

Tilburg University

Fiscal Policy, Monopolistic Competition and Finite Lives

Heijdra, B.J.; Ligthart, J.E.

Publication date:
2005

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Heijdra, B. J., & Ligthart, J. E. (2005). *Fiscal Policy, Monopolistic Competition and Finite Lives*. (CentER Discussion Paper; Vol. 2005-126). Macroeconomics.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Center



Discussion Paper

No. 2005–126

FISCAL POLICY, MONOPOLISTIC COMPETITION, AND FINITE LIVES

By Ben J. Heijdra, Jenny E. Ligthart

December 2005

ISSN 0924-7815

Fiscal Policy, Monopolistic Competition, and Finite Lives*

Ben J. Heijdra[‡]

University of Groningen and Netspar

Jenny E. Ligthart[‡]

Tilburg University and University of Groningen

Revised: December 2, 2005

Abstract

The paper studies the short-run, transitional, and long-run output effects of permanent and temporary shocks in public consumption under various financing methods. To this end, a dynamic macroeconomic model for a closed economy is developed, which features a perfectly competitive final goods sector and a monopolistically competitive intermediate goods sector. Finitely lived households consume final goods, supply labor, and save part of their income. Amongst the findings for a permanent rise in public consumption are: (i) monopolistic competition increases the absolute value of the balanced-budget output multiplier; (ii) positive long-run output multipliers are obtained only if the generational turnover effect is dominated by the intertemporal labor supply effect; (iii) short-run output multipliers under lump-sum tax financing are smaller than long-run output multipliers if labor supply is elastic; and (iv) bond financing reduces the size of long-run output multipliers as compared to lump-sum tax financing and may give rise to non-monotonic adjustment paths if labor supply is sufficiently elastic and the speed of adjustment of lump-sum taxes is not too high. Temporary bond-financed fiscal shocks are shown to yield: (i) permanent effects on output; and (ii) negative long-run output multipliers.

JEL codes: E12, E63, L16.

Keywords: Fiscal policy, output multipliers, Yaari-Blanchard model, overlapping generations, monopolistic competition, love of variety, temporary fiscal shocks.

*The authors would like to thank Leon Bettendorf, Peter Broer, Peter Ireland, and an anonymous referee for helpful comments. Jenny Ligthart gratefully acknowledges financial support from the Dutch Ministry of Finance and the hospitality of CESifo while working on the project.

[‡]Department of Economics, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands, Phone: +31-50-363-7303, Fax: +31-50-363-7207, E-mail: b.j.heijdra@rug.nl.

[‡]CentER and Department of Economics, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands, Phone: +31-13-466-8755, Fax: +31-13-466-3042, E-mail: j.ligthart@uvt.nl; and Department of Economics, University of Groningen.

1 Introduction

The recent economic downturns in Europe and the United States have revived the long-standing debate on the role fiscal policy can play in stimulating economic activity. The widely held belief among both academics and policy makers—firmly rooted in the traditional Keynesian view—is that public spending multipliers exceed unity. Empirical studies yield short-run public spending multipliers typically ranging from 0.6 to 1.4, whereas long-run multipliers are generally smaller, reflecting crowding out effects.¹ The simple Keynesian view assumes perfect competition on goods and labor markets, exogenously imposed price rigidity, and excess capacity, so that output is demand determined. In the last decade, however, a number of authors have stressed that output and employment multipliers in an imperfectly competitive environment are likely to exceed those under perfect competition.² These studies typically employ static models of monopolistic competition in the New Keynesian tradition, featuring explicit price setting and clearing labor and goods markets.

The main objective of this paper is to study theoretically the dynamic macroeconomic effects of fiscal policy. To this end, we develop a dynamic (non-stochastic) monopolistically competitive model, featuring overlapping generations (OLG) in the tradition of Yaari (1965) and Blanchard (1985)—in which households face a constant probability of death—and endogenous intertemporal labor supply. Our framework builds on the twin pillars of the Keynesian view—monopolistic competition and the failure of Ricardian equivalence. We employ an extended Yaari-Blanchard model to investigate in what way productivity effects associated with monopolistic competition and wealth effects related to finitely lived agents affect the *size* and the *sign* of balanced-budget output multipliers.³ In keeping with the recent literature, we reserve a central role for exit and entry of firms, which makes it possible to relate our findings to those found of models based on infinitely lived households. We consider various modes of financing (lump-sum taxes and government bonds) of two types of shocks (permanent and temporary) to public consumption. The analysis of temporary fiscal shocks allows us to link our results with those found in Vector Autoregressive (VAR) studies.⁴

Studies analyzing the long-run output effects of fiscal policy generally develop dynamic general equilibrium models with New Keynesian features. Most authors work in the RBC

¹Based on a review of simulations with calibrated large-scale macroeconomic models. See Hemming, Kell, and Mahfouz (2002).

²See, for example, Startz (1989), Molana and Moutos (1992), and Heijdra and van der Ploeg (1996). The latter have shown that free entry of firms may have important productivity enhancing effects.

³Simple Keynesian multipliers measure the effect on output of an exogenous increase in public spending not taking into account its financing, implying a deterioration of the fiscal balance. Unlike the simple Keynesian view, our fiscal multipliers are ‘balanced budget’ under lump-sum taxation and explicitly take into account the intertemporal government budget constraint under bond financing.

⁴Burnside, Eichenbaum, and Fisher (2004), Fatás and Mihov (2001), and Mountford and Uhlig (2002), employ VAR models to study the dynamic effects of fiscal policy and to compare the results with those of calibrated general equilibrium models.

tradition by allowing for stochastic shocks, a notable exception is the deterministic framework of Heijdra (1998). The literature is still relatively small. Early work by Devereux, Head, and Lapham (1996b) and Heijdra (1998)—who assume infinitely lived households—abstracts from nominal rigidities, whereas more recent contributions, including Galí, López-Salido, and Vallés (2004), Linnemann and Schabert (2003), Linnemann (2004), and Coenen and Straub (2004) model some form of price or wage stickiness.⁵

The reason we focus on monopolistic competition is that it is the key pillar of New Keynesian economics. A number of authors working in the real business cycle (RBC) tradition⁶ have shown—employing dynamic stochastic macroeconomic models—that monopolistic competition plays a vital role in explaining output persistence. Numerical results by Chatterjee and Cooper (1993) indicate that a model with free entry and exit of firms exhibits a slower adjustment speed—and thus more output persistence—than a perfectly competitive model or an increasing-returns-to-scale model with a fixed number of firms. Since the lack of a quantitatively significant propagation mechanism is widely considered to be an important weakness of RBC models (Stadler, 1994, p. 1769), monopolistic competition may therefore have a useful role to play in the analysis of fiscal spending shocks, not only in RBC frameworks but also in non-stochastic dynamic general equilibrium models more generally.

We extend the Yaari-Blanchard framework to incorporate endogenous (intertemporal) labor supply, which allows us to: (i) generate meaningful short-run output effects in a model that features a predetermined capital stock; and (ii) model a shock propagation mechanism, which is crucial to the multiplier mechanism of fiscal spending. The early studies show that an unanticipated and permanent increase in government consumption financed in a lump-sum fashion gives rise to a negative wealth effect that increases labor supply. The saving-investment accelerator propagates the shock and thus helps to explain a positive effect on output and employment. The productivity effects associated with the free entry and exit of firms magnifies this labor supply effect so that even relatively small economies of scale can make a major difference to the size of the long-run output effect of government spending. In contrast, under exogenous labor supply and infinitely lived households, long-run output effects of fiscal policy are zero, reflecting full crowding out of private by public consumption.

The novelty of our approach is the introduction of a population of finitely lived households in a modified representation of Heijdra’s (1998) framework.⁷ The Blanchard-Yaari framework has also been used in RBC models of a small open economy to obtain a unique and stationary

⁵We do not model price adjustment costs, given that our model does not include a monetary sector. Because of the absence of price stickiness, our work is most closely related to Devereux, Head, and Lapham (1996b) and Heijdra (1998). Coto-Martínez (2006) also follows this tradition, but his work focuses on public capital instead of public consumption.

⁶See the work of Chatterjee and Cooper (1993), Devereux, Head, and Lapham (1993, 1996a-b), Bénassy (1996b), Rotemberg and Woodford (1995), and Woodford (2001).

⁷Leith and Wren-Lewis (2000) also employ the Yaari-Blanchard framework, which they use to study the interactions between monetary and fiscal policy rules.

steady state. See Cardia (1991) and, more recently, Cavallo and Ghironi (2002).⁸ We consider a closed economy, in which the rate of interest is endogenous, implying that finite lives are not needed as a stationarity inducing device. More importantly, the finite lives extension is of interest because it has substantive implications of its own. Ricardian equivalence breaks down, so that bond financing has now a meaningful role in generating results that differ from those under lump-sum taxes. A number of new results emerge.

Under bond financing of a permanent fiscal impulse the size of long-run output multipliers is significantly reduced as compared to lump-sum tax financing. Bond-financed fiscal shocks may give rise to non-monotonic transition paths, irrespective of the type of shock, if labor supply is sufficiently elastic and the adjustment speed of lump-sum taxes is not too high. Temporary shocks are shown to have permanent effects, whereas in an infinite horizon framework the economy is left unaffected in the long run. Generally, we find a negative correlation between private consumption and output in the short run. However, a temporary rise in public consumption combined with an initial cut in lump-sum taxes, generates a positive correlation between private consumption and output in the medium and long run, providing a partial underpinning of the evidence found in VAR studies.

Under lump-sum tax financing of a permanent fiscal impulse, finite lives lowers the size of output multipliers, and possibly even changes their sign as compared to infinite horizons. Such a sign change occurs if private consumption and investment are crowded out in the long run, owing to a generational-turnover effect that dominates the conventional labor supply effect. If labor supply is exogenous, a negative long-run output effect is sure to materialize against a zero value obtained for the infinite horizon model. Numerical evidence, however, suggests that the generational-turnover effect is relatively weak for plausible parameter values, and as such is unlikely to overturn results derived under an infinite lives assumption.

Another contribution of our paper is that we are able to characterize analytically the short-run, transitional, and long-run effects of permanent and temporary fiscal shocks. In contrast, Devereux, Head, and Lapham (1996b) and many other RBC studies only obtain analytical results for the long-run effects, and have to resort to numerical simulations to compute the impact and transitional effects. We log-linearize the model and then solve it by making use of the Laplace transform technique pioneered by Judd (1982, 1998). We are able to trace out theoretical impulse responses of public spending shocks at business cycle frequencies. The impulse response functions can be shown to depend in a simple way on the structural parameters of the model. For permanent fiscal shocks, we have developed a simple diagrammatic apparatus to present the adjustment paths after a policy change and to

⁸For the existence of a stationary equilibrium in the representative agent framework, the (fixed) pure rate of time preference must be equal to the real rate of interest, which is exogenous in a small open economy (i.e., determined on world capital markets). With finite lives the world rate of interest need not be equal to pure rate of time preference (see Blanchard, 1985, pp 230-231). See also Schmitt-Grohé and Uribe (2003) for an overview of alternative ways to induce stationarity.

demonstrate the pivotal role of the intertemporal elasticity of labor supply.

The paper is structured as follows. Section 2 develops the basic dynamic OLG model, featuring a perfectly competitive final goods sector and a monopolistically competitive intermediate goods sector. Section 3 solves the model both analytically and graphically and analyzes the dynamic properties of the model. Section 4 studies analytically the output effects of a permanent rise in government consumption financed by lump-sum taxes. In addition, it presents some numerical exercises to demonstrate the quantitative workings of the model. Section 5 analyzes how the results under lump-sum taxation are affected by bond financing of permanent fiscal shocks. It also studies numerically the effects of temporary fiscal shocks. Finally, Section 6 concludes and provides directions for further research.

2 A Two-Sector Model

2.1 Firms

The economy consists of two types of firms. There are monopolistically competitive firms, each of which produces a slightly unique variety of an intermediate input, and perfectly competitive firms, which produce final output using intermediate goods. This way of modeling the firm sector is a modified representation of Hornstein (1993).

2.1.1 Final Goods Sector

Technology in the final goods sector is described by a Dixit-Stiglitz (1977) aggregator function:

$$Y(t) = N(t)^{\eta-\mu} \left[\int_0^{N(t)} Z_i(t)^{1/\mu} di \right]^\mu, \quad \eta \geq 1, \mu > 1, \quad (1)$$

where $Y(t)$ denotes aggregate output of final goods, $Z_i(t)$ is the quantity of variety i of the differentiated intermediate good, $N(t)$ is the number of input varieties, and t denotes time. The parameter η regulates the productivity effect of increased input variety, and $\mu/(\mu - 1)$ is the constant elasticity of substitution between any pair of input varieties.⁹ The production function of the final goods sector (1) implies *external* economies of scale, owing to increasing diversity, provided $\eta > 1$. This is the basic Ethier (1982) effect: more diversity in the differentiated goods sector enables final goods producers to use a more roundabout production process, which lowers unit cost.¹⁰

⁹Our production function (1) is more general than the one used by Hornstein (1993) and Devereux, Head, and Lapham (1996b) in that the diversity effect (η) and the price elasticity of input demand ($\mu/(1 - \mu)$) are parameterized separately. Ethier (1982), Heijdra and Van der Ploeg (1996), Devereux, Head, and Lapham (1996a), Bénassy (1996a-b), and Dixit and Stiglitz ([1975], 2004) also explicitly distinguish the two conceptually different effects.

¹⁰Note that these external scale economies only become effective if the number of firms is allowed to change. Holding constant $N(t)$, the technology (1) features constant returns to scale in the $Z_i(t)$ inputs.

The representative producer in the final goods sector minimizes the cost of producing a given quantity of final goods, $p(t)Y(t)$, by choosing the optimal mix of different input varieties, where $p(t)$ is unit cost:

$$p(t) \equiv N(t)^{\mu-\eta} \left[\int_0^{N(t)} p_i(t)^{1/(1-\mu)} di \right]^{1-\mu}, \quad (2)$$

and $p_i(t)$ is the price of input variety i . The input demand functions are obtained by applying Shephard's lemma to (2):

$$Z_i(t) = N(t)^{(\eta-\mu)/(\mu-1)} Y(t) \left(\frac{p(t)}{p_i(t)} \right)^{\mu/(\mu-1)}, \quad (3)$$

and feature a constant elasticity of demand. Output of the final goods sector is either consumed (by households or the government) or invested to augment the aggregate capital stock.

2.1.2 Intermediate Goods Sector

The intermediate goods sector consists of $N(t)$ monopolistically competitive firms, each of which produces a single differentiated input. The typical firm i rents capital, $K_i(t)$, and labor, $L_i(t)$, from the household sector. The *gross* production function of a firm, $F(\cdot)$ is given by:

$$Z_i(t) + \Phi = F(L_i(t), K_i(t)) \equiv L_i(t)^{\varepsilon_L} K_i(t)^{1-\varepsilon_L}, \quad 0 < \varepsilon_L < 1, \quad (4)$$

where $Z_i(t)$ is *net* marketable production of input variety i , Φ are fixed costs modeled in terms of the own output of firm i . The firm's cost function is:

$$TC_i(t) \equiv \left(\frac{w(t)}{\varepsilon_L} \right)^{\varepsilon_L} \left(\frac{r(t) + \delta}{1 - \varepsilon_L} \right)^{1-\varepsilon_L} p(t) (Z_i(t) + \Phi), \quad (5)$$

where $w(t)$ is the real wage rate, $r(t)$ is the real rate of interest, and δ is the rate of depreciation of capital. Each firm in the intermediate goods sector faces a downward-sloping demand curve for its own input variety from producers in the final goods sector (see (3)). Firms maximize profits—by choosing their price and factor demands—subject to (3) and (5). As a result, the price of input variety i is set equal to a constant markup, μ , over marginal cost:

$$p_i(t) = \mu \left(\frac{\partial TC_i(t)}{\partial Z_i(t)} \right) = \left(\frac{\mu}{\rho_i(t)} \right) \left(\frac{TC_i(t)}{Z_i(t)} \right), \quad (6)$$

where $\rho_i(t) \equiv (Z_i(t) + \Phi)/Z_i(t) > 1$ measures (local) internal increasing returns to scale due to the existence of fixed costs. Furthermore, the factor demands by firm i are determined by the usual marginal productivity conditions for labor and capital:

$$\frac{\partial Z_i(t)}{\partial L_i(t)} = \mu w(t), \quad \frac{\partial Z_i(t)}{\partial K_i(t)} = \mu (r(t) + \delta). \quad (7)$$

Under Chamberlinian monopolistic competition exit and entry of firms occurs instantaneously, so that all excess profits are eliminated. As a result, the intermediate input price equals average cost:

$$p_i(t) = \frac{TC_i(t)}{Z_i(t)}. \quad (8)$$

By combining (6) and (8), we obtain $\mu = \rho_i(t)$, which implies an equilibrium firm size in the intermediate goods sector of:

$$Z_i(t) = \bar{Z} \equiv \frac{\Phi}{\mu - 1}. \quad (9)$$

2.2 Households

2.2.1 Individual Households

In keeping with Blanchard (1985), there is a fixed population of agents each facing a constant probability of death ($\beta \geq 0$). Generations are disconnected because there are no bequests. During their entire life agents have a time endowment of unity, which they allocate to labor and leisure. The utility functional at time t of the representative agent born at time v is denoted by:

$$\Lambda(v, t) \equiv \int_t^\infty [\varepsilon_C \log C(v, z) + (1 - \varepsilon_C) \log [1 - L(v, z)]] e^{(\alpha + \beta)(t - z)} dz, \quad (10)$$

where $C(v, t)$ and $L(v, t)$ denote private consumption and labor supply in period t by an agent born in period v , respectively, α is the pure rate of time preference ($\alpha > 0$), and ε_C is a preference parameter ($0 < \varepsilon_C < 1$). The agent's budget identity is:

$$\dot{A}(v, t) = [r(t) + \beta] A(v, t) + w(t)L(v, t) - T(t) - C(v, t), \quad (11)$$

where $\dot{A}(v, t) \equiv dA(v, t)/dt$, $A(v, t)$ are real financial assets, $r(t)$ is the real rate of interest, $w(t)$ is the real wage rate (assumed to be age independent), and $T(t)$ are real *net* lump-sum taxes. The final good is used as the numeraire ($p(t) = 1$).

The representative agent, endowed with perfect foresight, chooses a time profile for $C(v, t)$ and $L(v, t)$ in order to maximize $\Lambda(v, t)$ subject to its budget identity (11) and a no-Ponzi-game (NPG) solvency condition. The household's optimal time profile of consumption is:

$$\frac{\dot{C}(v, t)}{C(v, t)} = r(t) - \alpha, \quad (12)$$

and labor supply is linked to private consumption and the wage rate according to (T1.7) in Table 1. Since the aggregate stock of financial assets is positive ($A(t) > 0$), the steady-state rate of interest must exceed the pure rate of time preference, that is, $r > \alpha$.¹¹

¹¹The rising individual consumption profiles (see (12)) ensure that financial wealth is transferred—via life-insurance companies—from deceased to surviving generations in the steady state.

2.2.2 Aggregate Household Sector

A key feature of Blanchard's (1985) model is its simple demographic structure, which enables the analytical aggregation over all currently alive households. At each instant of time a large cohort of size βF is born and βF agents die. Normalizing F to unity, the size of the population is constant and equal to unity. Aggregate variables can be calculated as the weighted sum of the values for the different generations. Aggregate financial wealth is, for example, $A(t) \equiv \int_{-\infty}^t A(v, t) \beta e^{\beta(v-t)} dv$. Similarly, the aggregate values for $C(t)$, $L(t)$, and $T(t)$ can be derived.

The main equations describing the behavior of the aggregate household sector are given by (T1.2) and (T1.7) in Table 1. Equation (T1.2) is the aggregate Euler equation modified for the existence of finitely lived agents. It has the same form as the Euler equation for individual households (equation (12)), except for a correction term, which represents the distributional effects caused by the turnover of generations. Optimal individual consumption *growth* is the same for all generations since they face the same rate of interest. But old generations have a higher private consumption *level* than young generations because they are wealthier. Since existing generations are continually being replaced by newborns, who possess no financial wealth, aggregate consumption growth falls short of individual consumption growth.

2.3 Government

The government's periodic budget identity is given in (T1.3), where $B(t)$ is real government debt at time t . The government consumes $G(t)$ units of the final good. Government spending consists of goods consumption and debt service, which is financed by issuing debt, $\dot{B}(t)$, or by changing the lump-sum tax, $T(t)$. Since the government must remain solvent, the NPG condition is $\lim_{z \rightarrow -\infty} B(z) \exp[-\int_t^z r(s) ds] = 0$, so that (T1.3) can be integrated forward to derive the government's intertemporal budget restriction:

$$B(t) = \int_t^{\infty} [T(z) - G(z)] \exp\left[-\int_t^z r(s) ds\right] dz. \quad (13)$$

Solvency of the government implies that the present value of current and future primary surpluses must be equal to the pre-existing level of government debt.

2.4 Symmetric Equilibrium

The model is symmetric and can thus be expressed in aggregate terms. Equation (9) shows that all existing firms in the intermediate sector are of equal size, \bar{Z} , and hence (by (6)) charge the same price and (by (7)) demand the same amounts of capital and labor, that is, $K_i(t) = \bar{K}(t)$ and $L_i(t) = \bar{L}(t)$. Equation (1) yields the expression for aggregate output in the final goods sector, that is, $Y(t) = N(t)^\eta \bar{Z}$. Hence, aggregate output of final goods is an iso-elastic function of the number of input varieties, $N(t)$.

The main equations of the model are reported in Table 1. Equations (T1.1)-(T1.3) describe the dynamics of the model. The aggregate physical capital stock evolves according to (T1.1), which shows that net investment equals gross investment minus replacement of the worn-out capital stock. The movement of consumption is described by equation (T1.2), which is the aggregate Euler equation corrected for the turnover of generations. We have used the fact that financial wealth is the sum of equity shares¹² and government bonds. The government budget identity is given in (T1.3).

Aggregate demand for labor and capital is given by (T1.4) and (T1.5), respectively. The equilibrium condition for the final goods market is presented in (T1.6), and aggregate labor supply is given in (T1.7). The equilibrium number of firms and the aggregate production function for the final goods sector are both given in (T1.8). Given the equilibrium number product varieties, there are constant returns to scale with respect to the production factors, but increasing returns to scale for aggregate output.¹³

Finally, (T1.4) and (T1.5) can be substituted in (T1.8) to obtain the factor price frontier (T1.9). On the left-hand side of this expression are the factors leading to an outward shift of the factor price frontier, namely traditional productivity gains (a rise in Ω_0) and Ethier productivity effects (a rise in $Y(t)$ if $\eta > 1$).

3 Model Solution

3.1 Stability

The local stability of the model can be studied by log-linearizing it around an initial steady state in which there is no government debt ($B(0) = 0$). Appendix Table 1 presents the main expressions. The state variables are the aggregate physical capital stock (a predetermined variable) and private consumption (a jump variable). By using labor demand (AT1.4), labor supply (AT1.7), and the aggregate production function (AT1.8)—all taken from Appendix Table 1—a ‘quasi-reduced form’ for aggregate output, $\tilde{Y}(t)$, is obtained:

$$\tilde{Y}(t) = \eta\phi(1 - \varepsilon_L)\tilde{K}(t) - (\phi - 1)\tilde{C}(t), \quad (14)$$

where a tilde denotes a relative change, for example, $\tilde{Y}(t) \equiv dY(t)/Y$, and ϕ represents the intertemporal labor supply effect as given in Definition 1.

Definition 1 *The intertemporal labor supply elasticity—as magnified by the diversity effect,*

¹²The market value of claims on the capital stock (that is, shares) is equal to the replacement value of the capital stock, owing to free entry and exit of firms. As a result, $K(t)$ measures both the physical capital stock and the real value of shares.

¹³Foreshadowing the discussion on short-run output multipliers somewhat, equation (T1.8) shows clearly that, as capital is predetermined, output effects occur at impact only if there is a labor supply response.

η —is defined as:

$$\phi \equiv \frac{1 + \theta}{1 + \theta(1 - \eta\varepsilon_L)} \geq 1, \quad (15)$$

where $\theta \equiv (1 - L)/L \geq 0$ is the ratio of leisure to labor (which is the intertemporal substitution elasticity of aggregate labor supply). Note that $\phi = 1$ if labor supply is exogenous.¹⁴ Two cases concerning η can be distinguished:

- (i) If $\eta\varepsilon_L \leq 1$, the sign restriction on ϕ is automatically implied since $\theta \geq 0$. If $\eta\varepsilon_L < 1$, ϕ is a concave function of θ with a positive asymptote of $(1 - \eta\varepsilon_L)^{-1}$ as $\theta \rightarrow \infty$. If $\eta\varepsilon_L = 1$, then $\phi = 1 + \theta \geq 1$; and
- (ii) If $\eta\varepsilon_L > 1$, ϕ has a vertical asymptote at $\theta = (\eta\varepsilon_L - 1)^{-1}$, and for $0 < \theta < (\eta\varepsilon_L - 1)^{-1}$, ϕ is a convex and increasing function of θ exceeding unity.

In order to cover the case of $\eta\varepsilon_L > 1$, we make the following assumption regarding the range of admissible values for θ .

Assumption 1 If $\eta\varepsilon_L > 1$, it is assumed that $0 \leq \theta < \bar{\phi} \equiv 1/(\eta\varepsilon_L - 1)$.

By using (14), (AT1.5), and (AT1.6) in (AT1.1)-(AT1.2), the dynamic system can be written as:

$$\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{C}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{K}(t) \\ \tilde{C}(t) \end{bmatrix} - \begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix}. \quad (16)$$

The Jacobian matrix (with typical element δ_{ij}) is defined as:

$$\Delta \equiv \begin{bmatrix} y[\eta\phi(1 - \varepsilon_L) - \omega_I] & -y(\omega_C + \phi - 1) \\ -(r - \alpha) - (r + \delta)[1 - \eta\phi(1 - \varepsilon_L)] & (r - \alpha) - (r + \delta)(\phi - 1) \end{bmatrix}, \quad (17)$$

where ω_C and ω_I denote the output shares of private consumption and investment, respectively, and $\gamma_K(t)$ and $\gamma_C(t)$ are shock terms (see (20) below). Saddle-point stability holds provided the determinant of Δ is negative (Proposition 1).

Proposition 1 (i) Under finite horizons $\chi \equiv 1 - \eta(1 - \varepsilon_L) > \omega_C/\phi$ is a sufficient condition for saddle-point stability, whereas under infinite horizons $\chi > 0$ is sufficient; (ii) The characteristic roots of the stable case are $r^* > r - \alpha + \omega_C(r + \delta) > 0$ and $-h^* < 0$; and (iii) The accelerator for time-invariant shocks takes the form $\tilde{I}(0) = (h^*/\delta)\tilde{K}(\infty)$.

Proof See Appendix A.3.

¹⁴Under exogenous labor supply, $L = 1$, which implies that $\theta = 0$. From (15) it follows that $\phi = 1$.

The intuition behind the sufficient condition $\chi > 0$ is that there should be diminishing returns to the aggregate capital stock (see (T1.8)). If households have infinite lives, labor supply is elastic and government spending is positive, the negative wealth effect in labor supply of a rise in the capital stock ensures that the marginal product of capital falls even if $\chi = 0$. As households get wealthier, they consume more leisure, which reduces the marginal productivity of capital. With both finite horizons and elastic labor supply, the sufficient condition depends on the values of the parameters. To simplify the discussion we impose:

Assumption 2 $\chi \equiv 1 - \eta(1 - \varepsilon_L) > \omega_G/\phi$,

which is very mild for plausible parameter values. Based on the parameter values of Section 4.3, we obtain $\chi = 0.61$, which easily satisfies the sufficient condition for exogenous labor supply ($\phi = 1$), and a fortiori for endogenous labor supply ($\phi > 1$).

3.2 Fiscal Policy Shocks

The rise in public spending can be permanent or temporary. In formal terms:

$$\tilde{G}(t) = \tilde{G}e^{-\xi_G t}, \quad \xi_G \geq 0, \quad (18)$$

where ξ_G is the adjustment speed of public consumption, and $\tilde{G} > 0$. A permanent spending increase implies $\xi_G = 0$ so that $\tilde{G}(0) = \tilde{G}(\infty) = \tilde{G}$. For $0 < \xi_G \ll \infty$, the spending shock is temporary so that we get:

$$\tilde{G}(\infty) \equiv \begin{cases} \tilde{G} & \text{for } \xi_G = 0 \\ 0 & \text{for } 0 < \xi_G \ll \infty. \end{cases} \quad (19)$$

Using (18), we can write the shock terms corresponding to (16) as:

$$\begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix} = \begin{bmatrix} y\omega_G\tilde{G}e^{-\xi_G t} \\ ((r - \alpha)/\omega_A)\tilde{B}(t) \end{bmatrix}, \quad (20)$$

which can be potentially time varying depending on the parameter setting. Temporary spending shocks give rise to a time varying $\gamma_K(t)$. Bond financing causes $\gamma_C(t)$ to be time-varying provided $r \neq \alpha$, that is, if Ricardian equivalence fails.

Proposition 2 summarizes some properties of the model that are useful in discussing the policy shocks.

Proposition 2 *For a given initial output share of public consumption (ω_G):*

- (i) r , y , ω_C , ω_I , and θ are independent of η ; and
- (ii) r , y , ω_C , and θ are increasing in β and ω_I is decreasing in β .

Proof See Appendix A.4.

3.3 Graphical Apparatus

In order to facilitate the discussion of the model, it is summarized graphically by means of two schedules plotted in Figure 1. The first schedule is the Capital Stock Equilibrium (CSE) curve, which represents all points for which the goods market is in equilibrium with a constant capital stock ($\dot{K}(t) = 0$). The CSE curve is obtained by rewriting the first equation in (16) in steady-state form; it is unambiguously upward sloping in $(\tilde{C}(t), \tilde{K}(t))$ space, that is, $\tilde{C}(t) = -(\delta_{11}/\delta_{12})\tilde{K}(t) + (1/\delta_{12})\gamma_K(t)$, where $\delta_{12} < 0$ (which is apparent from Definition 1) and $\delta_{11} > 0$.¹⁵ The dynamic forces operating on the economy off the CSE curve are as follows. Since $\delta_{12} < 0$, points above the CSE curve are associated with a falling capital stock over time because both goods consumption is too high and labor supply (and hence production) is too low. Consequently, investment is too low to replace the depreciated part of the capital stock. The opposite is the case for points below the CSE curve.

The MKR curve is the Modified Keynes-Ramsey rule, which represents the steady-state aggregate Euler equation augmented for endogenous labour supply and the turnover of generations ($\dot{C}(t) = 0$). The MKR curve is obtained by using the steady-state version of the second equation of (16), that is, $\tilde{C}(t) = -(\delta_{21}/\delta_{22})\tilde{K}(t) + (1/\delta_{22})\gamma_C(t)$. The slope of the MKR curve is ambiguous because the sign of both δ_{21} and δ_{22} depends on two effects that work in opposite directions, that is, the *generational turnover* (GT) effect and the *labor supply* (LS) effect. The intuition behind these two effects can best be explained by looking at the two polar cases.

Labor Supply Effect With Infinite Lives The pure LS effect is isolated by studying the model with endogenous labor supply and infinitely lived representative agents (RA), that is, $\phi > 1$ and $\beta = 0$. In that case, the MKR curve represents points for which the real interest rate equals the rate of time preference, $r[C, K] = \alpha$, so that the slope of MKR depends on the partial derivatives $\partial r/\partial C$ and $\partial r/\partial K$. To provide an intuitive explanation of the partial derivatives, Figure 2 depicts the situation on the markets for production factors. The demand for capital (K^D) is obtained by combining (AT1.5) and (AT1.8):

$$\left(\frac{r}{r + \delta}\right) \tilde{r}(t) = \eta\varepsilon_L \tilde{L}(t) - [1 - \eta(1 - \varepsilon_L)] \tilde{K}(t). \quad (21)$$

In terms of Figure 2(a), K^D is downward sloping in view of Assumption 2, and an increase in employment shifts K^D to the right. For a given stock of capital, the real interest rate clears the rental market for capital. In the infinite horizon model, the long-run supply curve of capital is horizontal and coincides with the dotted line in Figure 2(a).

By using (AT1.4) and (AT1.8), the demand for labor (L^D) can be written as:

$$\tilde{w}(t) = (\eta\varepsilon_L - 1)\tilde{L}(t) + \eta(1 - \varepsilon_L)\tilde{K}(t). \quad (22)$$

¹⁵From the information on steady-state shares we derive that $\omega_A = (1 - \varepsilon_L) - \omega_I$. It follows that $\eta\phi(1 - \varepsilon_L) - \omega_I = (\eta\phi - 1)(1 - \varepsilon_L) + \omega_A > 0$ since $\eta \geq 1$, and $\phi \geq 1$. Hence, $\delta_{11} > 0$.

In terms of Figure 2(b), an increase in the stock of capital shifts L^D to the right, but the slope of L^D is ambiguous and depends on the strength of the diversity effect. If $\eta\varepsilon_L \lesseqgtr 1$, L^D is downward sloping, horizontal or upward sloping. Labor supply, L^S , is upward sloping and shifts to the left if private consumption rises (see (AT1.7)). This is the wealth effect in labor supply, as private consumption is itself proportional to total wealth. Finally, Assumption 1 ensures that the labor supply curve is steeper (with respect to the wage axis) than the labor demand curve. The larger θ the steeper the labor supply curve and thus the more elastic labor supply.

An increase in private consumption (from C_0 to C_1) shifts the labor supply curve to the left, say from $L^S(w, C_0)$ to $L^S(w, C_1)$ in Figure 2(b), and for a given capital stock, employment falls from L_0 to L_1 .¹⁶ This reduces the marginal product of capital, shifts the demand for capital to the left, say from $K^D(r, L_0)$ to $K^D(r, L_1)$ in Figure 2(a), and causes a fall in the rate of interest. This explains why $\partial r/\partial C < 0$ and thus $\delta_{22} < 0$.

An increase in the capital stock (from K_0 to K_1) has two effects. First, the *direct effect* shifts the capital supply curve rightward, which reduces the rental price of capital for a given level of employment. In terms of Figure 2(a), the direct effect is represented by the shift from E_0 to B. There is also an *indirect effect* because the increase in the capital stock raises labor demand, say from $L^D(w, K_0)$ to $L^D(w, K_1)$ in Figure 2(b). For a given level of private consumption, employment expands from L_0 to L_2 , which is represented by the shift from E_0 to B. Because of the increase in employment, capital demand increases, say from $K^D(r, L_0)$ to $K^D(r, L_2)$ in Figure 2(a). The indirect effect thus represents the shift from point B to point C directly above it. For a small value of the intertemporal substitution elasticity of labor supply (θ close to 0) or a weak diversity effect ($\eta \approx 1$), the labor supply parameter is small ($1 < \phi < \bar{\phi}$), which we label moderately elastic labor supply.¹⁷ In that case, the direct effect of the capital stock dominates the employment-induced effect, so that the rate of interest depends negatively on the capital stock, $\partial r/\partial K < 0$ and $\delta_{21} < 0$. As a result, the MKR curve in Figure 1 is downward sloping. Points to the left of the curve are associated with a low capital stock, a high rate of interest, and a rising consumption profile.

For a high enough value of the labor supply parameter ($\phi = \bar{\phi}$), however, the rate of interest does not depend on the capital stock as the two effects exactly cancel. Figure 2(a) shows that the employment expansion shifts the demand for capital all the way to intersect supply in point D, implying a horizontal MKR curve. For points above the MKR curve,

¹⁶Conversely, a fall in consumption shifts the labor supply curve from $L^S(w, C_0)$ to $L^S(w, C_2)$ so that employment expands. The moves from E_0 to C and from C to D in Figure 2(b) represent, respectively, the *wealth effect* and the *substitution effect* in labor supply.

¹⁷Depending on the magnitude of ϕ , three labor supply cases can be distinguished that are all consistent with saddle point stability: (i) $\phi = 1$ (inelastic); (ii) $1 < \phi < \bar{\phi} \equiv 1/(\eta(1 - \varepsilon_L))$ (moderately elastic); and (iii) $\phi > \bar{\phi}$ (highly elastic). As labor supply becomes more elastic, the MKR curve rotates counter clockwise. Saddle point stability prescribes that the CSE curve is steeper than the MKR curve. The second case is drawn in Figure 1.

private consumption is too high, and both labor supply and the rate of interest are too low (that is, $r(t) < \alpha$). As a result, the consumption profile is downward sloping. For an even higher value of the labor supply parameter ($\phi > \bar{\phi}$), the employment-induced effect dominates the direct effect so that capital demand shifts all the way to intersect capital supply at point E of Figure 2(a). The rate of interest now depends positively on the capital stock, $\partial r / \partial K > 0$ and $\delta_{21} > 0$. Accordingly, the MKR curve is upward sloping. Points to the left of the MKR curve are associated with a low rate of interest, and a falling consumption profile.

Generational Turnover Effect With Exogenous Labor Supply The pure GT effect is isolated by studying the model with exogenous labor supply and finitely lived agents, for which $\phi = 1$ and $\beta > 0$. In that case, the MKR curve represents points for which the tilt to the consumption profile of individual households is precisely sufficient to compensate for the turnover of financial assets across generations, $r(K) - \alpha = \beta(\alpha + \beta)K/C$, where r now does not depend on private consumption because labor supply is exogenous. From Figure 2(a) it is clear that with a fixed labor supply, only the direct effect of a change in K remains so that $\partial r / \partial K < 0$.

The MKR curve is upward sloping because of the turnover of generations. Its slope can be explained by appealing directly to equation (T1.2) (with $\varepsilon_C = 1$, $A = K$ and $B = 0$) and Figure 3(a). Suppose that the economy is initially on the MKR curve, say at point E_0 . Now consider a lower level of private consumption, say at point B. For the same capital stock ($\tilde{K}(0) = 0$), both points feature the same rate of interest. Accordingly, individual consumption growth, $\dot{C}(v, t)/C(v, t) [= r - \alpha]$, coincides in the two points.

Equation (23) indicates, however, that aggregate consumption growth depends not only on individual growth but also the *proportional* difference between average consumption, $C(t)$, and consumption by a newly born generation, $C(t, t)$:¹⁸

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{C}(v, t)}{C(v, t)} - \beta\varepsilon_C \left(\frac{C(t) - C(t, t)}{C(t)} \right). \quad (23)$$

Since newly born generations start without any financial capital, the *absolute* difference between average consumption and consumption of a newly born household depends on the average capital stock and is thus the same in the two points. Since the level of aggregate consumption is lower in B, this point features a larger *proportional* difference between average and newly born consumption, thereby decreasing aggregate consumption growth. To restore a zero growth of aggregate consumption, the capital stock must fall (to point E_1), which not only raises individual consumption growth—by increasing the rate of interest—but also lowers the drag on aggregate consumption growth due to the turnover of generations. Because a smaller capital stock narrows the gap between average wealth (that is, the wealth of the

¹⁸We use the fact that $C(t) = \varepsilon_C(\alpha + \beta)[A(t) + H(t)]$ and $C(t, t) = \varepsilon_C(\alpha + \beta)H(t)$, where $H(t)$ is human wealth, that is, the after-tax value of the household's time endowment: $H(t) \equiv \int_t^\infty [w(z) - T(z)] \exp[-\int_t^z [r(s) + \beta] ds] dz$.

generations that pass away) and wealth of the newly born, the generational turnover effect is smaller.

For points above (below) the MKR curve, the GT effect is weak (strong), so that the aggregate consumption profile is rising (falling) over time. In terms of Figure 3(a), steady-state equilibrium is attained at the intersection of the CSE and MKR curves at point E_0 . Given the configuration of arrows, it is clear that this equilibrium is saddle-point stable, and that the saddle path, SP_0 , is upward sloping and steeper than the CSE curve.

4 Lump-Sum Tax Financing of Fiscal Shocks

The base case concerns an *unanticipated* and *permanent* impulse (at $t = 0$) to government consumption (that is, $\tilde{G} > 0$ and $\xi_G = 0$), which is financed by lump-sum taxes only ($\tilde{B}(t) = 0$, for all $t \geq 0$, so that $\tilde{G}(t) = \tilde{T}(t)$). We refer to this case as that of ‘pure lump-sum tax financing.’ In terms of (16), the shock terms are $\gamma_K(t) = y\omega_G\tilde{G} > 0$ and $\gamma_C(t) = 0$ for all $t \geq 0$. Hence, the MKR curve is unaffected and the CSE curve shifts down in Figures 1 and 3. Intuitively, increasing government consumption withdraws resources from the economy. To maintain the same capital stock in equilibrium, private consumption must fall. Before turning to the results of the most general model, we first study the case of exogenous labor supply.

4.1 Exogenous Labor Supply

In order to focus on the pure GT effect, this section deals with the case of exogenous labor supply ($\phi = 1$, see (15)). The GT effect ensures that the MKR curve is upward sloping as shown in Figure 3(a). A permanently higher level of government spending shifts the CSE curve down. Since the physical capital stock is fixed initially, the adjustment consists of a jump from E_0 to A on the new saddle path, SP_1 , followed by a gradual reduction of private consumption and capital toward the new equilibrium at E_1 . Table 2 summarizes the qualitative results under exogenous labor supply in Panel (b), which we compare with those for the RA model in Panel (a).¹⁹ It shows impact effects (at $t = 0$ when the policy is implemented) and long-run effects (at $t \rightarrow \infty$ when the new steady state is reached). The intuition behind the adjustment to the new steady state is discussed below.

At impact, all existing generations experience a reduction in human wealth—defined as the present discounted value of the household’s time endowment—because they are faced

¹⁹Unlike the RA results, the OLG results assume exogenous labor supply to focus on the generational turnover effect. Furthermore, considering exogenous labor supply in the RA model would not yield any long-run output effect (see below). We do not discuss in detail the results for the RA model (covering Panel (a)) given that these are analyzed for the monopolistically competitive case by Devereux, Head, and Lapham (1996b) and Heijdra (1998) and for the perfectly competitive case (covering Panels (a) and (c)) by Baxter and King (1993).

with a lump-sum tax increase, a gradual fall in wages, and a gradual increase in the interest rate, all of which prompts existing generations to cut consumption. Consequently, aggregate consumption falls at impact, though by less than one for one ($-1 < dC(0)/dG < 0$), since human wealth is discounted at the higher ‘risk-of-death’ adjusted discount rate, $r + \beta$.

During transition, the decline in the capital stock reduces the importance of the GT effect, reflecting a reduction in the difference between aggregate and new born consumption (as discussed in Section 3.3). As a result, aggregate consumption growth and savings fall. In the new steady state, the capital stock, private consumption, investment, output, and wages have all fallen and the interest rate has risen. Crowding out of private consumption by public consumption is more than one for one in the long run ($dC(\infty)/dG < -1$). Accordingly, the long-run output multiplier is negative:

$$\frac{dY(\infty)}{dG} = -\frac{(r - \alpha)\eta(1 - \varepsilon_L)}{\chi\omega_C(r + \delta) + (r - \alpha)(\chi - \omega_G)} < 0, \quad (24)$$

where the denominator is positive due to saddle point stability. By appealing to Proposition 2, it is straightforward to show that the output multiplier is decreasing in the diversity effect, that is, $\partial[dY(\infty)/dG]/\partial\eta < 0$. Hence, crowding out of private consumption by public consumption is more severe under monopolistic competition than under perfect competition. From (24) it can also be clearly seen that under exogenous labor supply in the infinite horizon RA model—featuring $\beta = 0$ so that $r = \alpha$ —a fiscal impulse yields full crowding out and thus cannot affect long-run output. Proposition 3 summarizes the main findings.

Proposition 3 *Consider the OLG model with exogenous labor supply (that is, $\phi = 1$). A pure lump-sum tax financed increase in public consumption gives rise to: (i) unchanged output, but a fall in private consumption in the short run; and (ii) a fall in both output and private consumption in the long run.*

Proof See text and Appendix A.5.

4.2 General Model

In the general model, agents have finite lives and labor supply is endogenous so both the LS and GT effects are operative. At impact, the general model behaves qualitatively in a similar fashion to a spending shock as the infinite horizon model—that is, private consumption falls and output and employment rise—with the exception being the investment response, for which the result is ambiguous because the LS and GT effects work in opposite directions. Investment at impact is:

$$\tilde{I}(0) = (h^*/\delta)\tilde{K}(\infty) \stackrel{\leq}{\geq} 0 \quad \Leftrightarrow \quad \phi \stackrel{\leq}{\geq} 1 + \gamma,$$

where $\gamma \equiv (r - \alpha)/(r + \delta) > 0$ summarizes the relative importance of the GT effect and h^* denotes the adjustment speed to the new steady state. So if the LS effect is dominated by the

GT effect, investment falls at impact despite that labor supply is endogenous. Conversely, if the LS effect is strong, say $\phi > 1 + \gamma$ (Figure 1), investment may rise, particularly if there is a strong diversity effect.²⁰ Hence, whereas finite lives help to explain crowding out of capital, the diversity effect gives rise to ‘crowding in’ of capital provided labor supply is endogenous.

The spending shock is followed by a transition period during which the capital stock gradually adjusts toward its new equilibrium value. However, in the general model it is possible for this transition to be absent. Indeed, if $\phi = 1 + \gamma$, the long-run capital stock is unaffected by the shock so that short-run and long-run effects for all variables are the same. Intuitively, the GT effect exactly matches the LS effect so that the MKR curve is vertical.

In view of the discussion in Section 4.1, it is not surprising that the sign of the long-run output effect is ambiguous. Intuitively, a strong LS effect ensures a positive long-run output effect whereas a strong GT effect works in the opposite direction. Nevertheless, the following condition can be derived:

$$\tilde{Y}(\infty) \begin{matrix} \leq \\ \geq \end{matrix} 0 \quad \Leftrightarrow \quad \left(\frac{\varepsilon_L}{1 - \varepsilon_L} \right) \left(\frac{\theta}{1 + \theta} \right) \begin{matrix} \leq \\ \geq \end{matrix} \frac{\gamma}{1 + \gamma}, \quad (25)$$

which follows from the output expression in Appendix Table 2. Equation (25) says that a high value for θ implies a strong LS effect whereas a high value for γ implies a strong GT effect. The most important observation is, however, that the diversity parameter η does not feature in (25). In view of Proposition 2, θ and r , and thus γ , are independent of η so that the *sign* of the output multiplier is unaffected by whether or not there exists a diversity effect in production. Of course, as the results in Sections 4.1 and 4.3 show, the *magnitude* of the long-run output multiplier is critically affected by the strength of the diversity effect.

Proposition 4 *Consider the OLG model with endogenous labor supply ($\phi > 1$) and let $\gamma \equiv (r - \alpha)/(r + \delta) > 0$. A pure lump-sum tax financed increase in public consumption has the following features: (i) output rises and private consumption falls at impact; and (ii) in the long run, output rises if ϕ is large enough (if $\phi > 1 + \gamma$) and private consumption falls if $1 < \phi < 1 + \gamma$.*

Proof *See text and Appendix A.5.*

How do these results compare to the RA model? Under finite horizons, taking the case of a sufficiently elastic labor supply, private consumption falls by less than in the infinite horizon framework because households internalize less of the additional tax burden associated with the fiscal impulse. Accordingly, labor supply expands by less and thus the increase in the capital stock will also be smaller. Steady-state output effects are thus smaller too.

²⁰For a plausibly calibrated version of the model, it can be shown that the LS effect dominates the GT effect, even if unrealistically high values of the birth rate are allowed. See Section 4.3 for a further discussion.

4.3 Numerical Exercises

To illustrate the quantitative significance of returns to scale, the intertemporal labor supply effect, and the birth rate on the size of the output multipliers, this section presents the results of simulations with a calibrated example of the model.

The parameters that are kept fixed throughout the simulations are the following. The rate of pure time preference (α) is assumed to be 3 percent. The rate of depreciation (δ) is set to 7 percent a year and the output share of labor income (ε_L) is set equal to 0.7 (which corresponds roughly to the value found for EU countries). Government spending as a share of GDP (ω_G) is 20 percent. In the simulations β , η , and θ are varied. Once these are set, all other information on output shares can be derived. In the benchmark case, $\beta = 0.05$, $\eta = 1.3$, and $\theta = 2$.

Table 3 reports the impact and long-run multipliers for output and private consumption as well as the adjustment speed of the economy (h^*) for different values of θ (across columns) and different values of β (across rows). In line with the analytical results, the output multipliers are increasing in θ and decreasing in β . Interestingly, the magnitude of θ is much more important to the size of the output multiplier than β . For example, even for a very high birth rate, say $\beta = 0.50$, a relatively modest value of θ suffices to explain a positive long-run output multiplier. Only for very small values of θ does the GT effect dominate the LS effect, suggesting that it is difficult to generate large OLG effects in models of this type.²¹

Table 4 shows the interaction between the birth rate (across columns) and the diversity effect (across rows). The first row corresponds to the perfectly competitive case ($\eta = 1.0$). The results suggest that the diversity effect exerts a much stronger effect on the output multipliers than the birth rate. Both the impact and the long-run output multipliers are increasing in η . For high values of β or high values of θ or both, the short-run output multiplier exceeds the long-run output multiplier.

Tables 3-4 demonstrate that the adjustment speed of the economy is increasing in the birth rate. Hence, an economy populated with finitely lived agents shows much less output persistence than an economy characterized by infinitely lived agents. The adjustment speed, however, is decreasing in η , indicating that the diversity effect can help explain output persistence.

5 Debt Financing of Fiscal Shocks

A well-known result from the traditional literature on the effectiveness of fiscal policy is that output multipliers are larger under debt financing than under lump-sum tax financing provided the model is stable (see Blinder and Solow (1973)). Intuitively, the rise in the

²¹This is also supported by the work of Ríos-Rull (1996). The business cycle statistics that he finds for a calibrated life-cycle economy are roughly in line with those found for standard RBC models as discussed in Cooley and Prescott (1995).

debt stock causes a wealth effect in private consumption that helps bring about Blinder and Solow’s result. Adherents of the Ricardian equivalence theorem have argued, however, that government debt and lump-sum taxes are equivalent. A key question is whether the classic Blinder-Solow result upholds in an OLG setting in which Ricardian equivalence fails. We first study permanent fiscal shocks and subsequently analyze temporary fiscal shocks.

5.1 Bond Path

The notion of debt financing is modeled as follows. At impact, government consumption rises—while keeping constant or cutting initial lump-sum taxes—so that a fiscal deficit emerges, which is financed by issuing government bonds during transition. Gradually, lump-sum taxes start to rise to finance the redemption of government debt. In the new steady state, the fiscal deficit is closed again. Formally, the path of lump-sum taxes is postulated as follows:

$$\tilde{T}(t) = \begin{cases} \tilde{G}(t) & \text{for } \xi_T = 0 \\ -\tilde{T}_0 + [1 - e^{-\xi_T t}] \tilde{T}_\infty & \text{for } \xi_T > 0 \end{cases}, \quad (26)$$

where $\tilde{T}_0 > 0$ if there is an initial lump-sum tax cut, ξ_T is the adjustment speed of lump-sum taxes, and subscripts to variables are used to denote time-invariant components. Policies involving bond financing are described by $\xi_T > 0$ whereas for $\xi_T = 0$ there is no bond financing.

In the absence of initial public debt, the government solvency condition (13) can—upon loglinearization—be written in general terms as $\mathcal{L}\{\tilde{T}(t), r\} = \mathcal{L}\{\tilde{G}(t), r\}$, where \mathcal{L} denotes the Laplace transform operator.²² Government solvency implies that the long-run increase in lump-sum taxes equals $T(\infty) = \tilde{T}_\infty - \tilde{T}_0$, where \tilde{T}_∞ is given by:

$$\tilde{T}_\infty = \left(\frac{r + \xi_T}{\xi_T} \right) \left(\tilde{T}_0 + \left(\frac{r}{r + \xi_G} \right) \tilde{G} \right) > \tilde{T}_0, \quad \text{for } \xi_T > 0. \quad (27)$$

Intuitively, in the long run, lump-sum taxes must rise by enough to cover additional government spending on goods *plus* the interest payments on the public debt that is accumulated during the transition period. Accordingly, future generations face a larger lump-sum tax burden than present generations.

By combining (18), (27), and (AT1.3), the bond path is obtained:

$$\tilde{B}(t) = \omega_G \left(\frac{r}{r + \xi_G} \right) \left[1 - e^{-\xi_G t} + \left(\frac{r}{\xi_T} \right) (1 - e^{-\xi_T t}) \right] \tilde{G} + \omega_G \frac{r}{\xi_T} (1 - e^{-\xi_T t}) \tilde{T}_0, \quad (28)$$

with $\xi_T > 0$ (of course, for $\xi_T = 0$, $\tilde{B}(t) \equiv 0$ for all t). The exogenous parameters in (28) are \tilde{G} , \tilde{T}_0 , ξ_G , and ξ_T , whereas the implied value \tilde{T}_∞ keeps the government solvent. By only gradually raising lump-sum taxes, the government allows for a smooth build-up of public debt

²² $\mathcal{L}\{x, s\}$ is the Laplace transformation of $x(t)$ evaluated at s , which is given by $\mathcal{L}\{x, s\} \equiv \int_0^\infty x(t)e^{-st} dt$. Intuitively, $\mathcal{L}\{x, s\}$ represents the present value of $x(t)$ using $s = r$ as the discount rate.

from an initial position of zero ($B(0) = 0$) to a long-run level of $B(\infty) > 0$. More formally, the long-run change in public debt is given by:

$$\tilde{B}(\infty) = \omega_G \left[\left(\frac{r + \xi_T}{\xi_T} \right) \left(\frac{r}{r + \xi_G} \right) \tilde{G} + \frac{r}{\xi_T} \tilde{T}_0 - \tilde{G}(\infty) \right]. \quad (29)$$

Provided $\xi_T > 0$ the resulting debt process is stable. For a given increase in public spending, the lower the value for ξ_T , the slower is the adjustment of lump-sum taxes, and the larger is the resulting long-run debt stock.

By using (28) in (20) the dynamic system can be written as in (16) with the following shock terms:

$$\begin{aligned} \gamma_K(t) &\equiv y\omega_G \tilde{G} e^{-\xi_G t}, & (30) \\ \gamma_C(t) &\equiv (r - \alpha)y\omega_G \left[\left(1 - e^{-\xi_G t} + \frac{r}{\xi_T} (1 - e^{-\xi_T t}) \right) \frac{\tilde{G}}{r + \xi_G} + (1 - e^{-\xi_T t}) \frac{\tilde{T}_0}{\xi_T} \right]. & (31) \end{aligned}$$

From (31) it appears that under finite horizons ($r > \alpha$), Ricardian equivalence fails, so that government debt has real effects at impact, during transition, and in the long run.²³

5.2 Permanent Fiscal Shocks

We consider two types of permanent fiscal shocks ($\xi_G = 0$) financed by public debt ($\xi_T > 0$): (i) moderate fiscal policy; and (ii) drastic fiscal policy.

Moderate Fiscal Policy We consider a bond-financed permanent rise in public spending $\gamma_K(t) = \tilde{G} > 0$ for all t , while keeping initial lump-sum taxes constant ($\tilde{T}_0 = 0$), so that $\gamma_C(\infty) > 0$ (from (31)). Subsequently, we will discuss the case of exogenous and endogenous labor supply.

The case of exogenous labor supply can be illustrated by Figure 3(b). At impact, the CSE curve shifts down from CSE_0 to CSE_1 , but the MKR curve is unaffected ($\gamma_C(0) = 0$). Gradually over time, the MKR curve shifts to the left, say from MKR_0 to MKR_1 ($\gamma_C(t) > 0$ for $t > 0$), owing to households accumulating government bonds in their portfolios. The adjustment is from E_0 to A' at impact followed by a gradual movement (the speed of which is governed by both $h^* > 0$ and $\xi_T > 0$) from A' to E'_1 . Under pure lump-sum tax financing, the adjustment is from E_0 to A on impact, followed by a gradual transition from A to E_1 , showing a larger impact effect and a smaller long-run effect on private consumption than under bond financing.

Compared to the pure lump-sum tax case, the *qualitative* effects of bond financing on all variables are similar, which is summarized in Panels (b) and (d) of Table 2. The results set

²³Clearly, the RA model satisfies the conditions for Ricardian equivalence. Since $r = \alpha$ it follows from (31) that $\gamma_C(t) = 0$ for all $t \geq 0$. Indeed, in Table 2 the results in Panels (a) and (c) are identical. Only the paths of (individual and aggregate) financial assets are affected by the financing method employed.

out in Proposition 3 continue to hold. The *quantitative* effects differ though—bond-financed impact effects are less pronounced and long-run effects are more pronounced than under lump-sum tax financing. Intuitively, the use of bond policy shifts the burden of additional lump-sum taxes from present to future generations. As a result, the reduction in human wealth at impact is less severe so that the fall in consumption is also smaller. In the long run, however, lump-sum taxes and the rate of interest are higher and the capital stock (and thus the wage) is lower than in the pure lump-sum case. Consequently, long-run crowding out of private consumption and capital formation is more severe under bond financing. In sum, Blinder and Solow’s result does not hold in an OLG setting with exogenous labor supply. The intergenerational redistribution of the tax burden under bond financing renders the long-run output multiplier smaller (instead of larger) than under pure lump-sum tax financing.

As was argued in Section 4.2, both the GT and LS effects are operative in the general model, so that the slope of the MKR curve is ambiguous. Two cases—differing in the transition paths to the new steady state—can be distinguished depending on the relative strengths of the GT and LS effects. First, if the GT effect is dominant ($1 < \phi < 1 + \gamma$),²⁴ adjustment to the new steady state after the spending shock is monotonic. The initial fall in private consumption is followed by a further fall in consumption ($\tilde{C}(\infty) < \tilde{C}(0) < 0$) and physical capital gradually declines to its lower steady-state level. This result is similar to that of Propositions 3 and 4.

Second, if the LS effect dominates the GT effect ($\phi > 1 + \gamma$), transition in both private consumption and the capital stock is non-monotonic. The impact effect on investment is ambiguous:

$$\tilde{I}(0) = \left([\phi - (1 + \gamma)] - \frac{(r - \alpha)(\phi + \omega_C - 1)}{(1 - \varepsilon_L)(r^* + \xi_T)} \right) \left(\frac{(r + \delta)\omega_G}{\omega_I r^*} \right) \tilde{G}. \quad (32)$$

For the case with $\phi > 1 + \gamma$, the term in square brackets on the right-hand side of (32) is positive. It dominates the (positive) second term of (32) provided ξ_T is not very small. In that case, investment rises on impact. This is drawn in Figure 3(c). The impact effect is a move from point E_0 to point A which lies below both the CSE_1 and MKR_0 lines. Consequently, the capital stock and private consumption start to rise, say from point A to point B, reflecting the increase in investment and rise in household wealth, respectively. Beyond point B the capital stock starts to fall again. As public debt starts to accumulate during transition, the MKR shifts down and meets the stable trajectory at point C, after which private consumption falls along with the capital stock toward the new equilibrium at point E_1 . The capital stock in the new equilibrium is smaller than that in the old equilibrium. All this is summarized in Proposition 5.

²⁴If the LS effect exactly matches the GT effect ($\phi = 1 + \gamma$), yielding a vertical MKR curve, there are transitional dynamics under bond financing (against no transitional dynamics under lump-sum tax financing). Public debt crowds out physical capital formation in the long run, causing the MKR curve to shift to the left. Adjustment to the new steady state is monotonic.

Proposition 5 *Consider the OLG model model with endogenous labor supply ($r > \alpha$, $\beta > 0$, $\phi > 1$) and let $\gamma \equiv (r - \alpha)/(r + \delta)$. An increase in government consumption financed by public debt has qualitatively similar effects to those of lump-sum tax financing if the generational turnover effect dominates the labor supply effect. The adjustment paths for private consumption and the capital stock may be non-monotonic if ξ_T is small and labor supply is sufficiently elastic ($\phi > 1 + \gamma$).*

Proof See text and Appendix A.5.

Drastic Fiscal Policy Drastic fiscal policy under bond financing is represented by $\gamma_K(t) = \tilde{G} > 0$ for all t and an initial cut in lump-sum taxes ($\tilde{T}_0 > 0$). The position of the MKR curve in the new steady state is determined by $\gamma_C(\infty) > 0$, which follows from (31) for $t \rightarrow \infty$. Here, the case of moderately elastic labor supply is studied.

Figure 3(c) can again be used to show that the CSE curve shifts down from CSE_0 to CSE_1 , whereas the MKR curve is unaffected at time $t = 0$. The initial fall in private consumption is smaller than under moderate fiscal policy, because the fall in households' after-tax human wealth is smaller. Over time, the MKR curve shifts down as households accumulate bonds in their portfolios. Note that the leftward shift of the MKR curve is larger than under moderate fiscal policy, which is explained by the initial cut in lump-sum taxes. The new steady state (E_2) is thus to the left of the steady state obtained for moderate fiscal policy (E_1). Consequently, the steady-state capital stock and the level of private consumption level are lower.

5.3 Temporary Fiscal Shocks

This section studies temporary fiscal shocks with a view to relate our work to the fiscal impulses found in VAR studies. To analyze the dynamic adjustment to a temporary increase in public spending, numerical examples are used. Various financing scenarios are distinguished to demonstrate the effect of the OLG model structure on the results of fiscal shocks.

Approach and Parameters We have first analytically derived (A.12) and (A.13), which are then used in the simulations. The parameter setting of the benchmark model is employed. Labor supply is thus moderately elastic in all scenarios considered: $\phi < \bar{\phi} \equiv 2.56$. The parameter of the path of the public consumption shock, ξ_G , is set to 0.10 (see (19)), implying high persistence in the public spending shock (the half-life of the adjustment is $\ln 2/\xi_G \approx 7$ years). In the benchmark case, the parameter of the lump-sum tax path under bond financing is set to $\xi_T = 0.05$.

Numerical Results Table 5 shows the numerical results for five different scenarios. Column (1) presents results for the RA model. Because the increase in public consumption is

temporary, the economy returns to its initial steady state, which is a widely accepted result in the literature. Obviously, in the new steady state factor prices have not changed. What is of interest here is the transitional dynamics. In the short run, investment, employment, and output increase but private consumption falls.²⁵ Wages fall on impact due a larger supply of labor, but gradually rise once labor demand increases.

Columns (2)-(5) present the OLG cases. Column (2) considers pure lump-sum tax financing ($\xi_T = 0$) of a fiscal impulse. The impulse responses are virtually identical to those of the RA model, showing only small quantitative differences. By comparing Tables 3 and 5, it is evident that—in line with received wisdom—temporary fiscal shocks yield smaller short-run output multipliers than permanent fiscal shocks.

Results are strikingly different, once we allow for debt financing in the OLG framework (Columns (3)-(5)). Column (3) assumes $\xi_T = 0.1$ and $\tilde{T}_0 = 0$, implying a bond-financed fiscal shock that leaves initial lump-sum taxes unchanged. A key result is that a temporary shock leads to hysteresis in macroeconomic variables. Private consumption not only falls in the short run but also in the long run. In addition, we find negative long-run output multipliers for a temporary shock, reflecting crowding out of private consumption and investment by public consumption. Intuitively, bond financing shifts the burden of taxation forward in time thereby reducing the net human capital of future generations. Accordingly, they will consume less. Column (4) sets $\xi_T = 0.05$, which shifts more of the burden of lump-sum taxation to future generations. Not surprisingly, there is a larger negative effect on private consumption and output in the new steady state.

Column (5) studies a public spending shock combined with a cut in initial lump-sum taxes ($\tilde{T}_0 = 0.1$). The short-run fall in private consumption is smaller than in Column (4) because households enjoy additional tax relief at impact. However, future generations pay the price of debt redemption, reducing their long-run consumption by more than under scenarios (3) and (4). In addition, the long-run output multiplier is (in absolute terms) the largest of all scenarios.

Links to VAR Evidence In line with VAR studies, we find that temporary fiscal shocks can have long-lasting effects on macroeconomic variables. The OLG model structure together with bond financing gives rise to non-zero steady-state effects, providing a shock propagation mechanism that has not been analyzed in the literature yet.

VAR studies find that in the short run a positive fiscal shock raises both private consumption and output. Our model finds a negative correlation between private consumption and output in the initial phase after the shock. However, the scenario of drastic fiscal policy (Column (5)) produces a positive correlation between $Y(t)$ and $C(t)$ in the medium run (already after 10 time periods) and long run. To also bring the initial phase of the transition path in

²⁵However, the crowding out of private consumption by public consumption is smaller than under permanent fiscal policy.

line with VAR evidence, new elements should be introduced into the model, which is beyond the scope of this paper.

6 Conclusions

The micro-founded dynamic macroeconomic framework explored here proves rich in further understanding the output effects of fiscal policy under various financing scenarios and settings of the structural parameters. The richness of the results naturally reflects the comprehensive nature of our model. Support is found for the claim that under certain conditions the activist (New)Keynesian view on fiscal policy may be too optimistic about the potency of fiscal policy in boosting output. In more detail, the results are as follows.

Take first the case of lump-sum tax financing of an unanticipated and permanent spending impulse. The effect of introducing overlapping generations of mortal agents is to lower the size of long-run output multipliers, and even possibly to change their sign—that is, they can now become negative rather than positive. Under lump-sum tax financing, public spending multipliers may turn negative if the generational turnover effect is sufficiently strong to dominate the intertemporal substitution effect in labor supply. For exogenous labor supply, this condition is evidently satisfied. However, for plausible parameter values, the generational turnover effect is unlikely to dominate the labor supply effect. In this context, short-run output multipliers are smaller than long-run multipliers.

Chamberlinian monopolistic competition, featuring zero excess profits due to instantaneous entry or exit of firms, increases the absolute value of balanced-budget output multipliers. Under lump-sum taxation and exogenous labor supply, Ethier-style productivity effects help to explain larger crowding out effects of public spending—and consequently more negative output multipliers—than under perfect competition. Under lump-sum taxation and sufficiently elastic labor supply, however, large crowding in effects may result, giving rise to positive long-run multipliers.

Bond financing of permanent fiscal shocks has real effects in an overlapping generations setting because Ricardian equivalence fails. Under bond financing of a rise in public consumption, long-run output multipliers are smaller than under lump-sum tax financing. Debt financing may give rise to non-monotonic adjustment paths if labor supply is moderately elastic and the speed of adjustment of lump-sum taxes is not too high. A bond-financed fiscal impulse combined with an initial cut in lump-sum taxes magnifies the reduction in long-run output multipliers as compared to multipliers under lump-sum tax financing.

A lump-sum tax financed temporary rise in public consumption does not have any long-run effects on macroeconomic variables, in line with results in an infinitely lived household framework. Bond financing, however, gives rise to strikingly different results; temporary fiscal shocks have permanent effects on output and other macroeconomic variables. Negative long-run output multipliers are obtained, caused by a large crowding out effect of private

investment by public consumption than under permanent shocks. Our model is partially able to produce evidence in line with that of VAR studies. Bond financing combined with a cut in lump-sum taxes produces a positive correlation between output and private consumption in the medium and long run.

There are of course many aspects of fiscal policy that have not been addressed here, such as the effects of anticipated fiscal shocks, other forms of financing the fiscal impulse (for example, labor taxation), and the optimal provision of public goods. Furthermore, the model could easily be turned into a full-fledged RBC model by including stochastic public spending shocks and menu-cost driven price stickiness. We leave these extensions for further research.

Appendix: Model Solution

In this appendix we show how the main results of Sections 3-5 were derived.

A.1 Log-linearization

We log-linearize the model of Table 1 around an initial steady state, using the following notational conventions. A tilde ($\tilde{\cdot}$) denotes a relative change, for example, $\tilde{x}(t) \equiv dx(t)/x$, for most of the variables, except for: (i) time derivatives: $\dot{\tilde{x}}(t) \equiv d\tilde{x}(t)/x$ for $x \in \{K, L, N, Y, C, I, w, r, T, G\}$, and $\dot{\tilde{B}}(t) \equiv rd\dot{B}(t)/Y$; and (ii) financial assets (that is, $A(t)$, $B(t)$), which are scaled by steady-state output and multiplied by r , for example, $\tilde{B}(t) \equiv rdB(t)/Y$. The results of the log-linearization are reported in Appendix Table 1.

The model can be reduced to a two-dimensional system of first-order differential equations in the capital stock, $\tilde{K}(t)$, and private consumption, $\tilde{C}(t)$. Conditional on the state variables and the policy shocks, the static part of the log-linearized model, consisting of equations (AT1.4)-(AT1.8) in Appendix Table 1, can be used to derive the following ‘quasi-reduced form’ expressions:

$$\tilde{Y}(t) = \eta\phi(1 - \varepsilon_L)\tilde{K}(t) - (\phi - 1)\tilde{C}(t), \quad (\text{A.1})$$

$$\omega_I\tilde{I}(t) = \tilde{Y}(t) - \omega_C\tilde{C}(t) - \omega_G\tilde{G}(t), \quad (\text{A.2})$$

$$\eta\varepsilon_L\tilde{L}(t) = \tilde{Y}(t) - \eta(1 - \varepsilon_L)\tilde{K}(t), \quad (\text{A.3})$$

$$\eta\varepsilon_L\tilde{w}(t) = (\eta\varepsilon_L - 1)\tilde{Y}(t) + \eta(1 - \varepsilon_L)\tilde{K}(t). \quad (\text{A.4})$$

A.2 Solution Method

Equations (A.1)-(A.2) and (AT1.5) can be combined with (AT1.1)-(AT1.2) to derive the dynamic system given in (16) in the main text. Taking the Laplace transform of (16) and noting that the capital stock is predetermined ($\tilde{K}(0) = 0$), we obtain:²⁶

$$\Lambda(s) \begin{bmatrix} \mathcal{L}\{\tilde{K}, s\} \\ \mathcal{L}\{\tilde{C}, s\} \end{bmatrix} = \begin{bmatrix} -\mathcal{L}\{\gamma_K, s\} \\ \tilde{C}(0) - \mathcal{L}\{\gamma_C, s\} \end{bmatrix}, \quad (\text{A.5})$$

where $\Lambda(s) \equiv sI - \Delta$, where I is the identity matrix. The characteristic roots of Δ are denoted in general terms by $-\lambda_1 < 0$ and $\lambda_2 > 0$. The jump in $\tilde{C}(0)$ is such that:

$$\text{adj } \Lambda(\lambda_2) \begin{bmatrix} -\mathcal{L}\{\gamma_K, \lambda_2\} \\ \tilde{C}(0) - \mathcal{L}\{\gamma_C, \lambda_2\} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (\text{A.6})$$

where $\text{adj } \Lambda(\lambda_2)$ is the adjoint matrix of $\Lambda(\lambda_2)$, which has rank 1. Using the first row of (A.6), we get:

$$\tilde{C}(0) = \mathcal{L}\{\gamma_C, \lambda_2\} + \left(\frac{\lambda_2 - \delta_{22}}{\delta_{12}} \right) \mathcal{L}\{\gamma_K, \lambda_2\}. \quad (\text{A.7})$$

²⁶The details of the solution method are set out in an accessible form in Heijdra and Van der Ploeg (2002, pp. 684-690).

The Laplace transforms of the shocks can be written as:

$$\mathcal{L}\{\gamma_K, s\} = \gamma_K^0 \frac{1}{s + \xi_G}, \quad (\text{A.8})$$

$$\mathcal{L}\{\gamma_C, s\} \equiv \gamma_C^0 \left(\frac{1}{s} - \frac{1}{s + \xi_G} \right) + \gamma_C^1 \left(\frac{1}{s} - \frac{1}{s + \xi_T} \right), \quad (\text{A.9})$$

where γ_C^0 , γ_C^1 , and γ_K^0 are defined as:

$$\begin{aligned} \gamma_C^0 &\equiv \frac{(r - \alpha)y\omega_G \tilde{G}}{r + \xi_G}, \\ \gamma_C^1 &\equiv \frac{(r - \alpha)y\omega_G}{\xi_T} \left[\left(\frac{r}{r + \xi_G} \right) \tilde{G} + \tilde{T}_0 \right], \\ \gamma_K^0 &\equiv y\omega_G \tilde{G}. \end{aligned} \quad (\text{A.10})$$

Using (A.8)-(A.9) in (A.7), we can thus write $\tilde{C}(0)$ as:

$$\tilde{C}(0) = \frac{\xi_G \gamma_C^0}{\lambda_2 (\lambda_2 + \xi_G)} + \frac{\xi_T \gamma_C^1}{\lambda_2 (\lambda_2 + \xi_T)} + \left(\frac{\lambda_2 - \delta_{22}}{\delta_{12}} \right) \frac{\gamma_K^0}{\lambda_2 + \xi_G}. \quad (\text{A.11})$$

The root inequality $\lambda_2 > \bar{\lambda}$ (see Section A.3) implies that $\lambda_2 > \delta_{22}$.

Solving for the first row of (A.5) yields the transition path for $\tilde{K}(t)$:

$$\begin{aligned} \tilde{K}(t) &= \frac{\delta_{12}(\gamma_C^0 + \gamma_C^1)}{\lambda_1 \lambda_2} \text{A}(\lambda_1, t) - \left[\frac{(\delta_{22} + \xi_G)\gamma_K^0 + \delta_{12}\gamma_C^0}{\lambda_2 + \xi_G} \right] \text{T}(\xi_G, \lambda_1, t) \\ &\quad - \frac{\delta_{12}\gamma_C^1}{\lambda_2 + \xi_T} \text{T}(\xi_T, \lambda_1, t), \end{aligned} \quad (\text{A.12})$$

whereas the second row of (A.5) gives rise the transition path for $\tilde{C}(t)$:

$$\begin{aligned} \tilde{C}(t) &= \tilde{C}(0) [1 - \text{A}(\lambda_1, t)] - \frac{\delta_{11}(\gamma_C^0 + \gamma_C^1)}{\lambda_1 \lambda_2} \text{A}(\lambda_1, t) \\ &\quad + \frac{\delta_{21}\gamma_K^0 + (\delta_{11} + \xi_G)\gamma_C^0}{\lambda_2 + \xi_G} \text{T}(\xi_G, \lambda_1, t) + \frac{(\delta_{11} + \xi_T)\gamma_C^1}{\lambda_2 + \xi_T} \text{T}(\xi_T, \lambda_1, t), \end{aligned} \quad (\text{A.13})$$

where $\text{A}(\lambda_1, t) \equiv 1 - e^{-\lambda_1 t}$ is an adjustment term. Note that $\text{T}(\xi_i, \lambda_1, t)$ for $i = \{C, T\}$ is a non-negative, bell-shaped temporary transition term of the following form:

$$\text{T}(\alpha_1, \alpha_2, t) \equiv \begin{cases} \frac{e^{-\alpha_2 t} - e^{-\alpha_1 t}}{\alpha_1 - \alpha_2} & \text{for } \alpha_1 \neq \alpha_2 \\ t e^{-\alpha_1 t} & \text{for } \alpha_1 = \alpha_2, \end{cases}$$

where $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$ are parameters.

A.3 Stability

Saddle-point stability holds provided the determinant of Δ is negative:

$$|\Delta| = -(r + \delta)y[\omega_G(\phi - 1) + \omega_C\phi\chi + \gamma[\phi\chi - \omega_G]], \quad (\text{A.14})$$

where $\chi \equiv 1 - \eta(1 - \varepsilon_L)$ and $\gamma \equiv (r - \alpha) / (r + \delta)$. Proposition 1(i) is proved as follows. With finite lives, $\beta > 0$, $r > \alpha$, so that $\gamma > 0$ and it follows from (A.14) that $\phi\chi \geq \omega_G$ is sufficient for saddle-point stability. With infinite horizons, $\beta = 0$, $r = \alpha$, and $\gamma = 0$. If $\omega_G(\phi - 1) = 0$ then stability holds iff $\chi > 0$. If $\omega_G(\phi - 1) > 0$ then $\chi > 0$ is sufficient but not necessary for stability.

Proposition 1(ii) is proved as follows. Since $|\Delta| = -\lambda_1\lambda_2 < 0$, Δ has distinct roots $-\lambda_1 \equiv -h^* < 0$ and $\lambda_2 \equiv r^* > 0$. To prove the inequality for the unstable root we must show that $\Lambda(\bar{\lambda}) < 0$, where $\bar{\lambda} \equiv r - \alpha + \omega_C(r + \delta) > 0$. After some manipulations we get:

$$\Lambda(\bar{\lambda}) = -y(\phi + \omega_C - 1) (\omega_G(r + \delta) + \bar{\lambda}[1 - (1 - \varepsilon_L)]) < 0,$$

which proves Proposition 1(ii). Proposition 1(iii) follows from the expressions for $\tilde{I}(0)$ and $\tilde{K}(\infty)$ in Appendix Table 2 and noting that $\tilde{G} + \tilde{T}_0 = 0$ for the case under consideration.

A.4 Changing β and η

The proof of Proposition 2 is as follows. By using (T1.1)-(T1.2) in steady state and (T1.4)-(T1.7), we get:

$$(r - \alpha)\omega_C = \beta\varepsilon_C(\alpha + \beta)\kappa, \tag{A.15}$$

$$1 - \omega_G = \omega_C + \delta\kappa, \tag{A.16}$$

$$(r + \delta)\kappa = 1 - \varepsilon_L, \tag{A.17}$$

$$\theta = \frac{(1 - \varepsilon_C)\varepsilon_L}{\varepsilon_C}\omega_C, \tag{A.18}$$

where $\kappa \equiv K/Y \equiv 1/y$. By substituting (A.16) into (A.15) and noting (A.17), we get a two-equation system in r and κ only:

$$r - \alpha = \frac{\beta\varepsilon_C(\alpha + \beta)\kappa}{1 - \omega_G - \delta\kappa}, \tag{A.19}$$

$$r + \delta = \frac{1 - \varepsilon_L}{\kappa}. \tag{A.20}$$

Clearly, $\omega_C > 0$ so we have from (A.15) and (A.16) that $0 < \kappa < (1 - \omega_G) / \delta$. Accordingly, equation (A.19) gives rise to an upward sloping curve in the (r, κ) space, whilst (A.20) is downward sloping. There is a unique equilibrium, κ^* , which is the positive root of the quadratic equation:

$$[\delta(\alpha + \delta) - \beta\varepsilon_C(\alpha + \beta)]\kappa^2 - [\delta(1 - \varepsilon_L) + (\alpha + \delta)(1 - \omega_G)]\kappa + (1 - \varepsilon_L)(1 - \omega_G) = 0. \tag{A.21}$$

Part (i) of Proposition 2 can be proved as follows. Since (A.21) does not contain η , κ^* and thus (via (A.17)) r do not depend on η . From this, (A.16), and (A.18), it follows that ω_C and θ are not affected either. Part (ii) of Proposition 2 follows from (A.19). An increase in β , rotates (A.19) counterclockwise, increases r and decreases κ . Hence, ω_C and θ increase whilst ω_I decreases.

A.5 Comparative Dynamics of Permanent Fiscal Shocks

By using (17) and (A.10) in (A.12) and (A.13) and noting that $\xi_G = 0$, we obtain the impact, transition, and long-run effects of a permanent rise in public consumption on the capital stock and private consumption. By also using (A.1)-(A.4) and (AT1.9) the results for the remaining variables are obtained.

The results in Section 4 of the main text are obtained by setting $\gamma_C^0 = \gamma_C^1 = 0$ in (A.11), (A.12), and (A.13), and choosing the appropriate parameter settings in Appendix Table 2: exogenous labor supply ($\phi = 1$) or endogenous labor supply ($\phi > 1$). The results in Section 5.2 are derived by setting $0 < \xi_T \ll \infty$ and using the values for γ_C^0 and γ_C^1 from (A.10).

References

- Baxter, M., King, R. G., 1993. Fiscal policy in general equilibrium. *American Economic Review* 83, 315–334.
- Bénassy, J.-P., 1996a. Monopolistic competition, increasing returns to specialisation, and output persistence. *Economics Letters* 52, 187–191.
- Bénassy, J.-P., 1996b. Taste for variety and optimum production patterns in monopolistic competition. *Economics Letters* 52, 41–47.
- Blanchard, O.-J., 1985. Debt, deficits, and finite horizons. *Journal of Political Economy* 93, 223–247.
- Blinder, A. S., Solow, R. M., 1973. Does fiscal policy matter? *Journal of Public Economics* 2, 319–337.
- Burnside, C., Eichenbaum, M., Fisher, J. D. M., 2004. Fiscal shocks and their consequences. *Journal of Economic Theory* 115, 89–117.
- Cardia, E., 1991. The dynamics of a small open economy in response to monetary, fiscal, and productivity shocks. *Journal of Monetary Economics* 28, 411–434.
- Cavallo, M. and Ghironi, F., 2002. Net foreign assets and the exchange rate: Redux revived. *Journal of Monetary Economics* 49, 1057–1097.
- Chatterjee, S., Cooper, R. W., 1993. Entry, exit, product variety, and the business cycle. NBER Working Paper 4562, National Bureau of Economic Research.
- Coenen, G., Straub, R., 2004. Non-Ricardian households and fiscal policy in an estimated DSGE model of the euro area. Mimeo, European Central Bank, Frankfurt.
- Cooley, T. F., Prescott, E. C., 1995. Economic growth and business cycles, in: Cooley, Th. F. (Ed.), *Frontiers of Business Cycle Research*, Princeton University Press, Princeton, NJ., pp. 1-38.
- Coto-Martínez, J., 2006. Public capital and imperfect competition. *Journal of Public Economics*, forthcoming.
- Devereux, M. B., Head, A. C., Lapham, B. J., 1993. Monopolistic competition, technology shocks, and aggregate fluctuations. *Economics Letters* 41, 57–61.
- Devereux, M. B., Head, A. C., Lapham, B. J., 1996a. Aggregate fluctuations with increasing returns to specialisation and scale. *Journal of Economic Dynamics and Control* 20, 627–656.

- Devereux, M. B., Head, A. C., Lapham, B. J., 1996b. Monopolistic competition, increasing returns, and the effects of government spending. *Journal of Money, Credit, and Banking* 28, 233–253.
- Dixit, A., Stiglitz, J. E., 1977. Monopolistic competition and optimum product diversity. *American Economic Review* 67, 297–308.
- Dixit, A., Stiglitz, J. E., 2004. Monopolistic competition and optimum product diversity [February 1975], in: Brakman, S., Heijdra, B. J. (Eds.), *The Monopolistic Competition Revolution in Retrospect*, Cambridge, Cambridge University Press, pp. 89-120.
- Ethier, W. J., 1982. National and international returns to scale in the modern theory of international trade. *American Economic Review* 72, 389–405.
- Fatás, A., Mihov, I., 2001. The effects of fiscal policy on consumption and employment: Theory and evidence. CEPR Working Paper 2760, Centre for Economic Policy Research.
- Galí, J., López-Salido, J. D., Vallés, J., 2004. Understanding the effects of government spending on consumption. ECB Working Paper 339, European Central Bank, Frankfurt.
- Heijdra, B. J., 1998. Fiscal policy multipliers: The role of monopolistic competition, scale economies, and intertemporal substitution in labor supply. *International Economic Review* 39, 659–696.
- Heijdra, B. J., Van der Ploeg, F., 1996. Keynesian multipliers and the cost of public funds under monopolistic competition. *Economic Journal* 106, 1284–1296.
- Heijdra, B. J., Van der Ploeg, F., 2002. *Foundations of Modern Macroeconomics*. Oxford University Press, Oxford.
- Hemming, R., Kell, M., Mahfouz, S., 2002. The effectiveness of fiscal policy in stimulating economic activity: A review of the literature. IMF Working Paper WP/02/208, International Monetary Fund, Washington.
- Hornstein, A., 1993. Monopolistic competition, increasing returns to scale, and the importance of productivity shocks. *Journal of Monetary Economics* 31, 299–316.
- Judd, K. L., 1982. An alternative to steady-state comparisons in perfect foresight models. *Economics Letters* 10, 55–59.
- Judd, K. L., 1998. *Numerical Methods in Economics*. MIT Press, Cambridge, MA.
- Leith, C., Wren-Lewis, S., 2000. Interactions between monetary and fiscal policy rules. *Economic Journal* 110, C93–C108.

- Linnemann, L., 2004. Tax base and crowding-in effects of balanced-budget fiscal policy. *Scandinavian Journal of Economics* 106, 273–297.
- Linnemann, L., Schabert, A., 2003. Fiscal policy in the new neoclassical synthesis. *Journal of Money, Credit, and Banking* 35, 911–929.
- Molana, H., Moutos, T., 1992. A note on taxation, imperfect competition, and the balanced-budget multiplier. *Oxford Economic Papers* 44, 68–74.
- Mountford, A., Uhlig, H., 2002. What are the effects of fiscal policy shocks? Mimeo, Humboldt University, Berlin.
- Ríos-Rull, J.-V., 1996. Life-cycle economies and aggregate fluctuations. *Review of Economic Studies* 63, 465–489.
- Rotemberg, J. J., Woodford, M., 1995. Dynamic general equilibrium models with imperfectly competitive product markets, in: Cooley, Th. F. (Ed.), *Frontiers of Business Cycle Research*. Princeton University Press, Princeton, NJ., pp. 243–293
- Schmitt-Grohé, S., Uribe, M., 2003. Closing small open economy models. *Journal of International Economics* 61, 163–185.
- Stadler, G. W., 1994. Real business cycles. *Journal of Economic Literature* 32, 1750–1783.
- Startz, R., 1989. Monopolistic competition as a foundation for Keynesian macroeconomic models. *Quarterly Journal of Economics* 104, 737–752.
- Woodford, M., 2001. The Taylor rule and optimal monetary policy. *American Economic Review* 91, 345–370.
- Yaari, M. E., 1965. Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies* 32, 137–150.

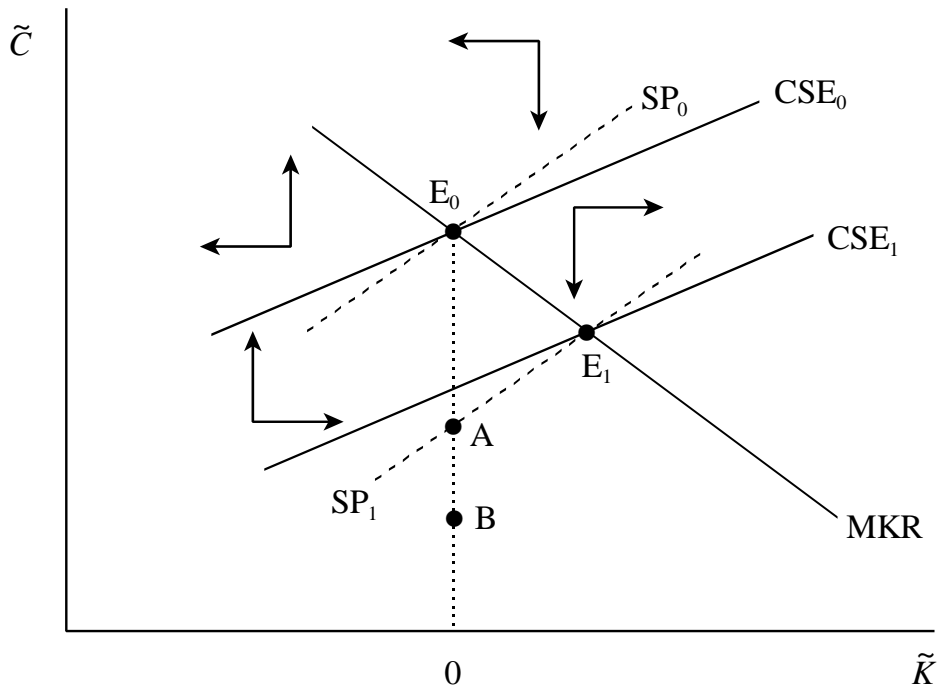
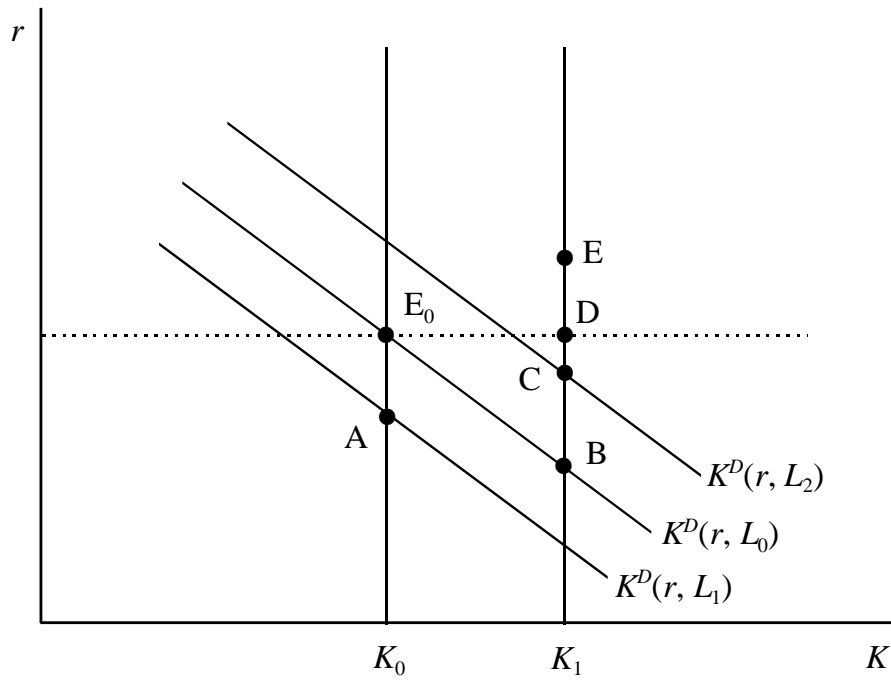


Figure 1: Lump-Sum Taxes and Endogenous Labor Supply

Panel (a): Rental Market for Capital



Panel (b): Labor Market

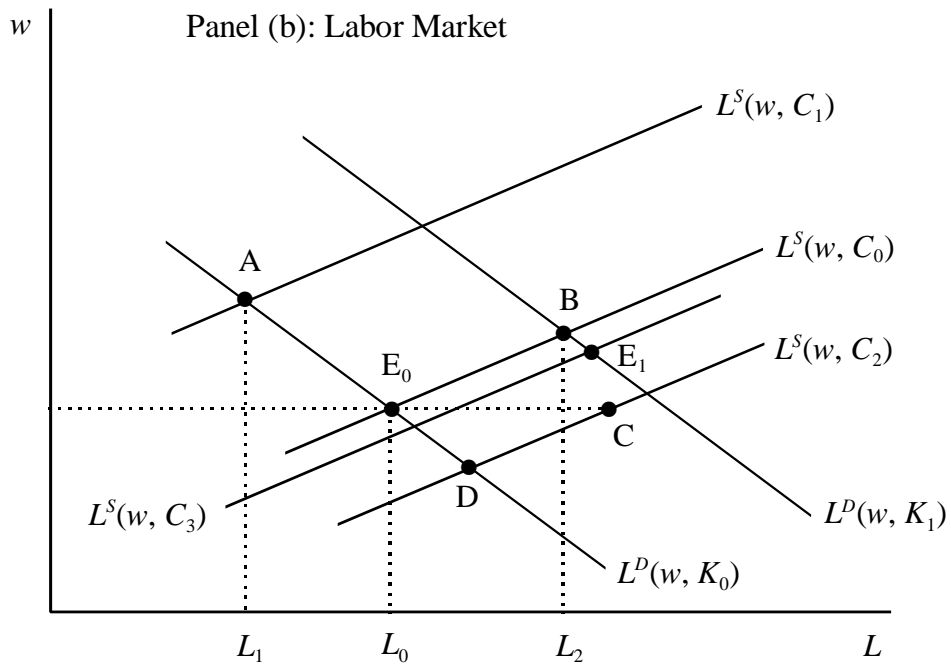
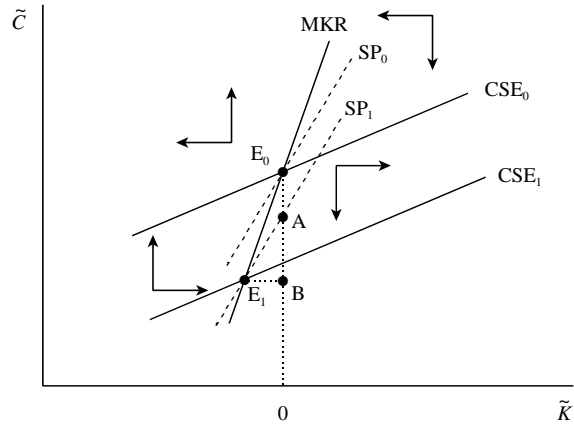


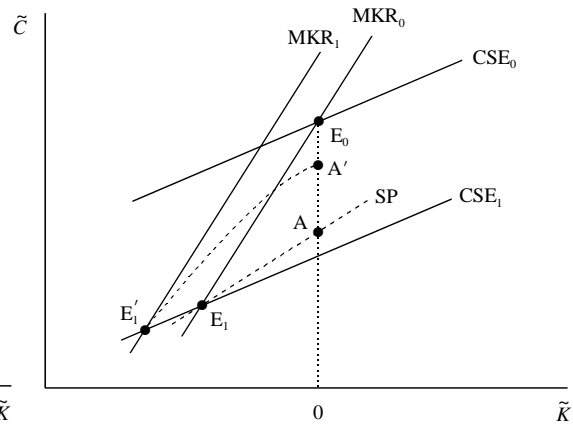
Figure 2: The Factor Markets

Figure 3: Permanent Public Consumption Shocks in the Overlapping Generations Model

Panel (a)
Lump-sum tax financing ($\phi = 1$)



Panel (b)
Bond financing ($\phi = 1$)



Panel (c)
Bond financing ($\phi > 1$)

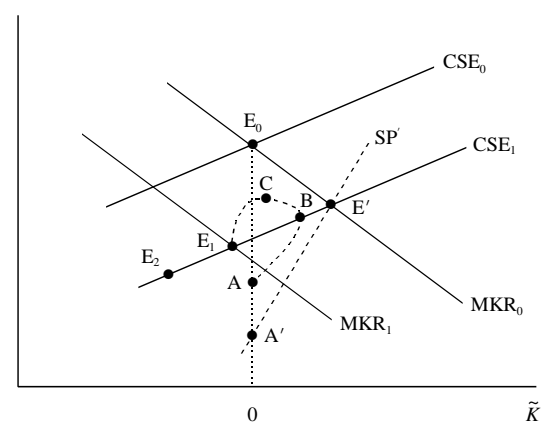


Table 1: Main Model Equations

$$\dot{K}(t) = I(t) - \delta K(t) \quad (\text{T1.1})$$

$$\dot{C}(t) = [r(t) - \alpha]C(t) - \beta\varepsilon_C(\alpha + \beta)[K(t) + B(t)] \quad (\text{T1.2})$$

$$\dot{B}(t) = r(t)B(t) + G(t) - T(t) \quad (\text{T1.3})$$

$$w(t)L(t) = \varepsilon_L Y(t) \quad (\text{T1.4})$$

$$[r(t) + \delta]K(t) = (1 - \varepsilon_L)Y(t) \quad (\text{T1.5})$$

$$Y(t) = C(t) + I(t) + G(t) \quad (\text{T1.6})$$

$$L(t) = 1 - \frac{(1 - \varepsilon_C)C(t)}{\varepsilon_C w(t)} \quad (\text{T1.7})$$

$$Y(t) = \left(\frac{\Phi}{\mu - 1} \right) N(t)^\eta = \Omega_0 L(t)^{\eta\varepsilon_L} K(t)^{\eta(1-\varepsilon_L)} \quad (\text{T1.8})$$

$$\Omega_0^{1/\eta} Y(t)^{(\eta-1)/\eta} = \left(\frac{w(t)}{\varepsilon_L} \right)^{\varepsilon_L} \left(\frac{r(t) + \delta}{1 - \varepsilon_L} \right)^{1-\varepsilon_L} \quad (\text{T1.9})$$

Note: $\Omega_0 \equiv \mu^{-\eta} [(\mu - 1)/\Phi]^{(\eta-1)}$

Table 2: Qualitative Effects of Permanent Fiscal Policy

Policy Measure	Time Period ¹	Representative Agent ²							Overlapping Generations ³						
		Y	C	I	L	K	w	r	Y	C	I	L	K	w	r
<i>Pure Lump-Sum Tax</i>		(a)							(b)						
Perfect Competition	0	+	-	+	+	0	-	+	0	-	-	0	0	0	0
	∞	+	-	+	+	+	0	0	-	-	-	0	-	-	+
Monopolistic Competition	0	+	-	+	+	0	? ⁴	+	0	-	-	0	0	0	0
	∞	+	? ⁵	+	+	+	+	0	-	-	-	0	-	-	+
<i>Public Debt</i>		(c)							(d)						
Perfect Competition	0	+	-	+	+	0	-	+	0	-	-	0	0	0	0
	∞	+	-	+	+	+	0	0	-	-	-	0	-	-	+
Monopolistic Competition	0	+	-	+	+	0	? ⁴	+	0	-	-	0	0	0	0
	∞	+	? ⁵	+	+	+	+	0	-	-	-	0	-	-	+

Notes: (1): $t = 0$, impact effect, $t = \infty$, long-run effect; (2): Endogenous labor supply; (3): Exogenous labor supply; (4): Sign of $(\eta\varepsilon_L - 1)$ and (5): See Appendix Table 2.

Table 3: Output Multipliers, Birth Rates, and Labor Supply Elasticities

		Value of θ				
		0	0.5	1	2	5
$\beta = 0$	$\frac{dY(0)}{dG}$	0	0.526	0.815	1.149	1.589
	$\frac{dC(0)}{dG}$	-1	-0.712	-0.576	-0.440	-0.299
	$\frac{dY(\infty)}{dG}$	0	0.721	1.009	1.261	1.482
	$\frac{dC(\infty)}{dG}$	-1	-0.430	-0.203	-0.004	0.171
	h^*	0.084	0.110	0.129	0.156	0.200
$\beta = 0.01$	$\frac{dY(0)}{dG}$	0	0.520	0.809	1.145	1.586
	$\frac{dC(0)}{dG}$	-0.987	-0.708	-0.574	-0.439	-0.299
	$\frac{dY(\infty)}{dG}$	-0.021	0.706	0.998	1.253	1.479
	$\frac{dC(\infty)}{dG}$	-1.010	-0.437	-0.207	-0.007	0.170
	h^*	0.086	0.112	0.131	0.157	0.200
$\beta = 0.05$	$\frac{dY(0)}{dG}$	0	0.484	0.769	1.109	1.564
	$\frac{dC(0)}{dG}$	-0.916	-0.680	-0.559	-0.433	-0.297
	$\frac{dY(\infty)}{dG}$	-0.127	0.613	0.920	1.197	1.448
	$\frac{dC(\infty)}{dG}$	-1.069	-0.479	-0.240	-0.029	0.159
	h^*	0.100	0.126	0.144	0.168	0.208
$\beta = 0.10$	$\frac{dY(0)}{dG}$	0	0.441	0.714	1.052	1.520
	$\frac{dC(0)}{dG}$	-0.850	-0.647	-0.539	-0.423	-0.294
	$\frac{dY(\infty)}{dG}$	-0.223	0.506	0.818	1.109	1.390
	$\frac{dC(\infty)}{dG}$	-1.138	-0.535	-0.286	-0.064	0.138
	h^*	0.124	0.152	0.170	0.192	0.226
$\beta = 0.50$	$\frac{dY(0)}{dG}$	0	0.342	0.562	0.846	1.270
	$\frac{dC(0)}{dG}$	-0.733	-0.577	-0.490	-0.393	-0.280
	$\frac{dY(\infty)}{dG}$	-0.380	0.291	0.576	0.840	1.105
	$\frac{dC(\infty)}{dG}$	-1.326	-0.695	-0.432	-0.196	0.024
	h^*	0.347	0.410	0.442	0.470	0.479

Table 4: Output Multipliers, Diversity Effect, and Birth Rates

		Value of β					
		0	0.01	0.05	0.10	0.50	1.00
$\eta = 1.0$	$\frac{dY(0)}{dG}$	0.867	0.863	0.830	0.777	0.597	0.554
	$\frac{dC(0)}{dG}$	-0.585	-0.583	-0.570	-0.550	-0.489	-0.476
	$\frac{dY(\infty)}{dG}$	0.922	0.916	0.871	0.802	0.596	0.553
	$\frac{dC(\infty)}{dG}$	-0.272	-0.274	-0.288	-0.313	-0.417	-0.450
	h^*	0.164	0.165	0.176	0.199	0.467	0.826
$\eta = 1.1$	$\frac{dY(0)}{dG}$	0.961	0.956	0.922	0.866	0.675	0.629
	$\frac{dC(0)}{dG}$	-0.538	-0.536	-0.526	-0.509	-0.459	-0.448
	$\frac{dY(\infty)}{dG}$	1.031	1.024	0.976	0.900	0.673	0.625
	$\frac{dC(\infty)}{dG}$	-0.186	-0.188	-0.205	-0.234	-0.347	-0.381
	h^*	0.161	0.162	0.173	0.197	0.468	0.830
$\eta = 1.3$	$\frac{dY(0)}{dG}$	1.149	1.145	1.109	1.052	0.845	0.793
	$\frac{dC(0)}{dG}$	-0.440	-0.439	-0.433	-0.423	-0.393	-0.386
	$\frac{dY(\infty)}{dG}$	1.261	1.253	1.197	1.109	0.840	0.783
	$\frac{dC(\infty)}{dG}$	-0.004	-0.007	-0.029	-0.064	-0.196	0.233
	h^*	0.156	0.157	0.168	0.192	0.470	0.840
$\eta = 1.5$	$\frac{dY(0)}{dG}$	1.137	1.332	1.300	1.244	1.037	0.982
	$\frac{dC(0)}{dG}$	-0.338	-0.338	-0.335	-0.331	-0.318	-0.316
	$\frac{dY(\infty)}{dG}$	1.507	1.499	1.435	1.336	1.027	0.959
	$\frac{dC(\infty)}{dG}$	0.191	0.187	0.162	0.121	-0.026	-0.065
	h^*	0.149	0.151	0.162	0.187	0.472	0.852

Table 5: Temporary Fiscal Shocks

	RA	OLG			
	(1)	(2)	(3)	(4)	(5)
$\frac{dY(0)}{dG}$	0.832	0.825	0.798	0.797	0.766
$\frac{dY(\infty)}{dG}$	0	0	-0.130	-0.164	-0.424
$\frac{dC(0)}{dG}$	-0.318	-0.322	-0.311	-0.311	-0.299
$\frac{dC(\infty)}{dG}$	0	0	-0.069	-0.087	-0.224
$\frac{dI(0)}{dG}$	0.150	0.147	0.109	0.108	0.064
$\frac{dI(\infty)}{dG}$	0	0	-0.061	-0.078	-0.200
$\frac{\tilde{L}(0)}{\tilde{G}}$	0.183	0.181	0.175	0.175	0.168
$\frac{\tilde{L}(\infty)}{\tilde{G}}$	0	0	-0.002	-0.003	-0.007
$\frac{\tilde{K}(0)}{\tilde{G}}$	0	0	0	0	0
$\frac{\tilde{K}(\infty)}{\tilde{G}}$	0	0	-0.062	-0.078	-0.201
$\frac{\tilde{w}(0)}{\tilde{G}}$	-0.017	-0.016	-0.016	-0.016	-0.015
$\frac{\tilde{w}(\infty)}{\tilde{G}}$	0	0	-0.024	-0.030	-0.078
$\frac{dr(0)}{\tilde{G}}$	0.017	0.017	0.017	0.017	0.017
$\frac{dr(\infty)}{\tilde{G}}$	0	0	0.004	0.005	0.012

Notes: (1): Representative agent; (2)-(5): overlapping generations;
(2): no debt financing; (3)-(5): with debt financing;
(3)-(4): moderate fiscal policy; and (5): drastic fiscal policy.

Appendix Table 1: The Log-linearized Model

$$\dot{\tilde{K}}(t) = y\omega_I [\tilde{I}(t) - \tilde{K}(t)] \quad (\text{AT1.1})$$

$$\dot{\tilde{C}}(t) = r\tilde{r}(t) + (r - \alpha) [\tilde{C}(t) - \tilde{K}(t) - (1/\omega_A)\tilde{B}(t)] \quad (\text{AT1.2})$$

$$\dot{\tilde{B}}(t) = r [\tilde{B}(t) + \omega_G\tilde{G}(t) - \omega_T\tilde{T}(t)] \quad (\text{AT1.3})$$

$$\tilde{L}(t) = \tilde{Y}(t) - \tilde{w}(t) \quad (\text{AT1.4})$$

$$\tilde{K}(t) = \tilde{Y}(t) - \left(\frac{r}{r + \delta}\right) \tilde{r}(t) \quad (\text{AT1.5})$$

$$\tilde{Y}(t) = \omega_C\tilde{C}(t) + \omega_I\tilde{I}(t) + \omega_G\tilde{G}(t) \quad (\text{AT1.6})$$

$$\tilde{L}(t) = \theta [\tilde{w}(t) - \tilde{C}(t)] \quad (\text{AT1.7})$$

$$\tilde{Y}(t) = \eta\tilde{N}(t) = \eta [\varepsilon_L\tilde{L}(t) + (1 - \varepsilon_L)\tilde{K}(t)] \quad (\text{AT1.8})$$

$$\left(\frac{\eta - 1}{\eta}\right) \tilde{Y}(t) = \varepsilon_L\tilde{w}(t) + \left(\frac{r(1 - \varepsilon_L)}{r + \delta}\right) \tilde{r}(t) \quad (\text{AT1.9})$$

Definitions:

$\varepsilon_L \equiv wL/Y$: Share of before-tax wage income in real output; $\omega_A \equiv rK/Y$: Share of income from financial assets in real output; $\omega_G \equiv G/Y$: Share of government spending in real output; $\omega_C \equiv C/Y$: Share of private consumption in real output; $\omega_I \equiv I/Y$: Share of investment spending in real output; $\theta \equiv (1 - L)/L$: Ratio of leisure to labor; $\omega_T \equiv T/Y$: Share of lump-sum taxes in real output; η : Diversity effect; and $y \equiv Y/K$: Initial output-capital ratio.

**Appendix Table 2: Pure Lump-Sum Tax Financing and Debt Financing
of a Permanent Fiscal Shock**

$$\begin{aligned}
\tilde{C}(0) &= - \left(\frac{\lambda_2 - (r - \alpha) + (\phi - 1)(r + \delta)}{\lambda_2(\phi + \omega_C - 1)} \right) \omega_G \tilde{G} + \frac{(r - \alpha)y}{\lambda_2(\lambda_2 + \xi_T)} \omega_G [\tilde{G} + \tilde{T}_0] \\
\tilde{I}(0) &= \left(\frac{(\phi - 1)(r + \delta) - (r - \alpha)}{\lambda_2 \omega_I} \right) \omega_G \tilde{G} - \frac{(r - \alpha)(\phi + \omega_C - 1)y}{\lambda_2(\lambda_2 + \xi_T)\omega_I} \omega_G [\tilde{G} + \tilde{T}_0] \\
\tilde{K}(\infty) = \tilde{I}(\infty) &= \left(\frac{(\phi - 1)(r + \delta) - (r - \alpha)}{\lambda_1 \lambda_2} \right) y \omega_G \tilde{G} - \frac{(r - \alpha)(\phi + \omega_C - 1)y^2}{\xi_T \lambda_1 \lambda_2} \omega_G [\tilde{G} + \tilde{T}_0] \\
\tilde{C}(\infty) &= - \left(\frac{(r - \alpha) + [1 - \eta\phi(1 - \varepsilon_L)](r + \delta)}{\lambda_1 \lambda_2} \right) y \omega_G \tilde{G} - \frac{(r - \alpha)[\eta\phi(1 - \varepsilon_L) - \omega_I]y^2}{\xi_T \lambda_1 \lambda_2} \omega_G [\tilde{G} + \tilde{T}_0] \\
\tilde{Y}(\infty) &= \left(\frac{-(r - \alpha)\eta\phi(1 - \varepsilon_L) + (\phi - 1)[r - \alpha + r + \delta]}{\lambda_1 \lambda_2} \right) y \omega_G \tilde{G} \\
&\quad - \frac{(r - \alpha)[\omega_C \eta\phi(1 - \varepsilon_L) + (\phi - 1)\omega_I]y^2}{\xi_T \lambda_1 \lambda_2} \omega_G [\tilde{G} + \tilde{T}_0] \\
\tilde{L}(\infty) &= \frac{(\phi - 1)\chi y [r - \alpha + r + \delta]}{\eta \varepsilon_L \lambda_1 \lambda_2} y \omega_G \tilde{G} - \frac{(r - \alpha)(\phi - 1)[(1 - \omega_C)\chi - \omega_G]y^2}{\xi_T \eta \varepsilon_L \lambda_1 \lambda_2} \omega_G [\tilde{G} + \tilde{T}_0] \\
\tilde{w}(\infty) &= \left(\frac{-\eta^2 \phi \varepsilon_L (1 - \varepsilon_L)(r - \alpha) + (\phi - 1)(\eta - 1)[r - \alpha + r + \delta]}{\eta \varepsilon_L \lambda_1 \lambda_2} \right) y \omega_G \tilde{G} \\
&\quad - \frac{(r - \alpha)[\eta^2 \varepsilon_L \phi \omega_C (1 - \varepsilon_L) + (\phi - 1)[(\eta - 1)\omega_I + \eta(1 - \varepsilon_L)\omega_G]]y^2}{\xi_T \eta \varepsilon_L \lambda_1 \lambda_2} \omega_G [\tilde{G} + \tilde{T}_0] \\
\left(\frac{r}{r + \delta} \right) \tilde{r}(\infty) &= \frac{(r - \alpha)y\phi\chi}{\lambda_1 \lambda_2} \omega_G \tilde{G} - \frac{(r - \alpha)[\phi\chi\omega_C + (\phi - 1)\omega_G]y^2}{\xi_T \lambda_1 \lambda_2} \omega_G [\tilde{G} + \tilde{T}_0]
\end{aligned}$$

Notes:

Moderate fiscal policy: $\tilde{G} > 0, \tilde{T}_0 = 0$; Drastic fiscal policy: $\tilde{G} > 0, \tilde{T}_0 > 0$; The case of pure lump-sum financing is obtained by setting $\tilde{T}_0 = -\tilde{G} > 0$. Note further that $\chi \equiv 1 - \eta(1 - \varepsilon_L) > 0$; $-\lambda_1 < 0$ and $\lambda_2 > 0$ are the characteristic roots of $|\Delta|$ defined in (17).
