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Smulders, J.A.; van der Werf, E.H.

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CLIMATE POLICY AND THE OPTIMAL EXTRACTION OF HIGH- AND LOW-CARBON FOSSIL FUELS

By Sjak Smulders, Edwin van der Werf

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Climate policy and the optimal extraction of high- and low-carbon fossil fuels*

Sjak Smulders[†], Edwin van der Werf[‡]

Abstract

We study how restricting CO₂ emissions affects resource prices and depletion over time. We use a Hotelling-style model with two non-renewable fossil fuels that differ in their carbon content (e.g. coal and natural gas) and that are imperfect substitutes in final good production. We study both an unexpected constraint and an anticipated constraint. Both shocks induce intertemporal substitution of resource use. When emissions are unexpectedly restricted, it is cost-effective to use high-carbon resources relatively more (less) intensively on impact if this resource is relatively scarce (abundant). If the emission constraint is anticipated, it is cost-effective to use relatively more (less) of the low-carbon input before the constraint becomes binding, in order to conserve relatively more (less) of the high-carbon input for the period when climate policy is active in case the high-carbon resource is relatively scarce (abundant).

JEL Classification: O13, Q31, Q43

Keywords: Climate policy, non-renewable resources, input substitution

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[†]Department of Economics and CentER, Tilburg University. Email: J.A.Smulders@uvt.nl

[‡]Corresponding author. Department of Economics and CentER, Tilburg University, Warandelaan 2, P.O. Box 90153, 5000 LE Tilburg, the Netherlands. Phone: +31 13 466 8730. Email: E.H.vanderWerf@uvt.nl

1 Introduction

Substitution from high-carbon to low-carbon energy sources may allow an economy to reduce carbon dioxide (CO_2) emissions at lower cost. For example, a country can build gas-fuelled powerplants instead of coal-fuelled powerplants to meet the demand for energy and still reduce polluting emissions. Questions that arise in the presence of a limit to CO_2 emissions are whether there should be a transition towards a 'low-carbon economy', and whether we should leave the stocks of coal in the ground, at least for a while. In a standard static partial equilibrium setting, a CO_2 emission tax affects the user cost of high-carbon energy more than that of low-carbon energy and substitution will take place towards the low-carbon energy. We show that in the more appropriate dynamic setting, with energy coming from non-renewable resource stocks, the results are quite different. Extending the canonical non-renewable resource model with a second resource, we find that a binding CO_2 emission constraint not necessarily calls for substitution towards low-carbon in the short run, but - depending on a well-defined measure of scarcity of the two resources - may instead call for relatively more intensive high-carbon use in the short-run and less of it in the long run.

Taking the current global policy regarding global warming as a starting point, we study the effect of a constant ceiling on carbon dioxide emissions ('Kyoto forever') on fossil fuel extraction when the government uses a cost-effective instrument. We not only study the effect of an unexpected emissions ceiling but we also investigate an anticipated emission constraint, which comes closer to the case of the Kyoto Protocol where the first commitment period in which countries are supposed to have reduced their emissions is 2008-2012. We build a model that is as close as possible to the standard non-renewable resource model and distinguish between two non-renewable resources that are imperfect substitutes in production, for example coal and natural gas. In addition the two resources differ in carbon content (per unit

of energy), as in the process of energy generation more carbondioxide per unit of energy is emitted with the use of coal than with the use of oil or gas.

We build our arguments on the fact that high-carbon and low-carbon inputs are imperfect substitutes at an aggregate level. Substitution between different types of products implies indirect substitution between energy types and types of fossil fuels. If the transport sector shifts from road transport to rail, this is implicitly a change in the fossil fuel mix as trucks use oil-based products while the railsector uses electricity, which can be generated by gas-fuelled powerplants. Also the powerplants themselves can substitute between fossil fuel types: although for an individual powerplant the choice between coal and gas is a binary one, the point of indifference between the two inputs may differ for several powerplants, leading to imperfect substitution at the industry level.

We show that relative extraction in the constrained economy not only depends on the carbon content of the two inputs, but also on their relative productivity and availability. Because the emissions come from the use of non-renewable resources, an emission constraint ceases to be binding when the remaining resource stocks have become sufficiently small. The best way to cope with an emission constraint is to intertemporally reallocate the extraction of the two given resource stocks such that production per unit of carbondioxide emissions is relatively high at the time the emission constraint is binding, and low when the constraint no longer (or - in the case of an anticipated constraint - not yet) binds. Hence the constrained economy uses the resource with the lowest amount of emissions per unit of output relatively more intensively, as compared to an unconstrained economy and this resource is not necessarily the resource with lowest amount of carbon per unit of energy.

The option of substituting low-carbon for high-carbon fuels to meet climate targets has been studied analytically in Chakravorty et al. (2005b)

and numerically in Chakravorty et al. (1997). The latter paper develops a numerical integrated assessment model with several non-renewables (oil, coal and natural gas), multiple energy demand sectors and a clean renewable resource. The authors simulate three scenarios for technical change with optimal climate policy and conclude that "under any reasonable scenario for technological change, most of the earth's coal resources will never be used. Oil and natural gas, however, are both completely exhausted in all three situations." (Chakravorty et al., 1997, p. 1225). However, they do not report the paths of extraction of the individual resources, nor do they identify the forces underlying their results. In Chakravorty et al. (2005b) climate policy consists of an exogenous ceiling on the stock of pollution. A high- and a low-carbon fossil fuel are perfect substitutes, together with a clean backstop technology, in energy generation. Unless it has been exhausted before, only the low-carbon input is used until the ceiling on the stock of pollution is achieved. From this instant on first the low-carbon input is used, possibly followed by a period in which both inputs are used simultaneously, and finally only the high-carbon input is used.

Most theoretical papers studying climate policy and fossil fuel extraction use a single (polluting) non-renewable resource. Withagen (1994) extends the standard Hotelling (1931) model with stock externalities from resource use and a utility function that is separable in utility from resource use and disutility from the stock of pollution. A social planner assures that the marginal benefits of fuel use equals the marginal costs, including environmental damage. Withagen shows that the optimal, monotonically decreasing extraction path has initially less extraction than in the pure mining model, but more extraction later on. Grimaud and Rougé (2005) treat pollution as a flow and extend the model with an innovating sector to have endogenous growth. They confirm Withagen's conclusions.

A second branch of theoretical papers has both a polluting non-renewable

and a non-polluting backstop technology. Tahvonen (1997) extends Withagen's model with extraction costs and a backstop and shows that, if the initial stock of externalities is low enough, the extraction path of the non-renewable may have an inverted U-shape form. In a related paper, Chakravorty et al. (2005a) study the effects of an exogenous ceiling on the stock of emissions on the use of the non-renewable resource and the backstop technology during and after the period that the constraint is binding. The exogenous price of the backstop relative to the emission constraint determines the optimal allocation. Few papers study imperfect substitution between non-renewable resources. Exceptions are Beckmann (1974) and Hartwick (1978), but these early studies are not concerned with carbon emissions.

The only paper, to our knowledge, that studies the effect of an announced emission constraint is by Kennedy (2002). Using a two-period model without resources he shows that it may be optimal for a small country to reduce emissions before the 2008-2012 commitment period, either because of co-benefits (e.g. reductions in emissions of other pollutants than CO₂ that go together with a reduction in fossil fuel combustion) or because early investments in physical capital help reducing adjustment costs.

The remainder of the paper develops as follows. After presenting our model in section 2, we study the economy without any form of climate policy. In section 4 we study an unexpected and initially binding constant CO₂ emission ceiling. Section 5 presents the effects of an announced constraint. We conclude in section 6. The appendix contains proofs of propositions and technical details.

2 The model

Our economy consists of three groups of agents, who all take prices as given. Consumers maximize intertemporal utility by buying final goods and trading assets, which are claims to the resource stocks. Producers maximize

profits by buying two resource inputs to produce and sell a homogeneous final good. In addition each producer has a number of tradable emission permits. Resource extractors maximize intertemporal profits by extracting (at zero extraction cost) the two non-renewable resources, which are sold to producers in the final goods industry. Although we present the results for the decentralized economy with regulation through tradable pollution permits, it can be shown that the planner that wishes to maximize utility subject to an exogenous emission constraint chooses exactly the same allocation. Hence, the setting we study is one of cost-effective environmental regulation.

The representative consumer derives utility from final good Y and faces an intertemporal budget constraint: $dV(t)/dt = r(t)V(t) - Y(t)$. Here $V(t)$ is wealth and $r(t)$ is the market interest rate, at time t . The consumer maximizes intertemporal utility:

$$U(t) = \int_t^\infty \ln Y(\tau) \cdot e^{-\rho\tau} d\tau, \quad (1)$$

where ρ is the utility discount rate. Maximizing (1) subject to the intertemporal budget constraint implies the following Ramsey rule:

$$\hat{Y}(t) = r(t) - \rho. \quad (2)$$

where, as in the remainder of this paper, the hat denotes the growth rate ($\hat{Y} = d \ln Y / dt$).

The competitive final goods industry produces Y from two fossil fuel inputs, H and L , both scaled to units of energy, according to the following constant returns to scale CES technology (we suppress the time argument when no confusion arises):

$$Y = \left(\eta_H R_H^{\frac{\sigma-1}{\sigma}} + \eta_L R_L^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (3)$$

where R_i is the amount extracted of resource $i \in \{H, L\}$, η_H and η_L are positive technology parameters and $\sigma \in (0, \infty)$ is the constant elasticity of substitution. The use of fossil fuels causes emissions of carbon dioxide. The two inputs differ in their CO₂ emission intensity per unit of energy and we denote the (constant) CO₂ emission coefficients of H and L by ε_H and ε_L respectively, with $\varepsilon_H > \varepsilon_L$ so that H is the relatively dirty or high-carbon input. The total amount of emissions is denoted by Z .¹ If the economy is subject to an emissions constraint, total emissions cannot exceed a maximally allowed amount \bar{Z} , according to the following constraint:

$$\varepsilon_H R_H(t) + \varepsilon_L R_L(t) = Z(t) \leq \bar{Z}. \quad (4)$$

As we are interested in the reaction of the economy to the constraint rather than in optimal climate policy itself, we assume that the constraint \bar{Z} is exogenous. The government allocates tradable emission permits over producers in the final goods industry, who trade them at a market price p_Z and buy resources of type i at price p_{Ri} . The price of the final good is normalized to one for every period. Firms maximize profits and the first order conditions for resource use read (from (3) and (4)):

$$\eta_i \left(\frac{Y}{R_i} \right)^{\frac{1}{\sigma}} = p_{Ri} + \varepsilon_i p_Z. \quad (5)$$

This equation states that the marginal revenue from resource input i (the marginal product at the left-hand side) equals its marginal cost (the user price at the right-hand side), which consists of the price of the resource augmented with the cost of pollution in case the constraint is binding.²

¹Our notation is consistent with the measurement of R_i in units of energy and Z in units of carbon. By rescaling R_i and/or Z it is possible to normalize - without loss of generality - three of the four parameters ε_L , ε_H , η_L , and η_H , to unity. However, for the sake of ease of interpretation we do not apply this normalisation.

²Note that we will always have an interior solution. If $R_i = 0$ we would have $Y = 0$

The two fossil fuels are extracted from stocks of non-renewable resources, S_H and S_L respectively, according to

$$dS_i/dt = -R_i, \quad (6)$$

$$\int_0^\infty R_i dt \leq S_{i0},$$

where S_{i0} is the initial stock of resource i . Resource owners maximize the net present value of profits from exploiting the non-renewable resource stocks, taking resource price p_{Ri} as given. Extraction costs are assumed to be zero so the resource price is a pure scarcity rent. For each of the resources this results in the familiar Hotelling rule:

$$dp_{Ri}/dt = r \cdot p_{Ri}. \quad (7)$$

From this we see that the relative resource rent p_{RH}/p_{RL} will be constant over time, as both rents grow at the same rate.

We are now ready to study extraction of the two resources. We first study extraction in an economy without a CO₂ emission constraint and then move to a constrained but otherwise identical economy.

3 The economy without (the prospect of) climate policy

Suppose that from some instant T (possibly equal to 0) on the economy is unconstrained and does not expect future climate policy. In this case the economy is described by a pure depletion or cake eating model from $t = T$ on (see e.g. Heal, 1993). If we write the firms' optimality conditions (5) with

for $\sigma \leq 1$, and $\partial Y/\partial R_i = \eta_i (Y/R_i)^{1/\sigma} \rightarrow \infty$ for $\sigma > 1$ which violates (5) for finite p_{Ri} and p_Z .

$p_Z = 0$ in growth rates and combine them with the Hotelling and Ramsey rules we find the growth rates of extraction:

$$\hat{R}_H = \hat{R}_L = \hat{Y} - r = -\rho \quad \forall t \geq T, \quad (8)$$

That is, extraction and emissions decrease at a rate equal to the utility discount rate. After integrating (8) and imposing the constraint that forward-looking resource owners anticipate that eventually all reserves will be sold, we find that the extraction rates of the two resources can be expressed as:

$$R_i(t) = \rho S_i(t) \quad \forall t \geq T. \quad (9)$$

Consequently total emissions equal

$$Z(t) = \rho \cdot (\varepsilon_H S_H(t) + \varepsilon_L S_L(t)) \quad \forall t \geq T \quad (10)$$

(see (4)). According to (8) and (9), relative extraction is constant over time and equal to instant T 's relative stock:

$$\frac{R_H(t)}{R_L(t)} = \frac{S_H(T)}{S_L(T)} \quad \forall t \geq T. \quad (11)$$

From the first order conditions (5) and equilibrium relative extraction (11) we find the equilibrium relative scarcity rent:

$$\frac{p_{RH}(t)}{p_{RL}(t)} = \frac{\eta_H}{\eta_L} \left(\frac{S_H(T)}{S_L(T)} \right)^{-1/\sigma} \quad \forall t. \quad (12)$$

These results reveal that as long as the economy is unconstrained and does not expect future climate policy, relative extraction in the unconstrained economy is constant and equals relative stocks at each point in time. Since conservation of both resource stocks requires that resource owners earn the same return on the two resources, both resource prices grow at

the common rate r in equilibrium. Hence, the relative price is constant over time and the CES production function then implies that relative demand is constant as well. As resource owners want to fully exploit the available reserves, stock dynamics require relative extraction to equal relative stocks which implies that the initial relative scarcity rent in an unconstrained economy is determined by initial availability of the resources.

4 An unexpected emission constraint

We now introduce an unexpected ceiling on emissions in the model. The ceiling is introduced at time $t = 0$ and is binding by then. It will stay at the level \bar{Z} forever, which is known by all agents ('Kyoto forever'). The constraint will not bind forever, though, since resource stocks, from which emissions stem, are depleted over time (cf. (10)). We define $t = T$ as the instant from which onward total emissions cease to be constrained. Hence, $p_Z(t) = 0$ for any $t \geq T$ and, from (4) and (9), $\varepsilon_H \rho S_H(T) + \varepsilon_L \rho S_L(T) = \bar{Z}$. The total amount of CO₂ that will be emitted from $t = 0$ on can be written as

$$\begin{aligned} \varepsilon_H S_{H0} + \varepsilon_L S_{L0} &= [\varepsilon_H (S_{H0} - S_H(T)) + \varepsilon_L (S_{L0} - S_L(T))] \\ &\quad + \varepsilon_H S_H(T) + \varepsilon_L S_L(T). \end{aligned}$$

Using the fact that the term in square brackets represents total emissions in the period that the economy is constrained (which equals $T\bar{Z}$), and that from the definition of T the sum of the other terms at the right-hand side equals \bar{Z}/ρ , we find the following solution for T :

$$T = \frac{\varepsilon_H S_{H0} + \varepsilon_L S_{L0}}{\bar{Z}} - \frac{1}{\rho}. \quad (13)$$

Clearly, a larger initial stock or a stricter environmental policy implies a longer period of being restricted. A lower discount rate, and hence more patient consumers, implies that the economy is suffering the constraint for a shorter period as the economy tends to extract and pollute less (see (10)).

The economy's behaviour after the constraint ceases to be binding is described in the previous section. Therefore we can now focus on how the two stocks of fossil fuels will be exploited when the CO₂ emission constraint binds. The development of relative extraction in the constrained economy with an unannounced emission constraint is summarized in the following proposition:

Proposition 1. *Suppose a CO₂ emission constraint is unexpectedly introduced at time $t = 0$. Let T denote the instant at which the constraint ceases to be binding and define $\bar{S} = (\eta_H \varepsilon_L / \eta_L \varepsilon_H)^\sigma$. Then*

1. *if $S_{H0}/S_{L0} < \bar{S}$,*

(a) $S_H(t)/S_L(t) < R_H(t)/R_L(t) < \bar{S}$ for all $t \in (0, T)$;

(b) $p_{RH}(t)/p_{RL}(t) > (\eta_H/\eta_L)(S_{H0}/S_{L0})^{-1/\sigma} > \varepsilon_H/\varepsilon_L$ for all $t \geq 0$;

(c) $d(S_H(t)/S_L(t))/dt < 0$ for all $t \in (0, T)$;

(d) $d(R_H(t)/R_L(t))/dt < 0$ for all $t \in (0, T)$;

(e) $R_H(T)/R_L(T) = S_H(T)/S_L(T) < S_{H0}/S_{L0}$;

(f) $\lim_{t \downarrow 0} R_H(t)/R_L(t) > \lim_{t \uparrow 0} R_H(t)/R_L(t) = S_{H0}/S_{L0}$;

2. *if $S_{H0}/S_{L0} > \bar{S}$, all strict inequalities in part 1 are reversed;*

3. *if $S_{H0}/S_{L0} = \bar{S}$, all strict inequalities in part 1 are replaced by equalities;*

4. *if $S_{H0}/S_{L0} \neq \bar{S}$,*

(a) $\lim_{t \uparrow 0} Z(t)/Y(t) > \lim_{t \downarrow 0} Z(t)/Y(t)$;

- (b) $\lim_{t \uparrow 0} Z(t)/Y(t) < Z(T)/Y(T)$;
- (c) $d(Z(t)/Y(t))/dt > 0$ for all $t \in (0, T]$;
- (d) $d(Z(t)/Y(t))/dt = 0$ for all $t > T$.

Proof. See Appendix. □

The proposition states that if the high carbon resource is physically relatively scarce, in particular if $S_{H0}/S_{L0} < \bar{S}$, then at the instant that emissions become unexpectedly restricted, the relative scarcity rent of this scarce high-carbon input (p_{RH}/p_{RL}) jumps up, and its relative use (R_H/R_L) first jumps up and then gradually declines to a level that is lower than before (see 1(e)). As a consequence the high-carbon resource is physically even scarcer in the long run, which explains the upward jump in the relative scarcity rent. If the high carbon resource is relatively abundant, in particular if $S_{H0}/S_{L0} > \bar{S}$, the reverse happens. Nevertheless, in both cases, on impact the restriction on emissions results in a lower emission intensity, Z/Y , which then gradually increases.

We illustrate the paths of extraction for the case in which $S_{H0}/S_{L0} < \bar{S}$, corresponding to part 1 of the proposition, by the thick arrows in Figure 1. The constrained economy moves along line \bar{Z} , at which emissions are at the imposed ceiling and which is defined by $R_H = (\bar{Z} - \varepsilon_L R_L)/\varepsilon_H$. Since the economy moves to lower production isoquants, pollution per unit of GDP increases over time. The unconstrained economy, which according to (9) extracts a constant fraction of each available stock, moves down along a ray from the origin with slope S_{H0}/S_{L0} .

The proposition shows that on impact substitution is not necessarily to the 'environmentally clean' low-carbon input but to the relatively scarce input, and that this substitution reduces pollution per unit of output (part 4 of the proposition). In particular, on impact substitution is towards the low-carbon resource if $S_{H0}/S_{L0} > \left(\frac{\eta_H/\eta_L}{\varepsilon_H/\varepsilon_L}\right)^\sigma \equiv \bar{S}$ (that is, if high carbon

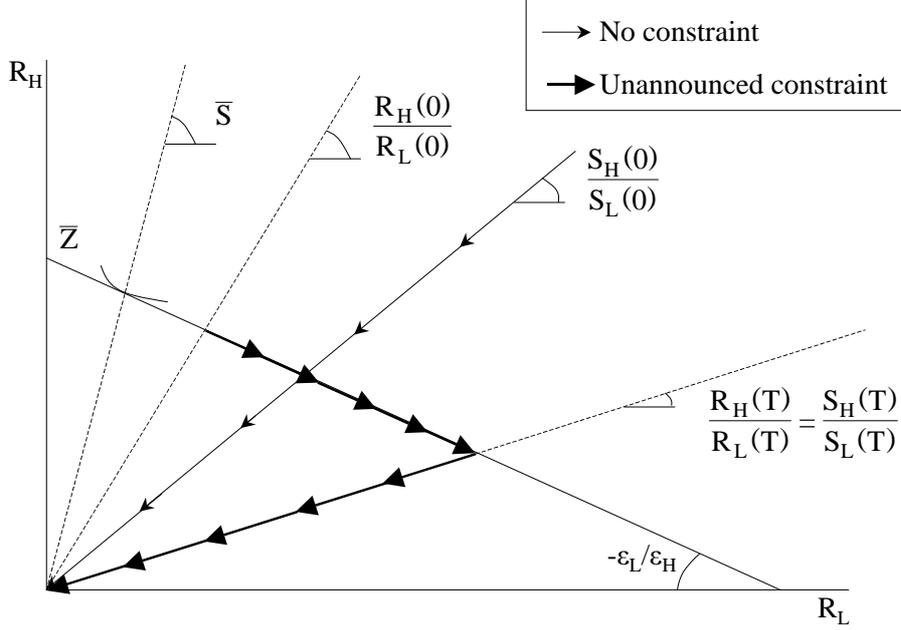


Figure 1: Extraction paths for $\bar{S} > S_H(0)/S_L(0)$: the unconstrained economy and the economy with an unannounced constraint

resources are relatively abundant), but towards the high-carbon resource if $S_{H0}/S_{L0} < \left(\frac{\eta_H/\eta_L}{\varepsilon_H/\varepsilon_L}\right)^\sigma \equiv \bar{S}$ (that is, if high carbon resources are relatively scarce).³ In this sense, we can define the 'economically clean' input to be the relatively scarce input j for which $S_{j0}/S_{-j0} < (\eta_j\varepsilon_{-j}/\eta_{-j}\varepsilon_j)^\sigma$, as opposed to the environmentally clean input i for which $\varepsilon_i < \varepsilon_{-i}$. Intuitively, the scarce resource cannot unlimitedly be used intensively because it is available in relatively low quantities. When used less intensively, its marginal productivity is high and substituting towards it increases output per unit of

³Note that although the proposition only specifies how relative extraction evolves over time, extraction of each of the two resources follows directly: when constrained total emissions equal \bar{Z} , so that R_H and R_H/R_L move in the same direction, while R_L and R_H/R_L move in opposite directions.

emissions. The downside of such substitution is that the resource becomes even scarcer so that in the future its use has to be reduced and pollution intensity will rise. However, this occurs when the cost of emissions is low or (after T) even zero, so that this intertemporal substitution of resource use reduces the cost of complying with the carbon constraint.

Since no non-renewable resource can be forever used more intensively, the direction of substitution between resource use after the ceiling is imposed contrasts with the substitution on impact. If the emission constraint on impact triggers faster extraction of one resource, this resource must be extracted at a slower pace later on, since a given stock restricts cumulative extraction. As a result, climate change policy cannot make production less high-carbon intensive forever, as is stated in the following corollary:

Corollary 1. *If $S_{H0}/S_{L0} \neq \bar{S}$, there is always a period when the constraint is binding and at the same time the high-carbon input is used relatively more intensively in the economy with an emission constraint than in the economy without.*

Proof. Proposition 1, (a) and (e) in part 1 and 2 imply $R_H(0)/R_L(0) \leq S_{H0}/S_{L0} \leq R_H(T)/R_L(T)$ in an economy with binding emission constraint, while (11) implies $R_H(t)/R_L(t) = S_{H0}/S_{L0}$ in an economy without. \square

To explore the mechanisms behind our results, we divide (5) for the low-carbon input by that for the high-carbon input and rewrite the result as:

$$\frac{\eta_H}{\eta_L} \left(\frac{R_H}{R_L} \right)^{-1/\sigma} = (1 - \zeta) \frac{p_{RH}}{p_{RL}} + \zeta \frac{\varepsilon_H}{\varepsilon_L} \quad (14)$$

where $\zeta = p_Z \varepsilon_L / (p_{RL} + p_Z \varepsilon_L)$ is the share of pollution costs in the user price of low-carbon resources. This equation reveals that relative demand for energy sources depends on the relative user price, which is a weighted average of relative scarcity rents and relative pollution costs (right-hand side

expression of (14)). As a result, relative energy use is naturally bounded by relative scarcity rents (which reflect the physical availability of the resources) and relative pollution costs.

Over time, the share of pollution cost in the user price ζ falls, since scarcity rents increase and the price of pollution permits falls. Relative scarcity rents (p_{RH}/p_{RL}) and pollution costs ($\varepsilon_H/\varepsilon_L$) are constant over time (see (12)). Therefore, the relative user price of high-carbon resources may rise or fall over time depending on the sign of $\varepsilon_H/\varepsilon_L - p_{RH}/p_{RL}$ (see (14)). If $\varepsilon_H/\varepsilon_L < p_{RH}/p_{RL}$, as in part 1 of the proposition, the relative user price of high-carbon resources increases over time. Intuitively, with this inequality the high-carbon resource is relatively costly mainly because of scarcity cost rather than pollution cost, and this resource benefits the least from declines in pollution costs. Users then gradually substitute towards the low-carbon resource during the period that the emissions constraint is binding. In the opposite situation with ($\varepsilon_H/\varepsilon_L > p_{RH}/p_{RL}$) the high-carbon resource mainly benefits from pollution price reductions and users gradually substitute to the high-carbon resource.⁴

The following corollary summarizes the implication of our results that climate policy does not always increase the relative user price of the high-carbon input: climate policy only increases the relative user price of carbon inputs if the high-carbon input is physically relatively scarce.

Corollary 2. *Define the user price of resource i as $p_{Ri} + \varepsilon_i p_Z$. Suppose a binding emission constraint is unexpectedly introduced. Then*

1. *if $S_{H0}/S_{L0} < \bar{S}$, the relative user price of high-carbon resources drops*

⁴The relative scarcity rent is endogenous, so that we cannot directly determine the sign of $\varepsilon_H/\varepsilon_L - p_{RH}/p_{RL}$. However, the proposition shows that if equilibrium relative resource rent without constraint falls short of relative pollution costs, this applies a fortiori in the constrained economy: using notation as defined at the start of the appendix, we have $p(0^-) = \eta S_0^{-1/\sigma} < \varepsilon \Leftrightarrow S_0 < \bar{S}$ from (12), so that $p(0^+) > \varepsilon$ from proposition 1, part 1(b).

on impact and increases while the economy is constrained.

2. if $S_{H0}/S_{L0} > \bar{S}$, the relative user price of high-carbon resources jumps up on impact and decreases while the economy is constrained.
3. if $S_{H0}/S_{L0} = \bar{S}$, the relative user price is constant over time.

Proof. Follows from (14) and parts 1-3(d) of Proposition 1. \square

As climate policy is a delicate political topic it is likely to be subject to changes in its stringency over time. If the emission constraint becomes tighter, pollution costs become a more important determinant in the cost of resource use as compared to scarcity rents, *ceteris paribus*. As a consequence the relative extraction rate jumps closer towards \bar{S} (where \bar{S} is the level that would apply if scarcity did not matter), as is stated by the following claim:

Proposition 2. *Suppose a binding CO_2 emission constraint \bar{Z}_1 that ceases to be binding at $t = T_1$ is introduced at time $t = 0$. Suppose that at time $t = t_2 \in (0, T_1)$ the constraint is unexpectedly changed to \bar{Z}_2 and still binding. If $S_{H0}/S_{L0} \neq (\eta/\varepsilon)^\sigma$ and $\bar{Z}_2 < \bar{Z}_1$, then⁵*

1. the economy is unconstrained at

$$t \geq T_2 \equiv \frac{[\varepsilon_L S_{L0} + \varepsilon_H S_{H0}]}{Z_2} - \frac{1}{\rho} - t_2 \frac{\bar{Z}_1 - \bar{Z}_2}{Z_2} > T_1;$$

2. relative extraction jumps according to

$$\left| \lim_{t \downarrow t_2} R_H(t) / R_L(t) - \bar{S} \right| < \left| \lim_{t \uparrow t_2} R_H(t) / R_L(t) - \bar{S} \right|;$$

3. relative extraction when unconstrained changes according to

$$\left| R_H(T_2) / R_L(T_2) - \bar{S} \right| > \left| R_H(T_1) / R_L(T_1) - \bar{S} \right|;$$

4. relative resource rents jump according to

$$\left| \lim_{t \downarrow t_2} p_{RH}(t) / p_{RL}(t) - \varepsilon_H / \varepsilon_L \right| > \left| \lim_{t \uparrow t_2} p_{RH}(t) / p_{RL}(t) - \varepsilon_H / \varepsilon_L \right|;$$

⁵If $\bar{Z}_2 > \bar{Z}_1$, then all inequality signs in parts 1-5 are reversed.

5. *the pollution intensity of production jumps down:*

$$\lim_{t \downarrow t_2} Z(t)/Y(t) < \lim_{t \uparrow t_2} Z(t)/Y(t).$$

Proof. See Appendix. □

When climate policy suddenly becomes more stringent, the constraint is binding for a longer period (part 1 of the proposition). With a more stringent constraint less units of carbondioxide are extracted so that it takes longer before unconstrained emissions are below the level of the ceiling. In reaction to tighter environmental policy the economy further increases the productivity of the inputs in terms of emissions per unit of GDP (part 5). The resulting relative extraction rate and relative resource rent are closer to the results that would apply in an economy in which pollution only (rather than scarcity) would matter.

5 Announcement effects

We now investigate how the economy reacts to an emission constraint in the case that agents anticipate the actual implementation of the policy. In particular, we study the path of resource extraction for the situation in which the carbon constraint starts to be effective at time $t_K > 0$, but is announced at time $t = 0$, so that preparations can be made over the period $t \in (0, t_K)$.

Agents maximize the same objective functions subject to the same constraints as in the previous section, with the only difference that the constraint (4) is now binding from $t = t_K$ instead of $t = 0$. For an announced constraint, the path of relative extraction can be characterized by the following proposition:

Proposition 3. *Suppose a CO_2 emission constraint is announced at time $t = 0$ and introduced at time $t_K > 0$. Let $t = T_A$ be the instant at which the constraint ceases to be binding. Then,*

1. when S_{i0} is replaced by $S_i(t_K)$ and $R_i(T)$ and $S_i(T)$ are replaced by $R_i(T_A)$ and $S_i(T_A)$ respectively, parts 1-3(a), (c) and (d) of Proposition 1 hold for all $t \in (t_K, T_A)$ and part (b) of parts 1-3 holds for all $t \geq 0$;
2. if $S_{H0}/S_{L0} < \bar{S}$,
 - (a) $R_H(t)/R_L(t) = R_H(T_A)/R_L(T_A) < S_{H0}/S_{L0}$ for all $t \in (0, t_K)$;
 - (b) $S_H(t)/S_L(t) > S_{H0}/S_{L0}$ for all $t \in (0, t_K)$;
 - (c) $d(S_H(t)/S_L(t)) dt > 0$ for all $t \in (0, t_K)$;
 - (d) $\lim_{t \uparrow t_K} R_L(t) > \lim_{t \downarrow t_K} R_L(t)$;
 - (e) $\lim_{t \uparrow t_K} R_H(t) < \lim_{t \downarrow t_K} R_H(t)$;
3. if $S_{H0}/S_{L0} > \bar{S}$, all inequalities in part 2 are reversed;
4. if $S_{H0}/S_{L0} = \bar{S}$, all inequalities in part 2 are replaced by equalities;
5. (a) if $S_{H0}/S_{L0} \geq \bar{S}$, then $\lim_{t \uparrow t_K} Z(t) > \lim_{t \downarrow t_K} Z(t)$ and $\lim_{t \uparrow t_K} Z(t)/Y(t) > \lim_{t \downarrow t_K} Z(t)/Y(t)$;
- (b) if $S_{H0}/S_{L0} = \bar{S}$, then $\lim_{t \uparrow t_K} Z(t) = \lim_{t \downarrow t_K} Z(t)$ and $\lim_{t \uparrow t_K} Z(t)/Y(t) = \lim_{t \downarrow t_K} Z(t)/Y(t)$.

Proof. See Appendix. □

The proposition implies that the announcement of an emission constraint at a future date immediately causes a drop in the rate of extraction of the relatively more productive resource (in terms of GDP per unit of emissions). As a consequence more of this resource is conserved so that the constrained period starts with (relatively) more of the productive resource, as compared to the situation without announcement. At the instant the constraint becomes binding the extraction rate of the productive input jumps up and

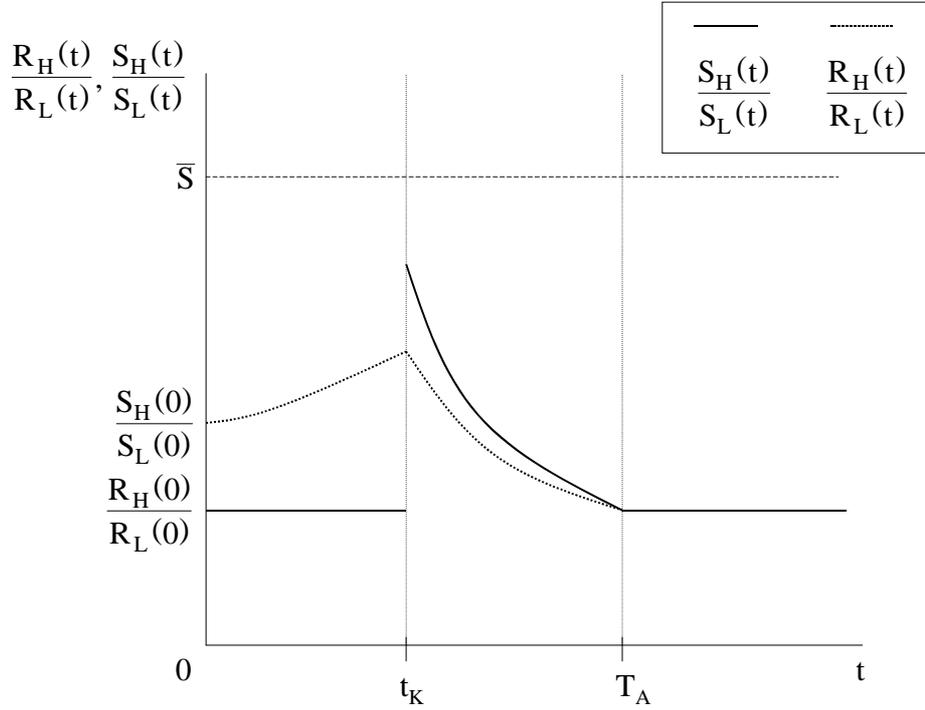


Figure 3: Development of relative extraction and relative stocks with announced constraint, for $\bar{S} > S_H(0)/S_L(0)$

is introduced. With an anticipated constraint the economy substitutes the more productive resource (in terms of GDP per unit of CO_2) for the less productive one at the instant the constraint becomes binding, while keeping output constant (a jump in relative extraction along the production isoquant from $R_H(0)/R_L(0)$ to $R_H(t_K)/R_L(t_K)$ in Figure 2). As a consequence, the economy's pollution intensity Z/Y decreases. Since the introduction of the constraint is expected and fully anticipated, consumption cannot jump and substitution takes place along a production isoquant, changing emissions but not the level of output of the final good. This is in contrast with the case without announcement in which both emissions and output jump at the instant the constraint is introduced.

Comparing part 1 of Proposition 2 with parts 2-4 we see that the less productive resource is used more intensively before the constraint becomes binding, while the more productive resource is used more intensively from the moment that the constraint becomes binding on. If $S_{H0}/S_{L0} < \bar{S}$ depletion of the more dirty resource is relatively slow at first, but it is relatively fast once the economy is constrained.

6 Concluding remarks

In reaction to a ceiling on the amount of carbondioxide emissions an economy may want to substitute between high-carbon and low-carbon fuels. We have shown that in the standard Hotelling model extended with a second, imperfectly substitutable resource, the economy optimally decreases the level of carbondioxide per unit of GDP. However, this is not always obtained through substitution of high-carbon for low-carbon inputs (e.g. natural gas for coal). Initially resource users substitute towards the input which, at the margin, has the highest level of output per unit of carbondioxide, and this may be the input with highest level of emissions per unit of energy. Since the total available resource stock is fixed, more intensive use of a resource early on has to be followed by less intensive use later on and vice versa. Thus the economy, although constrained in its emissions of carbondioxide, either sooner or later uses relatively more of the "dirty" input as compared to the case where it does not face a ceiling on emissions. With an anticipated constraint the economy switches towards the less productive input (in terms of GDP per unit of carbon) before the constraint becomes binding and jumps towards a relatively more intensive use of the more productive input when the emission ceiling becomes binding.

The interaction between the threat of future scarcity and the cost of current pollution is driving our results. On the one hand, scarcity of a particular resource results in a high relative price, low extraction, and high

productivity. On the other hand, when emissions are costly, producers substitute towards the resource with highest productivity per unit of emissions, which is the relatively scarce factor. Hence, in the *short run* (for given resource stocks), tighter emission constraints imply substitution to the relatively *scarce* resource. But then, this resource becomes even scarcer over time and in the *long run*, especially when emission levels are low enough to make the emissions constraint irrelevant, the economy must eventually use relatively more of the initially *abundant* resource. The overall result is that an emission constraint causes "frontloading" of extraction of the scarce resource. This scarce resource is not necessarily the high-carbon resource: we have established how emission and productivity coefficients, as well as the substitution elasticity, determine which of the two resource stocks should be considered as relatively scarce.

As long as an emission constraint is expected to be implemented in the future, pollution costs do not directly but indirectly affect relative extraction. Agents anticipate that in the future the currently scarce resource will be even scarcer so that they already start using less of the relatively scarce factor now. This anticipatory action makes the scarce and productive resource more abundant at the time emissions are costly and allows the economy to save cost.

Our results suggest that in order to cope with climate change, energy policies should not necessarily be directed to a fast transition to low-carbon energy sources. In addition to relative pollution content, scarcity of resources, in particular expected scarcity in the post-Kyoto period, as well as relative productivity of different energy resources should be taken into account. Although CO₂ emissions per unit of energy are larger for coal than for oil, and although coal is much more abundant, it is at considerable disadvantage compared to its imperfect substitutes oil or natural gas in for example transport activities. Hence, it remains to be seen whether a

shift away from coal leads to lower emissions per unit of GDP. This is an empirical question that is left to future research.

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A Appendix

We simplify notation using variables without subscripts to denote high-carbon to low-carbon ratios: $R(t) \equiv R_H(t)/R_L(t)$, $S(t) \equiv S_H(t)/S_L(t)$ and $p(t) \equiv p_{RH}(t)/p_{RL}(t)$, and similarly $\eta \equiv \eta_H/\eta_L$, $\varepsilon \equiv \varepsilon_H/\varepsilon_L$, and $S_0 \equiv S_{H0}/S_{L0}$. For any variable X we define $X(\tau^-) \equiv \lim_{t \uparrow \tau} X(t)$ and $X(\tau^+) \equiv \lim_{t \downarrow \tau} X(t)$.

Before proving the propositions, we present and prove the following lemma, which summarizes the dynamics of relative extraction R over three relevant time periods, first when the constraint is announced but not effective, (ii) when the constraint binds, (iii) when the constraint is not binding.

Lemma 1. *Let t_K be the instant at which the constraint becomes binding and T_U the instant at which the constraint ceases to be binding. Then*

$$R(t) = S(T_U), \quad \forall t \in (0, t_K) \quad (\text{A.1})$$

$$\frac{dR}{dt} = f(R) \left[R^{1/\sigma} - \frac{\eta}{\varepsilon} \right], \quad \forall t \in (t_K, T_U), \quad (\text{A.2})$$

$$\int_t^{T_U} \left(\frac{1}{1 + \varepsilon R(\tau)} - \frac{1}{1 + \varepsilon S(t)} \right) d\tau = 0, \quad \forall t \in (t_K, T_U) \quad (\text{A.3})$$

$$R(t) = \lim_{t' \uparrow T_U} R(t'), \quad \forall t \geq T_U. \quad (\text{A.4})$$

where f is a function of R and parameters with $f > 0$ and $\partial f / \partial \bar{Z} = 0$ for all $R > 0$.

Proof. For all $t \in [0, t_K) \cup [T_U, \infty)$ we have $p_Z = 0$ and, from (5), $p(t) = \eta(R(t))^{-1/\sigma}$. For all $t \geq T_U$, we have, from (11), $R(t) = S(t)$. Since p is constant over time (see (7)), we find $p(t) = \eta(S(T_U))^{-1/\sigma} \forall t$; this proves (A.1).

Absent unexpected shocks, no resource will be left unexploited so that we can write:

$$\begin{aligned} \int_t^\infty \frac{R_H(\tau)}{S_H(t)} d\tau &= 1 = \int_t^\infty \frac{R_L(\tau)}{S_L(t)} d\tau \\ &\Leftrightarrow \int_t^\infty \left(\frac{R_H(\tau)}{S_H(t)} - \frac{R_L(\tau)}{S_L(t)} \right) d\tau = 0 \\ &\Leftrightarrow \int_t^{T_U} \left(\frac{\varepsilon_H R_H(\tau)}{\varepsilon_H S_H(t)} - \frac{\varepsilon_L R_L(\tau)}{\varepsilon_L S_L(t)} \right) d\tau + \int_{T_U}^\infty \left(\frac{R_H(\tau)}{S_H(t)} - \frac{R_L(\tau)}{S_L(t)} \right) d\tau = 0. \end{aligned}$$

From (11) the second integral in the last line equals zero. Substituting, from (4), $\varepsilon_H R_H = \bar{Z} - \varepsilon_L R_L$ in the first integral in the last line, and rewriting, we find:

$$\int_t^{T_U} \left[\frac{\varepsilon_L S_L(t) + \varepsilon_H S_H(t)}{\varepsilon_L S_L(t) \varepsilon_H S_H(t)} \bar{Z} \right] \left(\frac{\varepsilon_L S_L(t)}{\varepsilon_L S_L(t) + \varepsilon_H S_H(t)} - \frac{\varepsilon_L R_L(\tau)}{\bar{Z}} \right) d\tau = 0$$

Dividing out the term in brackets, substituting (4) to eliminate \bar{Z} , and rewriting, we find (A.3).

Prices cannot jump in absence of unexpected events due to arbitrage. Then R can only jump if output Y jumps (see (5)), which is ruled out by the concavity of the utility function. This proves (A.4).

Finally we need to derive (A.2). We rewrite (5) as

$$\theta_i = \frac{p_{Ri}R_i}{Y} + \lambda_i q \quad (\text{A.5})$$

where $\lambda_i \equiv \varepsilon_i R_i/Z \in (0, 1)$ is the low-carbon's share in emissions, $\theta_i \equiv [1 + (\eta_{-i}/\eta_i)(R_{-i}/R_i)^{(\sigma-1)/\sigma}]^{-1} \in (0, 1)$ is the low-carbon's production elasticity, and $q \equiv p_Z Z/Y$ is the share of expenditures on emission permits in GDP. Differentiating (A.5) with respect to time, using the Hotelling rules (7), the Ramsey rule (2), using the fact that $R_i/R_j = (\varepsilon_j/\varepsilon_i)\lambda_i/(1 - \lambda_i)$ when differentiating θ_i , and using the fact that $\widehat{Z} = 0$ implies $\widehat{\lambda}_i = \widehat{\varepsilon_i R_i/Z} = \widehat{R}_i$, we find for $t \in (t_K, T_U)$:

$$\left(\frac{\theta_i(1 - \theta_i)(\sigma - 1) - \sigma\theta_i(1 - \lambda_i)}{(1 - \lambda_i)\sigma} \right) \widehat{\lambda}_i = (\theta_i - \lambda_i q) \rho + \lambda_i q \widehat{q}. \quad (\text{A.6})$$

We divide (A.6) by λ_i and subtract the resulting expression for the high-carbon input from the resulting expression for the low-carbon input to eliminate q and rewrite to obtain:

$$\widehat{\lambda}_L = \frac{(1 - \lambda_L)\sigma\rho}{\theta_L(1 - \theta_L) + (\lambda_L - \theta_L)^2\sigma} (\lambda_L - \theta_L). \quad (\text{A.7})$$

The sign depends on the sign of $\lambda_L - \theta_L$. Using the definitions of λ_i and θ_i , we rewrite this as:

$$\begin{aligned} \lambda_L - \theta_L &= \lambda_L \theta_L \left[\left(\frac{1}{\theta_L} - 1 \right) - \left(\frac{1}{\lambda_L} - 1 \right) \right] \\ &= \lambda_L \theta_L \left[\left(\frac{\theta_H}{\theta_L} \right) - \left(\frac{\lambda_H}{\lambda_L} \right) \right] \\ &= \lambda_L \theta_L \left[\frac{\eta_H}{\eta_L} \left(\frac{R_H}{R_L} \right)^{1-1/\sigma} - \left(\frac{\varepsilon_H R_H}{\varepsilon_L R_L} \right) \right] \end{aligned} \quad (\text{A.8})$$

Since by definition $\lambda_L/(1 - \lambda_L) = \varepsilon_L R_L/\varepsilon_H R_H = 1/\varepsilon R$, the left-hand side

of (A.7) can be written as:

$$\hat{\lambda}_L = -(1 - \lambda_L) \hat{R} \quad (\text{A.9})$$

Substituting (A.8) and (A.9) into (A.7), and noting that both λ_L and θ_L are functions of relative extraction R and parameters only, we find (A.2). \square

A.1 Proof of Proposition 1

Equation (A.3) implies that if R monotonically decreases over time, then $R(t)$ must first exceed, but eventually fall short of $S(t)$. More generally, for $\forall t \in (t_K, T_U)$, we have: if $dR(\tau)/d\tau \leq 0$, $\forall \tau \in (t, T_U)$, then $R(t) \geq S(t) \geq R(T_U)$. Equation (A.2) shows that, indeed, dR/dt cannot switch sign between t_K and T_U . Hence we have:

$$dR(t)/dt \leq 0 \Leftrightarrow (\eta/\varepsilon)^\sigma \geq R(t) \geq S(t) \geq R(T_U), \forall t \in (t_K, T_U).$$

This proves part 1-3, (a) and (d). From stock dynamics (6) we derive

$$\frac{dS}{dt} = \frac{R_L}{S_L} (S - R). \quad (\text{A.10})$$

Combined with part (a) of the proposition, this proves (c) in part 1-3. Part (e) then follows from (A.1) and part (a) of the proposition. Part (b) follows from (e) and (12). Part (f) follows from (11) and part (a) of the proposition. This completes the proof of parts 1-3 of proposition 1.

Finally we need to prove part 4. From (3) and (4) we find that Z/Y is a function of R only. Taking the first order derivative, we find:

$$\frac{d(Z/Y)}{dR} = \frac{\varepsilon - \eta R^{-1/\sigma}}{\frac{1}{\varepsilon_L} \frac{1}{\eta_L} [\eta_H R^{\frac{\sigma-1}{\sigma}} + \eta_L]^{\frac{\sigma}{\sigma-1} + 1}}$$

so that Z/Y reaches a minimum for $R = (\eta/\varepsilon)^\sigma \equiv \bar{S}$ and increases in $|R - \bar{S}|$.

From 1(a) and (11), we have $|R(0^-) - \bar{S}| = |S_0 - \bar{S}| > |R(0^+) - \bar{S}|$ and this proves part 4(a). From 1(e) and (11), we have $|R(0^-) - \bar{S}| = |S_0 - \bar{S}| < |R(T) - \bar{S}|$. This proves part 4(b). Part 4(c) follows from 1(a) and 1(d), and 4(d) follows from (11).

A.2 Proof of Proposition 2

We prove part 1 by using the procedure we used in the main text and derive T_2 from (9), (10), and (A.4) in the following way:

$$\begin{aligned}
\varepsilon_H S_{H0} + \varepsilon_L S_{L0} &= [\varepsilon_H (S_{H0} - S_H(t_2) + S_H(t_2) - S_H(T_2)) \\
&\quad + \varepsilon_L (S_{L0} - S_L(t_2) + S_L(t_2) - S_L(T_2))] \\
&\quad + \varepsilon_H S_H(T_2) + \varepsilon_L S_L(T_2) \\
&= t_2 \bar{Z}_1 + (T_2 - t_2) \bar{Z}_2 + \varepsilon_H \frac{R_H(T_2)}{\rho} + \varepsilon_L \frac{R_L(T_2)}{\rho} \\
\frac{\varepsilon_H S_{H0} + \varepsilon_L S_{L0}}{\bar{Z}_1} - \frac{1}{\rho} &= t_2 + (T_2 - t_2) \frac{\bar{Z}_2}{\bar{Z}_1} + \frac{\bar{Z}_2}{\bar{Z}_1} \frac{1}{\rho} - \frac{1}{\rho} \\
T_1 - T_2 &= \left(t_2 - T_2 - \frac{1}{\rho} \right) \frac{\bar{Z}_1 - \bar{Z}_2}{\bar{Z}_1}
\end{aligned}$$

This explicitly solves for T_2 . Since by assumption the new constraint is binding when introduced, we must have $t_2 < T_2$, and hence $T_2 \gtrless T_1 \iff \bar{Z}_1 \lesseqgtr \bar{Z}_2$, which proves part 1.

For part 2-4 we focus on the case where $S_{H0}/S_{L0} < (\eta/\varepsilon)^\sigma \equiv \bar{S}$, the proofs for the other cases are analogous. We continue the notation of the proof of Proposition 1. Since $\partial f/\partial \bar{Z} = 0$ in (A.2), a decline in \bar{Z} affects the equilibrium path of $R(t)$ only through an increase in T . Write $R^o(t)$ and $R^n(t)$ for relative extraction with the old and the new value for \bar{Z} respectively. Suppose the unexpected change in the constraint would not on impact change relative extraction, i.e. $R^n(t_2^+) = R^o(t_2^+)$. Then, from (A.2), $R^n(t) = R^o(t) \forall t \in (t_2, T_1]$, but $R^n(t) < R^o(t) \forall t \in (T_1, T_2)$ and the integral at the left-hand side of (A.3) with $R = R^n, t = t_2$ and $T_U = T_2$

exceeds the integral with $R = R^o, t = t_2$ and $T_U = T_1$. But this violates the equality in (A.3) for the new path. If $R^n(t_2^+) < R^o(t_2^+)$, then the integral for the new path is positive a fortiori. Hence, we must have $R^n(t_2^+) > R^o(t_2^+)$, which proves part 2 of the proposition.

We prove part 3 in a similar way. Suppose $R^n(T_2) = R^o(T_1)$, then $R^n(t) = R^o(t - T_2 + T_1)$ for $t \in (t_2 + T_2 - T_1, T_2)$ and $R^n(t) > R^o(t_2)$ for $t \in (t_2, t_2 + T_2 - T_1)$. But then (A.3) is violated on the new path since the integral becomes positive. A fortiori (A.3) is violated with $R^n(T_2) > R^o(T_1)$. Hence we must have $R^o(T_1) > R^n(T_2)$.

Combining the results in part 3 with (A.1), we find $R^o(T_1) = S^o(T_1) > S^n(T_2) = R^n(T_2)$. From (12), we then have $p^o(T_1) < p^n(T_2)$. Since $p > \varepsilon$ (from proposition 1, part 1b), and relative resource rents are constant over time (from (7)), we prove part 4.

Part 5 directly follows from part 4 of Proposition 1.

A.3 Proof of Proposition 3

The proof of the first part of the proposition follows the proof of Proposition 1 and is omitted.

In part 2-4, (a), the equality follows from (11) and (A.1); the inequality follows from (e) in part 1-3 in proposition 1.

Assume that $S(t_K) < (\eta/\varepsilon)^\sigma \equiv \bar{S}$. Then, from part 1 of the proposition and (A.10), we have

$$S(t_K) > S(T_A). \tag{A.11}$$

Suppose $S_0 \leq R(0^+)$. Then from (A.1), (A.4), and (A.10) the relative stock has to jump up at $t = t_K$ for (A.11) to hold, which violates continuity of stocks. So $S_0 > R(0^+) = S(T_A)$. It follows from (A.10) that $dS/dt > 0 \forall t \in (0, t_K)$ so that $S_0 < S(t_K)$. Since we started from the assumption

$S(t_K) < \bar{S}$, we must have $S_0 < \bar{S}$. This completes the proofs of parts 2(a)-(c); the proofs of parts 3 and 4, (a)-(c) are analogous and are omitted.

Arbitrage prevents resource rents to jump, so from (7) r is finite and from (2) income cannot jump. Since the emissions constraint starts to be binding at $t = t_K$ by construction, either emissions jump down and hence emissions per unit of income jump down at $t = t_K$, or emissions do not jump and neither do emissions per unit of output. The latter requires that relative extraction does not jump, which requires $S_0 = \bar{S}$, see part 4(a)-(c). This proves parts 4(d) and (e) and 5(b). The former, a jump down in emissions with continuous income, requires a jump along a production isoquant. Hence Z/Y falls and R_L and R_H move in opposite directions. Consider the case $S_0 < \bar{S}$. Then from part 2, (a), we have $R(0^+) = R(t_K^-) = S(T_A) < \bar{S}$, and from part 1 of this proposition $R(t_K^+) > S(t_K) > S(T_A)$, so that $R(t_K^+) > R(t_K^-)$. That is, at time t_K , R jumps up, so that R_H jumps up and R_L jumps down. Using the same procedure for the case $S_0 > \bar{S}$, we find that R_H jumps up and R_L jumps down. This proves parts (d) and (e) of parts 2-3 and (a) of part 5.