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Endogenous preemption on both sides of a market

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Abstract

We study a market in which both buyers and sellers can decide to preempt and set their quantities before market clearing. Will this lead to preemption on both sides of the market, only one side of the market, or to no preemption at all? We find that preemption tends to be asymmetric in the sense that it is restricted to only one side of the market (buyers or sellers).

Keywords: Preemption; Endogenous timing

JEL classification: C72; D43; L11

1. Introduction

Starting with Saloner (1987) and Hamilton and Slutsky (1990), there has been a growing literature that analyzes endogenous timing in oligopolistic markets. Generally, these models allow for endogenous timing on the supply side of the market only (e.g., Anderson and Engers, 1992; van Damme and Hurkens, 1999; Matsumura, 1999). In this paper we analyze a simple model that allows for endogenous timing on both sides of the market. Both buyers and sellers can decide whether or not to preempt. The main question is what the pattern of preemption will be. Will there be preemption by both sellers and

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buyers? Will only one side of the market preempt? Will all traders on one side of a market preempt, or only a subset of traders? Or will there perhaps be no preemption at all?

We consider a homogeneous market with complete information so that there can be only one price. Quantities demanded and supplied are the decision variables which can be determined earlier or later. In order to prevent rationing we assume a competitive fringe which ensures market clearing. Each trader outside the competitive fringe is a flexible trader and can either precommit to a certain quantity (move early) or refrain from doing so (move late). In the latter case, the trader joins the competitive fringe and acts as a price taker. The assumption that only preempting traders act strategically is not innocuous. It implies that traders who move early consider how their quantity affects the price, while the other traders do not.

We solve the market equilibrium for given numbers of preempting buyers and sellers and analyze then the stable configurations of (numbers of) preempting traders. We find that an equilibrium in which both buyers and sellers preempt exists only if there is at most one flexible trader on each side of the market. In all other cases, the only equilibrium outcome is for all flexible traders on one side of the market to precommit and for all traders on the other side of the market to abstain.

2. The market model

Let $\mathcal{S}$, resp. $\mathcal{B}$, denote the set of sellers, resp. buyers on a homogenous market. The number of sellers (buyers) is denoted by $S(B)$ where S, $B \geq 2$. Each seller's payoff function is given by

$$\pi = \left(p - \frac{y}{2\gamma}\right)y \text{ with } \gamma > 0$$

where p denotes the market price and $y(\geq 0)$ the individual sales amount of a given seller. Each buyer's payoff function is

$$u = \left(\frac{\alpha}{\beta} - \frac{x}{2\beta} - p\right)x \text{ with } \alpha, \beta > 0$$

where $x(\geq 0)$ is a buyer's individual demand. These payoff functions imply individual supply functions

$$y = y^*(p) = \gamma p \tag{1}$$

and individual demand functions

$$x = x^*(p) = \alpha - \beta p. \tag{2}$$

To render the analysis tractable we set $\alpha = \beta = \gamma = 1$.

2.1. The preemption game

Can a non-empty subgroup of traders on each market side gain by precommitting to what they will trade? We consider a two-stage commitment game with observable delay and two production periods (Hamilton and Slutsky, 1990). It is assumed that all traders but one on each market side have flexibility in the timing of production. Thus there are $B-1$ flexible buyers who can choose to state their demand early (in period 1) or late (in period 2). Likewise, there are $S-1$ flexible sellers who can choose to produce early (in period 1) or late (in period 2). The inflexible traders on each market side represent the competitive fringe which guarantees market clearing. Without loss of generality, we assume that it is seller S (buyer B) who is inflexible.

The preemption game with observable delay has the following stages:

Stage 0: Flexible sellers and buyers choose the period (period 1 or 2) in which they set their quantities.
Stage 1: Flexible sellers and buyers, who chose period 1, decide about their quantity; others wait.
Stage 2: Flexible traders, who chose period 2, as well as the inflexible traders act as price takers and set their quantities competitively. The market clears and payoffs are realized.

2.1.1. Solution of stage 2

Assume that $s \leq S-1$ sellers and $b \leq B-1$ buyers are precommited. Let $Y(X)$ be the sum of quantities that the s committed sellers (b committed buyers) have chosen in period 1. In period 2 non-committed players (as well as the two "fringe" traders) choose their quantities competitively such that the market clears. Thus, using (1) and (2) it must hold that

$$Y + (S-s)p = X + (B-b)(1-p)$$

or

$$p = \frac{B-b+X-Y}{S-s+B-b}.$$

2.1.2. Solution of stage 1

Anticipating the results of the second stage, a committing seller's and buyer's payoff are

$$\pi(y) = \left(p - \frac{y}{2}\right)y = \left(\frac{Y-X-B+b}{s-S+b-B} - \frac{y}{2}\right)y$$

$$u(x) = \left(1 - \frac{x}{2} - p\right)x = \left(1 - \frac{x}{2} - \frac{Y-X-B+b}{s-S+b-B}\right)x$$

From $\frac{\partial}{\partial y}\pi = 0$ and $\frac{\partial}{\partial x}u = 0$ as well as from the obvious symmetry of the equilibrium one gets:[1]

$$y^{c}(s,b) = \frac{B(B+S-b-s+1)-b}{(B+S+1)(B+S-b-s+1)} \text{ if } s \geq 1$$

$$x^{c}(s,b) = \frac{S(B+S-b-s+1)-s}{(B+S+1)(B+S-b-s+1)} \text{ if } b \geq 1$$

and a market price of

$$p = \frac{B(B+S-b-s+1)-b}{(B+S+1)(B+S-b-s)}.$$

The individual sales quantity of a non-committed seller is equal to the price p, or

$$y^{nc}(s,b) = \frac{B(B+S-b-s+1)-b}{(B+S+1)(B+S-b-s)} \text{ if } s \leq S-2.$$

[1] Note that these payoff functions are strictly concave in y and x, respectively, such that the first-order conditions are sufficient.

The individual quantity of a non-committed buyer is

$$x^{\mathrm{nc}}(s,b)=\frac{S(B+S-b-s+1)-s}{(B+S+1)(B+S-b-s)}\ \text{if}\ b\le B-2.$$

The total quantity sold and bought is

$$\begin{aligned}&sy^{\mathrm{c}}+(S-s)y^{\mathrm{nc}}=bx^{\mathrm{c}}+(B-b)x^{\mathrm{nc}}\\&=\frac{(S(S+B-s-b+1)-s)(B(S+B-s-b+1)-b)}{(S+B+1)(S+B-s-b)(S+B-s-b+1)}.\end{aligned}$$

A committed seller earns

$$\pi^{\mathrm{c}}(s,b)=\frac{1}{2}\frac{(B+S-b-s+2)(B(B+S-b-s+1)-b)^2}{(B+S+1)^2(B+S-b-s+1)^2(B+S-s-b)}\ \text{if}\ s\ge 1$$

and a non-committed seller

$$\pi^{\mathrm{nc}}(s,b)=\frac{1}{2}\frac{(B(B+S-b-s+1)-b)^2}{(B+S-s-b)^2(B+S+1)^2}\ \text{if}\ s\le S-2.$$

A committed buyer earns

$$u^{\mathrm{c}}(s,b)=\frac{1}{2}\frac{(S(B+S-b-s+1)-s)^2(B+S-b-s+2)}{(B+S+1)^2(B+S-b-s+1)^2(B+S-b-s)}\ \text{if}\ b\ge 1$$

and a non-committed buyer

$$u^{\mathrm{nc}}(s,b)=\frac{1}{2}\frac{(S(B+S-b-s+1)-s)^2}{(B+S-b-s)^2(B+S+1)^2}\ \text{if}\ b\le B-2.$$

Note that $y^{\mathrm{c}}(s,b)<y^{\mathrm{nc}}(s,b)$ and $x^{\mathrm{c}}(s,b)<x^{\mathrm{nc}}(s,b)$. Due to the assumption of a homogeneous market, this implies $\pi^{\mathrm{nc}}(s,b)>\pi^{\mathrm{c}}(s,b)$ and $u^{\mathrm{nc}}(s,b)>u^{\mathrm{c}}(s,b)$. Hence, taking s and b as given, both, sellers and buyers, would prefer to be non-committed. However, when deciding whether or not to precommit, a trader cannot take s and b as given. If a seller (buyer) decides not to commit $s(b)$ will be reduced by 1. This simple fact determines the equilibrium values for s and b.

2.2. Precommitment in stage 0

With the help of the results above we can derive the equilibrium numbers b^* and s^* (with $0\le b^*\le B-1$ and $0\le s^*\le S-1$) of committing buyers and sellers. For an inner equilibrium, that is for $1\le s\le S-2$ and $1\le b\le B-2$ the following four conditions have to be satisfied:

Committed seller: $\pi^{\mathrm{c}}(s,b)\ge\pi^{\mathrm{nc}}(s-1,b)$ or

$$\frac{1}{2}\frac{B(B-2b)(B+S-b-s+2)+2b^2}{(B+S+1)^2(B+S-b-s+1)^2(B+S-b-s)}\ge 0$$

Non−committed seller: $\pi^{\mathrm{nc}}(s,b)\ge\pi^{\mathrm{c}}(s+1,b)$ or

$$-\frac{1}{2}\frac{B(B-2b)(B+S-b-s+1)+2b^2}{(B+S+1)^2(B+S-b-s)^2(B+S-b-s-1)}\ge 0$$

$$\text{Committed buyer:}\quad u^{\mathrm{c}}(s,b)\geq u^{\mathrm{nc}}(s,b-1)\text{ or }$$
$$\frac{1}{2}\frac{S(S-2s)(B+S-b-s+2)+2s^2}{(B+S+1)^2(B+S-b-s+1)^2(B+S-b-s)}\geq 0$$

$$\text{Non-committed buyer:}\quad u^{\mathrm{nc}}(s,b)\geq u^{\mathrm{c}}(s,b+1)\text{ or }$$
$$-\frac{1}{2}\frac{S(S-2s)(B+S-b-s+1)+2s^2}{(B+S+1)^2(B+S-b-s)^2(B+S-b-s-1)}\geq 0.$$

Since all denominators are strictly positive for $0\leq s\leq S-1$ and $0\leq b\leq B-1$, these four conditions are equivalent to

$$\text{Committed seller: } B(B-2b)(B+S-b-s+2)+2b^2\geq 0 \tag{3}$$

$$\text{Non-committed seller: } -B(B-2b)(B+S-b-s+1)-2b^2\geq 0 \tag{4}$$

$$\text{Committed buyer: } S(S-2s)(B+S-b-s+2)+2s^2\geq 0 \tag{5}$$

$$\text{Non-committed buyer: } -S(S-2s)(B+S-b-s+1)-2s^2\geq 0 \tag{6}$$

From these conditions we derive

Proposition. *The only equilibrium configurations* (s^*, b^*) *of the commitment game are:*

(i) If $S=B=2$ then $s^*=1$ and $b^*=1$, i.e., the two flexible traders (one on each side of the market) precommit.
(ii) If $S\geq 3$ or $B\geq 3$ then [$s^*=S-1$ and $b^*=0$] or [$s^*=0$ and $b^*=B-1$].

Proof. There are nine possible equilibrium configurations, with $s^*=0$, $1\leq s^*\leq S-2$ or $s^*=S-1$ and $b^*=0$, $1\leq b^*\leq B-2$ or $b^*=B-1$. The proof proceeds by checking these configurations. We illustrate this by checking three. The others follow along similar lines. First, to check whether there is an inner solution as defined above, note that adding inequalities (3) and (4) as well (5) and (6) yields the conditions $B(B-2b)\geq 0$ and $S(S-2s)\geq 0$. Thus, necessary conditions for an inner solution are $b\leq B/2$ and $s\leq S/2$. But for these restrictions on s and b it is straightforward that inequalities (4) and (6) cannot be satisfied. Thus, there is no inner equilibrium. Second, consider the possibility that no flexible trader precommits (i.e. $s=0$ and $b=0$). In this case conditions (4) and (6) have to be satisfied. They reduce to $-(B+S+1)B^2\geq 0$ and $-(B+S+1)\ S^2\geq 0$. These conditions are never fulfilled. Hence, there is no equilibrium in which no trader precommits. Finally, consider the possibility that all flexible traders precommit (i.e., $s=S-1$ and $b=B-1$): In this case conditions (3) and (5) have to be satisfied. They reduce to $-2(B^2-2B-1)\geq 0$ and $-2(S^2-2S-1)\geq 0$ for the committed sellers and buyers respectively. These conditions will be satisfied simultaneously if and only if $S\leq 2$ and $B\leq 2$. □

The Proposition states that if $S\geq 3$ or $B\geq 3$ then all flexible traders on one side of the market precommit while no trader on the other side of the market precommits. Traders who preempt set lower quantities than those who do not preempt. Preempting sellers raise the price; preempting buyers lower the price. The marginal benefit of an effectuated price change decreases with the quantity traded, however. If many traders on the other side of the market preempt the equilibrium quantity is low which discourages attempts to change the price by the other side of the market. Thus, preemption on one market side causes the other side to abstain (and vice versa).

3. Conclusion

We analyze endogenous preemption on both sides of a market and show that preemption tends to be restricted to one side of the market. Either the buyers or the sellers preempt, but not both sides of the markets at the same time. Also it is not an equilibrium for no trader to preempt.

To simplify matters we relied on a symmetric model with quadratic utility and cost functions. More crucial is our assumption that traders who do not preempt join the competitive fringe. This suggests an alternative interpretation of our model as one that endogenizes the number of strategic traders in a market. It could be interesting to analyze how results change when flexible traders, who do not preempt, act strategically rather than competitively.

References

Anderson, S., Engers, M.P., 1992. Stackelberg vs. Cournot oligopoly equilibrium. International Journal of Industrial Organization 10, 127–135.

Hamilton, J.H., Slutsky, S.M., 1990. Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. Games and Economic Behavior 2, 29–46.

Matsumura, T., 1999. Quantity setting oligopoly with endogenous sequencing. International Journal of Industrial Organization 17, 289–296.

Saloner, G., 1987. Cournot duopoly with two production periods. Journal of Economic Theory 42, 183–187.

van Damme, E., Hurkens, S., 1999. Endogenous Stackelberg leadership. Games and Economic Behavior 28, 105–129.