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9. Spatial Evolution of Social Norms in a Common-Pool Resource Game

JOËLLE NOAILLY[1,*], CEES A. WITHAGEN[2,3], AND JEROEN C. J. M. VAN DEN BERGH[2,4]

[1]*CPB Netherlands Bureau for Economic Policy Analysis, The Hague, The Netherlands;*
[*]*Author for correspondence (e-mail: noailly@cpb.nl; phone +31-70-338-3498; fax +31-70-338-3350)*
[2]*Department of Spatial Economics, Faculty of Economics and Business Administration, Free University, Amsterdam, The Netherlands;*
[3]*Faculty of Economics and Business Administration, Tilburg University, Tilburg, The Netherlands;*
[4]*Institute for Environmental Studies, Free University, Amsterdam, The Netherlands*

We study the conditions for the emergence of cooperation in a spatial common-pool resource (CPR) game. We consider three types of agents: cooperators, defectors and enforcers. The role of enforcers is to punish defectors for overharvesting the resource. Agents are located on a circle and they only observe the actions of their two nearest neighbors. Their payoffs are determined by both local and global interactions and they modify their actions by imitating the strategy in their neighborhood with the highest payoffs on average. Using theoretical and numerical analysis, we find a large diversity of equilibria to be the outcome of the game. In particular, we find conditions for the occurrence of equilibria in which the three strategies coexist. We also derive the stability of these equilibria. Finally, we show that introducing resource dynamics in the system favors the occurrence of cooperative equilibria.

Key words: common property, cooperation, evolutionary game theory, local and global interaction game, self-organization

1. Introduction

The common-pool resource (CPR) game is an excellent vehicle to study social dilemmas. A social dilemma is a situation in which the pursuit of individual interest comes at the expense of the collective goals. In the context of the management of common-pool resources, such a social dilemma results in overexploitation and inefficiency compared to the Pareto optimum.

Are people's actions always governed by selfish behavior? Recent evidence has led economists to reconsider their assumptions on behavior. In practice, a certain proportion of the population often exhibits cooperative behavior that seems in contradiction with a rational, selfish agent perspective. Such behavior is especially common when social norms prevail. These can operate in a decentralized way through a system of mutual trust, reward or punishment.

L. Bretschger and S. Smulders (eds.), Sustainable Resource Use and Economic Dynamics, 191–216.

Ostrom (1990) collected a large range of case studies of rural communities in which the presence of social norms led to sustainable management of common-pool resources. An example that has received much attention is the lobster industry in Maine. In this community, fishermen were assigned a spatial territory to spread their traps. In order to increase their catch, free-riders tried to expand their territory. Every fisherman, however, was allowed to defend his territory using different degrees of sanctions ranging from reprimands to opening or destroying the traps of the free-riders (Acheson, 1988). In other settings, ceasing cooperation with rather than punishment of free-riders has also proved effective. For example, Japanese villagers, Irish fishermen and inhabitants of the Solomon islands chose to cut contact with other members of the community who were overfishing (Taylor, 1987; McKean 1982; Hviding and Baines 1994). In this way, free-riders are deprived of the benefits provided by cooperation in other economic activities.

Next to case studies, there is also much experimental evidence that supports the persistence of cooperation. This literature is too large to be reviewed here. Seminal work has been done by Ostrom et al. (1994) and Fehr and Gächter (2001). The latter study shows that often a small proportion of 'altruistic punishers' in the population is sufficient to enforce cooperation in the group. Van Soest and Vyrastekova (2004) provide an application in the field of renewable resources.

A key theoretical question that follows from this is: Why does cooperative behavior emerge in the first place? Compared to the real world evidence there is not so much theory on this subject. Fehr and Schmidt (1999) develop a theoretical model of inequity aversion. They assume that a small proportion of people is willing to sacrifice material payoffs if this leads to more 'fair' and equitable outcomes. Sethi and Somanathan (1996) discuss the view expressed by Dasgupta (1993), who offers three possible explanations.

1. Small communities can be considered as mini states with the capacity to force members of the community to accept rules of behavior. Sethi and Somanathan (1996) do not find this a strong argument, because it cannot explain the fact that sanctioning by private individuals can be spontaneous and may entail destructive actions that are often prohibited at the state level.
2. Rationality in a repeated game can be reconciled with cooperation. This is the well-known Folk theorem. But the problem here is of course that the set of potential equilibrium outcomes is very large and that alternating periods of cooperation and defection can arise, contradicting observed persistence of strategies.
3. Social norms are internalized through "communal living, role modeling, education and through experimenting rewards and punishments" (Dasgupta, 1993, p. 208). They can then thus motivate agents to do what they do.

To address the problem associated with explanation 2 and analyze the solution offered under 3, adopting an evolutionary game setting is a promising option. By tracing the evolution of cooperation (and defection) it can help to determine which hypothetical equilibria with or without cooperation are actually feasible from a dynamic as well as from a disaggregate (population) perspective.

Theoretical models to explain or analyze the role of social norms to sustain cooperation in a resource setting are rare. Sethi and Somanathan (1996) aim to analyze which norms, as mentioned under point 3 above, can be internalized, using an evolutionary game theoretic framework. In their model, agents can choose between three strategies: defection, cooperation or enforcement. Agents who choose to be enforcer punish defectors, even though they incur a cost for doing so. The sanction level and the cost of sanctioning borne by defectors and enforcers depend on the number of defectors and enforcers in the population. Payoffs are related to the size of the resource stock and, for defectors (and enforcers), to the sanction (punishing) cost level. The agents can modify their strategy over time through a process of social learning. They learn by imitating the strategy that yields above average profits in the population. This is modeled by a replicator dynamics that mimics the evolution of social norms in the population. Sethi and Somanathan identify two main equilibria: a population composed of only defectors and a population composed of only cooperators and enforcers.

Another theoretical study of the role of social norms in solving social dilemmas is Eshel et al. (1998), who consider a model of local interactions between altruistic and egoistic agents. Although they do not deal with a resource, they nevertheless suggest relevant elements for our approach. In the first place, they assume that agents imitate the strategy in their direct neighborhood with the highest average profit. Second, they are able to derive analytical results for a setting in which agents are spatially distributed on a circle and interact only with their two nearest neighbors.

In the present paper, we consider a spatial evolutionary CPR game that combines both local and global interactions. Agents can be cooperators, defectors or enforcers, and imitate the strategy yielding above average payoffs in their neighborhood. We model space just like in Eshel et al. (1998) by assuming a circle with agents that only observe their two nearest neighbors. This is a logical conceptual-analytical starting point, while it also provides a quite accurate picture of how interactions occur in a large range of CPR issues, for example irrigation problems. Indeed, in many rural communities experiencing water conflicts, the monitoring of water quotas is exerted by the farmer located upstream or downstream of the water flow (see Ostrom 1990; Smith 2000), suggesting a linear (or circular to avoid edge problems) model. In line with this, we assume in our model that enforcers can only punish defectors located in their immediate neighborhood, which implies local interaction. Payoffs further depend on the aggregate harvesting effort and on the evolution of the stock of the resource, which means global interactions. In other words, our model combines local and global interactions. We derive theoretical and numerical results on type of limit states that emerge in such a system. We obtain two main innovative results compared to previous work. First, equilibria in which the three types of strategies coexist survive in the long-run. Second, the emergence of such equilibria, and of cooperative equilibria in general, is facilitated when resource dynamics is introduced.

The paper is organized as follows. Section 2 presents the standard CPR game and its evolutionary version. Section 3 sets out the main results obtained with our model for the case without resource dynamics. Section 4 discusses the stability of equilibria. Section 5 presents the results with resource dynamics. Section 6 concludes.

2. The CPR Game

We consider the performance of three types of agents: cooperators, defectors and enforcers. They play a game that involves the exploitation of a common pool of a renewable natural resource. Cooperators and enforcers are supposed to display social behavior, meaning that they restrict the level of harvesting effort exercised. Defectors, however, are only interested in their own profits, and harvest with a relatively high effort level, thereby possibly harming the other players. In order to be more precise with regard to these concepts we introduce here briefly the standard CPR game as a benchmark (see e.g., Dasgupta and Heal 1979; Chichilnisky, 1994; or Ostrom et al. 1994). We consider first the case of no resource dynamics. Subsequently we discuss the case where the natural resource changes over time. Then we introduce the evolutionary CPR game.

2.1. THE STANDARD CPR GAME

A fixed population of $n (n > 1)$ agents has access to a common pool of resources. Initially, we assume that the size of the pool is constant over time. The exploitation of the resource leads to harvest. The individual effort level of agent i is denoted by $x_i (i = 1, 2, \ldots, n)$. The individual cost of effort is denoted by w. Total effort is:

$$X = \sum_{i=1}^{n} x_i. \tag{1}$$

Harvest depends on individual as well as aggregate effort. When aggregate effort is X total harvest is equal to $F(X)$. It is assumed that F is strictly concave and increasing, $F(0) = 0$, $F'(0) > w$, and $F'(\infty) < w$. The harvested commodity is taken as the numeraire. Each agent i receives a share of total revenues equal to his share in aggregate effort. Individual profits are then given by:

$$\pi_i(x_i, X) = \frac{x_i}{X} F(X) - w x_i. \tag{2}$$

Aggregate profits are:

$$\Pi(X) = \sum_{i=1}^{n} \pi_i(x_i, X) = F(X) - wX. \tag{3}$$

The Pareto efficient, aggregate profit maximizing, level of effort is defined by $F'(X_\mathrm{P}) = w$. The zero profit level of efforts is defined by $F(X_0) = wX_0$. The symmetric Nash equilibrium aggregate effort follows from

$$\frac{(n-1)}{n}\frac{F(X_\mathrm{C})}{X_\mathrm{C}} + \frac{1}{n}F'(X_\mathrm{C}) = w. \tag{4}$$

Clearly $X_0 > X_\mathrm{C} > X_\mathrm{P}$. So, the Nash equilibrium is suboptimal, but yields positive rents.

In the case of resource dynamics the social optimum can be described in several ways. One option (in continuous time) is to consider the maximization of the present value of total profits

$$\max \int_0^\infty e^{-rt}[F(X(t), N(t)) - wX(t)]dt$$

subject to

$$\dot{N}(t) = G(N(t)) - F(X(t), N(t)), N(0) = N_0.$$

Here r is the social discount rate, $N(t)$ denotes the resource stock at time t, G is the natural growth function, and F is the harvest function, increasing in aggregate effort as well as in the existing stock. Social behavior can then be defined as behavior consistent with a dynamic extraction path that follows from present value maximization. The Nash equilibrium is the solution to the differential game where each agent takes the time path of efforts of all other players as given and maximizes his own total discounted profits.

2.2. THE EVOLUTIONARY CPR GAME

In the evolutionary CPR game a distinction is made between cooperators, defectors and enforcers. Defectors do not behave according to the social norm, and may be punished by enforcers. We first introduce the set of strategies. Next, we discuss the payoffs. Then, we go into the spatial structure of the game. Finally, we introduce replicator dynamics.

2.2.1. *Strategies*

In our evolutionary framework agents have a fixed strategy reflecting bounded rationality. The individual effort by cooperators and enforcers is denoted by x_L and the effort by individual defectors is x_H.

For the case of no resource dynamics it is assumed that these effort rates are constant and satisfy

$$X_\mathrm{P} \leqslant nx_\mathrm{L} < nx_\mathrm{H}. \tag{5}$$

Hence, if all players (n) are cooperators or enforcers they end up more closely to the Pareto efficient outcome than when all players are defectors.[1]

For the case of resource dynamics there are several plausible ways of modeling effort by individual agents. As suggested above, cooperation can be modeled by assuming that if all agents were cooperators, they would mimic the present value maximizing extraction path. A feature common to evolutionary approaches, however, is that agents use rules of thumb rather than adopt individually or socially optimal strategies. One way to capture this is to assume that effort rates of agents are constants, that may, however, differ across types of agents. For example, the individual effort of cooperators and enforcers is x_{L} with nx_{L} close to X^{pv}, defined as the steady state effort of the present value maximizing program, whereas effort by defectors is larger: $x_{\mathrm{H}} > x_{\mathrm{L}}$. If $nx_{\mathrm{L}} = X^{\mathrm{pv}}$ and all agents are cooperators, convergence to the present value optimal steady state ocurs. An alternative approach allows for the strategy to depend on the existing stock, in line with the work of Sethi and Somanathan (1996). They assume that all players can observe the existing resource stock, or are informed about the stock by an agency. Then one can define $x_{\mathrm{L}}(t) = \alpha_{\mathrm{L}} N(t)$ and $x_{\mathrm{H}}(t) = \alpha_{\mathrm{H}} N(t)$ with α_{L} and α_{H} positive constants with $\alpha_{\mathrm{H}} > \alpha_{\mathrm{L}}$. In particular, α_{L} can be chosen such that convergence occurs to N^{pv}, the present value maximizing steady state resource stock. It need not be the case that the socially optimal steady state coincides with the steady state arising from present value maximization. Other objectives than present value maximization can be pursued as well.

2.2.2. *Payoffs*

The numbers of cooperators, defectors and enforcers are denoted by n^{C}, n^{D} and n^{E}, respectively. All cooperators and enforcers exercise an effort level of $x_{\mathrm{L}}(N)$ (obviously the argument N can be suppressed when resource dynamics is not taken into account) each. Enforcers punish defectors, at a cost γ per detected defector. Defectors make an effort $x_{\mathrm{H}}(N)$ and pay a sanction δ per enforcer that detects them. Define $Z(X, N) = F(X, N)/X - w$, which can be interpreted as aggregate profit per unit of effort. Individual profits, can be written as follows:

$$\pi^{\mathrm{C}}(X, N) = x_{\mathrm{L}}(N) Z(X, N), \tag{6}$$

$$\pi_k^{\mathrm{D}}(X, N) = x_{\mathrm{H}}(N) Z(X, N) - \delta k, \tag{7}$$

$$\pi_m^{\mathrm{E}}(X, N) = x_{\mathrm{L}}(N) Z(X, N) - \gamma m. \tag{8}$$

Here $\pi_k^{\mathrm{D}}(X, N)$ denotes the profits of a defector punished k times and $\pi_m^{\mathrm{E}}(X, N)$ is the payoff of an enforcer punishing m times.

2.2.3. *Spatial structure*

Sethi and Somanathan (1996) assume that all enforcers in the population can detect all defectors and punish them. Formally, this means that $k = n^{\mathrm{E}}$ and $m = n^{\mathrm{D}}$. Obviously, the spatial structure is irrelevant then. In contrast, we assume that an enforcer can only detect and punish a defector in his immediate neighborhood. This calls for a definition of neighborhood. There are several straightforward ways to do so. Eshel et al. (1998) describe players as located on a circle, implying that every agent has exactly two direct neighbors. Hence k and m take the values 0,1, or 2. One could extend the notion of neighborhood to two positions on the circle at each side. Then k and m run from 0 to 4. Another convenient way of defining neighborhood is on a torus. A torus is a two dimensional lattice whose corners are pasted together to ensure that all cells are connected, so that there are no edge effects. Then an agent's neighbors are, for example, those to the west, east, north and south. In this case k and m run from 0 to 4. One could include also those to the north–east etc., at the cost of higher complexity. In the present paper we focus on the circle with each agent having two neighbors, because this allows us to derive interesting theoretical results that are much more difficult to obtain for the torus. For an extensive numerical analysis on the two-dimensional torus, using a different learning rule as well, we refer to Noailly et al. (2004).

The sanctioning cost falling upon an enforcer is proportional to the number of defectors detected and punished, which expresses the efforts made by the enforcer. Similarly, in our setup it matters by how many enforcers a defector is detected. In the case of two enforcers, the cost to the defector is twice as high as in the case of only one enforcer. This can be regarded either as reflecting the sum of the damages inflicted upon the defector by individual enforcers or as the level of punishment being dependent on the amount of evidence provided by all enforcers together.

2.2.4. *Replicator dynamics*

A common element of evolutionary game theory is replicator dynamics, describing when, how and why agents switch strategies. In Sethi and Somanathan (1996) agents are assumed to be able to observe their own profits and the average profits in the population. The decision to change strategy is based on the comparison of these profits. This gives rise to a replicator dynamics equation of the following form:

$$\dot{n}^j = n^j(\pi^j - \overline{\pi}),\ j = \mathrm{C, D, E} \tag{9}$$

where $\overline{\pi} = (n^{\mathrm{C}}\pi^{\mathrm{C}} + n^{\mathrm{D}}\pi^{\mathrm{D}} + n^{\mathrm{E}}\pi^{\mathrm{E}})/n$, the average payoff in the entire population at time t. Therefore, agents do not necessarily switch to the most profitable strategy instantaneously. It follows that an equilibrium with all three strategies, a so-called CDE-equilibrium (with Cooperators, Defectors and Enforcers) will never prevail, because in such an equilibrium enforcers would do strictly worse than cooperators. In contrast to Sethi and Somanathan we explicitly take into account that agents do not observe the payoffs of the entire population. We make the more realistic assumption that agents only observe the payoffs of all agents in their neighborhood, including themselves. The aggregate replicator dynamics formulation then has to be dropped. Several alternative imitation or selection mechanisms can be adopted. One is that an agent imitates the strategy in his neighborhood with the highest payoff. The advantage of this rule is its simplicity. But it can lead to outcomes that may be considered implausible. Consider, for example, the case where a cooperator is surrounded by two defectors, one not being punished (and better off than the cooperator) and the other one severely punished, paying a very high sanction. In such a case it might not be considered very plausible for the cooperator to switch to defection. On the torus, with a cooperator surrounded by three defectors, one of which is not punished and the other three severely punished, the example might even be more appealing. However, there are no fundamental objections against modeling the imitation dynamics in this way. An alternative approach is to switch to the strategy that is doing best on average in the neighborhood. This implies a certain degree of rationality on behalf of the agent. Applying this rule to the previous example, the cooperator becomes a defector if on average the defectors in the cooperator's neighborhood do better than the cooperator. This is the rule employed by Eshel et al. (1998) and we will use it the present paper too.

3. No Resource Dynamics

This section deals with the case where resource dynamics is not taken into account. Consequently, the variable N, denoting the resource stock, is suppressed. At any instant of time τ the system is characterized by the number of agents of each type, $n^{\mathrm{C}}(\tau)$, $n^{\mathrm{D}}(\tau)$ and $n^{\mathrm{E}}(\tau)$, summing up to the given number n, and by the location of each agent on the circle. For convenience, we fix one position on the circle and call it position 1. Then a state of the system can be represented by a vector of length n consisting of ordered C's, D's and E's. So, with $n = 5$, the notation CDEDE means that there is a cooperating agent at position 1, there are defectors at positions 2 and 4, and enforcers at positions 3 and 5 (note, however, that this state is essentially the same as DEDEC). Time is considered discrete. At time $\tau+1$ the system finds itself in a new state, as a consequence of agents switching from one strategy to another. In first instance strategy changes occur only on the basis of replicator dynamics. Mutation is studied in Section 4. The questions we address in the present concern the limiting behavior of the system, as τ goes to infinity.

We have been able to identify a rich set of limit states. First of all there are equilibria. A state is called an equilibrium if no agents wants to change strategy. Second, there are blinkers. A state is called a blinker if agents change strategy, but the new resulting state is a rotation of the original state. For example: the state characterized by CDEED is a blinker, if, after all agents have made their choice of strategy, the new state is DCDEE. So, essentially neither the numbers of cooperators, defectors and enforcers, nor their relative positions on the circle have changed. We also found cycling, where composition of the population of strategies as well as locations change over time, but where after one period the system reproduces.

As shown by the profit equations given in the previous section, payoffs are affected by both local and global factors, namely sanctioning among neighbors and aggregate efforts, respectively. The combination of these two types of factors is an innovative feature of the present paper. However, it entails the inconvenience to render the model much more complex to analyze. Under some assumptions with regard to the ranking of profits, general theoretical results can be derived for equilibria and blinking. With regard to cycling we restrict ourselves to providing an example to show that it can actually occur.

3.1. EQUILIBRIA AND BLINKERS

We aim to derive conditions for the existence of certain types of equilibria and blinkers. Profit rankings are not unambiguous: we might have $\pi_1^{\mathrm{E}}(X) < \pi_1^{\mathrm{D}}(X)$ for some values of X and $\pi_1^{\mathrm{E}}(X) > \pi_1^{\mathrm{D}}(X)$ for other values. This complicates a theoretical analysis and makes it difficult to obtain clear-cut results. Therefore, we concentrate on unambiguous profit rankings here. To avoid clutter we omit the argument X when there is no danger of confusion. For example, $\pi_0^{\mathrm{D}} > \pi^{\mathrm{C}}$ means $\pi_0^{\mathrm{D}}(X) > \pi^{\mathrm{C}}(X)$ for all relevant X (i.e., $nx_{\mathrm{L}} \leqslant X \leqslant nx_{\mathrm{H}}$). To allow for a theoretical approach, we assume that the following three sets of profit rankings hold:

1.
$$\pi_0^{\mathrm{D}} > \pi^{\mathrm{C}} = \pi_0^{\mathrm{E}}.$$
$$\pi_0^{\mathrm{E}} > \pi_1^{\mathrm{E}} > \pi_2^{\mathrm{E}}.$$
$$\pi_0^{\mathrm{D}} > \pi_1^{\mathrm{D}} > \pi_2^{\mathrm{D}}.$$

   These rankings derive from the fact that we neglect the case of negative profits. This rules out the possibility that defectors do worse than cooperators even if they are not punished. Profits from harvesting are nonnegative if $Z(nx_{\mathrm{H}}) \geqslant 0$, because Z is decreasing and $X \leqslant nx_{\mathrm{H}}$.

2.
$$\pi^{\mathrm{C}} > \pi_1^{\mathrm{D}}.$$

   In order to have an interesting game, being punished should not be uniformly more profitable than cooperation. Several choices are open regarding the number of punishments needed to make cooperation more profitable than defection. For simplicity, we assume that being punished once is already worse than being cooperative.

3.
$$\pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}} \text{ implies } \pi_1^{\mathrm{E}} > \pi_2^{\mathrm{D}} > \pi_2^{\mathrm{E}}.$$
$$\pi_1^{\mathrm{E}} > \pi_1^{\mathrm{D}} \text{ implies } \pi_1^{\mathrm{D}} > \pi_2^{\mathrm{E}} > \pi_2^{\mathrm{D}}.$$

   Therefore, if being punished once is better than punishing once, then being punished twice is worse than punishing twice, and vice versa. Hence, in the former case, being a defector is not too advantageous.

We get analytical results for the set of parameter values that satisfy these assumptions, but the simulations suggest that the results we obtain analytically also hold for a much broader class of parameter values.

Since the imitation rule that we employ is based on comparison of average payoffs by agents, an additional distinction can be made. A defector punished once is doing better than an enforcer punishing once, with a non-punishing enforcer in his neighborhood, or this ranking is the other way around. To illustrate the intuition, consider the following complete string EEEDD, where the second defector is next to the first enforcer. The first and the third enforcers, both located next to a defector that is punished once, change to defection when the sanction rate is sufficiently low. However, with what we will call a moderately low sanction rate they stay enforcers.

From the above discussion and assumptions, we get the following profits orderings as stated in Definition 1.

Definition 9.1

(i) The sanction rate is relatively low if:
$\pi_0^{\mathrm{D}} > \pi^{\mathrm{C}} = \pi_0^{\mathrm{E}} > \pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}} > \pi_2^{\mathrm{D}} > \pi_2^{\mathrm{E}}$.

(ii) The sanction rate is relatively very low if:
$\pi_0^{\mathrm{D}} > \pi^{\mathrm{C}} = \pi_0^{\mathrm{E}} > \pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}} > \pi_2^{\mathrm{D}} > \pi_2^{\mathrm{E}}$ and $\pi_1^{\mathrm{D}} > \frac{1}{2}\left(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}}\right)$.

(iii) The sanction rate is relatively moderately low if:
$\pi_0^{\mathrm{D}} > \pi^{\mathrm{C}} = \pi_0^{\mathrm{E}} > \pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}} > \pi_2^{\mathrm{D}} > \pi_2^{\mathrm{E}}$ and $\pi_1^{\mathrm{D}} < \frac{1}{2}\left(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}}\right)$.

(iv) The sanction rate is relatively high if:
$\pi_0^{\mathrm{D}} > \pi^{\mathrm{C}} = \pi_0^{\mathrm{E}} > \pi_1^{\mathrm{E}} > \pi_1^{\mathrm{D}} > \pi_2^{\mathrm{E}} > \pi_2^{\mathrm{D}}$. So, the sanction rate is relatively low if $\pi_k^{\mathrm{D}} > \pi_k^{\mathrm{E}}$ for $k = 1, 2$. It is relatively high if $\pi_k^{\mathrm{D}} < \pi_k^{\mathrm{E}}$ for $k = 1, 2$. It should be noted that the wording, including 'relatively,' is chosen on purpose. For example, the sanction rate could be called absolutely low if $\pi_2^{\mathrm{D}} > \pi_1^{\mathrm{E}}$, or even $\pi_2^{\mathrm{D}} > \pi_0^{\mathrm{E}}$. We will consider such cases later on in this paper when performing simulations. Below we derive a set of sufficient conditions for each of the two rankings to hold, thereby showing that the definitions are not void.

Lemma 1

(i) *Suppose $\gamma > \delta$ and $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < 2\delta - \gamma$. Then the sanction rate is relatively low.*

(ii) *Suppose $\gamma > \delta$, and $\delta - \frac{1}{2}\gamma < (x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{H}}) < (x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < 2\delta - \gamma$. Then the sanction rate is relatively very low.*

(iii) *Suppose $\gamma > \delta$ and $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < \delta - \frac{1}{2}\gamma$. Then the sanction rate is relatively moderately low.*

(iv) *Suppose $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < \delta - \gamma$ and $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{H}}) > \delta - 2\gamma$. Then the sanction rate is relatively high.*

Proof. The proof of the lemma is given in the appendix.

The proof of the lemma is rather technical, but the idea behind it is easily explained. Consider, for example, statement (i). If the cost of sanctioning γ is higher than the sanction δ, then a defector being punished k times is better off than an enforcer punishing k times for all k, because profits from harvesting are higher for a defector, and the defector incurs a lower sanction than the cost the enforcer has to make to punish. Moreover, if $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < 2\delta - \gamma$, then $x_{\mathrm{H}}Z(X) - 2\delta < x_{\mathrm{L}}Z(X) - \gamma < 0$ for all $X \leqslant nx_{\mathrm{H}}$ and hence $\pi_1^{\mathrm{E}} > \pi_2^{\mathrm{D}}$. All the other proofs follow the same approach.

A further distinction suggests itself: a relatively very high versus a moderately high sanction rate, according to $\frac{1}{2}\left(\pi_0^{\mathrm{D}}+\pi_1^{\mathrm{D}}\right)$ being smaller or larger than π_1^{E}, respectively. However, this distinction is not meaningful, as can be seen as follows. The inequality $\frac{1}{2}\left(\pi_0^{\mathrm{D}}+\pi_1^{\mathrm{D}}\right) < \pi_1^{\mathrm{E}}$ requires $(x_{\mathrm{H}}-x_{\mathrm{L}})Z(X) < \frac{1}{2}\delta-\gamma$ for all $X \leqslant nx_{\mathrm{H}}$, so that it is necessary that $\frac{1}{2}\delta-\gamma > 0$. But the inequality $\pi_1^{\mathrm{D}} > \pi_2^{\mathrm{E}}$ requires $(x_{\mathrm{H}}-x_{\mathrm{L}})Z(X) > \delta-2\gamma = 2(\frac{1}{2}\delta-\gamma)$. This is a contradiction. Also, note that the relatively high sanction rate implicitly assumes that $\delta > \gamma$, since $(x_{\mathrm{H}}-x_{\mathrm{L}})Z(nx_{\mathrm{L}}) > 0$.

Next we establish several propositions regarding the existence and the characteristics of equilibria and blinkers, assuming that the profit ranking satisfies one of the definitions given above. States with only cooperators (‘allC’), only defectors (‘allD’), only enforcers (‘allE’), and only cooperators and enforcers (‘CE’), are always an equilibrium. A state with only defectors and cooperators (‘CD’) cannot be an equilibrium, because a cooperator next to a defector will change to defection. Therefore, we concentrate on the DE and CDE equilibria. A cluster in an equilibrium is a string of adjacent agents playing identical strategies. To start with we prove a lemma that turns out to be rather helpful.

Lemma 2 *Suppose $n \geqslant 3$.*

(i) *A string composed as CED cannot occur in an equilibrium.*
(ii) *A string composed as CD cannot occur in an equilibrium.*
(iii) *A string composed as DED cannot occur in an equilibrium.*
(iv) *A string composed as EDE cannot occur in an equilibrium.*

Proof.

(i) With CED, the punishing enforcer switches to cooperation, if not to defection.
(ii) With CD the defector switches to cooperation or the other way around.
(iii) and (iv) Obviously, DED cannot occur under a relatively low sanction rate, and EDE is ruled out in the case of a relatively high sanction rate. If DED would occur in an equilibrium with a relatively high sanction rate, the defectors surrounding the enforcer would not be punished twice, since EDE is ruled out. But then the enforcer would switch to defection. To exclude EDE in the relatively low sanction case, the same type of argument holds.

Proposition 1. *Suppose the sanction rate is relatively very low.*

(i) *There exists neither a DE nor a CDE equilibrium.*
(ii) *There exists neither a DE nor a CDE blinker.*

Proof.

(i) Suppose there exists an equilibrium with $n^{\mathrm{E}} > 0$ and $n^{\mathrm{D}} > 0$. There must be at least one enforcer next to a defector, because the equilibrium does not consist of defectors only, and if a defector is not punished, he cannot be a neighbor of a cooperator, because then the cooperator switches to defection. If a defector next to an enforcer is punished only once the enforcer will switch to defection, because $\pi_1^{\mathrm{D}} > \frac{1}{2}(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}})$, a contradiction. Hence every defector is punished twice, contradicting lemma 2(iv).

(ii) Suppose there is a blinker with $n^{\mathrm{E}} > 0$ and $n^{\mathrm{D}} > 0$. At least one agent switches to enforcement. This is not a cooperator. So, a defector should switch to enforcement. He will only do so if he is punished twice: so we have EDE. In order for the first enforcer in this string to switch to defection, we need DEDE, because with EEDE he will stay an enforcer. But now the first defector in the row will never switch to enforcement. This proves statement (ii) of proposition 1.

Proposition 2. *Suppose the sanction rate is relatively moderately low.*

(i) *For a DE-equilibrium to obtain it is necessary that $n \geqslant 5$. If $n = 5$ the equilibrium configuration is given by EEEDD. In any DE-equilibrium enforcers occur in clusters of minimal length 3.*

(ii) *For a CDE-equilibrium to obtain it is necessary that $n \geqslant 9$. If $n = 9$ the equilibrium configuration is given by CEEEDDEEE. In any CDE equilibrium any enforcer adjacent to a defector is part of a cluster of at least 3 enforcers.*

(iii) *There exists neither a DE nor a CDE blinker.*

Proof. The proof of the proposition is given in the appendix.

The intuition behind the proposition is straightforward. Since, by definition, $\pi_1^{\mathrm{E}} < \pi_1^{\mathrm{D}} < \frac{1}{2}\left(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}}\right)$, punishing enforcers need to be 'protected' by non-punishing enforcers. This leads to clusters of three enforcers. Protection by cooperators does not work, because, in an equilibrium, a punishing enforcer can never be located next to a cooperator. This also explains why a minimal number of players is required. Obviously, it might be the case that in a CDE-equilibrium the majority of agents is defecting.

Proposition 3. *Suppose the sanction rate is relatively high.*

(i) *For a DE-equilibrium to obtain it is necessary that $n \geqslant 5$. If $n = 5$ the equilibrium configuration is given by EEDDD. In any DE-equilibrium defectors occur in clusters of minimal length 3.*

(ii) *For a CDE-equilibrium to obtain it is necessary that $n \geqslant 8$. If $n = 8$, the equilibrium configuration is given by CEEDDDEE. In any CDE-equilibrium any defector adjacent to an enforcer is part of a cluster of at least 3 defectors.*

(iii) *There exist no DE blinkers. There do exist CDE blinkers. A necessary condition is $n \geqslant 4$. If $n = 4$, the blinker is CDDE.*

Proof.

(i) and

(ii) The proof of statements (i) and (ii) follows the lines of the proof of the previous proposition. It will not be given here.

(iii) Non-existence of DE blinkers is obvious. Suppose $n = 3$ and there is a CDE blinker. Then the cooperator remains a cooperator. Both the enforcer and the defector turn into cooperators. Hence there is no blinking in this case. Suppose $n = 4$. In a CDE blinker a cooperator never becomes an enforcer. Hence, at least one cooperator should turn into a defector. This can only be the case if he is next to a defector who is not punished. In the present case we cannot have CDCE because both cooperators will become defectors. Hence the only equilibrium candidate is CDDE. It is easily verified that this is a blinking equilibrium.

At this stage, we can summarize the main existence properties of the equilibria. We have established that the states C, D, E and CE are always part of equilibria, while CD never is. We also have proved that DE and CDE equilibria only occur for a moderately low sanction rate and for a sufficiently large population. Finally, we have shown that DE blinkers only occur for a high sanction rate and a sufficiently large population.

3.2. CYCLING

To illustrate the phenomenon of cycling in the present setting, consider the following initial state: DDDDEE. The defectors in positions 2 and 3 will not change strategy. The first and fourth defector change strategy if the average payoff of the defectors in their neighborhood is smaller than the payoff of an enforcer punishing once:

$$\frac{1}{2}\left[\pi_0^{\mathrm{D}}(X) + \pi_1^{\mathrm{D}}(X)\right] < \pi_1^{\mathrm{E}}(X). \tag{10}$$

If this inequality holds, for $X = 2x_{\mathrm{L}} + 4x_{\mathrm{H}}$, the enforcers stick to enforcement since then also

$$\pi_1^{\mathrm{D}}(X) < \pi_1^{\mathrm{E}}(X). \tag{11}$$

Therefore, if (10) holds, the new state becomes EDDEEE. The enforcers at positions 1 and 6 in the new state switch to defection if

$$\frac{1}{2}\left[\pi_0^{\mathrm{E}}(X) + \pi_1^{\mathrm{E}}(X)\right] < \pi_1^{\mathrm{D}}(X) \tag{12}$$

for $X = 4x_{\mathrm{L}} + 2x_{\mathrm{H}}$. When this condition holds, the defectors stay defectors. Now set $x_{\mathrm{L}} = 100, x_{\mathrm{H}} = 120, F(X) = 13.25X^{1/2}, w = 0.5, \gamma = 0.1, \delta = 0.525$. Then all conditions are satisfied. Therefore cycling between the two states indicated above, occurs with a period of one. It may be noticed that the range of the sanction δ, given the other parameter values, is rather small. This small range is also found in various other numerical examples with different parameter values for $x_{\mathrm{L}}, x_{\mathrm{H}}$ and the parameters of F. It suggests that cycling does not occur for a wide range of parameter values. Obviously, this does not matter, since the aim was just to provide an example. Moreover, it would be relatively easy to induce cycling if we allow profits from harvesting to be negative: $Z(X) < 0$. In this case the incentive of defectors to change strategy is much larger for defectors, because they earn less from harvesting than enforcers (they incur greater losses). In our example we took care that profits, even including sanctions and the cost of sanctioning, are positive. The importance of the example is that it shows that the system is not only steered through local interaction, but that global interaction through aggregate efforts plays a role too.

Comparing the results in this section with those obtained by Sethi and Somanathan, we observe that we not only have more types of limit states (cycling, blinking and equilibria), but within the class of equilibria, we have equilibria with cooperation surviving next to defection, which is a novel finding as well. This phenomenon occurs for sanction levels that can be deemed realistic. So, it turns out that the spatial structure of the game is pivotal in the characterization of potential equilibria.

4. Stability

In the previous section we have established the existence of equilibria where cooperators survive in groups with many defectors. This result is due to the spatial structure of our model. It would be less interesting if the occurrence of these equilibria would merely be a coincidence, namely for very specific spatial constellations, or if the equilibria would easily be disrupted by players making mistakes in choosing their strategies. In the present section we investigate this issue. We first make use of an approach common in applications of evolutionary game theory. Then we discuss and explore an alternative route, relying on numerical simulations with stochastic features.

In evolutionary game theory stability of equilibria is tied to mutations, meaning that players may make mistakes in deciding on their strategy. This then leads to the notion of stochastic stability. Before dealing with stochastic stability in detail we illustrate the concept by means of an example. Suppose we start with a configuration of only cooperators. This configuration will persist if all players strictly follow the imitation rule. However, suppose that each player has a given small probability of making a mistake. At some instant of time this probability materializes and a player becomes a defector. Then defection will infect a large part of the population within finite time: many cooperators will be eradicated. And it is highly unlikely that the stochastic process of mutation will restore the ‘allC’ equilibrium. This is essentially why this equilibrium is not stochastically stable.

One way to assess the stochastic stability or instability of equilibria is outlined in Young (1998) and in Eshel et al. (1998). We briefly sketch the procedure, merely to illustrate the difficulties encountered in its application. As was stated before, at any instant of time τ the state of the system is characterized by the number of agents of each type, $n^{\mathrm{C}}(\tau)$, $n^{\mathrm{D}}(\tau)$ and $n^{\mathrm{E}}(\tau)$, summing up to the given number of agents n, and by the location of each agent on the circle. Such a representation may be misleading, however. If two states are identical up to rotation or taking the mirror image, they should be considered as identical states. For example: the state CCDDEEE is essentially the same as CDDEEEC (each player is moved one position) and as EEEDDCC (we 'read' the circle in the opposite direction). So, in the sequel, we restrict ourselves to unique states. The state space is the finite set of all possible states. The matrix P of transition probabilities p_{ij} from state i to state j, is completely determined by the imitation dynamics. To keep things simple, we assume that a situation where a player has two equivalent strategies to choose from does not occur. Then the transition matrix consists of zeros and ones only. Next, we introduce mutation. After the transition to a new state a player has a probability $\frac{1}{2}\alpha$ of not adopting the strategy that is optimal according to the imitation rule, but, instead, going to pursue either of the two alternative strategies. So, a player who just became a cooperator, according to the imitation rule, will actually act as a defector or an enforcer, each with probability $\frac{1}{2}\alpha$. This yields another matrix of probabilities denoted by Q with a typical element q_{ij} denoting the probability of transition from state i to state j, as a consequence of the mutations that happen to take place in state i. The overall transition matrix is then γ with $\gamma_{ij} = \sum_k p_{ik} q_{kj}$. Let μ be the solution of the following system $\mu\,\gamma = \mu$, where μ is on the unit simplex: $\mu \geqslant 0$ and $\sum_i \mu_i = 1$. The vector μ is the unique stationary distribution of the process for a given mutation rate. Element μ_i indicates that as time gets large, state i will occur during a proportion μ_i of time. Finally, one considers the limit of μ for the mutation rate approaching zero.

It is clear from the exposition given above that in the case at hand it is almost unsurmountable to derive general results on the stochastic stability of CDE equilibria in our model. Already for the minimal number of agents in the low sanction case the set of possible states amounts to hundreds. Eshel et al. (1998) were able to derive results on stochastic stability, thanks to the fact that their analysis only involves two strategies. Moreover, Sethi and Somanathan (1996) do not inquire into stochastic stability, arguing that: "Given the time scales relevant for this paper, the introduction of stochastic perturbations is therefore unlikely to affect our main inferences." Like in the case of Sethi and Somanathan, one might consider our model as applying to fisheries. The time scales can be interpreted as referring to seasons, while updating occurs once per season. If an equilibrium would not persist after, say, 1000 seasons, then this should not be considered as a sign of instability because it concerns an extremely long time horizon for the system considered. In other words, if it takes thousands of seasons and thus years before a certain type of equilibrium (e.g., CDE) has completely vanished, then from a practical perspective this should not be regarded as a serious case of instability. Indeed, many other, directed factors will then have ample time to exercise their influence on the system and its stability, negating the relevance of the stochastic factors.

In view of the previous argument we investigate stability of the different equilibria, and in particular of CDE-equilibria, using numerical simulations. We employ the harvest function given by $F(N, X) = N^{1/2} X^{1/2}$ and consider a population of $n = 100$ agents. The other parameter values are

$$w = 5, \quad N_0 = 10^6, \tag{13}$$

$$x_{\mathrm{H}} = 120, \quad x_{\mathrm{L}} = 100, \tag{14}$$

$$\delta = 280, \quad \gamma = 300. \tag{15}$$

These parameters are chosen such that $nx_{\mathrm{L}} = X_{\mathrm{P}}$, implying that when all agents harvest low the social optimum is reached. Further, we have $Z(nx_{\mathrm{H}}) > 0$, so that in the absence of sanctioning all players enjoy positive profits. In a first step, we illustrate the above statement of Sethi and Somanathan (1996) by studying the time scales on which cooperative equilibria cease to occur. We start from a fixed spatial configuration, namely a CDE initial state with $n^{\mathrm{C}} = 25, n^{\mathrm{D}} = 25$ and $n^{\mathrm{E}} = 50$. The agents are positioned in the following order: 25 cooperators, 25 enforcers, 25 defectors and 25 enforcers. In the absence of mutation and with $\delta = 280$, this initial state is a CDE equilibrium. How does the frequency of CDE equilibria evolve when we introduce mutations? We assume that in every round each agent has a probability of making a mistake of $\alpha = 5/1000$, meaning that, at the beginning of every round, the agent has a chance of α to deviate from the decision rule. We record the population configuration at the end of every round. We conduct 100 simulation runs for different time horizons and compute the average time spent in each possible population configuration. The results are reported in Table I.

After 10,000 rounds, the system spent on average 24% of the time in a CDE-configuration. As expected, as the time horizon increases, i.e., as the number of mutations rises, the frequency of CDE-equilibria decreases. Eventually, as $\tau \to \infty$, the frequency will tend to zero. Nevertheless, this frequency decreases by only 1% per additional 10,000 rounds. After 30,000 rounds, the system spends still 22% of the time in a CDE-equilibrium. This suggests that the time scales over which CDE disappears may be very long and irrelevant for applications with seasonal updating. Note also that the mutation rate is kept constant in this experiment, whereas it should converge to zero in a proper test for stochastic stability.

Our approach with spatial interaction lends itself to examine stability of equilibria in an alternative manner, namely to look at the emergence of equilibria and the frequency of the different types of equilibria when we randomize over the initial shares of strategies as well as their distribution over the circle. For a given sanction rate δ, we vary:

1. the initial shares of each strategy in the population. To reduce the number of runs necessary to cover all the possible combinations of initial shares, only

Table I. Percentage of time spent in each equilibrium in the presence of mutations

τ	D-equil.	DE-equil.	CE-equil.	CDE-equil.
100	0.00	0.01	0.00	0.99
500	0.13	0.27	0.00	0.60
10,000	0.58	0.18	0.00	0.24
20,000	0.59	0.18	0.00	0.23
30,000	0.56	0.22	0.00	0.22

strategy shares that are multiples of 0.05 are considered. The set of initial coordinates $Z = ((1; 0; 0), (0.95; 0.05; 0) \dots (0; 0; 1))$ is composed of coordinates $z_0 = (n^{\mathrm{C}}/n, n^{\mathrm{D}}/n, n^{\mathrm{E}}/n)$. Further, we eliminate initial strategy shares composed of only cooperators and defectors, and of only cooperators and enforcers, as the outcomes can be easily predicted in these cases.[2] This leaves us with 190 potential initial shares,

2. the initial spatial distribution of strategies. For every z_0, we perform 100 so-called runs of 200 time-steps.[3] Each run starts with a draw from a uniform random spatial distribution, such that the probability of a position on the circle being occupied by a player of type j equals $n^j/n (j = \mathrm{C, D, E})$. This means that for each z_0, we consider 100 random spatial arrangements and register the resulting equilibrium. We find that on average 32% of the runs (out of 19,000) converge to a D-equilibrium, 4% converge to a CE-equilibrium, 33% to a DE-equibrium and 29% to a CDE-equilibrium. Cycling occurred in the CDE-configuration in 2% of the cases. We found no occurrence of blinker states. This is in line with our theoretical results since the sanction level $\delta = 280$ corresponds to a relatively moderately low sanction rate. What can we conclude from the fact that in almost 30% of the cases convergence to a CDE-equilibrium occurs? Formally, it does not prove the stochastic stability of this type of equilibrium. But the procedure followed strongly suggests that CDE-equilibria are not a mere coincidence. In an environment that is stochastic with respect to initial shares and initial locations, cooperation will survive in a large number of cases.

Additionally, these simulations provide two other types of insights on how the system works. First, we gain insights on how the initial distribution affects equilibria. Figure 1 shows the frequency of convergence to each equilibrium for the different initial shares combinations. In each graph, each z_0 is represented by a dot. The grey-black scale indicates the result of simulations with 100 random spatial distributions after 200 time steps. A black colored coordinate indicates that, starting with the respective z_0, all runs converge to the given type of equilibrium.[4] As expected, D-equilibria are more easily achieved for initial populations with few enforcers and, inversely, CE-equilibria are more likely to be reached for initial populations composed of many enforcers. CDE-equilibria are most frequently achieved for middle-range initial shares with a slight majority of enforcers.

Second, we gain insights on the effects of the initial location of strategies over space. Figure 2 shows the evolution of strategy shares over time starting from three identical share vectors $z_0 = (0.30; 0.30; 0.40)$ but with different initial spatial arrangements. The evolution of strategy shares is governed by two forces. First, enforcers who punish a lot imitate defectors in their neighborhood. In some sense, enforcers are then being eliminated by defectors. Second, enforcers who punish at least one defector switch to cooperation when cooperators are located in their neighborhood. So, we see that enforcers have a hard life. On the other hand, they

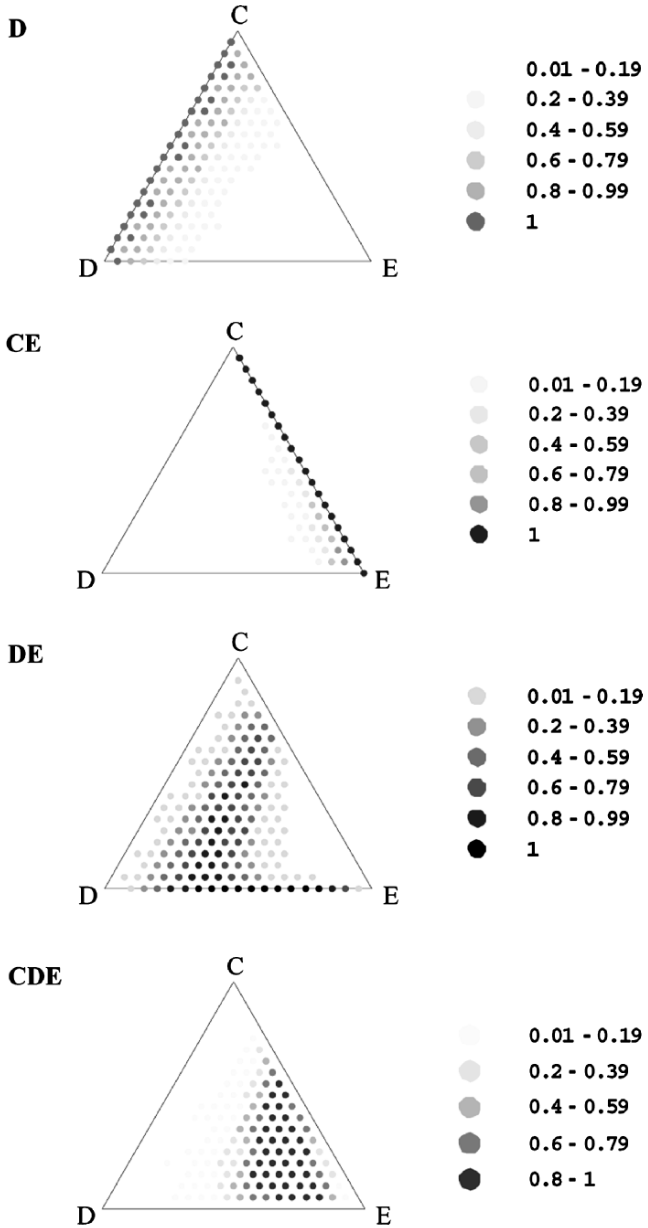

Figure 1. Frequency of equilibria for different initial shares multiple of 0.05, $\delta = 280$.

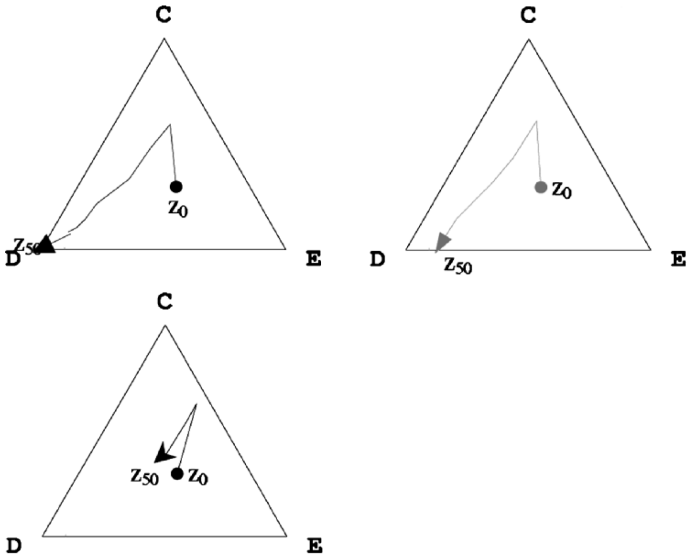

Figure 2. Evolution of strategy shares over time, for $z_0 = (0.30; 0.30; 0.40)$, $\delta = 280$.

eliminate defectors if they punish hard enough. In all of the approach paths we see the number of enforcers decrease; the number of defectors increases in the final steps.

Finally, to complete our analysis of stability and to confirm further that the occurrence of CDE-equilibria is not a mere coincidence, we run simulations for various sanction levels. Given our parameter values, the definition of a relatively very low sanction is satisfied for $200 < \delta < 232$. The sanction rate is relatively moderately low if $232 < \delta < 341$. It is relatively high if $400 < \delta < 680$. We also performed simulations for sanction rates outside the ranges that imply an unambiguous ordering of profits. For each sanction level, we performed 19,000 simulation runs and computed the average frequence of occurrence of each equilibrium. The results are displayed in Figure 3. The exact frequencies for each type of equilibrium can be found in Table B.1 in Appendix B.

As expected, the frequency of D-equilibria decreases as the sanction rises. Inversely, the frequency of CE-equilibria increases with the sanction level. The largest frequency of CDE-equilibria is found for $\delta = 700$. Beyond $\delta = 800$, the frequency of CE-equilibria rises sharply and it becomes almost impossible for defectors to survive in the population, as shown by the fall in the frequency of CDE- and D-equilibria. As expected from proposition 3, we also find blinkers in the range of relatively high sanction rates, even if the occurrence of this phenomenon is relatively rare (see Table B.1 in Appendix). Recall that for a CDE blinker to occur, the sanction level should be high and a single enforcer should be located between a cooperator and a defector. In large populations this is unlikely to happen. We also find that the occurrence of cycling CDE-equilibria is quite rare. The main conclusion we can draw from these exercises is that equilibria with cooperation have a high probability of survival.

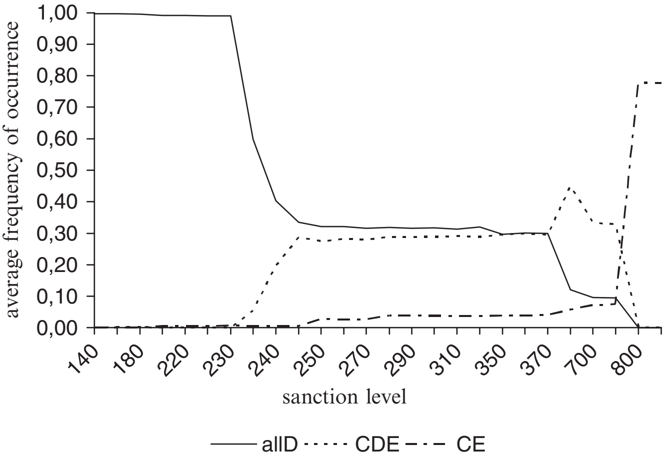

Figure 3. Average frequency of D-, CE-, and CDE-equilibria for different sanction levels.

5. Resource Dynamics

The role of resource dynamics on harvesting behavior is often neglected in the literature on common-pool issues. Experiments and games developed by Ostrom et al. (1994) do not pay any attention to resource dynamics. In real-world situations, however, harvesters are likely to reconsider and actually modify their strategies on the basis of observed changes in the resource stock. Feedback effects are present from harvesting activities to the natural resource and vice versa. Resource dynamics raises the issue of the dynamic development of the resource itself and the impact of varying resource stock level on harvest. In addition, a new dynamic issue is relevant in the present context, namely how resource dynamics affects the occurrence of cooperation. We start the analysis by postulating a logistic natural growth function:

$$G(N) = \rho N \left(1 - \frac{N}{K}\right) \tag{16}$$

with ρ the intrinsic growth rate and K the carrying capacity. Harvest is oftentimes represented by the Shaefer function where the harvest rate is effort multiplied by the resource stock. Alternatively, we assume that

$$F(X, N) = X^{\beta} N^{1-\beta} \tag{17}$$

with $0 < \beta < 1$. Updating of the resource stock after each round follows the usual pattern:

$$N_{t+1} = N_t + G(N_t) - F(X_t, N_t). \tag{18}$$

The steady state of the system is then the solution of

$$\rho N \left(1 - \frac{N}{K}\right) = X^{\beta} N^{1-\beta}. \tag{19}$$

We follow Sethi and Somanathan (1996) and assume that individual effort is proportional to the existing resource stock in the following manner:

$$x_{\mathrm{H}} = a_{\mathrm{H}} N \tag{20}$$
$$x_{\mathrm{L}} = a_{\mathrm{L}} N. \tag{21}$$

What is the effect of the introduction of resource dynamics on the limit states? It is to be expected that the qualitative nature of the limit states will not change: blinkers, cycling and equilibria can still occur. In Section 3.2 we saw that cycling resulted from the fact that payoffs are affected by aggregate harvest. Similarly, resource dynamics will influence payoffs, increasing the number of situations under which profit reversal and thus cycling will occur. In other words, given that an additional global interaction mechanism is operative, cycling is likely to become more frequent. We further expect that the likelihood of the occurrence of CDE-equilibria will not decrease. Overharvesting as a consequence of higher effort levels by defectors does not only reduce harvesting profits per unit of effort but also through the resulting smaller resource stock itself. Therefore, with a given effort rate of defectors, being a defector becomes relatively less rewarding when there are many defectors.

In the case of resource dynamics we can write:

$$\pi^{\mathrm{C}} = a_{\mathrm{L}} N \left[\left(\frac{1}{n^{\mathrm{D}}(a_{\mathrm{H}} - a_{\mathrm{L}}) + n a_{\mathrm{L}}} \right)^{\beta} - w \right], \tag{22}$$
$$\pi_k^{\mathrm{D}} = a_{\mathrm{H}} N \left[\left(\frac{1}{n^{\mathrm{D}}(a_{\mathrm{H}} - a_{\mathrm{L}}) + n a_{\mathrm{L}}} \right)^{\beta} - w \right] - k\delta, \tag{23}$$
$$\pi_m^{\mathrm{E}} = a_{\mathrm{L}} N \left[\left(\frac{1}{n^{\mathrm{D}}(a_{\mathrm{H}} - a_{\mathrm{L}}) + n a_{\mathrm{L}}} \right)^{\beta} - w \right] - m\gamma. \tag{24}$$

Consider π_k^{D}. We see that if n^{D} increases, two things happen. First, aggregate profits from harvesting given by the term in square brackets in (22) decrease. This is similar to the no resource dynamics case: it is a consequence of higher efforts, given the stock. Second, the stock decreases (after some time). This also leads to smaller profits as an additional effect. The stock effect can be assessed by realizing that the steady state with n^{D} enforcers equals:

$$N(n^{\mathrm{D}}) = K \left(1 - \frac{(n^{\mathrm{D}}(a_{\mathrm{H}} - a_{\mathrm{L}}) + n a_{\mathrm{L}})^{\beta}}{\rho} \right). \tag{25}$$

So, the stock effect comes in addition to the effort effect.

We run simulations with a_{H} and a_{L} fixed so that we can compare the average frequency of occurrence of equilibria with the case without resource dynamics. We fix $a_{\mathrm{L}} = 0.0001$ and take $\delta = 300$, $K = 2{*}10^6$ and $\rho = 0.2$. For the rest we employ the same parameters as before. This yields a steady state stock of 10^6 if all players were cooperators or enforcers. The parameter value $a_{\mathrm{L}} = 0.0001$ corresponds with $x_{\mathrm{L}} = 100$ while $a_{\mathrm{H}} = 0.0002$ corresponds with $x_{\mathrm{H}} = 200$ in the

Table II. Average frequency of convergence with and without resource dynamics, $\delta = 300$

With resource dynamics $x_H = a_H N$	D-equil.	DE-equil.	CDE-equil.
200	1.00	0.00	0.00
300	0.77	0.19	0.03
350	0.60	0.34	0.07
400	0.51	0.40	0.09
No resource dynamics			
200	1.00	0.00	0.00
300	1.00	0.00	0.00
350	0.65	0.32	0.03
400	0.55	0.39	0.07

case without resource dynamics. We calculate the frequency of equilibria for these parameter values with resource dynamics as well as without resource dynamics. In both cases D-equilibria occur with probability one. Similarly we performed the simulations for higher values of a_H. The results are given in Table II. We find that for identical x_H, resource dynamics leads to increasing occurrence of CDE-equilibria, as expected.[5]

Finally, we can show that in the case of fixed effort rates, the same type of results is to be expected. With fixed effort rates x_L and x_H we get

$$\pi^C = x_L \left[\left(\frac{N}{n^D(x_H - x_L) + n x_L} \right)^\beta - w \right]$$

$$\pi_k^D = x_H \left[\left(\frac{N}{n^D(x_H - x_L) + n x_L} \right)^\beta - w \right] - k\delta$$

$$\pi_m^E = x_L \left[\left(\frac{N}{n^D(x_H - x_L) + n x_L} \right)^\beta - w \right] - m\gamma$$

Now the steady state stock is a bit less straightforward to calculate. It satisfies

$$\rho N \left(1 - \frac{N}{K} \right) = N^\beta (n^D(x - \mathrm{H} - x_L) + n x_L)^\beta .$$

It is not clear beforehand that this N is increasing in n^D. In fact it is increasing if and only if $\frac{N}{K} < \frac{1}{3}$. For this reason the case at hand is slightly more complicated. But, under this condition, essentially we see the same mechanism at work. Higher n^D decreases aggregate profits directly through the effort effect, and, in addition, decreases aggregate profits through its effect on the stock. All this implies that the difference $\pi_k^D - \pi_m^E$ decreases when n^D increases, and more than in the absence of resource dynamics.

6. Conclusions

This paper has studied the emergence of cooperation in a particular spatial CPR game, namely with space modeled as a circle. The combination of evolution, space and resource dynamics can lead to a complex model system that easily defies analytical solutions. Here we proposed a model that allowed derivation of various analytical results, while additional conjectures were supported by a large number of numerical simulations.

The major contribution of the present paper is that in the CPR game a cooperative strategy can survive, even when the majority of agents is defecting. This result runs counter to Sethi and Somanathan (1996). Our finding is due to the assumption that agents base their actions on the observed profitability of strategies employed by neigboring agents. In such a setting cooperators and enforcers can in some sense protect each other. By means of several types of simulations we were able to establish support for the view that cooperative equilibria are likely to persist, even in stochastically changing environments. Introducing resource dynamics reinforces our results.

From a conceptual perspective, the approach adopted here can be understood as combining local and global interactions. Virtually all related, analytical work in the literature has focused solely on local interactions, which evidently renders much simpler models. The global interactions in this case are due to two factors. First, profits are affected by aggregate harvest, to which all agents contribute. Second, profits depend on the resource stock, which changes due to the composition of harvesting strategies in the population of agents. The presence of global feedback means that profit rankings of strategies are not necessarily fixed over time. Indeed, due to changes in the composition of the population of strategies the aggregate harvest and resource stock change, which in turn may alter the conditions under which the agents interact. The important implication is that resource dynamics combined with spatial evolution increases the frequency of stable equilibria in which resource use is sustainable.

The analytical results apply mainly to the case without global interactions. The alternative case was illustrated by a combination of analytical results, illustrative examples and systematic numerical simulations. Evidently, future work might concentrate on extending the boundary of analytical findings.

Future research may be devoted to examining alternative redistribution schemes of the fines collected, at least if this is the interpretation given to the sanctions rather than damages incurred. It has been assumed thusfar that redistribution in lump sum. An alternative assumption would be that enforcers get some kind of compensation. Another item worth investigating in more detail is the distinction between a cooperator and a non-punishing enforcer. In the present approach the distinction cannot be made on the basis of actual behavior or payoffs. But for the analysis it does make a difference whether an agent is a cooperator or an enforcer. Therefore, this line of research would investigate the issue of signaling characteristics.

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Notes

1. One could be more specific by assuming for example that $X_P = nx_L < X_C \leqslant nx_H \leqslant X_0$.
2. When there are no enforcers in the population, defectors always earn more than cooperators and will spread quickly through the population. When there are no defectors in the population, cooperators and enforcers earn the same payoffs and stick to their strategies so that there is no further evolution of strategies.
3. Convergence to equilibria always occurred within 200 time steps.
4. For illustration purposes, we add the frequencies in all the extreme cases in which the initial population is composed of two strategies only.
5. With the given parameters, CE-equilibria do not occur.

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Appendix A. Proofs

Proof Lemma 1

(i) Since $Z(nx_{\mathrm{H}}) > 0$, it follows that $\gamma > \delta$ implies $x_{\mathrm{H}}Z(X)-\delta > x_{\mathrm{L}}Z(X)-\gamma$ for all $X \leqslant nx_{\mathrm{H}}$. Hence $\pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}}$ and, a forteriori, $\pi_2^{\mathrm{D}} > \pi_2^{\mathrm{E}}$. If $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < 2\delta - \gamma$ then $x_{\mathrm{H}}Z(X) - 2\delta < x_{\mathrm{L}}Z(X) - \gamma < 0$ for all $X \leqslant nx_{\mathrm{H}}$ and hence $\pi_1^{\mathrm{E}} > \pi_2^{\mathrm{D}}$. If $\gamma > \delta$ and $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < 2\delta - \gamma$ then $x_{\mathrm{H}}Z(X) - \delta < x_{\mathrm{L}}Z(X)$ for all $X \leqslant nx_{\mathrm{H}}$ and hence $\pi_0^{\mathrm{E}} > \pi_1^{\mathrm{D}}$.

(ii) If $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) > \delta - \frac{1}{2}\gamma$ then $x_{\mathrm{H}}Z(X) - \delta > x_{\mathrm{L}}Z(X) - \frac{1}{2}\gamma < 0$ for all $X \leqslant nx_{\mathrm{H}}$ implying $\pi_1^{\mathrm{D}} > \frac{1}{2}(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}})$. Moreover, the sanction rate is relatively low.

(iii) If $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < \delta - \frac{1}{2}\gamma$ then $x_{\mathrm{H}}Z(X) - \delta < x_{\mathrm{L}}Z(X) - \frac{1}{2}\gamma < 0$ for all $X \leqslant nx_{\mathrm{H}}$ implying $\pi_1^{\mathrm{D}} < \frac{1}{2}(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}})$. A forteriori $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(X) < 2\delta - \gamma$ for all $X \leqslant nx_{\mathrm{H}}$, so that the sanction rate is relatively low.

(iv) If $(x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{L}}) < \delta - \gamma$ then $x_{\mathrm{H}}Z(X) - \delta < x_{\mathrm{L}}Z(X) - \gamma$ for all $X \leqslant nx_{\mathrm{H}}$, implying $\pi_1^{\mathrm{E}} > \pi_1^{\mathrm{D}}$. Then also $\pi_2^{\mathrm{E}} > \pi_2^{\mathrm{D}}$ because $\delta > \gamma$. If $\delta - 2\gamma < (x_{\mathrm{H}} - x_{\mathrm{L}})Z(nx_{\mathrm{H}})$ then $0 < x_{\mathrm{H}}Z(X) - \delta > x_{\mathrm{L}}Z(X) - 2\gamma$ for all $X \leqslant nx_{\mathrm{H}}$, implying $\pi_1^{\mathrm{D}} > \pi_2^{\mathrm{E}}$.

Proof Proposition 2

(i) Number the positions on the circle clockwise. Put an enforcer on position 1 and, without loss of generality, a defector on position 2. Suppose $n = 2$. This is not an equilibrium because $\pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}}$. Suppose $n = 3$. This case is ruled out by lemma 3(iii) or lemma 3(iv). Suppose $n = 4$. At number 3 there is a defector in view of lemma 3(iv). At number 4 there is an enforcer in view of lemma 3(iii). But this cannot be an equilibrium because $\pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}}$. Suppose $n = 5$. At number 3 there is a defector in view of lemma 3(iv). At number 4 there is an enforcer in view of lemma 3(iii). At number 5 there is an enforcer because of lemma 3(iv). So the equilibrium candidate looks like: EDDEE. This is indeed an equilibrium. The defectors will remain defectors since $\pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}}$ and the enforcers will remain enforcers since $\pi_1^{\mathrm{D}} < \frac{1}{2}(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}})$. Next we show that the minimal length of an E-cluster is equal to three. Suppose there exists a DE equilibrium (with $n \geqslant 5$) with only two adjacent enforcers, surrounded by defectors: DEED. Then, because of lemma 3 we must also have DEEDD. This cannot be (part of) an equilibrium because $\pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}}$.

(ii) Consider a CDE configuration. Put the cooperator closest to a defector on position 1. Suppose the first defector is at number 2. This contradicts lemma 3(ii). Suppose the first defector is at number 3. There is an enforcer at number 2 by construction. This cannot be an equilibrium in view of lemma 3(i). Suppose the first defector is at number 4. There are enforcers at numbers 2 and 3 by construction. This cannot be an equilibrium because the enforcer at number 2 will turn into a cooperator since $\pi^{\mathrm{C}} > \frac{1}{2}(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}})$. Suppose the first defector is at number 5. At numbers 2, 3 and 4 there are enforcers by construction. There cannot be an enforcer at number 6 because of lemma 3(iv). There cannot be a cooperator at number 6 by construction. Hence is a defector at number 6. Because of symmetry there are enforcers at numbers 7, 8, and 9. It is easily verified that this is an equilibrium. Therefore the minimal number of players necessary for a CDE equilibrium is 9. Suppose there is a CDE equilibrium with a string ED. We cannot have CED in view of lemma 3(i), nor DED (lemma 3(iii)). So, we have a string EED. We cannot have DEED by the following reasoning. If the further extension could be written as DEEDD then this cannot be an equilibrium because $\pi_1^{\mathrm{D}} > \pi_1^{\mathrm{E}}$, implying that the second enforcer in the row turns into a defector. Lemma 3(ii) rules out the further extension DEEDC. And the extension DEEDE is not allowed in view of lemma 3(iv). Therefore, DEED cannot be part of an equilibrium. Consider, therefore, CEED. Again the further extension cannot be CEEDD, CEEDC or CEEDE. Hence we should have EEED. Therefore, the minimal string of enforcers is 3 if an enforcer is adjacent to a defector.

(iii) In a blinker an enforcer will never switch to defection, for the following reason. An enforcer next to a defector will switch to defection only if it punishes twice: with CED the enforcer switches to cooperation, and with EED the (second) enforcer stays an enforcer since $\pi_2^{\mathrm{D}} < \pi_1^{\mathrm{D}} < \frac{1}{2}(\pi_0^{\mathrm{E}} + \pi_1^{\mathrm{E}})$. Therefore, we must have DED. But the first defector will not switch to enforcement since $\pi_2^{\mathrm{D}} > \pi_2^{\mathrm{E}}$. It follows that DE blinkers do not exist. In a CDE blinker a cooperator will never switch to enforcement. Therefore, there should be a defector switching to enforcement. A necessary condition is that we have EDE. But the first enforcer will not switch to defection.

Appendix B. Average Frequencies of Equilibria for Different Sanction Levels

Table B.1. Average frequency of convergence for different sanction levels

sanction	D-equil.	DE-equil.	CE-equil.	CDE-equil.	CDE-equil. (blinking)	CDE-equil. (cycling)	E-equil.
140	1.00	0.00	0.00	0.00	0.00	0.00	0.00
160	1.00	0.00	0.00	0.00	0.00	0.00	0.00
180	1.00	0.00	0.00	0.00	0.00	0.00	0.00
200	0.99	0.00	0.01	0.00	0.00	0.00	0.00
220	0.99	0.00	0.01	0.00	0.00	0.00	0.00
225	0.99	0.00	0.01	0.00	0.00	0.00	0.00
230	0.99	0.00	0.01	0.00	0.00	0.00	0.00
235	0.60	0.34	0.01	0.05	0.00	0.00	0.00
240	0.40	0.39	0.01	0.20	0.00	0.00	0.00
245	0.34	0.36	0.01	0.29	0.00	0.01	0.00
250	0.32	0.34	0.03	0.28	0.00	0.03	0.00
260	0.32	0.34	0.03	0.28	0.00	0.03	0.00
270	0.32	0.35	0.03	0.28	0.00	0.03	0.00
280	0.32	0.33	0.04	0.29	0.00	0.02	0.01
290	0.32	0.33	0.04	0.29	0.00	0.02	0.01
300	0.32	0.33	0.04	0.29	0.00	0.02	0.01
310	0.31	0.33	0.04	0.29	0.00	0.02	0.01
320	0.32	0.33	0.04	0.29	0.00	0.02	0.01
350	0.30	0.34	0.04	0.29	0.00	0.02	0.01
360	0.30	0.33	0.04	0.30	0.00	0.02	0.01
370	0.30	0.34	0.04	0.30	0.00	0.02	0.01
500	0.12	0.34	0.06	0.45	0.01	0.01	0.01
700	0.10	0.44	0.07	0.33	0.02	0.01	0.02
750	0.09	0.45	0.07	0.33	0.03	0.01	0.02
800	0.00	0.01	0.78	0.00	0.01	0.00	0.20
900	0.00	0.00	0.78	0.00	0.01	0.00	0.21