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THE COLLECTIVE MODEL OF HOUSEHOLD CONSUMPTION: A NONPARAMETRIC CHARACTERIZATION

BY LAURENS CHERCHYE, BRAM DE ROCK, AND FREDERIC VERMEULEN[1]

We provide a nonparametric characterization of a general collective model for household consumption, which includes externalities and public consumption. Next, we establish testable necessary and sufficient conditions for data consistency with collective rationality that only include observed price and quantity information. These conditions have a similar structure as the generalized axiom of revealed preference for the unitary model, which is convenient from a testing point of view. In addition, we derive the minimum number of goods and observations that enable the rejection of collectively rational household behavior.

KEYWORDS: Collective household models, intrahousehold allocation, revealed preferences, nonparametric analysis.

1. INTRODUCTION

TRADITIONALLY, HOUSEHOLD CONSUMPTION BEHAVIOR is crammed into the so-called unitary approach, which assumes that a household acts as if it were a single decision maker; it maximizes a well behaved (single) utility function subject to a household budget constraint. The collective model, which was first presented by Chiappori (1988, 1992), differs from the unitary model in that it explicitly recognizes that the individual household members have own, possibly diverging, rational preferences. These individuals are assumed to engage in a bargaining process that results in a Pareto efficient intrahousehold allocation.

Browning and Chiappori (1998) provided a characterization of a general collective model. They start from the "minimalistic" assumptions that the empirical analyst cannot determine which goods are privately and/or publicly consumed within the household, and that the quantities that are privately consumed by the different household members cannot be observed. In addition, they considered general individual preferences that allow for altruism and other externalities. Their core result for two-person households is that under collectively rational household behavior the pseudo-Slutsky matrix can be written as the sum of a symmetric negative semidefinite matrix and a rank 1 matrix. Browning and Chiappori showed necessity of this condition; Chiappori and Ekeland (2006) addressed the associated sufficiency question.

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Browning and Chiappori focused on a so-called *parametric* setting, which requires some (nonverifiable) functional structure that is imposed on the household decision process (i.e., the household members' preferences and the intrahousehold bargaining process). In this paper, we follow a *non*parametric approach, which analyzes household behavior without imposing any parametric structure on, for example, preferences; see Afriat (1967), Varian (1982), and, more recently, Blundell, Browning, and Crawford (2003). This nonparametric approach was first adapted to the collective model by Chiappori (1988), who restricted attention to a labor supply setting that involves a number of convenient simplifications for the empirical analyst (e.g., observability of household members' leisure/labor supply and no public consumption).

We aim to generalize Chiappori's work by providing a nonparametric characterization of the collective consumption model of Browning and Chiappori, which includes both public consumption and (*in casu* positive) externalities. In Section 2, we derive necessary and sufficient nonparametric conditions for data consistency with this general model. As we will discuss, these conditions imply *unobservable* (household member-specific) quantity and price information. In Sections 3 and 4, we subsequently establish necessary and sufficient conditions that only require *observed* prices and aggregate household quantities. Interestingly, this implies nonparametric tests for collective rationality that are finite in nature and do not require finding a solution to a system of (nonlinear) inequalities.[2] As a by-product, we derive the minimum number of goods and observations that enable rejection of collective rationality. Section 5 contains some concluding remarks. The Appendix contains the proofs of our results, and presents (finite) testing algorithms for the necessary and sufficient collective rationality conditions that are expressed in terms of observed prices and quantities.

2. A CHARACTERIZATION OF COLLECTIVE RATIONALITY FOR TWO-PERSON HOUSEHOLDS

We consider a two-member (1 and 2) household. (Generalizations for M-member households are found in Sections 1–3 of Cherchye, De Rock, and Vermeulen (2007).) The household purchases the (nonzero) n-vector of quantities $\mathbf{q} \in \Re^n_+$ with corresponding prices $\mathbf{p} \in \Re^n_{++}$. All goods can be consumed privately, publicly, or both. Generally, we have $\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^h$ for $\mathbf{q}$ the (ob-

[2]We see at least two important differences between our approach and that of Snyder (2000), who addresses a similar research question for Chiappori's (1988) original labor supply model. First, Snyder focuses on a more restricted model that includes egoistic agents and observable leisure. Second, we do not make use of semialgebraic theory for quantifier elimination. A well known limitation of these latter techniques is that they become computationally cumbersome for large data sets. For example, Snyder restricts to settings of only two observations, while we consider the general case of T observations.

served) aggregate quantities, $\mathbf{q}^1$ and $\mathbf{q}^2$ the (unobserved) private quantities of each household member, and $\mathbf{q}^h$ the (unobserved) public quantities.

Following Browning and Chiappori (1998), we consider general preferences for the household members that may depend not only on the own private and public quantities, but also (positively) on the other individual's private quantities; this allows for altruism and/or externalities.[3] Formally, this means that the preferences of each household member m ($m = 1, 2$) can be represented by a utility function of the form $U^m(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^h)$ that is nondecreasing in its arguments $\mathbf{q}^1$, $\mathbf{q}^2$, and $\mathbf{q}^h$. Throughout, we focus on nonsatiated utility functions.

Suppose T observations of the household. For each observation j we use $\mathbf{p}_j$ and $\mathbf{q}_j$ to denote the (observed) aggregate prices and quantities, respectively, while $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ represents the set of observations. For observed aggregate quantities $\mathbf{q}_j$, we define *feasible personalized quantities* $\widehat{\mathbf{q}}_j$ as

$$(2.1) \qquad \widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h) \quad \text{with} \quad \mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h \in \Re^n_+ \quad \text{and} \quad \mathfrak{q}_j^1 + \mathfrak{q}_j^2 + \mathfrak{q}_j^h = \mathbf{q}_j.$$

Each $\widehat{\mathbf{q}}_j$ captures a feasible decomposition of the aggregate quantities $\mathbf{q}_j$ into private quantities ($\mathfrak{q}_j^1$ and $\mathfrak{q}_j^2$) and public quantities ($\mathfrak{q}_j^h$). One possible specification of these personalized quantities $\mathfrak{q}_j^1$, $\mathfrak{q}_j^2$, and $\mathfrak{q}_j^h$ is the true quantities $\mathbf{q}_j^1$, $\mathbf{q}_j^2$, and $\mathbf{q}_j^h$, but, of course, these latter quantities are not observed. Using this concept, we can now define the condition for a collective rationalization of a set of observations S.

DEFINITION 1: Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ be a set of observations. A pair of utility functions U^1 and U^2 provides a *collective rationalization* of S if for each observation j there exist feasible personalized quantities $\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h)$ and $\mu_j \in \Re_{++}$ such that

$$U^1(\widehat{\mathbf{q}}_j) + \mu_j U^2(\widehat{\mathbf{q}}_j) \geq U^1(\widehat{\mathbf{z}}) + \mu_j U^2(\widehat{\mathbf{z}})$$

for all $\widehat{\mathbf{z}} = (\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h)$ with $\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h \in \Re^n_+$ and $\mathbf{p}_j'(\mathfrak{z}^1 + \mathfrak{z}^2 + \mathfrak{z}^h) \leq \mathbf{p}_j'\mathbf{q}_j$.

Thus, a collective rationalization of S requires that there exist, for each observation j, feasible personalized quantities $\widehat{\mathbf{q}}_j$ that maximize a weighted sum

[3]This setting generalizes Chiappori's (1988) altruistic model in two ways: it does not assume the observability of private and/or public consumption of any good, and it allows for public consumption. Admittedly, the assumption of positive externalities, which is not needed in a parametric setting (see Browning and Chiappori (1998)), may be restrictive in some instances. However, its restrictive nature should not be overestimated. Even though a negative externality may be associated with, for example, tobacco consumption, the nonsmoker's positive valuation of the smoker's utility generated by smoking might well outweigh that negative externality. In addition, within-household mechanisms may be instituted that decrease or even eliminate the negative externalities; see, for example, the widespread practice of smoking outside in households that consist of smokers as well as nonsmokers.

of household member utilities U^1 and U^2 for the given household budget $\mathbf{p}_j'\mathbf{q}_j$. This optimality condition reflects the Pareto efficiency assumption regarding observed household consumption in the collective model. Each weight μ_j represents the "bargaining power" of the household members for observation j; see Browning and Chiappori (1998) for a detailed discussion.

In view of our further exposition, it is interesting to compare the collective rationality condition in Definition 1 with the standard unitary rationality condition. According to Varian's (1982, p. 946) definition, a *unitary rationalization* of the observed set S requires a collective rationalization with $\mu_j = 0$ and $\mathrm{q}_j^1 = \mathbf{q}_j$ (or, equivalently, $\mathrm{q}_j^2 = \mathrm{q}_j^h = \mathbf{0}$) for each observation j.[4] In that presentation, unitary rationalization boils down to collective rationalization with one household member (*in casu* member 1) as the "dictator" in the household. This interpretation of the unitary model as a *dictatorship* model will return in our discussion in Section 4.

Before presenting nonparametric conditions for a collective rationalization, it is useful to briefly recapture the nonparametric conditions for a unitary rationalization. To do so, we define two relationships that will be used in the following discussion.

DEFINITION 2: For a set of observations $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$: if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, then $\mathbf{q}_i R_0 \mathbf{q}_j$, and if $\mathbf{q}_i R_0 \mathbf{q}_k, \mathbf{q}_k R_0 \mathbf{q}_l, \ldots, \mathbf{q}_z R_0 \mathbf{q}_j$ for some (possibly empty) sequence $(k, l, \ldots, z)$, then $\mathbf{q}_i R \mathbf{q}_j$.

In the unitary model, R_0 is commonly referred to as the *direct revealed preference* relation, while its transitive closure R is known as the *revealed preference* relation. Using Definition 2, we can define the *generalized axiom of revealed preference* (GARP).

DEFINITION 3: A set of observations $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ satisfies GARP if $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ whenever $\mathbf{q}_i R \mathbf{q}_j$.

Varian (1982) demonstrated that a unitary rationalization of a set of observations S is possible if and only if S satisfies the GARP. The GARP provides the basis for a test of data consistency with the unitary model. Essentially, this test proceeds in two steps: one first recovers the relations R_0 and R, and then

[4]Strictly speaking, $\mu_j = 0$ is excluded in Definition 1. As for that definition, we note that the requirement $\mu_j \in \Re_{++}$ pertains to the Pareto efficiency interpretation of household consumption, which is, of course, irrelevant if there is only one (dictator) household member. In fact, it can be shown that unitary rationality requires a collective rationalization for μ_j constant over all observations j, but we prefer the dictatorship interpretation of the unitary model in view of our following discussion. [Compare with Browning and Chiappori (1998); see also Browning, Chiappori, and Lechene (2006).] Furthermore, the fact that we can use $\mathrm{q}_j^1 = \mathbf{q}_j$ to obtain the unitary rationalization condition illustrates that the distinction between public and private consumption becomes irrelevant in the unitary model; this contrasts with the collective model.

subsequently checks the upper cost bound condition in Definition 3. This two-step structure will return in the collective rationality condition that we present in the next section.

Using Definitions 2 and 3, we can now establish nonparametric conditions for a collective rationalization of a set S. To do so, we first define *feasible personalized prices* $(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2)$ for observed aggregate prices $\mathbf{p}_j$, as follows:

$$\text{(2.2)} \qquad \widehat{\mathbf{p}}_j^1 = (\mathfrak{p}_j^1, \mathfrak{p}_j^2, \mathfrak{p}_j^h) \quad \text{and} \quad \widehat{\mathbf{p}}_j^2 = (\mathbf{p}_j - \mathfrak{p}_j^1, \mathbf{p}_j - \mathfrak{p}_j^2, \mathbf{p}_j - \mathfrak{p}_j^h)$$
$$\text{with} \quad \mathfrak{p}_j^1, \mathfrak{p}_j^2, \mathfrak{p}_j^h \in \Re_+^n \quad \text{and} \quad \mathfrak{p}_j^c \leq \mathbf{p}_j \quad (c = 1, 2, h).$$

This concept complements the concept of feasible personalized quantities in (2.1): $\widehat{\mathbf{p}}_j^1$ and $\widehat{\mathbf{p}}_j^2$ capture the fraction of the price for the personalized quantities $\widehat{\mathbf{q}}_j$ that is borne by, respectively, member 1 and member 2; $\mathfrak{p}_j^1$ and $\mathfrak{p}_j^2$ pertain to private quantities and $\mathfrak{p}_j^h$ pertains to public quantities.[5] Based on (2.1) and (2.2), we define a *set of feasible personalized prices and quantities*

$$\text{(2.3)} \qquad \widehat{S} = \{(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2; \widehat{\mathbf{q}}_j); j = 1, \dots, T\}.$$

We then have the following result.

PROPOSITION 1: *Let* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \dots, T\}$ *be a set of observations. The following conditions are equivalent*:

(i) *There exists a pair of concave and continuous utility functions* U^1 *and* U^2 *that provide a* collective rationalization *of* S.

(ii) *There exists a set of feasible personalized prices and quantities* $\widehat{S}$ *such that the sets* $\{(\widehat{\mathbf{p}}_j^1; \widehat{\mathbf{q}}_j); j = 1, \dots, T\}$ *and* $\{(\widehat{\mathbf{p}}_j^2; \widehat{\mathbf{q}}_j); j = 1, \dots, T\}$ *both satisfy* GARP.

(iii) *There exists a set of feasible personalized prices and quantities* $\widehat{S}$, *numbers* $U_j^m > 0$ *and* $\lambda_j^m > 0$ $(m = 1, 2)$ *such that for all* $i, j \in \{1, \dots, T\}$: $U_i^1 - U_j^1 \leq \lambda_j^1 (\widehat{\mathbf{p}}_j^1)'(\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j)$ *and* $U_i^2 - U_j^2 \leq \lambda_j^2 (\widehat{\mathbf{p}}_j^2)'(\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j)$.

The nonparametric conditions (ii) and (iii) have a structure similar to the unitary model; see Varian (1982) for an extensive discussion of the nonparametric requirements for unitary rationalization. The essential difference is that the conditions for collective rationalization are expressed in terms of a set of feasible personalized prices and quantities $\widehat{S}$. For a given specification of this set, Proposition 1 states nonparametric conditions *at the level of the household members 1 and 2* that are analogous to the unitary rationalization conditions *at the level of the aggregate household*. Contrary to the unitary case, the true personalized prices and quantities are unobserved. Therefore, it is only imposed that there must exist at least one $\widehat{S}$ that satisfies the conditions.

[5]It is easily verified that $(\widehat{\mathbf{p}}_j^1 + \widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_i = \mathbf{p}_j'\mathbf{q}_i$ for any i and j.

A final note pertains to the interpretation of the nonparametric conditions in Proposition 1. Following Chiappori (1988), we can interpret the different goods as "public" goods, given that they all enter both members' utility functions. In that interpretation, the personalized prices $(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2)$ may be understood as Lindahl prices: they must add up (over members 1 and 2) to the observed market prices so as to be consistent with Pareto efficiency. Thus, no qualitative distinction should be made between public and private quantities (where private quantities may be associated with externalities). Yet, there is a clear quantitative difference: household members may accord another marginal valuation to private consumption than to public consumption.

3. TESTABLE NECESSITY RESTRICTIONS

The (necessary and sufficient) conditions for a collective rationalization in Proposition 1 can be difficult to use in practice, because they are nonlinear in terms of feasible personalized prices $(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2)$ and quantities $\widehat{\mathbf{q}}_j$; see, for example, Watson, Bartholomew-Biggs, and Ford (2000) for a discussion of similar nonlinearity problems. In what follows we present testable conditions for collective rationality that solely use (observed) aggregate prices $\mathbf{p}_j$ and quantities $\mathbf{q}_j$. This section develops a necessary condition for a collective rationalization of a set of observations S that has a two-step structure similar to the unitary GARP (see our discussion following Definition 3). The next section presents the complementary sufficiency condition.

We first define the analogues of the relations R_0 and R for members 1 and 2 in the collective model.

DEFINITION 4: Let $\widehat{S} = \{(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2; \widehat{\mathbf{q}}_j); j = 1, \dots, T\}$ be a set of feasible personalized prices and quantities. Then for $m = 1, 2$: if $(\widehat{\mathbf{p}}_i^m)'\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i^m)'\widehat{\mathbf{q}}_j$, then $\widehat{\mathbf{q}}_i R_0^m \widehat{\mathbf{q}}_j$, and if $\widehat{\mathbf{q}}_i R_0^m \widehat{\mathbf{q}}_k, \widehat{\mathbf{q}}_k R_0^m \widehat{\mathbf{q}}_l, \dots, \widehat{\mathbf{q}}_z R_0^m \widehat{\mathbf{q}}_j$ for some (possibly empty) sequence $(k, l, \dots, z)$, then $\widehat{\mathbf{q}}_i R^m \widehat{\mathbf{q}}_j$.

Of course, different specifications of the set $\widehat{S}$ generally imply different relations R_0^m and R^m. To establish our testable necessary condition for collectively rational behavior, we derive restrictions on the relations R_0^m and R^m without reference to a specific $\widehat{S}$. In this respect, the next lemma specifies a useful relationship between R_0^m and R_0, which is defined in terms of the set of observations S.

LEMMA 1: *Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \dots, T\}$ be a set of observations. We have $\mathbf{q}_i R_0 \mathbf{q}_j$ if and only if, for all sets $\widehat{S}$ of feasible personalized prices and quantities, $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$.*

The intuition of this result pertains to the Pareto efficient nature of household behavior in the collective model. Specifically, if the household has chosen $\mathbf{q}_i$ when $\mathbf{q}_j$ was equally available (i.e., $\mathbf{q}_i R_0 \mathbf{q}_j$, which means $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$), then we always have that, *independently of the specification of the set* $\widehat{S}$, at least one household member must prefer the former (personalized) quantities to the latter (i.e., $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$). As a result, if we want to avoid selecting specific feasible personalized prices and quantities (because we lack information to do so), then we can start from the relation R_0 for specifying restrictions on the relations R_0^1 and R_0^2. Moreover, the equivalence result in Lemma 1 implies that we cannot do better when using only the set of observations S (rather than some $\widehat{S}$).

Lemma 1 provides the starting point for our testable necessity condition for collective rationality. We sketch the basic intuition of that condition by means of the next simple example.

EXAMPLE 1: Consider the case of three observations and three goods with prices and quantities

$$\mathbf{q}_1 = (8\ 2\ 1)', \quad \mathbf{q}_2 = (2\ 1\ 8)', \quad \mathbf{q}_3 = (1\ 8\ 2)';$$
$$\mathbf{p}_1 = (5\ 2\ 1)', \quad \mathbf{p}_2 = (2\ 1\ 5)', \quad \mathbf{p}_3 = (1\ 5\ 2)'.$$

This specific data structure implies that

$$\mathbf{p}_1'\mathbf{q}_1 > \mathbf{p}_1'(\mathbf{q}_2 + \mathbf{q}_3), \quad \mathbf{p}_2'\mathbf{q}_2 > \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3), \quad \text{and} \quad \mathbf{p}_3'\mathbf{q}_3 > \mathbf{p}_3'(\mathbf{q}_1 + \mathbf{q}_2),$$

so that for all observations $i, j \in \{1, 2, 3\}$ we have $\mathbf{q}_i R_0 \mathbf{q}_j$. Using Lemma 1, we therefore conclude

$$\forall i, j \in \{1, 2, 3\}, \quad \widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j \quad \text{or} \quad \widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j. \tag{3.1}$$

Given this, one possible specification of the relations R_0^1 and R_0^2 is

$$\widehat{\mathbf{q}}_1 R_0^1 \widehat{\mathbf{q}}_2, \widehat{\mathbf{q}}_2 R_0^1 \widehat{\mathbf{q}}_3 \quad \text{and} \quad \widehat{\mathbf{q}}_3 R_0^2 \widehat{\mathbf{q}}_2, \widehat{\mathbf{q}}_2 R_0^2 \widehat{\mathbf{q}}_1. \tag{3.2}$$

Intuitively, this specification means that member 1 prefers (personalized) $\widehat{\mathbf{q}}_1$ over $\widehat{\mathbf{q}}_2$ while member 2 prefers $\widehat{\mathbf{q}}_3$ over $\widehat{\mathbf{q}}_2$. In that case, the choice of the (aggregate) quantities $\mathbf{q}_2$ can be rationalized only if it is not more expensive than the sum of $\mathbf{q}_1$ and $\mathbf{q}_3$, which requires that $\mathbf{p}_2'\mathbf{q}_2 \leq \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3)$. However, this is inconsistent with $\mathbf{p}_2'\mathbf{q}_2 > \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3)$. Because the same argument can be repeated for any other possible specification of the relations R_0^1 and R_0^2 instead of (3.2), we conclude that a collective rationalization of this set of observations is impossible.[6]

[6] At this point, it is important that we can exclude for all $i, j \in \{1, 2, 3\}$ with $i \neq j$: $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ *and* $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$. Intuitively, the latter specification of the relations R_0^1 and R_0^2 means that both members 1 and 2 prefer (personalized) $\widehat{\mathbf{q}}_i$ over $\widehat{\mathbf{q}}_j$. In that case, the choice of (aggregate) $\mathbf{q}_j$ can be rationalized only if it is not more expensive than $\mathbf{q}_i$, which is inconsistent with $\mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i$. The formal argument is based on Lemma 2 (rule (iv)).

The basic structure of the collective rationalization test in this example parallels the two-step structure of the unitary GARP test. Specifically, we first specified the relations R_0^1 and R_0^2 in (3.2), and subsequently verified the corresponding upper cost bound condition (*in casu* $\mathbf{p}_2'\mathbf{q}_2 \leq \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3)$), which is not met for this particular set of observations.

To generalize these ideas, we first specify some further restrictions that must hold if a collective rationalization of the set of observations S is possible in terms of Proposition 1. In that case, there exists a set of feasible personalized prices and quantities $\widehat{S}$ such that the corresponding R_0^1 and R_0^2 satisfy the following conditions in relation to their transitive closures R^1 and R^2, aggregate prices $\mathbf{p}_j$, and quantities $\mathbf{q}_j$:

LEMMA 2: *Suppose that there exists a pair of utility functions* U^1 *and* U^2 *that provide a* collective rationalization *of the set of observations* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \dots, T\}$. *Then there exists a set of feasible personalized prices and quantities* $\widehat{S}$ *that defines the relations* R_0^m *and* R^m *for each member* $m \in \{1, 2\}$ *such that*:
(i) *if* $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ *and* $\widehat{\mathbf{q}}_j R^m \widehat{\mathbf{q}}_i$, *then* $\widehat{\mathbf{q}}_i R_0^l \widehat{\mathbf{q}}_j$ (*with* $m \neq l$);
(ii) *if* $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$ *and* $\widehat{\mathbf{q}}_{j_1} R^m \widehat{\mathbf{q}}_i$, *then* $\widehat{\mathbf{q}}_i R_0^l \widehat{\mathbf{q}}_{j_2}$ (*with* $m \neq l$);
(iii) *if* $\widehat{\mathbf{q}}_{i_1} R^1 \widehat{\mathbf{q}}_j$ *and* $\widehat{\mathbf{q}}_{i_2} R^2 \widehat{\mathbf{q}}_j$, *then* $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'(\mathbf{q}_{i_1} + \mathbf{q}_{i_2})$;
(iv) *if* $\widehat{\mathbf{q}}_i R^1 \widehat{\mathbf{q}}_j$ *and* $\widehat{\mathbf{q}}_i R^2 \widehat{\mathbf{q}}_j$, *then* $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$.

The interpretation of this result pertains to the very nature of the collective model, which—recall—explicitly recognizes the multiperson nature of the household decision process. More specifically, the four rules in Lemma 2 relate to *rationality across household members* for a given specification of the feasible personalized prices and quantities. First, rule (i) expresses that if member m prefers (personalized) $\widehat{\mathbf{q}}_j$ over $\widehat{\mathbf{q}}_i$ for (aggregate) $\mathbf{q}_j$ not more expensive than $\mathbf{q}_i$, then the choice of $\mathbf{q}_i$ can be rationalized only if the other member l prefers $\widehat{\mathbf{q}}_i$ over $\widehat{\mathbf{q}}_j$. Next, the meaning of rule (ii) is that if (aggregate) $\mathbf{q}_i$ is more expensive than the sum of $\mathbf{q}_{j_1}$ and $\mathbf{q}_{j_2}$, while member m prefers (personalized) $\widehat{\mathbf{q}}_{j_1}$ over $\widehat{\mathbf{q}}_i$, then the only possibility for rationalizing the choice of $\mathbf{q}_i$ is that the other member l prefers $\widehat{\mathbf{q}}_i$ over $\widehat{\mathbf{q}}_{j_2}$.

Rules (i) and (ii) define restrictions on the relations R_0^m and R^m. For a specification of these relations, rules (iii) and (iv) define the corresponding upper cost bound conditions. First, rule (iii) complements rule (ii): if members 1 and 2 prefer, respectively, (personalized) $\widehat{\mathbf{q}}_{i_1}$ and $\widehat{\mathbf{q}}_{i_2}$ over $\widehat{\mathbf{q}}_j$, then the choice of (aggregate) $\mathbf{q}_j$ can be rationalized only if it is not more expensive than the sum of $\mathbf{q}_{i_1}$ and $\mathbf{q}_{i_2}$. Finally, rule (iv) considers the special case where both members prefer the same (personalized) quantities $\widehat{\mathbf{q}}_i$ over $\widehat{\mathbf{q}}_j$, in which case, under the prices $\mathbf{p}_j$ the quantities $\mathbf{q}_j$ cannot be associated with a strictly higher expenditure level than $\mathbf{q}_i$.

Lemma 2 states that if a collective rationalization of the set of observations S is possible, then *there exists* a set of feasible personalized prices and quantities $\widehat{S}$ that is consistent with the rules (i)–(iv). To recall, Lemma 1 states that if

$\mathbf{q}_i R_0 \mathbf{q}_j$ (or, equivalently, $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$) then *for any* specification of the set $\widehat{S}$ we must have $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$. That is,

$$\text{(3.3)} \qquad \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \quad \Rightarrow \quad \widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j \quad \text{or} \quad \widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j.$$

Using this, we can specify restrictions on the relations R_0^1 and R_0^2 in terms of the set of observations S, that is, without explicit reference to a set of feasible personalized prices and quantities $\widehat{S}$. If there does not exist a specification of the relations R_0^1 and R_0^2, and corresponding transitive closures R^1 and R^2 that are consistent with (3.3) and at the same time meet rules (i)–(iv) in Lemma 2, then a collective rationalization of the set of observations S is impossible. Alternatively, a necessary condition for a collective rationalization of the set S to be possible is that there exists a specification of R_0^m and R^m ($m = 1, 2$) that is consistent with (3.3) and rules (i)–(iv) in Lemma 2. This idea underlies our testable necessity condition for collective rationality that is expressed directly in terms of the set of observations S of aggregate prices and quantities; the condition essentially combines the results in Lemmas 1 and 2.

To formalize the idea, we introduce some additional notation. First, referring to (3.3), for $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ we use $\mathbf{q}_i H_0^1 \mathbf{q}_j$ if we *hypothesize* $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ and use $\mathbf{q}_i H_0^2 \mathbf{q}_j$ if we *hypothesize* $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$. Let H^1 and H^2 denote the transitive closures of these *hypothetical* relations H_0^1 and H_0^2. The existence of a set of feasible personalized prices and quantities $\widehat{S}$ that satisfies the conditions in Proposition 1 implies that there exist relations H_0^m and H^m consistent with the analogues of rules (i)–(iv) in Lemma 2.

PROPOSITION 2: *Suppose that there exists a pair of utility functions U^1 and U^2 that provide a* collective rationalization *of the set of observations $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$. Then there exist* hypothetical *relations H_0^m and H^m for each member $m \in \{1, 2\}$ such that*:

(i) *if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, then $\mathbf{q}_i H_0^1 \mathbf{q}_j$ or $\mathbf{q}_i H_0^2 \mathbf{q}_j$*;

(ii) *if $\mathbf{q}_i H_0^m \mathbf{q}_k, \mathbf{q}_k H_0^m \mathbf{q}_l, \ldots, \mathbf{q}_z H_0^m \mathbf{q}_j$ for some (possibly empty) sequence $(k, l, \ldots, z)$, then $\mathbf{q}_i H^m \mathbf{q}_j$*;

(iii) *if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ and $\mathbf{q}_j H^m \mathbf{q}_i$, then $\mathbf{q}_i H_0^l \mathbf{q}_j$ (with $m \neq l$)*;

(iv) *if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$ and $\mathbf{q}_{j_1} H^m \mathbf{q}_i$, then $\mathbf{q}_i H_0^l \mathbf{q}_{j_2}$ (with $m \neq l$)*;

(v) *if $\mathbf{q}_{i_1} H^1 \mathbf{q}_j$ and $\mathbf{q}_{i_2} H^2 \mathbf{q}_j$, then $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'(\mathbf{q}_{i_1} + \mathbf{q}_{i_2})$*;

(vi) *if $\mathbf{q}_i H^1 \mathbf{q}_j$ and $\mathbf{q}_i H^2 \mathbf{q}_j$, then $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$.*

The intuition of the different rules follows immediately from our discussion of Lemmas 1 and 2 when replacing the relations R_0^m and R^m by their hypothetical counterparts H_0^m and H^m. More specifically, rule (i) refers to the result in Lemma 1. Rule (ii) defines the transitive closures H^1 and H^2 of the relations H_0^1 and H_0^2 (compare with Definition 4). Finally, rules (iii)–(vi) comply with rules (i)–(iv) in Lemma 2.

To illustrate the proposition, we recapture our Example 1.

EXAMPLE 1—Continued: The first step of our argument in Example 1 pertains to rule (i) in Proposition 2. Specifically, we can rephrase (3.1) in terms of the hypothetical relations H_0^1 and H_0^2 as

$$\forall i, j \in \{1, 2, 3\}, \quad \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \quad \Rightarrow \quad \mathbf{q}_i H_0^1 \mathbf{q}_j \quad \text{or} \quad \mathbf{q}_i H_0^2 \mathbf{q}_j.$$

Similarly, (3.2) complies with

$$\mathbf{q}_1 H_0^1 \mathbf{q}_2, \quad \mathbf{q}_2 H_0^1 \mathbf{q}_3 \quad \text{and} \quad \mathbf{q}_3 H_0^2 \mathbf{q}_2, \quad \mathbf{q}_2 H_0^2 \mathbf{q}_1.$$

Rule (v) in Proposition 2 then requires $\mathbf{p}_2'\mathbf{q}_2 \leq \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3)$, and this upper cost bound condition is not met by this set of observations. A similar inconsistency result holds for any other specification of the hypothetical relations H_0^m and H^m $(m = 1, 2)$: one can verify that any such specification that is consistent with rules (i)–(iv) cannot meet the corresponding upper cost bound conditions (v) and (vi).

Interestingly, Example 1 implies that it is sufficient to have three goods and three observations for rejecting collective rationality of observed household behavior. The following proposition states that this is also necessary.

PROPOSITION 3: *There do not always exist utility functions U^1 and U^2 that provide a* collective rationalization *of the set of observations $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ if and only if* (i) *the number of goods $n \geq 3$ and* (ii) *the number of observations $T \geq 3$.*

We only sketch the basic idea for the necessity result.[7] First, consider that there are only two goods ($n = 2$) and T (≥ 2) observations. In that case, a collective rationalization of the set of observations S is always achieved for the following specification of feasible personalized prices and quantities (for $(\mathbf{x})_e$ the eth entry of the vector $\mathbf{x}$):

$$\forall j, \quad \mathfrak{p}_j^1 = \mathbf{p}_j \quad \text{and} \quad \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0};$$
$$(\mathfrak{q}_j^1)_1 = (\mathbf{q}_j)_1 \quad \text{and} \quad (\mathfrak{q}_j^2)_2 = (\mathbf{q}_j)_2.$$

In words, goods 1 and 2 are allocated exclusively to, respectively, member 1 and member 2; for each observation j we have $(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j = (\mathbf{p}_j)_1(\mathbf{q}_j)_1$ and $(\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_j = (\mathbf{p}_j)_2(\mathbf{q}_j)_2$. It is easily verified that this specification of the feasible personalized quantities obtains consistency with the nonparametric conditions (ii) and (iii) in Proposition 1.

[7]The following arguments concentrate on $n = 2$ (for $T \geq 2$) and on $T = 2$ (for $n \geq 2$). If the necessity result holds in these cases, then it certainly also holds for $n < 2$ and $T < 2$.

Next, consider that there are only two observations ($T = 2$) and n (≥ 2) goods. In that case, a collective rationalization of the set of observations S is always achieved for

$$\mathfrak{p}_j^1 = \mathbf{p}_j \quad \text{and} \quad \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0} \quad \text{for} \quad j = 1, 2;$$

$$\mathfrak{q}_1^1 = \mathbf{q}_1 \quad (\text{or } \mathfrak{q}_1^2 = \mathfrak{q}_1^h = \mathbf{0}) \quad \text{and} \quad \mathfrak{q}_2^2 = \mathbf{q}_2 \quad (\text{or } \mathfrak{q}_2^1 = \mathfrak{q}_2^h = \mathbf{0}).$$

In words, members 1 and 2 are the dictators in, respectively, observation 1 (as $\mathfrak{q}_1^1 = \mathbf{q}_1$ and $(\widehat{\mathbf{p}}_1^1)'\widehat{\mathbf{q}}_1 = \mathbf{p}_1'\mathbf{q}_1$) and observation 2 (as $\mathfrak{q}_2^2 = \mathbf{q}_2$ and $(\widehat{\mathbf{p}}_2^2)'\widehat{\mathbf{q}}_2 = \mathbf{p}_2'\mathbf{q}_2$). Again, it is easy to verify consistency with conditions (ii) and (iii) in Proposition 1 for this specification of the feasible personalized prices and quantities.

Thus, the collective model can be rejected (or empirical testing is meaningful) as soon as there are at least three goods and three observations. Note that the lower bound of three goods is below the lower bound derived by Browning and Chiappori (1998) in their parametric setting: empirical falsification of their collective model necessitates at least five goods, because they focus on pseudo-Slutsky symmetry, which requires at least five goods for testable implications. By contrast, their parametric model equally needs only three goods to test pseudo-Slutsky negativity.[8]

To conclude, because the necessary condition in Proposition 2 requires only aggregate prices $\mathbf{p}_j$ and quantities $\mathbf{q}_j$, it enables an operational collective rationality test that applies to the general case of T observations. The Appendix presents a finite algorithm for verifying the condition and contains some further discussion regarding the practicality of the approach. Of course, this algorithm also applies to any subset of the set of observations S, thus implying weaker collective rationality tests.

4. TESTABLE SUFFICIENCY RESTRICTIONS

Although the condition in Proposition 2 is necessary for a collective rationalization, it is in general not sufficient.[9] This follows from Example 2, which contains data that satisfy the condition but cannot be collectively rationalized in the sense of Proposition 1.

EXAMPLE 2: We prove in the Appendix that a collective rationalization cannot be obtained for a set of seven observations with

$$\forall i \in \{1, \ldots, 7\}, \quad \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j \quad \text{for all } j \in \{1, \ldots, 7\} \backslash \{i\},$$

[8]We are grateful to an anonymous referee for pointing this out.

[9]In fact, it can be verified that the necessary condition in Proposition 2 is also sufficient for $T \leq 4$ (for compactness, we abstract from a formal statement). Although Example 2 uses $T = 7$ for mathematical elegance of the proof, it is worth stressing that similar (but less elegant) arguments can be established for $4 < T < 7$.

$$\forall i \in \{1, 7\}, \quad \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'(\mathbf{q}_j + \mathbf{q}_k) \quad \text{for all} \quad j, k \in \{1, \dots, 7\} \backslash \{i\} \quad \text{with} \quad j \neq k,$$

$$\forall i \in \{2, \dots, 6\}, \quad \mathbf{p}_i'\mathbf{q}_i = \mathbf{p}_i'(\mathbf{q}_j + \mathbf{q}_k) - \varepsilon \quad \text{for all} \quad j, k \in \{1, \dots, 7\} \backslash \{i\} \quad \text{with} \quad j \neq k,$$

where $(\min_{i,e}(\mathbf{p}_i)_e \min_{i,e}(\mathbf{q}_i)_e)/6 > \varepsilon > 0$ ($i \in \{1, \dots, 7\}$ and $e \in \{1, \dots, n\}$). For example, such a structure applies to $\mathbf{q}_i, \mathbf{p}_i \in \Re^7$ with

$$\forall i \in \{1, \dots, 7\}, \quad (\mathbf{q}_i)_i = 3 \quad \text{and} \quad (\mathbf{q}_i)_e = 1 \quad \text{if} \quad e \neq i,$$

$$\forall i \in \{1, 7\}, \quad (\mathbf{p}_i)_i = 11 \quad \text{and} \quad (\mathbf{p}_i)_e = 1 \quad \text{if} \quad e \neq i,$$

$$\forall i \in \{2, \dots, 6\}, \quad (\mathbf{p}_i)_i = 10 - \varepsilon \quad \text{and} \quad (\mathbf{p}_i)_e = 1 \quad \text{if} \quad e \neq i,$$

where $(1/6) > \varepsilon > 0$.

We next present a sufficient condition for a collective rationalization that solely uses observed (aggregate) prices and quantities. Essentially, as compared to the necessary condition in Proposition 2, this sufficient condition requires some additional structure in these prices and quantities, so that we can *always* conceive a household decision model (and corresponding feasible personalized prices and quantities) consistent with the collective rationality restrictions in Proposition 1; we explain the particular decision model subsequently. Like before, this condition implies (*in casu* sufficiency) tests for collective rationality that hold for the general case of T observations. A finite testing algorithm is presented in the Appendix.

PROPOSITION 4: *Suppose that for the set of observations* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \dots, T\}$ *there exist* hypothetical *relations* H_0^m *and* H^m *for each member* $m \in \{1, 2\}$ *that satisfy rules* (i)–(vi) *in Proposition* 2 *and*, *in addition*, *allow for constructing sets* S^1 *and* S^2 *with* $S^1 \subseteq S$ *and* $S^2 = S \backslash S^1$ *such that*

(vii) $S^m = \{(\mathbf{p}_j; \mathbf{q}_j) \in S \mid \mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ *whenever* $\mathbf{q}_i H^m \mathbf{q}_j\}$;

(viii) *for each* $(\mathbf{p}_i; \mathbf{q}_i), (\mathbf{p}_j; \mathbf{q}_j) \in S^m$, $\mathbf{q}_i H_0^m \mathbf{q}_j$ *whenever* $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$.

Then there exists a pair of utility functions U^1 *and* U^2 *that provide a* collective rationalization *of the set* S.

Referring to the interpretation of the unitary model as a dictatorship model (see Section 2), we can interpret this result in terms of a *situation-dependent dictatorship* model. Specifically, we prove in the Appendix that under conditions (i)–(viii) we can obtain consistency with the nonparametric condition (ii) in Proposition 1 for the following specification of the feasible personalized quantities and prices:

$$\text{if} \quad (\mathbf{p}_j; \mathbf{q}_j) \in S^1, \quad \text{then} \quad \mathrm{q}_j^1 = \mathbf{q}_j;$$

$$\text{if} \quad (\mathbf{p}_j; \mathbf{q}_j) \in S^2, \quad \text{then} \quad \mathfrak{q}_j^2 = \mathbf{q}_j;$$

$$\mathfrak{p}_j^1 = \mathbf{p}_j, \quad \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0} \quad \text{for all } (\mathbf{p}_j; \mathbf{q}_j) \in S.$$

For all observations j such that $(\mathbf{p}_j; \mathbf{q}_j) \in S^1$, member 1 is the dictator because $\mathfrak{q}_j^1 = \mathbf{q}_j$ (or, equivalently, $\mathfrak{q}_j^2 = \mathfrak{q}_j^h = \mathbf{0}$) and $(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j = \mathbf{p}_j'\mathbf{q}_j$. Similarly, member 2 is the dictator for the other observations.[10] Put another way, *the identity of the dictator depends on the observation or situation at hand*. In that interpretation, the statement $\mathbf{q}_i H^1 \mathbf{q}_j$ means that the (situation-dependent) dictator 1 prefers the (aggregate) $\mathbf{q}_i$ over $\mathbf{q}_j$; a directly similar interpretation holds for $\mathbf{q}_i H^2 \mathbf{q}_j$. Rule (vii) then specifies that the situation-dependent dictators 1 and 2 must respect the corresponding upper cost bounds. The additional rule (viii) indicates that if member m (1 or 2) is the dictator in situations i and j, then the choice of $\mathbf{q}_i$ when $\mathbf{q}_j$ was equally obtainable under the prices $\mathbf{p}_i$ can be rationalized only if member m prefers (aggregate) $\mathbf{q}_i$ over $\mathbf{q}_j$ (or $\mathbf{q}_i H_0^m \mathbf{q}_j$).

This situation-dependent dictatorship model can be regarded as a direct "collective" extension of the unitary decision model. Specifically, in contrast to the latter model, the former model implies *two* separate decision makers in the household, who are each (*fully*) responsible for a disjoint subset of the T observed aggregate quantities. Consequently, the sufficiency condition implies that there must exist a partitioning of the observed set S into two subsets that each individually meet the unitary GARP; that is, each individual dictator must act consistent with the unitary rationality condition *for those quantities for which she or he is (fully) responsible*. It is this interpretation that underlies the testing algorithm in the Appendix.

In summary, violation of the necessary condition in Proposition 2 means that a collective rationalization is impossible, while consistency with the sufficient condition in Proposition 4 entails the opposite conclusion. As for data that meet the necessity but not the sufficiency condition, we cannot directly tell from the observed (aggregate) prices and quantities whether a collective rationalization of the data is effectively possible.[11] For instance, the proof of the inconsistency result in Example 2 starts from the necessity condition (which, like

[10]We note that, technically, this specification of the feasible personalized quantities and prices is consistent with $\infty > \mu_j > 0$ for all j (see Section 4 of Cherchye, De Rock, and Vermeulen (2007) for details). An interpretation in terms of bargaining power is as follows (for the given specification of the personalized prices): for $(\mathbf{p}_j; \mathbf{q}_j) \in S^1$, the value of the bargaining weight μ_j (>0) of member 2 is too small to obtain $\mathfrak{q}_j^1 \neq \mathbf{q}_j$; conversely, for $(\mathbf{p}_j; \mathbf{q}_j) \in S^2$, the value of μ_j ($<\infty$) is too large to obtain $\mathfrak{q}_j^2 \neq \mathbf{q}_j$. Furthermore, we stress that the given specification of the feasible personalized prices and quantities should not be the unique one that obtains consistency with condition (ii) in Proposition 1 (and, thus, other interpretations of the sufficiency result are equally possible).

[11]At this point, it is worth emphasizing the subtle difference between collective rationality of household behavior and a collective rationalization of a set of household observations S. On the one hand, impossibility of a collective rationalization of S (e.g., inconsistency with the necessity condition in Proposition 2) necessarily implies collectively irrational behavior. On the other hand,

the unitary GARP, focuses on the full consumption bundles) to subsequently consider the construction of feasible personalized prices and quantities for individual goods. Such practice generally boils down to checking the inequalities in Proposition 1 that are nonlinear in these feasible personalized prices and quantities. (We avoid this in our proof of the result in Example 2 only because of our specific condition for ε.)

Still, even though the necessary condition should not generally coincide with the sufficient condition, we may expect the two conditions to become equally powerful (or to converge) when the sample size increases.[12] Specifically, for each observation j we have that $\min_{\mathbf{q}_i}\{\mathbf{p}_j'\mathbf{q}_i|\mathbf{q}_i H^1\mathbf{q}_j \text{ and not } \mathbf{q}_i H^2\mathbf{q}_j\}$ or $\min_{\mathbf{q}_i}\{\mathbf{p}_j'\mathbf{q}_i|\mathbf{q}_i H^2\mathbf{q}_j \text{ and not } \mathbf{q}_i H^1\mathbf{q}_j\}$ will generally get closer to zero for larger T. Hence, the requirement $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'(\mathbf{q}_{i_1} + \mathbf{q}_{i_2})$ whenever $\mathbf{q}_{i_1} H^1\mathbf{q}_j$ and $\mathbf{q}_{i_2} H^2\mathbf{q}_j$ in Proposition 2 (rule (v)) will approach the condition $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ whenever $\mathbf{q}_i H^m\mathbf{q}_j$ for $m = 1$ or 2 in Proposition 4 (rule (vii)).[13]

The associated convergence rate will then of course depend (positively) on the variation in the observed prices and quantities, and hence we may expect it to increase with the number of goods. For a given number of goods, the speed of convergence will vary with the specific data generating process that underlies the aggregate prices and quantities, which in turn depends on the household member utilities and on the characteristics of the within-household bargaining process. However, in general, we can safely argue that the empirical implications of the fairly rudimentary situation-dependent dictatorship solution (see the sufficient condition) will get closer to those of any more refined intrahousehold decision process (see the necessary condition) when the sample size increases.

5. CONCLUDING REMARKS

To conclude, we recall that the collective model under study considers general member-specific preferences and assumes only that the empirical analyst observes the aggregate household consumption quantities and prices. Attractively, the model encompasses a large variety of alternative behavioral models as special cases, which include additional prior information that implies extra restrictions regarding the feasible personalized quantities and prices (see

possibility of a collective rationalization of S (e.g., consistency with the sufficiency condition in Proposition 4) does not necessarily imply collectively rational behavior; it only means that we cannot reject collective rationality on the basis of the available set of observations.

[12]See, for example, Bronars (1987) for power notions in the context of nonparametric rationality tests.

[13]Note that the necessary condition (rule (vi)) and the sufficient condition (rule (vii)) both require $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ whenever $\mathbf{q}_i H^1\mathbf{q}_j$ *and* $\mathbf{q}_i H^2\mathbf{q}_j$. Also observe that the empirical restrictions that follow from rule (iv) in Proposition 2 imply those of rule (viii) in Proposition 4 when, for each observation j, $\min_{\mathbf{q}_i}\{\mathbf{p}_j'\mathbf{q}_i|\mathbf{q}_i H^1\mathbf{q}_j \text{ and not } \mathbf{q}_i H^2\mathbf{q}_j\}$ or $\min_{\mathbf{q}_i}\{\mathbf{p}_j'\mathbf{q}_i|\mathbf{q}_i H^2\mathbf{q}_j \text{ and not } \mathbf{q}_i H^1\mathbf{q}_j\}$ gets close to zero for large T.

(2.1) and (2.2) for the general model under study). For example, such additional structure may pertain to observability of private and/or public consumption quantities or to the nature of the individual members' preferences (namely, egoistic rather than altruistic). Notable cases are the traditional unitary model and the collective model of Chiappori (1988). For each of these special cases, we may expect more stringent testable necessary and sufficient conditions for collective rationalization that solely use observed prices and quantities. (These conditions can be obtained along similar lines as in the proofs of Propositions 2 and 4. The associated testing algorithms can proceed in the same way as those presented in the Appendix.)

As a final note, we recall that the testable collective rationality conditions in Propositions 2 and 4 have a structure analogous to the (unitary) GARP, which allows for easy adaptations of the existing power and goodness-of-fit measures for nonparametric consumption analysis (see, respectively, Bronars (1987) and Varian (1990)). Specifically, using the necessary and sufficient conditions, one can generate upper and lower bounds for each of these measures. (If these upper and lower bounds are situated close to each other, one possible interpretation is that the empirical content of the necessary and sufficient conditions is practically the same for the set of observations under study.)

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APPENDIX

PROOF OF PROPOSITION 1: Varian (1982) proved the equivalence between conditions (ii) and (iii) of the proposition. Therefore, it suffices to prove equivalence between (i) and (iii).[14]

[14]This proof generalizes that of Chiappori (1988), who focused on the specific case of household labor supply. Another difference is that Chiappori focused on (a strong version of) the *strong axiom of revealed preference* (SARP) conditions while our proof uses the (less stringent) GARP conditions. It is worth pointing out that all our results for the GARP can be adapted to apply for the (strong) SARP.

(i) $\Rightarrow$ (iii) Under condition (i), for each observation j there exists $\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h)$ that solves the problem (for $\widehat{\mathbf{z}} = (\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h)$ with $\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h \in \Re_+^n$)

$$\max_{\widehat{\mathbf{z}}} U^1(\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h) + \mu_j U^2(\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h) \quad \text{s.t.} \quad \mathbf{p}_j'(\mathfrak{z}^1 + \mathfrak{z}^2 + \mathfrak{z}^h) \leq \mathbf{p}_j'\mathbf{q}_j.$$

Given concavity, both individual utility functions are subdifferentiable, which carries over to their weighted sum $U^1 + \mu_j U^2$.[15] An optimal solution to the above maximization problem must therefore satisfy (for η_j the Lagrange multiplier associated with the budget constraint)

$$U^1_{\mathfrak{q}_j^c} + \mu_j U^2_{\mathfrak{q}_j^c} \leq \eta_j \mathbf{p}_j,$$

where $U^m_{\mathfrak{q}_j^c}$ $(m = 1, 2)$ is a subgradient of the utility function U^m defined for the vector $\mathfrak{z}^c \in \Re_+^n$ and evaluated at $\mathfrak{q}_j^c$ $(c = 1, 2, h)$. Letting $\mathfrak{p}_j^c = U^1_{\mathfrak{q}_j^c}/\eta_j$, $\lambda_j^1 = \eta_j$, and $\lambda_j^2 = \eta_j/\mu_j$ thus gives

$$\text{(A.1)} \qquad U^1_{\mathfrak{q}_j^c} = \lambda_j^1 \mathfrak{p}_j^c \quad \text{and} \quad U^2_{\mathfrak{q}_j^c} \leq \lambda_j^2(\mathbf{p}_j - \mathfrak{p}_j^c).$$

Next, concavity of the functions U^1 and U^2 implies $(m = 1, 2)$

$$\text{(A.2)} \qquad U^m(\widehat{\mathbf{q}}_i) - U^m(\widehat{\mathbf{q}}_j) \leq \sum_{c=1,2,h} U^m_{\mathfrak{q}_j^c}(\mathfrak{q}_i^c - \mathfrak{q}_j^c).$$

Substituting (A.1) into (A.2) and setting $U_k^m = U^m(\widehat{\mathbf{q}}_k)$ $(m = 1, 2;\ k = i, j)$ obtains condition (iii) of the proposition.

(iii) $\Rightarrow$ (i) Under condition (iii), we can define for any $\widehat{\mathbf{q}} = (\mathfrak{q}^1, \mathfrak{q}^2, \mathfrak{q}^h)$ such that $\mathbf{p}_j'(\mathfrak{q}^1 + \mathfrak{q}^2 + \mathfrak{q}^h) \leq \mathbf{p}_j'\mathbf{q}_j$,

$$\text{(A.3)} \qquad U^1(\widehat{\mathbf{q}}) = \min_{i \in \{1,\dots,T\}} [U_i^1 + \lambda_i^1 (\widehat{\mathbf{p}}_i^1)'(\widehat{\mathbf{q}} - \widehat{\mathbf{q}}_i)]$$

and

$$\text{(A.4)} \qquad U^2(\widehat{\mathbf{q}}) = \min_{i \in \{1,\dots,T\}} [U_i^2 + \lambda_i^2 (\widehat{\mathbf{p}}_i^2)'(\widehat{\mathbf{q}} - \widehat{\mathbf{q}}_i)].$$

Varian (1982) proved that $U^1(\widehat{\mathbf{q}}_j) = U_j^1$ and $U^2(\widehat{\mathbf{q}}_j) = U_j^2$. Next, given $\mu_j \in \Re_{++}$, we have that

$$U^1(\widehat{\mathbf{q}}) + \mu_j U^2(\widehat{\mathbf{q}}) \leq U_j^1 + \lambda_j^1 (\widehat{\mathbf{p}}_j^1)'(\widehat{\mathbf{q}} - \widehat{\mathbf{q}}_j) + \mu_j [U_j^2 + \lambda_j^2 (\widehat{\mathbf{p}}_j^2)'(\widehat{\mathbf{q}} - \widehat{\mathbf{q}}_j)].$$

[15]To be precise, $-U^m$ $(m = 1, 2)$ is convex and therefore subdifferentiable. This, of course, does not affect our argument.

Without losing generality, we concentrate on $\mu_j = (\lambda_j^1/\lambda_j^2)$, which obtains

$$U^1(\widehat{\mathbf{q}}) + \mu_j U^2(\widehat{\mathbf{q}}) \leq U_j^1 + \mu_j U_j^2 + \lambda_j^1(\mathbf{p}_j)'(\mathbf{q} - \mathbf{q}_j),$$

where $\mathbf{q} = (\mathfrak{q}^1 + \mathfrak{q}^2 + \mathfrak{q}^h)$.

Because $\mathbf{p}_j'\mathbf{q} \leq \mathbf{p}_j'\mathbf{q}_j$, we thus have

$$U^1(\widehat{\mathbf{q}}) + \mu_j U^2(\widehat{\mathbf{q}}) \leq U_j^1 + \mu_j U_j^2 = U^1(\widehat{\mathbf{q}}_j) + \mu_j U^2(\widehat{\mathbf{q}}_j),$$

which proves that $\widehat{\mathbf{q}}_j$ maximizes $U^1(\widehat{\mathbf{q}}) + \mu_j U^2(\widehat{\mathbf{q}})$ subject to $\mathbf{p}_j'(\mathfrak{q}^1 + \mathfrak{q}^2 + \mathfrak{q}^h) \leq \mathbf{p}_j'\mathbf{q}_j$. We conclude that the functions U^1 and U^2 in (A.3) and (A.4) provide a collective rationalization of S. These functions satisfy the conditions in part (i) of the proposition (compare with Varian (1982)). *Q.E.D.*

PROOF OF LEMMA 1—Necessity: We first derive that $\mathbf{q}_i R_0 \mathbf{q}_j$ implies $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$ for any set $\widehat{S}$. The result follows from the fact that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ (or $\mathbf{q}_i R_0 \mathbf{q}_j$) is incompatible with the existence of some $\widehat{S}$ such that $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j$ and $(\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_j$. Indeed, summing these last inequalities immediately yields $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j$.

Sufficiency: We next derive that if, for all sets of feasible personalized prices and quantities $\widehat{S}$, $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$, then $\mathbf{q}_i R_0 \mathbf{q}_j$. The result is obtained by noting that $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j$ implies $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i + (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j + (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_j$ for all $\widehat{S}$. It is then easy to see that if $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j$, then there exists $\widehat{S}$ such that $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j$ and $(\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_j$ (i.e., we have *neither* $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ *nor* $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$); for example, one may use $\mathfrak{p}_k^1 = (1/2)\mathbf{p}_k$ and $\mathfrak{q}_k^1 = \mathbf{q}_k$ $(k = i, j)$. Hence, we have for all sets $\widehat{S}$ that $\widehat{\mathbf{q}}_i R_0^1 \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$ only if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, that is, $\mathbf{q}_i R_0 \mathbf{q}_j$. *Q.E.D.*

PROOF OF LEMMA 2: Given that a collective rationalization of the set of observations S is possible, we consider a set $\widehat{S}$ that is consistent with condition (ii) in Proposition 1. Using Definition 4, this set $\widehat{S}$ defines relations R_0^m and R^m $(m = 1, 2)$. We will show that these relations satisfy rules (i)–(iv) in Lemma 2.

As for rule (i), we establish that if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ and $\widehat{\mathbf{q}}_j R^1 \widehat{\mathbf{q}}_i$, then $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$ (the argument for the other case is directly analogous). For $\widehat{\mathbf{q}}_j R^1 \widehat{\mathbf{q}}_i$, consistency with condition (ii) in Proposition 1 requires $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j$. Given $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, this last inequality implies $(\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_j$, which gives the result.

To derive rule (ii), suppose that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$ in combination with $\widehat{\mathbf{q}}_{j_1} R^1 \widehat{\mathbf{q}}_i$ while not $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_{j_2}$. On the one hand, not $\widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_{j_2}$ means that $(\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_{j_2}$. On the other hand, $\widehat{\mathbf{q}}_{j_1} R^1 \widehat{\mathbf{q}}_i$ requires that $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_{j_1}$ for the consistency with condition (ii) in Proposition 1. Combining these two inequalities would imply $\mathbf{p}_i'\mathbf{q}_i < (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_{j_1} + (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_{j_2} \leq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$, which contradicts $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$. Thus, we conclude that $(\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2}) \wedge \widehat{\mathbf{q}}_{j_1} R^1 \widehat{\mathbf{q}}_i) \Rightarrow \widehat{\mathbf{q}}_i R_0^2 \widehat{\mathbf{q}}_{j_2}$. A directly analogous argument holds for the other case.

As for rules (iii) and (iv), under $\widehat{\mathbf{q}}_{i_1} R^1 \widehat{\mathbf{q}}_j$ and $\widehat{\mathbf{q}}_{i_2} R^2 \widehat{\mathbf{q}}_j$ consistency with condition (ii) in Proposition 1 is obtained only if $(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_{i_1}$ and $(\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_{i_2}$. This last result immediately yields $\mathbf{p}_j'\mathbf{q}_j \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_{i_1} + (\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_{i_2} \leq \mathbf{p}_j'(\mathbf{q}_{i_1} + \mathbf{q}_{i_2})$ if $\mathbf{q}_{i_1} \neq \mathbf{q}_{i_2}$ and, similarly, $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ if $\mathbf{q}_{i_1} = \mathbf{q}_{i_2} = \mathbf{q}_i$. *Q.E.D.*

PROOF OF PROPOSITION 2: The result follows immediately from combining Lemmas 1 and 2, replacing the relations R_0^m and R^m with their hypothetical counterparts H_0^m and H^m. Rule (i) follows from Lemma 1. Rule (ii) defines the transitive closures H^1 and H^2 of the relations H_0^1 and H_0^2; compare with Definition 4. Finally, rules (iii)–(vi) follow from rules (i)–(iv) in Lemma 2. *Q.E.D.*

PROOF OF THE RESULT IN EXAMPLE 2: For the specific data structure, consistency with the condition in Proposition 2 implies that there exist hypothetical relations that must satisfy, for all $i, j \in \{1, \dots, 7\}$, $i \neq j$, $\mathbf{q}_i H^m \mathbf{q}_j$ and not $\mathbf{q}_i H^l \mathbf{q}_j$ for $m \neq l$; and we cannot have $\mathbf{q}_i H^1 \mathbf{q}_k$ and $\mathbf{q}_j H^2 \mathbf{q}_k$ for $k \in \{1, 7\}$ and for all $i, j \in \{1, \dots, 7\} \backslash \{k\}$. Given this, one possible specification of the relations H_0^m and H^m is[16]

$$\forall i, j \in \{1, \dots, 7\}, \quad (i > j \Rightarrow \mathbf{q}_j H^1 \mathbf{q}_i) \quad \text{and} \quad (i < j \Rightarrow \mathbf{q}_j H^2 \mathbf{q}_i).$$

Combining the corresponding requirements that follow from condition (ii) in Proposition 1 obtains, for all $i \in \{2, \dots, 6\}$ and $j \in \{1, \dots, 7\}$,

$$(i > j \Rightarrow \mathbf{p}_i'\mathbf{q}_j - \varepsilon \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \mathbf{p}_i'\mathbf{q}_j) \quad \text{and} \quad (i < j \Rightarrow 0 \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \varepsilon). \tag{A.5}$$

Next, because $(\mathbf{q}_j)_e = (\mathfrak{q}_j^1)_e + (\mathfrak{q}_j^2)_e + (\mathfrak{q}_j^h)_e$ and $\mathfrak{p}_i^c \leq \mathbf{p}_i$ $(c = 1, 2, h)$, we obtain that $\mathbf{p}_i'\mathbf{q}_j - \varepsilon \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \mathbf{p}_i'\mathbf{q}_j$ implies, for all $e \in \{1, \dots, n\}$,

$$(\mathbf{p}_i)_e(\mathbf{q}_j)_e - \varepsilon \leq \sum_{c \in \{1,2,h\}} (\mathfrak{p}_i^c)_e(\mathfrak{q}_j^c)_e \leq (\mathbf{p}_i)_e(\mathbf{q}_j)_e,$$

which in turn entails, for all $c \in \{1, 2, h\}$ with $(\mathfrak{q}_j^c)_e > 0$,

$$(\mathbf{p}_i)_e - \frac{\varepsilon}{(\mathfrak{q}_j^c)_e} \leq (\mathfrak{p}_i^c)_e \leq (\mathbf{p}_i)_e.$$

Similarly, the restriction $0 \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \varepsilon$ requires

$$\left[0 \leq \sum_{c \in \{1,2,h\}} (\mathfrak{p}_i^c)_e(\mathfrak{q}_j^c)_e \leq \varepsilon\right] \Rightarrow \left[\forall c \in \{1, 2, h\} : 0 \leq (\mathfrak{p}_i^c)_e \leq \frac{\varepsilon}{(\mathfrak{q}_j^c)_e}\right].$$

[16]The following argument can be repeated for any alternative specification of the relations H_0^m and H^m that meets the necessity condition in Proposition 2.

Let us concentrate on $e = 1$ and consider $0 < \sigma = \min_{j\in\{1,\dots,7\},e\in\{1,\dots,n\}}(\mathbf{q}_j)_e$. The pigeon hole principle implies $\forall j \in \{1,\dots,7\}$ that $\exists c_j \in \{1,2,h\}$, $(\mathfrak{q}_j^{c_j})_1 \geq (\sigma/3)$, so that we get

$$[\mathbf{p}_i'\mathbf{q}_j - \varepsilon \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \mathbf{p}_i'\mathbf{q}_j] \Rightarrow \left[\exists c_j \in \{1,2,h\} : (\mathbf{p}_i)_1 - \frac{3\varepsilon}{\sigma} \leq (\mathfrak{p}_i^{c_j})_1 \leq (\mathbf{p}_i)_1\right]$$

and

$$[0 \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \varepsilon] \Rightarrow \left[\exists c_j \in \{1,2,h\} : 0 \leq (\mathfrak{p}_i^{c_j})_1 \leq \frac{3\varepsilon}{\sigma}\right].$$

Note that $(\min_{j,e}(\mathbf{p}_j)_e \min_{j,e}(\mathbf{q}_j)_e)/6 > \varepsilon$ implies $(\mathbf{p}_i)_1 - \frac{3\varepsilon}{\sigma} > \frac{3\varepsilon}{\sigma}$. Using this, the preference structure in (A.5) obtains, $\forall i \in \{2,\dots,6\}$,

$$\text{(A.6)} \qquad \forall j_1, j_2 \in \{1,\dots,7\}, \quad (i > j_1 \wedge i < j_2 \Rightarrow c_{j_1} \neq c_{j_2});$$

the reasoning is that $(i > j_1 \Rightarrow (\mathbf{p}_i)_1 - \frac{3\varepsilon}{\sigma} \leq (\mathfrak{p}_i^{c_{j_1}})_1 \leq (\mathbf{p}_i)_1)$ and $(i < j_2 \Rightarrow 0 \leq (\mathfrak{p}_i^{c_{j_2}})_1 \leq \frac{3\varepsilon}{\sigma})$, which excludes $c_{j_1} = c_{j_2}$. Inconsistency with the collective rationalization conditions in Proposition 1 follows because (A.6) implies $c_{j_1} \neq c_{j_2}$ for all $j_1, j_2 \in \{1,3,5,7\}$, $j_1 \neq j_2$; and this contradicts $c_j \in \{1,2,h\}$ $\forall j \in \{1,\dots,7\}$. *Q.E.D.*

PROOF OF PROPOSITION 4: Suppose that we can construct sets S^1 and S^2 in Proposition 4. Then we can construct a set of feasible prices and quantities $\widehat{S}$ that meets condition (ii) in Proposition 1. Specifically, define $\widehat{S}$ such that

$$\text{if} \quad (\mathbf{p}_j;\mathbf{q}_j) \in S^1, \quad \text{then } \mathfrak{q}_j^1 = \mathbf{q}_j \quad (\text{and thus } \mathfrak{q}_j^2 = \mathfrak{q}_j^h = \mathbf{0})$$

$$\text{if} \quad (\mathbf{p}_j;\mathbf{q}_j) \in S^2, \quad \text{then } \mathfrak{q}_j^2 = \mathbf{q}_j \quad (\text{and thus } \mathfrak{q}_j^1 = \mathfrak{q}_j^h = \mathbf{0});$$

$$\mathfrak{p}_j^1 = \mathbf{p}_j, \quad \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0} \quad \text{for all } (\mathbf{p}_j;\mathbf{q}_j) \in S.$$

We restrict attention to household member 1, but a directly analogous reasoning applies to member 2. Condition (ii) in Proposition 1 states that $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_k, \dots, (\widehat{\mathbf{p}}_z^1)'\widehat{\mathbf{q}}_z \geq (\widehat{\mathbf{p}}_z^1)'\widehat{\mathbf{q}}_j$ for some (possibly empty) sequence $(k,\dots,z)$ implies $(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_i$. As a preliminary step, we note that under the preceding specification of the set $\widehat{S}$ we have for all $(\mathbf{p}_{l_1};\mathbf{q}_{l_1}) \in S^1$ that $(\widehat{\mathbf{p}}_{l_1}^1)'\widehat{\mathbf{q}}_{l_2} = 0$ if $(\mathbf{p}_{l_2};\mathbf{q}_{l_2}) \in S^2$. This mean that the only interesting case is $(\mathbf{p}_l;\mathbf{q}_l) \in S^1$ for all $l = i,j,k,\dots,z$. Hence, obtaining $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_k, \dots, (\widehat{\mathbf{p}}_z^1)'\widehat{\mathbf{q}}_z \geq (\widehat{\mathbf{p}}_z^1)'\widehat{\mathbf{q}}_j \Rightarrow (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_i$ boils down to verifying $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_k, \dots, \mathbf{p}_z'\mathbf{q}_z \geq \mathbf{p}_z'\mathbf{q}_j \Rightarrow \mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ for any possible sequence of $(i,k,\dots,z,j)$ with $(\mathbf{p}_l;\mathbf{q}_l) \in S^1$ for all $l = i,j,k,\dots,z$.

Using rule (viii) in Proposition 4, we have $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_k, \ldots, \mathbf{p}_z'\mathbf{q}_z \geq \mathbf{p}_z'\mathbf{q}_j \Rightarrow \mathbf{q}_i H_0^1 \mathbf{q}_k, \ldots, \mathbf{q}_z H_0^1 \mathbf{q}_j$, which in turn implies $\mathbf{q}_i H^1 \mathbf{q}_j$. Rule (vii) in Proposition 4 consequently guarantees $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$, that is, condition (ii) in Proposition 1 is met for member 1. *Q.E.D.*

Testing Algorithms

We first present an algorithm for checking the *necessary condition for a collective rationalization* of the set of observations S in Proposition 2. Before doing so, we introduce some additional notation. First, we define the set

$$D_j = \{(\mathbf{q}_i; \mathbf{p}_i) | \mathbf{q}_i R_0 \mathbf{q}_j\}.$$

Next, we use the notion that every specification of the hypothetical relations H_0^1 and H_0^2 (and the corresponding transitive closures H^1 and H^2) defines the sets ($m = 1, 2$)

$$D_j^m = \{(\mathbf{q}_i; \mathbf{p}_i) | \mathbf{q}_i H_0^m \mathbf{q}_j\} \quad \text{and} \quad ID_j^m = \{(\mathbf{q}_i; \mathbf{p}_i) | \mathbf{q}_i H^m \mathbf{q}_j\}.$$

The following algorithm will be expressed in terms of the sets D_j^m and ID_j^m rather than the relations H_0^m and H^m:

Step 1: For all $j \in \{1, \ldots, T\}$, construct the set D_j and set $C_j = \emptyset$. (Each set C_j captures all possible specifications of the sets D_j^1 and D_j^2 or, equivalently, the relations H_0^1 and H_0^2 that the algorithm considers in the successive iterations.)

Step 2: (See rule (i) in Proposition 2.) For all $j \in \{1, \ldots, T\}$, construct (D_j^1, D_j^2) such that (a) $D_j^m \subseteq D_j$ ($m = 1, 2$), (b) $D_j^1 \cup D_j^2 = D_j$, and (c) $(D_j^1, D_j^2) \notin C_j$. If for any j such (D_j^1, D_j^2) does not exist, then STOP the algorithm: *a collective rationalization of the set S is impossible*.

Step 3: (See rule (ii) in Proposition 2.) For all $j \in \{1, \ldots, T\}$, construct (ID_j^1, ID_j^2) using Warshall's algorithm (Varian (1982, p. 949)).

Step 4: For $j = 1, \ldots, T$, verify rule (iii) in Proposition 2. If OK, then go to $j + 1$ unless $j = T$, in which case then go to Step 5; else (a) $C_j = C_j \cup (D_j^1, D_j^2)$, (b) go to Step 2.

Step 5: For $j = 1, \ldots, T$, verify rule (iv) Proposition 2. If OK, then go to $j + 1$ unless $j = T$, in which case then go to Step 6; else (a) $C_j = C_j \cup (D_j^1, D_j^2)$, (b) go to Step 2.

Step 6: For $j = 1, \ldots, T$, verify rules (v) and (vi) in Proposition 2 for the constructed (ID_j^1, ID_j^2). If OK, then go to $j + 1$ unless $j = T$, in which case then STOP the algorithm—*the set S meets the necessary condition for a collective rationalization*; else (a) $C_j = C_j \cup (D_j^1, D_j^2)$, (b) go to Step 2.

This algorithm is clearly finite in nature and is on the order of $3^{|D_1|+|D_2|+\cdots+|D_T|}$. Specifically, for any $(\mathbf{q}_i; \mathbf{p}_i) \in D_j$ we must (maximally) consider three possibilities: $(\mathbf{q}_i; \mathbf{p}_i) \in D_j^1$, $(\mathbf{q}_i; \mathbf{p}_i) \in D_j^2$, and $(\mathbf{q}_i; \mathbf{p}_i) \in D_j^1 \cap D_j^2$. For each $j \in \{1, \ldots, T\}$, this gives us $3^{|D_j|}$ possible specifications of the sets D_j^m. We have $3^{|D_1|+|D_2|+\cdots+|D_T|} \leq 3^{T^2}$ for T observations, which gives us a finite upper bound for the number of specifications to be checked. (Hence, the upper bound 3^{T^2} applies only if $D_j = S$ for all observations j, which is of course an extreme scenario.)

We next consider the *sufficient condition for a collective rationalization* of the set of observations S in Proposition 4. This condition can be checked by means of the following algorithm:

Step 1: For the given set S, define $S^* = \{(S^1, S^2) | S^1 \subseteq S \text{ and } S^2 = S \backslash S^1\}$. (The set S^* captures all possible specifications of S^1 and S^2.)

Step 2: For $(S^1, S^2) \in S^*$ verify GARP for S^1 and S^2 (separately). If OK for some $(S^1, S^2) \in S^*$, then STOP the algorithm—*a collective rationalization of the set S is possible*. If not OK for any $(S^1, S^2) \in S^*$, then STOP the algorithm—*the set S does not meet the sufficient condition for a collective rationalization*.

Again, this algorithm is finite in nature: we maximally have to consider all possible subsets of S, which is exactly of magnitude 2^T for T observations.

To conclude, it is worth stressing that strategies exist that considerably enhance the computational efficiency of the testing algorithms. For example, Cherchye, De Rock, and Vermeulen (2005) showed that one may exclude from the testing exercise observations that are not involved in a (unitary) GARP-violating sequence of observations. In addition, they suggest so-called *mutually independent* subsets of observations for which the tests may be carried out separately. Finally, for each subset of, say, k $(\leq T)$ observations, one can exploit that a collective rationalization is possible for the first l $(\leq k)$ observations only if it is possible for the first $l-1$ observations. Hence, one may successively apply the testing algorithms to larger l (starting from $l = 3$), while each time respecting the feasibility restrictions associated with the (preceding) $l - 1$ case (i.e., regarding possible specifications (D_j^1, D_j^2) for the necessity test and (S^1, S^2) for the sufficiency test). We refer to Cherchye, De Rock, and Vermeulen (2005) for a more detailed discussion on the practicality of the tests, including an illustrative application to real-life data.

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