OPTIMAL PRIVATIZATION USING QUALIFYING AUCTIONS

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June 2005
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June 13, 2005

Abstract

This paper explores the use of auctions for privatizing public assets. In our model, a single "insider" bidder (e.g. incumbent management of a government-owned firm) possesses information about the asset’s risky value. In addition, bidders are privately informed about their costs of exploiting the asset. Due to the insider’s presence, uninformed bidders face a strong winner’s curse in standard auctions with devastating consequences for revenues. We show that the optimal mechanism discriminates against the informationally advantaged bidder to ensure truthful information revelation. The optimal mechanism can be implemented via a simple two-stage “qualifying auction.” In the first stage of the qualifying auction, non-binding bids are submitted to determine who enters the second stage, which consists of a standard second-price auction augmented with a reserve price.

JEL codes: D440, D820, L330

Key words: privatization, qualifying auction, winner's curse, information advantage

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1. Introduction

The World Bank’s “toolkit” or “practitioners’ guide” to privatization describes the following two-stage auction procedure for privatizing public assets (Welch and Frémont, 1998, p. 32): in the first stage non-binding expressions of interest are received from interested buyers. Based on these expressions of interest and a review of the financial capacity of potential bidders a short list of potential buyers is selected. These bidders then move to the second stage of the process, which consists of a more traditional auction with binding bids.

Welch and Frémont (1998) mention several practical advantages of this procedure. In this paper, we abstract away from these practical issues and investigate how the mechanism performs in terms of more traditional economic criteria such as efficiency and revenue. An increase in efficiency is often cited as one of the main goals of privatization but in many cases generating high revenues is at least as important. For instance, European countries that face the straightjacket of the Stability and Growth Pact have massively turned to privatization of public assets to reduce deficits and government debt. Similar trends are seen outside the European Union, where revenue is an equally important objective of privatization. Moreover, the International Monetary Fund and the World Bank explicitly mention using the revenue from privatization to reduce government debt and enhance fiscal stability in their Guidelines for Public Debt Management.

In our model, bidders’ valuations for the asset consist of both a private and a common value component. The former corresponds to the bidder’s cost of exploiting the asset, which is privately known to the bidder. In addition, one “insider” bidder (e.g. the state-owned firm’s incumbent management) is better informed about the asset’s common value. Besides privatization, there are many other situations of interest where exactly one insider bidder is better informed than the seller and other bidders. For example, when a company decides to outsource its catering division it may solicit bids from outside catering companies as well as from its current in-house caterer. Similarly, in management and employee buyouts (MEBOs) the firm’s current management has a clear informational advantage relative to other investors. As a final example, consider licenses for spectrum usage, which typically have a lease period

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1See also Gibbon (1996) and other “How-to-Guides” published by Privatization.org.
2For instance, we do not model potential problems with bidders’ financial capacities.
3See, for instance, the Economist (2002b), the Economist (2004), and the OECD (2003).
5See the International Monetary Fund and World Bank (2002).
of eight to fifteen years. When such a license is re-auctioned the current user has superior
information about its profitability.\footnote{A related example is the auctioning of gas stations along the Dutch highways. In this auction, the current owner competes with other oil companies for the rights to exploit the gas station in the next lease period.}

We are interested in situations where privatization involves substantial risk, i.e. when uncertainty about the asset’s common value is large relative to private-value cost differences. For example, when bidding for a contract to collect garbage, differences in fleet operating costs are likely small while the contract’s common value may vary depending on people’s willingness to voluntarily sort their garbage into several categories (paper, plastic, glass, etc.). Likewise, when market licenses are auctioned among firms with access to a common technology, private value differences are negligible compared to uncertainty about common market characteristics. Finally, this type of common value risk also plays a role in other more familiar settings. For instance, when a high-visibility art work is sold, bidders’ valuations depend crucially on its authenticity. This common value aspect introduces a strong winner’s curse element especially if one of the bidders has inside information.\footnote{Consider the following excerpt taken from http://www.maineantiquedigest.com/articles/vangogh.htm. “Want to take a chance on a Van Gogh? The bidding starts at $150,000 and there are no guarantees. You have to act fast. The painting is referred to as ‘Mystery Vincent Van Gogh painting’ and as ‘Sunflowers and Oleanders.’ The attorney for the trustee in the bankruptcy case already has a bid for $125,000 so the bidding will have to go to $150,000 to top that and then can proceed in $10,000 increments. What’s the catch? Opinion is definitely divided on whether the painting is a Van Gogh, whether it’s signed, and lots else. If you get a $15 million painting for $150,000, good for you. If you get a nice decoration for $150,000, good for the owners.”}

We show that in such risky environments, standard auctions perform poorly in terms of expected revenue due to the possibility of a winner’s curse. Both the second price auction and the English auction are dominated by the “qualifying auction” we study, which is modeled after the two-stage procedure employed by the World Bank. In the first stage of the qualifying auction, bidders place non-binding bids and all but the lowest bidder are allowed to participate in the second stage, which is a standard second price auction. We prove that the qualifying auction augmented with a reserve price implements the optimal (revenue-maximizing) mechanism.

The reason why the qualifying auction outperforms standard auctions is that it eliminates the adverse effects of the winner’s curse. Indeed, there exists an equilibrium of the qualifying auction where in the first stage every bidders bids the unconditional expected value for the asset. The intuition is that since first-stage bids are non-binding, the expected value does not have to be conditional on winning. If the insider bidder places a very low bid in the first stage, uninformed bidders observe the negative news about the asset’s common value and account for this via their second-stage bids. Since the bidder with the lowest first-stage bid is not allowed
to participate in the second stage, there is no incentive for the informed bidder to signal bad news if, in fact, she possesses good news about the asset’s common value.

Qualifying auctions were first studied by Ye (2004) who refers to it as a process of “indicative bidding.” Ye’s important contribution focuses on a different aspect of the two-stage qualifying auction, i.e. costly entry. In his model, bidders have some preliminary but inconclusive information about the asset for sale. In the first stage, they submit non-binding bids, which are used to select a few bidders for the second stage. Those that qualify for the second stage incur a large cost in determining their true, or final, value for the asset. Ye shows efficient entry cannot be guaranteed with indicative bidding and he proposes alternative two-stage formats to avoid this problem. The main difference with our paper is that in Ye’s (2004) model, bidders are symmetric and entry, or acquiring information, is costly. In contrast, in our model a single insider bidder possesses information that affects all while entry is costless. We show that qualifying auctions perform quite well in these situations.

Our paper is related to the work of Bulow, Huang, and Klemperer (1999) who consider pure common-value takeover auctions where a single bidder has a (small) private-value toehold in the company being acquired. They find that in the ascending auction the existence of a single toehold bidder can have disastrous effects for revenue because other bidders face a strong winner’s curse. This is akin to the situation that uninformed bidders face in our model. We show that the qualifying model solves the winner’s curse problem by discriminating against the informationally advantaged bidder.

Finally, there are a few papers that show technical similarities. Larson (2005) provides a thorough investigation of existence and uniqueness issues in pure common value auctions. By introducing small private value disturbances that vanish in the limit, Larson is able to pin down a unique equilibrium for the pure common value case. Hernando-Veciana and Tröge (2004) determine circumstances under which the insider bidder in an English auction is better off disclosing her information. Their analysis of the English auction is parallel to that of section 2. Hernando-Veciana (2004) shows that in pure common value second-price auctions, uninformed bidders may have higher expected payoffs than the single insider bidder. None of these papers determine the optimal mechanism for this context nor do they discuss the qualifying auction, which is the main focus of this paper.

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9More precisely, bidder $i$’s first-stage (second-stage) information is $X_i$ ($Y_i$) and her value for the asset is $X_i + Y_i$. To enter the second stage and learn $Y_i$, qualifying bidders have to pay a large entry cost.
The paper is organized as follows. The next section introduces the privatization model and shows that standard auctions perform poorly in terms of revenue. Section 3 solves for the optimal mechanism that maximizes revenue from privatization. Section 4 shows that the optimal mechanism can be implemented using a qualifying auction augmented with a reserve price. Section 5 concludes and the Appendix contains most of the proofs.

2. Standard Auctions for Risky Privatization

We assume that bidder $i$’s value for the asset consists of a common value component $V + \Theta$, with $V$ known and $\Theta$ unknown, minus a private cost $c_i$. In the “good” state of the world, $G$, the common value is $V + \frac{1}{2} \theta$ while in the “bad” state of the world, $B$, the common value is $V - \frac{1}{2} \theta$. There are $n$ bidders competing for the asset, $n - 1$ of which are uninformed about the common value. In particular, these uninformed bidders possess only the prior information that both states are equally likely, $P(G) = P(B) = \frac{1}{2}$. In contrast, the insider bidder receives a noisy signal $\vartheta \in \{g, b\}$ about the state of the world, where $P(g|G) = P(b|B) = q > \frac{1}{2}$ and $P(b|G) = P(g|B) = 1 - q$.

To simplify notation we define an uninformed bidder’s private value as $s_i \equiv V - c_i$, for $i = 2, \ldots, n$, where the $s_i$ have distribution function $F(.)$ with associated density $f(.)$ defined on the support $[\underline{s}, \overline{s}]$. This way the uninformed bidders’ values for the asset can be written as $s_i + \Theta$. The private value for the insider bidder depends on whether privatization occurs. For instance, empirical evidence shows that privatization of a government-owned firm enhances efficiency even when the current management remains in place (e.g., Hilke, 1993; Chapters 8, 9, and 10 in Shleifer and Vishny, 1998). Explanations include the introduction of a hard budget constraint for managers after privatization and the reduced influence of politicians on how the firm is run. We assume the insider’s private value for the asset is given by $s_1 = \underline{s}$ without privatization, and $s_1$ is a draw from $F(.)$ defined on $[\underline{s}, \overline{s}]$ otherwise.\footnote{We assume that $\underline{s} > \theta$ so that bidders’ values are strictly positive.}

\begin{assumption}
Assumption 1 The density $f(.)$ is symmetric and log-concave on $[\underline{s}, \overline{s}]$.
\end{assumption}

Log-concavity implies among other things that the density is single-peaked and that the hazard
rate \( f(s)/(1 - F(s)) \) is non-decreasing.\(^{13}\) Symmetry implies that the mean and median of the distribution are given by \((\bar{s} + \bar{s})/2\).

Our main interest concerns the case where the asset to be privatized carries substantial risk, which occurs when the variation in the common-value is large relative to the variation in private costs. In other words, we consider “almost common-value” auctions as in Bulow, Huang, and Klemperer (1999).\(^{14}\) Our model is also related to that of Larson (2005) who focuses on the case where the private value differences are vanishingly small.\(^{15}\)

**Assumption 2** The asset is “risky,” i.e. \((q - \frac{1}{2})\theta > \bar{s} - \bar{s}.

The definition of risky assets captures the idea that it is mainly the common-value component that determines the asset’s value. Note that the expected value of the asset to the least efficient informed bidder when receiving good news, \(\vartheta = g\), is given by \(\bar{s} + q\left(\frac{1}{2}\theta\right) + (1 - q)(-\frac{1}{2}\theta)\). Likewise, the expected value to the most efficient bidder after receiving bad news, \(\vartheta = b\), is \(\bar{s} + q(-\frac{1}{2}\theta) + (1 - q)(\frac{1}{2}\theta)\). Hence, in expected terms, the risky asset is worth more to the least efficient bidder who received good news than to the most efficient bidder who received bad news. This separation of expected values by the insider’s common-value signal is one (technical) reason to focus on risky projects. More importantly, the applications we have in mind (see the Introduction) are all characterized by a large degree of common-value uncertainty while private-value differences are small.

We first determine bidding behavior in an English auction. The optimal bidding strategy for the insider is easy to characterize: drop out at the net expected value \(b_I(s_I|\vartheta = g) = s_I + (q - \frac{1}{2})\theta\) in case of good news and at \(b_I(s_I|\vartheta = b) = s_I - (q - \frac{1}{2})\theta\) in case of bad news. For the uninformed bidders the optimal bidding strategy takes the form of a cut-point rule: bid low when \(s_U < s^*\) and high when \(s_U > s^*\), see Figure 1. The following lemma characterizes this strategy and the cut-point \(s^*\) (see also Hernando-Veciana and Tröge (2004) who derive a similar result for a related context).

\(^{13}\)Note that

\[
\frac{f(s)}{1 - F(s)} \geq -\frac{\int_{s}^{\bar{s}} f'(u)du}{\int_{s}^{\bar{s}} f(u)du} = -\frac{\int_{s}^{\bar{s}} (f'(u)/f(u)) f(u)du}{\int_{s}^{\bar{s}} f(u)du} \geq -\frac{f'(s)}{f(s)},
\]

where the final inequality holds because \(f(s)\) is log-concave so \(f'(s)/f(s)\) is non-increasing in \(s\). Hence, \(f(s)^2 + f'(s)(1 - F(s)) \geq 0\) or \((f(s)/(1 - F(s)))' \geq 0\).

\(^{14}\)The main difference is that the latter paper considers the effects of small value differences (due to “toeholds”) that are commonly known by the bidders. In contrast, we consider the effects of large informational differences.

\(^{15}\)See also Hernando-Veciana (2004) and Hernando-Veciana and Tröge (2004).
Lemma 1 The following constitutes an equilibrium of the English auction. The insider bidder drops out at \( b_I(s_I|\vartheta) \). An uninformed bidder drops out at

\[
\begin{align*}
\bar{s} + (q - \frac{1}{2})\theta \\
\dot{s} + (q - \frac{1}{2})\theta \\
\ddot{s} + (q - \frac{1}{2})\theta \\
\dddot{s} + (q - \frac{1}{2})\theta
\end{align*}
\]

when the insider is still active. If the insider dropped out at some price level, \( p \), an uninformed bidder drops out at \( p + \max(0, s_U - s_I) \) where \( s_I \) solves \( p = b_I(s_I|\vartheta) \) for \( \vartheta = b \) or \( \vartheta = g \).

Note that the English auction does not necessarily result in an efficient allocation. For example, when \( s_I < s_U < s^* \), inefficiencies occur when the insider receives good news about the asset’s value. Likewise, when \( s^* < s_U < s_I \), inefficiencies occur when the insider receives bad news about the asset’s value. These inefficiencies are small in magnitude as they involve differences of bidders’ private values, which are negligible for large \( \theta \). We next show that the impact of a single insider on the auction’s revenue is more profound.

Proposition 1 When a risky asset is privatized using the English auction, the loss in revenue due to the presence of a single insider grows linearly with \( \theta \) as \( \theta \) grows large.
What about the second price auction? With one informed and one uninformed bidder the uninformed bidder’s strategy is the same as in Lemma 1 as there is no opportunity for the uninformed bidder to learn about the asset’s common value. With additional uninformed bidders, however, the winner’s curse problem is exacerbated compared to the English auction. In the latter, active uninformed bidders can quit immediately after the insider drops out at a low price. In contrast, in the second price auction, an uninformed bidder may have to pay another uninformed bidder’s high bid when the insider’s information is bad. To avoid such a costly scenario, uninformed bidders will have to bid more cautiously in a sealed-bid second-price auction compared to the English auction.

**Lemma 2** The following constitutes an equilibrium of the second-price auction. The insider bids $b_I(s_I|\vartheta)$. An uninformed bidder bids

$$b_U(s_U) = \begin{cases} 
  s - (q - \frac{1}{2})\theta & \text{if } s_U < s^{**} \\
  B(s_U) & \text{if } s_U \geq s^{**}
\end{cases}$$

with $B(s_U)$ strictly increasing in $s_U$ and $\bar{s} + (q - \frac{1}{2})\theta < B(s_U) < s_U + (q - \frac{1}{2})\theta$ for all $s^{**} < s_U < \bar{s}$ and $s^{**} > s^* = \frac{1}{2}(\bar{s} + \bar{s})$.

Note that uninformed bidders’ optimal bids in the second price auction are more conservative than those in the English auction in two ways. First, high bids in the English auction exceed those in the second price auction.\(^{16}\) Second, the probability that an uninformed bidder bids high in the second price auction is lower than in the English auction ($1 - F(s^{**}) < \frac{1}{2}$). The latter effect becomes dominant when uncertainty about the common value grows.

**Lemma 3** With two or more uninformed bidders $\lim_{\theta \to \infty} s^{**} = \bar{s}$.

In other words, as the uncertainty about the common value grows the loss in revenue for the seller is proportional to $\theta$ irrespective of the state of the world. Recall that in the English auction, a loss proportional to $\theta$ occurs only in the bad state of the world or when all uninformed bidders have below average private values. We thus have:

**Proposition 2** When a risky asset is privatized using the second price auction, the loss in revenue due to the presence of a single insider is even worse than in the English auction when the uncertainty about the common value grows large.

\(^{16}\)We say an uninformed bidder’s bid is “high” when it exceeds the insider’s optimal bid in case of good news with positive probability.
While our focus is on revenue it is interesting to note that also efficiency is lower in the second price auction when uncertainty about the common value grows large.\footnote{It is straightforward to show that total surplus in the second price auction limits to}

$$\lim_{\theta \to \infty} W_{2nd}(\theta) = \frac{1}{2}(E(Y_1^n) + s^*)$$

and with a little more work the following lower bound for the English auction can be established

$$\lim_{\theta \to \infty} W_E(\theta) \geq \frac{1}{2}(E(Y_1^n) + s^*) + \frac{1}{2}(n-1) \int_{s^*}^{\hat{s}} (y-s^*) y F^{n-1}(y) f(y) dy.$$  \footnote{The next section discusses a practical implementation of the optimal mechanism, which does not require this assumption.}

3. Optimal Mechanism

Here we derive the optimal (revenue-maximizing) mechanism for risky privatization. We assume the designer can observe the insider’s identity, e.g. in an auction it is known which bid is placed by the state-owned firm’s incumbent management.\footnote{We consider direct revelation mechanisms where each uninformed bidder truthfully reveals her one-dimensional type, $s_i$, and the informed player 1 truthfully reveals her two-dimensional type $(\vartheta, s_1)$. Let $x_i(s_1, \cdots, s_n|\vartheta)$ denote the probability that player $i$ is awarded the asset as a function of bidders’ reports, $t_i(s_1, \cdots, s_n|\vartheta)$ her payment, and $u_i(s_1, \cdots, s_n|\vartheta)$ her utility. For $i \geq 2$ we define bidder $i$’s expected probability of winning $x_i(s_i) = E_{S_{-i}} \{x_i(s_1, \cdots, s_n|\vartheta)\} = \frac{1}{2} \int_{S_{-i}} (x_i(s_1, \cdots, s_n|b) + x_i(s_1, \cdots, s_n|g)) dF(S_{-i})$, where $S_{-i} = (s_1, \cdots, s_{i-1}, s_{i+1}, \cdots, s_n)$. Bidder $i$’s expected payment $t_i(s_i)$ and expected utility $u_i(s_i)$ are defined analogously. Let $x_1(s_1|\vartheta) = E_{S_{-1}} \{x_1(s_1, \cdots, s_n|\vartheta)\}$ denote the expected probability of winning for the informed bidder 1, with similar definitions for $t_1(s_1|\vartheta)$ and $u_1(s_1|\vartheta)$.}

An uninformed bidder $i \geq 2$ announces type $\hat{s}$ that maximizes

$$u_i(s_i) = \max_{\hat{s}} E_{\vartheta} \{x_i(\hat{s}|\vartheta)(s_i + \frac{1}{2} (P(G|\vartheta) - P(B|\vartheta)) \vartheta) - t_i(\hat{s}|\vartheta)\}.$$

Using the envelope theorem we have

$$\frac{\partial u_i(s_i)}{\partial s_i} = E_{\vartheta} \{x_i(s_i|\vartheta)\} \equiv x_i(s_i),$$
which can be integrated to give

\[ u_i(s_i) = u_i(s) + \int_{s}^{s_i} x_i(s)ds. \quad (1) \]

This necessary condition for truth telling is also sufficient if \( x_i(s) \) is non-decreasing in \( s \).

Next we turn to the informed bidder 1. The same steps leading up to (1) show that for the insider to truthfully reveal \( s_1 \) we must have, for \( \vartheta \in \{b, g\} \),

\[ u_1(s_1|\vartheta) = u_1(s|\vartheta) + \int_{s}^{s_1} x_1(s|\vartheta)ds. \quad (2) \]

In addition, for bidder 1 to truthfully reveal her common-value signal \( \vartheta \) we must have:

\[ u_1(s_1|\vartheta) = \max_{\hat{s}, \hat{\vartheta}} \{ x_1(\hat{s}|\hat{\vartheta})(s_1 + \frac{1}{2}(P(G|\vartheta) - P(B|\vartheta))\theta) - t_1(\hat{s}|\hat{\vartheta}) \}. \]

Lemma 4 provides sufficient conditions for bidders to participate and truthfully reveal their private information. The proof uses the following insight: if the insider lies about her common value information after receiving good news she also reports a private value \( \hat{s} = \bar{s} \) so as to maximize her probability of winning. Likewise, if the insider lies about her common value information after receiving bad news she reports a private value \( \hat{s} = g \). The conditions of Lemma 4 ensure that neither deviation is profitable.

**Lemma 4** Bidders participate and truthfully reveal their types if (1) and (2) hold and

(i) \( u_i(s) \geq 0 \) for \( i \geq 2 \), \( u_1(s|b) \geq 0 \) and

\[ u_1(s|g) = u_1(s|b) + x_1(s|b)((2q - 1)\theta - (\bar{s} - s)). \quad (3) \]

(ii) \( x_i(s_i) \) is non-decreasing in \( s_i \) for \( i \geq 2 \), \( x_1(s_1|\vartheta) \) is non-decreasing in \( s_1 \) for \( \vartheta \in \{b, g\} \) and

\[ x_1(s|g) = x_1(s|b). \quad (4) \]

Equations (3) and (4) patch together the insider’s expected utility and probability of winning across the two information signals. They are sufficient but not necessary conditions, i.e. the
equality signs in (3) and (4) can be relaxed to some degree.\textsuperscript{19} Strict equalities follow, however, when we restrict attention to revenue-maximizing mechanisms, which we discuss next.

The seller’s expected revenue, $R$, equals the sum of the expected transfers from the bidders plus the status quo revenues that occur when no privatization takes place. Using standard manipulations (e.g. Myerson, 1981), the expected revenue can be written as

$$R = \frac{1}{2} s + \frac{1}{2} \sum_{i=1}^{n} \int_{\frac{s_i}{2}}^{s_i} \int_{\frac{s_i}{2}}^{s_i} \left( s_i - \frac{1 - F(s_i)}{f(s_i)} \right) x_i(s_1, \ldots, s_n | b) dF(s_1) \cdots dF(s_n)$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \int_{\frac{s_i}{2}}^{s_i} \int_{\frac{s_i}{2}}^{s_i} \left( s_i - \frac{1 - F(s_i)}{f(s_i)} \right) x_i(s_1, \ldots, s_n | g) dF(s_1) \cdots dF(s_n)$$

$$- \frac{1}{2} \left( u_1(s | b) + u_1(s | g) \right) - \sum_{i=2}^{n} u_i(s). \quad (5)$$

The seller chooses the assignment functions, $x(\cdot | b)$ and $x(\cdot | g)$, and the utilities of the lowest types, $u_i(s)$ for $i \geq 2$ and $u_1(s | b)$ and $u_1(s | g)$, so as to maximize expected revenue subject to the incentive compatibility and participation constraints of Lemma 4.

Let $s = s_{\text{min}}$ denote the solution to

$$s - \frac{1 - F(s)}{f(s)} = \bar{s}. \quad (6)$$

Assumption 1 ensures the left side is strictly increasing in $s$ so the solution $s_{\text{min}}$ is unique. We say the insider reports good (bad) news if the reported common-value signal is $\hat{\vartheta} = g$ ($\hat{\vartheta} = b$).

**Proposition 3** If the informed bidder reports bad news, the optimal mechanism assigns the risky asset to the uninformed bidder with the highest reported private value if this value exceeds $s_{\text{min}}$; otherwise no privatization takes place. If the informed bidder reports good news, the optimal mechanism assigns the risky asset to the bidder with the highest reported private value if this value exceeds $s_{\text{min}}$; otherwise no privatization takes place.

**Proof.** First, note from (5) that it is optimal to set $u_i(s) = 0$ for $i \geq 2$ and $u_1(s | b) = 0$. Equation (3) becomes

$$u_1(s | g) = \int_{\frac{s}{2}}^{s} x_1(s_1 | b) ds_1 + x_1(\bar{s} | b)((2q - 1)\bar{\theta} - (\bar{s} - \bar{s})), \quad (3)$$

\textsuperscript{19}Equations (3) and (4) can be relaxed to $x_1(s | g) \geq x_1(\bar{s} | b)$ and $x_1(\bar{s} | b)\left( (2q-1)\bar{\theta}-(\bar{s}-\bar{s}) \right) \leq u_1(s | g) - u_1(s | b) \leq x_1(s | g)\left( (2q-1)\bar{\theta}-(\bar{s}-\bar{s}) \right)$. 


and since \( x_1(s|b) \) is non-decreasing in \( s \), we have \( x_1(\bar{s}|b) \geq \int_{\frac{s}{2}}^{\bar{s}} x_1(s_1|b) dF(s_1) \), so

\[
u_1(\bar{s}|g) \geq \int_{\frac{s}{2}}^{\bar{s}} \left( \frac{1}{f(s_1)} + (2q - 1)\theta - (\bar{s} - \bar{s}) \right) x_1(s_1|b) dF(s_1).
\]

Combining this with expression (5) for the expected revenue, we derive the following upper bound on revenue

\[
R \leq \frac{s}{2} + \frac{1}{2} \sum_{i=1}^{n} \int_{\frac{s}{2}}^{\bar{s}} (\bar{MR}_i(s_i) - \bar{s}) x_i(s_1, \ldots, s_n|b) dF(s_1) \cdots dF(s_n)
\]

\[
+ \frac{1}{2} \sum_{i=1}^{n} \int_{\frac{s}{2}}^{\bar{s}} (MR_i(s_i) - \bar{s}) x_i(s_1, \ldots, s_n|g) dF(s_1) \cdots dF(s_n),
\]

where we defined

\[
MR_i(s_i) = s_i - \frac{1 - F(s_i)}{f(s_i)}
\]

for \( i = 1, \ldots, n \), and

\[
\bar{MR}_1(s_1) = s_1 - \frac{2 - F(s_1)}{f(s_1)} - ((2q - 1)\theta - (\bar{s} - \bar{s})),
\]

and \( \bar{MR}_i(s_i) = MR_i(s_i) \) for \( i \geq 2 \).

Figure 2 shows \( MR_i(s) - \bar{s} \) in case of good news (right panel) and \( \bar{MR}_1(s) - \bar{s} \) in case of bad news (left panel). If \( \bar{MR}_1(s_1) - \bar{s} \) were everywhere negative then the seller should obviously never allocate the asset to the insider. Suppose \( \bar{MR}_1(s_1) - \bar{s} \) is positive for some values of \( s_1 \) as shown in the figure. At the upper end \( s_1 = \bar{s} \) we have

\[
\bar{MR}_1(\bar{s}) - \bar{s} = 2(\bar{s} - \bar{s}) - \frac{1}{f(\bar{s})} - (2q - 1)\theta < -\frac{1}{f(\bar{s})} < 0,
\]

where the first inequality follows from symmetry of \( f(\cdot) \) and the fact that the project is risky, i.e. \( (q - 1)\theta > (\bar{s} - \bar{s}) \). So when the marginal revenue \( \bar{MR}_1(s_1) \) is positive for some values of \( s_1 \) it is non-monotonic. Since the assignment function \( x_1(s_1|\vartheta) \) has to be non-decreasing in \( s_1 \) we need to “iron out” the marginal revenue curve (see, for instance, Bulow and Roberts, 1989). This yields the \( \mu''\mu''\mu'' \) curve in Figure 2. The main point is that the part of the curve that pertains to bad news lies everywhere below the zero line that connects \( \bar{s} \) and \( s \). To understand this result, note that \( \int_{\frac{s}{2}}^{\bar{s}} MR_i(s)f(s)ds = \bar{s} \). (To sell to all types, revenue has to equal \( \bar{s} \) since
that it is the maximum price at which all types are willing to buy.) The weighted $D, C$ areas (weighted by the density function $f(\cdot)$) are therefore equal: $D - C = 0$. Obviously, $D > A - B$ so $A - B - C < 0$. The line connecting $\mu'$ to $\mu''$ thus has to lie below the zero to replicate this negative area. As a result, the seller should never award the asset to the insider when the insider reports bad news. In other words, $x_1(s_1|b) = 0$ for all $s_1 \in [\underline{s}, \bar{s}]$.

This implies $u_1(s|g) = 0$ (see (3)) and the inequality in (7) becomes an equality. Hence, revenue is maximized when the right side of (7) is, which requires allocating the project to the bidder with the highest $\bar{MR}_i(s_i) - \bar{s}$ in case of bad news and to the bidder with the highest $MR_i(s_i) - \underline{s}$ in case of good news conditional on these values being positive. The log-concavity Assumption 1 ensures $MR_i(s_i)$ is an increasing function and it is positive if and only if $s > s_{min}$. So in case of bad news the project is assigned to the uninformed bidder with the highest private value if this private value exceeds $s_{min}$ otherwise no privatization takes place. In case of good news the project is assigned to the bidder with the highest private value (whether or not the bidder is informed) if this value exceeds $s_{min}$, otherwise no privatization takes place.  

\textit{Q.E.D.}
Note that the optimal mechanism discriminates against the informationally advantaged bidder to ensure truthful information revelation and protect uninformed bidders from the winner’s curse. This is akin to Myerson’s (1981) solution for the case of asymmetric value distributions where bidding credits are assigned to “weaker” bidders to enhance competition and force the advantaged bidder to bid closer to her true value.

4. Qualifying Auctions

In this section we describe a practical implementation of the optimal mechanism, consisting of a two-stage qualifying auction. In the first stage all \( n \) bidders place a bid. The \( n - 1 \) highest bids qualify for the second stage, which consists of a standard second price auction augmented with a reserve price. All qualifying bidders learn the lowest first-stage bid (but no other bids) as does the seller who sets an optimal reserve price based on this information. The following lemma describes an equilibrium in this context.

**Lemma 5** *In the first stage of the qualifying auction, optimal bids are*

\[
\begin{align*}
b_I(s_I|\vartheta) &= s_I + \frac{1}{2}(P(G|\vartheta) - P(B|\vartheta))\theta, \\
b_U(s_U) &= s_U.
\end{align*}
\]

*All but the lowest bidder qualify for the second stage. The seller and qualifying bidders get to know the lowest first-stage bid from which they can perfectly infer the insider’s information \( \vartheta \). The reserve price is set at \( r = b_I(s_{\min}|\vartheta) \) and qualifying bidders’ optimal second-stage bids are*

\[
b(s) = \begin{cases} 
b_I(s|\vartheta) & \text{if } s \geq s_{\min}, \\
\text{“no bid”} & \text{if } s < s_{\min},
\end{cases}
\]

*with \( s_{\min} \) the unique solution to (6).*

**Proof.** Since all bidders bid their unconditional expected values in the first stage, the lowest bid is either in the range \( [s - (q - \frac{1}{2})\theta, \bar{s} - (q - \frac{1}{2})\theta] \) or in the range \( [\underline{s}, \bar{s}] \), which are disjoint intervals by the risky asset assumption \( (q - \frac{1}{2})\theta > \bar{s} - s \). In the former case, the seller and qualifying bidders know \( \vartheta = b \) while in the latter case they know \( \vartheta = g \).

To show that the above strategies constitute an equilibrium note that, given first-stage behavior, qualifying bidders know \( \vartheta \) and hence it is optimal for them to bid their expected
values in the second stage. Now consider the qualifying stage. Suppose the insider’s information is \((\vartheta = g, s_I)\): can the insider gain by bidding something different then \(b_I(s_I|\vartheta)\)? There are two possibilities: (i) the deviating bid is the lowest first-stage bid and (ii) it is not. In the former case, the insider will forgo the opportunity to bid in the second stage, which cannot be profitable. In the latter case, the lowest first-stage bid is determined by an uninformed bidder (as it would be in equilibrium), in which case the seller and qualifying bidders infer that \(\vartheta = g\). Again such a deviation by the insider is not profitable. What if the insider’s information is \((\vartheta = b, s_I)\)? In this case she might deviate such that her bid is no longer the lowest first-stage bid. But then the seller and other qualifying bidders infer \(\theta = g\), in which case the insider will not be able to profitably win the second stage.

Finally, given the second price nature of the second stage auction, it is easy to show that uninformed bidders have no incentive to deviate from truthful bidding in the first stage. Q.E.D.

Note that the qualifying auction implements the optimal mechanism of the previous section and its revenue is independent of the degree of common value uncertainty unlike that of the standard auctions discussed above. Importantly, the qualifying auction is revenue maximizing when the insider’s identity is unknown or cannot be (legally) used. Moreover, the qualifying auction is robust in the sense that it is optimal also when there is no insider bidder. In this case, all bidders are symmetric and the first stage simply gets rid of the bidder with the lowest private value with no consequences for seller revenues.

**Proposition 4** The qualifying auction implements the optimal mechanism in situations with and without informational asymmetries whether or not the insider’s identity can be observed.

5. Conclusion

High-stakes auctions for privatizing public assets are often plagued by substantial bidder asymmetries, which may have devastating effects for the auction’s revenue. Previous literature has typically assumed that these asymmetries take the form of (known) private value differences (e.g. Myerson, 1981; Bulow, Huang, and Klemperer, 1999). In contrast, in this paper we analyze the effects of *informational asymmetries* that arise when one insider bidder has superior information about the asset’s common value while private value differences are small. To illustrate, consider the sale of the Los Angeles license in the 1995 FCC spectrum auction for
mobile-phone broadband licenses. The following discussion is taken from Klemperer (2002).

While the license value was hard to estimate, it was probably worth similar amounts to several bidders. But Pacific Telephone, which already operated the local fixed-line telephone business in California, had distinct advantages from its database on potential local customers and its familiarity with doing business in California.

Before bidding for the California phone license, Pacific Telephone announced in the Wall Street Journal that “if somebody takes California away from us, they’ll never make any money” (Cauley and Carnevale, 1994, p.A4). Pacific Telephone also hired one of the world’s most prominent auction theorists to give seminars to the rest of the industry to explain the winner’s curse argument that justifies this statement, and reinforced the point in full page advertisements that ran in newspapers of cities where their major competitors were headquartered (Koselka, 1995, p. 63).

The auction was a standard ascending (English) auction. And the result was that the bidding stopped at a very low price. In the end, the Los Angeles license yielded only $26 per capita. In Chicago, by contrast, the main local fixed-line provider was ineligible to compete and the auction yielded $31 per capita even though Chicago was thought less valuable than Los Angeles because of its lower household incomes, lower expected population growth, and more dispersed population.

We interpret the California example as follows: Pacific Telephone had inside information about the market they were already operating in, and, hence, they knew much better than other bidders how much the license would be worth. With differences in private valuations being small, Pacific Telephone could thus credibly claim that if its bid would be topped the winning bidder would have to fall prey to the winner’s curse. As a result the license sales price remained low (see Proposition 1) especially after a renowned auction theorist explained the logic behind the winner’s curse to other bidders, making sure uninformed bidders would opt for a cautious strategy (see Lemma 1).

The Chicago example shows that one way to resolve this problem is to exclude the insider bidder. This solution, however, is not always legally possible nor is it optimal. Practical literature concerning the divestment of government-owned assets, such as the World Bank’s “How-to-Guide” for privatization (Welch and Frémont, 1998), suggests a completely different approach based on a “qualifying auction.” This auction consist of two stages. In the first stage, bidders place non-binding bids and all but the lowest bidder are allowed to participate in the second stage, which is a standard second price auction augmented with a reserve price.
In this paper, we demonstrate that this simple format implements the revenue-maximizing mechanism in situations where a single insider bidder has superior information about the asset’s common value. The reason why the qualifying auction outperforms other formats is that it eliminates the adverse effects of the winner’s curse. Indeed, there exists an equilibrium of the qualifying auction where in the first stage every bidder bids the unconditional expected value for the asset (see Lemma 5). The intuition is that since first-stage bids are non-binding, the expected value does not have to be conditional on winning. If the insider places a very low bid in the first stage, uninformed bidders observe the negative news about the asset’s common value and account for this via their second-stage bids. And since the bidder with the lowest first-stage bid is not allowed to bid in the second stage, there is no incentive for the insider to signal bad news if, in fact, she possesses good news.

Through the addition of a qualifying stage the auction discriminates against the informationally advantaged bidder to ensure truthful information revelation (see Proposition 3), which is reminiscent of Myerson’s (1981) recipe for how to deal with value-advantaged bidders. An important difference, however, is that the qualifying auction treats bidders in a symmetric manner and neither the seller nor the bidders need to know the identity of the insider. Moreover, qualifying auctions remain optimal in situations where informational asymmetries are small or non-existent (Proposition 4). Their long-time use in the sales of complex and risky assets lends further credence to their effectiveness in combatting the adverse effects of large informational asymmetries.
Appendix A. Proofs

**Proof of Lemma 1.** Uninformed bidders learn nothing about the asset’s common value from the drop-out levels of other uninformed bidders. This implies that an uninformed bidder’s optimal strategy is independent of the number of uninformed bidders. Consider the case of one informed and one uninformed bidder. When the uninformed bidder with private value $s_U$ stays in until $s_U' - (q - \frac{1}{2})\theta$ she can win only when the insider receives bad news and her expected payoff is

$$\frac{1}{2} \int_{s_U}^{s_U'} \left( (s_U - (q - \frac{1}{2})\varepsilon) - (s_I - (q - \frac{1}{2})\varepsilon) \right) f(s_I) ds_I = \frac{1}{2} \int_{s_U}^{s_U'} (s_U - s_I) f(s_I) ds_I. \quad (A.1)$$

Likewise, when the uninformed bidder with private value $s_U$ stays in until $s_U' + (q - \frac{1}{2})\theta$ her expected payoff is

$$\frac{1}{2} \int_{s_U}^{s_U'} (s_U - s_I) f(s_I) ds_I + \frac{1}{2} \int_{s_U}^{s_U'} (s_U - s_I) f(s_I) ds_I = \frac{1}{2} (s_U - s^*) + \frac{1}{2} \int_{s_U}^{s_U'} (s_U - s_I) f(s_I) ds_I. \quad (A.2)$$

The first term on the left side corresponds to the case where the insider received bad news (and, hence, the uninformed bidder always wins) and the second term on the left side corresponds to the case of good news. Obviously, the uninformed bidder’s optimal choice is to set $s_U' = s_U$. Furthermore, the expected payoff of bidding low (A.1) is greater (smaller) than that of bidding high (A.2) when $s_U$ is smaller (greater) than the average private value $s^*$. Q.E.D.

**Proof of Lemma 2.** We say the uninformed bid is high (low) when it has a positive (zero) chance of winning against the equilibrium bid of an insider who received good news. The expected payoff of an uninformed bidder with private value $s_U$ when bidding low, $s_U' - (q - \frac{1}{2})\theta$, equals

$$\frac{1}{2} \int_{s_U}^{s_U'} \left( (s_U - (q - \frac{1}{2})\varepsilon) - (s - (q - \frac{1}{2})\varepsilon) \right) dF(s) n^{-1}.$$ 

Clearly, it is optimal to set $s_U' = s_U$ in this case.

Optimal high bids can be derived from the following marginal argument: in equilibrium, $B(s_U)$ follows from the condition that the costs and benefits of raising the bid to $B(s_U + \epsilon)$ cancel. Note that such an increase has an effect only when it turns the uninformed bidder into a winner while she was previously losing. This occurs when (i) the insider received bad news and
by raising her bid the uninformed bidder just beats the highest of the other uninformed bidders, (ii) the insider received good news and by raising her bid the uninformed bidder just beats the highest of the other uninformed bidders, and (iii) the insider received good news and by raising her bid the uninformed bidder just beats the insider. Case (i) occurs when the highest of the other uninformed bidders’ private values lies between \( s_U \) and \( s_U + \epsilon \) while the insider’s private value can be anything. This event occurs with probability \( \epsilon(n-2)f(s)F(s)^{n-3} \) and the net gain in this case is \( s_U - (q - \frac{1}{2})\theta - B(s_U) \). Similarly, case (ii) occurs with probability \( \epsilon(n-2)f(s)F(s)^{n-3}F(B(s_U) - (q - \frac{1}{2})\theta) \) where the extra term is included to capture the probability that the insider’s bid in case of good news, \( s_I + (q - \frac{1}{2})\theta, \) is less than \( B(s_U) \). The net gain in this case is \( s_U + (q - \frac{1}{2})\theta - B(s_U) \). Finally, case (iii) arises when all other uninformed bidders have private values less than \( s_U \) and the insider’s private value lies between \( B(s_U) - (q - \frac{1}{2})\theta \) and \( B(s_U + \epsilon) - (q - \frac{1}{2})\theta \). The probability of this event is \( \epsilon F(s)^{n-2}B'(s_U)f(B(s_U) - (q - \frac{1}{2})\theta) \) and the net gain in this case is \( s_U + (q - \frac{1}{2})\theta - B(s_U) \).

Adding the different scenarios yields the following differential equation for \( B(s_U) \)

\[
0 = (n-2)f(s_U)F(s_U)^{n-3}(s_U - (q - \frac{1}{2})\theta - B(s_U)) \\
+ (n-2)F(B(s_U) - (q - \frac{1}{2})\theta)f(s_U)F(s_U)^{n-3}(s_U + (q - \frac{1}{2})\theta - B(s_U)) \\
+ B'(s_U)f(B(s_U) - (q - \frac{1}{2})\theta)F(s_U)^{n-2}(s_U + (q - \frac{1}{2})\theta - B(s_U)) \\
(A.3)
\]

with boundary condition \( B(\bar{s}) = \bar{s} + (q - \frac{1}{2})\theta \).

Note that for \( n = 2 \), (A.3) implies \( B(s) = s + (q - \frac{1}{2})\theta \), which is the result of Lemma 1. For \( n \geq 3 \), we do not have an explicit solution to (A.3) but some insight can be gleaned as follows. First, the requirement that the bid is high implies \( B(s_U) > \bar{s} + (q - \frac{1}{2})\theta \). Hence, by the risky asset Assumption 2, \( B(s_U) > s_U \) and (A.3) thus implies that \( B'(s_U) > 0 \). Furthermore, (A.3) implies \( B(s_U) < s_U + (q - \frac{1}{2})\theta \) for \( s_U < \bar{s} \) since otherwise all terms on the right side are negative. Finally, the boundary condition \( B(\bar{s}) = \bar{s} + (q - \frac{1}{2})\theta \) is derived as follows. Suppose, in contradiction, \( B(\bar{s}) < \bar{s} + (q - \frac{1}{2})\theta \). Since \( B'(s_U) > 0 \), \( B(\bar{s}) > B(s) \) for all \( s < \bar{s} \) so raising \( B(\bar{s}) \) leaves unchanged the probability of winning against an informed bidder but raises the probability of winning against an informed bidder with private value \( s < \bar{s} \) who received good news, which is profitable. Hence \( B(\bar{s}) = \bar{s} + (q - \frac{1}{2})\theta \).

The differential equation in (A.3) thus characterizes a well-defined increasing bidding function. Our final task is to determine the set of private values for which an uninformed bidder will bid low or high. Let \( s^{**} \) denote the private value for which the uninformed bidder is indifferent.
between a low and high bid. This cut-off value can be determined as follows: define the low
bid \( L \equiv s^{**} - (q - \frac{1}{2})\theta \) and the high bid \( H \equiv B(s^{**}) \) where \( B(s_U) \) is determined by (A.3). The
equilibrium expected payoffs of an uninformed bidder with private value \( s^{**} \) from bidding low is
\[
\pi(L) = \frac{1}{2} \int_{s_U}^{s^{**}} (s^{**} - s) dF(s)^{n-1},
\]
and the equilibrium expected payoff from bidding high is
\[
\pi(H) = \pi(L) + \frac{1}{2} F(s^{**})^{n-2} \left\{ \int_{s_U}^{s^{**}} (s^{**} - s_I) dF(s_I) + \int_{s_U}^{H - (q - \frac{1}{2})\theta} (s^{**} - s_I) dF(s_I) \right\}.
\]
The additional terms on the right side arise as follows. In equilibrium, an uninformed bidder
with cut-off private value, \( s^{**} \), never wins against other uninformed bidders who bid high, which
explains the \( F(s^{**})^{n-2} \) term. Furthermore, the first (second) term in the curly brackets pertains
to the case where the insider received bad (good) news. An uninformed bidder with private
value \( s^{**} \) is indifferent if and only if the terms in the curly brackets cancel, which implies
\[
s^{**} = s^* + \int_{B(s^{**}) - (q - \frac{1}{2})\theta}^{s^{**}} (s^{**} - s) dF(s), \tag{A.4}
\]
where \( s^* \equiv E(s) = \frac{1}{2}(\bar{s} + \bar{s}). \) This defines a negatively sloped curve in \((s^{**}, B(s^{**}))\) space that
runs from \((s^*, s^* + (q - \frac{1}{2})\theta)\) to \((\bar{s}, \bar{s} + (q - \frac{1}{2})\theta)\). The differential equation (A.3) defines a
positively sloped curve in \((s^{**}, B(s^{**}))\) space (since \( B'(\cdot) > 0 \)) that starts below \( s^* + (q - \frac{1}{2})\theta \) at
\( s^{**} = s^* \) (since \( B(s_U) < s_U + (q - \frac{1}{2})\theta \) for all \( s_U < \bar{s} \)) and ends at \((\bar{s}, \bar{s} + (q - \frac{1}{2})\theta)\). The unique
intersection of these two curves defines the cut-off value \( s^{**} \). Since \( B(s_U) < s_U + (q - \frac{1}{2})\theta \) for
all \( s_U < \bar{s} \), (A.4) implies \( s^{**} > s^* \).

**Proof of Proposition 1.** The inefficiencies mentioned in the main text may cause the bidder
with the highest private value to determine the price (instead of the bidder with the second
highest private value) with positive effects on revenue. A lower bound for revenue loss results
by computing revenue as if it is always the bidder with the highest private value that determines
the price. If \( Y^k_n \) denotes the \( k^{th} \) highest from \( n \) private value draws the sales price is then based
on \( Y^1_n \) (instead of \( Y^2_n \)). The sales price is raised by \( (q - \frac{1}{2})\theta \) when the insider receives good
news and the highest of the uninformed bidders’ private values exceeds \( s^* \), and it is lowered
by \( (q - \frac{1}{2})\theta \) otherwise. The former event occurs with probability \( \frac{1}{2}(1 - (\frac{1}{2})^{n-1}) \). Hence, a
lower bound for the loss in revenue is
\[
(q - \frac{1}{2})\theta - \frac{1}{2} \left( 1 - \frac{1}{\theta} \right) (n-1) \left( q - \frac{1}{2} \right) \theta - (E(Y^1_n) - E(Y^2_n)) = \left( \frac{1}{\theta} \right)^{n-1} (q - \frac{1}{2}) \theta - (E(Y^1_n) - E(Y^2_n)),
\]
which is linearly increasing in \( \theta \).

Q.E.D.

**Proof of Lemma 3.** Suppose, in contradiction, \( \lim_{\theta \to \infty} s^{**} = \tilde{s} < \bar{s} \). For large \( \theta \), the probability that an uninformed bidder with private value \( s > \tilde{s} \) pays another uninformed bidder’s high bid in case of bad news is strictly positive (\( \frac{1}{2} (F(s)^{n-2} - F(\tilde{s})^{n-2}) > 0 \)), resulting in a large loss. The uninformed bidder is better off bidding low, contradicting Lemma 2. Q.E.D.

**Proof of Lemma 4.** Note that bidders’ expected payoffs are non-negative so participation is guaranteed. To ensure bidder 1 does not report \( \hat{\vartheta} = b \) when in fact \( \vartheta = g \) we require
\[
u_1(s_1|g) \geq \max_{\hat{s}} \{ x_1(\hat{s}|b)(s_1 + (q - \frac{1}{2})\theta) - t_1(\hat{s}|b) \}. (A.5)
\]
The incentive compatibility constraint for bidder 1 of type \((\vartheta, s_1) = (b, \tilde{s})\) implies, for all \( \hat{s} \),
\[
x_1(\hat{s}|b)(\tilde{s} - (q - \frac{1}{2})\theta) - t_1(\hat{s}|b) \geq x_1(\hat{s}|b)(\tilde{s} - (q - \frac{1}{2})\theta) - t_1(\hat{s}|b).
\]
The risky asset Assumption 2 implies \( (2q - 1)\theta > (\tilde{s} - s_1) \) for all \( s_1 \), and since \( x_1(\cdot|b) \) is non-decreasing we have, for all \( \hat{s} \),
\[
x_1(\hat{s}|b)((2q - 1)\theta - (\tilde{s} - s_1)) \geq x_1(\hat{s}|b)((2q - 1)\theta - (\tilde{s} - s_1)).
\]
Adding the previous two inequalities yields, for all \( \hat{s} \),
\[
x_1(\hat{s}|b)(s_1 + (q - \frac{1}{2})\theta) - t_1(\hat{s}|b) \geq x_1(\hat{s}|b)(s_1 + (q - \frac{1}{2})\theta) - t_1(\hat{s}|b).
\]
Hence, the maximization problem in \((A.5)\) is solved by \( \hat{s} = \bar{s} \) for all types \( s_1 \). In other words, if the insider lies about her common-value information by reporting \( \vartheta = b \) while, in fact, \( \vartheta = g \), she also reports \( \bar{s} \) as her private value to maximize her probability of winning. Condition \((A.5)\) reduces to
\[
u_1(s_1|g) \geq x_1(\hat{s}|b)(s_1 + (q - \frac{1}{2})\theta) - t_1(\hat{s}|b)
= u_1(\bar{s}|b) + x_1(\hat{s}|b)((2q - 1)\theta - (\tilde{s} - s_1)). (A.6)
\]

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To prove this inequality, note that for $\vartheta = g$ the incentive compatibility equation (2) yields

$$u_1(s_1|g) = u_1(s|g) + \int_{\bar{s}}^{s_1} x_1(s|g)ds$$

$$\geq u_1(s|g) + x_1(s|g)(s_1 - s) = u_1(s|g) + x_1(\bar{s}|b)(s_1 - s), \quad (A.7)$$

where we used that $x_1(\cdot|g)$ is increasing and (4). Inequality (A.6) now follows from (A.7) and (3).

Finally, to guarantee that an informed bidder with common-value signal $\vartheta = b$ never reports $\hat{\vartheta} = g$, we require

$$u_1(s_1|b) \geq \max_{\hat{s}} \{x_1(\hat{s}|g)(s_1 - (q - \frac{1}{2})\theta) - t_1(\hat{s}|g)\}.$$ 

Proceeding in an analogous manner as above shows that the solution to the maximization problem is given by $\hat{s} = \bar{s}$ for all $s_1$, so the condition becomes

$$u_1(s_1|b) \geq x_1(s|g)(s_1 - (q - \frac{1}{2})\theta) - t_1(s|g)$$

$$= u_1(s|g) - x_1(s|g)((2q - 1)\theta - (s_1 - \bar{s})).$$

Using (3) and (4) the expression in the second line can be rewritten as $u_1(s|b) + \int_{\bar{s}}^{s_1} x_1(s|b)ds - x_1(s|b)(\bar{s} - s_1) \leq u_1(s|b) + \int_{\bar{s}}^{s_1} x_1(s|b)ds = u_1(s_1|b)$ where the inequality follows since $x_1(s|b)$ is non-decreasing in $s$. 

Q.E.D.
References


