Implementing Cooperative Solution Concepts

Ju, Y.; Wettstein, D.

Publication date:
2006

Citation for published version (APA):
Ju, Y., & Wettstein, D. (2006). *Implementing Cooperative Solution Concepts: A Generalized Bidding Approach*. (CentER Discussion Paper; Vol. 2006-42). Microeconomics.

CentER

Discussion Paper

No. 2006–42

IMPLEMENTING COOPERATIVE SOLUTION CONCEPTS: A GENERALIZED BIDDING APPROACH

By Yuan Ju, David Wettstein

April 2006

ISSN 0924-7815

TILBURG UNIVERSITY

Implementing Cooperative Solution Concepts: a Generalized Bidding Approach

Yuan Ju[1] David Wettstein[2]

April 2006

[1]School of Economic and Management Studies, Keele University, Keele, Staffordshire, ST5 5BG, UK. Tel: +44-1782-583095, Fax: +44-1782-717577, E-mail: Y.Ju@keele.ac.uk.

[2]Corresponding author. Department of Economics, Ben-Gurion University of the Negev, Beer-Sheva, 84105, Israel. Tel: +972-8-6472291, Fax: +972-8-6472941, E-mail: wettstn@bgu.ac.il.

Abstract

This paper provides a framework for implementing and comparing several solution concepts for transferable utility cooperative games. We construct bidding mechanisms where players bid for the role of the proposer. The mechanisms differ in the power awarded to the proposer. The Shapley, consensus and equal surplus values are implemented in subgame perfect equilibrium outcomes as power shifts away from the proposer to the rest of the players. Moreover, an alternative informational structure where these solution concepts can be implemented without imposing any conditions of the transferable utility game is discussed as well.

JEL classification codes: C71; C72; D62.
Keywords: implementation; bidding mechanism; Shapley value; consensus value; equal surplus value.

1 Introduction

Cooperation among individuals, firms or countries generates benefits to be shared and costs to be imputed. The analysis of these problems proceeded both axiomatically, studying the implications of normative issues and strategically, deriving the likely outcomes of maximizing behavior by the parties involved. The merging of both approaches lies at the core of the Nash program (Nash (1953)) calling for a non-cooperative (strategic) foundation to cooperative (normative) solution concepts.

We provide a non-cooperative foundation to several cooperative solution concepts by using a class of bidding mechanisms that differ in the power awarded to the proposer chosen through a bidding process. The mechanisms constructed are related to the bidding mechanisms first constructed by Pérez-Castrillo and Wettstein (2001, 2002). The bidding for the role of the proposer is the same as in the previous mechanisms, however the role itself varies from one mechanism to another. Whereas previously the proposer was the only player allowed to make offers and once declined she was removed from the game, we now allow for a second round of offers. In this manner we are able to implement a continuum of cooperative solution concepts.

We construct explicit mechanisms implementing the Shapley value (Shapley (1953)), the equal surplus value (cf. Driessen and Funaki (1991) and Moulin (2003)) and the consensus value (Ju et. al. (2004)). In all mechanisms, the players first participate in a bidding procedure to determine a proposer. The proposer announces an offer to all the other players. If the offer is accepted, the proposer pays out according to it and collects the value generated by the grand coalition. If the offer is rejected the other players engage again in the same game. The difference between the mechanisms is in what happens when the other players have finished the game. In all the mechanisms we construct the proposer and the other players have the right to make, accept and reject a second set of offers. The precise rules as to who makes the offer and who has a right to reject or accept vary according to the solution implemented.

The Shapley value is implemented when the proposer chosen first can make a second offer to the other players. The equal surplus value emerges as an equilibrium outcome when the other players can make the proposer (who was "left out") an offer to join them. The consensus value is the equilibrium outcome when the proposer and the rest of the players bid for the right to make another offer.

This option of "re-entering" the game after being rejected is very reasonable. Even in the absence of such an explicit option, players in any "real-life" situation may try to exercise it through a mutual agreement. This argument leads to the study of implementation with renegotiation (Maskin and Moore (1999) and Baliga and Brusco (2000)). Clearly, suitably modified versions of the general constructions in these papers as well as those in the usual implementation literature using sequential mechanisms (Moore and Repullo (1988) and Maniquet (2003)) would provide a non-cooperative foundation to the solution concepts we discuss. However, these mechanisms appropriate for general environments would be highly complex, requiring the transmission of large amounts of information, compared to our, as well as, previous mechanisms constructed to realize cooperative solution concepts.

Furthermore, we offer an alternative specification of the cooperative environment, where a coalition can, if necessary, prove what is the amount it can generate for its members to share. One such instance is the situation where the players have to share among themselves a given estate with well documented claims on the part of every coalition. In this setting we show that suitably defined generalized bidding mechanisms implement the solution concepts, previously discussed, for any transferable utility (TU) game.

Several previous papers have indeed dealt with providing non-cooperative foundations to cooperative solution concepts. Gul (1989, 1999) suggested a bargaining procedure that leads to the Shapley value. Hart and Mas-Collel (1996) constructed a bargaining procedure that leads to the Shapley value in TU games and the Nash bargaining solution for pure bargaining problems. Krishna and Serrano (1995) provided further results regarding this procedure. Hart and Moore (1990), Winter (1994), and Dasgupta and Chiu (1998) constructed games that lead to the Shapley value.[1] Vidal-Puga and Bergantiños (2003) introduced a coalitional bidding mechanism, as an extension of the bidding mechanism defined by Pérez-Castrillo and Wettstein (2001), and implemented the Owen value (1977). By considering the possibility of the breakdown of negotiations when rejecting an offer, Ju et. al. (2004) designed a two-level bidding mechanism and provided an implementation of the consensus value.

The generalized bidding approach, using the same basic game with different "end-games" appended to it to implement a variety of values, highlights the different "non-cooperative" rationales underlying the various values. This approach provides a structured algorithm to design mechanisms for implementing cooperative solution concepts. It should be noted that the generalized bidding mechanisms introduced in this paper yield the actual values implemented rather than implementing them in expected terms.

Moreover, this approach can be used to implement solution concepts in other cooperative environments such as partition function form games, games with a coalition structure and primeval games (cf. Ju and Borm (2005)). Being able to apply the same extensive

[1] An extensive discussion of these implementations of the Shapley value can be found in Pérez-Castrillo and Wettstein (2001) which offers an implementation of the Shapley value via a bidding mechanism. For the implementations of other cooperative solutions and a general view of the research area, we refer to Serrano (2005).

form to varied domains of cooperative games is one of the objectives of the Nash program as stated in Hart and Mas-Colell (1996) and Serrano (2005).

In the next section, we present the environment and the solution concepts to be implemented. In Section 3, we describe the basic mechanism and show that suitably defined variants of it implement the different value concepts. Section 4 presents the alternative interpretation of the environment and the modified mechanisms. The last section concludes by discussing several possible extensions and applications of the approach, which suggests further directions of research.

2 The cooperative model and the values

We denote by $N = \{1, ..., n\}$ the set of players, and let $S \subseteq N$ denote a coalition of players. A cooperative game in characteristic form is denoted by (N, v) where $v : 2^N \to \mathbb{R}$ is a characteristic function satisfying $v(\emptyset) = 0$. Throughout the paper, $|S|$ denotes the cardinality of S, and in particular, when no confusion arises, let $|N| = n$. For a coalition S, $v(S)$ is the total payoff that the members in S can obtain if S forms. For notational simplicity, given $i \in N$, we use $v(i)$ instead of $v(\{i\})$ to denote the stand-alone payoff of player i. A *value* is a mapping f which associates with every game (N, v) a vector in $\mathbb{R}^n$. A value determines the payoffs for every player in the game.

Given a cooperative game (N, v) and a subset $S \subseteq N$, we define the subgame $(S, v|_S)$ by assigning the value $v|_S(T) \equiv v(T)$ for any $T \subseteq S$.

We denote by ϕ the Shapley value for game (N, v) which is defined by

$$\phi_i(N, v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} [v(S \cup \{i\}) - v(S)]$$

for all $i \in N$. It is the unique value that satisfies efficiency, additivity, symmetry and the null player property.

The equal surplus value, denoted by ϕ^{es}, is a more straightforward value and allocates to each player, besides her stand-alone payoff generated by her singleton coalition, an equal share of the surplus (in excess of the sum of all players' stand-alone payoffs) generated by the grand coalition. Formally, it is defined by

$$\phi_i^{es}(N, v) = v(i) + \frac{1}{n} \left(v(N) - \sum_{j \in N} v(j) \right)$$

for all $i \in N$. The equal surplus value fails to satisfy the null player property. However, this solution concept can be well motivated from an egalitarian perspective. For axiomatizations of the equal surplus value, we refer to van den Brink and Funaki (2004).

Ju et. al. (2004) proposed a recursive two-sided negotiation procedure to establish cooperation and share the payoff of the grand coalition. This procedure leads to a new value, the consensus value, denoted by ψ. It is shown that the consensus value equals the middle point between the Shapley value and the equal surplus value. That is,

$$\psi_i(N, v) = \frac{1}{2}\phi_i(N, v) + \frac{1}{2}\phi_i^{es}(N, v)$$

for all $i \in N$. The consensus value is the unique solution concept that satisfies efficiency, additivity, symmetry and the neutral null player property. Alternative characterizations for this value using an equal welfare loss property or by means of individual rationality and a type of monotonicity can be found in Ju et. al. (2004) and van den Brink et. al. (2005), respectively.

From a cooperative (normative) point of view, the applications and suitability of these solution concepts in different contexts can be further elaborated on based upon the four fundamental principles of distributive justice discussed in Moulin (2003): compensation, reward, exogenous rights, and fitness.

3 The generalized bidding mechanisms

In this section, we construct the family of bidding mechanisms that will implement the various cooperative solutions. These mechanisms provide a convenient benchmark to evaluate and compare these values from a non-cooperative perspective.

The basic *bidding mechanism* can be described informally as follows: At stage 1 the players bid to choose a proposer. Each player bids by submitting an $(n-1)$-tuple of numbers (positive or negative), one number for each player (excluding herself). The player for whom the net bid (the difference between the sum of bids made by the player and the sum of bids the other players made to her) is the highest, is chosen as the proposer. Before moving to stage 2, the proposer pays to each player the bid she made. So in this stage, the net bids are used to measure players' willingness to become the proposer. As a reward to the chosen proposer for her effort (represented by her net bid), she has the right to make a scheme how to split $v(N)$ among all the players at the next stage.

At stage 2 the proposer offers a vector of payments to all other players in exchange for joining her to form the grand coalition. The offer is accepted if all the other players agree. In case of acceptance the grand coalition indeed forms and the proposer receives $v(N)$ out of which she pays out the offers made. In case of rejection the proposer "waits" while all the other players go again through the same game.

The mechanism described thus far implements the Shapley value[2] as shown in Pérez-Castrillo and Wettstein (2001). We now add further bidding stages in case of rejection to the mechanism to obtain what we term a *generalized bidding mechanism*. In these additional stages the first proposer (in fact, the rejected proposer) and the proposer chosen among the remaining players (when an agreement is reached within themselves) bid and accept further offers (note that these stages are also present in the game played by any remaining group of players).

The first variant implementing the Shapley value has the first proposer (denoted for simplicity by a) make an offer to the proposer chosen among the remaining players (denoted for simplicity by b). The offer is for a to form the grand coalition rather than b. If the offer is accepted the grand coalition forms, a receives $v(N)$ and pays the offer, b receives the offer from a and pays all the commitments made by him, and all the other players receive what they were promised. In this variant a retains the right to make offers.

The second variant implementing the equal surplus value has b make an offer to a. If the offer is accepted the grand coalition forms, a receives the offer, b receives $v(N)$ and pays the offer to a as well as what he owes to the remaining players. In this variant a loses the right to make offers.

In the third variant implementing the consensus value a and b bid for the right to make an offer. If a wins the game proceeds as in the first variant and if b wins the second variant goes into effect.

We now formally describe the bidding games and start by describing the mechanism implementing the Shapley value.

Mechanism A1. If there is only one player $\{i\}$, she simply receives $v(i)$. When there are two or more players, the mechanism is defined recursively. Given the rules of the mechanism for at most $n-1$ players, the mechanism for $N = \{1, \ldots, n\}$ proceeds in five stages.

Stage 1: Each player $i \in N$ makes $n-1$ bids $b_j^i \in \mathbb{R}$ with $j \neq i$. Hence, at this stage, a strategy for player i is a vector $(b_j^i)_{j \neq i}$.
For each $i \in N$, define the *net bid* to player i by $B^i = \sum_{j \neq i} b_j^i - \sum_{j \neq i} b_i^j$. Let $i^* = \text{argmax}_i(B^i)$ where an arbitrary tie-breaking rule is used in case of a non-unique maximizer. Once the winner i^* has been chosen, player i^* pays every player $j \in N \backslash \{i^*\}$, $b_j^{i^*}$.

[2] In the case where the rejected proposer gets her stand-alone payoff instead of "waiting".

Stage 2: Player i^* makes a vector of offers $x_j^{i^*} \in \mathbb{R}$ to every player $j \in N\backslash\{i^*\}$.

Stage 3: The players other than i^*, sequentially, either accept or reject the offer. If at least one player rejects it, then the offer is rejected. Otherwise, the offer is accepted.
If the offer is accepted, which means that all players agree with the proposer on the scheme of sharing $v(N)$, then each player $j \in N\backslash\{i^*\}$ receives $x_j^{i^*}$ at this stage, and player i^* receives $v(N) - \sum_{j \neq i^*} x_j^{i^*}$. Hence, the final payoff to player $j \neq i^*$ is $x_j^{i^*} + b_j^{i^*}$ while player i^* receives $v(N) - \sum_{j \neq i^*} x_j^{i^*} - \sum_{j \neq i^*} b_j^{i^*}$.
If the offer is rejected, all players other than i^* proceed to play a similar game with one player less, i.e., with the set of players $N\backslash\{i^*\}$, bargaining over a "conditional" pie, the size of which is determined in the following stages of the game (in this mechanism the size of this pie in any subgame perfect equilibrium is $v(N\backslash\{i^*\})$. Once the players in $N\backslash\{i^*\}$ have reached an agreement (e.g., the offer made by a proposer j^* chosen among the set of players $N\backslash\{i^*\}$ is immediately accepted by all players in $N\backslash\{i^*, j^*\}$, or reuniting these players by renegotiation as shown below at stages 4 and 5) so that the coalition $N\backslash\{i^*\}$ forms and a payoff scheme $(y_j^3)_{j \neq i^*}$, where the superscript 3 denotes stage 3, is "conditionally generated", the game moves to stage 4. In case no agreement is reached by $N\backslash\{i^*\}$ and thereby coalition $N\backslash\{i^*\}$ does not emerge, player i^* loses the option of renegotiating with $N\backslash\{i^*\}$ and is indeed left alone and gets her stand-alone payoff $v(i^*)$ at this stage.

Stage 4: Player i^* makes an offer $\widetilde{x}_{j^*}^{i^*}$ in $\mathbb{R}$, to the proposer j^* chosen among the set of players $N\backslash\{i^*\}$. (The offer is to let i^* form the grand coalition instead of player j^*.)

Stage 5: Player j^* accepts or rejects the offer. If the offer is accepted then at this stage each player $k \in N\backslash\{i^*, j^*\}$ receives y_k^3, player j^* receives $\widetilde{x}_{j^*}^{i^*} - \sum_{k \in N\backslash\{i^*, j^*\}} y_k^3$, and player i^* receives $v(N) - \widetilde{x}_{j^*}^{i^*}$. Hence, the final payoff to player $k \in N\backslash\{i^*, j^*\}$ is $y_k^3 + b_k^{i^*}$; player j^* receives $\widetilde{x}_{j^*}^{i^*} - \sum_{k \in N\backslash\{i^*, j^*\}} y_k^3 + b_{j^*}^{i^*}$, player i^* receives $v(N) - \widetilde{x}_{j^*}^{i^*} - \sum_{j \neq i^*} b_j^{i^*}$. If the offer is rejected each player $j \neq i^*$ finally receives $y_j^3 + b_j^{i^*}$ and player i^* receives $v(i^*) - \sum_{j \neq i^*} b_j^{i^*}$.

We will show that for any zero-monotonic game (N, v) (i.e., $v(S) \geq v(S\backslash\{i\}) + v(\{i\})$ for all $S \subseteq N$ and $i \in S$), the subgame perfect equilibrium (SPE) outcomes of Mechanism A1 coincide with the payoff vector $\phi(N, v)$ as prescribed by the Shapley value.

Theorem 3.1 *Mechanism A1 implements the Shapley value of a zero-monotonic game* (N, v) *in SPE.*

Proof.
Let (N, v) be a zero-monotonic game. The proof proceeds by induction on the number of players n. It is easy to see that the theorem holds for $n = 1$. We assume that it holds for all $m \leq n - 1$ and show that it is satisfied for n.

First we show that the Shapley value is an SPE outcome. We explicitly construct an SPE that yields the Shapley value as an SPE outcome. Consider the following strategies:

At stage 1, each player $i \in N$, announces $b_j^i = \phi_j(N, v) - \phi_j(N \backslash \{i\}, v|_{N\backslash\{i\}})$ for every $j \in N \backslash \{i\}$.

At stage 2, a proposer, player i^*, offers $x_j^{i^*} = \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ to every $j \in N\backslash\{i^*\}$.

At stage 3, any player $j \in N\backslash\{i^*\}$ accepts any offer which is greater than or equal to $\phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ and rejects any offer strictly less than $\phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$.

At stage 4, player i^* makes an offer $\widetilde{x}_{j^*}^{i^*} = v(N\backslash\{i^*\})$ to any selected proposer $j^* \in N\backslash\{i^*\}$.

At stage 5, player j^*, the proposer of the set of players $N\backslash\{i^*\}$, accepts any offer greater than or equal to $v(N\backslash\{i^*\})$ and rejects any offer strictly less than it.

Clearly these strategies yield the Shapley value for any player who is not the proposer, since the game ends at stage 3 and $b_j^{i^*} + x_j^{i^*} = \phi_j(N, v)$, for all $j \neq i^*$. Moreover, given that following the strategies the offer is accepted by all players, the proposer also obtains her Shapley value.
Note that all net bids equal zero by the balanced contribution property for the Shapley value (Myerson (1980)).
To show that the previous strategies constitute an SPE, note first that the strategies at stages 2, 3, 4, and 5 are best responses: In case of rejection at stage 3 proposer i^* can obtain $v(N) - v(N\backslash\{i^*\})$ in the end (it pays her to make an offer that is accepted at stage 4, by zero-monotonicity), and all other players play the bidding mechanism with player set $N\backslash\{i^*\}$ and payoff $v(N\backslash\{i^*\})$. By the induction hypothesis, we have the Shapley value as the outcome of this game. That is, each player $j \in N\backslash\{i^*\}$ gets $\phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$.
Consider now the strategies at stage 1. If player i^* increases her total bid, then she will be chosen as the proposer with certainty, but her payoff will decrease. If she decreases her total bid another player will propose and player i^*'s payoff would still equal her Shapley value. Finally, any change in her bids that leaves the total bid constant will influence the identity of the proposer but will not affect player i^*'s payoff.

The proof that any SPE yields the Shapley value proceeds by a series of claims.

Claim (a). In any SPE, at stage 5, any player j^* (the proposer from the set of players $N\backslash\{i^*\}$), accepts any offer greater than or equal to $v(N\backslash\{i^*\})$ and rejects any offer strictly less than it. Hence in any SPE at stage 4, the proposer i^* will offer any player j^* exactly the amount $v(N\backslash\{i^*\})$.

This claim can be readily verified due to zero-monotonicity.

Claim (b). In any SPE, at stage 3, all players other than the proposer i^* accept the offer if $x_j^{i^*} > \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ for every $j \neq i^*$. Otherwise, if $x_j^{i^*} < \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ for at least some $j \neq i^*$, then the offer is rejected.

Note that if an offer made by the proposer i^* is rejected at stage 3, all other players, $N\backslash\{i^*\}$, by Claim (a), will get exactly $v(N\backslash\{i^*\})$. Consequently, in case of rejection at stage 3, by the induction hypothesis, the payoff to a player $j \neq i^*$ is $\phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$. We denote the last player that has to decide whether to accept or reject the offer by β. If the game reaches β, i.e., there has been no previous rejection, her optimal strategy involves accepting any offer higher than $\phi_\beta(N\backslash\{i^*\})$ and rejecting any offer lower than $\phi_\beta(N\backslash\{i^*\})$. The second to last player, denoted by $\beta - 1$, anticipates the reaction of player β. So, $\beta - 1$ will accept the offer when the game reaches him with $x_{\beta-1}^{i^*} > \phi_{\beta-1}(N\backslash\{i^*\})$ and $x_\beta^{i^*} > \phi_\beta(N\backslash\{i^*\})$. If $x_{\beta-1}^{i^*} < \phi_{\beta-1}(N\backslash\{i^*\})$ and $x_\beta^{i^*} > \phi_\beta(N\backslash\{i^*\})$, player $\beta - 1$ will reject the offer. If $\beta - 1$ observes $x_\beta^{i^*} < \phi_\beta(N\backslash\{i^*\})$, he will be indifferent to accepting or rejecting any offer $x_{\beta-1}^{i^*}$. Following this argument till the first player, Claim (b) is constructed.

Claim (c). If $v(N) > v(N\backslash\{i^*\}) + v(i^*)$, for the game that starts at stage 2 there exist two types of SPE. Firstly, an obvious SPE is as follows: At stage 2, player i^* offers $x_j^{i^*} = \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ to all $j \neq i^*$; at stage 3, every player $j \neq i^*$ accepts any offer $x_j^{i^*} \geq \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ and rejects the offer otherwise. Secondly, any set of strategies where, the proposer offers, at stage 2, $x_j^{i^*} \leq \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ to some players $j \neq i^*$ and at stage 4 offers $y_{j^*}^{i^*} = v(N\backslash\{i^*\})$ to j^*, and at stage 3, any player $j \in N\backslash\{i^*\}$ rejects any offer $x_j^{i^*} < \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ and at stage 5 the representative j^* for $N\backslash\{i^*\}$ accepts any offer greater than or equal to $v(N\backslash\{i^*\})$, also constitutes an SPE. There could be no other equilibrium: it cannot be that an offer is rejected at stage 3 and, furthermore, the offer made at stage 4 is also rejected. If this were to happen, the player who made an offer at stage 4 can obtain, due to zero-monotonicity, a better outcome by making instead an offer that must be accepted. If $v(N) = v(N\backslash\{i^*\}) + v(i^*)$, there exist another type of SPE in addition to the above two types. Any set of strategies where, the proposer offers, at stage 2, $x_j^{i^*} \leq \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ to some players $j \neq i^*$ and at stage 4 offers $\widetilde{x}_{j^*}^{i^*} \leq v(N\backslash\{i^*\})$ to j^*, and at stage 3, any player $j \in N\backslash\{i^*\}$ rejects any offer $x_j^{i^*} \leq \phi_j(N\backslash\{i^*\}, v|_{N\backslash\{i^*\}})$ and at stage 5 the representative j^* for $N\backslash\{i^*\}$ rejects any offer less than or equal to $v(N\backslash\{i^*\})$, constitutes an SPE as well.

One can readily see that the proposed strategies constitute SPE by checking that the proposer has no incentive to increase any offer, given that all offers are no lower than $\phi_j(N\backslash\{i^*\})$ for all $j \neq i^*$, to a level higher than $\phi_k(N\backslash\{i^*\})$ to any particular player $k \neq i^*$, and verifying that in all the SPE of this subgame the final payoffs to the proposer i^* and every other player $j \neq i^*$ are $v(N) - v(N\backslash\{i^*\}) - \sum_{j\in N\backslash\{i^*\}} b_j^{i^*}$ and $\phi_j(N\backslash\{i^*\}) + b_j^{i^*}$, respectively.

Claim (d). In any SPE, $B^i = B^j$ for all $i, j \in N$, and hence $B^i = 0$ for all $i \in N$.

Denote $\Omega = \{i \in N | B^i = \max_{j\in N}(B^j)\}$. If $\Omega = N$ the claim is satisfied since $\sum_{i\in N} B^i = 0$. Otherwise, we can show that any player i in Ω has the incentive to change her bids so as to decrease the sum of payments in case she wins. Furthermore, these changes can be made without altering the set Ω. Hence, the player maintains the same probability of winning and obtains a higher expected payoff. Take some player $j \notin \Omega$. Let player $i \in \Omega$ change her strategy by announcing ${b'}_k^i = b_k^i + \epsilon$ for all $k \in \Omega\backslash\{i\}$, and ${b'}_j^i = b_j^i - |\Omega|\epsilon$ for j, and ${b'}_l^i = b_l^i$ for all $l \notin \Omega \cup \{j\}$. Then, the new net bids are $B'^i = B^i - \epsilon$, $B'^k = B^k - \epsilon$ for all $k \in \Omega\backslash\{i\}$, $B'^j = B^j + |\Omega|\epsilon$ and $B'^l = B^l$ for all $l \notin \Omega \cup j$. If ϵ is small enough so that $B^j + |\Omega|\epsilon < B^i - \epsilon$, then $B'^l < B'^i = B'^k$ for all $l \in \Omega$ (including j) and for all $k \in \Omega$. Therefore, Ω does not change. However, $\sum_{h\neq i} b_h^i - \epsilon < \sum_{h\neq i} b_h^i$.

Claim (e). In any SPE, each player's payoff is the same regardless of whom is chosen as the proposer.

This claim can be readily proved by contradiction. If some player can get extra payoff given a specific identity of the proposer, then this player will have incentive to adjust her bids accordingly, which contradicts Claim (d).

Claim (f) In any SPE, the final payment received by each of the players coincides with each player's Shapley value.

We know that if player i is the proposer, her final payoff will be $v(N) - v(N\backslash\{i\}) - \sum_{j\neq i} b_j^i$. In case of player $j \neq i$ becoming the proposer, player i's final payoff will be $\phi_i(N\backslash\{j\}) + b_i^j$. We can now proceed as in Pérez-Castrillo and Wettstein (2001) to show that each player's payoff coincides with her Shapley value. ■

In order to arrive at the Shapley value the proposer chosen through bidding at stage 1 has the power to make another offer, following the rejection of her initial offer, before the conclusion of the game. An equally plausible scenario is that the proposer chosen at stage 1 forfeits the right to make another offer once rejected. It is the proposer chosen in the following stage who has the right to make a second offer before the game ends. Hence we have a new generalized bidding mechanism, described in what follows, which is shown to implement the equal surplus value.

Mechanism A2. Stages $1, 2$ and 3 are the same as in Mechanism A1 up to the point where an offer is rejected. When an offer made at stage 3 is rejected, all players other than i^* proceed to play the same game where the set of players is $N\backslash\{i^*\}$ and they bargain over a "conditional" pie, the size of which is determined in the last stage of the game (in this mechanism the size of this pie in any subgame perfect equilibrium is $v(N) - v(i^*)$). Once the players in $N\backslash\{i^*\}$ have reached the "stage 3 conditionally generated" $(y_j^3)_{j\neq i^*}$ payoff scheme which also implies that the coalition $N\backslash\{i^*\}$ forms, the game moves to stage 4. Otherwise, the game stops and proposer i^* obtains $v(i^*)$ at this stage.

Stage 4: Player j^*, the proposer chosen among the set of players $N\backslash\{i^*\}$ makes an offer $\widetilde{x}_{i^*}^{j^*}$ in $\mathbb{R}$, to player i^*. (The offer is to pay i^* this amount for joining in to form the grand coalition).

Stage 5: Player i^* accepts or rejects the offer. If the offer is accepted then at this stage each player $k \in N\backslash\{i^*, j^*\}$ receives y_k^3, player j^* receives $v(N) - \widetilde{x}_{i^*}^{j^*} - \sum_{k\in N\backslash\{i^*,j^*\}} y_k^3$, and player i^* receives $\widetilde{x}_{i^*}^{j^*}$. Hence, the final payoff to player $k \in N\backslash\{i^*, j^*\}$ is $y_k^3 + b_k^{i^*}$; player j^* receives $v(N) - \widetilde{x}_{i^*}^{j^*} - \sum_{k\in N\backslash\{i^*,j^*\}} y_k^3 + b_{j^*}^{i^*}$, player i^* receives $\widetilde{x}_{i^*}^{j^*} - \sum_{j\neq i^*} b_j^{i^*}$. If the offer is rejected each player $j \neq i^*$ finally receive $y_j^3 + b_j^{i^*}$ and player i^* receives $v(i^*) - \sum_{j\neq i^*} b_j^{i^*}$.

Theorem 3.2 *Mechanism A2 implements the equal surplus value of a zero-monotonic game (N, v) in SPE.*

Proof.
The proof is similar to that of Theorem 3.1. The differences are in the construction of the SPE strategies and in Claim (f). Hence, we explicitly construct an SPE that yields the equal surplus value as an SPE outcome and show that the counterpart of Claim (f) (that payoffs must coincide with the equal surplus value) holds as well.

To constuct an SPE, consider the following strategies.

At stage 1, each player $i \in N$, announces $b^i_j = \phi^{es}_j(N, v) - \phi^{es}_j(N\backslash\{i\}, v^{-i})$,[3] for every $j \in N\backslash\{i\}$.

At stage 2, a proposer, player i^*, offers $x^{i^*}_j = \phi^{es}_j(N\backslash\{i^*\}, v^{-i^*})$ to every $j \in N\backslash\{i^*\}$.

At stage 3, any player $j \in N\backslash\{i^*\}$ accepts any offer which is greater than or equal to $\phi^{es}_j(N\backslash\{i^*\}, v^{-i^*})$ and rejects any offer strictly less than $\phi^{es}_j(N\backslash\{i^*\}, v^{-i^*})$.

At stage 4, a proposer within $N\backslash\{i^*\}$, player j^* makes an offer $\widetilde{x}^{j^*}_{i^*} = v(i^*)$ to i^*.

At stage 5, player i^*, the "waiting" proposer for the set of players N, accepts any offer greater than or equal to $v(i^*)$ and rejects any offer strictly less than it.

One can readily verify that these strategies yield the equal surplus value for any player and constitute an SPE.

To show that in any SPE the final payment received by each of the players coincides with each player's equal surplus value, we note that if i is the proposer, her final payoff will be $v(N) - (v(N) - v(i)) - \sum_{j\neq i} b^i_j$, whereas if $j \neq i$ is the proposer, i will get final payoff $\phi^{es}(N\backslash\{j\}, v^{-j}) + b^j_i = (v(i) + \frac{v(N)-v(j)-\sum_{k\neq j} v(k)}{n-1}) + b^j_i$. Hence the sum of the payoffs to player i over all possible choices is (recall that all net bids are zero)

$$\begin{aligned} & v(N) - (v(N) - v(i)) - \sum_{j\neq i} b^i_j + \sum_{j\neq i}\left(v(i) + \frac{v(N) - v(j) - \sum_{k\neq j} v(k)}{n-1} + b^j_i\right) \\ = \quad & nv(i) + \left(v(N) - \sum_{l\in N} v(l)\right) \\ = \quad & n \cdot \phi^{es}_i(N, v). \end{aligned}$$

Since the payoffs are the same regardless of who is the proposer (by the same reason as discussed in Claim (e) of the proof for Theorem 3.1) we see that the payoff of each player in any equilibrium must coincide with the equal surplus value. ■

[3]The game $(N\backslash\{i\}, v^{-i})$ is defined by $v^{-i}(N\backslash\{i\}) = v(N) - v(i)$ and $v^{-i}(S) = v(S)$, for any $S \subset N\backslash\{i\}$.

The Shapley and equal surplus values resulted from a “zero-one” decision, either the first stage proposer or the subsequently chosen proposer have the right to make a second offer. It is also of interest to know what happens if the power to make a second offer is somehow shared between the two. One could randomize giving each an equal probability to have the right to make another offer. Alternately the two could bargain via a Rubinstein alternating offer game (Rubinstein (1982)). We adopt again a bidding approach letting the two bid for the right to make a further offer. The mechanism is formally described in what follows and is shown to implement the consensus value.

Mechanism A3. The rules of stages 1, 2 and 3 are as before except that, at stage 3, due to the changes in the subsequent stages, in case of rejection the conditional pie being bargained over within $N\backslash\{i^*\}$ is $v(N\backslash\{i^*\})+\frac{v(N)-v(N\backslash\{i^*\})-v(i^*)}{2}$. We now describe the game from stage 4 onwards.

Stage 4: When an offer made by i^* has been rejected at stage 3, player j^*, the proposer chosen among the set of players $N\backslash\{i^*\}$ and player i^* bid for the right to take the role of the proposer (the game played, in fact, coincides with the stage 1 game with $n = 2$). Player i^* and player j^* simultaneously submit bids $\widetilde{b}_{j^*}^{i^*}$ and $\widetilde{b}_{i^*}^{j^*}$ in $\mathbb{R}$. The player with the larger net bid pays the bid to the other player and assumes the role of the proposer. In case of identical bids the proposer is chosen randomly.

Stage 5: Depending on whether the proposer is i^* or j^*, the game proceeds as in Mechanism A1 (when i^* is the proposer) or Mechanism A2 (when j^* is the proposer). The payoffs are adjusted to take into account the bidding at stage 4.

Theorem 3.3 *Mechanism A3 implements the consensus value of a zero-monotonic game* (N, v) *in SPE.*

Proof.

The proof is again similar to that of Theorem 3.1. The differences are once more in the construction of the SPE strategies and in Claim (f). Hence, we explicitly construct an SPE that yields the consensus value and show that the Claim (f) (that payoffs must coincide with the consensus value) also holds.

To construct an SPE yielding the consensus value consider the following strategies.

At stage 1, each player $i \in N$ announces $b_j^i = \psi_j(N,v) - \psi_j(N\backslash\{i\}, \widehat{v}^{-i})$,[4] for every $j \in N\backslash\{i\}$.

At stage 2, a proposer, player i^*, offers $x_j^{i^*} = \psi_j(N\backslash\{i^*\}, \widehat{v}^{-i})$ to every $j \in N\backslash\{i^*\}$.

At stage 3, any player $j \in N\backslash\{i^*\}$ accepts any offer which is greater than or equal to $\psi_j(N\backslash\{i^*\}, \widehat{v}^{-i^*})$ and rejects any offer strictly less than $\psi_j(N\backslash\{i^*\}, \widehat{v}^{-i^*})$.

At stage 4, player i^* announces $\widetilde{b}_{j^*}^{i^*} = v(N\backslash\{i^*\}) + \frac{v(N)-v(N\backslash\{i^*\})-v(i^*)}{2} - v(N\backslash\{i^*\}) = \frac{v(N)-v(N\backslash\{i^*\})-v(i^*)}{2}$ while player j^* announces $\widetilde{b}_{i^*}^{j^*} = v(i^*) + \frac{v(N)-v(i^*)-v(N\backslash\{i^*\})}{2} - v(i^*) = \frac{v(N)-v(i^*)-v(N\backslash\{i^*\})}{2}$.

At stage 5, player i^* makes an offer $\widetilde{x}_{j^*}^{i^*} = v(N\backslash\{i^*\})$ to j^* and player i^* makes an offer $\widetilde{x}_{i^*}^{j^*} = v(i^*)$ to i^*. Moreover, i^* accepts any offer greater than or equal to $v(i^*)$ and rejects any offer strictly less than it. Similarly, j^* accepts any offer greater than or equal to $v(N\backslash\{i^*\})$ and rejects any offer strictly less than it.

One can readily verify that these strategies yield the consensus value for any player and constitute an SPE.

To show that in any SPE each player's final payoff coincides with her consensus value, we note that if i is the proposer her final payoff is given by $v(N)-(v(N\backslash\{i\})+\frac{v(N)-v(N\backslash\{i\})-v(i)}{2})-\sum_{j\neq i} b_j^i$ whereas if $j \neq i$ is the proposer, the final payoff of i is $\psi_i(N\backslash\{j\}, \widehat{v}^{-j}) + b_i^j$.

Hence the sum of payoffs to player i over all possible choices of the proposer is (note that all net bids are zero, which can be proved by the equal welfare loss property of the consensus value (Ju et. al. (2004)))

$$
\begin{aligned}
& v(N) - \left(v(N\backslash\{i\}) + \frac{v(N) - v(N\backslash\{i\}) - v(i)}{2}\right) - \sum_{j\neq i} b_j^i + \sum_{j\neq i} \left(\psi_i(N\backslash\{j\}, \widehat{v}^{-j}) + b_i^j\right) \\
= \quad & \frac{v(N) - v(N\backslash\{i\}) + v(i)}{2} + \sum_{j\neq i} \left(\frac{1}{2}\phi_i(N\backslash\{j\}, \widehat{v}^{-j}) + \frac{1}{2}\phi_i^{es}(N\backslash\{j\}, \widehat{v}^{-j})\right) \\
= \quad & \frac{v(N) - v(N\backslash\{i\}) + v(i)}{2} + \frac{1}{2}\sum_{j\neq i} \left(\phi_i(N\backslash\{j\}, v|_{N\backslash\{j\}}) + \frac{\frac{v(N)-v(N\backslash\{j\})-v(j)}{2}}{n-1}\right) \\
+ \quad & \frac{1}{2}\sum_{j\neq i} \left(v(i) + \frac{\frac{v(N)+v(N\backslash\{j\})-v(j)}{2} - \sum_{k\in N\backslash\{j\}} v(k)}{n-1}\right) \\
= \quad & \frac{1}{2}\left(v(N) - v(N\backslash\{i\}) + \sum_{j\neq i} \phi_i(N\backslash\{j\}, v|_{N\backslash\{j\}})\right) + \frac{1}{2}\left(nv(i) + \left(v(N) - \sum_{l\in N} v(l)\right)\right)
\end{aligned}
$$

[4]The game $(N\backslash\{i\}, \widehat{v}^{-i})$ is formally defined by $\widehat{v}^{-i}(N\backslash\{i\}) = v(N\backslash\{i\}) + \frac{v(N)-v(N\backslash\{i\})-v(i)}{2}$ and $\widehat{v}^{-i}(S) = v(S)$, for all $S \subset N\backslash\{i\}$.

$$
\begin{aligned}
&= n\left(\frac{1}{2}\phi_i(N,v) + \frac{1}{2}\phi_i^{es}(N,v)\right) \\
&= n\psi_i(N,v).
\end{aligned}
$$

Since the payoffs are the same regardless of who is the proposer, the payoff of each player in any equilibrium must coincide with the consensus value. ■

As discussed earlier Mechanism A3 requires both proposers to compete for the right to make a further proposal and *a priori* both have equal power. However, what happens if the mechanism treats the players asymmetrically: bids made by one player are "worth more" than those made by the other. Such a mechanism implements the α-consensus value (cf. Ju et. al. (2004)) of a zero-monotonic game in SPE.

The mechanisms constructed can be adapted in several ways. One option is to vary the treatment of a proposer in case she makes an offer that is rejected. We could make it less attractive to make an offer that is rejected, steering the players to end the game sooner rather than later. In the mechanisms to implement the Shapley value, the new rule would allow for any arbitrary payoff $\theta^{i^*} \leq v(i^*)$ to be given to the proposer i^* at stage 5 in case no agreement is reached, whereas the rest of the players still obtain $v(N\backslash\{i^*\})$ if coalition $N\backslash\{i^*\}$ forms. The difference between $v(i^*)$ and θ^{i^*} may be interpreted as a punishment. This mechanism would encourage the players to make acceptable offers and lead to larger coalitions similar to Moldovanu and Winter (1994) where it is stated that "we assume that each player prefers to be a member of large coalitions rather than smaller ones provided that he earns the same payoff in the two agreements" and Hart and Mas-Colell (1996) "both proposers and respondents break ties in favor of quick termination of the game".

The extreme case is where the proposer receives zero in case an offer is rejected and stages 4 and 5 are the same as in Mechanism A2. This mechanism implements the egalitarian solution.[5] Moreover, one can implement any convex combination of the egalitarian solution and the Shapley value using a construction similar to that implementing the α-consensus value.

4 Implementation in "better informed" environments

In the previous section the players were fully informed as regards the characteristic function v, whereas the "designer" of the mechanism had no knowledge of what different coalitions

[5] For a TU game (N,v), the egalitarian solution, denoted by ϕ^{eg}, is defined by $\phi_i^{eg}(N,v) = \frac{v(N)}{n}$ for all $i \in N$.

can achieve. In this section we adopt a different informational structure. This serves two purposes. The first is that it provides a much wider scope for applying our mechanisms and shows how they can be easily adapted to handle versatile environments. The second is that as more information is made available, the solution concepts we discussed can be implemented without imposing any further conditions, such as zero-monotonicity, on the environment.

The different informational structure is introduced by assuming that the players in addition to being fully informed with respect to the characteristic function, can also, if necessary, prove what each coalition of players can obtain. Put differently, the value of each coalition can not only be observed but also verified by an outside authority if needed. The designer having the ability to verify coalition values if necessary, can design slightly different bidding mechanisms to work in such environments. One such conceivable scenario is where a set of players (heirs), $N = \{1, ..., n\}$, have to divide a sum (estate) of known size, $v(N)$. Furthermore, each subset of the players can prove what part of the sum they are entitled to (have documented claims regarding their part of the estate).

The basic bidding mechanism we now construct, can be informally described as follows: Stages 1, 2 and 3 are the same as in previous mechanisms up to the point where an offer is rejected at stage 3. In case of rejection all the players other than the proposer play a similar game with one player less. The different mechanisms will however have them sharing pies of different sizes.

In the first variant, yielding the Shapley value, the remaining players bargain over their prescribed coalitional payoff, and the rejected proposer receives the difference between $v(N)$ and that coalitional payoff.

In the second variant, yielding the equal surplus value, the rejected proposer, say i, gets her stand-alone payoff $v(i)$ and all other players play the same game again, bargaining over what remains of $v(N)$, i.e., $v(N) - v(i)$.

The third variant takes, as before, a less extreme approach and shares the benefits of rejoining to form the grand coalition between the rejected proposer and the other players. Once an offer is rejected, we move from the status-quo outcome where proposer i gets $v(i)$ and the remaining players bargain over $v(N\backslash\{i\})$ to a new starting point where the rejected proposer receives $v(i) + \frac{1}{2}(v(N) - v(i) - v(N\backslash\{i\}))$. and the remaining players bargain over $v(N\backslash\{i\}) + \frac{1}{2}(v(N) - v(i) - v(N\backslash\{i\}))$. Hence each obtains half of the surplus generated by rejoining to form the grand coalition.

As one can see from above, this approach requires a game to proceed in three stages only. Below we formally describe the bidding games, focusing only on the rules in case where the offer made by a proposer (chosen in the bidding stage) has been rejected. All other rules of these games are the same.

Mechanism B1. At stage 3, if the offer made by i^* is rejected, all players other than i^* proceed to play a similar game where the set of players is $N \backslash \{i^*\}$ and they will bargain over $v(N \backslash \{i^*\})$, and player i^* leaves the game and receives $v(N) - v(N \backslash \{i^*\})$ from this stage. The final payoff to player i^* is then $v(N) - v(N \backslash \{i^*\}) - \sum_{j \neq i^*} b_j^{i^*}$. The final payoff to any player $j \neq i^*$ is the payoff he obtains in the game played by $N \backslash \{i^*\}$ plus the bid $b_j^{i^*}$.

Theorem 4.1 *Mechanism B1 implements the Shapley value of an arbitrary cooperative game (N, v) in SPE.*

Proof.
Since the proof follows the same line as that of Theorem 3.1, we will skip most of it and stress just two aspects to illustrate the way the proof proceeds. First, to show that the Shapley value is an SPE outcome, one can consider the strategies of the first three stages provided in Theorem 3.1. Second, we explicitly provide Claim (c) below to describe the full set of SPE.

Claim (c). For the game that starts at stage 2 there exist two types of SPE. One is that at stage 2 player i^* offers $x_j^{i^*} = \phi_j(N \backslash \{i^*\})$ to all $j \neq i^*$ and, at stage 3, every player $j \neq i^*$ accepts any offer $x_j^{i^*} \geq \phi_j(N \backslash \{i^*\})$ and rejects the offer otherwise. The other is that at stage 2 the proposer offers $x_j^{i^*} \leq \phi_j(N \backslash \{i^*\})$ to some players $j \neq i^*$ and, at stage 3, any player $j \in N \backslash \{i^*\}$ rejects any offer $x_j^{i^*} \leq \phi_j(N \backslash \{i^*\})$. ■

As one can see, the key feature of Mechanism B1 (implementing the Shapley value) is that it specifies a rule giving $v(N) - v(N \backslash \{i^*\})$ to proposer i^* if her offer is rejected at stage 3 and the rest of the players are guaranteed with bargaining over $v(N \backslash \{i^*\})$. Is this rule acceptable in practice? How about the other possible ways in dealing with the situation when an offer is rejected? Different contexts may call for different treatments. An opposite choice to Mechanism B1 may follow this argument: In return to the highest net bid made by proposer i^*, she should be guaranteed with her stand-alone payoff $v(i^*)$ in case of the offer rejected so that the remaining players get the residual, i.e., $v(N) - v(i^*)$.

Mechanism B2. At stage 3, if the offer is rejected, proposer i^* leaves the game with $v(i^*)$ from this stage, whereas all other players proceed to play a similar game where the set of players is $N\backslash\{i^*\}$ and they bargain over $v(N)-v(i^*)$. The final payoff to player i^* is then $v(i^*)-\sum_{j\neq i^*} b_j^{i^*}$. The final payoff to any player $j\neq i^*$ is the payoff he obtains in the game played by $N\backslash\{i^*\}$ plus the bid $b_j^{i^*}$.

Theorem 4.2 *Mechanism B2 implements the equal surplus value of an arbitrary cooperative game* (N,v) *in SPE.*

Proof.
The proof is analogous to that of Theorem 3.2. ∎

Following the same reasoning as in the previous section one is naturally led to another mechanism described as follows.

Mechanism B3. When an offer is rejected at stage 3, both parties, proposer i^* and the remaining players $N\backslash\{i^*\}$ first get their *status quo* payoffs, and then share the surplus $v(N)-v(i^*)-v(N\backslash\{i^*\})$ equally. That is, i^* leaves the game with her stand-alone payoff $v(i^*)$ plus half of the surplus, i.e., $\frac{v(N)-v(i^*)-v(N\backslash\{i^*\})}{2}$, from this stage, whereas all other players proceed to play a similar game with the set of players $N\backslash\{i^*\}$ and bargain over $v(N\backslash\{i^*\})+\frac{v(N)-v(i^*)-v(N\backslash\{i^*\})}{2}$. The final payoff to player i^* is then $v(i^*)+\frac{v(N)-v(i^*)-v(N\backslash\{i^*\})}{2}-\sum_{j\neq i^*} b_j^{i^*}$. The final payoff to any player $j\neq i^*$ is the payoff he obtains in the game played by $N\backslash\{i^*\}$ plus the bid $b_j^{i^*}$.

Theorem 4.3 *Mechanism B3 implements the consensus value of an arbitrary game* (N,v) *in SPE.*

Proof.
Adopting the same idea as that for proving Theorem 3.1 and Theorem 3.3, the proof can be readily constructed. ∎

We want to note that, by suitable modifications, other results in section 3 can be obtained in this environment as well.

5 Conclusion

In this paper we provided a unified framework to implement and study values for transferable utility environments. The main building block is a bidding mechanism that starts by having the players bid for the role of the proposer. The proposer makes an offer to all the remaining players, if the offer is accepted the game ends. In case of rejection the remaining players play the same game again. Once this process ends, the first proposer "re-enters" the game, to play against the proposer ("second proposer") chosen form the remaining players. From here onwards the mechanisms differ. In order to implement the Shapley value the original proposer has the right to make another offer before the game ends. To achieve the equal surplus value the second proposer is awarded that right. The consensus value is implemented when the two proposers bid for the right to make another offer. In effect, any average of the Shapley and equal surplus values can be achieved by suitably adjusting the rules of the mechanism for the two proposers' interaction. These results are valid for any transferable utility game satisfying zero-monotonicity. We also showed that in case where the payoffs that different coalitions can obtain are verifiable by an outside party, the mechanism can be modified to implement the above solution concepts in any transferable utility environment.

The design of a single basic mechanism to implement several cooperative solution concepts serves twin purposes. On one hand it provides a robust non-cooperative foundation for the application of various solutions and on the other hand it makes it possible to examine them critically by the rules needed to implement them. This might provide important insights as the rules of the game are "quite detached" from the axioms generating these values.

There are several possible extensions of the "generalize bidding" approach to other cooperative environments and solution concepts. For games in partition function form, the use of similar mechanisms can complement results obtained by Maskin (2003) and Macho-Stadler et. al. (2005) by implementing values proposed by Pham Do and Norde (2002) and Ju (2004). For games with a coalition structure, these mechanisms can serve as an alternative way of implementing the Owen value (Owen 1977) which was implemented by Vidal-Puga and Bergantiños (2003) for strictly superadditive games. Recently, Ju and Borm (2005) introduced a new class of games, primeval games, to model inter-individual externalities and analyze compensation rules from a normative point of view. The implementability of these compensation rules via generalized bidding mechanisms is another interesting direction of research.

Moving away from general cooperative environments, the mechanisms constructed in this paper can also resolve distributional problems in many concrete settings such as cost-sharing environments, bankruptcy disputes and dissolution of partnerships.

References

[1] Baliga, S. and S. Brusco (2000), Collusion, Renegotiation and Implementation, *Social Choice and Welfare* 17: 69-83.

[2] Brink, R van den and Y. Funaki (2004), Axiomatizations of a Class of Equal Surplus Sharing Solutions for Cooperative Games with Transferable Utility, *Tinbergen Institute Discussion Paper*, TI 2004-136/1, Tinbergen Institute, Amsterdam/Rotterdam, the Netherlands.

[3] Brink, R van den, Y. Funaki and Y. Ju (2005), Axiomatizations of the Convex Combinations of a Class of Equal Surplus Solutions and the Shapley Value, *Manuscript*, Free University Amsterdam, the Netherlands.

[4] Dasgupta, A. and Y.S. Chiu (1998), On Implementation via Demand Commitment Games, *International Journal of Game Theory* 27: 161-189.

[5] Driessen, T.S.H. and Y. Funaki (1991), Coincidence of and Collinearity between Game Theoretic Solutions, *OR Spektrum* 13: 15-30.

[6] Gul, F. (1989), Bargaining Foundations of Shapley Value, *Econometrica* 57: 81-95.

[7] Gul, F. (1999), Efficiency and Immediate Agreement: a Reply to Hart and Levy, *Econometrica* 67: 913-918.

[8] Hart, S. and A. Mas-Colell (1996), Bargaining and Value, *Econometrica* 64: 357-380.

[9] Hart, O. and J. Moore (1990), Property Rights and the Theory of the Firm, *Journal of Political Economy* 98: 1119-1158.

[10] Ju, Y. (2004), The Consensus Value for Games in Partition Function Form, *CentER Discussion Paper*, 2004-60, Tilburg University. To appear in *International Game Theory Review.*

[11] Ju, Y., P. Ruys and P. Borm (2004), The Consensus Value: a New Solution Concept for Cooperative Games, *CentER Discussion Paper*, 2004-50, Tilburg University, the Netherlands.

[12] Ju, Y. and P. Borm (2005), Externalities and Compensation: Primeval Games and Solutions, *Keele Economics Research Paper*, 2005-05, Keele University, UK.

[13] Krishna, V. and R. Serrano (1995), Perfect Equilibria of a Model of n-person Noncooperative Bargaining, *International Journal of Game Theory* 27: 259-272.

[14] Macho-Stadler, I., D. Pérez-Castrillo and D. Wettstein (2005), Efficient Bidding with Externalities, *CREA Working Paper*, 159, Universitat Autònoma de Barcelona, Spain. To appear in *Games and Economic Behavior*.

[15] Maniquet, F. (2003), Implementation under Perfect Information in Economic Environments, *Social Choice and Welfare* 21: 323-346.

[16] Maskin, E. (2003), Bargaining, Coalitions and Externalities, presidential address of the Econometric Society.

[17] Maskin, E. and J. Moore (1999), Implementation and Renegotiation, *Review of Economic Studies* 66: 39-56.

[18] Moldovanu, B. and E. Winter (1994), Core Implementation and Increasing Returns to Scale for Cooperation, *Journal of Mathematical Economics* 23: 533-548.

[19] Moore, J. and R. Repullo (1988), Subgame Perfect Implementation, *Econometrica* 58: 1083-1099.

[20] Moulin, H. (2003), *Fair Division and Collective Welfare*, MIT Press.

[21] Myerson, R.B. (1980), Conference Structures and Fair Allocation Rules, *International Journal of Game Theory* 9: 169-182.

[22] Nash, J.F. (1953), Two Person Cooperative Games, *Econometrica* 21: 128-140.

[23] Owen, G. (1977), Values of Games with a priori Unions, in Henn, R. and O. Moeschlin (eds.) *Essays in Mathematical Economics and Game Theory*, Springer-Verlag, Berlin, pp. 76-88.

[24] Pérez-Castrillo, D. and D. Wettstein (2001), Bidding for the Surplus: a Non-Cooperative Approach to the Shapley Value, *Journal of Economic Theory* 100: 274-294.

[25] Pérez-Castrillo, D. and D. Wettstein (2002), Choosing Wisely: a Multi-bidding Approach, *American Economic Review* 92: 1577-1587.

[26] Pham Do, K.H. and H. Norde (2002), The Shapley Value for Partition Function Form Games, *CentER Discussion Paper*, No. 2002-04, Tilburg University. To appear in *International Game Theory Review*.

[27] Rubinstein, A. (1982), Perfect Equilibrium in a Bargaining Model, *Econometrica* 50: 97-109.

[28] Serrano, R. (2005), Fifty Years of the Nash Program, 1953-2003, *Investigaciones Economicas* 29: 219-258.

[29] Shapley, L. S. (1953), A Value for n-Person Games, in Kuhn, H.W. and A.W. Tucker (eds.) *Contributions to the Theory of Games II*, Princeton University Press, Princeton, NJ, pp. 307-317.

[30] Vidal-Puga, J. and G. Bergantiños (2003), An Implementation of the Owen Value, *Games and Economic Behavior* 44: 412-427.

[31] Winter, E. (1994), The Demand Commitment Bargaining Game and Snowballing Cooperation, *Economic Theory* 4: 255-273.