COMMUNITY RATING IN HEALTH INSURANCE: 
TRADE-OFF BETWEEN COVERAGE AND SELECTION

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Community rating in health insurance: trade-off between coverage and selection

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Abstract

We analyze the role of community rating in the optimal design of a risk adjustment scheme in competitive health insurance markets when insurers have better information on their customers’ risk profiles than the sponsor of health insurance. The sponsor offers insurers a menu of risk adjustment schemes to elicit this information. The optimal scheme includes a voluntary reinsurance option. Additionally, the scheme should sometimes be complemented by a community rating requirement. The resulting inefficient coverage of low-cost types lowers the sponsor’s cost of separating different insurer types. This allows the sponsor to redistribute more rents from low-cost to high-cost consumers.

Keywords: health insurance, cherry picking, risk adjustment, mechanism design

JEL classification: I13, D02, D47

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1. Introduction

Many countries with private health insurance markets feature premium rate restrictions in the form of community rating (CR), see Gale (2007) for an overview. This means that insurers have to accept any customer and charge the same price to each customer for a given contract.\footnote{This is also referred to as ‘pure community rating’. Less restrictive forms might allow for some rate differentiation according to, for instance, age.} Policy makers’ motivation for CR is to enforce solidarity among high-risk and low-risk consumers on the health insurance market. In the absence of CR, insurers can engage in third degree price discrimination, also known as risk rating (RR), and charge high (low) prices to high (low) risk consumers.

Health economists tend to see CR as a misguided policy, because it “requires insurers to ignore information about risk that they actually do have when they set the premiums” (Pauly, 2008, pp. 121). If insurers are not allowed to use this information in setting prices, they use other means, such as quality, waiting lists for providers in the network, or co-payments to separate risk types. As we know from Rothschild and Stiglitz (1976) this reduces welfare.\footnote{See Van de Ven and Schut (2011, pp. 394) for further arguments why CR remains so popular with policy makers. They argue that risk adjustment is a better instrument than CR to achieve solidarity between risk types. We derive conditions under which it is optimal to combine CR and risk adjustment.}

In this paper we argue that CR can be part of a second-best policy if two conditions are met. First, insurers are able to risk select insured along dimensions that the sponsor of health insurance, an employer or the government, cannot contract on. Second, CR is combined with a form of risk adjustment that combines both ex ante and ex post risk adjustment (sometimes referred to as prospective and retrospective risk adjustment).

Ex ante risk adjustment taxes or subsidizes the insurer based on observable characteristics of its insured that provide a signal of expected health costs. Ex ante risk adjustment requires verifiable data that is relatively easy to obtain for the sponsor of health insurance. If all dimensions along which the insurer can risk-select were contractible, the sponsor could force insurers to only offer the welfare maximizing contract. A sponsor who values solidarity between high risk and low risk types, could then require that this contract is offered at the same price to each customer, and compensate insurers for cost differences using perfect ex ante risk adjustment.

In practice, the insurer usually has more information on its insured than the sponsor,\footnote{Also, the sponsor may not want to use some variables correlated with expected health care costs for ethical reasons, think of ethnicity or religion, or because including them would decrease insurers’ incentives to reduce} and...
can select on this information in a way that is not contractible for the sponsor. An insurer selectively contracting some providers can exclude the ones that are specialized in certain chronic conditions or contract providers with long waiting lists for particular treatments. The insurer’s help desk could reply more promptly to certain queries than to others. As these dimensions are not contractible for the sponsor, the insurer has an advantage which can be used to game the system. In particular, the insurer tries to cherry pick insured whose expected costs are low within their (ex ante) risk adjustment class (see e.g. Brown et al., 2011, who document such strategic behavior for Medicare Advantage in the US).

In contrast, ex post risk adjustment is based on realized costs. It compensates insurers for consumers that turn out to be costly ex post by repaying part of the realized costs. This is a form of risk sharing or reinsurance, with the risk adjuster playing the role of the reinsurer (see e.g. Swartz, 2003; Dow, Fulton and Baicker, 2010). Both forms of risk adjustment reduce the underlying cost differences and therefore the insurance contracts vary less with risk type.

Ex post risk adjustment leaves no scope for selection to game the system, because costs are known. Instead, the downside of ex post risk adjustment is that insurers’ incentives for cost containment are muted when the regulator acts as a reinsurer (Dow, Fulton and Baicker, 2010). Moreover, ex post risk adjustment does more than is needed: it averages out all variance in costs, while removing selection incentives requires the removal of predictable differences in expected costs only.

An optimal risk adjustment system will contain a combination of both ex ante and ex post elements. Systems which combine ex ante and ex post risk adjustment indeed exist (see for instance Van de Ven et al., 2003, who describe how ex ante payments are combined with risk sharing in various European countries). In the US, the Health Insurance Exchanges that are being established under the Patient Protection and Affordable Care Act combine a transitory reinsurance program, and a risk adjustment scheme that will take current period’s diagnoses as inputs (HHS, 2012). The Dutch risk adjustment system contains both types, though the explicit ex post component is currently being gradually phased out to stimulate insurers to contain health care costs.

As noted by Cutler and Reber (1998, pp. 464) “we do not know the optimal combination of prospective and retrospective risk adjustment”. In this paper we explore this issue when costs, see Van de Ven and Ellis (2000) and Van de Ven and Schut (2011, pp. 384) for a discussion.

Note that providing such incentives is often the reason for having private competitive health insurance in the first place.
insurers have private information on cost types within observable risk adjustment classes. We ask whether, in this second-best world, a CR requirement can be an efficient component of such a risk adjustment scheme with ex ante and ex post compensation.

To design an optimal risk adjustment scheme, we take a mechanism design approach: how can the sponsor optimally elicit the insurers’ private information on their consumers’ expected costs? We consider a two-tiered contracting model with perfectly competitive insurers who offer a menu of contracts to consumers in Rothschild-Stiglitz fashion. The insurers’ incentives for attracting high or low cost consumers are in turn determined by the sponsor’s risk adjustment mechanism. We show that, within classes of publicly observable cost predictors, the sponsor can use ex post risk adjustment to screen insurers on the privately observable part of expected costs. We then explore whether the sponsor’s objective function, a weighted sum of high and low type consumers’ surpluses, can be higher under a CR requirement than with RR.

We find that optimal risk adjustment is qualitatively similar under CR and RR: it offers the insurer a choice whether or not to buy some reinsurance for their customers. The scheme therefore involves subjective risk adjusting as in Sappington and Lewis (1999). Foregoing some ex ante payments in exchange for high ex post reinsurance is attractive for an insurer who knows his customers have high expected health care costs relative to their observed risk class. Conversely, for an insurer who faces customers with low expected health care costs compared to their publicly observable characteristics, the costs of reinsurance are higher than the benefits. This insurer prefers ex ante adjustment only, and is in fact willing to contribute to the risk adjustment fund, subsidizing the high types. In this way, optimal risk adjustment targets the information advantage of the insurers vis-a-vis the sponsor, and allows the sponsor to tax low risk types to subsidize the high-risk types.\(^5\)

Quantitatively, the optimal schemes under CR and RR as well as the equilibrium outcomes differ. We find that CR dominates RR if (i) the inefficiency of ex-post reinsurance is not too big and (ii) the sponsor has a bias towards the high-risk type consumers.

The intuition why CR can raise welfare is that under CR, low-risk types get less generous coverage than high-risk types. This reduces expected expenditures on these types for insurers.

\(^5\)Van de Ven and van Vliet also suggested a risk adjustment scheme involving subjective risk adjustment: “Let an insurer himself decide –within certain boundaries– for which patients, or for which types of care, or to what extent he wants to share the risk with the Central Fund. (…) An important advantage of such a flexible form of risk sharing would be that the additional information the insurer might have about the residual predictable risk that is not accounted for in the capitation payment, will not be employed for cream skimming, but will be reflected in the preferred form of risk sharing.” (Van de Ven and van Vliet, 1992, italics are in the original text).
As the expected costs are lower, the benefits of reinsurance decrease for low risk types. This makes it easier for the sponsor to screen insurers with low-risk consumers: their benefits of mimicking the high-type insurers are lower under CR than under RR. Hence, the sponsor can reduce the inefficiency of ex post risk adjustment under CR and raise welfare.

Our paper relates to several strands of literature. First, our model builds on the adverse selection framework of Rothschild and Stiglitz (1976). On top of this asymmetric information problem between insurers and insured, we add a second layer of asymmetric information: between insurers and the health insurance sponsor. In Rothschild-Stiglitz, CR induces inefficient under-insurance of low risk types. Buchmueller and DiNardo (2002) observe this theoretical prediction in the real world. They document a decrease in coverage for the (healthier) young when the state of New York imposed a CR mandate. Moreover, if insurers cannot reduce coverage, or other dimensions of generosity, sufficiently to separate high from low cost types, the insurance market may enter a death-spiral, where lower cost types drop out of the market entirely, see Cutler and Reber (1998). Chetty and Finkelstein (2012) provide a recent overview of selection effects in insurance markets.

Second, our paper connects to the risk adjustment literature; see Van de Ven and Ellis (2000) and Ellis (2008) for overviews of this literature. Ex ante and ex post risk adjustment have been analyzed separately before. Our paper uses a mechanism design approach in which ex ante and ex post are combined to elicit the insurer’s private information about customer types, as in Sappington and Lewis (1999). Whereas they focus on providers who can engage in high-risk patient dumping, our model focuses on insurers who compete and offer insurance to all consumers. Selection here takes the form of reduced generosity (e.g. coverage) for low-cost types. In contrast to Sappington and Lewis (1999), in our work the coverage distortion is endogenous, and takes place on the second tier in a hierarchical contracting model.

Also in this strand of the literature is Glazer and McGuire (2000). They have a model where both the sponsor and the insurers have an imperfect signal about a customer’s type. They show how this imperfect signal should be incorporated in the risk adjustment system to get an efficient outcome. In our model, the sponsor has no signal about the types in a risk class. In other words, to the sponsor these types are observationally equivalent. But the insurer does have more information on these types, and this allows it to game the system. We show how the sponsor can address this problem by offering combinations of ex ante and ex post risk adjustment.
Finally, our model connects to the literature on contracting hierarchies. We consider a two-tiered model of insurers designing contracts for consumers, which in turn depend on risk adjustment contracts designed by the sponsor for the insurers. Related models are those of DeMarzo, Fishman and Hagerty (2005) and Faure-Grimaud, Laffont and Martimort (1999), who look at models of an intermediary supervising an agent, and ask how the principal’s arrangements with the intermediary affect those downstream contracts. Bijlsma, Boone and Zwart (2014) consider a health insurance model with risk adjustment, and analyze how risk adjustment parameters shape insurance contracts when insurers compete imperfectly. In the current work, we are more explicit about the information asymmetry between principal and intermediaries (the insurers), and solve for the optimal mechanism both upstream and downstream.

This paper is organized as follows. We first describe the two-tier model of perfectly competitive insurers offering incentive-compatible contracts to consumers and the sponsor offering incentive-compatible risk-adjustment contracts to insurers. We then analyze optimal risk adjustment under both RR and CR requirements. We derive when and why CR outperforms RR. Proofs can be found in the appendix.

2. The model

We present a two-tier model of risk-adjustment in health insurance. Risk averse consumers, who can have either high or low expected health-care cost, buy health insurance from perfectly competitive insurers who compete in contracts à la Rothschild and Stiglitz. A regulator or sponsor offers a menu of risk adjustment contracts to these insurers, in an effort to reduce distortions in the insurance market (efficiency) and to bring prices for high and low-risk consumers closer together (solidarity). There is information asymmetry: the sponsor does not observe consumer health types, while insurers know their own consumers’ types. The sponsor may either allow insurers to use their information to explicitly price discriminate between consumers, or ban explicit price discrimination and impose that consumers may buy any contract (with a given price) that the insurer offers. The former situation we refer to as RR and the latter as CR. Under CR, the insurers engage in second-degree price discrimination to separate the two types of consumer.

We follow Rothschild and Stiglitz and assume that the sponsor cannot use contracts to prohibit second-degree price discrimination. That is, we assume that the variables used to
separate types are not (all) contractible.

We next describe the agents’ objectives, the structure of the insurance contracts and the properties of the risk adjustment system that the sponsor designs. We conclude this section by summarizing the timing of the model.

2.1. Consumers

We consider consumers within an observable risk class, which is defined on the basis of characteristics contractible for the sponsor. These typically include age, sex, and previous treatments (Van de Ven and Ellis, 2000). Within such a risk class—say, female, age 40, no previous treatments—a fraction \( \phi \in [0,1] \) of consumers has low expected health care costs (denoted l-consumers), while the remaining \( 1 - \phi \) consumers have high expected costs (denoted h-consumers). Type h and l-consumers need treatment with probabilities \( \theta^h \) and \( \theta^l \) resp., with \( \theta^h > \theta^l \). A patient who needs treatment incurs treatment costs \( y \). The expected treatment costs in case of treatment \( \bar{y} > 0 \) are the same for both types.\(^6\)

Consumers purchase health insurance contracts \((p, \gamma)\) that are characterised by a price (premium) \( p \) and an inverse measure of generosity \( \gamma \geq 0 \). The expected cost for the insurer when a \( \theta \)-type buys contract \((p, \gamma)\) equals \( \theta(\bar{y} - \gamma) \); that is, \( \gamma \) is the cost saving for the insurer of reduced generosity. It may help the reader to follow the exposition, to think of \( \gamma \) as a copayment, similar to the Rothschild and Stiglitz framework.

As explained in the introduction, we assume that \( \gamma \) is not contractible for the sponsor. In other words, the sponsor cannot prohibit second degree price discrimination in case of CR. Other interpretations of \( \gamma \) include, narrow provider network (which allows the insurer to bargain lower treatment prices), waiting lists (which reduces expected costs by postponing treatments).

Assume that all consumers participate in the insurance market; either because health insurance is mandatory or because the sponsor is willing to subsidize health insurance such that everyone prefers buying insurance.

A \( \theta \)-type consumer who accepts a contract \((p, \gamma)\) has utility \( u(p, \gamma, \theta) \) with the following standard and intuitive properties. First, if insurance is fairly priced, that is \( p = \theta(\bar{y} - \gamma) \), then consumer surplus is maximized at full insurance, \( \gamma = 0 \).\(^7\) At \( \gamma = 0 \), utility falls with a marginal

\(^{6}\)Appendix B explicitly treats the case where insurers can invest effort \( e \) to reduce health care costs and introduces a distribution \( F(y|e) \) of treatment costs. We abstract from this here to keep the presentation simple.

\(^{7}\)In other words, if \( \gamma \) denotes a copayment, we assume that moral hazard on the part of patients plays no role and focus on moral hazard on the part of the insurer. Allowing for patient moral hazard—either over-
increase in $\gamma$ at the rate $\theta$ with which $\gamma$ is incurred by the insured. Intuitively, for copayment $\gamma$ close to 0, risk aversion is only a second-order effect. Second, for $\gamma > 0$ this effect is bigger due to risk aversion. Third, at full insurance the types do not differ in their marginal utility of consumption at a given price $p$. Finally, the marginal utility of consumption is decreasing. Formally, these properties can be stated as

\[-u_p(p, 0, \theta)\theta + u_\gamma(p, 0, \theta) = 0\]
\[-u_p(p, \gamma, \theta)\theta + u_\gamma(p, \gamma, \theta) < 0 \text{ for } \gamma > 0.\]
\[u_p(p, 0, \theta^h) = u_p(p, 0, \theta^l).\]
\[u_{pp}(p, \gamma, \theta) \leq 0.\]

When analyzing the CR case, we analyze second-degree price discrimination by the insurers. To guarantee that the single-crossing condition is satisfied in our model, and separation of $h$ and $l$-consumers is possible, we make the following assumption on the utility function.

**Assumption 1.** Consider two insurance contracts $(p_1, \gamma_1), (p_2, \gamma_2)$. Then $\gamma_1 < \gamma_2$ if and only if

\[u(p_1, \gamma_1, \theta^h) - u(p_2, \gamma_2, \theta^h) > u(p_1, \gamma_1, \theta^l) - u(p_2, \gamma_2, \theta^l).\]

This assumption states that the $h$-consumer, who has high probability of incurring $\gamma$, is always willing to pay more than the $l$-consumer for a decrease in $\gamma$. In other words, lower $\gamma$ is preferred by both types, but more so by the $h$-consumer.

A simple example of a utility function satisfying these assumptions is mean-variance risk aversion

\[u(p, q, \theta) = w - p - \gamma\theta - \frac{1}{2}r\theta(1 - \theta)\gamma^2.\]

with $\theta^l < \theta^h < 1/2$. Here, $w$ is the consumer’s initial wealth, the insurance premium is $p$, the expected co-payment equals $\gamma\theta$. The consumer is then exposed to health costs with variance $\theta(1 - \theta)\gamma^2$. The disutility due to risk aversion equals $\frac{1}{2}r\theta(1 - \theta)\gamma^2$, where $r > 0$ is a measure of risk aversion. At any $(p, \gamma)$ the indifference curve $p(\gamma)$ defined by $u(p(\gamma), \gamma, \theta) = C$ has a more negative slope for $h$ than for $l$-consumers. This implies assumption 1. A second example of preferences satisfying these assumptions is Von Neumann-Morgenstern utility $u(p, \gamma, \theta) =$ consumption of health care or underinvestment in prevention—would redefine efficient insurance from $\gamma = 0$ to some $\gamma > 0$.

See, for instance, Fudenberg and Tirole (1991, chapter 7) for a discussion of the single-crossing condition.
\[(1 - \theta)u(w - p) + \theta u(w - p - \gamma)\] with \[u' > 0, u'' < 0.\] Note that \(\gamma > 0\) can also be interpreted as a disutility due to restricted provider choice or due to delayed treatment (waiting list).

### 2.2. Insurers and risk adjustment

We consider a perfectly competitive insurer market, as in Rothschild and Stiglitz (1976). Insurers compete in offering contracts \((p, \gamma)\) to consumers. These contract offers may be restricted to consumers of one particular type under RR, while insurers may try to screen consumers by offering menus of contracts with distorted coverage \(\gamma\) under CR.

The insurers’ costs of insuring a consumer consist of the consumer’s realized expenditures minus her copayment if she gets a health shock, \(y - \gamma\), plus the contribution from the risk-adjustment scheme implemented by the sponsor.

The sponsor offers insurers a risk-adjustment scheme \((t, x)\) consisting of an ex ante transfer \(t\) and ex post reimbursement indexed by \(x\). Negative \(t\) means an ex ante payment from the insurer to the sponsor. Within the contractible risk class, the sponsor does not observe whether an insurer has a contract with an \(h\) or \(l\)-consumer; we denote these cases by resp. \(h\)-insurer and \(l\)-insurer. The sponsor induces truthful revelation by offering a menu of contracts \((t^i, x^i)\), \(i = h, l\). In other words, risk adjustment is used to separate insurer types \(-l\) or \(h\)-insurer– by having different degrees of ex ante and ex post generosity.

We assume that an insurer’s expenditure \(y - \gamma\) is contractible for the sponsor, but not \(\gamma\) itself. If an insured needs treatment and the insurer spends \(z = y - \gamma\), then the insurer receives ex post reimbursement \(r(z, x)\). The reimbursement function \(r\) is assumed to be smooth, satisfies \(r(0, x) = 0\) and \(r_x \in [0, 1]\): expenditure \(z\) is neither taxed nor subsidized more than one-for-one. The index \(x \in [0, 1]\) parameterizing this family of reimbursement functions indicates the generosity of the ex post scheme, \(r_x \geq 0\). We normalize \(r(z, 0) = 0\) and \(r(z, 1) = z\). Thus, \(x = 0\) implies no ex post adjustment, while \(x = 1\) implies full reimbursement of realized costs. Further, \(r_{xz} \geq 0\), implying that a more generous scheme raises marginal reimbursement \(r_z\).

A simple example of a reimbursement function is proportional reimbursement of costs \(z\): \(r(z, x) = xz\). The analysis in the main text can be understood by keeping this proportional reimbursement in mind. Appendix B shows that the results hold for more general ex post schemes. For example, the sponsor could decide to reimburse only costs above a threshold \(z^*\). This could be optimal if high costs are less elastic with respect to insurer effort than low costs.

Define \(R(x, \gamma, \theta)\) as the expected costs, net of ex ante payment \(t\) and premium income
for an insurer of insuring a $\theta$-consumer with a contract $(p, \gamma)$, when the insurer faces ex post reinsurance $r(z, x)$. Expected costs equal expected net payment of treatment costs minus expected ex post reimbursement of those costs through the risk adjustment scheme. With proportional reimbursement we have $R(x, \gamma, \theta) = (1 - x)\theta(\bar{y} - \gamma)$.

Similarly, let $C(x, \gamma, \theta)$ denote the sponsor’s expected costs of financing an ex post scheme $r(.)$ with generosity $x$. By our normalization of $x$, we have that $C(0, \gamma, \theta) = 0$. With proportional ex post adjustment we have $C(x, \gamma, \theta) = x\theta(\bar{y} - \gamma)$.

The sponsor chooses a function $r(z, x)$ and offers a menu $(t^l, x^l), (t^h, x^h)$ to insurers, where the insurer truthfully reveals the type $(l, h)$ of its customer. For our analysis the function $r(.)$ is exogenous.\footnote{The optimal family of reimbursement functions $r$ will depend on the exact distribution of costs $y$. Kitmann and Lorenz (2011) study the design of reimbursement functions that optimally trade off efficiency and selection costs given a distribution of costs. Note that our analysis and results are valid for any family of functions $r$ that satisfies the general conditions, including the optimal family.}

Given this function, we derive the optimal $t^l, x^l$.

If the insurer claims to have an h-consumer, it receives $t^h$ ex ante and the ex post expected net expenditure equals $R(x^h, \gamma, \theta)$. Truthful revelation of the insurer’s private information $\theta$ requires the risk adjustment scheme to be incentive compatible, i.e., the choice of risk adjustment contract minimizes insurer’s costs:

$$-t^l + R^l \leq -t^h + \hat{R}^h$$
$$-t^h + R^h \leq -t^l + \hat{R}^l$$

where we use short-hand notation: $R^l = R^l(x^l, \gamma^l) = R(x^l, \gamma^l, \theta^l)$, $R^h = R^h(x^h, \gamma^h) = R(x^h, \gamma^h, \theta^h)$ and for an l-insurer who claims to be h: $\hat{R}^h = \hat{R}^h(x^h, \gamma^l) = R(x^h, \gamma^l, \theta^l)$. Similarly, an h-insurer who claims to be l has expected costs $\hat{R}^l = \hat{R}^l(x^l, \gamma^h) = R(x^l, \gamma^h, \theta^h)$\footnote{Note that in these expressions the lying insurer does not adjust $\gamma$. In the proofs we check that, indeed, when an insurer decides to lie about the type of an $i$-customer, there is no reason to offer such a type $\gamma \neq \gamma^i$.}

As we assume perfect competition, the prices $p^l$ and $p^h$ charged by insurers to l and h-customers reflect the sum of ex ante transfers and ex post net expected costs. If the risk adjustment scheme is incentive compatible, prices are are given by

$$p^l = -t^l + R^l$$
$$p^h = -t^h + R^h$$

The insurer’s incentive compatibility constraints can conveniently be written in terms of $p^l$ and
$p^h$ as

\[
p^h \geq p^l + R^h - \hat{R}^h \quad (IC^l_i)
\]
\[
p^l \geq p^h + R^l - \hat{R}^l \quad (IC^h_i)
\]

The binding insurer constraint will be $(IC^l_i)$: an l-insurer would like to pretend to have an h-customer to benefit from more generous ex post adjustment $x^h$. We work with a binding $(IC^l_i)$ constraint and in the proofs we verify that $(IC^h_i)$ is satisfied in the equilibrium.

An l-insurer earns a positive information rent:

\[
R^h - \hat{R}^h \geq 0 \quad (9)
\]

With full reimbursement $x^h = 1$, the rent disappears: $R^h(1, \gamma^h) = \hat{R}^h(1, \gamma^l) = 0$. Second, with $x^h = 0$ we have $R^h(0, \gamma^h) = \theta^h(\bar{y} - \gamma^h)$ and $\hat{R}^h(0, \gamma^l) = \theta^l(\bar{y} - \gamma^l)$. We assume the marginal effect of generosity $x$ to satisfy

\[
R^h_x(x, 0) < \hat{R}^h_x(x, \gamma^l) < 0 \quad (10)
\]

for each $\gamma^l \geq 0$. As $x$ increases, the sponsor covers a bigger share of the realized costs. The cost reduction is bigger for customers with higher expected costs. Increased generosity of the ex post scheme thus reduces the information rent $R^h - \hat{R}^h$. With proportional reimbursement we get $\frac{d}{dx}(1-x)\theta^h(\bar{y} - \gamma^h) < \frac{d}{dx}(1-x)\theta^l(\bar{y} - \gamma^l)$ because $\theta^h > \theta^l$ and $\gamma^l \geq \gamma^h = 0$ (as we show below). The intuition for (9) is that an l-insurer can always claim to have an h-customer: such a mimicking insurer has lower expected costs than one with a true h-customer.

As we will see, reducing the rent $R^h - \hat{R}^h$ reduces distortions in the health insurance market and thereby increases total welfare. Ex post reimbursement does this, but at a cost. In particular, ex post reimbursement leads to moral hazard on the insurers’ side. If the sponsor covers part of the realized costs, the incentives for insurers to keep health care costs low are reduced. Indeed, insurers face the full effort cost to keep expenditure low, but part of the benefits flows to the sponsor. If there is no moral hazard issue on the insurer side, we have

\[
R(x, \gamma, \theta) + C(x, \gamma, \theta) = \theta(\bar{y} - \gamma)
\]

Put differently, increasing $x$ just shifts costs from the insurer to the sponsor, leaving total costs
unaffected. In the presence of moral hazard, however, shifting part of the realized costs on to the sponsor raises total costs as the insurer invests less effort to keep treatment costs low. In the main text, we will include those moral hazard costs in a reduced form by assuming that

\[ R(x, \gamma, \theta) + C(x, \gamma, \theta) = \alpha(x, \gamma, \theta) \]

for some positive function \( \alpha \). This captures how more generous ex post adjustment leads to higher overall costs. Appendix B presents a simple model where insurers invest effort to reduce health care costs that underlies this equation and in addition derives three technical results that we need in the proofs below.\(^{11}\)

2.3. The sponsor’s objective

The sponsor’s problem is to design a menu of two risk adjustment schemes, \((t^l, x^l)\) and \((t^h, x^h)\), where insurers are required to assign each contracted consumer to one of the two schemes.\(^{12}\) We assume the sponsor is subject to a budget constraint with a budget normalized to 0,

\[ \phi(t^l + C^l) + (1 - \phi)(t^h + C^h) \leq 0 \] \hspace{2cm} (BC)

where \( C^l = C(x^l, \gamma^l, \theta^l) \) and \( C^h = C(x^h, \gamma^h, \theta^h) \) are the costs to the scheme of paying out the expected ex post reimbursements.

In the sections below, we analyze and compare the RR case, where insurers can explicitly price discriminate, with the CC case, where they can only engage in second-degree price discrimination. The sponsor cannot directly contract on the insurer price \( p^i \) and coverage \( \gamma^i \). These follow from the equilibrium in the insurance market, given the risk adjustment scheme and choice of RR or CR that the sponsor imposes.

We assume that the sponsor maximizes weighted welfare and attaches weight \( 1 - \omega \) to \( h\)-consumers’ utility and \( \omega \in [0, \phi] \) to \( l\)-consumers’ utility. If \( \omega = \phi \) then the sponsor maximizes total welfare. Since we assume perfect competition in the insurer market, insurer profits will

\(^{11}\)In particular, equations (41), (42), (43).

\(^{12}\)Alternatively, the insurer can reveal the fraction of \( h\)-customers. This leads to a similar analysis.
be zero and the sponsor solves the following program:

$$\max_{(t^i, x^i), (t^h, x^h)} W = \omega u(p^l, \gamma^l, \theta^l) + (1 - \omega) u(p^h, \gamma^h, \theta^h)$$

s.t. $IC_i^{h,l}$, $BC$ and

perfectly competitive insurance market with either RR or CR

Solidarity is captured in two ways. First, the possibility of giving a relatively higher welfare weight to $h$-consumers than their fraction in the population would justify ($\omega < \phi$), expresses a solidarity or equity motive on the part of the sponsor.$^{13}$ Second, because $u_{pp} < 0$ the planner tries to keep $p^h$ and $p^l$ close together.

2.4. Timeline

Figure 1 below summarizes the timing. First, the sponsor determines the menu of risk adjustment contracts $(t^i, x^i)$, and either allows RR, or forces insurers to use CR. Insurers observe the risk adjustment system and on that basis set contracts $(p^i, \gamma^i)$ that consumers choose from. Consumers pay their chosen premiums $p$ and insurers report their consumers’ types to the sponsor. Insurers receive (pay) the ex ante risk adjustment transfers $t$. Finally, consumers incur health costs and payments are settled.

Figure 1: Timeline

<table>
<thead>
<tr>
<th>Sponsor offers menu $(t^i, x^i)$ to insurers and allows RR or enforces CR.</th>
<th>Insurers offer menu $(p^i, \gamma^i)$ to consumers.</th>
<th>Consumers select an insurance contract.</th>
<th>Insurers select risk adjustment contracts.</th>
<th>Consumers fall ill with probability $\theta^i$ inducing costs $y$.</th>
<th>Payments are made.</th>
</tr>
</thead>
</table>

The stage in which insurers exert effort to reduce treatment costs is treated implicitly in the main text and modelled explicitly in appendix B.

$^{13}$Fleurbaey and Schokkaert (2011) give an overview of why the sponsor might have such equity considerations. See Bijlsma, Boone and Zwart (2014) in which a similar modelling approach is used to capture solidarity.
3. Risk rating

In this section we analyze optimal contracts in the case of RR, when insurers are allowed to explicitly price discriminate. Section 4 analyses CR. In that case, additional consumer incentive compatibility constraints make sure that consumers self-select into contracts designed for their type.

In either case, our analysis proceeds as follows. We first use the binding constraints of the sponsor’s optimization program \((P_\omega)\) to express equilibrium insurance contracts \((p^i, \gamma^i)\) in terms of the sponsor’s contract parameters \(t^i\) and \(x^i\). We show that optimal risk adjustment involves offering ex post reinsurance only to insurers of \(h\)-consumers, \(x^h > 0\), while setting \(x^l = 0\). We then consider the sponsor’s optimization, and in particular its dependence on the weight \(\omega\) and the moral hazard cost of reinsurance \(\alpha\).

In both this section and the next, the following lemma will be useful. The lemma gives the explicit expressions for \(p^{h,l}\) if insurer incentive compatibility \((IC^l_l)\) and the budget constraint \((BC)\) bind. It is then routine to verify that these two equations, together with expressions (7,8) for prices under perfect competition, result in the following.

**Lemma 1.** If \((IC^l_l)\) and \((BC)\) are binding, prices equal

\[
p^l = \phi(C^l + R^l) + (1 - \phi)(C^h + R^h) - (1 - \phi)(R^h - \hat{R}^h) \quad (12)
\]

\[
p^h = \phi(C^l + R^l) + (1 - \phi)(C^h + R^h) + \phi(R^h - \hat{R}^h) \quad (13)
\]

We can interpret these expressions in the following way. The sum of the first two terms in equations (12) and (13) equals the average price \(\phi p^l + (1 - \phi)p^h\), which also equals total costs because of perfect competition. The last term in the expressions for \(p^h\) and \(p^l\) equals the redistribution \((R^h - \hat{R}^h \geq 0)\) from the \(h\)-insurer to the \(l\)-insurer. Because an \(l\)-insurer would like to mimic an \(h\)-insurer, the sponsor has to give the \(l\)-insurer an information rent \(R^h - \hat{R}^h\) to induce truthful revelation of the customer’s type. Due to the budget constraint \((BC)\), this information rent raises \(p^h\) and reduces \(p^l\).

Lemma 1 makes clear that risk adjustment helps to implement cross subsidies from \(l\) to \(h\)-consumers. If risk adjustment is such that the information rent \(R^h - \hat{R}^h\) equals 0, prices equal the average costs in the population. Without risk adjustment, when \(x^h = x^l = 0\), there is no cross subsidy. Indeed, in that case \(C^l = C^h = 0\) and \(\hat{R}^h = R^l\). The lemma then shows
that prices equal expected cost, \( p^l = R^l(0, \gamma^l) \) and \( p^h = R^h(0, \gamma^h) \).

A sponsor that values solidarity wants to reduce \( R^h - \hat{R}^h \). Because getting part of their costs reimbursed ex post is more profitable for \( h \) than for \( l \)-insurers \( (R^h_x - \hat{R}^h_x < 0) \), this can be achieved by raising \( x^h \), that is, increasing ex post risk adjustment. Of course, the disadvantage of raising \( x^h \) is the inefficiency \( \alpha \) it induces. This will increase costs, and hence average prices, which reduces consumer utilities.

Whereas \( x^h \) can reduce \( p^h \), it is clear from equations (12,13) together with (11) that \( x^l \) can only raise prices \( p^{h,l} \), while it has no effect on the redistribution term. Hence, it is no surprise to find that the sponsor sets ex post risk adjustment for \( l \)-insurers to zero, \( x^l = 0 \). This is the standard “no distortion at the top” result. The following lemma characterizes the RR optimum.

**Lemma 2.** With RR, an insurance market equilibrium always exists and has \( \gamma^h = \gamma^l = 0 \) and \( x^l = 0 \). Further, in the sponsor’s problem, (BC) and (IC) hold with equality.

The intuition for \( \gamma^h = \gamma^l = 0 \) is straightforward. Under RR, the insurers can separate \( h \) and \( l \)-consumers directly and hence there is no reason to distort \( \gamma \). Consumers get efficient health insurance. Consequently, the sponsor can focus attention on a single goal: solidarity. How can the remaining risk adjustment parameters \( t^l \), \( t^h \) and \( x^h \) be chosen in order to lower \( p^h \geq p^l \)?

We define \( \bar{x}^h \) as the level of ex post risk adjustment such that the \( l \)-insurer’s information rent, \( R^h - \hat{R}^h \), disappears. That is, at \( x^h = \bar{x}^h \), the costs of an \( h \)-insurer equals the costs of an \( l \)-insurer claiming to have an \( h \)-consumer:

\[
R^h(\bar{x}^h, 0) - \hat{R}^h(\bar{x}^h, 0) = 0 \quad (14)
\]

To illustrate, with proportional ex post reimbursement, we find \((1 - \bar{x}^h)(\theta^h - \theta^l)\bar{y} = 0 \) if \( \bar{x}^h = 1 \). Consequently, at \( x^h = \bar{x}^h \), both types are charged the same price: \( p^h = p^l \). We assume that the sponsor does not want to redistribute more than \( p^h = p^l \) and hence chooses the optimal \( x^h \in [0, \bar{x}^h] \).\(^{14}\)

It is clear that in the absence of costs to ex post risk adjustment, if \( \alpha = 0 \) and moral hazard is absent, the sponsor will choose to fully redistribute, \( x^h = \bar{x}^h \), so that prices for both types are equal. Conversely, if ex post risk adjustment is very costly, the benefits of redistribution will never outweigh the costs of doing so, and we expect that the sponsor chooses \( x^h = 0 \). To interpolate between these extremes, we parametrize the insurer moral hazard function \( \alpha(x, 0, \theta^h) \)

\(^{14}\)The sponsor’s objective function in \((P')\) assumes that \( p^h \geq p^l \) and hence \( u^h \leq u^l \).
as follows, \( \alpha(x, 0, \theta^h) = \varepsilon \tilde{\alpha}(x) \) for some positive function \( \tilde{\alpha}(.) > 0 \). The following proposition summarizes how the optimal \( x^h \) with RR changes as we vary \( \varepsilon \).

**Proposition 1.** With risk-rating, the planner sets \( 0 \geq t^l \geq t^h \). Furthermore,

- If \( \varepsilon = 0 \) full ex post reinsurance obtains, \( x^h = \bar{x}^h \);
- If \( \omega = \phi \) and reinsurance is costly, \( \varepsilon > 0 \), partial reinsurance obtains, \( x^h < \bar{x}^h \);
- If \( \omega < \phi \) there exists an \( \bar{\varepsilon} > 0 \) such that \( x^h(\varepsilon) = \bar{x}^h \) for all \( \varepsilon < \bar{\varepsilon} \).

The proposition makes two main points. First, the analysis of changes in moral hazard costs shows when full ex post reinsurance is optimal. If the sponsor’s objective function is biased towards the h-consumer (\( \omega < \phi \)), it remains optimal for the sponsor to implement \( p^h = p^l \), as long as reinsurance is not too costly (\( \varepsilon < \bar{\varepsilon} \)). Thus, in that case the sponsor optimally sets \( x^h = \bar{x}^h \) for small moral hazard \( \varepsilon > 0 \).

Second, the proposition makes clear how the mechanism of combining ex post and ex ante risk adjustment works. The sponsor effectively offers insurers the choice between buying reinsurance \( x^h > 0 \) at a price \( |t^h| \) and paying (a tax) \( |t^l| < |t^h| \) without reinsurance (\( x^l = 0 \)). For h-insurers, reinsurance is attractive. They receive ex post risk adjustment with a probability \( \theta^h > \theta^l \) and their total compensation is positive. For l-insurers, however, the cost of reinsurance exceeds the benefit. They prefer paying a tax \( |t^l| \) instead. As a result, risk adjustment moves prices closer together and redistributes towards h-consumers.

4. Community Rating

Under CR insurers cannot discriminate based on consumers’ observable characteristics. This can arise because the sponsor does not allow insurers to use their information on relevant characteristics when selling insurance, or because insurers do not observe such characteristics when selling the contract. Hence, under CR the contracts \( (p^l, \gamma^l), (p^h, \gamma^h) \) that insurers offer have to satisfy consumer incentive compatibility \( (IC^h,c) \) to get truthful revelation by the insured consumers:

\[
\begin{align*}
    u(p^h, \gamma^h, \theta^h) &\geq u(p^l, \gamma^l, \theta^h) & (IC^h_c) \\
    u(p^l, \gamma^l, \theta^l) &\geq u(p^h, \gamma^h, \theta^l) & (IC^l_c)
\end{align*}
\]
We now have to analyze again how the equilibrium contracts that insurers offer to consumers depend on the risk adjustment scheme offered by the sponsor to the insurers. The consumer incentive compatibility constraints introduce additional distortions in coverage $\gamma$ that we have to take into account. For this purpose, taking the risk adjustment scheme as given, we define the CR competitive market equilibrium as in Rothschild and Stiglitz (1976):

**Definition 1.** Vector $(p^l, \gamma^l, p^h, \gamma^h)$ forms a CR equilibrium, given the risk adjustment scheme $(t^l, x^l, t^h, x^h)$, if

- contracts $(p^l, \gamma^l), (p^h, \gamma^h)$ satisfy consumer incentive compatibility conditions $(IC^h_c)$ and $(IC^l_c)$,
- each contract that is offered earns a non-negative profit and
- it is not possible to introduce a (new) contract which makes strictly positive profits.

From the definition, we can make a number of observations on the equilibrium.

**Lemma 3.** If a CR equilibrium exists, it satisfies the following conditions

1. high types get full coverage $\gamma^h = 0$,
2. $(IC^h_c)$ holds with equality and
3. insurers make zero profits.

These properties of the equilibrium, familiar from the Rothschild-Stiglitz context, carry over for any risk adjustment scheme. The intuition is that, because generosity $\gamma = 0$ is optimal—see equation (1)—the cost of a marginal reduction in $\gamma > 0$ is always less than the value the consumer attaches to such an increase in generosity. The only reason for not reducing $\gamma$ to zero is that a constraint is hit. While the $IC^h_c$ constraint bars $\gamma^l$ from being equal to zero, nothing stops $\gamma^h$ because the $l$-consumer does not want to mimic $h$: we find $\gamma^h = 0$.

With RR, lemma 2 guarantees existence of an equilibrium. It is well known that existence of a separating Rothschild-Stiglitz equilibrium is not guaranteed, as there may be a profitable deviation to a pooling equilibrium if $\phi$ is sufficiently high. For the remainder of this analysis we ignore this issue. We note that the sponsor can always design a risk adjustment scheme $(t^l, x^l), (t^h, x^h)$, such that an equilibrium exists. In particular, a (pooling) equilibrium exists by setting $x^h = \bar{x}^h, x^l = 0$. Then it is the case that $p^h = p^l$ and $\gamma^h = \gamma^l = 0$. 

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We now turn to explore the sponsor’s optimization problem, \((P_\omega)\), using the constraints that are summarized in lemma 3. The sponsor chooses a set of risk adjustment contracts, \((t^l, x^l), (t^h, x^h)\), such that the resulting market equilibrium \((p^l, \gamma^l), (p^h, \gamma^h)\) maximizes weighted consumer surplus. With RR we have \(x^l = 0\) (see lemma 2) because \(x^l\) raises prices \(p_h, t\), while it does not affect redistribution. A similar reasoning leads to \(x^l = 0\) with CR as well. Also, as in the RR case, the l-insurer incentive compatibility constraint and the budget constraint hold with equality:

**Lemma 4.** Assume that an equilibrium exists. In the optimum, IC\(_i^l\) and BC hold with equality, \(x^l = 0\) and prices are given by \((12,13)\).

We are now ready to analyze the sponsor’s optimization problem: substituting the results in lemma’s 3 and 4 in the binding constraints, we study the optimal choice of ex post risk adjustment \(x^h\) for h-insurers. Compared to the analysis under RR, we now also have to take into account that, in the CR market equilibrium, copayments \(\gamma^l\) may be positive. As in the case of RR, we again characterize optimal \(x^h\) as a function of the costs of reinsurance, using \(\alpha(x, 0, \theta^h) = \varepsilon \tilde{\alpha}(x)\). In particular, we are interested in how optimal risk adjustment changes in the neighborhood of the full-reinsurance point, \(x^h = \bar{x}^h\), as we allow positive reinsurance costs \(\varepsilon > 0\). We have the following result.

**Proposition 2.** With CR, the planner sets \(0 \geq t^l \geq t^h, \gamma^h = 0, \text{ and } \gamma^l \geq 0\) such that IC\(_c^h\) binds. Furthermore,

- if \(\varepsilon = 0\) full ex post reinsurance obtains, \(x^h = \bar{x}^h\) and \(\gamma^l = 0\);
- if \(\omega = \phi\) and reinsurance is costly, \(\varepsilon > 0\), partial reinsurance obtains, \(x^h < \bar{x}^h\), and the low-types’ coverage is distorted, \(\gamma^l > 0\);
- if \(\omega < \phi\), then there exists an \(\varepsilon > 0\) such \(x^h = \bar{x}^h\) and \(\gamma^l = 0\) for each \(\varepsilon < \bar{\varepsilon}\);
- if \(\varepsilon\) is sufficiently large, no risk adjustment obtains, \(t^h = t^l = x^h = x^l = 0\). Low type coverage is at the Rothschild-Stiglitz level, \(\gamma^l = \gamma^l_{RS} > 0\).

As in the RR case, h-insurers opt for reinsurance and pay less than the actuarial cost of such reinsurance through the ex ante payment \(|t^h|\). For l-insurers, ex post insurance is less valuable, with the consequence that they prefer paying an ex ante contribution \(|t^l|\). This again forces prices closer together than would be the case without risk adjustment. The benefit is not only
redistribution towards the h-consumers, but also a reduction in the distortion on l-consumer coverage \( \gamma^l \) that goes hand in hand with a reduction in the price difference.

If ex post risk adjustment does not lead to higher costs \( (\varepsilon = 0) \), the sponsor implements the efficient outcome with \( \gamma^l = 0 \). Equations \((IC^h)\) and \((IC^l)\) then imply that we need \( x^h = \bar{x}^h \) to obtain this outcome. If \( \varepsilon > 0 \) and the sponsor maximizes total welfare \( (\omega = \phi) \), the cost of ex post risk adjustment implies incomplete risk adjustment, and hence \( \gamma^l > 0 \) is optimal. However, as in the RR case, if the sponsor puts more weight on the unfortunate h-consumers \( (\omega < \phi) \), a range of \( \varepsilon > 0 \) exists such that the sponsor implements \( x^h = \bar{x}^h \) and \( \gamma^l = 0 \). For \( \varepsilon > 0 \) close enough to 0, increasing \( x^h \) reduces \( p^h \). This reduction in \( p^h \) at the expense of the l-consumers increases welfare if \( \omega < \phi \): l-consumers pay for the costs of ex post risk adjustment, and this cost has a lower weight in the sponsor’s objective function. For high values of \( \varepsilon \), also \( p^h \) will be increasing in \( x^h \); from that point onward, \( x^h < \bar{x}^h \) and \( \gamma^l > 0 \) become optimal.

If the inefficiency of ex post reinsurance is high enough, the sponsor does not use risk adjustment at all \( (x^h = x^l = 0 \text{ and } t^h = t^l = 0 \text{ because of } (BC)) \). The standard Rothschild-Stiglitz equilibrium then obtains. We know that welfare is higher with RR than it is with CR in the Rothschild-Stiglitz equilibrium. Indeed, without risk adjustment, the contracts for high types are the same under RR and CR while there is inefficient coverage for the low types \( (\gamma^l > 0) \) under CR. In contrast, for low costs of ex post reinsurance, the following section shows that CR can lead to higher welfare than RR.

5. When is CR optimal?

This section compares total welfare under RR and CR. As explained in the introduction, health economists tend to view CR as Pareto inferior to RR. This is true in a setting where the information asymmetry between sponsor and insurers is not explicitly addressed. In our set up, the sponsor combines ex ante and ex post risk adjustment to induce insurers to truthfully reveal their private information. In this second-best case, CR can lead to higher welfare than RR. In particular, if the sponsor’s objective is biased towards high types and the moral hazard inefficiency is positive but not too high, we find that CR dominates RR.

Let \( W_{RR}(\varepsilon) \) denote the optimal value of \( (P_\omega) \) in the RR case and \( W_{CR}(\varepsilon) \) in the CR case as a function of \( \varepsilon \) with \( \alpha(x, 0, \theta^h) = \varepsilon \tilde{\alpha}(x) \) as before. We have the following proposition.

**Proposition 3.** If \( \omega < \phi \) and assuming that \( x^h_{RR}(\varepsilon) = \bar{x}^h \) is globally optimal whenever it is
locally optimal. Then:

1. \( W_{CR}(\varepsilon) = W_{RR}(\varepsilon) \) for \( \varepsilon \geq 0 \) small enough;
2. \( W_{CR}(\varepsilon) > W_{RR}(\varepsilon) \) for \( \varepsilon > 0 \) in a middle range;
3. \( W_{RR}(\varepsilon) > W_{CR}(\varepsilon) \) for \( \omega > 0 \) and \( \varepsilon \) big enough.

As we make few assumptions on second derivatives, we cannot be sure that the optimal \( x^h \) is well behaved as a function of \( \varepsilon \). For our result that CR can dominate RR, we only need that \( x^h \) under RR remains at its maximum level until that ceases to be locally optimal. A simple set up with (i) \( r(x, z) = xz \), (ii) mean variance utility and (iii) \( \bar{\alpha}'(x) \geq 0 \) is enough to get this behavior (see footnote 16 in the appendix).

It follows from propositions 1 and 2 above that for \( \varepsilon \geq 0 \) small enough and \( \omega < \phi \), the solutions under RR and CR are the same: \( x^h = \bar{x}^h, \gamma_l^t = 0 \) and prices are the same as well. Hence, \( W_{CR} = W_{RR} \) for these values of \( \varepsilon \). Further, if \( \varepsilon \) is big enough that no risk adjustment is used (\( x^h = t^h = t^l = 0 \)) we know that RR strictly dominates CR (if \( \omega > 0 \)) in the Rothschild and Stiglitz outcome.

The interesting case is in between these two extremes. The proposition shows that a range of \( \varepsilon \) exists such that CR dominates RR in terms of weighted welfare \( W \). CR, with its distortive coverage \( \gamma_l^t > 0 \), can outperform RR which has efficient coverage for both types. To understand this result, first note that, as the moral hazard parameter \( \varepsilon \) increases, \( x^h_{CR} \) falls below \( \bar{x}^h \) before \( x^h_{RR} \) does. The sponsor screens insurer types with ex post reinsurance \( x^h \) and extracts rents from \( l \)-consumers to redistribute to the \( h \)-consumers. Having \( \gamma_l^t > 0 \), which relaxes insurer incentive compatibility, \( IC_1^l \), increases the potential for rent extraction. Indeed, the difference in benefits of reinsurance between both insurer types is larger because expected expenses for \( l \)-insurers are even lower if \( \gamma_l^t > 0 \). As consumers pay part of the cost if \( \gamma \) denotes a copayment,\(^{15}\) the benefit of reinsurance for mimicking \( l \)-insurers is reduced. Consequently, the sponsor can extract larger ex ante payments from \( l \)-insurers without violating their incentive compatibility constraint. These payments are passed on to consumers, bringing prices for both consumer types closer together. The reduction in \( x^h \) by increasing \( \gamma_l^t \) is therefore initially more attractive under CR compared to RR. The second step is then a revealed preference argument (with a slight abuse of notation):

\[
W_{CR}(\varepsilon) = W_{CR}(x^h_{CR}(\varepsilon)) > W_{CR}(\bar{x}^h) = W_{RR}(\bar{x}^h) = W_{RR}(\varepsilon)
\]

\(^{15}\)Or accept reduced provider choice at a lower premium with lower expected costs.
for values of $\varepsilon$ where $x_{CR}^h(\varepsilon) < \bar{x}^h = x_{RR}^h(\varepsilon)$. For these values of $\varepsilon$, $\bar{x}^h$ could be implemented under CR in which case welfare would be the same as under RR. But $x_{CR}^h < \bar{x}^h$, hence $W_{CR}$ is higher than under $\bar{x}^h$ and consequently it is higher than $W_{RR}$.

CR thus outperforms RR if the cost of reinsurance is small but not too small. Inefficiency $\gamma^l > 0$ has the benefit of reducing $\rho^h$ which is beneficial for $h$-consumers which get higher weight $(1 - \omega > 1 - \phi)$ in the sponsor’s objective. In fact, with $\omega = 0$ CR always (weakly) outperforms RR (as there is no welfare cost of $\gamma^l > 0$). In this sense, policy makers’ preference for CR can be better founded than health economists tend to acknowledge. A bias in favour of $h$-consumers because the sponsor values solidarity together with risk adjustment as an instrument to screen insurers can motivate a choice for CR.

6. Conclusion

We studied optimal risk adjustment in competitive health insurance markets when insurers have better information on their customers’ risk profiles than the sponsor of the health insurance scheme. This occurs if the insurer observes more consumer characteristics than the ex ante system corrects for. Such information advantage allows insurers to game the system by cherry picking insured whose expected costs are low within their (ex ante) risk adjustment class.

A risk adjustment system with both ex ante and ex post components can avoid this cost by inducing insurers to truthfully reveal their private information. By offering some ex post reinsurance to separate insurer types, the sponsor can redistribute from low to high risk consumers. If reinsurance does not induce costs due to moral hazard, the sponsor can achieve both optimal efficiency as well as solidarity by completely reinsuring insurers. In practice, however, such ex post risk adjustment is likely to be costly due to moral hazard and the sponsor only offers partial reinsurance.

The sponsor can improve on this equilibrium by adding one more distortion. A distortion on the low-risks’ contracts’ generosity reduces not only their welfare, but also their information rent. The lower information rent here appears as redistribution to the high types. In health insurance, contract generosity is typically hard to contract on for the sponsor. We show that in the presence of such incontractibility, the sponsor can gain by enforcing an equilibrium in which this distortion emerges endogenously by imposing CR.

Although CR is never optimal in the absence of a motivation to screen insurers, when risk
adjustment tries to elicit the insurers’ private information, CR can raise total weighted surplus if the preference for redistribution is sufficiently high. The resulting distortions in consumer contracts, which in themselves are socially costly, relax the incentive compatibility constraint of the insurers: by reducing the insurers’ expected costs for low risk consumers, CR makes reinsurance a less attractive option for these insurers. This makes it easier for the sponsor to separate insurer types and hence allows more redistribution from low to high risk consumers, raising welfare.
References


A. Proofs of results

Proof of lemma 2 The proof has the following steps. First, we argue that $\gamma_l = \gamma_h = 0$. Then, we show that $x^l = 0$ assuming that $(IC^l_i)$ and (BC) are binding. Finally, we check that $(IC^h_i)$ is satisfied as well.

Because insurers can risk-rate, insurance contracts do not have to satisfy consumer incentive compatibility constraints (as with CR in section 4). Suppose in equilibrium one of the insurers offers a contract with $\gamma > 0$. Because consumers are risk-averse and the insurer is risk-neutral, consumers value increased insurance (lower $\gamma$) more than it costs the insurer to provide that insurance (where we use that $r_z \in [0, 1]$). Thus the insurer can raise profits by reducing $\gamma$. Hence both $h$ and $l$-consumers get offered efficient insurance contracts ($\gamma = 0$). Note that $\gamma_h = \gamma_l = 0$ holds irrespective of whether the insurer reports the customer’s type truthfully to the sponsor.

Equation (BC) holds with equality. If it would be slack, it is possible to raise $t^h, t^l$ keeping $IC^l_i$ and $IC^h_i$ binding. This unambiguously increases the sponsor’s objective function. Next consider the insurer IC constraints. At least one of these is binding. Suppose not, i.e. both $(IC^l_i)$ and $(IC^h_i)$ are slack. Then we can reduce $p^h$ and increase $p^l$ (by adjusting $t^{h,l}$) in a way that satisfies (BC). This increases $W$ because $\omega \leq \phi$, $u_{pp} \leq 0$ and $(IC^l_i)$ together with (9) imply $p^h > p^l$. We assume here that $(IC^l_i)$ is binding and check afterwards that $(IC^h_i)$ is satisfied as well.

Because both $(IC^l_i)$ and (BC) hold with equality, we use lemma 1 for the prices $p^{h,l}$. Then the effect of $x^l$ on welfare can be written as follows.

$$\frac{\partial W}{\partial x^l} = \omega \frac{\partial u(p^l, 0, \theta^l)}{\partial x^l} + (1 - \omega) \frac{\partial u(p^h, 0, \theta^h)}{\partial x^l} = \omega \frac{\partial u}{\partial p}(p^l, 0, \theta^l) \frac{\partial p^l}{\partial x^l} + (1 - \omega) \frac{\partial u}{\partial p}(p^h, 0, \theta^h) \frac{\partial p^h}{\partial x^l} = \phi \alpha(x^l, 0, \theta^l) \left( \omega \frac{\partial u}{\partial p}(p^l, 0, \theta^l) + (1 - \omega) \frac{\partial u}{\partial p}(p^h, 0, \theta^h) \right) \leq 0$$

because $\frac{\partial u}{\partial p} < 0$ and $\frac{\partial (C^l + R^l)}{\partial x^l} = \alpha(x^l, \gamma, \theta^l) \geq 0$. It follows that $x^l = 0$.

We finish this proof by checking that $(IC^h_i)$ is satisfied. By adding the insurer’s incentive
constraints, it follows that

\[ R^l - \hat{R}^h \leq \hat{R}^l - R^h \]  

(16)

As \((IC^h_i)\) holds with equality, (16) implies that \((IC^l_h)\) holds as well. With \(x' = 0\), (16) can be written as

\[ R^l(0, 0) - \hat{R}^h(x^h, 0) \leq \hat{R}^l(0, 0) - R^h(x^h, 0) \]  

(17)

or equivalently –because \(\hat{R}^h(0, 0) = R^l(0, 0), \hat{R}^l(0, 0) = R^h(0, 0)\):

\[ \int_0^{x^h} |\hat{R}^h_x(x, 0)| dx \leq \int_0^{x^h} |R^h_x(x, 0)| dx \]  

(18)

which holds because of equation (10).

Q.E.D.

**Proof of proposition 1**

Given that \(x' = \gamma' = \gamma^h = 0\), we consider

\[ \frac{\partial W}{\partial x^h} = \omega \frac{\partial u(p', 0, \theta^l)}{\partial x^h} + (1 - \omega) \frac{\partial u(p^h, 0, \theta^h)}{\partial x^h} \]

\[ = \omega u_p \frac{\partial p^l}{\partial x^h} + (1 - \omega) u_p^h \frac{\partial p^h}{\partial x^h} \]

\[ = (1 - \phi)\alpha(x^h, 0, \theta^h)(\omega u_p^l + (1 - \omega)u_p^l) + |R^h_x - \hat{R}^h_x|(|u_p^h|\phi(1 - \omega) - |u_p^l|\omega(1 - \phi)) \]

where we used (10). The first term is negative as \(u_p < 0\) and \(\alpha(x^h, 0, \theta^h) = \varepsilon \hat{\alpha}(x^h) \geq 0\): this is the inefficiency due to ex post reimbursement. The second term captures the positive effect of ex post reimbursement on solidarity: reducing the redistribution from \(h\) to \(l\) by raising \(x^h\). This term is indeed positive for \(x^h \in [0, \bar{x}^h]\) because \(p^h \geq p'\) together with equations (2,3) and \(\omega \leq \phi\).

If \(\varepsilon = 0\) then \(\partial W/\partial x^h \geq 0\) and the sponsor equalizes prices by setting \(x^h = \bar{x}^h\).

If \(\varepsilon > 0\) and \(\omega = \phi\), then \(\partial W/\partial x^h |_{x^h = \bar{x}^h} < 0\) and hence \(x^h < \bar{x}^h\).

If \(\omega < \phi\), there exists \(\bar{\varepsilon} > 0\) as defined in the proposition.\(^\text{16}\)

\(^\text{16}\)If we assume \(r = xz\) and mean variance utility as in (5), we have \(|R^h_x - \hat{R}^h_x| = (\theta^h - \theta')y\) and \(u_p = -1\). Then we can write

\[ \frac{\partial W}{\partial x^h} = -(1 - \phi)\varepsilon \hat{\alpha}(x) + (\theta^h - \theta')y(\phi - \omega) \]

(19)
Finally, equation (6) implies

\[ t^l - t^h \geq R^l(0, 0) - \hat{R}^h(x^h, 0) \geq 0 \]  

(21)

where the last inequality follows from (41).

Q.E.D.

Proof of lemma 3

- Suppose to the contrary that high types are not fully covered, \( \gamma^h > 0 \). Then an insurer could offer a new, more profitable contract with slightly higher coverage, as, by inequality (1), the high type consumer’s value of such lower \( \gamma^h \) grows more strongly than the costs to the insurer, since \( r_\gamma(y - \gamma, x) \in [-1, 0] \). Such a new contract will certainly continue to satisfy \( IC^h_c \). If l-consumers decide to buy this contract as well, it becomes more profitable as their expected costs are lower. This profitable deviation contradicts definition 1. Hence \( \gamma^h = 0 \).

- Suppose –by contradiction– that \( IC^h_c \) is slack. We consider two cases:

  - \( \gamma^l > 0 \): because of equation (1), a new contract with \( \tilde{\gamma}^l < \gamma^l \) and \( \tilde{p}^l > p^l + (\gamma^l - \tilde{\gamma}^l)\theta^l \)
    can be introduced which l-consumers prefer (but h-consumers do not; as \( IC^h_c \) is slack by assumption) and which leads to strictly positive profits. This contradicts definition 1.
  
  - \( \gamma^l = 0 \): then we have

\[
\begin{align*}
    u(p^h, 0, \theta^h) &> u(p^l, 0, \theta^h) (\text{slackness of } IC^h_c) \\
    u(p^l, 0, \theta^l) &\geq u(p^h, 0, \theta^l) (\text{by } IC^l_c).
\end{align*}
\]

But this is impossible because \( u_p < 0 \).

Hence, in each case there is a violation and thus \( IC^h_c \) cannot be slack. With \( (IC^h_c) \) holding with equality, assumption 1 implies that \( (IC^l_c) \) is satisfied as well.

---

Then, if moreover \( \tilde{\alpha}'(x) \geq 0 \), \( W \) is concave in \( x^h \), and we find

\[
\bar{\epsilon} = \frac{(\theta^h - \theta^l)\bar{y}(\phi - \omega)}{(1 - \phi)\bar{\alpha}(\bar{x}^h)} > 0.
\]

(20)
Suppose an insurer makes positive profits in equilibrium. If positive profits are from h-consumers, an insurer can offer a new contract with slightly lower $p_h$ and make a strictly positive profit. Even if l-consumers choose this contract as well, it is profitable (as l-consumers have lower expected costs than h-consumers). This contradicts definition 1. Next, consider the case where the insurers make a profit on the l-consumers. Then one can construct a new profitable contract $\tilde{\gamma}_l > \gamma_l$, $\tilde{p}_l < p_l$ such that $IC^h_c$ remains satisfied and the new contract is more attractive to l-consumers. Again this contradicts definition 1. Q.E.D.

Proof of lemma 4

We first show that $IC^l_i$ binds. Assume to the contrary that $IC^l_i$ is slack. First, suppose that $\gamma > 0$. Then the sponsor can slightly increase $t^h$ and decrease $t^l$ without violating $IC^l_i$ (such a change cannot violate $IC^h_i$). Such a transfer from l to h-consumers is in itself beneficial for welfare, as welfare is biased towards h-consumers. The transfer increases $p_l$ and decrease $p_h$, which relaxes $IC^h_i$. This, in turn, allows $\gamma$ to fall in the resulting equilibrium, and hence $W$ is increased; a contradiction.

If, instead, $\gamma = 0$, we are in a pooling equilibrium, with necessarily $p_l = p_h$, or

$$-t^h + \hat{R}^h = p^h = p_l = -t^l + R^l < -t^h + \hat{R}^h$$

where the inequality represents slack $IC^l_i$. But this cannot hold since $R^h \geq \hat{R}^h$ by (9).

Next, we verify that $(IC^l_i)$ also holds for $\gamma \neq \gamma^l$, i.e.

$$-t^l + R^l(x^l, \gamma^l) \leq -t^h + \hat{R}^h(x^h, \gamma^l)$$ (22)

also when $\gamma$ in $\hat{R}^h(x^h, \gamma^l)$ is replaced by another $\gamma \geq 0$. First, consider $\gamma > \gamma^l$. As the l-consumer is risk averse, utility falls faster with $\gamma$ than the insurer’s costs because of equation (43) in appendix B. Hence, it is not a profitable deviation for the insurer to offer the l-consumer $\gamma > \gamma^l$ and then claim it is an h-consumer. Second, consider $\gamma < \gamma^l$ (in case $\gamma > 0$). To make this deviation contract $(p, \gamma)$ attractive for the l-consumer, it needs to be the case that $u(p, \gamma, \theta_l) \geq u(p^l, \gamma^l, \theta^l)$. Then a binding $(IC^h_c)$ together with assumption 1 implies that $u(p, \gamma, \theta^h) > u(p^h, 0, \theta^h)$: h-consumers buy this contract as
well. We find

$$\phi \hat{R}^h(x^h, \gamma) + (1 - \phi) R^h(x^h, \gamma) \geq \hat{R}^h(x^h, \gamma^l)$$

(23)

because $\hat{R}^h \leq 0$ (equation (60)) and $R^h \geq \hat{R}^h$ (equation (9)). Hence,

$$-t^l + R^l(x^l, \gamma^l) \leq -t^h + \hat{R}^h(x^h, \gamma^l) \leq -t^h + \phi \hat{R}^h(x^h, \gamma^l) + (1 - \phi) R^h(x^h, \gamma)$$

and this deviation is not profitable.

- Next, if (BC) is slack, it would be possible to raise $t^h, t^l$ and $q^l$, keeping $IC_l^t$ and $IC_c^h$ binding. This unambiguously increases welfare.

- With ($IC_i^t$) and (BC) binding, lemma 1 implies that $p^{h,l}$ are given by (12,13).

- Proof that ($IC_h^c$) is satisfied, is the same as in the proof of lemma 2; where now (18) becomes

$$\int_0^{x^h} |\hat{R}^h(x, \gamma^l)| dx \leq \int_0^{x^h} |R^h_0(x, 0)| dx$$

- Finally, we use figure 2 to show that $x^l = 0$. The curve shows an indifference curve for the l and h-consumer. The intersection of these two curves determines the contract $\gamma^l, p^l$.

Denote by $\gamma^l(x^l)$ the solution for $\gamma^l$ as a function of the choice of $x^l$. We have that $(p^h, \gamma^h = 0)$ and $(p^l, \gamma^l)$ should always be on the same h-consumer’s indifference curve, by binding $IC_c^h$. Furthermore, prices are determined by $\gamma^l$ and $x^l$; equations (12,13) imply that

$$\frac{\partial p^h}{\partial x^l} = \phi \alpha > 0$$

(24)

$$\frac{\partial p^h}{\partial \gamma^l} = -\phi \theta^l - \phi \hat{R}^h < 0$$

(25)

$$\frac{\partial p^l}{\partial x^l} = \phi \alpha > 0$$

(26)

$$\frac{\partial p^l}{\partial \gamma^l} = -\phi \theta^l + (1 - \phi) \hat{R}^h \begin{cases} < 0 \\ \geq -\theta^l \end{cases}$$

(27)

where $-\theta^l \leq \hat{R}^h \leq 0$ from (60).

Now we consider separately two cases, one in which equilibrium $\gamma^l$ decreases as $x^l$ grows,
and one in which $\gamma^l$ increases with $x^l$. We show that in both cases, $W$ is decreasing in $x^l$. Hence, $x^l = 0$ is optimal.

Case 1. Suppose first that $\frac{d\gamma^l}{dx^l} < 0$. Then

$$\frac{dp^h}{dx^l} = \frac{\partial p^h}{\partial x^l} + \frac{\partial p^h}{\partial \gamma^l} \frac{d\gamma^l}{dx^l} > 0,$$

so $p^h$ increases with rising $x^l$, and hence high types are worse off ($u^h$ decreases: indifference curve shifts upward in figure 2). Since $\gamma^l$ decreases by assumption, the new intersection $(p^l, \gamma^l)$ in the figure will be in region $A$ in the figure, and clearly $u^l$ also decreases. Hence, in this case, the increase in $x^l$ reduces both $u^h$ and $u^l$ and thus $W$.

Case 2. Suppose instead that $\frac{d\gamma^l}{dx^l} \geq 0$. Then

$$\frac{dp^l}{dx^l} = \frac{\partial p^l}{\partial x^l} + \frac{\partial p^l}{\partial \gamma^l} \frac{d\gamma^l}{dx^l} \geq -\theta^l \frac{d\gamma^l}{dx^l}$$

(28)

Since the l-consumer is risk averse, the indifference curve $u^l$ has slope less than $-\theta^l$: an increase in $\gamma^l$ should be compensated by a fall in $p^l$ that is bigger than $\theta^l$ (equation (1)). However, $|\partial p^l/\partial \gamma^l| \leq \theta^l$ by equation (27) and $u^l$ falls (indifference curve shifts upward). As $\gamma^l$ increases in this case, $(p^l, \gamma^l)$ is in region $B$ and again, both $u^h$ and $u^l$ decrease. Therefore, also in this case, $W$ falls with $x^l$. Q.E.D.

Proof of proposition 2 To find the welfare effects of $\gamma^l$ and $x^h$, we first derive how $\gamma^l$ and
\( x^h \) vary along \((IC^h_c)\) (holding with equality):

\[
\dot{u}_p^h \left( \frac{\partial p^l}{\partial x^h} \, dx^h + \frac{\partial p^l}{\partial \gamma^l} \, d\gamma^l \right) + \dot{u}_\gamma^h \, d\gamma^l = u_p^h \left( \frac{\partial p^h}{\partial x^h} \, dx^h + \frac{\partial p^h}{\partial \gamma^l} \, d\gamma^l \right)
\] (29)

where we used shorthand

\( \dot{u}^h = u(p^l, \gamma^l, \theta^h) \), \( u^h = u(p^h, 0, \theta^h) \).

From equations (12, 13) we find

\[
\begin{align*}
\frac{\partial p^h}{\partial x^h} &= (1 - \phi)\alpha + \phi(R_x^h - \hat{R}_x^h) \\
\frac{\partial p^h}{\partial \gamma^l} &= \phi(C_{\gamma}^l + R_{\gamma}^l) - \phi\hat{R}_{\gamma}^l \\
\frac{\partial p^l}{\partial x^h} &= (1 - \phi)\alpha - (1 - \phi)(R_x^h - \hat{R}_x^h) \\
\frac{\partial p^l}{\partial \gamma^l} &= \phi(C_{\gamma}^l + R_{\gamma}^l) + (1 - \phi)\hat{R}_{\gamma}^l
\end{align*}
\] (30, 31, 32, 33)

Using this and evaluating at \( x^h = \bar{x}^h, \gamma^l = 0 \), we find

\[
\begin{align*}
\frac{\partial W}{\partial x^h} &= (-u_p)[-(1 - \phi)\alpha(\bar{x}^h, 0, \theta^h) + (\phi - \omega)|R_x^h - \hat{R}_x^h|] \\
\frac{\partial W}{\partial \gamma^l} &= (-u_p)(\phi - \omega)(\hat{R}_{\gamma}^h + \theta^l) \\
\frac{d\gamma^l}{dx^h} &= \frac{R_x^h - \dot{X}_x}{\hat{R}_{\gamma}^h + \theta^l} < 0
\end{align*}
\] (34, 35, 36)

From this we derive

\[
\frac{dW}{dx^h} \bigg|_{x^h = \bar{x}^h, \gamma^l = 0} = (-u_p) \left[ -(1 - \phi)\varepsilon\tilde{\alpha}(\bar{x}^h) + (\phi - \omega)|R_x^h - \hat{R}_x^h| \left( 1 - \frac{\hat{R}_{\gamma}^h + \theta^l}{\theta^h + \hat{R}_{\gamma}^h} \right) \right]
\] (37)

From this the claims about \( \varepsilon \) in the proposition follow.
Finally, we consider $dW/dx^h$ at $x^h = 0$:

$$
\left. \frac{dW}{dx^h} \right|_{x^h=0} = \omega \left( u_p^l \left[ (1 - \phi)(\alpha - (R_{x}^h - \hat{R}_{x}^h))(\phi(C_{\gamma}^l + R_{\gamma}^l) + (1 - \phi)R_{\gamma}^l \frac{d\gamma^l}{dx^h} + u_{\gamma}^l \frac{d\gamma^l}{dx^h} \right]ight.
\left. + (1 - \omega)u_p^h \left[ (1 - \phi)\alpha + \phi(R_{x}^h - \hat{R}_{x}^h) + \phi(C_{\gamma}^l + R_{\gamma}^l - \hat{R}_{\gamma}^l) \frac{d\gamma^l}{dx^h} \right] \right)
\leq 0
$$

with $\alpha = \varepsilon \hat{\alpha}(0)$ for $\varepsilon$ high enough. \hfill Q.E.D.

**Proof of proposition 3** First note that $\varepsilon = 0$ implies $W_{RR}(0) = W_{CR}(0)$ because in this case $x^h = \bar{x}^h$, $p^l = p^h$ and $\gamma^l = \gamma^h = 0$ under both RR and CR. Further, propositions 1 and 2 imply that for $\omega < \phi$ there exists $\bar{\varepsilon}$ such that $W_{RR}(\varepsilon) = W_{CR}(\varepsilon)$ for each $\varepsilon \in [0, \bar{\varepsilon}]$.

By equation (43): $0 < \hat{R}_{\gamma}^l + \theta^l < \hat{R}_{\gamma}^h + \theta^h$ and hence

$$
\left. \frac{\partial W_{RR}}{\partial x^h} \right|_{x^h=\bar{x}^h} = (-u_p) \left[ - (1 - \phi)\varepsilon \hat{\alpha}(\bar{x}^h) + (\phi - \omega)|R_{x}^h - \hat{R}_{x}^h| \right]
$$

$$
> (-u_p) \left[ - (1 - \phi)\varepsilon \hat{\alpha}(\bar{x}^h) + (\phi - \omega)|R_{x}^h - \hat{R}_{x}^h| \left( 1 - \frac{\hat{R}_{\gamma}^h + \theta^l}{\theta^h + \hat{R}_{\gamma}^h} \right) \right] = \left. \frac{\partial W_{CR}}{\partial x^h} \right|_{x^h=\bar{x}^h}
$$

Thus, there exist values of $\varepsilon$ such that $\partial W_{RR}/\partial x^h|_{x^h=\bar{x}^h} > 0 > \partial W_{CR}/\partial x^h|_{x^h=\bar{x}^h}$. For these values of $\varepsilon$ we have $x^h_{CR}(\varepsilon) = \bar{x}^h > x^h_{CR}(\varepsilon)$. With a slight abuse of notation, we write welfare as $W(\varepsilon, x^h(\varepsilon))$. Then we have for these values of $\varepsilon$ that\(^{17}\)

$$
W_{CR}(\varepsilon, x^h_{CR}(\varepsilon)) > W_{CR}(\varepsilon, \bar{x}^h) = W_{RR}(\varepsilon, \bar{x}^h) = W_{RR}(\varepsilon, x^h_{RR}(\varepsilon))
$$

For $\varepsilon$ high enough, proposition 2 implies that $x^h_{CR} = 0$ and $\gamma^l_{CR} = \gamma^l_{RS}$. Then we know that $u^h_{CR} = u^h_{RR}$ while $u^l_{CR} < u^l_{RR}$. Hence, $\omega > 0$ implies that $W_{CR}(\varepsilon) < W_{RR}(\varepsilon)$ for such high values of $\varepsilon$. \hfill Q.E.D.

\(^{17}\)Note that we do not exclude the possibility where $x^h_{CR}$ drops discontinuously from $\bar{x}^h$ to some $x^h_{CR} < \bar{x}^h$. 

33
B. General ex post schemes

This appendix introduces a simple model where insurers invest effort to keep health expenditures low. We derive equations (9) and (11) from the main text:

\[ R^h - \hat{R}^h \geq 0 \]

\[ R_x(x, \gamma, \theta) + C_x(x, \gamma, \theta) = \alpha(x, \gamma, \theta) \geq 0 \]

and the following three results:

\[ R^l(0, \gamma^l) - \hat{R}^h \geq 0 \] (41)

\[ R^l(0, \gamma^l) = -\theta^l \] (42)

\[ |\hat{R}^l| \leq \theta^l \] (43)

The intuition for these three results is as follows. First, suppose that there is no ex post transfer for l-consumers: \( x^l = 0 \). If an l-insurer claims to be h, part of her costs are reimbursed ex post which lowers ex post costs compared to the case where it truthfully reveals the customer’s type. With proportional reimbursement this clearly holds: \( \theta^l(\bar{y} - \gamma^l) \geq (1 - x^h)\theta^l(\bar{y} - \gamma^l) \). Second, assuming \( x^l = 0 \), a small increase in \( \gamma^l \) reduces the costs of an l-insurer by the probability \( \theta^l \) that \( \gamma^l \) is paid to the insurer: \( \frac{d(\theta^l(\bar{y} - \gamma^l))}{d\gamma} = -\theta^l \). Third, when an l-insurer claims to be h, part of the expenditure is reimbursed ex post, hence the effect of an increase in \( \gamma \) is smaller. With proportional reimbursement \( |\frac{d}{d\gamma}(1 - x^h)\theta^l(\bar{y} - \gamma)| \leq |\frac{d}{d\gamma}\theta^l(\bar{y} - \gamma)| = \theta^l \).

Our claim is that a broad range of ex post risk adjustment systems satisfies these conditions. For the analysis here, these are the aspects of ex post risk adjustment that are important. The following model yields the required results.

Let \( Y = [y, \bar{y}] \) denote the range of expenditures \( y \), once an agent falls ill. When interpreting \( \gamma \) as copayment, we assume \( \underline{y} > \gamma \) over the relevant range of \( \gamma \).\(^{18}\) Insurers can invest effort \( e \) to reduce expenditure. Let \( F(y|e) \) denote the distribution function of \( y \), which is the same for both types \( l, h \). Then comparing two effort levels \( e_1 > e_2 \), we assume that \( F(y|e_2) \) first order stochastically dominates \( F(y|e_1) \). That is, \( F(y|e_2) \leq F(y|e_1) \). Expected expenditure

\(^{18}\)In fact, the highest \( \gamma \) that is relevant for our analysis is \( \gamma^l \) in the Rothschild-Stiglitz outcome. Hence, we assume \( \gamma^l_{RS} < \underline{y} \).
(conditionally on needing health care) is given by
\[
\bar{y}(e) = \int_y y dF(y|e)
\] (44)

First order stochastic dominance implies that \( \bar{y}(e) \leq 0 \). The expected ex post reimbursement can be written as
\[
\bar{r}(x, \gamma, e) = \int_y r(y - \gamma, x) dF(y|e)
\] (45)

where the exogenously given function \( r(z, x) \) is defined in the main text. As \( r_z \geq 0 \), first order stochastic dominance implies that \( \bar{r}_e \leq 0 \). Further, \( r_z(z, x) \in [0, 1] \) implies that \( \bar{r}_x(x, \gamma, e) \in [-1, 0] \).

We assume that insurers’ effort cost \( \psi(e) \) is incurred once an insured needs treatment. What we have in mind is the following: once a patient falls ill, the insurer can check whether treatments are necessary, try to guide the patient to a cheaper provider etc. The insurer chooses effort \( e \) that minimizes total costs:
\[
R(x, \gamma, \theta) = \min_e \theta[\bar{y}(e) - \gamma - \bar{r}(x, \gamma, e) + \psi(e)]
\] (46)

Let \( e(x, \gamma) \) denote the effort level that solves this minimization problem. In case of an interior solution, we have
\[
\bar{y}_e(e(x, \gamma)) - \bar{r}_e(x, \gamma, e(x, \gamma)) + \psi_e(e(x, \gamma)) = 0
\] (47)

It follows that \( de/dx \leq 0 \). This can be seen as follows:
\[
[\bar{y}_{ee}(e(x, \gamma)) - \bar{r}_{ee}(x, \gamma, e(x, \gamma)) + \psi_{ee}(e(x, \gamma))] \frac{de}{dx} = \bar{r}_{ex}(x, \gamma, e(x, \gamma))
\] (48)

The expression in square brackets is positive (second order condition) and the right hand side is non-positive (first order stochastic dominance with \( r_{xz}(z, x) \geq 0 \)). Hence, we find \( de/dx \leq 0 \): as the ex post reimbursement becomes more generous, insurers invest less effort.

The cost for the sponsor of implementing ex post insurance with generosity \( x \) is given by
\[
C(x, \gamma, \theta) = \theta \bar{r}(x, \gamma, e(x, \gamma))
\] (49)
Total cost can then be written as

\[ R(x, \gamma, \theta) + C(x, \gamma, \theta) = \theta(\bar{y}(e(x, \gamma)) - \gamma + \psi(e(x, \gamma))) \]  

(50)

Hence we find that

\[ R_x(x, \gamma, \theta) + C_x(x, \gamma, \theta) = \theta(\bar{y}_e(e(x, \gamma)) + \psi(e(x, \gamma)) \frac{de}{dx} \]  

(51)

\[ = \theta \bar{r}_e(x, \gamma, e(x, \gamma)) \frac{de}{dx} \geq 0 \]  

(52)

where the second equality follows from (47); the inequality follows from \( \bar{r}_e \leq 0 \) and \( de/dx \leq 0 \) derived above. Writing \( \alpha(x, \gamma, \theta) = \theta \bar{r}_e \frac{de}{dx} \geq 0 \), we find equation (11).

The following inequalities derive equation (9):

\[ R^h(x, 0) = R(x, 0, \theta^h) = \theta^h[\bar{y}(e(x, 0)) - \bar{r}(x, 0, e(x, 0)) + \psi(e(x, 0))] \geq \]  

(53)

\[ \theta^h[\bar{y}(e(x, 0)) - \bar{r}(x, 0, e(x, 0)) + \psi(e(x, 0))] \geq \]  

(54)

\[ \theta^h[\bar{y}(e(x, 0)) - \gamma^l - \bar{r}(x, \gamma^l, e(x, 0)) + \psi(e(x, \gamma^l))] \geq \]  

(55)

\[ \theta^h[\bar{y}(e(x, \gamma^l)) - \gamma^l - \bar{r}(x, \gamma^l, e(x, \gamma^l)) + \psi(e(x, \gamma^l))] = R(x, \gamma^l, \theta^l) = \hat{R}^h(x, \gamma^l) \]  

(56)

where the first inequality follows from \( \theta^h > \theta^l \), the second from \( \gamma^l \geq 0 \) and \( \bar{r}_\gamma \in [-1, 0] \), the third inequality from the fact that \( e(x, \gamma^l) \) minimizes the insurer’s cost with an \( l \)-insured.

With a similar reasoning we can prove (41):

\[ R^l(0, \gamma^l) = \theta^l[\bar{y}(e(0, \gamma^l)) - \gamma^l + \psi(e(0, \gamma^l))] \geq \]  

(57)

\[ \theta^l[\bar{y}(e(0, \gamma^l)) - \gamma^l - \bar{r}(x, \gamma^l, e(0, \gamma^l)) + \psi(e(0, \gamma^l))] \geq \]  

(58)

\[ \theta^l[\bar{y}(e(x, \gamma^l)) - \gamma^l - \bar{r}(x, \gamma^l, e(x, \gamma^l)) + \psi(e(x, \gamma^l))] = \hat{R}^h(x, \gamma^l) \]  

(59)

Finally,

\[ R_\gamma(x, \gamma, \theta^l) = \hat{R}^h_\gamma(x, \gamma) = -\theta^l(1 + \bar{r}_\gamma(x, \gamma, e)) \geq -\theta^l \]  

(60)

because \( \bar{r}_\gamma \in [-1, 0] \); which proves (43). Further, \( \bar{r}(0, \gamma, e) = 0 \) for all \( \gamma \geq 0 \); hence \( R_\gamma(0, \gamma, \theta^l) = -\theta^l \): equation (42).