen (1988) measures the importance of different factors through  $\gamma_j \bar{z}_j$  where  $\bar{z}_j$  is the midpoint of  $z_j$  in a sequence of local experiments  $s = 1, 2, \dots$  (so  $\gamma_j$  and  $\bar{z}_j$  should be indexed by  $s$ ).

Fourth, in the presence of interactions we must qualify our answer to the question "What is the effect of factor  $j$ ?: we must answer "the effect of factor  $j$ , if the values of the other factors are... ". So we should address the following issues:

- (i) "Are there significant interactions?" ( $\beta_{jj'}, \gamma_{jj'}, \delta_{jj'}$  have identical  $t$ -statistics; it does not matter which one is tested; see Kleijnen, van den Burg, and van der Ham (1979, p. 62)).
- (ii) "If the other factors are at their midpoints ( $z_{j'} = \bar{z}_{j'}$ ), then the estimated effect of a unit change in factor  $j$  is  $\hat{\delta}_j$ "; see (10). ( $\hat{\delta}_j$  equals  $\hat{\beta}_j/a_j$ ; the estimate  $\hat{\delta}_j$  is significant if and only if  $\hat{\beta}_j$  is (see (11)). If the standard design is orthogonal in  $x_j$  and the observations  $y_1$  are independent with common variance, then the  $\hat{\beta}_j$  are orthogonal;  $\text{cov}(\hat{\gamma}_j, \hat{\gamma}_{j'})$  equals  $b_j b_{j'} \text{cov}(\delta_{jj'})$  and is not zero.)

#### 4. RESOLUTION IV DESIGNS.

Box and Hunter(1961, p. 319) defined Resolution IV designs as experimental designs with no main effect confounded with any other main effect or two-factor interaction, but with two-factor interactions aliased with each other.



The statistical literature gives experimental designs expressed in standardized variables  $x_j$ . A Resolution IV design yields unbiased estimators  $\hat{\beta}_j$  of the standardized main effects  $\beta_j$ . These estimators  $\hat{\beta}_j$  do not give unbiased estimators of the original effects  $\gamma_j$ ; see (11) and (12) where no estimates of individual interactions are available. However, we saw that the  $\hat{\beta}_j$  do give unbiased estimators of  $\delta_j$ . The estimators  $\hat{\delta}_j$  are relevant, not the estimators  $\hat{\gamma}_j$ , since  $\delta_j$  measures the unit effect at the midpoint (see (10)) and the midpoint is of interest (not the zero point, except for very special situations; see Section 3). The  $\hat{\beta}_j$  are of interest since they measure the total effect over the range  $R_j$ ; see (14).

As a numerical example consider an s,S inventory system; that is, when at the end of a time period the inventory is below level  $s$ , an order is placed to increase the inventory up to level  $S$ . Demand during a time period is a random variable. We may simulate this system, in order to estimate the effects on the out-of-stock probability ( $y$ ) of the following six factors  $x_j$  ( $j=1, \dots, 6$ ):

Factor 1: demand distribution family. The symbol + in Table 1 denotes the exponential distribution with parameter  $\nu$  or mean  $1/\nu$ ; the symbol - denotes the uniform distribution with domain from  $1/(2\nu)$  to  $3/(2\nu)$  so that the mean remains  $1/\nu$ .

Factor 2: distribution of the delivery (or lead) time family: + refers to the exponential distribution with parameter  $\mu$ , whereas - denotes the Erlang distribution with parameters  $4\mu$  and 4 (sum of four exponential distributions); hence the mean lead time is  $1/\mu$ .

Factor 3: expected demand: - denotes the value 1; + denotes 5.

Factor 4: expected lead time: - means 3, and + denotes 7.

Factor 5: expected time between demands: - denotes 1, and + means 6. The distribution of these times is exponential.

Table 1. Input and output in s, S inventory example.

Run	Input factors $x_j$ with $j =$						output y
	1	2	3	4	5	6	
1	+	+	+	+	+	+	0.0542
2	+	+	-	+	-	-	0.0301
3	+	-	+	-	+	-	0.0502
4	+	-	-	-	-	+	0.0002
5	-	-	-	+	+	+	0.0000
6	-	-	+	+	-	-	0.7177
7	-	+	-	-	+	-	0.0000
8	-	+	+	-	-	+	0.1925
9	-	-	-	-	-	-	0.0000
10	-	-	+	-	+	+	0.0328
11	-	+	-	+	-	+	0.0328
12	-	+	+	+	+	-	0.1009
13	+	+	+	-	-	-	0.3120
14	+	+	-	-	+	+	0.0000
15	+	-	+	+	-	+	0.6572
16	+	-	-	+	+	-	0.0024

Factor 6: s value: - denotes 3, and + means 10. S is fixed at 25.

Here factors 1 and 2 are qualitative, while factors 3 through 6 are quantitative. We use a Resolution IV design, that is, we use sixteen runs (or factor combinations), namely a  $2^{6-2}$  design with generators  $5 = 23^4$  and  $6 = 13^4$  so that  $23 = 45$ ,  $24 = 35$ ,  $13 =$

Table 2. Standardized versus original versus centered scale in s,S inventory example: no interactions.

Number of the effect j	Estimated standardized main effect		Estimated original main effect		Estimated centered main effect	
	$\hat{\beta}_j$	$(\hat{\sigma}_{\hat{\beta}_j})$	$\hat{\gamma}_j$	$(\hat{\sigma}_{\hat{\gamma}_j})$	$\hat{\delta}_j$	$(\hat{\sigma}_{\hat{\delta}_j})$
0	0.136	(0.042)*	-0.036	(0.163)	0.136	(0.042)*
1	0.002	(0.042)	0.002	(0.042)	0.002	(0.042)
2	-0.046	(0.042)	-0.046	(0.042)	-0.046	(0.042)
3	0.128	(0.042)*	0.064	(0.021)*	0.064	(0.021)*
4	0.063	(0.042)	0.032	(0.021)	0.032	(0.021)
5	-0.106	(0.042)*	-0.043	(0.017)*	-0.043	(0.017)*
6	-0.115	(0.042)	-0.004	(0.012)	-0.004	(0.012)

\* Significant at level 0.05; see standard errors shown in parentheses (df=9).

46, 14 = 36, 12 = 56, 15 = 26, 25 = 34 = 16. Table 2 displays the estimated main effects  $\hat{\gamma}_j$  and the overall mean  $\hat{\gamma}_0$  assuming all interactions are zero. Table 3 gives results if the seven two-factor interactions in the left-hand side of the preceding confounding relations are assumed to be important. Table 4 gives results if the "right-hand" interactions are important (45, 35, 46, 36, 56, 26, and 34). The estimated standard errors, shown in Tables 2, 3, and 4, are computed from the residual sum of squares; to test significance we use the Student statistic  $t_y$ .

Table 3. Standardized versus original versus centered scale in s,S inventory example: "left-hand" interactions.

Number of the effect j(jj')	Estimated standardized main effect		Estimated original main effect		Estimated centered main effect	
	$\hat{\beta}_j(jj')$	$(\hat{\sigma}_{\hat{\beta}_j(jj')})$	$\hat{\gamma}_j(jj')$	$(\hat{\sigma}_{\hat{\gamma}_j(jj')})$	$\hat{\delta}_j$	$(\hat{\sigma}_{\hat{\delta}_j(jj')})$
0	0.136	(0.007)*	-0.036	(0.026)	0.136	(0.007)*
1	0.002	(0.007)	0.045	(0.023)	0.002	(0.007)
2	-0.046	(0.007)*	0.205	(0.023)*	-0.046	(0.007)*
3	0.128	(0.007)*	0.064	(0.003)*	0.064	(0.003)*
4	0.063	(0.007)*	0.032	(0.003)*	0.032	(0.003)*
5	-0.106	(0.007)*	-0.043	(0.003)*	-0.043	(0.003)*
6	-0.015	(0.007)*	-0.004	(0.002)	-0.004	(0.002)
23	-0.054	(0.007)*	-0.027	(0.003)*	-0.027	(0.003)*
24	-0.099	(0.007)*	-0.049	(0.003)*	-0.049	(0.003)*
13	0.002	(0.007)	0.001	(0.003)	0.001	(0.003)
14	-0.015	(0.007)	-0.008	(0.003)	-0.008	(0.003)
12	0.007	(0.007)	0.007	(0.007)	0.007	(0.007)
15	-0.005	(0.007)	-0.002	(0.003)	-0.002	(0.003)
25	0.055	(0.007)*	0.022	(0.003)*	0.022	(0.003)*

\* Significant at level 0.05; see standard errors shown in parentheses (df=2).

with degrees of freedom  $\nu$  equal to 16-7 and 16-14 respectively. Tables 2, 3, and 4 show that in a Resolution IV design the standardized main effects  $\hat{\beta}_1$  through  $\hat{\beta}_6$  and the centered main

Table 4. Standardized versus original versus centered scale in s,S inventory example: "right-hand" interactions.

Number of the effect j(jj')	Estimated standardized main effect $\hat{\beta}_j(jj')$ ( $\hat{\sigma}_{\hat{\beta}_j(jj')}$ )	Estimated original main effect $\hat{\gamma}_j(jj')$ ( $\hat{\sigma}_{\hat{\gamma}_j(jj')}$ )	Estimated centered main effect $\hat{\delta}_j$ ( $\hat{\sigma}_{\hat{\delta}_j(jj')}$ )
0	0.136 (0.007) *	-0.242 (0.061)	0.136 (0.007) *
1	0.002 (0.007)	0.002 (0.007)	0.002 (0.007)
2	-0.046 (0.007) *	-0.036 (0.014)	-0.046 (0.007) *
3	0.128 (0.007) *	0.079 (0.012) *	0.064 (0.003) *
4	0.063 (0.007) *	0.026 (0.010)	0.032 (0.003) *
5	-0.106 (0.007) *	0.065 (0.010) *	-0.043 (0.003) *
6	-0.015 (0.007)	-0.002 (0.007)	-0.004 (0.002)
45	-0.054 (0.007) *	-0.011 (0.001) *	-0.011 (0.001) *
35	-0.099 (0.007) *	-0.020 (0.001) *	-0.020 (0.001) *
46	0.002 (0.007)	0.000 (0.001)	0.000 (0.001)
36	-0.015 (0.007)	-0.002 (0.001)	-0.002 (0.001)
56	0.007 (0.007)	0.001 (0.001)	0.001 (0.001)
26	-0.005 (0.007)	-0.002 (0.002)	-0.002 (0.002)
34	0.055 (0.007) *	0.014 (0.002) *	0.014 (0.002) *

\* Significant at level 0.05; see standard errors shown in parentheses (df=2).

effects  $\hat{\delta}_1$  through  $\hat{\delta}_6$  are not affected by the inclusion of two-factor interaction; the original main effects  $\hat{\gamma}_1$  through  $\hat{\gamma}_6$ , however, are influenced.

Another example is found in Kleijnen, van den Burg, and van der Ham (1979) where a harbor was simulated: the  $t$  statistic (with 128 degrees of freedom) for one of the standardized factors is 30.85; for the original factor this value is only 0.06.

#### 5. CONCLUSION.

Experimenters should center their independent variables as in (6), because unit effects  $\delta_j$  at the midpoint  $\bar{z}$  are of interest, in general. Resolution IV designs give unbiased estimators of these "centered" main effects  $\delta_j$  (even if the factors are quantitative so that the standard designs must be linearly transformed as in (5)). These designs also give unbiased estimators of the standardized effects  $\beta_j$  which measure the total effects over the ranges  $R_j$ . When optimizing, the importance of different factors is measured through  $\gamma_j \bar{z}_j$  where  $\bar{z}_j$  is the midpoint of  $z_j$  in the local experiment, and  $\gamma_j$  is the local original effect.

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