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Published in:
Journal of Financial and Quantitative Analysis

Publication date:
2007

[Link to publication](#)

Citation for published version (APA):
Boes, M. J., Drost, F. C., & Werker, B. J. M. (2007). The impact of overnight periods on option pricing. *Journal of Financial and Quantitative Analysis*, 42(2), 517-534.

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The Impact of Overnight Periods on Option Pricing

Mark-Jan Boes, Feike C. Drost, and Bas J. M. Werker*

Abstract

This paper investigates the effect of closed overnight exchanges on option prices. During the trading day, asset prices follow the literature's standard affine model that allows for stochastic volatility and random jumps. Independently, the overnight asset price process is modeled by a single jump. We find that the overnight component reduces the variation in the random jump process significantly. However, neither the random jumps nor the overnight jumps alone are able to empirically describe all features of option prices. We conclude that both random jumps during the day and overnight jumps are important in explaining option prices, where the latter account for about one quarter of total jump risk.

1. Introduction

As a result of the shortcomings in the classical Black-Scholes model for option pricing, two streams of literature can be identified. The first stream extends the Black-Scholes framework to time-varying volatility and the occurrence of random jumps in the underlying stock price process. Hull and White (1987) derive option prices in a stochastic volatility model under the assumption that volatility risk is idiosyncratic. Heston (1993) gives closed-form option pricing formulas using a mean-reverting volatility process and an explicit volatility risk premium. Parallel to this, Merton (1976) motivates the occurrence of abnormal events by a jump component in the underlying stock price process. That paper discusses the implications for option pricing in case jumps are modeled as a compound Poisson process and under the assumption that jump risk is not priced in the market. The models derived in Heston (1993) and Merton (1976) can be merged in the affine jump diffusion framework of Duffie, Pan, and Singleton (2000), where asset returns and variances are driven by a finite number of state variables. The second

*Boes, mboes@feweb.vu.nl, Finance Group, Free University Amsterdam, De Boelelaan 1105, 1081 HV, Amsterdam, The Netherlands; Drost, f.c.drost@uvt.nl, and Werker, b.j.m.werker@uvt.nl, Econometrics and Finance Group, CentER, Tilburg University, P.O. Box 90153, 5000 LE, Tilburg, The Netherlands. This work was supported in part by the European Community's Human Potential Programme under contract HPRN-CT-2002-00232 [MICFINMA]. The authors gratefully acknowledge support of ABN-AMRO Asset Management The Netherlands and Netspar in conducting this research. Moreover, we are grateful to Robert Tompkins and Michael Johannes (the referee) for useful remarks.

stream of literature uses more general Lévy processes instead of Brownian motion and the compound Poisson process as driving factors for asset returns. If the parsimonious variance gamma process is assumed to be the stochastic process for underlying stock returns, Madan, Carr, and Chang (1998) derive closed-form expressions for the density of asset returns and option prices. Stochastic volatility models driven by Lévy processes are studied in Carr, Geman, Madan, and Yor (2003), among others.

From the empirical results concerning the aforementioned models, it is evident that jumps are important in explaining characteristics of asset returns and option prices (see, for example, Bakshi, Cao, and Chen (1997), Pan (2002), Andersen, Benzoni, and Lund (2002), and Madan, Carr, and Chang (1998)). Using a parametrically specified pricing kernel, Pan (2002) provides evidence that both volatility risk and jump risk are priced in the SPX options market. Coval and Shumway (2001) end up with a similar conclusion using returns on option positions. These positions are constructed such that the value of the positions is only sensitive to changes in the two risk factors. The Lévy literature also provides support for priced volatility and jump risk since the parameter estimates under the objective and the risk-neutral measure are generally significantly different. For instance, Madan, Carr, and Chang (1998) find significant negative skewness under the risk-neutral probability measure, while this is not present in their objective parameter estimates. The differences between the objective and the risk-neutral distributions are indicative of the presence of a price for crash risk in options markets. However, it is not always obvious how to infer market prices of risk from the estimation results because a parametric pricing kernel that defines risk prices is usually not specified in this literature. On the whole, it is clear from both streams of literature that jumps, next to stochastic volatility, are important in explaining observed patterns in asset returns and option prices.

The present paper considers the jump process in more detail by focusing on jumps in asset prices that are inherent to overnight market closure. Most of the empirical research cited above uses daily returns. These returns are calculated using the last tick price on the exchange of each trading day. However, the exchange is closed a large part of the day and information that arrives during the closing time cannot be immediately incorporated in stock and option prices. For instance, European investors use information revealed in U.S. stock markets by submitting orders to their exchange before the opening. This means that the opening price of the exchange reflects overnight information. The effect of market closure on stock (index) returns has been considered extensively in the literature.¹ Important findings are that i) open-to-open returns are more volatile than close-to-close returns (see, for instance, Amihud and Mendelson (1987), (1991), Stoll and Whaley (1990), and Cao, Choe, and Hatheway (1997)), ii) weekend returns are lower than weekday returns (see, for example, French (1980), Gibbons and Hess (1981), and Keim and Stambaugh (1984)), and iii) returns over trading periods are more volatile than returns over nontrading periods (see, among others, Fama (1965),

¹In most studies that treat the effects of market closure on stock returns, nontrading periods are exogenously determined by the closing time of the exchanges. An alternative is provided in Jones, Kaul, and Lipson (1994), who define nontrading periods as periods where exchanges are open, but traders choose not to trade.

French and Roll (1986), Oldfield and Rogalski (1980), and Amihud and Mendelson (1991)). However, the influence of market closure on option pricing has not been treated yet.

In this paper, we stress the difference in information by using alternative processes driving intraday and overnight returns, respectively. In particular, in the spirit of Andersen, Benzoni, and Lund (2002), we assume a continuous part with stochastic volatility (reflecting the normal vibrations in the stock price) and a jump part (modeling the arrival of important new information) during the day. Furthermore, the “normal” overnight change in the stock price is modeled by means of a single jump. We investigate the theoretical and empirical implications of this added factor on option prices. This paper is related to Dubinsky and Johannes (2005) who propose a model that includes jumps at fixed points in time. Those time points are the earnings announcement dates of firms. As a consequence, the analysis in the paper is performed for single stock options. The paper finds increased volatility around these announcement dates, which affects option prices. Although we do not provide a formal analysis along these lines, it seems that both effects are complimentary.

We find, for the SPX market over two separate periods, that both random jumps and overnight jumps are important for option pricing. In particular, the overnight jump component accounts for approximately one quarter of total jump variation. Moreover, the inclusion of overnight jumps leads to different parameter estimates for the stochastic volatility and random jump part of the stock price process. This will have important consequences for hedging these risks.

The organization of the paper is as follows. Section II provides the theoretical formulation and motivation of the model under both the objective and risk-neutral measure. We also give a closed-form option pricing formula in the spirit of Heston (1993). Section III describes the data and discusses the estimation procedure. In Section IV, the empirical results are presented. Section V concludes. Mathematical details are gathered in the Appendix.

II. The Overnight Jump Model

A. Stock Price Process

Worldwide financial markets do not allow for continuously trading stocks, interest rates products, and derivatives. Trading usually starts in the local time morning hours and ends in the late afternoon or in the evening. Of course, it is possible for individual and institutional investors to do 24-hour trading all over the world: by the time London closes, Wall Street is already open and when the U.S. markets stop trading, Asian exchanges have already opened their doors. Due to increasing globalization and financial market integration, economies and firms from various countries are interrelated. As a consequence, changes in the value of financial instruments on different exchanges are not independent. This does not only hold if exchanges are open simultaneously, but also if one market is closed. In case an exchange is closed, relevant news cannot be immediately incorporated in prices. For instance, a high closing of stocks traded on the Dow Jones usually has a positive effect on stock price openings in Europe. All news that is important

for the value of a particular stock should ideally be processed in the opening price of the stock. The difference between the closing price and the opening price the next day can be seen as a measure of the revealed information all over the world during the overnight period.²

Up to now, the overnight period in financial markets has not been considered in the derivative pricing literature. This paper tries to fill this gap by explicitly modeling this period through an additional jump process.

Asset managers tend to leave their active cash on an overnight bank account. Overnight accounts are used for two reasons. First, asset managers earn interest on their cash position. Second, asset managers have the cash (almost) always available. The specifics of overnight deposits on borrowings differ across markets worldwide. For instance, in the eurozone, asset managers are allowed to drop their cash until 5PM on an overnight account. The cash can be withdrawn again immediately after midnight. The overnight rate is the shortest term interest rate. Overnight interest rates are determined by supply and demand together with the central bank's repo rate and the liquidity supply of the central bank. In this paper, a separate overnight rate is not explicitly incorporated in the price processes because the overnight rate is not exactly an interest rate that is earned between the closing and the opening of the stock exchange. Moreover, it is well known that interest rates only marginally affect the prices of the plain vanilla options we are interested in. Therefore, we assume that the money market process is given by

$$(1) \quad \frac{dB_t}{B_t} = rdt,$$

i.e., $B_t = \exp(rt)$.

In this paper, we use the equivalent martingale method for pricing options. In comparison to the standard Black and Scholes (1973) framework, there are added risk factors that make the market incomplete with respect to the traded financial securities. A consequence is the non-uniqueness of the equivalent martingale measure \mathbb{Q} . Motivated by, for example, the Breeden (1979) consumption-based model, the value process of the underlying in transaction time under the risk-neutral probability measure \mathbb{Q} is defined by

²There are important differences in market opening procedures among exchanges. Specifically, on the NYSE a stabilized auction market opens trading, while on the NASDAQ a quote-driven, dealer market mechanism is used for all transactions during the trading day. However, even though there is no formal call market opening on the NASDAQ, the open of trade is preceded by a pre-opening session that facilitates price discovery. Greene and Watts (1996) and Masulis and Shivakumar (1997) examine the differences in close-to-open price reaction to overnight news announcements across these markets. Greene and Watts (1996) find that the opening procedure on the NASDAQ leads to prices that incorporate more of the overnight information. In addition, Masulis and Shivakumar (1997) report that the NASDAQ reacts faster to overnight seasoned equity offering announcements. Cao, Ghysels, and Hatheway (2000) conclude that the more rapid price adjustment on the NASDAQ is a consequence of the pre-opening session.

$$\begin{aligned}
 (2) \quad \frac{dS_t}{S_{t-}} &= rdt + \sigma_t dW_t^S + d \sum_{i=1}^{N_t} (Y_i - 1) - \lambda \mu_{RJ} dt + d \sum_{i=1}^{\lfloor 252t \rfloor} (V_i - 1), \\
 \log Y_i &\sim N \left(\log(1 + \mu_{RJ}) - \frac{1}{2} \sigma_{RJ}^2, \sigma_{RJ}^2 \right), \\
 \log V_i &\sim N \left(-\frac{1}{2} \sigma_{OJ}^2, \sigma_{OJ}^2 \right),
 \end{aligned}$$

where $\{W_t^S\}$ is a standard Brownian motion independent of the Poisson process $\{N_t\}$ with

$$N_t \sim \text{Poisson}(\lambda t).$$

Both $\{W_t^S\}$ and $\{N_t\}$ are also assumed to be independent of sequences of jumps $\{Y_i\}$ and $\{V_i\}$. Note that the volatility model with jumps of Bakshi, Cao, and Chen (1997) and Andersen, Benzoni, and Lund (2002) is obtained by deleting the last sum covering the overnight jump part in (2). The time-varying volatility process $\{\sigma_t^2\}$ will be defined below.

The random jump distribution of the Y s is parametrized such that a single jump multiplies, in expectation, the price by $1 + \mu_{RJ}$. On a yearly basis, due to the random number of jumps, this implies an expected instantaneous drift term that needs to be compensated in (2) to keep the martingale property of the discounted price process. The expected number of *random* Y -jumps during one calendar year (in addition to the 252 V -jumps) is equal to λ .

Our contribution consists of an extra jump term that is added to the stock price process. For simplicity, we count weekends as a single night and we have 252 days a year. At each time, which is a multiple of $\frac{1}{252}$, an overnight period is inserted. Each overnight period results in an additional stock return that is reflected by the jump V_i . Finally, note as required that the \mathbb{Q} -expected yearly return on the stock price in our model is given by

$$E_t S_{t+1}/S_t = \exp(r).$$

The specification of the stochastic variance process in (2) is taken from Heston (1993),

$$\begin{aligned}
 (3) \quad d\sigma_t^2 &= -\kappa (\sigma_t^2 - \sigma^2) dt + \sigma_\sigma \sigma_t dW_t^V, \\
 \text{Corr}_t (dW_t^V, dW_t^S) &= \rho,
 \end{aligned}$$

where κ is the speed of mean reversion, σ^2 is the long-run mean of the variance, and σ_σ is the volatility of volatility. This specification allows for a negative premium for volatility risk (see, for example, Bakshi and Kapadia (2003)) for theoretical and empirical evidence. It has been often observed that a large decline in the stock price is accompanied by a positive shock in volatility levels. This is captured by means of a negative parameter ρ .

B. Option Pricing

Given the risk-neutral processes in (2), a standard plain vanilla call option can be priced using

$$C_t = B_t E_t^{\mathbb{Q}} \left(\frac{\max(S_T - X, 0)}{B_T} \right),$$

where T is the maturity and X is the strike price of the option. Following Heston (1993), we show in the Appendix that the pricing formulas for the value of a call option C and a put option P at time t can be simplified as

$$(4) \quad C(t, T) = S_t P_1 - X e^{-r(T-t)} P_2,$$

$$(5) \quad P(t, T) = X e^{-r(T-t)} (1 - P_2) - S_t (1 - P_1),$$

where the probabilities P_1 and P_2 are given by (7) and (8). The proof uses the independence of the overnight process and the intraday process and the fact that the trading day part of the model is an affine jump diffusion in the spirit of Duffie, Pan, and Singleton (2000).

III. Data and Estimation

In the previous section, we motivated the different processes describing the intraday and overnight returns. We focus on the S&P 500 index in two periods: a low volatility period from January 1, 1992 until August 27, 1997 and a high volatility period from July 9, 1999 until November 27, 2003.

To assess the effects of market closure in an intuitive and informal way, Table 1 shows the sample statistics of the close-to-close, open-to-close, and close-to-open returns series for the respective sample periods. From the standard deviations in Table 1, it is clear that the overnight return is an important part of the total daily return in both the first and the second period. As the sample standard deviation of the overnight returns is lower than the standard deviation of the intraday returns, one may conclude that information important for S&P stocks generally arrives during trading hours. Information of significant importance during the night often leads to a high, either positive or negative, return on the S&P 500, which explains the high kurtosis values of overnight returns in Table 1.

TABLE 1
Summary Statistics of S&P 500 Returns

	January 1992–August 1997			July 1999–November 2003		
	Close-Close	Open-Close	Close-Open	Close-Close	Open-Close	Close-Open
Average	13.2%	5.5%	7.7%	-4.3%	-3.5%	-0.8%
Std. dev.	10.5%	9.9%	2.7%	20.6%	18.9%	7.9%
Skewness	-0.28	-0.26	-2.54	0.13	0.21	0.25
Kurtosis	4.8	4.7	40.8	4.6	5.9	10.4

Finally, we have the daily closing option quotes of SPX options for both sample periods. These data are extracted from the ABN-AMRO Asset Management option database. Following Bakshi, Cao, and Chen (1997), for each day in the sample only the mid-price based on the last reported bid-ask quote (prior to 3:00PM CST) of each option contract is used for estimation. Of course, the aforementioned S&P 500 index levels are measured at the same time. Following Jackwerth and Rubinstein (1996), we assume that the dividend amount and timing expected by the market is identical to the dividends actually paid on the index. We use interpolated LIBOR rates as a proxy of the risk-free rate.

Table 2 provides descriptive statistics on call and put option prices (stated in terms of Black-Scholes implied volatilities) that i) have time to expiration of greater than or equal to six calendar days, ii) have a bid price of greater than or equal to \$3/8, iii) have a bid-ask spread of less than or equal to \$1, and iv) have a Black-Scholes implied volatility greater than zero and less than or equal to 0.70, and satisfy the arbitrage restriction,

$$C(t, T) \geq \max \left(0, S_t e^{-q(T-t)} - X e^{-r(T-t)} \right),$$

for call options and a similar restriction for put options. In this formula, X is the option exercise price, q is the dividend rate, and r is the continuously compounded intraday risk-free rate.

TABLE 2
Summary Statistics on SPX Call and Put Option-Implied Volatilities

The reported numbers are average implied volatilities of options on the S&P 500 index corresponding to the last tick before 3:00PM and the total number of observations for each maturity category. The sample periods are January 1, 1992–August 27, 1997 and July 9, 1999–November 27, 2003, respectively.

		January 1992–August 1997				July 1999–November 2003			
		Days to Expiration			Subtotal	Days to Expiration			Subtotal
Moneyiness		< 60	60–180	> 180		< 60	60–180	> 180	
<i>Calls</i>									
ITM	< 0.97	0.210 14,753	0.171 14,802	0.140 6,821	36,376	0.319 12,618	0.277 10,918	0.252 2,882	26,418
ATM	0.97–1.03	0.136 14,611	0.138 13,693	0.152 5,571	33,875	0.222 7,945	0.225 6,834	0.238 2,906	17,685
OTM	> 1.03	0.124 4,768	0.118 9,380	0.172 5,836	19,984	0.302 13,886	0.234 13,742	0.208 2,198	29,826
	Subtotal	34,132	37,875	18,228	90,235	34,449	31,494	7,986	73,929
<i>Puts</i>									
OTM	< 0.97	0.191 12,912	0.173 14,729	0.172 7,065	34,706	0.330 14,321	0.283 12,132	0.249 2,965	29,418
ATM	0.97–1.03	0.137 14,690	0.139 13,771	0.151 5,709	34,170	0.220 7,942	0.222 6,805	0.233 2,906	17,653
ITM	> 1.03	0.163 8,513	0.125 11,259	0.130 6,122	25,894	0.256 10,223	0.220 10,630	0.205 2,161	23,014
	Subtotal	36,115	39,759	18,896	94,770	32,486	29,567	8,032	70,085

From the numbers in Table 2, well-known patterns in implied volatilities across strikes and maturities are recognized. The volatility skew or smile is clearly present for all option categories but one. This exceptional category is probably

less frequently traded. From the return data in Table 1, it is clear that the 1992–1997 sample period can be characterized as a low volatility period and the 1999–2003 sample as a high volatility period. This characterization of both periods also becomes clear from the implied volatilities in Table 2 since they are consistently on a higher level across strike prices and maturities in the 1999–2003 sample period. Christensen and Prabhala (1998), among others, provide evidence for a high correlation between realized volatility and Black-Scholes implied volatility.

In this paper, we extract information about \mathbb{Q} -parameters from the option prices since our focus is on the influence of overnight jumps on these options. The practical implementation of the estimation procedure is straightforward and follows Bakshi, Cao, and Chen (1997). For a particular day t , a set of N options is chosen for which the closing price is observed. Henceforth, the i th option price in this set will be denoted by O_{it}^{obs} . For all options, related values such as strike price, remaining time to maturity, risk-free interest rates, and (dividend discounted) value of the underlying are observed as well. Subsequently, we have a model price of option i at time t , say O_{it}^{model} , that is a function of the structural \mathbb{Q} parameter vector $\theta = (\mu_{RJ}, \sigma_{RJ}, \lambda, \sigma_{OJ}, \kappa, \sigma, \sigma_\sigma, \rho)$ and the unobservable instantaneous variance σ_t^2 . For a particular time t , the estimated parameter vector is determined from

$$(6) \quad [\hat{\theta}_t, \hat{\sigma}_t^2] = \arg \min_{\theta, \sigma_t^2} \sum_{i=1}^N \left(\frac{O_{it}^{\text{model}} - O_{it}^{\text{obs}}}{O_{it}^{\text{obs}}} \right)^2.$$

This objective function implies that we focus on fitting the steepness of the observed (Black-Scholes) implied volatility skews or otherwise stated the tails of the market-implied risk-neutral distribution (see Britten-Jones and Neuberger (2000)). The procedure is repeated for each day in both samples resulting in two time series of estimators. The parameter estimates presented in Section IV are the averages of these estimates. Note that an advantage of such an approach is that possible time variation in the parameters is not excluded a priori. This is especially relevant for the overnight jump volatility σ_{OJ} as Dubinsky and Johannes (2005) relate this specifically to macroeconomic or earnings announcements that would lead to time variation. Similar procedures are applied to option pricing models in Bakshi, Cao, and Chen (1997) and Madan, Carr, and Chang (1998). In the implementation of the procedure above, we only use the out-of-the-money options (for low strikes put options and for high strikes call options) since these options are generally more liquid than in-the-money options.

IV. Empirical Results

This section provides the estimation results obtained by applying the data and estimation techniques as described in Section III to the model formulated in Section II. First, as a benchmark, results are presented for the standard stochastic volatility model (SV) and the stochastic volatility model with random jumps (SVRJ). These results are followed by a discussion of the results in the extended model including overnight jumps. The results are presented both in a setting with only stochastic volatility during the day (SVOJ) as well as in a setting where random jumps are possible (SVRJOJ).

A. Standard Option Pricing Models

We present the results for the SV and SVRJ models in order to make them comparable to, for instance, those of Bakshi, Cao, and Chen (1997). Their model specification and their estimation techniques are similar to the ones that are employed in this section. For both sample periods described in Section III, Table 3 gives an overview of the estimation results of the risk-neutral parameters.

For the SV model, Table 3 confirms that the average instantaneous volatility in the 1992–1997 sample is low in comparison to, for example, the estimated values in Bakshi, Cao, and Chen (1997) over the period June 1988 to May 1991. In the 1999–2003 sample, the average instantaneous volatility is higher. In comparison to Bakshi, Cao, and Chen (1997) and Broadie, Chernov, and Johannes (2005), the parameters σ_σ , κ , and σ are also estimated differently. For instance, Bakshi, Cao, and Chen (1997) estimate σ_σ equal to 0.39 while this parameter in Broadie, Chernov, and Johannes (2005) is estimated at a level of 2.82 in a stochastic volatility model. One obvious explanation for these differences is the different sample periods used. Furthermore, Bakshi, Cao, and Chen (1997) focus on absolute pricing errors while in this section relative pricing errors are considered (see (6)). By using relative pricing errors, the misspecification of the SV model becomes more apparent since a high value of σ_σ is necessary to fit empirically observed implied volatility curves.

TABLE 3
Implied Average Parameter Estimates in the SV, SVRJ, SVOJ, and SVRJOJ Models

Table 3 shows the implied average parameter estimates in the SV, SVRJ, SVOJ, and SVRJOJ models using option data on the S&P 500 from the low volatility period January 1, 1992–August 27, 1997 and the high volatility period July 9, 1999–November 27, 2003. Standard deviations of the daily parameter estimates are given in brackets.

	January 1992–August 1997				July 1999–November 2003			
	SV	SVRJ	SVOJ	SVRJOJ	SV	SVRJ	SVOJ	SVRJOJ
μ_{RJ}		-6.3% (3.9%)		-7.0% (3.6%)		-11.5% (7.6%)		-8.8% (6.0%)
σ_{RJ}		8.8% (4.2%)		6.8% (2.8%)		13.8% (10.0%)		11.2% (6.6%)
λ		0.60 (0.05)		0.56 (0.40)		0.63 (0.11)		1.21 (1.06)
σ_{OJ}			7.6% (2.9%)	5.0% (2.7%)			6.8% (5.0%)	7.5% (4.5%)
κ	1.67 (0.96)	3.55 (1.00)	1.64 (0.11)	3.30 (1.09)	1.60 (0.51)	3.90 (0.11)	1.74 (0.21)	3.33 (3.57)
σ	16.0% (4.3%)	11.6% (3.5%)	16.0% (0.9%)	11.4% (5.0%)	15.9% (1.5%)	11.3% (3.1%)	15.9% (1.5%)	12.1% (5.1%)
σ_σ	61.1% (18.7%)	40.0% (30.0%)	91.9% (31.3%)	51.8% (29.2%)	86.8% (18.7%)	39.3% (11.2%)	81.3% (36.3%)	55.5% (41.9%)
ρ	-0.69 (0.15)	-0.59 (0.20)	-0.90 (0.16)	-0.70 (0.21)	-0.64 (0.18)	-0.53 (0.07)	-0.89 (0.13)	-0.72 (0.22)
σ_t	14.4% (3.0%)	11.7% (3.2%)	11.8% (3.5%)	9.6% (3.4%)	24.8% (5.3%)	20.2% (5.0%)	22.0% (6.8%)	17.6% (7.5%)
SSE	0.70 (0.58)	0.16 (0.15)	0.39 (0.36)	0.12 (0.09)	0.70 (0.66)	0.22 (0.31)	0.48 (0.50)	0.11 (0.14)

To address this issue in more detail, consider the typical situation where the option-implied volatility curve for short-term options is downward sloping in

the strike price for low levels of the strike price.³ The steepness of the implied volatility curve provides information about the risk-neutral distribution of the underlying index at the maturity date. The steeper the implied volatility curve for a certain strike price region, the more probability mass in that particular region of the implied risk-neutral distribution.

There is an enormous literature on methodologies that extract information about the risk-neutral distribution from option prices (see, for example, Coutant, Jondeau, and Rockinger (1998), Jackwerth (1999), Britten-Jones and Neuberger (2000), Anagnou, Bedendo, Hodges, and Tompkins (2002), Bliss and Panigirtzoglou (2002), and Panigirtzoglou and Skiadopoulos (2004)). Because squared relative errors are minimized, the fit of cheaper options (short-term OTM puts and calls) is relatively more important compared to the more expensive options in the sample (long-term ATM and ITM puts and calls). Stated differently, the focus is more on the tails of the market-implied risk-neutral distribution. The negative slope of the implied volatility curve for short-term options forces the optimization algorithm to choose parameter values that are able to generate negative skewness in the risk-neutral distribution. The desired skewness can be obtained both from ρ and σ_σ . In more detail, the SV estimates would imply a volatility of volatility $\sigma_\sigma\sigma_t$ of 9% in the low volatility period and a volatility of volatility of 22% in the high volatility period, while using empirical data volatility of volatility is estimated around 5% in low volatility markets and 12% in high volatility markets.⁴ Although the estimate of σ_σ differs from the estimates in Bakshi, Cao, and Chen (1997), Bates (2000), and Broadie, Chernov, and Johannes (2005), the conclusion is the same: the volatility of volatility parameter σ_σ is estimated at too high a level to be consistent with time-series estimates in, for instance, Andersen, Benzoni, and Lund (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), and Eraker, Johannes, and Polson (2003). The latter study reports the highest estimate of $\sigma_\sigma = 14\%$ in a stochastic volatility model.

The estimation results show that part of the misspecification in the SV model is solved by adding random jumps to the option's underlying value. Compared to the SV estimates, the parameter estimates of σ_σ and ρ are much smaller in the SVRJ model, which is due to the appearance of (on average) negative jumps that capture (part of) the negative skewness in the implied risk-neutral distribution. A similar conclusion can be found in Bakshi, Cao, and Chen (1997) and Bates (2000). These studies also find that adding jumps to the risk-neutral return process leads to lower estimates of ρ and σ_σ . The three-parameter random jump size process combined with stochastic volatility is superior to the SV model in describing the tails of the market-implied risk-neutral distribution and fitting the option data.

Comparing the results for both sample periods, the parameter estimates show that the instantaneous volatility in the SVRJ models is lower on average than in the SV model. This is intuitively correct since the total variation in the underlying

³For shorter maturities, the option-implied volatility curve usually has a smile shape (see Table 2 and Tompkins (2001)) and, hence, the option-implied volatility curve is not downward sloping over the whole range of strike prices.

⁴These estimates are based on the standard deviation of the at-the-money Black-Scholes implied volatilities of the data described in Section III.

value is now divided in the variation of a jump component and the variation that stems from the stochastic volatility part of the model. The variance in the log return due to the jumps is given by

$$\text{var} \left(\sum_{i=1}^{N_{t+1}-N_t} \log Y_i \right) = \lambda \sigma_{RJ}^2 + \lambda \left(\log(1 + \mu_{RJ}) - \frac{1}{2} \sigma_{RJ}^2 \right)^2.$$

The full variance decomposition for the SVRJ model is presented in Table 4, which shows that the variance due to the random jump part is given by 0.007 and 0.023 in the respective sample periods. Taking σ_t^2 as a proxy of the variance of the continuous part of the underlying value process, approximately one third of the total variance is due to random jumps. Moreover, if the variance of the random jump part is added to the estimate of σ_t^2 , then for both samples the total variance in the SV model (0.021 in the first and 0.062 in the second sample period) is comparable to the total variance in the SVRJ model.

TABLE 4
Variance Decomposition of the SVRJ and SVRJOJ Models

Table 4 shows the variance decomposition of the SVRJ and SVRJOJ models for the 1992–1997 and 1999–2003 sample periods. The numbers are based on the implied parameter estimates of Table 3.

	1992–1997		1999–2003	
	SVRJ	SVRJOJ	SVRJ	SVRJOJ
Continuous part	0.014	0.009	0.041	0.031
Random jump part	0.007	0.006	0.023	0.027
Overnight jump part		0.003		0.006
Total	0.021	0.017	0.063	0.063
Volatility	11.7%	9.6%	20.2%	17.6%
Objective function	0.163	0.121	0.216	0.108

Summarizing, the results of this subsection show that the parameter estimates in the SVRJ model are in line with the findings of Bakshi, Cao, and Chen (1997), Bates (2000), and Broadie, Chernov, and Johannes (2005). The addition of the random jump component stabilizes the stochastic volatility parameters to more reasonable levels and, hence, reduces the misspecification of the model. Compared to time-series estimates, the volatility of volatility parameter is still estimated at too large a value. This indicates misspecification of the risk-neutral volatility process that possibly could be solved by adding jumps to the volatility process. Eraker, Johannes, and Polson (2003) find strong evidence for jumps in volatility by using index returns.

B. Option Pricing Models with Overnight Jumps

As the goal of the present subsection is to assess the importance of overnight trading halts for derivative pricing, the estimation results for the SVOJ and SVRJOJ models are compared with the results in the previous subsection.

Table 3 shows that the parameter estimates in the SVOJ model are quite similar to the ones resulting from the SV model. Again, just as discussed for the SV model, the parameters σ_σ and ρ are extreme in the SVOJ model. Unlike a model

with Poissonian jumps, overnight jumps at fixed times lead to a normal jump distribution without skewness. Note that the (objective) skewness and kurtosis estimates in Table 1 give (albeit indirect) evidence of a nonnormal jump distribution. Moreover, as already observed in the SVRJ model, the attributed proportion of the total variance due to jumps is approximately one third. Especially in the second sample period, the SVOJ model fails to reproduce this result. Taking σ_t as a proxy of the standard deviation of the continuous part, the total variance is given by $\sigma_t^2 + \sigma_{OJ}^2$. Using this it follows that the jump proportion of the variance is slightly less than one third (29%) in the first period, but that it is far too low (9%) in the second period. Since jumps play a more dominant role in high volatility periods, this once more indicates that the SVOJ model is misspecified. A final objection against the SVOJ model is the fit to the option data. Of course, the SVOJ model beats the classical SV model, but the increased fit due to overnight jumps although not negligible is low in comparison to the inclusion of random jumps as in the SVRJ model. All this leads to the conclusion that replacement of the random jumps in the SVRJ model by overnight jumps is not sufficient. However, the question of whether overnight jumps influence option prices remains open. This issue will be tackled now.

The SVRJOJ model clearly outperforms the models discussed before. In comparison to the SV, SVRJ, and SVOJ models, the SVRJOJ model considerably improves the fit of option prices in both sample periods. The addition of random jumps to the SVOJ model has the same effect on the parameters σ_σ and ρ as the addition of random jumps to the SV model. The reasoning is also the same: the random jump part captures (part of) the negative skewness in the risk-neutral distribution required to fit option prices that otherwise could only be captured by extreme values of σ_σ and ρ . Comparing the remaining parameters in the SVRJOJ model with the SVRJ model leads to several conclusions. Since overnight jumps are included, the parameter estimates of the random jump distribution are less dominant and since the total variance has to be divided over three terms, the estimated variance of the continuous part diminishes. One striking difference is the change in the estimated intensity λ . In the first sample period, the estimated value decreases as expected since additional jumps are added. However, in the high volatility period, the intensity is almost doubled compared to the SVRJ model. This effect is greatly offset by the much lower value of σ_{RJ} . Possibly, in high volatility periods the introduction of overnight jumps allows the model to fit many more small jumps. The addition of overnight jumps comes at the cost of a worse empirical identifiability of λ . This is reflected by the higher standard deviation of the estimate of λ in the SVRJOJ model compared to the SVRJ model.

In the same spirit as in the previous subsection, the total risk-neutral variance of the log return can be split into three parts: a first component from the stochastic volatility term σ_t and two remaining components from both the random jumps and the overnight jumps. The trading period's variance consists of the variance of the continuous component (stochastic volatility) and (part of) the random jump component. The nontrading overnight period variance is due to the remaining part of the random jump component and the overnight jumps.

Given the estimates of the SVRJOJ model in Table 3, the variance decomposition is provided in Table 4. The estimated variances due to the jumps are

0.009 and 0.032 in the respective periods. These values can be split into a variance of 0.006 (0.027) attributed to the random jumps and 0.003 (0.006) due to the overnight jumps in the first (second) sample period. The proportion of the total variance attributed to jumps has increased to around 50% in both sample periods compared to the SVRJ model. On average, one quarter of this part has to be attributed to the overnight jumps, once more indicating that the inclusion of overnight jumps is non-negligible.

This section shows that the most appealing model is clearly the SVRJOJ model, which allows for difference in intraday asset return variance and overnight asset return variance. The SVRJOJ model fits empirical option prices best in two different sample periods. Since this model contains the overnight jump part, which covers approximately one quarter of total jump variance, the estimation results show that overnight periods are important and have a considerable impact on option prices. The economic content of this result is that the risk of overnight closures is identifiable from option prices. Investors that have positions in these options are faced with an additional and undiversifiable source of risk that was previously attributed to random jump risk.

V. Conclusion

We present an option pricing model that explicitly models the influence of nontrading overnight periods on option prices. One of the main conclusions is that both random jumps during trading periods and the overnight jump are important in explaining observed option prices. We show that in two sample periods, of which the first can be characterized as a period of low volatility and the second as a period of high volatility, the added jump component covers a significant amount of the variation in the underlying value (risk-neutral) process. In more detail, the results show that the overnight jump part covers approximately one quarter of total jump variation. Moreover, 50% of the daily variance is explained by jumps that are either random or overnight. Furthermore, the empirical results reveal that models including the overnight jump component give a better fit of empirical option prices than the traditional pricing models. Finally, the results show that a model containing only overnight jumps in combination with stochastic volatility has the same problem as a pure stochastic volatility model: the estimated volatility of volatility is too large in comparison to the volatility of volatility extracted from volatility series. Hence, this paper concludes that total jump risk should be separated into random jump risk and overnight jump risk.

Appendix. Option Pricing Formula

We derive the theoretical formula for a plain vanilla call option given the risk-neutral process in (2). The put price follows similarly. Using Itô's Lemma, the stochastic differential of $\log S_t$ is

$$d \log S_t = \left(r - \frac{1}{2} \sigma_t^2 \right) dt + \sigma_t dW_t^S + d \left(\sum_{i=1}^{N_t} \log Y_i \right) - \lambda \mu_{RJ} dt + d \left(\sum_{i=1}^{\lfloor 252t \rfloor} \log V_i \right).$$

Following Scott (1997), the call option value formula is given by

$$\begin{aligned} C(t, T) &= B_t E_t \left(\frac{\max(S_T - X, 0)}{B_T} \right) \\ &= S_t P_1 - e^{-r(T-t)} X P_2, \end{aligned}$$

where

$$\begin{aligned} P_1 &= \int_X^\infty \frac{S_T}{E_t(S_T)} p_t(S_T) dS_T, \\ P_2 &= P_t(S_T > X). \end{aligned}$$

Since the probability density function is unknown under our assumptions regarding the evolution of stock and money market, Fourier inversion techniques are used to derive expressions for P_1 and P_2 (see Bakshi and Madan (2000)). For P_2 , this gives

$$(7) \quad P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \operatorname{Re} \left(\frac{\exp(-i\alpha \log X) \varphi(\alpha)}{i\alpha} \right) d\alpha,$$

where $\varphi(\alpha)$ denotes the characteristic function of the random variable $\log S_T$, i.e., $\varphi(\alpha) = E_t \exp(i\alpha \log S_T)$. The probability P_1 will be obtained later from P_2 . Given the process of $\log S_t$ above, $\varphi(\alpha)$ can be written with $\tau = T - t$ as

$$\begin{aligned} \varphi(\alpha) &= E_t \{ \exp(i\alpha \log S_T) \}, \\ &= E_t \left\{ \exp \left(i\alpha \left[\log S_t + r\tau - \frac{1}{2} \int_t^T \sigma_u^2 du + \int_t^T \sigma_u dW_u^S \right. \right. \right. \\ &\quad \left. \left. \left. + \sum_{i=N_t+1}^{N_T} \log Y_i - \lambda \mu_{RJ} \tau + \sum_{i=\lfloor 252t \rfloor + 1}^{\lfloor 252T \rfloor} \log V_i \right] \right) \right\}, \\ &= E_t \left\{ \exp \left(i\alpha \left[\log S_t + r\tau - \frac{1}{2} \int_t^T \sigma_u^2 du + \int_t^T \sigma_u dW_u^S \right] \right) \right\} \\ &\quad \times E_t \left\{ \exp \left(i\alpha \left[\sum_{i=N_t+1}^{N_T} \log Y_i - \lambda \mu_{RJ} \tau \right] \right) \right\} E_t \left\{ \exp \left(i\alpha \sum_{i=\lfloor 252t \rfloor + 1}^{\lfloor 252T \rfloor} \log V_i \right) \right\}. \end{aligned}$$

The characteristic functions of the various parts will be derived separately. The first part is equal to formula (17) in Heston (1993), i.e.,

$$\begin{aligned} &E_t \left\{ \exp \left(i\alpha \left(\log S_t + r\tau - \frac{1}{2} \int_t^T \sigma_u^2 du + \int_t^T \sigma_u dW_u^S \right) \right) \right\} \\ &= \exp \left(C(\tau; \alpha) + D(\tau; \alpha) \sigma_t^2 + i\alpha \log S_t \right), \end{aligned}$$

where

$$\begin{aligned} C(\tau; \alpha) &= r\alpha\tau + \frac{\kappa\sigma^2}{\sigma_\sigma^2} \left\{ (\kappa - \rho\sigma_\sigma\alpha + d)\tau - 2 \log \left(\frac{1 - ge^{d\tau}}{1 - g} \right) \right\}, \\ D(\tau; \alpha) &= \frac{\kappa - \rho\sigma_\sigma\alpha + d}{\sigma_\sigma^2} \frac{1 - e^{d\tau}}{1 - ge^{d\tau}}, \end{aligned}$$

and

$$g = \frac{\kappa - \rho\sigma_\sigma i\alpha + d}{\kappa - \rho\sigma_\sigma i\alpha - d},$$

$$d = \sqrt{(\rho\sigma_\sigma i\alpha - \kappa)^2 + \sigma_\sigma^2 (i\alpha + \alpha^2)}.$$

The random jump part of the model is described by means of a compensated compound Poisson process. The lognormal distribution of the jump sizes Y_i determines the characteristic function, still with $\tau = T - t$, as

$$E_t \left\{ \exp \left(i\alpha \left[\sum_{i=N_t+1}^{N_T} \log Y_i - \lambda\mu_{RJ}\tau \right] \right) \right\} = \exp \left\{ \lambda\tau \left[(1 + \mu_{RJ})^{i\alpha} \exp \left(\left(\frac{i\alpha}{2} \right) (i\alpha - 1) \sigma_{RJ}^2 \right) - 1 \right] - i\alpha\lambda\mu_{RJ}\tau \right\}.$$

The expression for the characteristic function of the fixed jump part is more tractable since (relative to the random jump part) one source of randomness disappears. The characteristic function then can be calculated, using the lognormal jump sizes V_i , as

$$E_t \left\{ \exp \left(i\alpha \sum_{i=[252t]+1}^{[252T]} \log V_i \right) \right\} = \exp \left(-\frac{1}{2} \alpha (\alpha + i) n \sigma_{OI}^2 / 252 \right),$$

where $n = [252T] - [252t]$. The characteristic function of the terminal stock price is determined and can be used to obtain P_2 in the option pricing formula.

In order to obtain P_1 , observe the following lemma with $Y = \log S_T$.

Lemma A.1. Let Y be a random variable whose distribution has density p and characteristic function φ and for which $E \{ \exp(Y) \} < \infty$. Define the distribution F by its survival function,

$$1 - F(z) = \int_z^\infty \frac{\exp(y)}{E \{ \exp(Y) \}} p(y) dy.$$

Then, F has characteristic function $\tilde{\varphi}$ with

$$\tilde{\varphi}(\alpha) = \frac{\varphi(\alpha - i)}{E \{ \exp(Y) \}}.$$

Proof. Let Z have distribution function F and density

$$f(z) = \frac{\exp(z)p(z)}{E \{ \exp(Y) \}}.$$

Now

$$\begin{aligned} \tilde{\varphi}(\alpha) &= E \exp(i\alpha Z) = \int_{-\infty}^\infty \exp(i\alpha z) \frac{\exp(z)p(z)}{E \{ \exp(Y) \}} dz \\ &= \int_{-\infty}^\infty \frac{\exp(i(\alpha - i)z)}{E \{ \exp(Y) \}} p(z) dz = \frac{E \exp \{ i(\alpha - i)Y \}}{E \{ \exp(Y) \}} \\ &= \frac{\varphi(\alpha - i)}{E \{ \exp(Y) \}}, \end{aligned}$$

which concludes the proof. \square

Comparable to (7), this leads to

$$(8) \quad P_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \operatorname{Re} \left(\frac{\exp(-i\alpha \log X) \varphi(\alpha - i)}{i\alpha \varphi(-i)} \right) d\alpha.$$

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