
Labor Supply, Income Taxes, and Hours Restrictions in The Netherlands

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ABSTRACT

In this paper, two models of individual labor supply are discussed. The first one is the by now classical Hausman-type model with convex piecewise linear budget constraints, in which both random preferences and optimization errors are incorporated by means of normally distributed random variables. Estimated coefficients are plausible but the model has the shortcoming that unemployment for males is not captured and that the simulated hours distribution misses the spikes in the sample distribution of working hours. Therefore, an alternative model is introduced which explicitly takes into account demand side restrictions on working hours. The difference with the standard model is the replacement of the optimization error by the assumption that each individual can choose from a finite set of wage hours packages and either picks the job offer yielding highest utility or decides not to work. It turns out that this model captures the sample distribution of working hours very well, for males as well as females. Wage and income elasticities according to the two models are similar and in line with other recent findings in The

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Netherlands. Dead weight loss calculations for the second model which explicitly take the hours restrictions into account, imply that the dead weight loss is much smaller than as calculated with the standard model.

I. Introduction

Due to the pioneering work of Jerry Hausman, the treatment of piecewise linear convex budget constraints in the analysis of labor supply is now a rather standard practice. See, for example, Hausman (1980, 1981a), Blomquist (1983) and Moffitt (1986). In Section II of this paper we replicate this type of analysis on data for The Netherlands. Both the labor supply of women and men is studied for a cross-section of Dutch households drawn in 1985. Some simulations based on the estimation results are performed to calculate elasticities and dead weight losses.

Although the standard model comes out with plausible coefficients and the results are well in line with earlier findings in The Netherlands, a simple simulation reveals that various features of the data are not reproduced. In particular, the model overpredicts employment¹ of males, whereas for females the employment percentage of various education groups is badly tracked by the model. A comparison of the distribution of hours worked in the sample with the hours distribution generated by the model makes clear that the simulated distribution is far too smooth. In particular, the model misses spikes at 40 hours a week for males and at 40, 20, and 32 hours a week for females.

All this suggests that, at least in The Netherlands, the assumption underlying the model that observed hours mainly reflect the outcome of unrestricted choices by individuals is incorrect. In Section III, we introduce a simple reduced form model of the demand side of the labor market, in which employers offer wage hours packages and each individual can choose from a limited number of such offers. Out of these jobs, either the individual chooses the one yielding the highest utility level, or he or she decides not to work. It is also possible that the person is unemployed because he or she has received zero offers.

It turns out that this adjustment of the standard model makes a dramatic difference in terms of the explanation of unemployment among males and

1. Throughout this paper, we use the following terminology. An individual is employed if he or she works any positive number of hours and unemployed if he or she works zero hours, either voluntarily or involuntarily. The employment and unemployment rates are defined as fractions of the sum of the numbers of employed and unemployed people (i.e., including nonparticipants).

the fit of the generated hours distribution to the observed distribution in the sample. Interestingly, in the calculation of elasticities we obtain results that are of the same order of magnitude as in the standard model. The results of the extended model suggest that for females the distribution of desired hours is situated to the left of the distribution of hours offered by the employers. This mismatch between both distributions is a possible explanation of the low participation rate of women in The Netherlands in comparison with most other developed countries. The results imply that if women could all work their desired number of hours, the employment rate of married females would rise from about 40 percent to about 80 percent, although most of them would choose to work 16 hours a week or less.

We introduce a generalized dead weight loss measure which explicitly takes the hours restrictions into account. Dead weight loss calculations for the extended model suggest that the efficiency loss due to the tax system is much smaller than as calculated with the standard model.

All these results should be interpreted with caution. Although the model in Section III in some respects certainly performs better than the model in Section II, it does not yield a perfect description of labor supply behavior in The Netherlands. Test results show that misspecification is still present. In Section IV, some possible future extensions and improvements of the model are suggested. The main contribution of the present paper is perhaps not a reliable conclusion about “the true labor supply elasticities” or a guideline for tax reforms. Much more, it is another warning against the temptation to stick to one particular model, without carefully investigating whether this model is able to explain certain features of the data to a sufficient extent.

II. Common Model

Starting point of the analysis is a modified version of the model introduced by Hausman (1981a):

$$(2.1) \quad h_{ij}^* = \beta w_{ij} + \delta N_{ij} + X_j' \alpha + \varepsilon_j$$

$$h_j^* = 0 \quad \text{if } h_{1j}^* < 0$$

$$(2.2) \quad = h_{ij}^* \quad \text{if } H_{i-1,j} \leq h_{ij}^* \leq H_{ij}, i = 1, \dots, s$$

$$= H_{ij} \quad \text{if } h_{ij}^* > H_{ij} \text{ and } h_{i+1,j}^* < H_{ij}, i = 1, \dots, s - 1$$

$$= T \quad \text{if } h_{sj}^* > H_{sj}$$

$$(2.3) \quad h_j = 0 \quad \text{if } h_j^* + v_j < 0 \text{ or } h_j^* = 0$$

$$= h_j^* + v_j \quad \text{if } h_j^* + v_j \geq 0 \text{ and } h_j^* > 0$$

where

h_{ij}^* = utility maximizing number of working hours for individual j on the line containing budget segment i

w_{ij} = virtual hourly wage rate of individual j on budget segment i

N_{ij} = virtual nonlabor income of individual j for budget segment i

X_j = vector of exogenous demographic variables of individual j

ε_j = random variable representing preference variation which is not explained by X_j

h_j^* = desired number of working hours

H_{ij} = kink-points of the budget constraint ($H_{0j} = 0, H_{sj} = T$)

T = total time available

h_j = observed number of working hours of individual j

v_j = random variable representing measurement or optimization errors

s = the number of budget segments

β, δ, α = parameters.

The main difference between this model and the one in the Hausman (1981a) paper is that random preferences (ε_j) are included in the constant term rather than in the income coefficient β . The error terms (ε_j, v_j)' are assumed to be drawn from the bivariate normal distribution with mean $(0, 0)'$ and covariance matrix

$$\begin{pmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix}.$$

The corresponding direct utility function and expenditure function are given by

$$(2.4) \quad U(h, c) = -\delta \frac{h - \delta c - X'\alpha - \varepsilon}{\beta - \delta h} - \log(\beta - \delta h)$$

and

$$(2.5) \quad e(w, u) = \{-e^{-\delta w - u} + \beta - \beta\delta w - \delta(X'\alpha + \varepsilon)\}/\delta^2$$

respectively, where c in (2.4) denotes consumption and u in (2.5) denotes the utility level. The direct utility function is defined and quasi-concave on the set $\{(h, c); \beta - \delta h > 0\}$, which contains the set $\{(h, c); h \geq 0\}$ if $\beta > 0$ and $\delta < 0$. In this case the model is also coherent in the sense that (2.1), (2.2), and (2.3) yield exactly one solution h_j for all ε_j and v_j . (See Gourieroux et al. 1980 and Van Soest et al. 1988 for a more general treatment of coherency of limited dependent variable models and the paper by MaCurdy in this issue, in which coherency of the kinked budget constraint model is discussed).

A. Data and Estimation Results

The data we used stem from a labor mobility survey carried out in The Netherlands in 1985 under auspices of the Organization of Strategic Labor Market Research (OSA). The sample contains information on 849 families consisting of at least husband and wife. Some sample statistics are mentioned in Table 1. The sample contains 315 families in which both partners are employed. In 486 families only the husband is employed and in 16 cases only the female works. In the remaining 32 families both partners are unemployed. After-tax wage rates of employed individuals were not directly observed but constructed from hours worked and after-tax labor income. See Appendix A for details on these variables. Before-tax wage rates were then calculated by using an approximation of the Dutch income tax system. Some simplifying assumptions were necessary because the data set did not contain all the necessary information on deductables, health insurance premiums, etc.

Making use of the computed before-tax wage rates of workers, a before-tax wage equation allowing for the possibility of selection bias was estimated for males and females separately. The explanatory variables used were $\log(\text{age})$ (*LAGEM* and *LAGEF* for males and females, respectively), $\log(\text{age})$ squared, dummies referring to different education levels (*EDM* and *EDF*), and an index variable referring to the sector of education (*EDSECM* and *EDSECF*: 2 = technical or business, 1 = semi-technical or semi-business, 0 = neither technical nor business).

Estimation results appear in Appendix A. Making use of actual before-tax wage rates for workers and predicted before-tax wage rates for unemployed individuals, a convex piecewise linear approximation of the budget set was constructed for each person.² Again, lack of information required that we simplify assumptions. Minor nonlinearities and nonconvexities due to, for example, thresholds in deductables were ignored, as well as unemployment benefits; only the basic system of at most eleven income brackets was explicitly taken into account.

The model was estimated by maximum likelihood using the algorithm of Berndt et al. (1974). The calculation of bivariate probabilities was based

2. Although this is not an uncommon procedure, there are various difficulties associated with it. Given the way in which wages are constructed, they are bound to suffer from measurement error (which may be correlated with measurement error in the observed hours). In particular, we ignore the possibility that overtime and second jobs are paid at a different rate than the primary job. As a result, the budget constraints suffer from measurement error. The use of predicted wages for individuals who do not work also leads to a misrepresentation of the true budget sets of these individuals. The solution to these problems is far from trivial, and beyond the scope of the present paper. See also Section IV.

Table 1
Sample Statistics

Variable	Mean	Standard Deviation	Minimum	Maximum	Number
<i>LOGFS</i> (logarithm of family size)	1.18	0.37	0.69	2.30	849
<i>DCH</i> < 6 (dummy children younger than 6)	0.26	0.44	0	1	849
<i>LAGEM</i> (logarithm of age, male)	3.65	0.26	3.00	4.14	849
<i>LAGEF</i> (logarithm of age, female)	3.58	0.27	2.89	4.11	849
<i>L2AGEM</i> (<i>LAGEM</i> -squared)	13.38	1.87	8.97	17.17	849
<i>L2AGEF</i> (<i>LAGEF</i> -squared)	12.88	1.94	8.35	16.90	849
<i>EDM</i> (education level male)	2.78	1.08	1	5	849
<i>EDF</i> (education level female)	2.35	1.01	1	5	849
<i>EDSECM</i> (education sector male)	0.95	0.99	0	2	849
<i>EDSECF</i> (education sector female)	0.29	0.70	0	2	849
<i>WRATM</i> (after-tax wage rate, male) ^a	15.97	5.80	6.87	59.47	801
<i>WRATF</i> (after-tax wage rate, female) ^a	12.54	4.53	5.81	39.38	331
<i>WRBTM</i> (before-tax wage rate, male) ^a	27.90	13.80	8.94	174.55	801
<i>WRBTF</i> (before-tax wage rate, female) ^a	19.27	8.06	7.35	60.65	331
<i>HM</i> (working hours per week, male) ^a	42.07	6.70	4	71	801
<i>HF</i> (working hours per week, female) ^a	27.29	12.52	2	60	331

a. Working individuals only.

upon a formula given by Abramowitz and Stegun (1970, p. 940). Estimation results are given in Table 2. The conditions $\beta > 0$ and $\delta < 0$ which are sufficient for coherency appeared not to be binding. The estimates for the wage rate coefficients are significantly positive, implying that labor supply is forward bending, for both males and females. The income effects have the expected negative sign, and for females the coefficient differs significantly from 0. Both the wage and the income effects are stronger for

Table 2
Estimation Results of the Common Model (standard errors in parentheses)

Parameter	Males		Females	
β (wage rate)	0.51	(0.17)	1.29	(0.44)
δ (unearned income)	-0.0055	(0.0035)	-0.0080	(0.0039)
α_0 (constant term)	-153	(104)	-489	(203)
α_1 (LOGFS)	0.18	(1.77)	-31.8	(7.0)
α_2 (DCH < 6)	-0.94	(1.33)	-21.4	(5.1)
α_3 (LAGE)	108	(58)	325	(120)
α_4 (L2AGE)	-15.3	(7.9)	-50.0	(17.3)
σ_ϵ (random preference)	12.93	(0.49)	19.19	(5.36)
σ_ν	3.22	(1.40)	12.77	(3.23)

females than for males.³ The impact of family characteristics (family size and the presence of young children) is insignificant for males and strongly significant for females. The estimates imply that, *ceteris paribus*, labor supply is maximal at 34 and 26 years of age for males and females, respectively. The estimated standard deviations of the random variables ϵ and ν are significantly different from zero. For males in particular, the results suggest that random preferences are the most important source of random variation of observed working hours.

B. Simulations and Computation of Dead Weight Loss

Table 3 provides sample means and a simulation of the actual 1985 situation in order to see to what extent the model is able to describe the data. This table gives means of hours worked and average employment rates for males and females divided into groups according to various individual characteristics. Sample means of hours worked (zeroes included) in Column 2 can be compared with simulated means (Column 3) in which both sources of random variation are taken into account.⁴ The results show, for

3. Since wage and income elasticities according to this model vary in a strongly nonlinear way, we do not discuss elasticities for specific individuals. Instead, we present elasticities of the average numbers of working hours and the average employment rates in the next subsection. These elasticities are computed from simulation results.

4. The figures in the table are based on 10 random drawings of ϵ and ν for each individual. The results appear to be very insensitive with respect to the chosen number of drawings per individual.

Table 3
Simulation of the Actual 1985 Situation

Males	1	2	3	4	5	6
All males	849	39.70	39.40	0.943	0.999	0.999
Education level						
1	134	37.69	39.31	0.903	0.999	0.999
2	167	37.05	38.94	0.910	0.999	0.998
3	342	40.54	39.05	0.965	0.999	0.999
≥ 4	206	41.75	40.42	0.961	1.000	1.000
Age						
<30	154	39.60	39.09	0.955	0.999	0.999
30-39	300	40.83	40.40	0.960	1.000	1.000
40-49	225	39.96	39.94	0.924	0.999	0.999
≥ 50	170	37.43	37.23	0.929	0.999	0.998
Family size						
2	263	39.98	39.18	0.958	1.000	1.000
3	140	38.71	39.30	0.929	0.999	0.998
4	282	38.85	39.75	0.929	0.999	0.999
≥ 5	164	41.52	39.25	0.957	0.999	0.999
Females						
All females	849	10.64	9.94	0.390	0.487	0.412
Education level						
1	217	6.73	7.75	0.290	0.403	0.332
2	223	6.94	8.77	0.278	0.452	0.374
3	322	13.26	11.30	0.450	0.522	0.452
≥ 4	87	20.18	13.38	0.701	0.660	0.557
Age						
<30	226	20.35	18.25	0.593	0.709	0.637
30-39	296	7.84	8.34	0.334	0.445	0.372
40-49	212	7.66	5.71	0.330	0.388	0.306
≥ 50	115	4.25	5.53	0.243	0.344	0.266
Family size						
2	263	23.32	21.30	0.711	0.821	0.741
3	140	6.71	7.64	0.329	0.486	0.390
4	282	5.65	5.24	0.277	0.371	0.289
≥ 5	164	2.25	1.76	0.122	0.153	0.113

Note: Column 1: number in the sample; 2: hours worked, sample mean; 3: hours worked, simulated; 4: employment, sample; 5: desired employment, simulated; 6: actual employment, simulated.

instance, that for females, the model captures the differences in hours worked between people of different levels of education to some extent but not completely. This may be a consequence of the fact that education was not included as an explanatory variable in the vector X of individual characteristics but it may also be due to the restrictive way in which hours are allowed to depend on the wage rate. The differences in the average numbers of hours worked for different age levels and family sizes appear to be well captured. For males, there are hardly any differences left to be explained.

The other columns of Table 3 refer to employment rates: the sample employment rate (Column 4), the simulated employment rates with only random preferences (Column 5), and with both sources of random variation taken into account (Column 6). The numbers in Column 5 may be interpreted as probabilities of desired employment. Actual observed employment may differ from this because of the error sources included in v , i.e., demand-side restrictions, measurement errors, suboptimal behavior, etc. For females, predicted employment is slightly larger than the observed employment in the sample. Again, the fact that differences in preferences between females of different education levels are not fully captured by the model becomes apparent. For males, the model appears unable to explain unemployment. The predicted employment rate exceeds 0.997 for all groups, whereas the actual employment rate in the sample is 0.944. This shortcoming of the model may be due to the fact that fixed costs of working are not incorporated or to the fact that demand-side restrictions as a source of involuntary unemployment are not explicitly taken into account.

Table 4 shows the consequences of a 10 percent increase of all after-tax wage rates. The aggregate average number of hours worked would increase by 6.6 percent for females and by 1.2 percent for males. In relative terms, the change is similar for groups with different education levels, age, or family size. The relatively large increase of female working hours is partly explained by the rise in the average employment probability of 5.6 percent. For males, the simulated employment rate was already almost equal to one before the wage increase and therefore hardly changes. A simulation of a 10 percent decrease of after-tax wage rates yields results that are almost symmetric to the results presented in Table 4. The aggregate average of hours worked falls by 6.3 percent for females and 1.2 percent for males.

Income elasticities are computed in the same way. A 10 percent rise of all unearned incomes (virtual incomes due to the tax system excluded) leads to a fall in average hours worked for males of only 0.1 percent. For females, the fall is 2.3 percent, mainly because the employment rate falls by 2.1 percent.

Table 4
Simulation of a 10% Increase of After-Tax Wage Rates

	Males			Females		
	1	2	3	1	2	3
All	39.89	0.999	0.999	10.59	0.513	0.434
Education level						
1	39.77	0.999	0.999	8.26	0.427	0.350
2	39.40	0.999	0.999	9.36	0.478	0.393
3	39.54	0.999	0.999	11.99	0.547	0.476
≥ 4	40.94	1.000	1.000	14.35	0.691	0.595
Age						
< 30	39.53	0.999	0.999	19.09	0.729	0.654
30–39	40.88	1.000	1.000	8.97	0.470	0.396
40–49	40.44	0.999	0.999	6.29	0.419	0.333
≥ 50	37.74	0.999	0.999	5.98	0.372	0.290
Family size						
2	39.64	1.000	1.000	22.23	0.837	0.762
3	39.78	0.999	0.999	8.36	0.524	0.424
4	40.25	0.999	0.999	5.81	0.403	0.314
5	39.76	0.999	0.999	2.04	0.174	0.126

Note: Column 1: hours worked, simulated; 2: desired employment, simulated; 3: actual employment, simulated

Whereas Table 3 contains information on average numbers of working hours, Figures 1 and 2 refer to the actual and simulated distributions of working hours. Frequencies of zero hours of work are not included in the figures (these frequencies can be obtained from Table 3). The figures present the actual sample distributions for all males and all females, respectively, and present two different simulated hours distributions: the distribution with all sources of random variation taken into account, i.e., the distribution of $h_j - s$ given by (2.3), and the distribution of the $h_j^* - s$ given by (2.2). The latter can be interpreted as the distribution of desired working hours, since it does not take into account the errors included in v_j , which reflect several sources of deviations from optimal behavior. The gap between desired hours frequencies and simulated actual frequencies for females reflects involuntary unemployment, as explained by this model: a negative realization of v_j implies that a person who prefers to

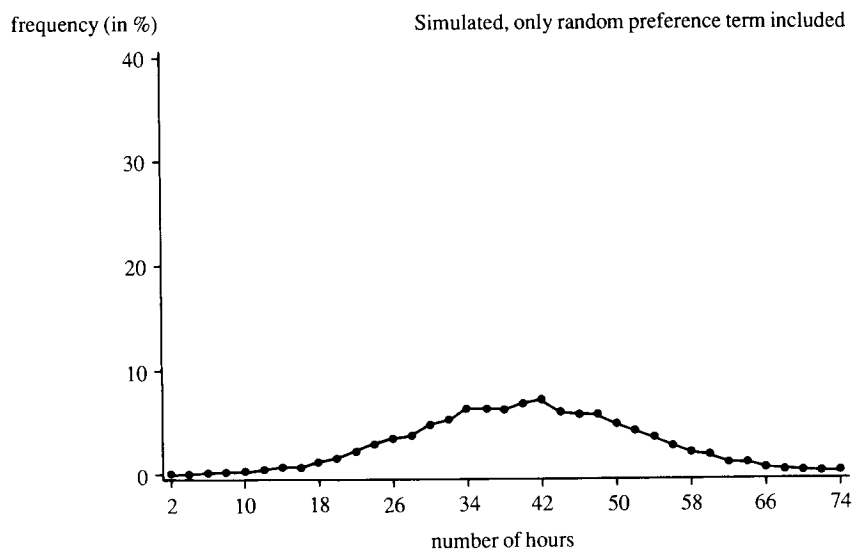
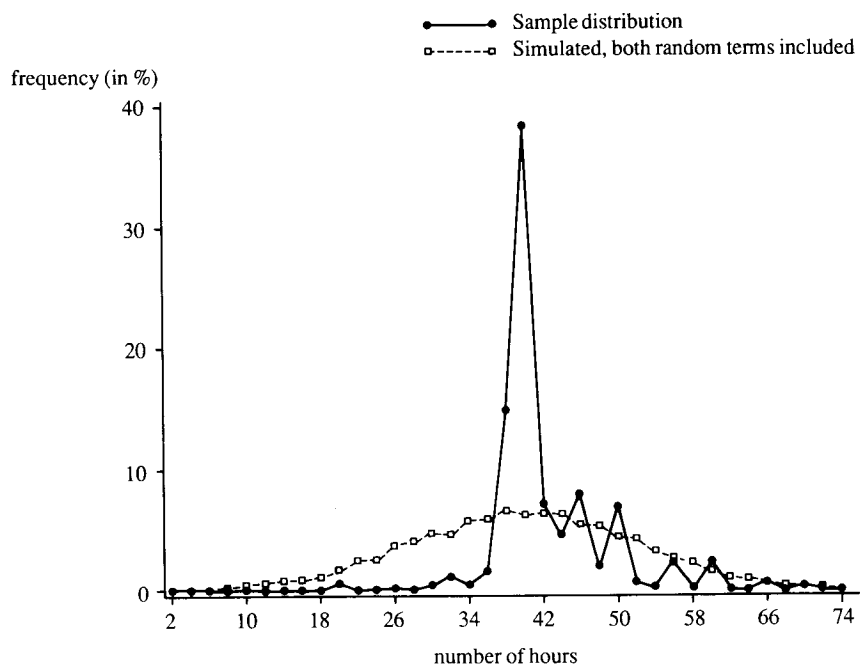


Figure 1
Distribution of working hours per week
Males, common model

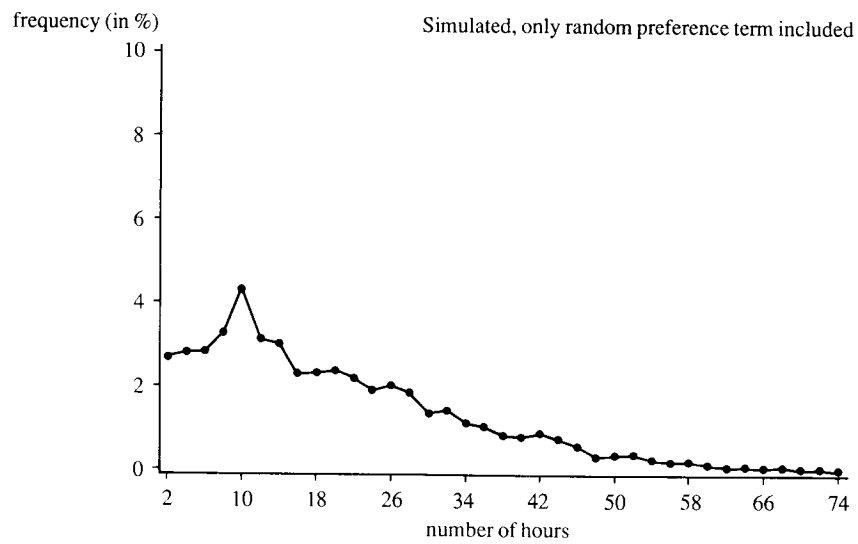
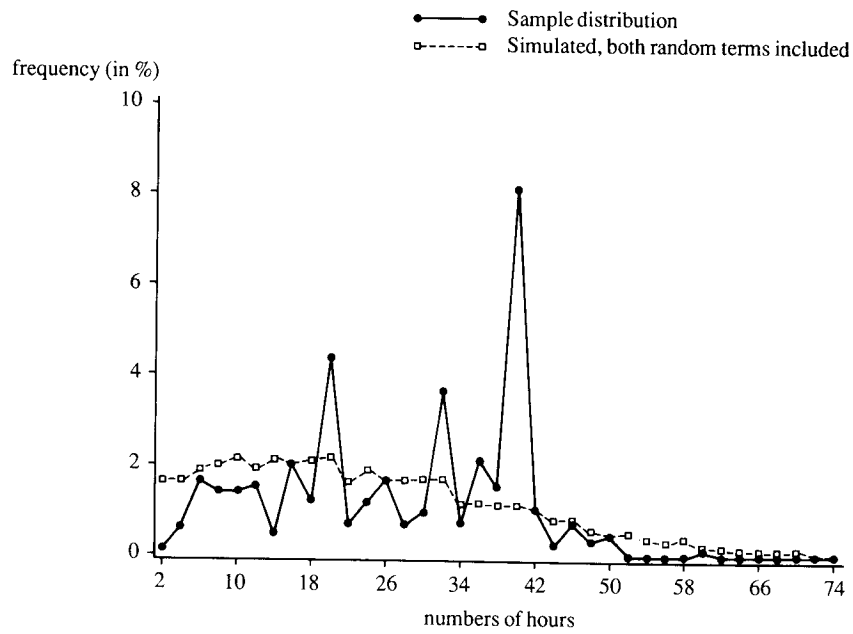


Figure 2
Distribution of working hours per week
Females, common model

work only a few hours a week does not actually work. In this model the probability of involuntary unemployment is thus a strongly decreasing function of desired labor supply.

The figures show that the model is not able to explain the spikes at 40 hours of work for males and females and at 20 and 32 hours of work for females. This is a usual shortcoming of empirical labor supply models in The Netherlands which do not take into account any forms of hours restrictions and it motivates the explicit modeling of such restrictions. In the next section such a model will be discussed.

Dead weight loss (*DWL*) calculations are based on the measure introduced by Kay (1980) and Pazner and Sadka (1980). *DWL* is defined as the equivalent variation (*EV*) associated with the tax system minus the revenue raised by the tax system, where *EV* is defined as the maximum lump sum the individual would be willing to pay instead of taxes on labor income (see, for example, Hausman 1981b). This definition of *EV* and *DWL* for the special case of piecewise linear progressive income taxation is illustrated in Figure 3. Here w_1 denotes the before-tax wage rate which is assumed to be equal to the after-tax wage rate along the first income bracket. N denotes the individual's nonlabor income, not including any virtual components due to the tax system. The maximum utility level that can be attained under the actual tax regime is $u_1 = U(h^*, c^*)$. The equivalent variation is given by $N - e(w_1, u_1)$, and *DWL* is the difference between *EV* and the amount of taxes paid under the actual system ("Tax"). Contrary to the measure based on the compensating variation introduced by Diamond and McFadden (1974), this *DWL* measure starts from the maximum utility level that can be attained under the actual system (u_1) and does not rely on the imaginary utility level which could be attained in a world without taxes.

The results of *DWL* calculations are mentioned in Table 5. *DWL* was calculated 10 times for each individual, with different random drawings of ϵ . The random variation through v was not taken into account. For a given individual and given ϵ , h^* , and corresponding consumption c^* were determined using (2.1) and (2.2). Then (2.4) and (2.5) were used to compute *EV*. The table contains the average *DWL* for groups with different characteristics, in absolute terms (*Dfl* per week, Column 6) as well as in relative terms, as a fraction of the tax revenues in the original system (Column 7). The average *DWL* is approximately 32.6 percent and 30.3 percent for males and females, respectively, of the average amount of taxes paid according to the actual system. As was to be expected, *DWL* is highest for people with a large labor supply, since their marginal tax rate is the largest. For the same reason, one would expect *DWL* for males to exceed *DWL* for females. This is true in absolute terms but not in relative terms, because female wage elasticity is larger than male wage elasticity. The

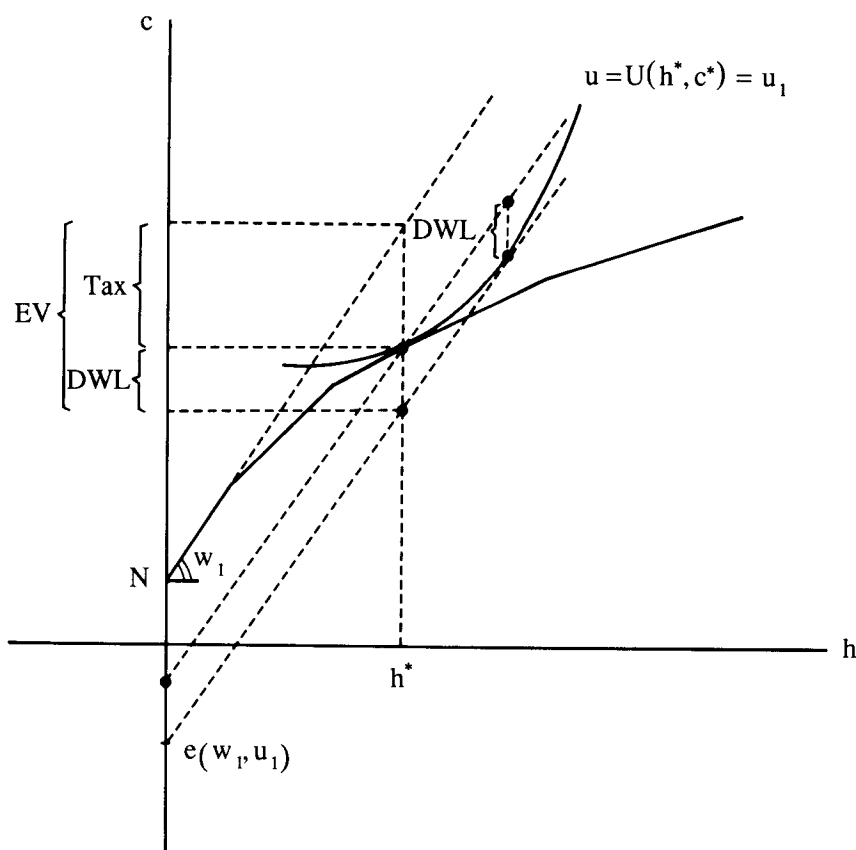


Figure 3
Dead weight loss in the common model

effect on hours worked of changing the actual tax system into a system of lump sum taxes is illustrated in Columns 2 and 3 of the table, which contain average predicted numbers of working hours for the actual and the lump sum system. The larger the differences between the numbers in these columns, the larger are the dead weight losses. *DWL* was also calculated ignoring random preference variation, with both ε and ν set equal to 0. The results were quite similar to those mentioned in Table 5.

C. Conclusion

The estimation results of the standard model are satisfactory in the sense that all parameter estimates have the expected signs. Moreover, esti-

Table 5
Dead Weight Loss Calculations

Males	1	2	3	4	5	6	7
All males	849	39.39	45.88	191	253	62	0.33
Education level							
1	134	39.26	43.35	111	132	21	0.19
2	167	38.90	43.31	120	141	21	0.18
3	342	39.03	44.90	169	213	44	0.26
≥ 4	206	40.45	51.25	336	489	153	0.45
Age							
< 30	154	39.08	42.91	110	124	14	0.12
30–39	300	40.34	47.16	203	268	65	0.32
40–49	225	39.89	47.96	245	343	98	0.40
≥ 50	170	37.32	43.58	171	225	54	0.32
Family size							
2	263	39.20	44.40	155	187	32	0.21
3	140	39.31	45.54	181	236	55	0.30
4	282	39.78	46.93	210	280	70	0.33
≥ 5	164	39.08	46.72	224	328	104	0.46
Females							
All females	849	9.33	11.11	16	21	5	0.30
Education level							
1	217	7.24	8.22	9	10	2	0.18
2	223	7.92	9.19	11	13	2	0.22
3	322	10.73	12.80	20	26	6	0.30
≥ 4	87	12.92	17.01	36	53	16	0.45
Age							
< 30	226	17.77	20.74	32	38	7	0.21
30–39	296	7.65	9.56	16	24	7	0.44
40–49	212	5.20	6.18	6	8	2	0.31
≥ 50	115	4.67	5.26	4	5	1	0.24
Family size							
2	263	20.75	24.63	41	52	11	0.26
3	140	6.75	8.17	10	14	5	0.53
4	282	4.54	5.46	5	8	2	0.46
≥ 5	164	1.44	1.66	1	1	0	0.34

Note: Column 1: number in the sample; 2: hours simulated, actual tax system; 3: hours simulated, lump sum taxes; 4: taxes, actual system (Tax) *Dfl* per week; 5: taxes, lump sum (EV) (*Dfl* per week); 6: dead weight loss (EV-Tax) (*Dfl* per week); 7: (EV-Tax)/Tax.

mated wage and income elasticities are largely in accordance with what we would expect intuitively. On the other hand, however, simulation of the actual situation and, in particular, the figures comparing sample distributions with simulated distributions reveal important shortcomings of the model: it does not capture the spikes in the male and female hours distribution and it cannot explain unemployment among males. In order to test the specification of the model formally, it was also estimated using information on employed individuals only (with a conditional likelihood function). For males as well as females, some of the resulting parameter estimates were quite different from the original ones and standard errors of the estimates were smaller instead of larger. Therefore, the formal Hausman test statistics (see Hausman 1978) were not computed, but the misspecification intuitively became more apparent. These estimation results are mentioned in Appendix B. The figures in this appendix show that these estimates still imply that the spikes in the male and female hours distribution are not explained.

Several extensions of the model can be suggested to overcome the shortcomings. In our opinion, the explicit incorporation of binding constraints on working hours seems a very important one, at least in The Netherlands. This approach is taken in the remainder of this paper.

III. A Model with Demand Side Restrictions

The labor supply model discussed in Section II takes account of tax laws in describing the budget constraint. In this section we present a model which also explicitly captures demand side restrictions by modeling the limited availability of jobs with different, distinct, numbers of hours. Other labor supply models taking into account hours restrictions and involuntary unemployment include, for example, Moffitt (1982), Ham (1982), and Blundell et al. (1987). The model studied here is largely based on a paper by Dickens and Lundberg (1985).

A. The Model

The starting point is the common model, described in Equations (2.1)–(2.2). The error term v , representing among other things deviations from preferred numbers of hours due to demand-side restrictions, is omitted. Instead, we model demand-side restrictions by means of distributional assumptions about job offers. Another difference with the common model is that the number of hours of work is no longer considered to be a continuous variable, but a discrete one. In this section we consider numbers of hours at four-hour intervals (i.e., 0, 4, 8, etc.). In what follows, we

use the subindex l to denote such hours points h . For example, $l = 0$ corresponds with $h = 0$, $l = 1$ with $h = 4$, $l = 2$ with $h = 8$ and so on. For clarity of presentation we omit both the index j , denoting the j th individual and the index i , denoting budget segment i [compare Equation (2.1)].

We assume that employers offer jobs with fixed numbers of hours. Workers face the market distribution of these employment opportunities. Furthermore, it is assumed that the market distribution of job offers is the same for all workers, such that the probability that one job offer involves $h_l (\neq 0)$ working hours is:

$$(3.1) \quad \Pr[\text{one job offer } h = h_l] = p_l, l = 1, \dots, m.$$

Here m is the number of different values of working hours $h_l > 0$. An individual receives N job offers which are not necessarily different; he may, for example, receive N job offers all requiring 40 hours per week. The probability of this event (conditional on N) is p_{10}^N ($l = 10$ corresponds to $h = 40$). The number of job offers an individual receives, N , is assumed to be a drawing from a binomial distribution $B(N_{\max}, P_{of})$. The maximum number of job offers N_{\max} is fixed at 10.⁵

In this context the labor supply decision becomes a discrete choice between N job offers, drawn from the market distribution of offers, or not working:

$$(3.2) \quad h = h_k \text{ iff } U(h_k, c_k) \geq U(h_l, c_l)$$

for all l in the range of received job offers and for $l = 0$. Here c_l is the consumption level corresponding to h_l . If there are no hours constraints, i.e., an individual can choose *any* number of hours, then (3.2) holds for all l . In general, the individual maximizes utility on a *subset* of possible numbers of hours only. For a nonworker this subset may contain only one element, zero hours, because N is assumed to be a random variable of which zero is one of the possible outcomes. In this way the model allows for involuntary unemployment. The main idea of the model is that an individual is only observed to work $h_k (> 0)$ hours if he or she received at least one job offer h_k and prefers this job offer to all different job offers which were received and to unemployment. The individual is unemployed if he prefers zero hours of work to all job offers he received or if he received zero offers.

The likelihood contribution of a given observation is a function of:

- parameters of the utility function ($\beta, \delta, \alpha - s, \sigma_\varepsilon$)

5. Conceivably, the probability of receiving a certain number of job offers may depend on an individual's characteristics. We have decided to ignore this possibility for the sake of (some) simplicity.

- probabilities of job offers with different numbers of hours ($p_k - s$)
- the parameter determining the number of job offers an individual receives (P_{of}).

In this model there are three sources of randomness, namely:

- ε , representing stochastic preferences,
- N , the number of job offers, and
- the offered numbers of hours.

Let $R_k(\varepsilon, N)$ be the conditional probability of observing $h = h_k$, for given ε and N ($k = 0, \dots, m$). It is straightforward to determine $R_k(\varepsilon, N)$ from (3.2) because if preferences (ε) are known, it is easy to check for each $l \neq k$ whether $U(h_k, c_k)$ exceeds $U(h_l, c_l)$ or not. Since the taste parameter ε is not observed, the likelihood of observing $h = h_k$ hours given N can be written as the mean of $R_k(\varepsilon, N)$:

$$(3.3) \quad L(h = h_k|N) = \int_{-\infty}^{\infty} f(\varepsilon)R_k(\varepsilon, N) d\varepsilon,$$

where f is the density function of ε ($\sim N(0, \sigma_\varepsilon^2)$). For random N the likelihood function (3.3) is given by:

$$(3.4) \quad L(h = h_k) = \sum_{N=0}^{N_{\max}} L(h = h_k|N)p(N)$$

where p is the probability function corresponding to $B(N_{\max}, P_{of})$. Since $L(h = h_k|N = 0) = 0$ if $h_k \neq 0$, for workers Equation (3.4) turns into

$$(3.5) \quad L(h = h_k) = \sum_{N=1}^{N_{\max}} L(h = h_k|N)p(N).$$

Since $L(h = 0|N = 0) = 1$, Equation (3.4) can be written for nonworkers as

$$(3.6) \quad L(h = 0) = p(0) + \sum_{N=1}^{N_{\max}} L(h = 0|N)p(N).$$

The two terms in (3.6) reflect that unemployment can either be due to the fact that no job offers are received or to the fact that all job offers received are less attractive than not working. For more details about the model and the likelihood function, see Tummers and Woittiez (1988). The main difference with the common model is the fact that the error term v is replaced by the job offer mechanism. Thus, an alternative explanation is

given for differences between actual and desired labor supply: Instead of assuming that these deviations are random drawings from a normal distribution, flexibility is added in the sense that correlation of deviations from desired behavior with the desired number of working hours is allowed for. As in the common model, the distinction between desired and actual labor supply hinges strongly on identifying assumptions, since no information on desired behavior is used. In the extended model, the main identifying assumption is that the job offer distribution does not depend on individual characteristics such as age, education, etc. Therefore, conclusions about desired labor supply should be interpreted with caution. Eventually, desired hours are only introduced as a tool to create a model which yields a reasonable description of the distribution of actual working hours.

This sort of comment also applies to other features of the model. For instance, our specification of the job-offer distribution is only one out of many; we assume that the job-offer distribution is fixed (not dependent on the tax system). We have chosen to explain the spikes in the hours distribution by demand side restrictions and a smooth distribution of preferences, rather than by spiked preferences. Without more information in the data, one cannot (nonparametrically) identify which of the various possibilities of specification is the correct one.

B. Estimation Results

Table 6 presents estimation results of the model described above, which we refer to as the extended model. It was estimated for males and females separately with the data described in Section II. As said before, the number of hours in this model is assumed to be a discrete variable. Each of the points with hours strictly greater than 0 corresponds to a probability that this number of hours is offered. These probabilities could be estimated freely, but to reduce the number of parameters to be estimated we have set the probabilities of various points equal to each other. We have, for example, assumed that jobs involving 4, 8, 12, or 16 hours per week are offered with the same probability. For the exact distributional assumptions of job offers see the upper panel of Table 6. The maximum number of hours offered is set equal to the sample maximum of hours worked, 72 for males ($m = 18$) and 60 for females ($m = 15$).

The number of job offers an individual receives is a random drawing from the binomial distribution $B(10, P_{of})$. The estimated value of P_{of} is 1 (upper bound) for males and 0.395 for females. This implies that a man always receives 10 offers, whereas a woman only receives about four job offers on average. It follows from the numbers in Table 6 that according to this model most job offers involve 40 or more hours per week. The estimates for $p_{12} = \dots = p_m$ seem rather large, for both men and women, and

Table 6
Estimation Results of the Extended Model (standard errors in parentheses)

	Males	Females
Job offers^a		
$p_1 = \dots = p_4$ (4, 8, 12, 16)	0.0009	0.012
p_5 (20)	0.002	0.030
$p_6 = p_7$ (24, 28)	0.001	0.015
p_8 (32)	0.003	0.050
p_9 (36)	0.006	0.036
p_{10} (40)	0.297	0.302
p_{11} (44)	0.309	0.130
$p_{12} = \dots = p_m$ (48, ..., 4m) ^b	0.054	0.094
P_{of}	1.0	0.395
		(0.0005)
		(0.001)
		(0.0004)
		(0.001)
		(0.001)
		(0.132)
		(0.062)
		(—) ^b
		(—) ^c
Preferences		
β (wage rate)	0.405	0.768
δ (unearned income)	-0.0007	-0.0041
α_0 (constant term)	-2.59	-172.1
α_1 (LOGFS)	-0.141	-14.1
α_2 (DCH < 6)	-0.899	-9.03
α_3 (LAGE)	167.	126.0
α_4 (L2AGE)	-23.7	-19.8
σ_ε (random preferences)	11.91	7.71
		(0.243)
		(0.0013)
		(85.5)
		(3.1)
		(2.22)
		(49.9)
		(7.2)
		(1.96)

a. The number(s) of working hours to which the probabilities correspond are given in parentheses.

b. $m = 18$ for males and $m = 15$ for females; $p_{12} = \dots = p_m$ is determined by the other probabilities because the probabilities add up to one. Therefore, no standard error is computed.

c. Since the estimate is at its upper bound, no standard error is computed.

imply that almost everyone has the opportunity to work 48 hours or more. These large numbers, however, do not necessarily imply that many people actually work so many hours, since preferences are such that these offers will rarely be accepted (see below).

The lower panel of Table 6 contains estimated parameter values of the utility function. It is striking that for females all estimated values are smaller in absolute value than the corresponding ones in the common model, but they are still significant. The presence of children strongly reduces (*ceteris paribus*) the female's desired number of working hours, whereas for males family characteristics only play a minor role. The age profiles of preferred hours do not differ much from those found in Section II. Hours rise with age until about 32 years for men and 25 years for women. There is a remarkable difference between the estimates for σ_ϵ for females in the two models. Apparently, part of the variation in actual hours worked which the common model explained by different preferences, is ascribed to differences in hours restrictions by the extended model.

C. Simulations

Table 7 provides simulation results of the extended model. This table is comparable with Table 3 with an extra column added representing simulated desired working hours. The main differences between the simulations with the common model (Table 3) and the extended model (Table 7) are:

- Differences in hours worked between people of different levels of education are better explained by the extended model (but not completely).
- Desired employment for females is much larger in the extended model than in the common model, implying that according to the extended model females are strongly restricted in their choice: There are not enough jobs with a limited number of working hours (see also Figure 5).
- The extended model performs better than the common model in the sense that it is capable of explaining the 6 percent unemployment of males. Since simulated desired employment is close to one, unemployment is explained by the hours constraints: All males receive 10 job offers, but some of them only receive offers involving an unattractive (large) number of working hours and thus they choose not to work.

Table 7
Simulation of the Actual 1985 Situation

Males	1	2	3	4	5	6	7
All males	849	39.70	37.01	40.25	0.943	0.999	0.944
Education level							
1	134	37.69	36.06	39.37	0.903	0.998	0.931
2	167	37.05	36.41	39.70	0.910	0.999	0.936
3	342	40.54	37.11	40.33	0.965	0.999	0.945
≥ 4	206	41.75	37.95	41.13	0.961	0.999	0.958
Age							
< 30	154	39.60	36.54	39.80	0.955	0.999	0.937
30-39	300	40.83	38.55	41.51	0.960	0.999	0.960
40-49	225	39.96	37.53	40.74	0.924	0.999	0.952
≥ 50	170	37.43	34.05	37.78	0.929	0.998	0.912
Family size							
2	263	39.98	36.36	39.67	0.958	0.999	0.935
3	140	38.71	36.69	39.96	0.929	0.999	0.940
4	282	38.85	37.62	40.78	0.929	0.999	0.952
≥ 5	164	41.52	37.29	40.50	0.957	0.999	0.948
Females							
All females	849	10.64	12.45	10.89	0.390	0.825	0.396
Education level							
1	217	6.73	10.18	8.01	0.290	0.753	0.313
2	223	6.94	11.75	9.69	0.278	0.822	0.366
3	322	13.26	13.50	12.35	0.450	0.850	0.435
≥ 4	87	20.18	16.05	15.72	0.701	0.925	0.534
Age							
< 30	226	20.35	18.74	20.11	0.593	0.928	0.636
30-39	296	7.84	11.34	9.17	0.334	0.816	0.354
40-49	212	7.66	9.60	6.50	0.330	0.794	0.283
≥ 50	115	4.25	8.24	5.27	0.243	0.707	0.238
Family size							
2	63	23.32	20.72	22.82	0.711	0.976	0.717
3	140	6.71	11.59	8.59	0.329	0.878	0.356
4	282	5.65	9.34	5.87	0.277	0.805	0.269
≥ 5	164	2.25	5.30	2.35	0.122	0.575	0.133

Note: Column 1: number in the sample; 2: actual hours worked, sample mean; 3: preferred hours, simulated; 4: actual hours worked, simulated; 5: employment, sample; 6: desired employment, simulated; 7: actual employment, simulated.

The consequences of a 10 percent increase in all after-tax wage rates are presented in Table 8. It shows a small increase (1.0 percent) in hours worked for men, and a larger increase (7.9 percent) for women. These results are similar to those in the common model. It is interesting to see that for females the elasticity of the average actual number of working hours with respect to their own wage rate is larger than the corresponding elasticity of desired hours (5.9 percent). This may be explained by the fact that the choice set is discrete. Some females will not react at all if their wage rate increases, but for others the discrete “jump” may exceed the rise in preferred hours. Apparently, the second effect slightly dominates the first.

Income elasticities are obtained in the same way. If unearned incomes rise with 10 percent, male labor supply hardly changes. For females, actual hours and employment decrease by 2.3 percent and 2.0 percent, respectively. Preferred hours and employment fall by 2.0 percent and 1.0 percent.

Table 8
Simulation of a 10 Percent Increase of After-Tax Wage Rates

	Males				Females			
	1	2	3	4	1	2	3	4
All	37.48	40.64	0.999	0.949	13.18	11.75	0.849	0.421
Education level								
1	36.49	39.75	0.999	0.936	10.81	8.71	0.779	0.335
2	36.84	40.08	0.999	0.941	12.46	10.52	0.846	0.392
3	37.57	40.72	0.999	0.950	14.25	13.24	0.873	0.460
≥ 4	38.47	41.55	1.000	0.963	16.97	16.97	0.942	0.568
Age								
< 30	36.94	40.15	0.999	0.942	19.45	21.03	0.939	0.658
30–39	39.01	41.88	1.000	0.964	12.11	10.10	0.842	0.383
40–49	38.01	41.13	0.999	0.957	10.35	7.33	0.824	0.311
≥ 50	34.54	38.25	0.998	0.920	8.84	5.92	0.734	0.260
Family size								
2	36.79	40.05	0.999	0.941	21.45	23.88	0.980	0.740
3	37.16	40.36	0.999	0.946	12.35	9.61	0.898	0.387
4	38.10	41.17	0.999	0.957	10.10	6.71	0.835	0.297
≥ 5	37.78	40.90	0.999	0.953	5.93	2.79	0.620	0.152

Note: Column 1: preferred hours, simulated; 2: actual hours worked, simulated; 3: desired employment, simulated; 4: actual employment, simulated.

Table 9
Simulation of an Increase of the Number of Part-Time Job Offers

	Males		Females	
	1	2	1	2
All	38.80	0.968	11.54	0.486
Education level				
1	37.89	0.959	8.90	0.399
2	38.23	0.963	10.58	0.461
3	38.89	0.968	12.80	0.523
≥ 4	39.71	0.976	15.94	0.631
Age				
< 30	38.34	0.963	19.39	0.711
30–39	40.18	0.977	10.14	0.451
40–49	39.31	0.973	7.85	0.382
≥ 50	36.12	0.948	6.54	0.326
Family size				
2	38.18	0.962	21.87	0.798
3	38.50	0.965	10.08	0.475
4	39.37	0.972	7.40	0.372
≥ 5	39.07	0.970	3.35	0.190

Note: Column 1: actual hours worked, simulated; 2: actual employment, simulated.

In a final simulation the consequences are studied of relaxing hours restrictions in the sense that more part-time jobs are offered, i.e., jobs involving 20 hours per week. According to the estimation results, the probability that at least one 20 hours a week job is offered is 2.3 percent for males and 11.4 percent for females. Table 9 shows what happens if the value of p_5 (the probability that one offer involves 20 hours) is increased in such a way that the probability of receiving at least one offer of 20 hours becomes 0.5.⁶

Because restrictions are relaxed, actual numbers move towards pre-

6. For males, this implies $p_5 = 0.067$, and as a consequence $p_{12} = \dots = p_{18}$ is reduced to 0.045. For females, $p_5 = 0.170$ and $p_{12} = \dots = p_{15} = 0.0592$.

ferred ones. Thus, many of those who prefer to work part-time but either did not work or worked full-time because there was no part-time job opportunity, will now be able to find a 20 hours job. Unemployment and full-time work will fall in favor of part-time work. For males, the fall in the number of full-time workers dominates, and average working hours decrease for all age and education categories. 12.5 percent of all males will choose to work 20 hours a week. For females, the fall in the number of people who do not work dominates and the average number of working hours rises for all groups. Unemployment falls by 15 percent, and 23.5 percent of all women will work 20 hours.

The parameter estimates in the lower panel of Table 7 were used to simulate the distribution of preferred hours, given in Figures 4 and 5 for men and women, respectively. These figures can be compared with the simulated hours distributions without measurement or optimization errors, presented in Figures 1 and 2. The two distributions are very similar apart from differences due to the different desired employment probabilities which we already discussed (see Table 8).

Combining the demand side of the model (the offers distribution) and the supply side (preferred hours) yields the distribution of actual working hours such as it is simulated with the extended model. Comparing the sample distribution of actual hours with the simulated distribution shows that the extended model predicts an hours distribution much more in line with the data than the common model. This improvement must be attributed to the different distributional assumptions. By assuming that both random variables in the common model are normally distributed, one forces an hours distribution which is too smooth. In the extended model this is no longer the case. Note that the extended model has a larger number of parameters, which makes it easier to produce a good fit.

Figures 4 and 5 only display information about working individuals. Let us now focus on the 61 percent of females and 5.6 percent of males who do not work. The extended model predicts an unemployment rate of 60.5 percent and 5.5 percent for females and males, respectively. Table 10 yields information on how these numbers come about. In the first column the simulated actual working hours distribution is given, and in the second the simulated preferred hours distribution. Only 17.5 percent of the females prefer not working to any positive number of working hours. The remaining 43 percent of predicted nonworkers is due to hours constraints. A large number of women, 54 percent of all females, would prefer to work between 4 and 16 hours per week. But jobs requiring such low number of hours are rarely offered: the last column contains the probability that the choice set contains the number of hours h_l , $1 - (1 - P_{ofpl})^{N_{\max}}$ ($l = 1, \dots, m$). This column again shows that for almost everyone the choice set contains the opportunity of working full-time or more, but that many do

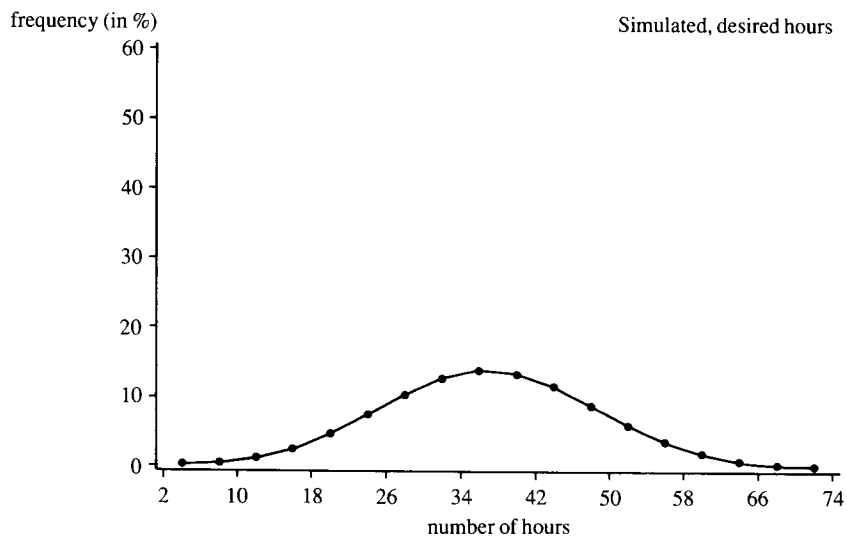
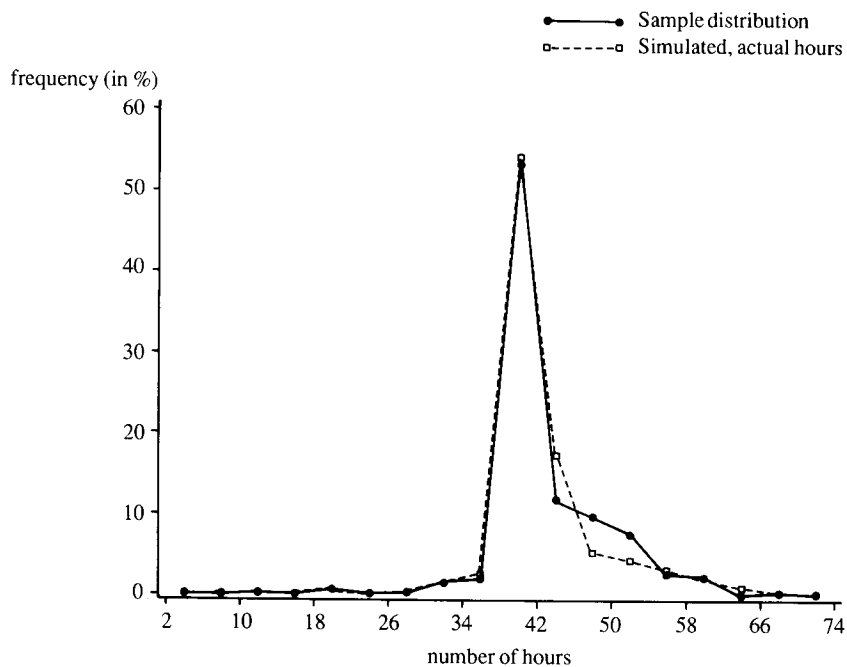


Figure 4
Distribution of working hours per week
Males, extended model

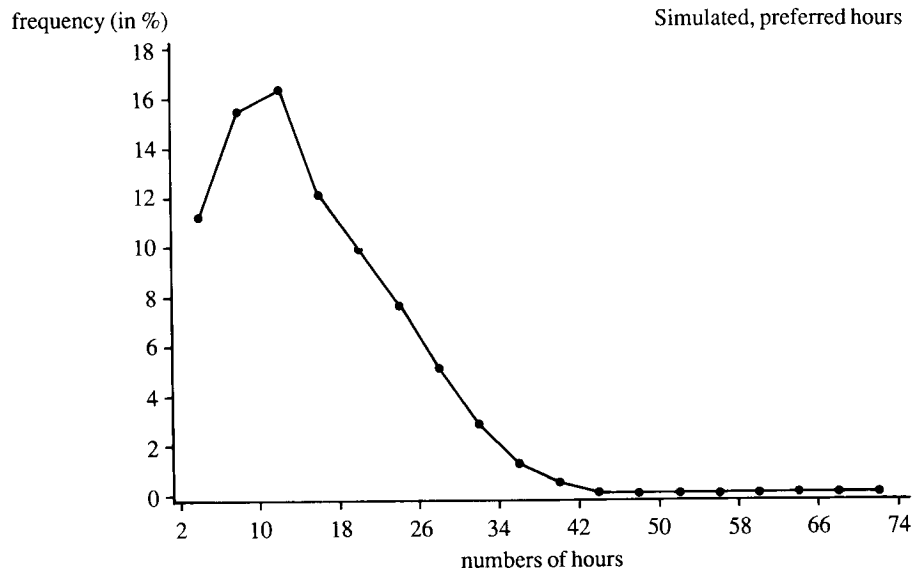
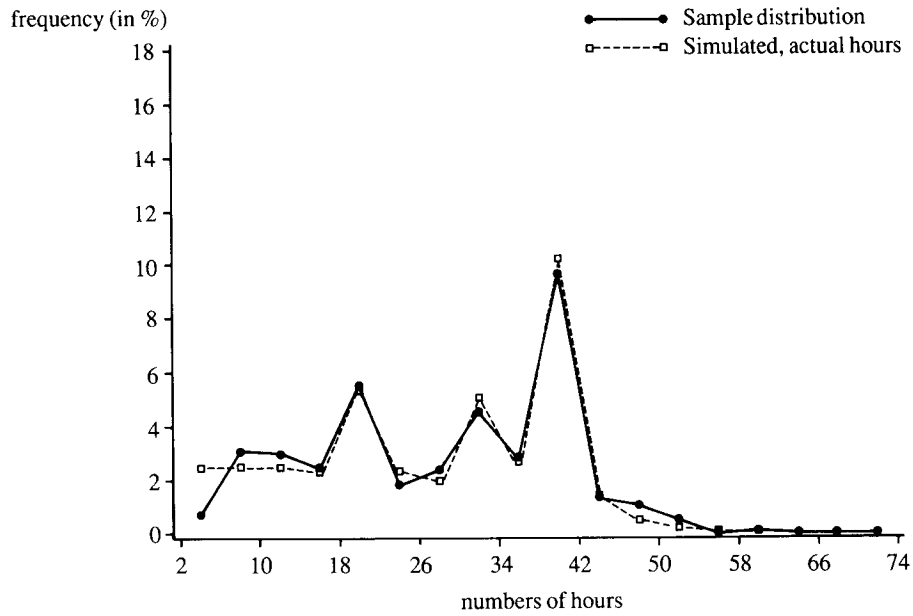


Figure 5
Distribution of working hours per week
Females, extended model

Table 10
Simulated Actual and Desired Hours Distributions and Job Offer Probabilities (probabilities $\times 100$)

Hours (h_i)	Males			Females		
	Actual	Preferred	Offered	Actual	Preferred	Offered
0	5.58	0.11	—	60.44	17.45	—
4	0.07	0.20	0.9	2.40	11.19	4.6
8	0.10	0.54	0.9	2.50	15.50	4.6
12	0.13	1.24	0.9	2.47	16.32	4.6
16	0.17	2.59	0.9	2.31	12.11	4.6
20	0.59	4.70	2.4	5.33	9.94	11.3
24	0.32	7.38	1.1	2.32	7.67	5.8
28	0.38	10.30	1.1	1.96	5.12	5.8
32	1.42	12.71	3.4	5.07	2.86	18.1
36	2.71	13.85	5.8	2.64	1.26	13.3
40	54.25	13.35	97.0	10.24	0.44	72.3
44	17.52	11.49	97.5	1.41	0.12	42.0
48	5.29	8.73	42.5	0.50	0.02	34.1
52	4.46	5.88	42.5	0.23	0.00	34.1
56	3.16	3.53	42.5	0.10	0.00	34.1
60	1.96	1.89	42.5	0.04	0.00	34.1
64	1.09	0.90	42.5	0.02	0.00	—
68	0.55	0.38	42.5	0.01	0.00	—
72	0.25	0.22	42.5	0.00	0.00	—

not have the option of working part-time. Intuitively, the fact that firms are reluctant to offer jobs with few hours a week can be explained by fixed employer costs for each separate employee. Finally, we note that women get offers with few hours more frequently than men. This is in line with the type of job often held by women. Women often work in the service sectors, where part-time jobs are more common.

In the extended model the rare occurrence of people working only a few hours is explained by hours constraints. Again, however, it should be stressed that this explanation hinges strongly on the identifying assumptions. An alternative explanation for the lack of part-time jobs may, for instance, be the existence of fixed costs at the supply side of the labor market, such as costs of child care, etc., which are not taken into account in either the common or the extended model. These fixed costs make it unattractive for an individual to work only a few hours.

D. Dead Weight Loss Calculations

When dead weight losses are calculated, restrictions on working hours are usually ignored. Since the extended model explicitly takes these restrictions into account, it seems natural to incorporate the hours restrictions also in the calculation of *DWL*. We therefore introduce a measure of *DWL* which is the appropriate substitute for the measure used in Section II in case of a finite set of job opportunities with corresponding consumption determined by the progressive piecewise linear tax system. The measure we introduce shares the characteristic of the measure used earlier in that the only indifference curve that matters is the one corresponding to the maximum utility level which can be attained under the actual tax system. In this way, the measure depends on the substitution effect only, and does not rely on income effects.

Let us first consider the case in which individual preferences are known with certainty (ε is given) and the finite set of job offers is given. The individual chooses the best point in the finite set $\{(h^0, c^0), (h^1, c^1), \dots, (h^N, c^N)\}$, where $0 = h^0 < h^1 < \dots < h^N$ and c^i denotes consumption corresponding to h^i according to the actual tax system.⁷ The two figures below show what *DWL* may look like. Figure 6 refers to a person with a choice set containing six points. The optimal choice is (h^2, c^2) , as can be seen from the form of the indifference curve corresponding to utility level $U(h^2, c^2)$, which is the curve through points *A*, *E*, and *C*. Taxes paid will thus equal T_1 . (Note that in the first tax bracket no taxes are paid, so the before-tax wage rate equals w_1 , the after-tax (marginal) wage rate corresponding to the first bracket.)

If the tax system were replaced by a lump sum tax of the same amount T_1 , the individual's choice set would consist of six points on the line through *A* and *B*. From the indifference curve $U = U(h^2, c^2)$, it can be seen that the individual would then choose to work either h^3 or h^4 hours. The way in which the other two indifference curves are drawn in the figure implies that he would prefer h^4 and would end up at point *B*. Thus, utility would rise and if the lump sum tax would be raised a bit above T_1 , he would still be better off than in case of the actual system. *DWL* is defined as the answer to the question "how much more would the taxed consumer *at most* be willing to pay as a lump sum rather than in the form of labor taxes (hours restrictions taken into account)?" If the lump sum is raised to $T_1 + BC$ (the distance between the points *B* and *C*), and if the individual would in that case still choose to work h^4 hours, then he would be just as well off as under the original system. But he can do better by

7. Note the distinction between super- and subindices: $(h^0, c^0) = (0, c_0)$ but in general $\{(h^1, c^1), \dots, (h^N, c^N)\}$ is a subset of $\{(h_1, c_1), \dots, (h_m, c_m)\}$.

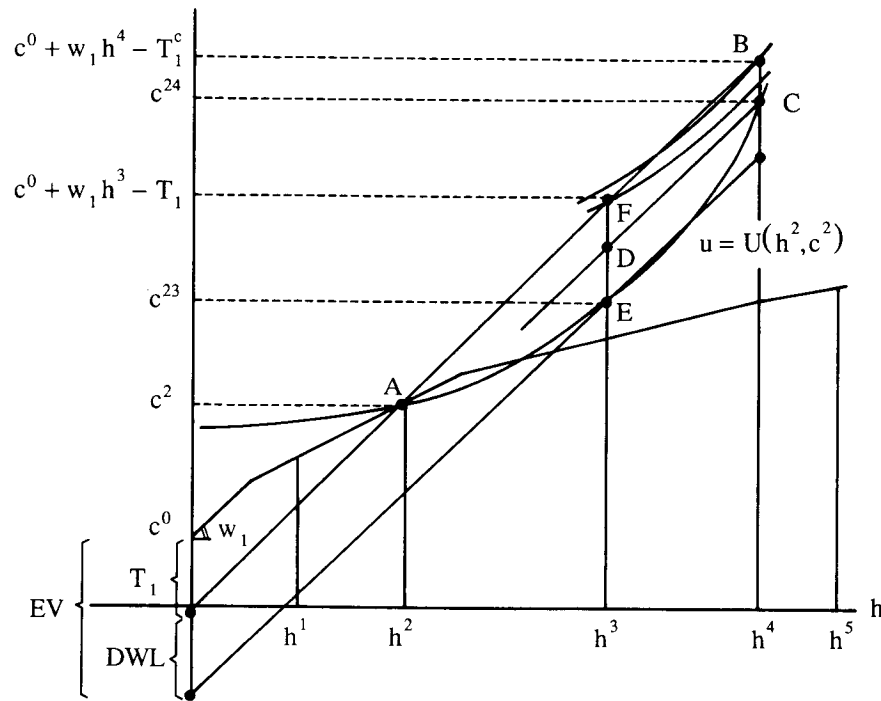


Figure 6
Dead weight loss for a finite set of job offers

choosing h^3 hours and moving to point D . Thus, if the lump sum is raised even further with the amount DE , the individual will still attain utility $U(h^2, c^2)$. EV is thus given by $T_1 + EF$ and DWL is given by EF , which is the maximum vertical distance between points corresponding to job opportunities on the line through A , F , and B (the line through the original optimum with slope w_1) and the indifference curve through A , E , and C (the indifference curve corresponding to the maximum utility level under the original tax regime). Let c^{2j} be the consumption level defined implicitly by

$$(3.7) \quad U(h^2, c^2) = U(h^j, c^{2j}) \quad (j = 2, \dots, N).$$

DWL is then given by

$$(3.8) \quad DWL = \text{Max}_{j=2, \dots, N} \{w_1 h^j + c^0 - c^{2j}\} - T_1.$$

In Figure 6, the maximum is attained for $j = 3$. Note that this definition of *DWL* does not depend on the pattern of indifference curves other than the one through the original optimum; the other two indifference curves in Figure 6 only serve as an illustration but do not affect the outcome of *EV* and *DWL*.

If in the case referred to by Figure 6 (h^3, c^3) and h^4, c^4) were omitted from the choice set, then *DWL* would be zero. In that case, the individual would still work h^2 hours if the tax system was replaced by a lump sum T_1 . This suggests that because of the inflexibility due to hours constraints *DWL* in presence of these constraints will tend to be lower than in case of free choice.

The reverse, however, is also possible, as is illustrated by Figure 7. This figure refers to someone with a choice set consisting of two points only. Confronted with the actual tax system, this person chooses not to work and pays no taxes, but still the dead weight loss is non-zero. Under the appropriate lump sum regime, the individual will work $h^1 > 0$ hours and pay a lump sum $w_1 h^1 + c^0 - c^1 > 0$. On the other hand, *DWL* would be zero if hours worked could be chosen freely, since in that case the individual would work h^* hours and changing the tax system would have no effect.

It is straightforward to extend the examples given above to the 'general case' with preferences given by (2.4), for known ϵ and given job offers $\{(h^0, c^0), (h^1, c^1), \dots, (h^N, c^N)\}$: For $i, j \in \{0, \dots, N\}$, $i \leq j$, let c^{ij} be defined by

$$(3.9) \quad U(h^i, c^i) = U(h^j, c^{ij}).$$

An explicit expression for c^{ij} is easily derived from (2.4):

$$(3.10) \quad c^{ij} = \frac{1}{\delta^2} [\{\beta - \delta h^j\} \{U(h^i, c^i) + \log(\beta - \delta h^j)\} + \delta \{h^j - X' \alpha - \epsilon\}].$$

Let (h^{i*}, c^{i*}) be the utility maximizing choice. *DWL* is given by

$$(3.11) \quad \begin{aligned} DWL &= \text{Max}_{j=i^*, \dots, N} \{w_1 h^j + c^0 - c^{i^*j}\} - \{w_1 h^{i^*} + c^0 - c^i\} \\ &= \text{Max}_{j=i^*, \dots, N} \{w_1 (h^j - h^{i^*}) + c^i - c^{i^*j}\}. \end{aligned}$$

If preferences or the set of job offers are not fixed, we assume that the lump sum can be adjusted exactly to each possible realization of the random preference term ϵ and to each set of job offers, and we are interested in the expectation of *DWL*, which can be written as

$$(3.12) \quad E\{DWL\} = E_\epsilon \{E\{DWL|\epsilon\}\} = \int_{-\infty}^{\infty} f(\epsilon) E\{DWL|\epsilon\} d\epsilon.$$

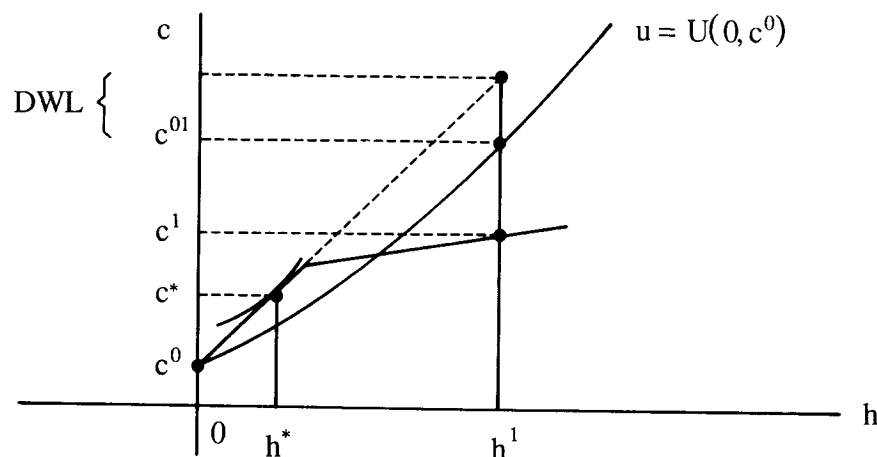


Figure 7

Dead weight loss for someone who does not work

Computation of $E\{DWL|\varepsilon\}$ involves taking the expectation with respect to the discrete distribution of job offer opportunities. This can be done analytically but involves some technical details, similar to those encountered in writing down the likelihood function of the model (see Tummers and Woittiez 1988). An analytical expression for $E\{DWL|\varepsilon\}$ is derived in Appendix C. Since this expectation is a complicated function of ε , it is not possible to compute the integral in (3.12) analytically. Therefore, for each individual 10 values $\varepsilon_j (j = 1, \dots, 10)$ are drawn randomly from a $N(0, \sigma_\varepsilon^2)$ -distribution and $E\{DWL\}$ is approximated by $1/10 \sum_{j=1}^{10} E\{DWL|\varepsilon_j\}$.

Mean dead weight losses for age and education categories are given in Table 11, which can be compared to Table 5. According to the extended model, DWL appears to be much smaller than according to the common model, for both males and females. On average, DWL in the extended model is 10.7 percent of taxes paid for males and 15.4 percent for females. In the common model these figures were 32.6 percent and 30.3 percent, respectively. Since wage rate elasticities are approximately the same in the two models, the differences must be due to the hours restrictions. The average change in hours worked (the difference between Columns 3 and 2) for males is much smaller in the extended model than in the common model, but for females this is not the case. Moreover, the change from the actual tax system to lump sum taxation in the model with hours restrictions has a positive impact on employment (the effect described in the

Table 11
Dead Weight Loss Calculations

Males	1	2	3	4	5	6	7	8	9
All males	849	40.28	42.71	203	224	22	0.11	0.944	0.954
Education level									
1	134	39.17	40.82	116	124	8	0.07	0.922	0.933
2	167	39.56	41.33	127	135	8	0.06	0.931	0.942
3	342	40.51	42.75	184	200	17	0.09	0.947	0.956
≥ 4	206	41.21	44.98	351	402	50	0.14	0.963	0.974
Age									
< 30	154	39.73	41.05	115	119	4	0.04	0.935	0.940
30–39	300	41.66	44.30	218	241	23	0.11	0.961	0.971
40–49	225	41.13	44.08	259	293	34	0.13	0.961	0.969
≥ 50	170	37.20	39.58	181	199	18	0.10	0.898	0.917
Family size									
2	263	39.54	41.45	161	172	11	0.07	0.933	0.943
3	140	39.85	42.15	189	208	19	0.10	0.935	0.944
4	282	40.75	43.46	225	250	25	0.11	0.952	0.963
≥ 5	164	41.03	43.90	243	278	35	0.15	0.955	0.965
Females									
All females	849	10.84	12.63	23	27	4	0.15	0.395	0.433
Education level									
1	217	8.39	9.45	12	13	1	0.09	0.321	0.345
2	223	9.23	10.82	16	18	2	0.13	0.354	0.391
3	322	12.37	14.18	27	30	3	0.13	0.434	0.471
≥ 4	87	15.43	19.46	57	71	14	0.25	0.533	0.615
Age									
< 30	226	19.88	21.84	38	41	3	0.08	0.633	0.666
30–39	296	9.20	11.30	26	31	6	0.22	0.353	0.399
40–49	212	6.34	7.98	13	15	3	0.22	0.278	0.319
≥ 50	115	5.59	6.51	8	9	1	0.13	0.247	0.270
Family size									
2	263	22.83	25.24	49	54	4	0.09	0.718	0.756
3	140	8.72	10.86	20	25	5	0.23	0.360	0.411
4	282	5.79	7.54	12	16	4	0.32	0.267	0.311
≥ 5	164	2.11	2.67	3	4	1	0.25	0.125	0.142

Note: Column 1: number in the sample; 2: hours simulated, actual tax system; 3: hours simulated, lump sum taxes; 4: taxes, actual system (*Dfl* per week) (Tax); 5: taxes, lump sum (*Dfl* per week) (EV); 6: dead weight loss (*Dfl* per week) (EV-Tax); 7: (EV-Tax)/Tax; 8: simulated employment, actual tax system; 9: simulated employment, lump sum taxes.

example illustrated in Figure 7), as can be seen from Columns 8 and 9 in Table 11. Particularly for females this seems to play an important role: employment increases by 9.7 percent. However, the effect of this on total taxes paid is only small, since those who change from unemployment to employment choose to work relatively few hours a week. The extra tax revenues should mainly come from those who already paid a large amount and apparently for these people the hours restrictions play the largest role.

IV. Conclusions

In Sections II and III, two models of individual labor supply are estimated, both of them based on the linear Hausman (1981) specification and accounting for a piecewise linear budget constraint. Although some of the parameter estimates seem substantially different in the two models, calculated elasticities are quite similar. In a survey paper, Theeuwes (1988) discusses eight other recent empirical studies of labor supply in The Netherlands. He presents eight wage-rate elasticities for hours worked of women, ranging from 0.20 to 3.23 with a mean of 1.39. Compared to this, our elasticities of 0.65 and 0.79 are low but not out of line. For males, Theeuwes mentions four wage elasticities, ranging from -0.25 to 0.27 with a mean of 0.07 . Our values (0.12 and 0.10) fit quite well in this range. The income elasticities that we find are also largely in accordance with previous Dutch findings.

Dead weight loss calculations yield quite different results for the two models. The *DWL* of 30 percent for females corresponds to values of 27 percent and 37 percent, which were obtained by Grift (1988) with Dutch 1983 data of married women. The substantially different *DWL*'s found in Section III should perhaps not be too surprising, since the *DWL* definition hinges strongly on the structure of the model, i.e., the hours constraints.⁸

The introduction of the extended model is motivated by the fact that the common model yields a poor description of the sample distribution of working hours. The results, in particular Figures 4 and 5, unquestionably show that in this respect the model in Section III is a success. This, however, does not mean that it is free of misspecification. Several White tests for different subvectors of the parameter vector were performed and, generally, the null-hypothesis of no misspecification was rejected.

8. Of course, it should be realized that *DWL* by definition will tend to overestimate the efficiency loss based on the tax system in practice, since the lump sum system is an alternative which has never been implemented in any society and for good and well-known reasons.

Rejection was strongest for the parameters referring to demand side restrictions, intuitively suggesting that this is where most misspecification is located. On the other hand, for females, the hypothesis that P_{of} does not depend on $LAGE$ and $L2AGE$ is accepted at a 5 percent level by a Lagrange multiplier test (a test statistic of 2.2 with critical value $\chi_{2;0.05} = 6.0$).

A number of extensions and improvements of the model certainly deserve more attention in future research. The specification of preferences is convenient but possibly restrictive. For instance, preliminary analysis of a labor supply equation involving a quadratic wage term, suggests a significant improvement. This extension would correspond to the utility specification suggested by Hausman and Ruud (1984). Another interesting example is given by Heckman (1974), who starts with a specification of the indifference curves.

A second point relates to the treatment of the budget constraint for non-workers. For them, the budget constraint is based on predicted before-tax wages. Random variation across individuals in before-tax wages is ignored. This calls for simultaneous estimation of a wage equation and a labor supply equation. Examples of such models are Moffitt (1984), who introduces hours dependent wage rates resulting in an S-shaped budget curve, and Tummers and Woittiez (1988), who combine Moffitt's model with demand side restrictions. Although there can be little doubt about the empirical relevance of such extensions, they would add an additional layer of complexity to an already intricate model. Hence, this is left for future work. In the third place, the modeling of job offers in a static framework has the merit of simplicity, but a more natural approach would be to allow for consecutive job offers and the possibility that individuals move from one job to another. Somewhat in the same spirit it should be noted that budget constraints are not exogenous in a dynamic world. For instance, the level of unemployment benefits in The Netherlands often depends both on the duration of the preceding spell of employment and on the duration of the current spell of unemployment.

Appendix

A. Data and Wage Equations

The data we use stem from the OSA Survey. Working hours are the answer to the question:

“On average, how many hours do you work per week?”

and thus include regular overtime and hours worked in second jobs.

Table A.1
Wage Equations

	Males Parameter	<i>t</i> -value ^a	Females Parameter	<i>t</i> -value ^a
Constant	-5.96	2.75	-12.71	4.49
<i>DED2</i> ^b	0.063	1.54	0.091	1.58
<i>DED3</i> ^b	0.144	3.93	0.160	3.24
<i>DED4</i> ^b	0.393	10.10	0.468	8.11
<i>LAGE</i>	4.624	3.85	8.681	5.35
<i>L2AGE</i>	-0.600	3.58	-1.214	5.25
<i>EDSEC</i> ^c	0.026	2.08	0.066	2.79
Lambda ^d	1.379	1.39	-0.013	0.27
Number of observations	801		331	
<i>R</i> ²	0.243		0.290	

a. *t*-values are not corrected for the possible selectivity bias.

b. *DED2*, *DED3*, *DED4*: dummy variables referring to the levels of education *EDM* (males) and *EDF* (females), ranging from 1 (lowest level) to 5 (highest level). *DED2* = 1 if *ED* = 2, *DED3* = 1 if *ED* = 3, *DED4* = 1 if *ED* = 4 or *ED* = 5.

c. *EDSEC*: index variable referring to the sector of education; *EDSED* = 2: technical or business, *EDSEC* = 1: semi-technical or semi-business, *EDSEC* = 0: neither technical nor business.

d. Lambda: the inverse of Mill's ratio.

The main component of after-tax labor income is the answer to the question:

“What is the net wage in your present job, i.e., the amount you receive without shiftwork allowance, overtime allowance, tips, travel allowance, entertainment expenses, etc.; taxes and premiums for welfare benefits are also excluded.”

The answer can be the amount per week, per four weeks, or per month, and is transformed into weekly income. Regular shiftwork allowance, overtime allowance, tips, etc., are given separately in the survey. The after-tax wage we use is the sum of the main component and these regular allowances.

In estimating the labor supply models of Sections II and III, unknown before-tax wage rates of nonworkers were replaced by predicted wage rates. Predictions are based on the following estimation results of the log-wage equation. These were estimated following the Heckman procedure

Table A.2
Estimation Results of the Common Model; Workers Only (standard errors in parentheses)

Parameter	Males		Females	
β (wage rate)	0.0	(—) ^a	0.91	(0.26)
δ (unearned income)	-0.0013	(0.0035)	-0.0061	(0.0028)
α_0 (constant term)	-154	(56)	25.1	(132)
α_1 (<i>LOGFS</i>)	0.32	(0.90)	-14.8	(2.6)
α_2 (<i>DCH < 6</i>)	-0.17	(0.68)	-0.97	(2.4)
α_3 (<i>LAGE</i>)	109	(31)	21.0	(76.1)
α_4 (<i>L2AGE</i>)	-15.2	(4.3)	-5.1	(10.8)
σ_ϵ (random preferences)	5.04	(11.6)	1.55	(4.98)
σ_v	4.30	(13.4)	10.53	(0.64)

a. The estimate is at its lower bound. This bound is imposed to avoid coherency problems.

to take account of possible selectivity bias. For both males and females, the selectivity bias is not significant. The meaning of the exogenous variables is explained in Section II of the main text.

B. Estimation of the Common Model Using Information on Workers Only

The model described in Section II was also estimated using information on workers only, taking into account selectivity bias due to truncation by using conditional maximum likelihood. Note that this estimation procedure has the advantage that imputation of predicted wage rates for non-workers is avoided. Estimation results are mentioned in Table A.2. Some of the estimates are substantially different from those mentioned in Table 2, pertaining to the estimation for both workers and non-workers. The large differences strongly suggest that the common model is severely misspecified.

Simulations based on these results can be compared with those mentioned in Tables 3 and 4. Predicted participation for males is almost equal to 1, as in Table 3. For females, the average simulated participation probability equals 0.957, which is quite out of line with both the actual sample participation and simulated participation in Table 3. The estimated average wage and income elasticities of actual working hours are both 0.0 for males. For females, they are 0.38 and -0.17, respectively.

Figures A.1 and A.2 are obtained in the same way as Figures 1 and 2

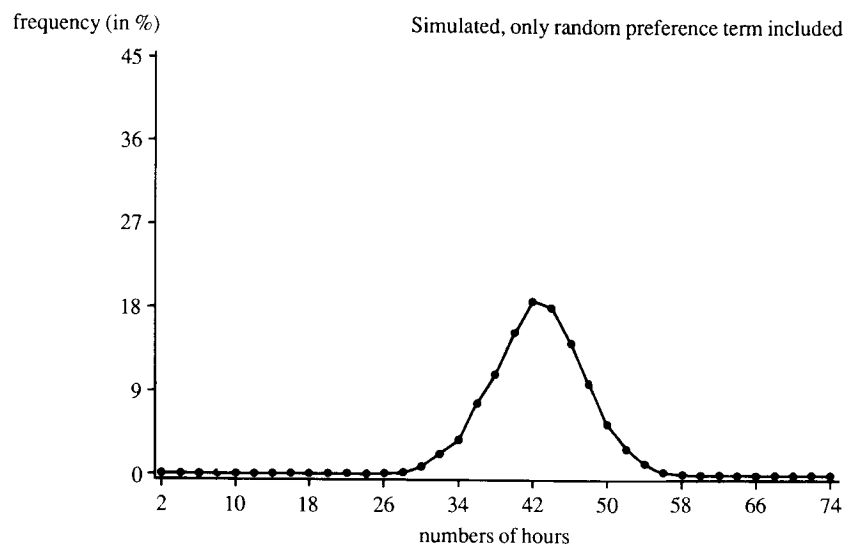
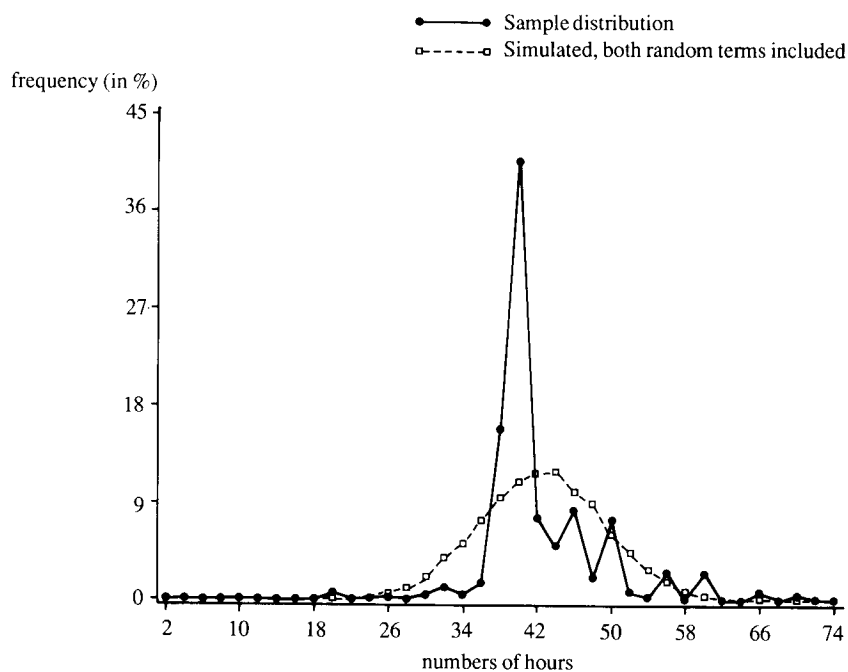


Figure A.1
Conditional distribution of working hours per week
Males, common model, workers only

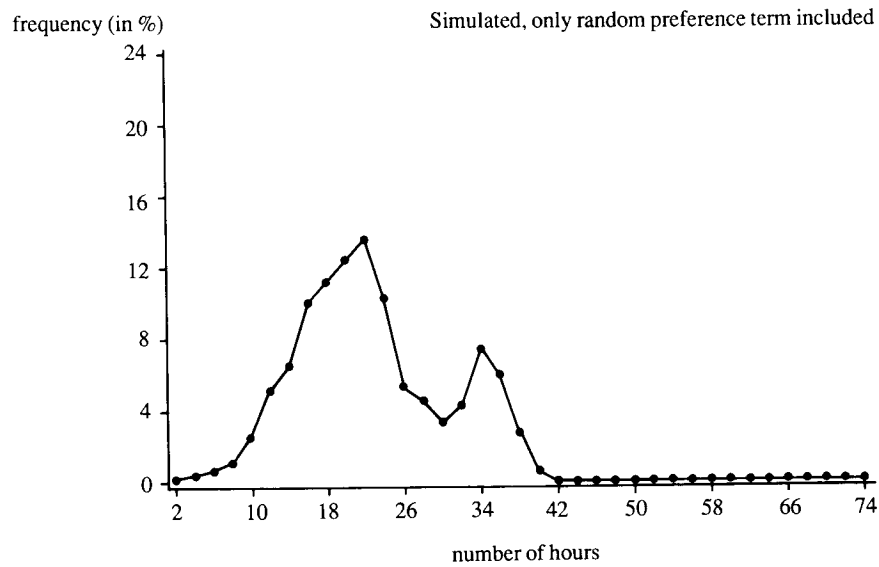
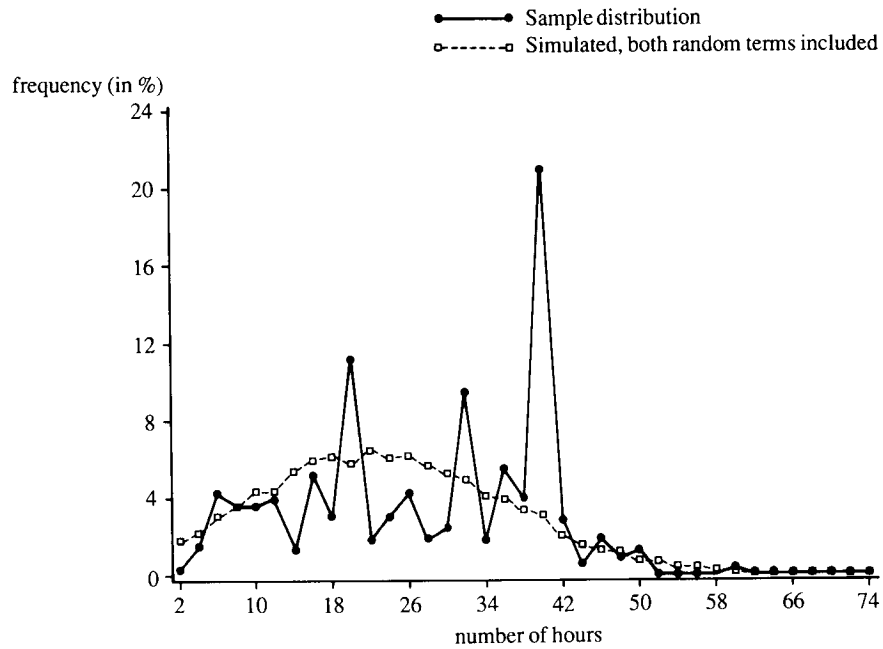


Figure A.2
 Conditional distribution of working hours per week
 Females, common model, workers only

and can be used to compare the sample distribution of actual working hours with the simulated distribution based on the estimates of Table A.2. The figures show that excluding non-workers does not solve the problem that the spikes in the sample distribution of actual working hours are not explained.

C. Derivation of $E\{DWL|\varepsilon\}$

In this appendix we derive an analytical expression for the conditional expectation of DWL for given ε , which can be used to compute the unconditional expectation with Equation (3.12). The technique used here is similar to the one needed for writing down the likelihood function of the extended model (see Tummers and Woittiez 1988, Section 2 and Appendix). We assume that ε is given but that job offers are random. For $i, j \in \{0, 1, \dots, m\}$, $i \leq j$, let

$$(A.1) \quad DWL_{ij} = \{w_1(h_j - h_i) + c_i - c_{ij}\},$$

where c_i is consumption corresponding to h_i and c_{ij} is defined in the same way as c^{ij} in (3.9). DWL_{ij} is the realization of DWL if in case of the actual tax system the optimal choice is (h_i, c_i) , which is (h^{i*}, c^{i*}) in (3.11), and in case of the “appropriate” lump sum taxation the optimal choice is (h_j, c_{ij}) which is (h^j, c^{i*j}) in (3.11). Since $DWL_{ii} = 0$, the expected dead weight loss conditional on ε is given by

$$(A.2) \quad E\{DWL|\varepsilon\} = \sum_{i=0}^m \sum_{j=i+1}^m \Pr\{DWL = DWL_{ij}\} DWL_{ij}.$$

The probabilities $\Pr\{DWL = DWL_{ij}\}$ can be calculated as follows: Let

$$(A.3) \quad \Omega_i = \{k \in \{0, \dots, m\}; U(h_k, c_k) \leq U(h_i, c_i)\} \quad (i = 0, \dots, m)$$

and

$$(A.4) \quad \Delta_{ij} = \{k \in \{0, \dots, m\}; k < i \text{ or } DWL_{ik} \leq DWL_{ij}\}.$$

$$(i, j \in \{0, \dots, m\}, i < j).$$

Thus, Ω_i corresponds to the offers which are not preferred to (h_i, c_i) , and Δ_{ij} corresponds to the offers which yield a dead weight loss that does not exceed DWL_{ij} , conditional on the fact that (h_i, c_i) is the optimal choice. Therefore, $DWL = DWL_{ij}$ if and only if for each job offer (h_k, c_k) we have $k \in \Omega_i \cap \Delta_{ij}$ and (h_i, c_i) as well as (h_j, c_j) are offered. For $A \subset \{0, 1, \dots, m\}$ let

$$(A.5) \quad q(A) = (1 - P_{of}) + \sum_{k \in A \setminus \{0\}} P_{of} p_k \quad \text{if } 0 \in A$$

$$= 0 \quad \text{if } 0 \notin A.$$

We avoid conditioning on the number of job offers N by interpreting the $N_{\max} - N$ “missed job offers” as offers of zero hours (which are of no importance, since zero hours of work can be chosen anyhow). Thus, if $0 \in A$ then $q(A)$ can be interpreted as the probability that one job offer belongs to A . For each $i, j \in \{0, 1, \dots, m\}$ with $i < j$ we can now write

$$(A.6) \quad \Pr[DWL = DWL_{ij}] = \{q(\Omega_i \cap \Delta_{ij})\}^{N_{\max}} - \{q(\Omega_i \cap \Delta_{ij} \setminus \{i\})\}^{N_{\max}}$$

$$- \{q(\Omega_i \cap \Delta_{ij} \setminus \{j\})\}^{N_{\max}}$$

$$+ \{q(\Omega_i \cap \Delta_{ij} \setminus \{i, j\})\}^{N_{\max}}.$$

This implies that $\Pr[DWL = DWL_{ij}] = 0$ if $U(h_i, c_i) < U(h_0, c_0)$ [i.e., $0 \notin \Omega_i$: the individual prefers $(0, c_0)$ to (h_i, c_i)] or if $U(h_i, c_i) < U(h_j, c_j)$ [i.e., $j \notin \Omega_i$: if (h_i, c_i) and (h_j, c_j) are both in the choice set, the individual chooses (h_j, c_j) , so (h_i, c_i) cannot be optimal]. The expected dead weight loss can now be found by substitution of (A.6) into (A.2).

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