Beggar Thy Thrifty Neighbour

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The international spillover effects of pensions under population ageing

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April 2006

Abstract

This paper explores the international spillover effects of ageing through capital markets when countries have different pension systems. We use a two-country two-period overlapping-generations model, where the two countries only differ in their pension schemes. Two forms of population ageing are considered, namely an increase in longevity and a fall in fertility. It is shown that in the long run a country using a funded pension system experiences negative spillovers from the fact that the other country uses a PAYG system. The short-run spillovers, however, are opposite to the spillovers in the long run.

JEL codes: F21, H55, J11

Keywords: ageing, pensions, spillovers

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1 Introduction

In the coming decades, differences in the extent and the timing of ageing in the developed world will lead to international spillover effects through the capital market. This has been established in an extensive literature, starting with a seminal paper by Cutler et al. (1990), followed by a number of papers presenting simulation experiments with large multi-country overlapping-generations models (see for example Brooks (2000), Fehr et al. (2003), Börsch-Supan et al. (2004), Domeij and Flodén (2004), and Attanasio and Violante (2005)).

Less attention, however, has been paid to a second possible reason for international spillover effects in case of ageing, i.e., differences in pension systems between countries. Within the EMU, for example, there are large differences in pension arrangements. Some countries, such as the Netherlands, have large funded schemes, while other countries, e.g., Italy and Germany, rely almost completely on pay-as-you-go (PAYG) systems. Together with differences in the institutional set-up of PAYG schemes this causes savings in the various countries to react differently to the ageing of the population, even if the pattern of ageing is identical across countries. In a common capital market, these differences in savings will spill over to other countries.

The aim of this paper is to explore the international spillover effects of a symmetric ageing shock when countries in a common capital market have different pension systems. In other words, we want to shed light on the question how, in case of population ageing, a country with an extensive funded pension scheme (e.g., the Netherlands) is affected by the fact that other EMU countries (such as Italy) rely to a relatively large extent on unfunded pensions, and vice versa.

Some studies (for example Casarico (2001), Groezen (2003), and Jousten and Legros (2002)) investigate what happens with capital flows when two countries that differ in the degree of funding of their pension systems integrate their capital markets. Casarico (2001) finds that the young and the future generations in the PAYG country are better off in the open economy equilibrium. In the funded country, however, welfare can either increase or decrease for the young and future generations. So it is not necessarily true, as one might expect, that both countries gain from the integration of capital markets. In contrast to the above-mentioned papers, we study the consequences of ageing given that countries have an integrated capital market but different pension arrangements. To our knowledge our paper is one of the first taking this approach. One exception is the paper by Börsch-Supan et al.

(2004) who also allow for differences in the generosity of public PAYG pension schemes. In contrast to that paper, we develop a simple model that enables us to derive an analytical solution of the transition path, in order to gain insight into the underlying mechanisms of the results.

We use a two-country two-period overlapping-generations model where one country has a PAYG pension system and the other country has a fully-funded retirement scheme. The countries are identical in all other respects and are hit by an identical demographic shock. Two typical variants of the PAYG scheme are distinguished: a defined-benefit scheme where the working generation bears the burden of an ageing population by paying higher taxes, and a fixed-contribution scheme where the burden of ageing lies with the elderly. Moreover, we consider two forms of population ageing, namely an increase in longevity and a decrease in fertility.

We find that, in general, a country with funded pensions is in the long run adversely affected by the existence of a PAYG scheme in the other country. This means that a country using a funded system is more vulnerable to an ageing shock when it has a common capital market with a country relying on PAYG pensions. In the short run, however, the spillover effects may be opposite: generations born in the country with the funded pension scheme around the time of the demographic shock may gain from the fact that the other country has a PAYG scheme. The reason for this is that the rise in the capital-labour ratio, resulting from the ageing shock, is smaller in case the funded country has integrated capital markets with a country that uses a PAYG pension scheme instead of a funded system. As a result the fall in the interest rate, which especially harms these initial generations, is less.

The rest of the paper is organized as follows. Section 2 presents the benchmark model. In Section 3 we discuss the spillover effects when ageing is modelled as an increase in longevity, first in case of a defined-benefit PAYG scheme and then shortly for the case of fixed contributions. Section 4 deals with some modifications of the model: the effects of a fertility decline (4.1), and we consider a more general type of utility function (4.2). The final section concludes.

2 The model

Following Buiter (1981), we use a two-period overlapping-generations model of an open economy. The world consists of two countries, country P and country F, and the only difference between the two countries is the way the pensions are financed. Country P uses a PAYG system and country F has a fully-funded retirement scheme.

Production

Production per young individual is described by a standard neoclassical constant-returns-to-scale production function, $f(k_t^i)$, where k_t^i stands for the amount of capital per young individual in period t in country i, $i = P,\ F$. Perfect competition among producers results in the usual equilibrium conditions, $r_t^i = f'(k_t^i) - \delta$ and $w_t^i = f(k_t^i) - k_t^i f'(k_t^i)$, where r_t^i is the interest rate, w_t^i denotes the real wage, and δ is the depreciation rate of capital. There is perfect capital mobility between the two countries, but labour is immobile. Since capital can freely move across countries, the interest rates will be equalized, i.e., $r_t^P = r_t^F = r_t,\ \forall t$. As both countries are endowed with the same production technology, we have $k_t^P = k_t^F = k_t$, and consequently $w_t^P = w_t^F = w_t$.

Demographics

Both economies are populated with non-altruistic, identical individuals who live for two periods. So in each period both a young and old generation are alive. The active population, L_t^i, is supposed to increase at the same exogenous rate n_t[1]. The number of active (young) people at time t is then:

$$L_t^i = L_0^i \cdot \prod_{j=0}^{t} (1 + n_j) \tag{1}$$

where L_0^i is the initial population size. As the countries may differ in their initial population size, the model allows for scale differences between the two countries. Define $\frac{L_0^F}{L_0^P} = \nu$, so if L_0^P is normalized to 1, then ν tells us the relative size of L_0^F. A decrease in n_t can be interpreted as a fall in the fertility rate.

We assume that an agent born at t lives throughout old age with probability ε_{t+1}. ε_t can be interpreted as average longevity; when ε_t rises, the expected lifespan of people is longer.

[1]Throughout this paper, both economies are assumed to be dynamically efficient, i.e., $r_t > n_t,\ \forall t$.

Government

The government in country P runs a PAYG pension system, that is, the pension benefits of the elderly (z_t^P) are covered by lump-sum taxes of the young (τ_t^P)[2]. Since, at time t, there are $\varepsilon_t L_{t-1}^P$ old agents and L_t^P young agents, a PAYG system satisfies:

$$z_t^P = \frac{1+n_t}{\varepsilon_t}\,\tau_t^P \tag{2}$$

In country F, the government invests the contributions of the young (τ_t^F) and returns them with interest in the next period in the form of transfers to the then old agents (z_{t+1}^F). As only a fraction of ε_{t+1} of young people born at t survives to period $t+1$, the contributions of those who deceased will fall to surviving contemporaries. In this case, we have:

$$z_{t+1}^F = \frac{1+r_{t+1}}{\varepsilon_{t+1}}\,\tau_t^F \tag{3}$$

Households

Expected lifetime utility of a representative individual born at t is given by the following utility function[3]:

$$E_t U\left(c_t^{y,i}, c_{t+1}^{o,i}\right) = \log\left(c_t^{y,i}\right) + \frac{1}{1+\rho}\,\varepsilon_{t+1}\log\left(c_{t+1}^{o,i}\right) \tag{4}$$

where $\rho > 0$ stands for the (constant) pure rate of time preference of an individual, $c_t^{y,i}$ is consumption when young, and $c_{t+1}^{o,i}$ is consumption in the second period of life.

Young agents inelastically supply one unit of labour. We assume perfect annuity markets, which implies that the assets of those who deceased are distributed among the people who survived. The total return on savings is therefore $\frac{1+r_{t+1}}{\varepsilon_{t+1}}$. The consolidated lifetime budget constraint is:

$$c_t^{y,i} + \frac{\varepsilon_{t+1}\, c_{t+1}^{o,i}}{1+r_{t+1}} = w_t - \tau_t^i + \frac{\varepsilon_{t+1}\, z_{t+1}^i}{1+r_{t+1}} \tag{5}$$

Maximizing lifetime utility with respect to the lifetime budget constraint gives the following

[2] It would be more realistic to assume proportional taxes, but this only complicates the analysis and does not qualitatively change our results.

[3] To test the robustness of the effects of a demographic shock on utility, we also considered the more general case where preferences are represented by a CES utility function. These results are discussed in Section 4.2.

expressions for individual optimal savings in both countries:

$$s_t^P = \frac{\varepsilon_{t+1}}{1+\rho+\varepsilon_{t+1}} \left[w_t - \tau_t^P \right] - \frac{1+\rho}{1+\rho+\varepsilon_{t+1}} \frac{\varepsilon_{t+1} \; z_{t+1}^P}{1+r_{t+1}} \tag{6}$$

$$s_t^F + \tau_t^F = \frac{\varepsilon_{t+1}}{1+\rho+\varepsilon_{t+1}} \, w_t \tag{7}$$

Note that optimal savings in country F do not depend on the interest rate. The reason is that, given a logarithmic utility function, the intertemporal substitution elasticity is equal to one[4]. For the same reason, optimal savings in country P only react to the interest rate because it changes the net present value of the pension benefit.

Equilibrium international capital market

Individuals invest their savings either in the home country or abroad. Their portfolios will be composed such that interest rates are equalized. Equilibrium in the international capital market is given by:

$$s_t^P + \nu \left(s_t^F + \tau_t^F \right) = (1+\nu)(1+n_{t+1})k_{t+1} \tag{8}$$

From equations (6) and (7), it can be seen that country F has higher savings than country P, implying that country F exports capital abroad.

3 Increase in longevity

One of the causes of population ageing in the industrialized world is that people live longer. Given an unchanged retirement age, this increase in longevity implies that pension income, be it based on PAYG or funding, has to finance a larger part of lifetime consumption. As a result, lifetime utility will decrease under both pension systems.

In this section, we analyze the international spillover effects of ageing when longevity increases permanently at $t = 0$. The demographic shock is *unexpected*, i.e., people do not adjust their behaviour in the period before the shock. Moreover, the size and the timing of the ageing shock is the same in both countries. As a result, ageing will induce capital flows between the two countries only because the pension systems differ.

[4]This is the reason why we also considered the case of CES utility, where the intertemporal elasticity of substitution is not necessarily equal to one. Savings *do* depend on the interest rate in that case.

We employ the method of comparative dynamics, adopted from Judd (1982), to calculate the effect of the longevity shock analytically. The probability of reaching the second period of life at time t is given by:

$$\varepsilon_t = \varepsilon + \pi h_t \tag{9}$$

where ε denotes the initial steady-state value[5], h_t describes the time pattern of a perturbation of this steady-state value and π reflects the magnitude of this perturbation. Ageing is reflected by a positive value of h_t[6]. To focus solely on the effects of an increase in longevity we assume that there is no population growth, i.e., $n_t \equiv 0\ \forall t$. The effects of an ageing shock can be traced by linearising the capital-accumulation equation (8) with respect to π around the initial steady state. The resulting first-order difference equation for k_t describes the capital-labour ratio changes over time and the determining factors. Given the change in the capital-labour ratio we can derive the changes in all other variables.

First, we explain the effects of an increase in longevity when the PAYG system is characterized by defined benefits. After that we shortly describe what changes when the PAYG system is characterized by fixed contributions instead.

3.1 Defined benefits

This section analyzes the international spillover effects of population ageing in case the PAYG scheme is characterized by defined benefits. In terms of the model this means that the pension benefit in country P is fixed at z^P ($z_t^P = z^P\ \forall t$). This implies that in response to a longer expected lifespan taxes have to increase to keep the PAYG scheme balanced. So the burden of ageing is entirely borne by the working population.

[5]Throughout the paper, we omit time subscripts to denote the (initial) steady-state value of the respective variable.

[6]Because we assume that longevity increases *permanently*, this means that $h_0 = h_1 = \ldots = h_\infty > 0$. However, as people do not anticipate the rise in longevity we have that $E(h_0) = 0$.

The change in the capital-labour ratio

Using the method described above we obtain the following first-order difference equation for the evolution of the capital-labour ratio[7]:

$$
\begin{aligned}
\frac{\partial k_{t+1}^{FP}}{\partial \pi} = & -\frac{\varepsilon f''(k)k}{\Psi}\frac{\partial k_t^{FP}}{\partial \pi} - \overbrace{\frac{z^P \varepsilon}{(1+\nu)\Psi}}^{3} h_t \\
& + \left[\overbrace{\frac{(1+\rho)(w-\frac{\tau^P}{1+\nu})}{\Psi(1+\rho+\varepsilon)}}^{1} - \overbrace{\frac{(1+\rho)^2 z^P}{(1+\nu)\Psi(1+\rho+\varepsilon)(1+r)}}^{2} \right] h_{t+1}
\end{aligned} \tag{10}
$$

where $\Psi \equiv (1+\rho+\varepsilon) - \frac{(1+\rho)\varepsilon z^P f''(k)}{(1+\nu)(1+r)^2} > 0$.
Equation (10) shows the change in the capital-labour ratio after an increase in longevity when the two economies have different pension schemes. To analyse the international spillover effects we derive the same kind of equations for the situation where the two economies use the same pension system. The first-order difference equation for the case where both countries have a PAYG scheme is given by:

$$
\frac{\partial k_{t+1}^{PP}}{\partial \pi} = -\frac{\varepsilon f''(k)k}{\Psi^P}\frac{\partial k_t^{PP}}{\partial \pi} - \overbrace{\frac{z^P \varepsilon}{\Psi^P}}^{3} h_t + \left[\overbrace{\frac{(1+\rho)(w-\tau^P)}{\Psi^P(1+\rho+\varepsilon)}}^{1} - \overbrace{\frac{(1+\rho)^2 z^P}{\Psi^P(1+\rho+\varepsilon)(1+r)}}^{2} \right] h_{t+1} \tag{11}
$$

while the expression for the change in k_{t+1} in case both economies use a funded system is given by:

$$
\frac{\partial k_{t+1}^{FF}}{\partial \pi} = -\frac{\varepsilon f''(k)k}{\Psi^F}\frac{\partial k_t^{FF}}{\partial \pi} + \overbrace{\frac{(1+\rho)w}{\Psi^F(1+\rho+\varepsilon)}}^{1} h_{t+1} \tag{12}
$$

with $\Psi^P \equiv (1+\rho+\varepsilon) - \frac{(1+\rho)\varepsilon z^P f''(k)}{(1+r)^2} > 0$ and $\Psi^F \equiv (1+\rho+\varepsilon) > 0$.
By comparing the capital-labour ratio changes in case the two countries have the same pension system (equations (11) and (12)) with the change in the capital-labour ratio when the two countries have different pension schemes (equation (10)), we derive the pure spillover effects of pensions and ageing in a common capital market[8].

[7] In Appendix A we show the derivation of this expression.
[8] To exclude the effects of integration it is assumed that the initial steady state is the same in all cases.

As the increase in longevity $t = 0$ is not anticipated, savings at $t = -1$ do not adjust and, as a result, the capital stock per worker does not change at the time of the shock, that is, $\frac{\partial k_0}{\partial \pi} = 0$. At $t = 0$, there are several effects that influence optimal savings in both countries in different directions. First of all, people have more incentives to save because the chance of reaching the second period of life is higher (this effect is indicated by a 1 in the first-order difference equations). In country P, however, the existence of PAYG pension benefits in the second period of life has a depressing effect on savings when longevity goes up (this effect is indicated by a 2). Moreover, with a defined-benefit PAYG pension scheme, savings decrease as contributions have to go up to keep the PAYG system balanced (this is the so-called *dependency-ratio effect*, indicated by a 3). So the effect on total savings, and the capital-labour ratio, is ambiguous. From equation (10) it follows that, given that $h_t = h_{t+1}$, the capital-labour ratio rises after a permanent increase in longevity if:

$$\frac{z^P}{1+\nu} < \frac{(1+\rho)(1+r)\,w}{\varepsilon(1+\rho)(1+r) + (1+\rho)^2 + \varepsilon(1+\rho+\varepsilon)(1+r)} \tag{13}$$

We assume that this condition is fulfilled[9]. The increase in the common capital-labour ratio at $t = 1$ leads to higher wages, which engenders higher savings in both countries (this is the first term in the three equations). Due to these higher savings the capital-labour ratio continues to rise.

Savings unambiguously rise more in the funded country than in the PAYG country, as effects 2 and 3[10] do not appear in equation (12). When the funded country has a common capital market with a PAYG country, part of its extra savings flow to country P. To illustrate the mechanics of the model we also show some numerical simulation experiments[11]. The change in the capital-labour ratio for the three different cases can be seen in Figure

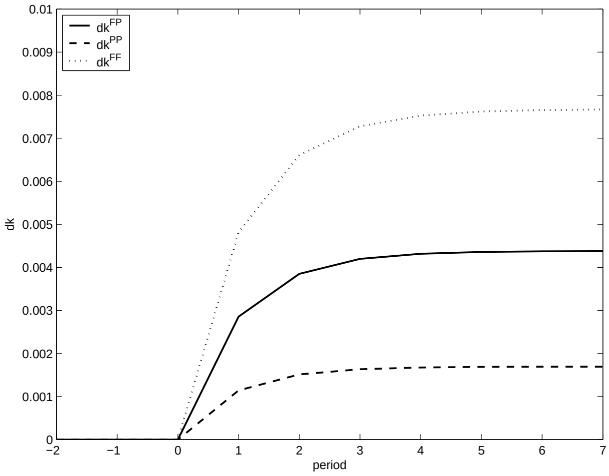
[9] As shown in Appendix B.1 this condition holds for realistic parameter values. The appendix also shows, however, that in case both countries have a PAYG scheme the contribution rate does not have to be unrealistically high for the capital-labour ratio to *decrease* after a longevity shock. This would not change the international spillover effects qualitatively, however.

[10] Assuming proportional taxes would result in a smaller dependency ratio effect, because the rise in taxes can be less as a result of the increase in wages. This in turn would lead to smaller spillover effects, but would not change our results qualitatively.

[11] The graphs are based on simulations with $f(k_t) = k_t^{0.3}$, $\nu = 1$, $\frac{\tau^P}{w} = 0.2$, $\varepsilon = 0.94$ and $h_t = 0.05$. Capital depreciates at 5% per year and assuming that one period is 30 years this means that $\delta = 1 - (0.95)^{30} = 0.7854$. Agents are relatively patient with a time preference rate of 1% per year, so that $\rho = (1.01)^{30} - 1 = 0.3478$. We also derived numerically the non-linear transition path and compared the numerical results with those found with the method of comparative dynamics. The relative error of the linearised path was one percent at most.

1[12].

Figure 1: Change in k_t



The change in utility

To infer whether a country gains or loses from being in an integrated capital market with a country that has another pension system, we compare the utility effects of ageing in the different cases. In Appendix C we derive the long-run change of utility in both countries, for the PAYG country we get:

$$\frac{\partial U_\infty^P}{\partial \varepsilon_\infty} = \frac{f''(k)}{c^{y,P}(1+r)}\left[s^P - k(1+r)\right]\frac{\partial k_\infty}{\partial \varepsilon_\infty} - \frac{1}{c^{y,P}}\left(z^P + \frac{s^P}{\varepsilon}\right) \tag{14}$$

The last term indicates that for a given production capacity, an increase in longevity leads to a lower utility as individuals have to share total life-time consumption with more people[13]. As explained in the appendix the whole term in front of $\frac{\partial k_\infty}{\partial \varepsilon_\infty}$ is positive. This means that the rise in the capital-labour ratio after an increase in longevity reduces the

[12] If the longevity shock was *expected* instead, the capital-labour ratio would already increase at $t = 0$ as a result of effects 1 and 2. And then from $t = 0$ we would have the dependency-ratio effect, which negatively affects the capital-labour ratio in the next period. This results in a smaller increase in k_t at $t = 1$, compared to $t = 0$, in case both countries use a PAYG system. The long-run effects do not change, however.

[13] In order to compare lifetime utility before and after ageing properly, we take the value of ε_t constant at unity. So we do not take into account that a higher life expectancy is actually something that is nice for people. If this is taken into account the overall utility effects of an increase in longevity are still negative, however. An increase in longevity only results in a higher utility if people can work longer. We leave this for future research.

direct negative utility effects. However, this indirect positive effect on utility is a second-order effect and, therefore, not large enough to compensate for the negative utility effects, as can also be seen in Figure 2.

The rise in the capital-labour ratio is larger in case the PAYG country is in a common capital market with a funded country instead of with a PAYG country. As a result, in the long run the PAYG country gains from having a common capital market with a country that has a fully-funded pension scheme. Notice, however, that the spillovers are exactly opposite for the generation born at the time of the ageing shock ($t = 0$). The reason for this is that the fall in the interest rate resulting from the rise in the capital-labour ratio, which especially harms this initial generation, is less in case both countries use a PAYG system. As these initial generations do not enjoy the gains that result from a larger increase in wages when the economies have different pension schemes, the negative utility effects of the lower interest rate are smaller in case the PAYG country does *not* have a common capital market with a country using a funded pension system.

The change in long-run utility in the funded country is given by:

$$\frac{\partial U_\infty^F}{\partial \varepsilon_\infty} = \frac{f''(k)}{c^{y,F}(1+r)}\left[(s^F+\tau^F) - k(1+r)\right]\frac{\partial k_\infty}{\partial \varepsilon_\infty} - \frac{1}{c^{y,F}}\frac{(s^F+\tau^F)}{\varepsilon} \tag{15}$$

Again there is the direct negative effect on utility because consumption has to be shared with more. For the funded country, however, it is not necessarily true that a higher capital-labour ratio results in a higher utility. As explained in the appendix, there are two opposing mechanisms, that can make the whole expression in front of $\frac{\partial k_\infty}{\partial \varepsilon_\infty}$ either negative or positive. For realistic values of the different parameters it is positive however. Because the capital-labour ratio increases less in case the funded country has a common capital market with a PAYG country rather than with a funded-pension country, the fall in utility in country F is larger in the former case. So the funded country experiences negative spillover effects from the PAYG country in the long run, as shown in Figure 3. In other words, as a funded pension system is better suited to deal with ageing than a PAYG scheme, individuals in the PAYG country gain and those living in the funded country lose from being in a common capital market.

As was the case for the PAYG country, the short-run spillovers are opposite to the spillovers in the long run. The reasoning is the same as before: the initial generations only incur the losses that result from a falling interest rate, they do not have the gains from the higher wages.

Figure 2: Change in U_t^P

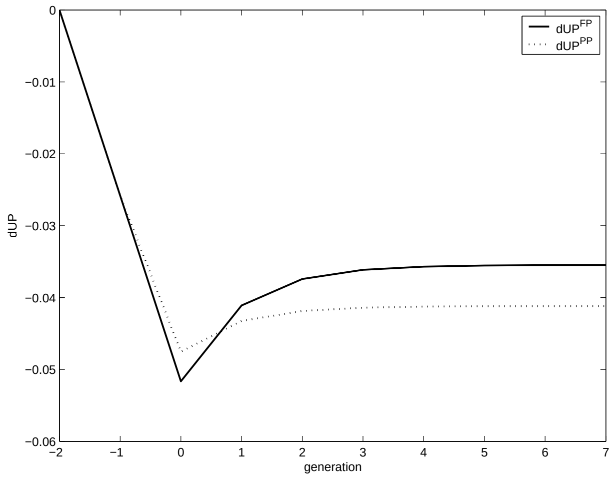
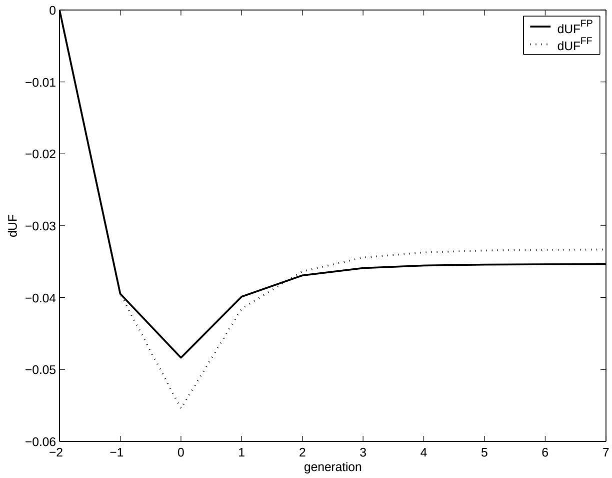


Figure 3: Change in U_t^F



In these simulation graphs we assumed that the two economies have the same size, that is, $\nu = 1$. However, as most EMU countries mainly use PAYG pension schemes, a country like the Netherlands, relying to a relatively large extent on funded pensions, can be considered as relatively small. In case the funded country is smaller than the PAYG country, that is, when $\nu < 1$, the negative spillover effects for the funded country become larger.

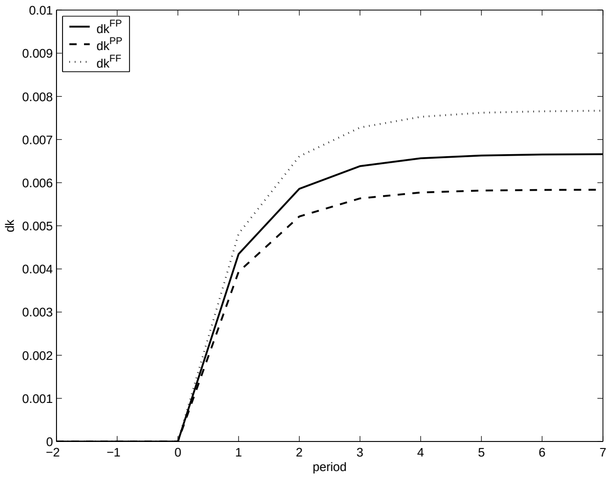
3.2 Fixed contributions

If the PAYG pension system is characterized by fixed contributions, the change in the capital-labour ratio in case of different pension schemes, is described by:

$$\frac{\partial k_{t+1}^{FP}}{\partial \pi} = -\frac{\varepsilon f''(k)k}{\Psi}\frac{\partial k_t^{FP}}{\partial \pi} + \left[\overbrace{\frac{(1+\rho)(w-\frac{\tau^P}{1+\nu})}{\Psi(1+\rho+\varepsilon)}}^{1} - \overbrace{\frac{(1+\rho)^2 z^P}{(1+\nu)\Psi(1+\rho+\varepsilon)(1+r)}}^{2} + \overbrace{\frac{(1+\rho)\,\tau^P}{(1+\nu)\Psi(1+r)\varepsilon}}^{3}\right] h_{t+1} \tag{16}$$

The only difference with equation (10) is the dependency-ratio effect (indicated by a 3 in both equations). In case of defined benefits, the higher dependency ratio led to less savings because contributions had to rise to keep the PAYG system balanced. With a fixed PAYG tax, however, pension benefits are reduced due to a higher dependency-ratio, which increases savings. This implies that the capital-labour ratio increases more in case the PAYG scheme is characterized by fixed contributions, resulting in a smaller decrease in utility after a longevity shock. However, as shown formally in Appendix D, savings in country F still increase more than in country P, so that the rise in the capital-labour ratio is smaller when country F has integrated capital markets with a country that uses a PAYG pension system. This can also be seen in Figure 4. In the long run, this results in positive spillover effects for the PAYG country and negative spillovers for the funded country[14]. It should be noticed, however, that the spillovers are smaller than in the defined-benefits case.

Figure 4: Change in k_t



Our main findings can be summarized as follows:

Result 1 *In case ageing is characterized by an unexpected increase in longevity:*

1. *Utility falls in both countries.*
2. *The capital-labour ratio increases for realistic parameter values.*
3. *Savings in the funded country unambiguously rise more than in the PAYG country, which leads to capital flows from the funded country to the PAYG country.*
4. *In the long run, the funded country experiences negative spillover effects from the PAYG scheme in the other country.*

[14] Assuming an *expected* longevity shock does not change these results qualitatively.

5. *The short-run spillovers are opposite to the spillovers in the long run.*
6. *The spillovers are smaller in case the PAYG pension system is characterized by fixed contributions compared to a defined-benefit PAYG scheme.*

4 Modifications

In this section the analysis presented above is modified in two ways to investigate the robustness of our results. First, we study the international spillover effects in case population ageing is caused by a lower fertility rate. Second, we consider a more general type of utility function. More precisely, we analyze whether the spillover effects change when preferences are represented by a CES utility function instead of a logarithmic utility function.

4.1 Decrease in fertility

The ageing of the population in the industrialized world is not only caused by the fact that people live longer. Another important reason is that women give birth to a smaller number of children, that is, the fertility rate is declining. In this section, we analyze the spillover effects of pensions and population ageing in case of a permanent decrease in fertility at $t = 0$. It is assumed that the demographic shock is anticipated before the shock actually hits the two economies[15]. This means that individuals born at $t = -1$ already take into account the economic consequences of the shock when they make their optimizing decisions. As before, we employ the method of comparative dynamics. Let the fertility rate at time t be given by $n_t = n + \pi g_t$. Ageing is reflected by a negative value of g_t[16]. To isolate the effects of a fertility decline, it is assumed that agents live throughout their old-age with certainty, i.e., $\varepsilon_t \equiv 1 \; \forall t$. And again we distinguish between a defined-benefit and a fixed-contribution PAYG scheme.

The main difference with a longevity shock is that a decline in the rate of population growth leads to a shrinking labour force, so that less investment is needed to keep the capital-labour ratio constant. This is the so-called *capital-thickening effect*, which has a direct positive effect on the capital-labour ratio. As a result of this capital-thickening effect, a fall in fertility can actually have positive utility effects for individuals.

[15] As children are actually born long before they enter the labour market, it is not unreasonable to assume that people foresee the decline in the workforce. Assuming an *unexpected* fertility shock would not change our main results.

[16] A *permanent* fall in fertility implies that $g_0 = g_1 = \ldots = g_\infty < 0$.

Defined benefits

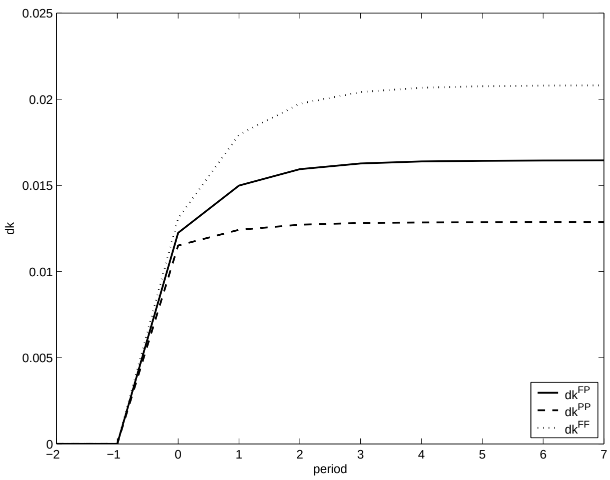
The first-order difference equation for the change in the capital-labour ratio is:

$$\frac{\partial k_{t+1}^{FP}}{\partial \pi} = -\frac{f''(k)\,k}{\Delta}\frac{\partial k_t^{FP}}{\partial \pi} + \overbrace{\frac{z^P}{(1+\nu)\Delta}}^{3} g_t - \overbrace{\frac{(2+\rho)k}{\Delta}}^{4} g_{t+1} \qquad (17)$$

with $\Delta \equiv (2+\rho) - \frac{(1+\rho)z^P f''(k)}{(1+\nu)(1+r)^2} > 0$.

The just described capital-thickening effect is indicated by a 4 in equation (17). As before the fall in fertility has a *dependency-ratio effect* in the PAYG country (effect 3), which affects savings negatively in case of a defined-benefit system[17]. Because the country with the funded pension system does not have the negative dependency-ratio effect, the capital-labour ratio increases more when this country is not integrated with country P. Figure 5 confirms these results[18].

Figure 5: Change in k_t



In the long run, individuals in country F experience negative spillover effects from the PAYG scheme in the other country (Figure 7), as this PAYG scheme leads to a smaller

[17] As in section 3.1 we can derive a condition that has to hold for the capital-labour ratio to rise after an ageing shock. From equation (17) it follows that, given $g_t = g_{t+1}$, k_t rises after a permanent decline in the fertility rate if $\frac{z^P}{1+\nu} < (2+\rho)k$. As shown in Appendix B.2 this condition holds for a large range of parameter values.

[18] In these simulation graphs it is assumed that the population size is constant in the initial steady state ($n = 0$). Assuming a positive population growth rate in the original steady state does not qualitatively change our results. We take $g_t = -0.1$.

increase in the capital-labour ratio after a fertility shock. People in the PAYG country, however, gain from having integrated capital markets with a country that uses a funded pension system (Figure 6). So the international spillover effects do not change qualitatively compared to the effects of the longevity shock. Notice that lifetime utility is higher in the funded country in the long run, which is due to the capital-thickening effect, while in the PAYG country the long-run utility effects are negative, as the dependency-ratio effect dominates the capital-thickening effect. In case the PAYG system is not too large, that is, when the dependency-ratio effect is not too large, people in the country P also experience positive utility effects of a rise in fertility in the long run. From simulations it follows that this is the case if $\tau^P < 0.13w$.

Figure 6: Change in U_t^P

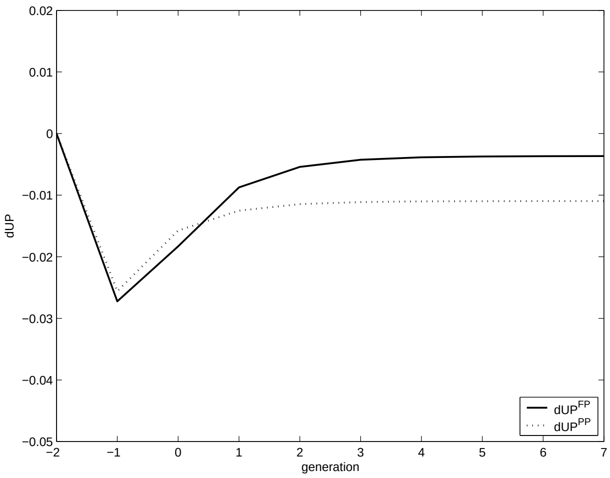
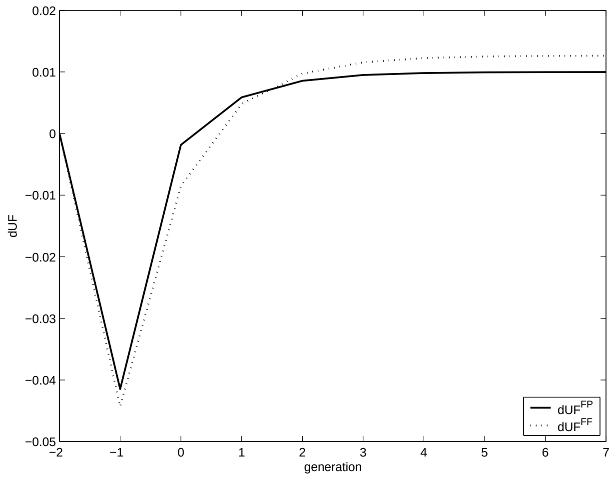


Figure 7: Change in U_t^F



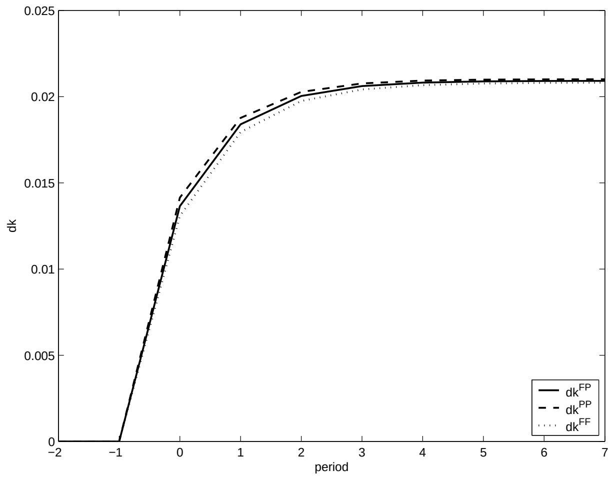
Fixed contributions

The first-order difference equation for $\frac{\partial k_{t+1}}{\partial \pi}$ in case of fixed contributions is:

$$\frac{\partial k_{t+1}^{FP}}{\partial \pi} = -\frac{f''(k)\,k}{\Delta}\,\frac{\partial k_t^{FP}}{\partial \pi} - \left[\overbrace{\frac{(2+\rho)k}{\Delta}}^{4} + \overbrace{\frac{(1+\rho)\,\tau^P}{(1+\nu)\Delta\,(1+r)}}^{3} \right] g_{t+1} \tag{18}$$

As in Section 3.2, the dependency-ratio effect (effect 3) works in the opposite direction as the defined-benefit case. As a result, this effect intensifies the capital-thickening effect, so that individuals in the PAYG country now also experience positive long-run utility effects of a fertility decline. Moreover, because the funded country has no dependency-ratio effect, the capital-labour ratio actually rises less in this country. This means that the spillover effects turn around, that is, citizens in country P suffer and those living in country F gain from having an integrated capital market with a country that uses another pension scheme. Figure 8, however, shows that the difference between the change in k_t in the two countries is very small, especially in the long run. This implies that these opposite spillovers are also very small. Moreover, the next section shows that these spillovers are not robust for changes in the intertemporal elasticity of substitution.

Figure 8: Change in k_t



4.2 CES utility function

The results presented until now are based on a logarithmic utility function, which means that the intertemporal elasticity of substitution (σ) is equal to one. In the literature, values taken for σ range from $\frac{1}{4}$ to $\frac{1}{2}$ [19], which implies that our assumption of $\sigma = 1$ is probably not very realistic. Therefore, we also simulated the model for smaller values of σ[20]. If $\sigma < 1$ the income effect is larger than the substitution effect, and a falling interest rate leads to higher savings. This positive effect on savings is largest in the funded country, because it has the highest savings in the initial steady state. This means that, if the spillover effects for the funded country are already negative for $\sigma = 1$, they will only be reinforced if $\sigma < 1$. This gives the following result:

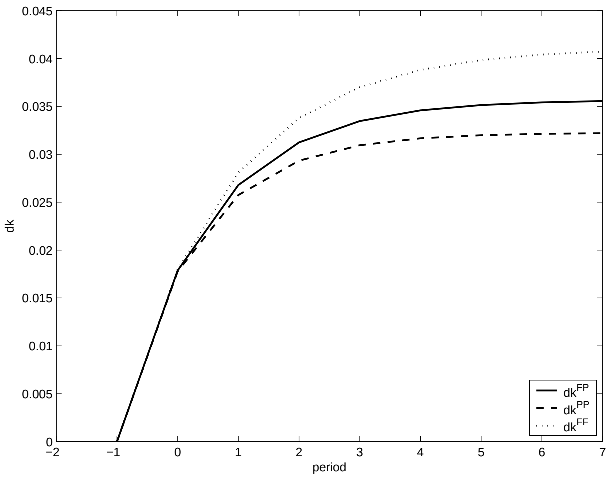
[19] Auerbach and Kotlikoff (1987) for example assume a value of $\frac{1}{4}$.

[20] For expositional clarity, we do not show the analytical expressions for the case of a CES utility function.

Result 2 *The findings stated in Result 1 do not change if the intertemporal elasticity of substitution is smaller than one.*

In the previous subsection it was shown that in case of a fertility shock *and* a fixed-contributions PAYG scheme, savings in country P increased more than in country F when $\sigma = 1$, resulting in small *positive* spillover effects for the funded country. For a small enough value of σ, however, the interest-rate effect of $\sigma < 1$ dominates the dependency-ratio effect, so that savings in country F increase more than in country P. This is illustrated below, where the change in k_t for $\sigma = \frac{1}{4}$ is shown.

Figure 9: Change in k_t



This means that the funded country experiences negative spillover effects of the PAYG scheme as in the other cases. So we can conclude that for smaller and probably more realistic values for the intertemporal elasticity of substitution the long-run spillovers for the funded country are negative in all considered cases.

The main results of a fertility decline are:

Result 3 *In case ageing is characterized by an expected decrease in fertility:*

1. *Utility rises in the funded country in the long run and may increase in the PAYG country, due to the capital-thickening effect.*

2. *For realistic values of the intertemporal elasticity of substitution the spillovers do not differ qualitatively from those in case of an increase in longevity: the PAYG scheme*

has negative spillovers in the long term for the funded country, while the short-run spillovers are positive.

5 Concluding remarks

In the coming decades, most developed countries will have an ageing population. Ageing arises as a result of changes in two determinants of demographic structure: the decline in fertility rates and the ongoing increase in longevity. In this paper we show that an increase in longevity has negative utility effects in the long run as individuals have to share total life-time consumption with more people. A fall in fertility, on the other hand, has positive utility effects in the long run in a country using a fully-funded pension scheme induced by the so-called capital-thickening effect. Countries relying on PAYG pensions, however, also have the so-called dependency-ratio effect, which weakens the capital-thickening effect in case of a defined-benefit scheme, so that the long-run utility effects of a fertility decline are negative. In case of a fixed-contributions PAYG scheme the dependency-ratio effect intensifies the capital-thickening effect and the long-run utility effects of a decrease in fertility are also positive for individuals living in a PAYG country.

European countries differ in the degree of funding of their pension systems. On the one hand there are countries like the UK and the Netherlands, that have an extensive funded pension system. Most other countries, on the other hand, mainly rely on unfunded pensions. These differences in pension schemes engender different saving responses when the population is ageing, which leads to capital flows. This paper focuses on the question how, in case of population ageing, countries are affected by the fact that other countries use other pension systems. From the literature it is known that a PAYG system is less suitable to cope with an ageing population than a funded pension scheme, that is, using a PAYG scheme makes a country more vulnerable to an ageing shock. We find that in the long run a PAYG country gains from having a common capital market with a country relying on funded pensions, the main reason being that in the funded country savings increase more in response to ageing. A country using a funded retirement scheme, on the other hand, experiences long-run negative spillover effects from the PAYG system in the other country. The short-run spillovers, however, are opposite to the spillovers in the long run. In other words, the initial generations in the funded country gain from the fact that the other country uses a PAYG scheme, whereas these initial generations in the PAYG country are negatively affected by the funded system in the other country.

Extensions of the analysis, that we leave for future research, would be to allow for taxes on wage income, endogenous labour supply, and the fact that an increase in longevity can also lengthen the working period of people. Moreover, this paper considers *balanced* PAYG pension systems, that is, in response to population ageing contributions or pension benefits adjust in such a way so that the PAYG scheme does not run a deficit. Governments in funded countries, however, are mainly concerned about the negative effects of *unbalanced* PAYG schemes. In other words, they worry how their country will be affected when countries with PAYG schemes use government debt to cope with the costs of ageing. In the future, we also want to extend the model presented in this paper to allow for government debt. Moreover, by including monetary policy, it is also possible to analyse the spillover effects in case governments put pressure on the central bank to accommodate, so that their debt burden is reduced. This paper, however, already shows that countries with funded pensions are mostly negatively affected by the fact that other countries have PAYG pension schemes, even when the government of the PAYG country keeps its pension system balanced.

A Derivation of the first-order difference equation of the capital-labour ratio

In this appendix we derive the first-order difference equation for the evolution of the capital-labour ratio given in equation (10). Linearising the capital-accumulation equation (8) with respect to π around the initial steady state gives:

$$\frac{\partial s_t^P}{\partial \pi} + \nu \frac{\partial (s_t^F + \tau_t^F)}{\partial \pi} = (1+\nu)\frac{\partial k_{t+1}}{\partial \pi} \tag{19}$$

Then we derive expressions for $\frac{\partial s_t^P}{\partial \pi}$ and $\frac{\partial (s_t^F+\tau_t^F)}{\partial \pi}$, using equations (6) and (7):

$$\begin{aligned}\frac{\partial s_t^P}{\partial \pi} &= \frac{(1+\rho)(w-\tau^P)}{(1+\rho+\varepsilon)^2}\frac{\partial \varepsilon_{t+1}}{\partial \pi} + \frac{\varepsilon}{1+\rho+\varepsilon}\left[\frac{\partial w_t}{\partial \pi} - \frac{\tau_t^P}{\partial \pi}\right] \\ &\quad + \frac{(1+\rho)\varepsilon z^P}{(1+\rho+\varepsilon)^2(1+r)}\frac{\partial \varepsilon_{t+1}}{\partial \pi} - \frac{(1+\rho)z^P}{(1+\rho+\varepsilon)(1+r)}\frac{\partial \varepsilon_{t+1}}{\partial \pi} \\ &\quad + \frac{(1+\rho)\varepsilon z^P}{(1+\rho+\varepsilon)(1+r)^2}\frac{\partial r_{t+1}}{\partial \pi}\end{aligned} \tag{20}$$

$$\frac{\partial (s_t^F+\tau_t^F)}{\partial \pi} = \frac{(1+\rho)w}{(1+\rho+\varepsilon)^2}\frac{\partial \varepsilon_{t+1}}{\partial \pi} + \frac{\varepsilon}{1+\rho+\varepsilon}\frac{\partial w_t}{\partial \pi} \tag{21}$$

$\frac{\partial w_t}{\partial \pi}$ and $\frac{\partial r_{t+1}}{\partial \pi}$ are given by:

$$\frac{\partial w_t}{\partial \pi} = -kf''(k)\frac{\partial k_t}{\partial \pi} \tag{22}$$

$$\frac{\partial r_{t+1}}{\partial \pi} = f''(k)\frac{\partial k_{t+1}}{\partial \pi} \tag{23}$$

Combining equations (19) - (23) and simplifying gives:

$$\begin{aligned}\frac{\partial k_{t+1}}{\partial \pi} &= -\frac{\varepsilon f''(k)k}{\Psi}\frac{\partial k_t}{\partial \pi} - \frac{\varepsilon}{(1+\nu)\Psi}\frac{\partial \tau_t^P}{\partial \pi} \\ &\quad + \left[\frac{(1+\rho)(w-\frac{\tau^P}{1+\nu})}{\Psi(1+\rho+\varepsilon)} - \frac{(1+\rho)^2 z^P}{(1+\nu)\Psi(1+\rho+\varepsilon)(1+r)}\right]\frac{\partial \varepsilon_{t+1}}{\partial \pi}\end{aligned} \tag{24}$$

with $\Psi \equiv (1+\rho+\varepsilon) - \frac{(1+\rho)\varepsilon z^P f''(k)}{(1+\nu)(1+r)^2}$.

Using equations (2) and (9) we know that:

$$\frac{\partial \tau_t^P}{\partial \pi} = z^P \, \frac{\partial \varepsilon_t}{\partial \pi} \tag{25}$$

$$\frac{\partial \varepsilon_{t+1}}{\partial \pi} = h_{t+1} \tag{26}$$

Substituting these last two expressions into equation (24) we obtain equation (10).

B Conditions for a long-run increase in k_t

B.1 Longevity shock

As explained in Section 3.1 the capital-labour ratio rises after a permanent increase in longevity as long as:

$$\frac{z^P}{1+\nu} < \frac{(1+\rho)(1+r)\,w}{\varepsilon(1+\rho)(1+r)+(1+\rho)^2+\varepsilon(1+\rho+\varepsilon)(1+r)} \tag{27}$$

In the same way we can obtain a condition for the case where the PAYG country does not have a common capital market with a country that uses a funded system:

$$z^P < \frac{(1+\rho)(1+r)\,w}{\varepsilon(1+\rho)(1+r)+(1+\rho)^2+\varepsilon(1+\rho+\varepsilon)(1+r)} \tag{28}$$

The two graphs below show how the sign of these two conditions changes for different parameter values[21]. For positive values on the vertical axis the above conditions hold, and an increase in longevity leads to an increase in the capital-labour ratio. The horizontal axis shows the relative size of the lump-sum PAYG tax compared to the wage.

Figure 10: Condition (27)

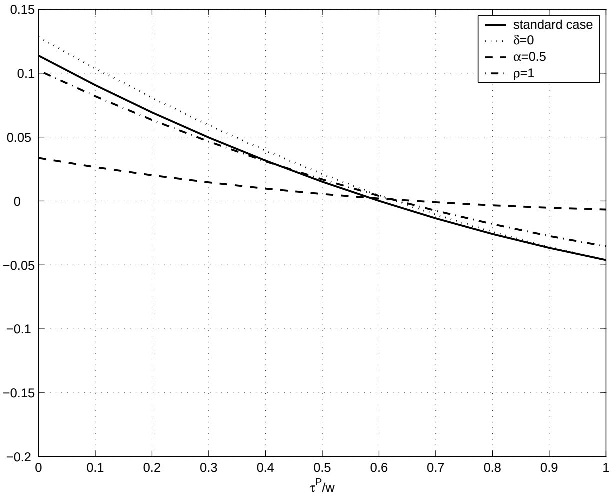
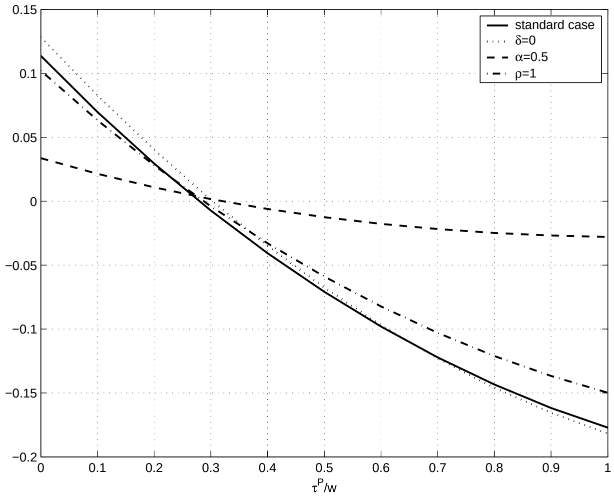


Figure 11: Condition (28)



As can be seen from Figure 10, condition (27) holds in the standard case as long as

[21] The standard case is characterized by $\alpha = 0.3$, $\varepsilon = 0.94$, $\nu = 1$, $\delta = 0.7854$, and $\rho = 0.3478$. See also footnote 11 in the main text.

$\tau^P < 0.6w$. For other parameter values the PAYG system can be even larger. Therefore we can say that for realistic parameter values the capital-labour ratio increases after a rise in longevity when one country has a PAYG scheme and the other country uses a funded system.

In case both countries use a PAYG system (see Figure 11), the capital-labour ratio only rises after a permanent increase in longevity as long as $\tau^P < 0.28w$. This is not an unrealistically high value for the contribution rate of the PAYG system, especially when expenditures on medical care are included, implying that the capital-labour ratio may actually fall after a rise in longevity when both countries use a PAYG scheme. This is the point Fehr et al. (2003) make. They argue that the increase in taxes needed to finance the benefits is so large that the capital-labour ratio falls after an ageing shock. Most other studies that use multi-country general equilibrium models, however, find that ageing increases the capital stock per worker, see Brooks (2000), Miles (2001), McMorrow and Röger (2003), Börsch-Supan et al. (2004), Domeij and Flodén (2004), and Attanasio and Violante (2005). It appears, however, that the results of Fehr et al. (2003) are mainly driven by the fact that the effective labour supply is rising due to labour-augmenting technical progress, which more than offsets the reduction in the labour force when the economy is ageing. But it is not obvious at all whether technical progress will be able to offset the negative effects on the supply of labour. Therefore, we follow the other studies and assume that an increase in longevity results in a higher capital-labour ratio if both countries rely on PAYG pensions when we present the simulation graphs in the main text[22]. The spillovers do not change qualitatively when k_t^{PP} falls instead, the reason being that the spillovers depend on the relative change of k_t^{FF} to k_t^{PP}. As the dependency-ratio effect is absent when both countries use funded pensions, the capital-labour ratio unambiguously increases more in that case, that is, we always have that $dk_t^{FF} > dk_t^{PP}$.

B.2 Fertility shock

As explained in footnote 17 the capital-labour ratio rises after a permanent decline in fertility if:

$$\frac{z^P}{1+\nu} < (2+\rho)k \tag{29}$$

[22]The simulation graphs are based on the assumption that $\tau^P = 0.2w$.

while this condition in case both countries have a PAYG pension scheme is equal to:

$$z^P < (2+\rho)k \tag{30}$$

As before we produce graphs that show how the sign of these two conditions changes for different parameter values:

Figure 12: Condition (29)

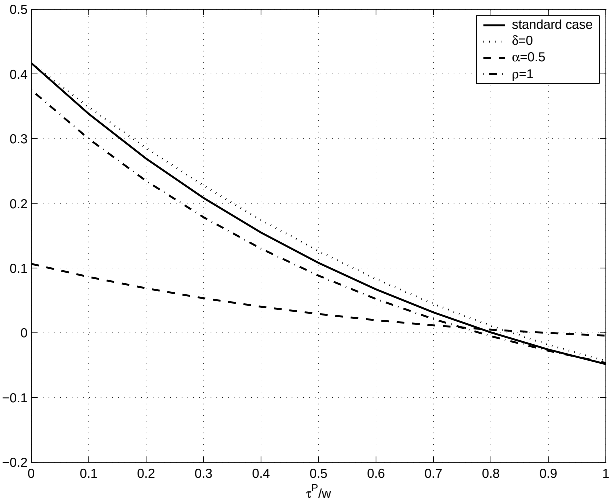


Figure 13: Condition (30)

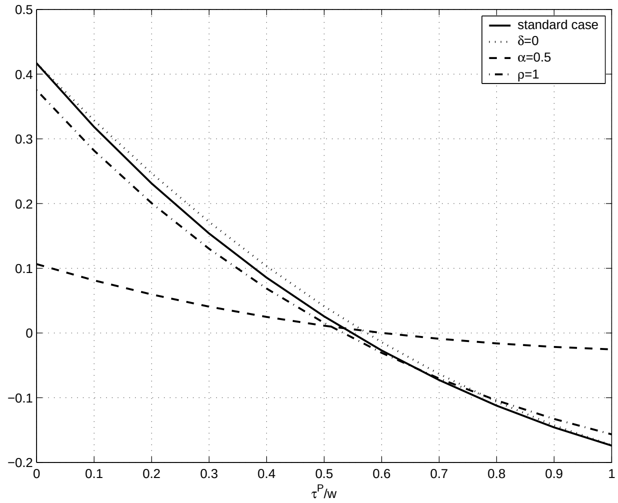


Figure 12 shows that condition (29) holds in the standard case as long as $\tau^P < 0.8w$, while we can see from Figure 13 that condition (30) holds if $\tau^P < 0.55w$. This means that in case ageing is characterized by a fall in the fertility rate, the capital-labour ratio only falls when the PAYG system is unrealistically large.

C Effects on utility

In this appendix we derive the expressions for the long-run change of utility in both countries. From these equations we can infer the relationship between the change in the capital-labour ratio and utility.

C.1 PAYG country

First we derive the change of long-run utility in country P. Therefore we first need to know what happens with consumption in both periods of life in the long run:

$$\frac{\partial c_{\infty}^{y,P}}{\partial \pi} = \frac{\partial w_{\infty}}{\partial \pi} - \frac{\partial \tau_{\infty}^{P}}{\partial \pi} - \frac{\partial s_{\infty}^{P}}{\partial \pi} \tag{31}$$

$$\frac{\partial c_{\infty}^{o,P}}{\partial \pi} = \frac{(1+r)}{\varepsilon}\frac{\partial s_{\infty}^{P}}{\partial \pi} + \frac{s^P}{\varepsilon}\frac{\partial r_{\infty}}{\partial \pi} - \frac{(1+r)s^P}{\varepsilon^2}\frac{\partial \varepsilon_{\infty}}{\partial \pi} + \frac{\partial z_{\infty}^{P}}{\partial \pi} \tag{32}$$

The long-run change in utility is[23]:

$$\frac{\partial U_{\infty}^{P}}{\partial \pi} = \frac{1}{c^{y,P}}\frac{\partial c_{\infty}^{y,P}}{\partial \pi} + \frac{\varepsilon}{1+\rho}\frac{1}{c^{o,P}}\frac{\partial c_{\infty}^{o,P}}{\partial \pi} \tag{33}$$

Using the Euler condition ($\frac{c^{y,P}}{c^{o,P}} = \frac{1+\rho}{1+r}$), equation (33) can be written as:

$$\frac{\partial U_{\infty}^{P}}{\partial \pi} = \frac{1}{c^{y,P}}\left(\frac{\partial c_{\infty}^{y,P}}{\partial \pi} + \frac{\varepsilon}{1+r}\frac{\partial c_{\infty}^{o,P}}{\partial \pi}\right) \tag{34}$$

Substituting equations (31) and (32) into equation (34) gives:

$$\begin{aligned}\frac{\partial U_{\infty}^{P}}{\partial \pi} =& \frac{1}{c^{y,P}}\left[\frac{\partial w_{\infty}}{\partial \pi} - \frac{\partial \tau_{\infty}^{P}}{\partial \pi} - \frac{\partial s_{\infty}^{P}}{\partial \pi}\right] \\ &+ \frac{1}{c^{y,P}}\left[\frac{\varepsilon}{1+r}\left(\frac{(1+r)}{\varepsilon}\frac{\partial s_{\infty}^{P}}{\partial \pi} + \frac{s^P}{\varepsilon}\frac{\partial r_{\infty}}{\partial \pi} - \frac{(1+r)s^P}{\varepsilon^2}\frac{\partial \varepsilon_{\infty}}{\partial \pi} + \frac{\partial z_{\infty}^{P}}{\partial \pi}\right)\right]\end{aligned} \tag{35}$$

Using that $\frac{\partial w_{\infty}}{\partial \pi} = -kf''(k)\,\frac{\partial k_{\infty}}{\partial \pi}$ and $\frac{\partial r_{\infty}}{\partial \pi} = f''(k)\,\frac{\partial k_{\infty}}{\partial \pi}$, we can write:

$$\frac{\partial U_{\infty}^{P}}{\partial \pi} = \frac{1}{c^{y,P}}\left[-kf''(k)\frac{\partial k_{\infty}}{\partial \pi} - \frac{\partial \tau_{\infty}^{P}}{\partial \pi} + \frac{f''(k)s^P}{(1+r)}\frac{\partial k_{\infty}}{\partial \pi} - \frac{s^P}{\varepsilon}\frac{\partial \varepsilon_{\infty}}{\partial \pi} + \frac{\varepsilon}{1+r}\frac{\partial z_{\infty}^{P}}{\partial \pi}\right] \tag{36}$$

[23] This equation shows the change in utility for a person living for two periods with certainty, i.e., $\varepsilon = 1$.

With defined benefits it holds that $\frac{\partial \tau_\infty^P}{\partial \pi} = z^P \frac{\partial \varepsilon_\infty}{\partial \pi}$ and $\frac{\partial z_\infty^P}{\partial \pi} = 0$, so that this equation can be rearranged to:

$$\frac{\partial U_\infty^P}{\partial \pi} = \frac{f''(k)}{c^{y,P}(1+r)} \left[s^P - k(1+r)\right] \frac{\partial k_\infty}{\partial \pi} - \frac{1}{c^{y,P}} \left(z^P + \frac{s^P}{\varepsilon}\right) \frac{\partial \varepsilon_\infty}{\partial \pi} \tag{37}$$

Dividing this expression by $\frac{\partial \varepsilon_\infty}{\partial \pi}$, we get an expression for the change in U^P in the long run when longevity changes (this is equation (14) in the text):

$$\frac{\partial U_\infty^P}{\partial \varepsilon_\infty} = \frac{f''(k)}{c^{y,P}(1+r)} \left[s^P - k(1+r)\right] \frac{\partial k_\infty}{\partial \varepsilon_\infty} - \frac{1}{c^{y,P}} \left(z^P + \frac{s^P}{\varepsilon}\right) \tag{38}$$

The following things can be said about the terms in front of $\frac{\partial k_\infty}{\partial \varepsilon_\infty}$:

1. $\frac{f''(k)}{c^{y,P}(1+r)} < 0$, because $f''(k) < 0$.
2. $s^P - k(1+r) < 0$, because we know that $s^P < k(1+n)$: country P is a capital importer in the initial steady state. And because we assumed dynamic efficiency ($r > n$), we know that $k(1+n) < k(1+r)$. So that $s^P < k(1+r)$.

These two points imply that an increase in the capital-labour ratio, after an increase in longevity, has positive utility effects. There are two reasons for this result:

1. A higher capital-labour ratio leads to a lower interest rate, which is good for the country that borrows money (the PAYG country).
2. Moreover, a lower interest rate means that the economy is closer to the Golden Rule point ($r = n$).

As the rise in the capital-labour ratio is larger in case the PAYG country has integrated capital markets with a country that uses a funded system instead of PAYG system, utility is also higher in that case.

C.2 Funded country

Unfortunately it is not possible to draw such a clear conclusion for the country that uses a funded pension system. The main intuition for this is as follows. Again we have that a higher capital-labour ratio is good for the funded country, because a lower interest rate causes the economy to be closer to its Golden Rule point. On the other hand, a lower interest rate is bad for the funded country because it is a lender of money. This can be shown more formally as follows (using the same kind of technique used above for the PAYG country). The expressions for the long-run change of consumption and utility are given by:

$$\frac{\partial c_{\infty}^{y,F}}{\partial \pi} = \frac{\partial w_{\infty}}{\partial \pi} - \frac{\partial (s_{\infty}^{F} + \tau_{\infty}^{F})}{\partial \pi} \tag{39}$$

$$\frac{\partial c_{\infty}^{o,F}}{\partial \pi} = \frac{(1+r)}{\varepsilon}\frac{\partial (s_{\infty}^{F} + \tau_{\infty}^{F})}{\partial \pi} + \frac{(s^{F} + \tau^{F})}{\varepsilon}\frac{\partial r_{\infty}}{\partial \pi} - \frac{(1+r)(s^{F} + \tau^{F})}{\varepsilon^{2}}\frac{\partial \varepsilon_{\infty}}{\partial \pi} \tag{40}$$

$$\frac{\partial U_{\infty}^{F}}{\partial \pi} = \frac{1}{c^{y,F}}\left(\frac{\partial c_{\infty}^{y,F}}{\partial \pi} + \frac{\varepsilon}{1+r}\frac{\partial c_{\infty}^{o,F}}{\partial \pi}\right) \tag{41}$$

Substituting the equations for consumption (39) and (40) into the equation for utility (41), and using the same expressions for $\frac{\partial w_{\infty}}{\partial \pi}$ and $\frac{\partial r_{\infty}}{\partial \pi}$ as before, the following expression for the change in utility in country F can be obtained:

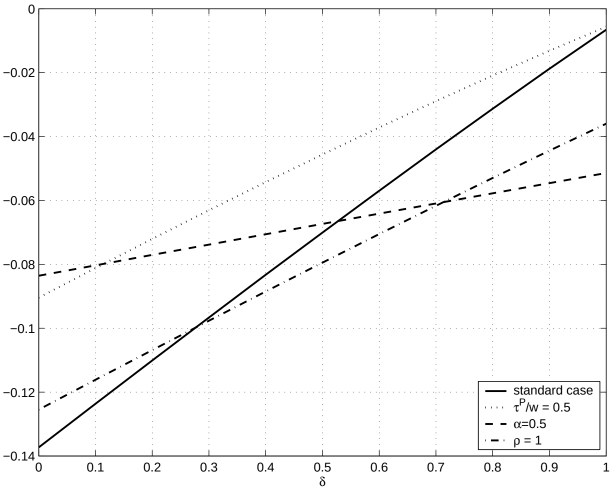
$$\frac{\partial U_{\infty}^{F}}{\partial \pi} = \frac{f''(k)}{c^{y,F}(1+r)}\left[(s^{F} + \tau^{F}) - k(1+r)\right]\frac{\partial k_{\infty}}{\partial \pi} - \frac{1}{c^{y,F}}\frac{(s^{F} + \tau^{F})}{\varepsilon}\frac{\partial \varepsilon_{\infty}}{\partial \pi} \tag{42}$$

Dividing by $\frac{\partial \varepsilon_{\infty}}{\partial \pi}$ we get (equation (15) in the text):

$$\frac{\partial U_{\infty}^{F}}{\partial \varepsilon_{\infty}} = \frac{f''(k)}{c^{y,F}(1+r)}\left[(s^{F} + \tau^{F}) - k(1+r)\right]\frac{\partial k_{\infty}}{\partial \varepsilon_{\infty}} - \frac{1}{c^{y,F}}\frac{(s^{F} + \tau^{F})}{\varepsilon} \tag{43}$$

which is comparable to equation (38) for the PAYG country. As before we have that the first term $\left(\frac{f''(k)}{c^{y,F}(1+r)}\right)$ is negative. But we cannot say anything about the sign of the second term $\left((s^{F} + \tau^{F}) - k(1+r)\right)$. Firstly, we know that in the initial steady state country F is a capital exporter, which means that $s^{F} + \tau^{F} > k(1+n)$. Secondly, it holds that $r > n$, because we assume dynamic efficiency, implying that $k(1+r) > k(1+n)$. So there are two opposing mechanisms, meaning that we do not know whether $s^{F} + \tau^{F} - k(1+r)$ will be negative or positive. The first effect reflects the fact that a lower interest rate is bad for a country that is a lender on the international capital market, the second effect is the Golden Rule effect. In case the first effect dominates the second, $s^{F} + \tau^{F} - k(1+r)$ is positive and the whole term in front of $\frac{\partial k_{\infty}}{\partial \varepsilon_{\infty}}$ is negative, implying that an increase in the capital-labour ratio after an increase in longevity actually affects utility negatively. On the other hand, when the second effect dominates the first, we have the same result as for the PAYG country: a higher capital-labour ratio leads to a higher utility in the funded country. This is the same conclusion as in Casarico (2001). Below we present a graph that shows how the sign of $s^{F} + \tau^{F} - k(1+r)$ (vertical axis) changes for different parameter values[24]. The vertical axis shows the depreciation rate (δ).

Figure 14: $s^F + \tau^F - k(1+r)$



This graph shows that $s^F + \tau^F - k(1+r) < 0$ for different values of the parameters, which means that the Golden Rule effect dominates the interest rate effect. This in turn implies that for realistic parameter values, a rise in the capital-labour ratio has positive utility effects in the funded country[25]. As the capital-labour ratio increases more in case the funded country has a common capital market with a country that also uses a funded pension scheme, opposed to a PAYG system, utility is higher in that case.

[24] In the standard case we have the following parameter values: $\rho = 0.3478$, $\frac{\tau^P}{w} = 0.2$, $\nu = 1$, $\alpha = 0.3$, and $\varepsilon = 0.94$.

[25] It is actually possible to have $s^F + \tau^F - k(1+r) > 0$. This is for example the case when $\rho = 0$ *and* $\delta > 0.87$. A time preference rate of zero is not very realistic, however. Therefore we conclude that for realistic parameter values it holds that $s^F + \tau^F - k(1+r) < 0$.

D Proof that $\mathrm{d}k_t^{PP} < \mathrm{d}k_t^{FF}$ in Section 3.2

In this appendix we proof that savings in the funded country still increase more than in the PAYG country when the PAYG scheme is characterized by fixed contributions. This implies that the capital-labour ratio in the funded country increases less in case it has a common capital market with a country using a PAYG pension system.

Savings in the funded country increase more than in the PAYG country after an increase in longevity in case the following condition holds:

$$\frac{(1+\rho)(w-\tau^P)}{\Psi^P(1+\rho+\varepsilon)} - \frac{(1+\rho)^2 z^P}{\Psi^P(1+\rho+\varepsilon)(1+r)} + \frac{(1+\rho)\tau^P}{\Psi^P(1+r)\,\varepsilon} < \frac{(1+\rho)\,w}{\Psi^F(1+\rho+\varepsilon)} \tag{44}$$

which can written as:

$$\frac{\varepsilon(1+\rho)(1+r)(w-\tau^P) - \varepsilon(1+\rho)^2\, z^P + (1+\rho)(1+\rho+\varepsilon)\,\tau^P}{\Psi^P(1+\rho+\varepsilon)(1+r)\,\varepsilon} < \frac{(1+\rho)\,w}{\Psi^F(1+\rho+\varepsilon)} \tag{45}$$

where:

$$\begin{aligned} \Psi^P &\equiv (1+\rho+\varepsilon) - \frac{(1+\rho)\,\varepsilon\, z^P\, f''(k)}{(1+r)^2} > 0 \\ \Psi^F &\equiv (1+\rho+\varepsilon) > 0 \end{aligned}$$

and $\Psi^P > \Psi^F$.
Simplifying and using the fact that $z^P = \frac{\tau^P}{\varepsilon}$ gives:

$$\frac{-\tau^P r}{\Psi^P(1+r)} < \frac{w}{\Psi^F} - \frac{w}{\Psi^P} \tag{46}$$

The left-hand side is always negative and because $\Psi^F < \Psi^P$ we know that $\frac{w}{\Psi^F} > \frac{w}{\Psi^P}$, so that the right-hand side is always positive. This implies that equation (46) always holds and that the positive dependency-ratio effect in case of fixed contributions is not large enough to compensate for the negative effects a PAYG system has in general, so that savings in the funded country increase more than in the PAYG country.

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