Optimal welfare and in-work benefits with search unemployment and observable abilities

Boone, J.; Bovenberg, A.L.

Published in:
Journal of Economic Theory

Publication date:
2006

Optimal welfare and in-work benefits with search unemployment and observable abilities*

Jan Boone[†] and Lans Bovenberg[‡]

Dept. of Economics

Tilburg University

P.O. Box 90153

5000 LE Tilburg

The Netherlands

Suggestion running title: In-work benefits and search

July 30, 2005

*We thank an anonymous referee for comments and suggestions.

†CentER, TILEC, Tilburg University, ENCORE, IZA and CEPR. email: j.boone@uvt.nl. Jan Boone gratefully acknowledges financial support from NWO, KNAW and VSNU through a Vernieuwingsimpuls grant.

‡CentER, NETSPAR, TILEC, Tilburg University and CEPR. email: a.l.bovenberg@uvt.nl

Abstract

This paper explores the optimal interaction between the tax system and social assistance in insuring people against the risks of involuntary unemployment and low ability. To that end, we introduce search unemployment in a model of optimal non-linear income taxation. The relationship between welfare benefits and the optimal level of in-work benefits is U-shaped. This explains why in-work benefits are called for both in countries that grant low welfare benefits and countries that provide high welfare benefits. An earned-income tax credit optimally induces all agents to look for work if job search is cheap and effective, agents are not very risk averse, and the least-skilled agents are relatively productive.

Key words: search, in-work tax benefits, unemployment compensation, redistribution, risk aversion.

Journal of Economic Literature Classification Numbers: H21, J64, J65

1. Introduction

In recent years many industrialized countries have employed tax policies to encourage unemployed persons to seek work. Following the example of the United States, several European countries have introduced or are considering in-work tax benefits in the form of an Earned Income Tax Credit (EITC). More generally, by lowering taxes on low-skilled work, governments increasingly stimulate low-skilled workers to look for jobs. These policies are part of the so-called 'welfare-to-work' strategy, where governments fight poverty by raising employment of low-skilled workers rather than by increasing welfare benefits for these workers.

To investigate the optimal interaction between tax policies and welfare payments, we introduce unemployment risk in a model of optimal non-linear income taxation in which agents feature heterogeneous abilities. In such a setting, social assistance is required to insure risk-averse agents against the risk of becoming unemployed. At the same time, by redistributing between high-skilled and low-skilled workers, a non-linear income tax protects these agents against the risk of being born without many skills. Indeed, inequities originate not only in heterogeneous abilities (as in [13]) but also in differences in employment status.

In introducing search unemployment in an optimal tax model, we focus on the case in which the government can verify a worker's ability. We adopt this informational assumption for four main reasons. First, it allows us to explore the arguments for in-work tax benefits in circumstances that are favorable to these tax subsidies. As argued below, in-work benefits are most attractive if the participation constraint rather than the incentive compatibility constraint for work effort is binding for high-ability agents. The second reason for our informational assumption is that this allows us to clearly identify how search incentives restrict the ability of the government to redistribute resources away from the most productive workers towards agents with lower consumption levels. Without the intensive decision margin (i.e. the selection of work effort after having found a job), only the search margin (i.e. the extensive decision margin or participation constraint) constrains redistribution. A third reason for assuming that types are observable is that it allows us to interpret high public benefits for low-ability households as disability benefits with which the government tags low-ability agents (see [1]). In this interpretation, the search costs are the information costs associated with the government verifying the ability level of these low-ability agents. The final reason for our informational assumption is that it simplifies the analysis and is a first step towards exploring a more complex model in which the intensive and extensive margins interact (see [4]). Indeed, our model is closely related to the small optimal tax literature that incorporates the endogenous decision to participate in the labor market but abstracts from endogenous labor supply on the intensive margin (see [5], [6] and [9]).

Our analysis is based on a special utility function in which leisure enters utility in a linear fashion. Whereas these preferences are more specific than the more general preferences in [5], [6], and [15], the additional structure on preferences allows for more analytical results. This sheds additional light on the determinants of the optimal tax schedule. Indeed, a substantial literature (see, e.g., [2], [7], [12], and [16]) has turned to quasi-linear preferences in leisure in order to obtain more intuition for the determinants of the optimal non-linear income tax. These preferences imply that a utilitarian government cares about the distribution of consumption rather than the distribution of work effort. Our approach is thus closely related to [11], who note that policy debates focus on raising consumption rather than reducing the work effort (or raising leisure) of the poor. [11], however, adopt a non-welfarist social welfare function. We, in contrast, continue in the welfarist tradition, but assume a special, quasi-linear utility function. A final reason for adopting this particular utility function is that it clearly illustrates the importance of the search margin in constraining the redistributive powers of the government. In the absence of the search margin, the government engages in extreme redistribution if it can observe a worker's ability: only the most skilled worker exerts work effort, as the distribution of work is determined by efficiency rather than equity considerations. We show that the search margin excludes this extremely redistributive policy.

We study issues similar to those explored by [15], who incorporates two labor-supply margins in an optimal tax model: namely, not only hours worked (the intensive margin) but also the participation decision (the extensive margin). Although the search margin in our model resembles the extensive margin in the model of Saez, and our model is close to the version of the Saez model in which work effort on the intensive margin is exogenous, our model differs from Saez' approach in two important respects. First, in the Saez model, agents voluntarily choose not to participate in the labor market. In our model, in contrast, agents are exposed to the risk of being involuntarily unemployed; agents thus face two risks: being born with low ability and being involuntarily unemployed. Second, our model, which puts more structure on the participation margin, is more explicit than [15] about the labor-market imperfections affecting the costs and effectiveness of labor-market search. This facilitates the welfare analysis of these imperfections and allows us to explore the optimal interaction between social assistance and non-linear income taxation as instruments to insure agents against the two risks of low ability and involuntary unemployment.

In our setting, we explore how the government can best address these two reasons for poverty (i.e. low skills and involuntary unemployment). Should the government rely mainly on welfare payments rather than on in-work benefits, or should it offer in-work tax benefits that exceed welfare benefits? Generous social assistance helps those who are poorest (i.e. the unemployed), but harms incentives to look for jobs. In-work benefits do not suffer from this latter drawback, but are less well targeted at those most in need. Moreover, in-work benefits for low-ability agents distort the hours that agents choose to work on the intensive margin by making it more attractive for high-ability agents to mimic lower ability agents. If the government can observe an agent's ability, however, the incentive compatibility constraint on hours worked can be ignored and the participation constraint (i.e. extensive margin) is binding also for high-ability types. We show that in that case (in which the incentive compatibility constraint is not binding), in-work benefits tend to be generous compared to welfare benefits. In fact, the government may find it optimal to offer in-work benefits that exceed optimal welfare benefits, even though the unemployed are poorer than agents with a job. The reason is that social assistance is a relatively ineffective way to fight poverty because it benefits not only the poorest but also the richest agents. In particular, if welfare benefits are increased, the government must reduce taxes on the most efficient agents in order to prevent these high-ability workers from leaving the labor market. The associated adverse distributional effect of reducing taxes for the richest agents may outweigh the benefit of alleviating poverty among the unemployed.

We also explore the case in which the government can optimize only the tax system and has to take the level of social assistance as given. Indeed, in practice, taxes and social assistance are often set by different agencies based on rather distinct considerations. Alternatively, one can view our analysis as exploring how the tax system can be employed to address the possibly sub-optimal aspects of social assistance. The relationship between exogenous social assistance and optimal in-work benefits appears to be U-shaped. As welfare benefits are raised from a low initial level, these benefits absorb the budgetary room for generous in-work benefits as an instrument to fight poverty. At low levels of social assistance, therefore, welfare benefits and in-work benefits are substitutes in fighting poverty. As welfare benefits are increased further, however, the participation constraint for marginal workers becomes binding and the government needs to raise in-work benefits to draw people out of unemployment into work. At high levels of social assistance, therefore, in-work benefits and social assistance become complements: in-work benefits help to offset the impact of higher social assistance benefits on the participation constraint. This U-shaped relationship between in-work benefits and welfare payments reveals that generous in-work benefits are called for in countries with low welfare benefits and in those with high welfare benefits, but for different reasons. In countries with relatively low levels of social assistance (such as the United States), in-work benefits are aimed at poverty alleviation. In countries with generous social assistance (such as most Western European countries), in contrast, in-work benefits protect the incentives to participate in the labor market. The distinct comparative static effects of larger welfare benefits correspond to a distinction between so-called low welfare and high welfare economies, both of which feature voluntary unemployment but for different reasons.

The rest of this paper is structured as follows. After section 2 formulates the model, section 3 sets up the optimization problem for the government and discusses the optimality conditions. Section 4 introduces two benchmark cases in which either both the search and work-effort margins constrain distribution or neither does. This sets the stage for the main analysis in which only the search margin limits redistribution. Under this informational assumption, section 5 explores optimal tax policy if welfare benefits b are set exogenously. We can interpret this case as the tax authorities optimizing the tax system, taking the welfare system as given. The case in which the government can simultaneously optimize the tax and social assistance systems is investigated in section 6. This section shows that a low welfare economy can emerge in this setting, thereby generalizing a result in [5]. Section 7 concludes. All proofs are contained in the appendix.

2. Model

Consider an economy with agents who feature heterogeneous skills. A worker of ability (or skill) n working y hours (or providing y units of work effort) supplies ny efficiency units of homogeneous labor. With a linear production function featuring a constant unitary labor productivity, these efficiency units are transformed into the same number of units of output. We select output as the numeraire. Hence, the before-tax wage per hour is given by the exogenous parameter n. Overall gross output (or gross income) $z(n)$ amounts to $z(n) \equiv ny(n)$. The density of agents of ability n is denoted by $f(n)$, while $F(n)$ represents the corresponding cumulative distribution function. The support of the distribution of abilities is given by $[n_0, n_1]$.

Workers exhibit homogeneous tastes. In particular, they share the following quasi-linear utility function over consumption x and hours worked (or work effort) y :

$$u(x, y) = v\left(x\right) - y,$$

where $v'\left(x\right) > 0, v''\left(x\right) < 0$ for all $x \geq 0$ while $\lim_{x \downarrow 0} v'\left(x\right) = \infty$ and $\lim_{x \longrightarrow +\infty} v'\left(x\right) = 0$. The concavity of $v(.)$ implies that agents are risk averse and thus want to obtain insurance against the risks of unemployment and low earning capacity n. The specific cardinalization of the utility function affects the distributional preferences of a utilitarian government. In particular, the concavity of $v(.)$ implies that a utilitarian government aims to fight poverty of both unemployed and low-ability agents.

As in [2], [7], [12] and [16], work effort y enters the utility function in a linear fashion. This has two major consequences. First, consumption x is not affected by income effects. A higher average tax rate thus induces households to work more rather than to cut consumption. Second, a utilitarian government cares only about overall work effort and not about the distribution of work effort over the various agents. Such a government thus aims at an equal distribution of consumption (i.e. the alleviation of poverty) rather than an equal distribution of work effort and welfare.

Instead of working with work effort $y(n)$ and consumption $x(n)$ as the instruments of the worker, we find it more convenient to write the utility function in terms of gross income (or output) $z(n) \equiv ny(n)$ and net income (or consumption) $x(n)$. Utility of type n is then written as $u(n) \equiv v\left(x(n)\right) - z(n)/n$.

Agents have to search for a job. In particular, by searching with intensity $s \in [0, 1]$, agents find a job with probability s.[1] Search costs $\gamma(s)$ are given by

$$\gamma(s) = \begin{cases} \gamma s & \text{if} \quad s \in [0, \bar{s}] \\ +\infty & \qquad \text{otherwise,} \end{cases}$$

where $\bar{s} < 1$ captures the idea that agents can fail to find a job, even if they search at full capacity $\bar{s}$. If an agent does not succeed in finding a job, (s)he receives a welfare benefit $b \geq 0$, which the agent takes as given. Since the government cannot observe the ability of unemployed agents, the welfare benefit does not depend on n. An agent of ability n thus selects search

[1]This formalization of the search margin in a static framework is similar to the one-period search model in [8].

intensity s to maximize expected utility

$$U(n) = \max_{s} \left\{ -\gamma(s) + su(n) + (1-s)\, v(b) \right\}.$$

Substituting the search cost function $\gamma(s)$ introduced above, one can easily verify that the optimal choice of s for type n amounts to

$$s(n) = \begin{cases} 0 & \text{if} \quad u(n) < \gamma + v(b) \\ \bar{s} & \text{if} \quad u(n) \geq \gamma + v(b). \end{cases} \tag{1}$$

The government has to satisfy its budget constraint, which states that aggregate tax revenues should equal the sum of aggregate welfare benefits and exogenously given exhaustive government expenditure g :

$$\int_{n_0}^{n_1} f(n)\, s(n)\, T(n)\, dn = b \int_{n_0}^{n_1} f(n)\,(1 - s(n)) dn + g, \tag{2}$$

where $T(n) = z(n) - x(n)$ denotes the tax paid by type n if this type finds work.

The utilitarian government maximizes ex-ante expected utility (i.e. expected utility before ability and labor market status have been revealed):

$$\max_{s(.), z(.), x(.)} \int_{n_0}^{n_1} f(n) \left[-\gamma s(n) + s(n)\, u(n) + (1 - s(n))\, v(b) \right] dn. \tag{3}$$

In optimizing the objective function, the government is able to verify a person's ability n after that person has found a job. As long as a person remains unemployed, however, the government cannot observe the ability of that person. By finding a job, a person thus reveals his ability. Hence, in contrast to social assistance, the tax system can discriminate across types.

3. The optimal tax problem

This section introduces the main ingredients for characterizing the optimal tax schedule if the government cannot observe agents' search effort but is able to observe the ability of workers.

We first rewrite the government's optimization problem by using two observations. First, due to the linearity of the cost function $\gamma(s)$, an agent should either search full time $\bar{s}$ or not at all (i.e. $s = 0$). Second, the highest ability types should work and therefore search for a job because these types are most efficient and thus feature the lowest labor costs. These two observations imply that the government selects a marginal type $n_s \in [n_0, n_1]$ such that types $n < n_s$ do not search, while all types $n \geq n_s$ search at full capacity. We can thus formulate the optimization problem of the utilitarian government as follows:

$$\begin{aligned}
&\max_{n_s, x(.), z(.)} F(n_s)\, v(b) + [1 - F(n_s)] \left(-\gamma\bar{s} + (1-\bar{s})\, v(b)\right) + \\
&+ \int_{n_s}^{n_1} \left\{ \begin{array}{c} \bar{s}\left(v(x(n)) - \frac{z(n)}{n}\right) f(n) + \lambda_E f(n)\bar{s}\,[z(n) - x(n)] \\ -\eta(n)\left(\gamma - v(x(n)) + \frac{z(n)}{n} + v(b)\right) \end{array} \right\} dn \\
&-\lambda_E \left\{ b\,[F(n_s) + (1 - F(n_s))(1-\bar{s})] + g \right\},
\end{aligned} \tag{4}$$

with complementary slackness conditions for the participation constraint

$$\begin{aligned}
v(x(n)) - \frac{z(n)}{n} &\geq v(b) + \gamma \text{ and } \eta(n) \geq 0, \\
\eta(n)\left(\gamma - v(x(n)) + \frac{z(n)}{n} + v(b)\right) &= 0
\end{aligned}$$

for all types $n \geq n_s$ whom the government wants to search and

$$\begin{aligned}
\int_{n_s}^{n_1} f(n)\bar{s}\,[z(n) - x(n) + b]\, dn &= b + g \text{ and } \lambda_E \geq 0, \\
\lambda_E \left(\int_{n_s}^{n_1} f(n)\bar{s}\,[z(n) - x(n) + b]\, dn - b - g \right) &= 0
\end{aligned}$$

for the government budget constraint. Here, $\eta(n)$ and λ_E represent the Lagrange multipliers for the participation constraint and the government budget constraint, respectively.

4. Setting the stage: two benchmark cases

This section explores two benchmark cases that set the stage for our main results in which the government can observe ability but not search. In particular, after investigating the case in which neither the search margin nor the intensive decision margin (i.e. the selection of work effort y after having found a job) limits redistribution, we explore the case in which both these margins constrain the ability of the government to redistribute resources.

4.1. No search

In order to understand the impact of the search margin on the optimal tax system, this subsection explores the case in which the search margin is absent (i.e. $\eta(n) = 0$). In that case, all work should be performed by the most productive type n_1 so that marginal production costs λ_E are determined by the marginal labor costs of this type (i.e. $\lambda_E = 1/n_1$).[2] All types feature the same consumption level $\bar{x}$ determined by $v'(\bar{x}) = \lambda_E = 1/n_1$. Intuitively, given the linear specification of the disutility of work effort $c(y) = y$, efficiency rather than equity considerations determine the distribution of work effort. Hence, all labor should be performed by the most efficient workers (i.e. the workers of type n_1). In contrast to the distribution of work effort, the distribution of consumption affects social utility in view of the concave nature of the utility of consumption $v(x)$. In particular, the government prefers an equal distribution of consumption, with the uniform consumption level being determined by the costs of supplying labor by the most efficient type n_1 (i.e $v'(\bar{x}) = 1/n_1$). Whereas the distribution of consumption is thus equal, the distribution of utility is highly unequal, as the most efficient workers perform

[2] To see why this holds, note that work effort $z(n)$ enters the optimization problem (4) in a linear way. If search is observable (as well as type), the Lagrange multiplier $\eta(n) = 0$ for all types n. Hence, it becomes optimal to concentrate all production where it is cheapest (i.e. at type n_1, where the marginal cost of production is only $\frac{1}{n_1}$).

all of the work. Indeed, the government levies a highly progressive tax $T(n_1)$ on these types, inducing these workers to produce sufficient resources for providing a consumption level $\bar{x}$ to all other agents. The non-negative tax level $T(n_1)$ is determined (from the government budget constraint (2)) by the costs of granting tax subsidies (i.e. $-T(n) = \bar{x}$) to all types $n < n_1$.

To exclude this extreme solution, this paper assumes that the government cannot verify search. In that case, the search margin prevents the government from shifting all work effort to the most productive type. In particular, having the most skilled type perform all work effort would discourage this type from looking for such a demanding job. In this way, the participation constraint $u(n) \geq v(b) + \gamma$ prevents the government from exploiting the most efficient types through an extremely redistributive tax system. Hence, work effort must be distributed more equally over the population.

4.2. Unobservable skills

In order to understand the conditions under which a search subsidy is optimal, we consider the case in which the government cannot verify working agents' skills. The optimal tax structure for this case is analyzed in [4], but here we are interested only in the sign of $T(n) + b$. The following result shows that search is necessarily taxed if the government cannot observe workers' ability.

Proposition 1 *If the government cannot observe workers' ability and b is optimally set, then $n_s > n_0$ implies that*

$$T(n) + b > 0$$

for each $n \geq n_s$.

If the government cannot observe workers' ability, then any feasible tax system has the property that in-work utility rises with ability (i.e. $u'(n) > 0$). For all non-marginal workers

$n > n_s$, $u(n_s) \geq \gamma + v(b)$ together with $u'(n) > 0$ implies that $u(n) > \gamma + v(b)$ for $n > n_s$, so that the participation constraint is binding only for the marginal worker n_s. For the non-marginal workers, the incentive compatibility constraint for work effort rather than the participation constraint is binding.

The reason why search should be taxed (i.e. $T(n) + b \geq 0$) can be stated as follows. The participation constraint $u(n) \geq \gamma + v(b)$ implies that consumption of workers exceeds the consumption of the unemployed. Hence, compared to in-work benefits, social assistance is a more effective way to fight poverty because welfare benefits accrue to the poorest agents. Since the participation constraint is binding only for the marginal worker n_s, the only efficiency cost of more generous welfare is that it harms the incentives of marginal workers to search for a job. In the optimum, therefore, the government balances the positive marginal distributional benefits of social assistance with positive marginal efficiency costs. Accordingly, the marginal external effect on the government budget constraint of the marginal worker n_s looking for a job should be positive (i.e. $T(n_s) + b \geq 0$). Non-marginal workers are taxed even heavier (i.e. $T(n) > T(n_s)$ for $n > n_s$) in view of the redistributive preferences of the government, so that the tax on search, $T(n) + b$, is positive for all workers. Below we show that this result no longer holds if the government has information about workers' skill.

5. The optimal tax system with search and observable types

This section turns to the case in which the government can observe ability but not search and faces an exogenous welfare benefit b.[3] An important element in characterizing the solution for

[3]For this exercise to be useful, g should not be so high that the government budget constraint cannot be met even without any welfare benefits (i.e. $b = 0$). Definition 2 in the appendix formally defines the upperbound on g such that positive welfare benefits can be financed. Throughout this paper g is assumed to lie below this

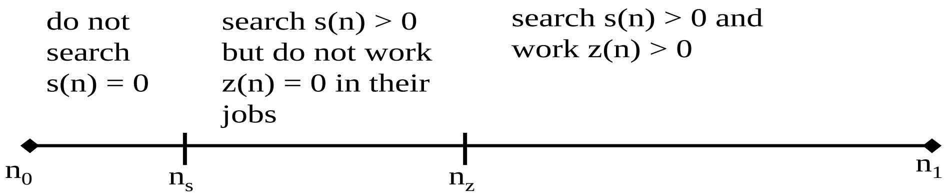

Figure 1: *Form of solution to optimal tax problem.*

the optimal tax problem is the least efficient type $\hat{n}$ that searches for a job (i.e. $u(\hat{n}) = v(b)+\gamma$) without a net search subsidy (or subsidy on work). This type $\hat{n}$ satisfies the following equation:

$$v\left(x\left(\hat{n}\right)\right) - \frac{x\left(\hat{n}\right) - b}{\hat{n}} = v\left(b\right) + \gamma, \tag{5}$$

where $x\left(\hat{n}\right)$ is the optimal consumption level for type $\hat{n}$ in the absence of taxes and subsidies:

$$x\left(\hat{n}\right) = \arg\max_{x} \left\{ v\left(x\right) - \frac{x}{\hat{n}} \right\}.$$

Hence $z\left(\hat{n}\right) = x\left(\hat{n}\right) - b$ and thus (using $z\left(n\right) = x\left(n\right) + T\left(n\right)$) $T\left(\hat{n}\right) + b = 0$. Definition 1 in the appendix provides a more formal characterization of $\hat{n}$. Types $n > \hat{n}$ search for a job even if the government does not subsidize search. Less efficient types $n < \hat{n}$, in contrast, must be paid a search subsidy to induce them to look for a job.

The solution to the optimal tax problem (4) takes a form as shown in figure 1. In particular, a type $n_s \geq n_0$ exists such that the lowest skills $n \in [n_0, n_s]$ do not search for a job (i.e. $s\left(n\right) = 0$) and draw social assistance. Another type $n_z \geq n_s$ exists such that skills $n \in \langle n_s, n_z]$ search for a job (i.e. $s\left(n\right) = \bar{s}$) but do not work in their jobs (if they find one), i.e. $z\left(n\right) = 0$. They enjoy a positive government transfer $-T(n) > 0$ financing the consumption level $x\left(n\right) = \bar{x}$, where $\bar{x}$ is the first-best consumption level for type n_z, i.e. $\bar{x} = \arg\max_x \left\{ v\left(x\right) - \frac{x}{n_z} \right\}$.[4] Finally, the highest skills $n \in \langle n_z, n_1]$ search for a job (i.e. $s\left(n\right) = \bar{s}$) and work positive hours if they find a job. In particular, production $z\left(n\right)$ for these types is chosen such that these skilled workers

upperbound.

[4] $-T\left(n\right) > b$ for these types $n \in \langle n_s, n_z]$ because $v\left(\bar{x}\right) \geq v\left(b\right) + \gamma$ and positive search costs ($\gamma > 0$) imply $\bar{x} > b$ and hence $T\left(n\right) + b = -\bar{x} + b < 0$.

are indifferent between searching for a job and staying unemployed:

$$z(n) = n\left[v(x(n)) - v(b) - \gamma\right],$$

where $x(n)$ is determined by the first-best outcome:

$$x(n) = \arg\max_{x}\left\{v(x) - \frac{x}{n}\right\}.$$

We can now formulate the solution to the optimization problem (4) with an exogenous level of social assistance b as follows.

Proposition 2 *For given g and density function $f(.)$ on $[n_0, n_1]$, values $\underline{b}, \bar{b}$ exist with $0 \leq \underline{b} \leq \bar{b} < +\infty$ such that for $b \in [0, \underline{b}\rangle$, the optimal tax system results in a so-called low welfare [LW] economy with $n_s = n_0, n_z > \hat{n}$ and $T(n_z) + b > 0$;*
for $b \in [\underline{b}, \bar{b}]$, the optimal tax system results in a so-called high welfare [HW] economy with $n_s \in [n_0, \hat{n}], n_z = \hat{n}$ and $T(n_z) + b = 0$;[5]
and for $b > \bar{b}$ the government budget constraint cannot be financed.

We first discuss the properties of the solution that hold in both the *low welfare* [LW] and *high welfare* [HW] economies before we turn to the properties that differ across these two economies. In both the low welfare and high welfare economies, a set of agents optimally searches for a job (and pays search costs in the process), even though these agents do not exert any effort in their jobs (i.e. $z(n) = 0$).[6] The reason why costly search for a job is optimal even though workers do not produce anything is that workers reveal their ability if they find

[5]The choice of the interval $[n_s, n_z\rangle$ in the solution for the high welfare economy above is somewhat arbitrary. Any other (set of) subinterval(s) of $[n_0, n_z\rangle$ with the same mass of agents is a solution as well.

[6]The reader may wonder why the government cannot observe the ability of unemployed agents but does observe the ability of employed agents working zero hours. In fact, these latter agents work $\varepsilon > 0$ hours, but the government minimizes ε so that it can just uncover a worker's type. We assume, in fact, that ε is infinitely small.

a job. This additional information has social value because it allows the government to better target its policy instruments aimed at poverty alleviation at the least skilled, most deserving, types by providing in-work benefits that are explicitly aimed at these types. In-work benefits are a relatively efficient means to fight poverty because, in contrast to social assistance, these benefits do not harm the search incentives of the more skilled types and therefore do not reduce the rents that can be extracted from the high skilled. Jobs are thus an effective way to get less skilled people out of poverty even though these agents do not produce anything in their jobs. The relative efficacy of in-work benefits as a poverty-fighting instrument compared to welfare benefits explains why a search subsidy (*i.e.* $T(n)+b<0$) is optimal for some people (although these people do not provide work effort if their job search is successful). This search subsidy resembles an EITC: agents collect higher public transfers in work than in unemployment. In other words, the marginal tax rate on searching for a job is negative.

We can interpret the jobs in which agents do not produce anything (i.e. $z(n)=0$) as disability insurance.[7] In this interpretation, the search costs correspond to the costs of uncovering the information about the ability level of low-ability agents $n<n_z$. Indeed, it does not matter for the optimal allocation whether the costs of tagging these agents are paid by the government directly or indirectly (i.e. by having to pay agents a sufficiently high in-work benefit to induce these agents to search for a job). We thus can interpret the search subsidies as disability benefits. In that interpretation, it seems most natural to provide these benefits to the agents with the lowest ability rather than to the more able agents $[n_s, n_z\rangle$.

5.1. The low welfare economy

In a low welfare economy, the government can afford to pay relatively generous search subsidies to all agents who do not search for a job without such a subsidy. Instead of welfare benefits,

[7]Another interpretation of these jobs is workfare for agents who are not productive enough to earn high wages in the private sector. High-ability agents are not eligible for these jobs.

therefore, an EITC insures agents against the risk of being born with low ability.[8] Jobs (with in-work benefits) rather than welfare benefits are thus the preferred route out of poverty due to low productivity. Social assistance insures people only against the risk of remaining unemployed after having actively searched for a job. This risk of involuntary unemployment (i.e. $1 - \bar{s}$) is the same for all skills. Indeed, social assistance is paid only to the population share $1 - \bar{s}$ that is involuntarily unemployed (i.e. those unfortunate agents who looked for a job and thus paid search costs, but nevertheless did not find one).

In the low welfare economy, the government budget enjoys a surplus if the government extracts the participation rents from all types that search without a net subsidy $n > \hat{n}$ $(z(n) = n(v(x(n)) - \gamma - v(b)))$, while at the same time providing not only welfare benefits to the part of the population $1 - \bar{s}$ that has not found work but also tax subsidies in the form of an EITC that allow all other types who found work to enjoy the same consumption level as the marginal worker $\hat{n}$ without having to exert any work effort (i.e. $-T(n) = x(\hat{n}) > b$ for $n < \hat{n}$). What is the optimal strategy for spending this surplus at a given level of welfare b? In view of the concavity of $v(.)$, the marginal utility of consumption is highest for the marginal worker $\hat{n}$ (and for all the types $n < \hat{n}$ who benefit from net search subsidies $-(T(n) + b) > 0$)). The government would thus like to employ the surplus to raise the consumption level of this worker. However, providing the marginal worker $\hat{n}$ with more consumption reduces the marginal utility of consumption $v'(x(\hat{n}))$ below the marginal utility costs of providing labor by this type, $1/\hat{n}$. Accordingly, the marginal worker $\hat{n}$ is better-off if the government reduces his work time $z(\hat{n})$ to zero instead of giving him more consumption. The government therefore utilizes the surplus

[8] In a low welfare economy, those born with low ability are better off (in terms of utility) than the more skilled types, but enjoy lower consumption levels. In a high welfare economy, in contrast, insurance against low ability is less effective since some agents with low ability do not benefit from generous in-work benefits and must rely instead on relatively low unemployment benefits instead. This still makes them as well-off (in terms of utility) as the most skilled.

to increase not only the consumption level $\bar{x}$ for all types that exert no work effort (by raising the EITC and thus the search subsidy $-(T(n)+b)$), but also the number of fortunate types who do not have to exert effort in their jobs. Hence, the surplus is spent by both raising the level of the search subsidy and the number of people who are eligible for it. The government thus pays search subsidies also to people who would search for a job even without search subsidies. As a direct consequence, the marginal worker is taxed on search (i.e. $T(n_z) + b > 0$ as shown in proposition 2). Intuitively, the government can afford to distort the search margin for the marginal worker n_z because all agents search (i.e. n_s has a corner solution). Indeed, the search margin is not binding for the marginal worker.

The government continues to spend its surplus in this way until the government budget constraint holds with equality. When spending the surplus, the government ensures that the marginal utility of consumption equals the marginal labor costs for the lowest type that exerts effort (i.e, $v'(\bar{x}) = \frac{1}{n_z}$ where n_z is the marginal worker).

5.2. The high welfare economy

In the high welfare economy, the government cannot afford to pay search subsidies to all types $[n_0, \hat{n}\rangle$ who are not productive enough to look for a job without search subsidies. As a result, a subset of these types $[n_0, n_s\rangle$ is not paid in-work tax benefits, so they do not search and remain unemployed. In a high welfare economy, therefore, the tax system (by paying in-work tax benefits in the form of an EITC) cannot fully protect less able agents against poverty. Also social assistance plays a role in insuring agents against the risk of being born with low ability n.

The subset $[n_s, n_z\rangle$ who are paid an EITC, $-T(n) > b$, without having to exert any effort in their jobs enjoy the highest utility level of all types.[9] Since $v(x(n)) \geq v(b) + \gamma$ for all

[9] The utility level of the low-ability agents who do not search $[n_0, n_s\rangle$ corresponds to that enjoyed by the

$n \geq n_s$, we have that $x(n) > b$ for $n \in [n_s, n_z\rangle$. Substituting the Taylor expansion $v(b) = v(\bar{x}) + v'(\bar{x})(b - \bar{x}) + 1/2v''(\xi)(b - \bar{x})^2$ (and noting that $v'(\bar{x}) = 1/\hat{n}$)) into (5), we find that $\gamma = -1/2v''(\xi)(\bar{x} - b)^2$. Hence, in deciding whether to provide one more search subsidy and thus relying more on the tax system rather than welfare benefits in fighting poverty, the government faces a trade-off between saving on the costs of search γ and fighting poverty by reducing the costs of unequal consumption levels (which are directly related to the concavity of utility $v(.)$). Large search costs γ imply that the agents have to pay relatively large costs to reveal their type by finding a job.[10] This makes the tax system a relatively expensive instrument to redistribute resources: relatively large search subsidies are required to induce agents to reveal their type. Encouraging more agents to search thus becomes less attractive. A concave utility function, in contrast, implies that unequal consumption levels $\bar{x}$ and b become costly. Hence, the government finds it attractive to fight poverty by taking people out of unemployment by paying them a relatively high in-work tax benefit $\bar{x} = -T(n) > b$.

For the marginal worker n_z, the in-work benefit corresponds to the welfare benefit so that the search subsidy goes to zero (i.e. $T(n_z) + b = 0$ in proposition 2). Search thus does not impose any first-order welfare effects, as the government pays this worker the same in unemployment and work: search is neither taxed nor subsidized. In other words, the marginal tax rate on search is zero and the search margin is not distorted. In contrast to the case without observable ability (see Proposition 1), the government can afford to eliminate the search distortion because it does not have to worry about generous in-work benefits for less skilled workers distorting the

most productive types $\langle n_z, n_1]$ who have found jobs in which they exert positive work effort. The consumption level of the unemployed, however, is lower than that of the more productive types who have found jobs. In terms of utility, the higher consumption level of these latter types is balanced by higher work effort and search costs.

[10] Lemma 9 in the appendix shows that, in the absence of search costs (i.e. $\gamma = 0$), all agents should search (i.e. $n_s = n_0$). Accordingly, the tax system (rather than unemployment benefits) insures people against the risk of being born with low ability.

intensive margin of more skilled workers.

5.3. Welfare and in-work benefits

This sub-section explores how the optimal tax system responds to exogenous changes in the welfare benefit b.

Proposition 3 *An increase in the welfare benefit b yields the following effects:*
[LW] for $b \in [0, \underline{b}\rangle$, $\frac{dn_z}{db} < 0, \frac{dT(n)}{db} > 0$ *for* $n \in [n_s, n_z\rangle$ *with* $n_s = n_0$,
[HW] for $b \in \langle \underline{b}, \bar{b}\rangle$, $\frac{dn_z}{db} > 0, \frac{dn_s}{db} > 0, \frac{dT(n_z)}{db} = -1$ *and* $\frac{dT(n)}{db} < 0$ *for* $n \in \langle n_s, n_z\rangle$.

In a low welfare economy, higher social assistance raises the number of workers exerting positive effort ($\frac{dn_z}{db} < 0$). The reason is the following. Higher welfare benefits impose two additional burdens on the government budget. First, they raise the expenditures on social assistance for the share of the population $(1 - \bar{s})$ that is involuntarily unemployed. Second, they reduce the tax revenues that can be extracted from the most skilled agents, as these types now find it more attractive to rely on the higher welfare benefits rather than searching for a job. These two additional budgetary burdens reduce the resources that can be spent on in-work tax benefits. Hence, higher welfare benefits reduce both the level of these search subsidies (thereby reducing the consumption of the least efficient agents) and the number of people who are eligible for these search subsidies (thereby increasing work effort for marginal workers). At the same time, in response to their lower average tax burden, the more efficient types who already exerted work effort reduce that work effort. Work effort is thus spread out more equally over various workers. Intuitively, by tightening the participation constraint of the most efficient workers, more generous welfare benefits allow for less redistribution through the tax system. Indeed, a higher welfare benefit pushes the low welfare economy away from the economy without a search margin (discussed in section 4.1) in which only the most productive worker provides positive work effort.

In a high welfare economy, in contrast, higher welfare benefits reduce the number of agents who work positive hours (i.e. $\frac{dn_z}{db} > 0$). Intuitively, higher social assistance raises the productivity requirements for workers who search without a search subsidy: with a better alternative option (namely not searching and collecting a higher welfare benefit), fewer workers are inclined to search for a job in the absence of search subsidies $-T(n) + b > 0$. The lowest type that searches for a job without a work subsidy must thus become more skilled. Since the marginal worker is more productive, marginal resource costs decline (i.e. $\frac{d\lambda_E}{db} = \frac{d\left(\frac{1}{n_z}\right)}{db} < 0$). These lower resource costs raise the in-work benefits of those who collect search subsidies (i.e. $\frac{dx(n)}{db} = \frac{dx(n_z)}{db} > 0$ for $n \in \langle n_s, n_z]$).

Combining these comparative static results for the low welfare and the high welfare economies, we find that the population that exerts work effort is largest at the point where a low welfare economy turns into a high welfare economy. Starting from a low welfare economy, a higher welfare benefit first reduces the population that can be allowed to enjoy leisure (by paying them generous in-work benefits). Eventually, a higher welfare benefit exhausts the surplus that can be spent on providing search subsidies to agents who would not search without being paid such a search subsidy. At that point, the low welfare economy turns into a high welfare economy. An even higher welfare benefit raises the productivity requirements for the worker that is indifferent to searching for a job in the absence of a search subsidy. This causes a drop in the number of workers who exert positive work effort. Accordingly, the relationship between the welfare benefit and the number of people who do not exert work effort is U-shaped.

Also the relationship between in-work benefits and welfare benefits is U-shaped. At low levels of social assistance, welfare benefits and the EITC are substitutes in fighting poverty because higher welfare benefits absorb the budgetary room for generous in-work benefits as an instrument to fight poverty. At high levels of welfare, however, the participation constraint for marginal workers is binding and higher welfare benefits induce the government to raise in-work benefits for marginal workers and to reduce taxes on more efficient workers in order to induce the latter agents to continue to look for work. In-work benefits thus help to offset the impact of higher welfare benefits on the participation constraint so that in-work and welfare benefits are complements. The relationship between in-work benefits and social assistance payments explains why generous in-work benefits are called for in both countries that grant low and countries that provide high welfare benefits. In countries with low levels of social assistance (such as the United States), in-work benefits are used to alleviate poverty. In countries with high levels of welfare (such as most Western European countries), in contrast, in-work benefits protect the incentives to participate in the labor market.

6. Optimal welfare benefits

This section turns to the case in which the government can optimally set not only the tax system but also social assistance. In this setting, it considers two related issues. First, if b is optimally chosen (instead of exogenously given), does optimal policy produce a high welfare or low welfare economy? Second, with optimal welfare benefits, proposition 1 in section 4.2 shows that the government should always tax search (i.e. $T(n) + b > 0$) if it cannot observe skills. With observable skills, in contrast, the current section shows that search may be subsidized rather than taxed (i.e. $T(n) + b < 0$).

We determine the optimal level of welfare by taking the first-order condition of (4) with respect to b. This yields

$$(1 - \frac{\lambda_E}{v'(b)})\left[F(n_s) + (1 - \bar{s})(1 - F(n_s))\right] = \int_{n_z}^{n_1} \eta(n)\, dn \tag{6}$$

This equation shows that fighting poverty by using high welfare benefits is constrained by the participation constraints for the high ability agents $n > n_z$. Hence, the more important these participation constraints are, the lower optimal social assistance becomes and thus the more likely one ends up in a low welfare economy.

The cost of using in-work benefits instead of welfare to fight poverty is that agents have to invest search costs γ to find a job and become eligible for in-work benefits. The following result shows that if search costs are low enough, optimal b implies a low welfare economy.

Proposition 4 *If $\gamma = 0$, the optimal b implies a low welfare economy. As long as $n_z < n_1$, we find that*

$$T(n) + b < 0$$

for $n \in [n_0, n_z]$.

By having all types search for a job ($n_s = n_0$ in a low welfare economy), the government relies maximally on in-work benefits to relieve poverty. Intuitively, the advantage of in-work benefits over social assistance as a redistributive device is that these benefits can better target the least skilled by using more information. This information, however, is not costless: agents can reveal this information only after having searched for jobs, thereby paying search costs $\gamma \geq 0$. Without these search costs ($\gamma = 0$), however, in-work benefits have only advantages so that the government maximally relies on in-work benefits.

The reason why the in-work benefit $-T(n)$ exceeds the welfare benefit b is the participation constraint for the high efficiency types. In particular, raising $-T(n)$ for the types $n \in [n_0, n_z]$ does not create distortions, while raising b harms work effort by high types. Therefore, the government sets the in-work benefit at a higher level than the welfare benefit. The following simple example shows that the result $T(n) + b < 0$ is also possible with positive search costs.

Example 1 *Consider an economy with only two types of agents: $n_0 = 0$ and $n_1 > 0$ with proportions f_0 and f_1 ($f_0 + f_1 = 1$). Assume that the search cost γ satisfies the following* *inequalities*

$$v(x) - \frac{x}{n_1 f_1} < \gamma < v(x),$$

where x is determined by $v'(x) = \frac{1}{n_1}$. In this economy, $z_0 = y_0 = 0$ because production by type n_0 does not yield any output. Clearly, the government would like to redistribute from the n_1 type to the n_0 type. Does this take the form of welfare benefits b or in-work benefits x_0? To determine this, the social planner solves

$$\begin{aligned}
&\max_{\substack{x_0,x_1,z_1,\\ 0\le s_0,s_1\le\bar{s}\\ b\ge 0}} \left\{ \begin{array}{c} f_0\left(-\gamma s_0 + s_0 v(x_0) + (1-s_0)v(b)\right) + \\ f_1\left(-\gamma s_1 + s_1\left[v(x_1) - \frac{z_1}{n_1}\right] + (1-s_1)v(b)\right) \end{array} \right\} + \\
&+\lambda_E\left\{-f_0 s_0 x_0 + f_1 s_1 (z_1 - x_1) - \left[f_0(1-s_0) + f_1(1-s_1)\right]b\right\} \\
&+\eta_0\left\{v(x_0) - \gamma - v(b)\right\} \\
&+\eta_1\left\{v(x_1) - \frac{z_1}{n_1} - \gamma - v(b)\right\}.
\end{aligned}$$

We derive conditions under which the optimal welfare benefit b equals zero, while the in-work benefit x_0 is positive. For this, we need only the first-order conditions for x_1 and z_1:

$$\begin{aligned}
f_1 s_1 v'(x_1) - \lambda_E f_1 s_1 + \eta_1 v'(x_1) &= 0, \\
-f_1 s_1 \frac{1}{n_1} + \lambda_E f_1 s_1 - \frac{\eta_1}{n_1} &= 0.
\end{aligned}$$

These two equations yield

$$\begin{aligned}
\lambda_E &= \frac{1}{n_1} + \frac{\eta_1}{n_1 f_1 s_1}, \\
v'(x_1) &= \frac{1}{n_1}.
\end{aligned}$$

Since we want to redistribute as much consumption from type n_1 to type n_0 as is feasible, type n_1 must work as hard as possible.[11] *The maximal amount is determined by the search constraint*

[11] The planner, however, should not redistribute so much that $x_0 > x_1$. To avoid this, the following condition must hold (note that z_1 is maximal if $b = 0$): $x_0 = \frac{f_1 \bar{s}(z_1 - x_1)}{f_0 s_0} \le \frac{f_1(n_1 v(x_1) - n_1\gamma - x_1)}{f_0} < x_1$. The last inequality is met due to the assumptions made on γ.

$v(x_1) - \frac{z_1}{n_1} \geq v(b) + \gamma$. Hence, we find

$$z_1 = n_1 (v(x_1) - v(b) - \gamma).$$

If b is raised, then social gains equal

$$[f_0 (1 - s_0) + f_1 (1 - s_1)] v'(b),$$

while the social costs equal the cost of additional expenses on b plus the loss in tax revenue (which equals $f_1 s_1 n_1 v'(b)$ because the expression for z_1 above implies that $\frac{dz_1}{db} = -n_1 v'(b)$)

$$\lambda_E \{f_0 (1 - s_0) + f_1 (1 - s_1) + f_1 s_1 n_1 v'(b)\}.$$

If at $b = 0$ the costs exceed the gains, the optimal welfare benefit level equals 0:

$$\lambda_E f_1 s_1 n_1 v'(0) + [f_0 (1 - s_0) + f_1 (1 - s_1)] (\lambda_E - v'(0)) > 0,$$

or equivalently (using $v'(0) = +\infty$ and $\lambda_E \geq \frac{1}{n_1}$ because $\eta_1 \geq 0$)

$$\bar{s} > \frac{1}{(1 + f_1)}.$$

The interpretation of this condition is that search should be relatively effective.

With observable ability, the participation constraint for high ability types is binding. This explains why $T(n) + b < 0$ is optimal in this case, whereas $T(n) + b > 0$ (see proposition 1 in section 4.2) if ability is not observable. In the latter case, high skilled agents earn an informational rent. This makes the participation constraint for these types slack. Hence, welfare, which tightens the participation constraint, does not affect the behavior of the high skilled. At the same time, in-work benefits do affect that behavior, as the high skilled may want to mimic the low skilled who collect generous in-work benefits. Whereas in-work benefits thus distort work effort of high skilled agents, welfare no longer directly impacts the behavior of these agents. Accordingly, compared to in-work benefits, welfare becomes a more attractive instrument for fighting poverty.

A low welfare economy is more likely if optimal social assistance is not generous. In order to obtain some intuition about when social assistance b is low compared to the in-work benefit $\bar{x}$, one can approximate the search subsidy $-T(n) - b = \bar{x} - b$ by the following expression[12]

$$\frac{\bar{x} - b}{\xi} = \frac{v'(b)}{-\xi v''(\xi)} \frac{\bar{s} \int_{n_z}^{n_1} f(n) \left[\frac{n}{n_z} - 1\right] dn}{\left[F(n_w) + (1 - \bar{s})(1 - F(n_w))\right]},$$

where $b < \xi < \bar{x}$. This expression indicates that b is low compared to $\bar{x}$ if search is effective (as indicated by a high level of $\bar{s}$) and agents are not particularly risk averse (as indicated by a low coefficient of relative risk aversion $-\xi v''(\xi)/v'(b)$). Intuitively, if not many people are involuntarily unemployed and the government does not care much about inequality in consumption, the government sets welfare b at a relatively low level.

We can conclude that a low welfare economy, which minimizes the dependency on social assistance and uses jobs (and in-work benefits) rather than welfare as a anti-poverty device, is more likely to be observed if search is cheap (i.e. γ is low) and effective (i.e. $\bar{s}$ is large), agents are productive (so that $\hat{n}$ is close to n_0 and the government does not need to pay many agents a search subsidy to induce them to search for a job), and agents are not particularly risk averse so that they do not mind poor insurance against unemployment risk.

Whereas we thus establish that in the social optimum an economy may feature low welfare, [5] find that their economy is necessarily high welfare if welfare benefits are set optimally. Choné and Laroque's result originates in their assumption of a Rawlsian welfare function, which implies very high risk aversion. If we were to adopt such a Rawlsian welfare function, we would arrive at the same conclusions as Choné and Laroque. The reason is that the participation constraint implies that the worst-off agent collects welfare benefits. Hence, a Rawlsian social

[12] This expression can be derived by using the Taylor expansion $v'(\bar{x}) - v'(b) = v''(\xi)(\bar{x} - b)$ (with $b < \xi < \bar{x}$) and substituting $v'(\bar{x}) = \lambda_E$ into equation (25) in the appendix.

planner maximizes these welfare benefits subject to the government budget constraint. Social assistance is therefore raised up to the point where a further increase would bankrupt the economy. This implies that the economy is necessarily high welfare rather than low welfare.

More generally, social assistance continues to play an important role in protecting agents against lack of skills if search is expensive and ineffective and if agents are not productive (so that using the tax system is expensive), especially if agents are risk averse. In that case, high welfare benefits reduce the budgetary room for paying in-work benefits to the large population that is not productive enough to search for a job without these generous in-work benefits. In particular, people must be paid high in-work benefits in order to overcome the search costs for revealing their type. Moreover, in-work benefits cannot reach those that are involuntarily unemployed (i.e. those who search but still remain unemployed). Intuitively, work is an expensive and ineffective way out of poverty so that in-work benefits are expensive (as search costs are high) and fail to reach the involuntary unemployed.

7. Conclusions

This paper has explored the interaction between the tax and social assistance systems in insuring people against the risks of involuntary unemployment and low ability. By assuming that the government can observe a worker's type, we have stacked the cards in favor of in-work tax benefits as an instrument to fight poverty. We showed, however, that even with this strong informational assumption, the redistributive power of the tax system is constrained if the government cannot verify job search. In particular, the financing of in-work benefits can be problematic if search costs are high and a large number of agents is not productive. This is especially so if high welfare benefits constrain the ability of the government to extract taxes from the more efficient workers. In this way, social assistance limits the ability of the tax system to finance in-work benefits.

If the government can optimally set both the tax system and the level of social assistance, it faces a trade-off in deciding on the relative importance of in-work and welfare benefits. Relying on in-work benefits allows effective targeting of benefits at workers with low ability without directly distorting the search incentives of more able workers. However, in-work benefits are expensive if search costs are high. Moreover, they do not reach the poorest agents who suffer from involuntary unemployment. Clearly, social assistance remains important in insuring agents against unemployment risk. The welfare system continues to play an important role also in protecting agents against lack of skills if search is expensive and ineffective and if agents are not productive. In that case, work is an expensive and ineffective way out of poverty.

References

[1] G. Akerlof, The Economics of 'Tagging' as Applied to the Optimal Income Tax, Welfare Programs, and Manpower Planning, *Amer. Econ. Rev.* **68** (1978), 8-19.

[2] R. Boadway, K. Cuff and M. Marchand, Optimal Income Taxation with Quasi-Linear Preferences Revisited, *Journal of Public Economic Theory* **2** (2000), 435-460.

[3] J. Boone and A.L. Bovenberg, Optimal Taxation with Observable Search, Mimeo, Tilburg University (2001).

[4] J. Boone and A.L. Bovenberg, The Optimal Taxation of Unskilled Labor with Job Search and Social Assistance,*J. Public Econ.* **88** (2004), 2227-2258.

[5] P. Choné and G. Laroque, Optimal Incentives for Labor Force Participation, INSEE Working Paper 2001-25, Paris.

[6] P.A. Diamond, Income Taxation with Fixed Hours of Work, *J. Public Econ.* **13** (1980), 101-110.

[7] U. Ebert, A Reexamination of the Optimal Nonlinear Income Tax, *J. Public Econ.* **49** (1992), 47-73.

[8] A.J. Hosios, On the Efficiency of Matching and Related Models of Search and Unemployment, *Rev. Econ. Stud.* **57** (1990), 279-298.

[9] M. Hungerbühler, E. Lehmann, A. Parmentier and B. van der Linden, Optimal income taxation in an equilibrium unemployment model: Mirrlees meets Pissarides, Mimeo CORE Louvain (2003).

[10] M.I. Kamien and N.L. Schwartz, "Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management," North-Holland, 1981.

[11] R. Kanbur, M. Keen and M. Tuomala, Optimal Non-Linear Income Taxation for the Alleviation of Income-Poverty, *Europ. Econ. Rev.* **38** (1994), 1613-1632.

[12] S. Lollivier and J. Rochet, Bunching and Second-Order Conditions: A Note on Optimal Tax Theory, *J. Econ. Theory* **31** (1983), 392-400.

[13] J.A. Mirrlees, An Exploration in the Theory of Optimal Income Taxation, *Rev. Econ. Stud.* **38** (1971), 175-208.

[14] J. Rochet and L. Stole, Nonlinear Pricing with Random Participation, *Rev. Econ. Stud.* **69** (2002), 277-311.

[15] E. Saez, Optimal Income Transfer Programs: Intensive Versus Extensive Labor Supply Responses, *Quart. J. Econ.* **117** (2002), 1039-1073.

[16] J.A. Weymark, Comparative Static Properties of Optimal Nonlinear Taxes, *Econometrica* **55** (1987), 1165-1185.

Appendix: Proofs of results

Proof of Proposition 1

With the government not being able to observe workers' ability, it has to respect incentive compatibility with respect to individual work effort. In other words, given a certain tax schedule $\tilde{T}(z)$ as a function of gross income, workers choose their production to maximize in-work utility $u(x,z) = v\left(z - \tilde{T}(z)\right) - \frac{z}{n}$. We use $u(n)$ to denote type n's maximized utility level:

$$u(n) = \max_{z} \left\{ v\left(z - \tilde{T}(z)\right) - \frac{z}{n} \right\}.$$

Using the envelope theorem, we find

$$u'(n) = \frac{z(n)}{n^2}, \tag{7}$$

where we used the first-order condition for z. To facilitate the inclusion of Eq. (7) into our optimization problem, we employ $u(n)$ as control variable instead of $x(n)$, as we did in (4). The optimization problem thus becomes

$$\begin{aligned}
&\max_{n_s, u(.), z(.)} F(n_s) v(b) + [1 - F(n_s)](-\gamma\bar{s} + (1-\bar{s}) v(b)) + \\
&+ \int_{n_s}^{n_1} \left\{ \bar{s}u(n)f(n) - \lambda_u(n)\left[u'(n) - \frac{z(n)}{n^2}\right] + \lambda_E [f(n)\bar{s}T(n)] \right\} dn \\
&-\lambda_E \{ b[F(n_s) + (1 - F(n_s))(1-\bar{s})] + g \} \\
&-\eta_s (\gamma - u(n_s) + v(b)),
\end{aligned}$$

where $T(n) = z(n) - x(n) = z(n) - v^{-1}\left(u(n) + \frac{z(n)}{n}\right)$. Furthermore, since $z(n) \geq 0$ for each type, Eq. (7) implies that $u'(n) \geq 0$ so that there is only one type n_s for which the restriction $\gamma - u(n_s) + v(b) \leq 0$ is binding. In fact, if $n_s > n_0$, it must be the case that

$$u(n_s) = \gamma + v(b).$$

To see this, note first that $u(n_s) < \gamma + v(b)$ is not possible as type n_s will not exert effort s to find a job. Also $u(n_s) > \gamma + v(b)$ cannot occur because that would provide an incentive to a type $n < n_s$ (but close to n_s) to mimic type n_s and find a job. This would violate incentive compatibility in terms of search.

The first-order conditions for $n_s, u(.)$ and $z(.)$ can be written as

$$\begin{aligned}
\eta_s u'(n_s) &= f(n_s)\left\{\bar{s}\left[\underbrace{-\gamma - v(b) + u(n_s)}_{=0} + \lambda_E(b + T(n_s))\right]\right\}, && (8)\\
\lambda_u'(n) &= \bar{s} f(n)\left(\frac{\lambda_E}{v'(x(n))} - 1\right), && (9)\\
0 &= -\frac{\lambda_u(n)}{n^2} + \lambda_E f(n)\,\bar{s}\left(\frac{1}{nv'(x(n))} - 1\right). && (10)
\end{aligned}$$

The transversality conditions are[13]

$$\begin{aligned}
\lambda_u(n_s) + \eta_s &= 0, && (11)\\
\lambda_u(n_1) &= 0, && (12)
\end{aligned}$$

and the government budget constraint amounts to

$$\int_{n_s}^{n_1} f(n)\bar{s}\left[z(n) - v^{-1}\left(u(n) + \frac{z(n)}{n}\right)\right] dn = \left[F(n_s) + (1 - F(n_s))(1 - \bar{s})\right] b + g. \qquad (13)$$

[13] [10] derive the transversality condition for the case in which the end value of a state variable can be freely chosen but has to satisfy an inequality constraint. Our condition above is the equivalent of that condition for a free starting point under an inequality constraint. The intuition is the following. If the constraint is binding, the Lagrange multiplier is strictly positive $\eta_s > 0$, and the shadow value $\lambda_u(n_s)$ is thus negative. In other words, one would like to reduce $u(n_s)$ (as $\lambda_u(n_s) < 0$), but cannot do so because of the constraint $u(n_s) \geq \gamma + v(b)$.

Eq. (10) can be written as

$$\frac{1}{v'(x(n))} = n + \frac{1}{n}\frac{\lambda_u(n)}{\lambda_E f(n)\bar{s}}. \tag{14}$$

Substituting this into (9) to eliminate $v'(x(n))$, we arrive at

$$\lambda_u'(n) = \frac{\lambda_u(n)}{n} + \bar{s} f(n)(\lambda_E n - 1). \tag{15}$$

We are now set to prove that $b + T(n_s) > 0$. Assume (by contradiction) $b + T(n_s) \le 0$. Then, Eq. (8) implies that $\eta_s \le 0$, and thus we find from (11) that

$$\lambda_u(n_s) \ge 0.$$

We now consider two subcases: (i) $\lambda_E n_s - 1 \ge 0$ and (ii) $\lambda_E n_s - 1 < 0$, and show that the result $\lambda_u(n_s) \ge 0$ leads to a contradiction in each case.

(i) If $\lambda_E n_s - 1 \ge 0$, then $\lambda_u(n_s) \ge 0$ together with Eq. (15) implies that $\lambda_u(n) > 0$ for each $n > n_s$. However, this contradicts the second transversality condition (12) that $\lambda_u(n_1) = 0$.

(ii) If $\lambda_E n_s - 1 < 0$, then using $\lambda_u(n_s) \ge 0$ together with (12), it must be the case that there exists $\tilde{n} \le n_1$ such that $\lambda_u(\tilde{n}) = 0$ and $\lambda_u'(\tilde{n}) \le 0$. The combination of $\lambda_u(\tilde{n}) = 0$ with $\lambda_u'(\tilde{n}) \le 0$ implies that

$$\lambda_E \tilde{n} - 1 \le 0. \tag{16}$$

Now we write the first-order condition for b as

$$[F(n_s) + (1 - F(n_s))(1 - \bar{s})] v'(b) - \lambda_E [F(n_s) + (1 - F(n_s))(1 - \bar{s})] = \eta_s v'(b).$$

Since our assumption (to be contradicted) that $T(n_s) + b \le 0$ implies that $\eta_s \le 0$, this equation implies $\lambda_E \ge v'(b)$. Furthermore, the participation constraint (with $\gamma > 0$) implies that $x(n) > b$ for $n \ge n_s$ and thus we must have $v'(b) > v'(x(n))$ for all $n \ge n_s$. Using the characterization of $\tilde{n} > n_s$ that says $\lambda_u(\tilde{n}) = 0$ together with Eq. (14), we establish

$$\lambda_E \ge v'(b) > v'(x(\tilde{n})) = \frac{1}{\tilde{n}}.$$

This, however, contradicts Eq. (16) above. Q.E.D.

Definition 1 *We define $\hat{n}$ as the biggest root to the equation $\zeta_\gamma(.,b) = 0$, that is,*

$$\hat{n} \equiv \max\{n \geq 0 | \zeta_\gamma(n,b) = 0\}, \tag{17}$$

with

$$\zeta_\gamma(n,b) \equiv n(v(x(n)) - v(b) - \gamma) - x(n) + b, \tag{18}$$

where $x(n)$ is determined by $v'(x(n)) = \frac{1}{n}$.

The following definition gives the upperbound on g such that the government budget constraint can be financed.

Definition 2 *For given $f(.), n_0$ and n_1 define $g^*(\gamma)$ as*

$$g^*(\gamma) = \int_{\hat{n}}^{n_1} f(n)\,\bar{s}\,(n(v(x(n)) - \gamma) - x(n))\,dn,$$

where $\hat{n}$ is defined in Eq. (17).

Proof of proposition 2

First, we define the values $\underline{b}$ and $\bar{b}$. Define $\underline{b}$ as the solution in b of the following equation:

$$\int_{\hat{n}(b)}^{n_1} f(n)\,\bar{s}\,(n[v(x(n)) - \gamma - v(b)] - x(n))\,dn + F(\hat{n}(b))\,\bar{s}\,(-x(\hat{n}(b))) = b(1-\bar{s}) + g,$$

where $x(n)$ is determined by $v'(x(n)) = \frac{1}{n}$ and $\hat{n}(b)$ is defined in Eq. (17). If there is no solution $b \geq 0$ to this equation, define $\underline{b} = 0$. Note that $\frac{d\hat{n}(b)}{db} > 0$ for the following reason. Differentiate $\zeta_\gamma(n,b)$ with respect to n and b. This yields

$$(v(x(\hat{n})) - v(b) - \gamma)\frac{d\hat{n}}{db} - (\hat{n}v'(b) - 1) = 0.$$

Therefore $\frac{d\hat{n}}{db} > 0$ because $v(x(\hat{n})) - v(b) - \gamma > 0$ and $v'(b) > \frac{1}{\hat{n}}$ (the last inequality follows from $b < x(\hat{n})$ as shown in the proof of lemma 8 below).

The left-hand side of this equation is decreasing in b, while the right-hand side is increasing in b. Hence, if there is a solution to the equation, it is unique.

Define $\bar{b}$ as the solution in b of the following equation

$$\int_{\hat{n}(b)}^{n_1} f(n)\,\bar{s}\,(n\,[v\,(x\,(n)) - \gamma - v\,(b)] - x\,(n))\,dn = b\,(1 - \bar{s}\,[1 - F\,(\hat{n}\,(b))]) + g$$

with $x\,(n)$ determined by $v'\,(x\,(n)) = \frac{1}{n}$. This value of $\bar{b}$ is always well defined because (i) the left-hand side of this equation is decreasing in b (use $\frac{d\hat{n}(b)}{db} > 0$ as derived above), (ii) the right-hand side is increasing in b, (iii) at $b = 0$ the left-hand side exceeds the right-hand side by the assumption that $g < g^*\,(\gamma)$ (as defined in definition 2 above) and (iv) the right-hand side exceeds the left-hand side at a value of b satisfying $\hat{n}\,(b) = n_1$.

The proposition is proved for the high welfare economy, using a number of lemmas derived below. Once the solution for the high welfare economy is proved, the extension to the low welfare economy is straightforward (see also the text in section 5.1).

Lemma 1 *If $z\,(n) > 0$ for some $n \in [n_0, n_1]$, then $z\,(n') > 0$ for each $n' > n$.*

Proof. First, consider the optimal level of before-tax income $z(n)$. Observe that the optimization problem (4) is linear in $z\,(n)$ with coefficient $\left\{-\bar{s}f\,(n)\,\frac{1}{n} + \lambda_E f\,(n)\,\bar{s} - \eta\,(n)\,\frac{1}{n}\right\}$. The optimal $z\,(n)$ is therefore determined as follows:[14]

$$\begin{aligned} z\,(n) &= n\,(v\,(x\,(n)) - \gamma - v\,(b)) \text{ only if } \left\{-\bar{s}f\,(n)\,\frac{1}{n} + \lambda_E f\,(n)\,\bar{s} - \eta\,(n)\,\frac{1}{n}\right\} = 0 \\ z\,(n) &= 0 \text{ if } \bar{s}f\,(n) + \eta\,(n) > \lambda_E f\,(n)\,\bar{s}n. \end{aligned} \tag{19}$$

Now suppose that the claim in the lemma is not correct. That is (from (19))

$$\left\{-\bar{s}f\,(n)\,\frac{1}{n} + \lambda_E f\,(n)\,\bar{s} - \eta\,(n)\,\frac{1}{n}\right\} = 0,$$

[14] We cannot have $-\bar{s}f\,(n)\,\frac{1}{n} + \lambda_E f\,(n)\,\bar{s} - \eta\,(n)\,\frac{1}{n} > 0$ because that would imply (by the linearity of $z\,(.)$ in the optimization problem) $z\,(n) = +\infty$ so that $z\,(n) > n\,(v\,(x\,(n)) - \gamma - v\,(b))$. This, however, would violate the participation constraint.

and there exists $n' > n$ such that

$$\left\{-\bar{s}f\left(n'\right)\frac{1}{n'}+\lambda_{E}f\left(n'\right)\bar{s}-\eta\left(n'\right)\frac{1}{n'}\right\}<0. \tag{20}$$

The first-order condition for maximizing (4) with respect to consumption $x\left(.\right)$ is given by

$$\eta\left(n\right)=\bar{s}f\left(n\right)\left[\frac{\lambda_{E}}{v'\left(x\left(n\right)\right)}-1\right] \tag{21}$$

for types $n \geq n_s$. Using this, the two equations above can be rewritten as

$$\begin{aligned} 1-\frac{1}{nv'\left(x\left(n\right)\right)} &= 0, \\ 1-\frac{1}{n'v'\left(x\left(n'\right)\right)} &< 0, \end{aligned}$$

which implies that $x\left(n'\right) > x\left(n\right)$ because $n' > n$. By assumption $z\left(n'\right) = 0$; hence, we get

$$v\left(x\left(n'\right)\right)>v\left(x\left(n\right)\right)-\frac{z\left(n\right)}{n}\geq v\left(b\right)+\gamma,$$

and thus $\eta\left(n'\right) = 0$, while $\eta\left(n\right) \geq 0$. However, this implies

$$-\frac{1}{n'}+\lambda_{E}-\eta\left(n'\right)\frac{1}{\bar{s}f\left(n'\right)n'}=-\frac{1}{n'}+\lambda_{E}>-\frac{1}{n}+\lambda_{E}-\eta\left(n\right)\frac{1}{\bar{s}f\left(n\right)n}=0,$$

which contradicts inequality (20). ■

Lemma 2 *If $z\left(n\right) > 0$ for some n, then it is the case that*

$$z\left(n'\right)=n'\left(v\left(x\left(n'\right)\right)-\gamma-v\left(b\right)\right)$$

for each $n' > n$.

Proof. Suppose not; that is, assume that for some $n' > n$ we have

$$z\left(n'\right)<n'\left(v\left(x\left(n'\right)\right)-\gamma-v\left(b\right)\right).$$

Then there exists $\varepsilon > 0$ such that the alternative function $\tilde{z}(n)$ which equals $z(n)$ everywhere except for the points n and n' where $\tilde{z}$ is determined by

$$\tilde{z}(n') = z(n') + \varepsilon,$$
$$\tilde{z}(n) = z(n) - \frac{f(n')}{f(n)}\varepsilon.$$

For ε small enough, this function $\tilde{z}$ satisfies all the constraints and $\tilde{z}$ is budget neutral by construction. It is routine to verify that $\tilde{z}$ yields higher welfare than z. ■

Define $\tilde{n}_z$ as follows

$$\tilde{n}_z \equiv \inf\{n \geq n_0 | z(n) > 0\}.$$

Then we find from the results above, (21) and (19), that for each $n > \tilde{n}_z$, it is the case that

$$\begin{aligned} \eta(n) &> 0, \\ z(n) &= n(v(x(n)) - \gamma - v(b)), \\ v'(x(n)) &= \frac{1}{n}. \end{aligned}$$

Now turn to the types below $\tilde{n}_z$. Note that the following result does not assume that the set $[n_s, \tilde{n}_z\rangle$ is non-empty. We will prove below that the set is in fact non-empty.

Lemma 3 *For each $n \in [n_s, \tilde{n}_z\rangle$ we have that $z(n) = 0$ and $x(n) = \tilde{x}$ for some $\tilde{x} > 0$.*

Proof. $z(n) = 0$ follows from the definition of $\tilde{n}_z$. The result that all these agents get the same consumption level follows from the concavity of $v(.)$. ■

Lemma 4 $v'(\tilde{x}) = \frac{1}{\tilde{n}_z}$.

Proof. Suppose not (in two parts):

(i) suppose (by contradiction) that $v'(\tilde{x}) < \frac{1}{\tilde{n}_z}$. This implies that $\tilde{x} > x(\tilde{n}_z)$ and hence

$$v(\tilde{x}) > v(x(\tilde{n}_z)) - \frac{z(\tilde{n}_z)}{\tilde{n}_z} \geq \gamma + v(b).$$

Then we can construct new functions $\tilde{x}(.)$ and $\tilde{z}(.)$ that are identical to $x(.)$ and $z(.)$ with the following exceptions

$$\tilde{x}(n) = \tilde{x} - \varepsilon,$$
$$\tilde{z}(\tilde{n}_z) = z(\tilde{n}_z) - \varepsilon\frac{f(n)}{f(\tilde{n}_z)}$$

for some type $n \in [n_s, \tilde{n}_z\rangle$ and $\varepsilon > 0$ small enough. These functions $\tilde{z}$ and $\tilde{x}$ are budget neutral and do not violate any constraint (as long as ε is small enough) and raise aggregate welfare.

(ii) suppose (by contradiction) that $v'(\tilde{x}) > \frac{1}{\tilde{n}_z}$. Then there exists a type $n < \tilde{n}_z$ such that for some $\varepsilon > 0$ we have

$$v(\tilde{x} + \varepsilon) - \frac{\varepsilon}{n} > v(\tilde{x}).$$

That is, the government can raise the utility of this type n without making anyone worse-off. ■

Lemma 5 *Assume that $n_s < \tilde{n}_z$, then $v'(\tilde{x}) = \lambda_E$.*

Proof. $\eta(n) \geq 0$ implies (from (21)) that $v'(\tilde{x}) \leq \lambda_E$. So the question is whether $v'(\tilde{x}) < \lambda_E$ is possible. Assume (by contradiction) that $v'(\tilde{x}) < \lambda_E$ is indeed possible. Then we have $\eta(n) > 0$ and thus (by complementary slackness and $z(n) = 0$) $v(\tilde{x}) = \gamma + v(b)$ so that $\tilde{x} > b$.

Now consider the first-order condition for maximizing (4) with respect to the least efficient type that searches for a job (the so-called marginal searcher n_s):

$$\bar{s}f(n_s)\left\{v(b) + \gamma - v(x(n_s)) + \frac{z(n_s)}{n_s} - \lambda_E(z(n_s) - x(n_s) + b)\right\} = 0, \tag{22}$$

where we have used that in the high welfare economy the optimal n_s is not a corner solution. In the low welfare economy we do have a corner solution here ($n_s = n_0$).

Substituting $v(\tilde{x}) = \gamma + v(b)$ into this equation yields

$$0 - \lambda_E(z(\tilde{n}_w) - \tilde{x} + b) = 0.$$

However, this implies that $z(\tilde{n}_w) = \tilde{x} - b > 0$, which contradicts $n_s < \tilde{n}_z$. ■

Corollary 1 *$\eta(n) = 0$ for all $n \in [n_s, \tilde{n}_z\rangle$.*

Lemma 6 $\lambda_E = \frac{1}{\tilde{n}_z}$.

Proof. Suppose not (in two parts)

(i) suppose (by contradiction) that $\lambda_E > \frac{1}{\tilde{n}_z}$. The equation for $z(\tilde{n}_z) > 0$ can be written as

$$-\frac{1}{\tilde{n}_z} + \lambda_E - \eta(\tilde{n}_z)\frac{1}{\bar{s}f(\tilde{n}_z)\tilde{n}_z} = 0.$$

But then there exists $n < \tilde{n}_z$ such that $-\frac{1}{n} + \lambda_E - 0 > 0$, which contradicts $z(n) = 0$.

(ii) suppose (by contradiction) that $\lambda_E < \frac{1}{\tilde{n}_z}$. Then there exists $n' > \tilde{n}_z$ such that $\lambda_E n' < 1$, which contradicts $\eta(n') = f(n')\bar{s}(\lambda_E n' - 1) \geq 0$ for this type $n' > \tilde{n}_z$. ■

Lemma 7 *$\tilde{n}_z = \hat{n}$ where $\hat{n}$ is defined in Eq. (17).*

Proof. Substituting $z(n_s) = 0$, $x(n_s) = \tilde{x}$ (with $v'(\tilde{x}) = \frac{1}{\tilde{n}_z}$), and $\lambda_E = \frac{1}{\tilde{n}_z}$into Eq. (22), we arrive at

$$v(b) + \gamma - v(\tilde{x}) - \frac{1}{\tilde{n}_z}(-\tilde{x} + b) = 0. \tag{23}$$

To analyze the solution to this equation, we write the function $\zeta_\gamma : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ as defined in Eq. (18) as

$$\begin{aligned} \zeta_\gamma(n, b) &= n(v(x(n)) - v(b) - \gamma) - x(n) + b, \\ \text{where } x(n) \text{ is defined by } v'(x(n)) &= \frac{1}{n}. \end{aligned}$$

Thus, Eq. (23) boils down to

$$\zeta_\gamma(n, b) = 0$$

for exogenously given b.

As the next lemma shows, this equation has at most two solutions.

Lemma 8 *For each $b \geq 0$, it is the case that*

(i) the equation $\zeta_0(n,b) = 0$ has a unique solution $\hat{n} \geq 0$ with $x(\hat{n}) = b$;

(ii) the equation $\zeta_\gamma(n,b) = 0$ with $\gamma > 0$ has exactly two solutions $0 < \hat{n}_1 < \hat{n}_2$ with

$$\begin{aligned} v(x(\hat{n}_1)) &< v(b) + \gamma, \\ v(x(\hat{n}_2)) &> v(b) + \gamma. \end{aligned}$$

Proof. First, note that ζ_γ is strictly convex in n. This follows from

$$\begin{aligned} \frac{\partial \zeta_\gamma(n,b)}{\partial n} &= v(x(n)) - v(b) - \gamma, \\ \frac{\partial^2 \zeta_\gamma(n,b)}{\partial n^2} &= v'(x(n)) \frac{dx(n)}{dn} > 0, \end{aligned}$$

because $v''(x(n)) \frac{dx(n)}{dn} = \frac{-1}{n^2}$ implies that $\frac{dx(n)}{dn} > 0$.

Second, note that $\zeta_\gamma(0,b) = b > 0$ (here, we use the assumption that $\lim_{x \downarrow 0} v'(x) = +\infty$), $\lim_{n \to +\infty} \zeta_\gamma(n,b) = +\infty$ and that

$$\min_{n \geq 0} \zeta_\gamma(n,b) = b - v^{-1}(v(b) + \gamma).$$

Now consider in turn the cases $\gamma = 0$ and $\gamma > 0$. If $\gamma = 0$, we see that $\min_{n \geq 0} \zeta_0(n,b) = 0$. So by the strict convexity of ζ_0, the value $\hat{n}$ for which this minimum is reached is the unique solution to $\zeta_0(n,b) = 0$. Further, using the first-order condition $\frac{\partial \zeta_0(n,b)}{\partial n} = 0$ for this minimum, we find that $v(x(\hat{n})) - v(b) = 0$ or equivalently $x(\hat{n}) = b$.

Next, consider the case where $\gamma > 0$. Then clearly $\min_{n \geq 0} \zeta_\gamma(n,b) < 0$, so (using $\zeta_\gamma(0,b) = b > 0$ and $\lim_{n \to +\infty} \zeta_\gamma(n,b) = +\infty$) we have two solutions to the equation $\zeta_\gamma(n,b) = 0$. At the smallest ($\hat{n}_1$) of these solutions, we have that ζ_γ is downward sloping (see figure 2):

$$\left. \frac{\partial \zeta_\gamma(n,b)}{\partial n} \right|_{n = \hat{n}_1} = v(x(\hat{n}_1)) - v(b) - \gamma < 0,$$

and thus $v(x(\hat{n}_1)) < v(b) + \gamma$. Similarly, at $\hat{n}_2$ the function ζ_γ is upward sloping (again see figure 2) and we have $v(x(\hat{n}_2)) > v(b) + \gamma$. ■

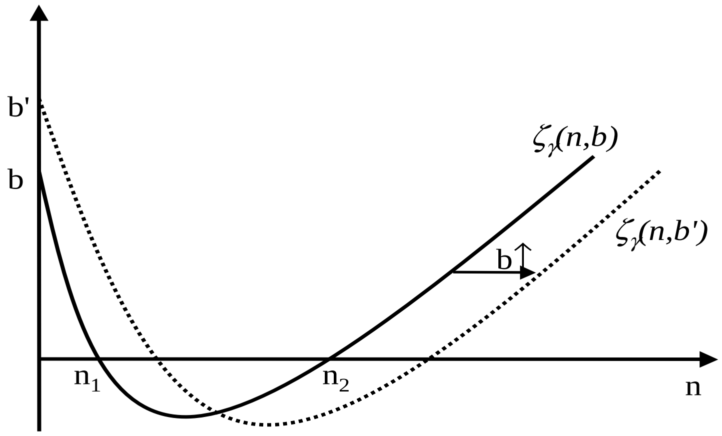

Figure 2: $\zeta_\gamma(n, b)$ *as a function of* n.

Using this result, we continue the proof that $\tilde{n}_z = n_z = \hat{n}$. If $\gamma = 0$, Eq. (23) has only one solution and hence it is indeed the case by definition 1 that $\tilde{n}_z = \max\{n \geq 0 | \zeta_0(n, b) = 0\} = \hat{n}$. If $\gamma > 0$, we need the solution to Eq. (23) which features $v(x(n)) - \gamma \geq v(b)$. That is, we need the solution $\hat{n}_2$ in the lemma above. So again we have $\tilde{n}_z = \max\{n \geq 0 | \zeta_\gamma(n, b) = 0\} = \hat{n}$.

Summarizing, in the high welfare economy we have found that $n_z = \hat{n}, \lambda_E = 1/n_z$. Further, for types $n \geq n_z$ we see that $s(n) = \bar{s}, z(n) = n(v(x(n)) - v(b) - \gamma)$ and $v'(x(n)) = \frac{1}{n}$. For types $n \in [n_s, n_z\rangle$ we find $s(n) = \bar{s}, z(n) = 0$ and $v'(x(n)) = \frac{1}{n_z}$. Finally, for $n \in [n_0, n_s\rangle$ we find $s(n) = z(n) = 0$ and $x(n) = b$. Substituting this into the government budget constraint shows that there exists a value of $n_s \in [n_0, \hat{n}]$ such that the government budget constraint holds with equality. This follows from the assumption that b lies between $\underline{b}$ and $\bar{b}$. Similarly, one can see that $b > \bar{b}$ cannot be financed by the economy.

Because $n_z = \hat{n}$ in a high welfare economy, we know for $n > n_z$ that

$$v(x(n)) - v(b) - \gamma + \frac{1}{n}(-x(n) + b) > 0, \tag{24}$$

where $v'(x(n)) = \frac{1}{n}$. Combining this with $z(n) = n(v(x(n)) - \gamma - v(b))$ for $n > n_z$, we find

$$\begin{aligned} T(n) + b &= z(n) - x(n) + b \\ &= n\left\{v(x(n)) - v(b) - \gamma + \frac{1}{n}(-x(n) + b)\right\} > 0 \end{aligned}$$

because of Eq. (24). Similarly, because by definition of $\hat{n}$ we have

$$v(x(\hat{n})) - v(b) - \gamma + \frac{1}{\hat{n}}(-x(\hat{n}) + b) = 0,$$

and $n_z = \hat{n}$, we also obtain that $T(n_z) + b = 0$.

In the low welfare economy, $n_z > \hat{n}$ and $z(n) = n(v(x(n)) - v(b) - \gamma)$ imply $T(n) + b > 0$ for all $n \geq n_z$. Q.E.D.

Proof of Proposition 3

[LW] To find $\frac{dn_z}{db}$ differentiate the government budget constraint in the low welfare economy with respect to b and n_z. This can be written as

$$-f(n_z)\bar{s}n_z[v(x(n_z)) - \gamma - v(b)]\frac{dn_z}{db} = \left[1 - \bar{s}F(n_z) - \int_{n_z}^{n_1} f(n)\bar{s}(-nv'(b) + 1)\right].$$

Using the results that $[v(x(n_z)) - \gamma - v(b)] > 0$ and $-nv'(b) + 1 < 0$ (because $b < x(n)$ implies $v'(b) > v'(x(n)) = \frac{1}{n}$ for $n > n_z$), we find $\frac{dn_z}{db} < 0$.

Finally,

$$-\frac{dT(n)}{db} = \frac{d\bar{x}}{db} = \frac{dx(n_z)}{dn_z}\frac{dn_z}{db} < 0$$

for $n \in [n_0, n_z\rangle$.

[HW] In the high welfare economy $n_z = \hat{n}$, and n_s is determined by the government budget constraint. Hence $\frac{dn_z}{db} = \frac{d\hat{n}}{db} > 0$, which was shown in the proof of proposition 2.

The effect that $\frac{dn_s}{db} > 0$ follows from the government budget constraint as follows:

$$\left[-\underbrace{f(\hat{n})\,\bar{s}\,(\hat{n}\,(v\,(x\,(\hat{n})) - \gamma - v\,(b)))}_{>0} - \underbrace{\bar{s}\,(F\,(\hat{n}) - F\,(n_s))\,\frac{dx\,(\hat{n})}{d\hat{n}}}_{>0}\right]\underbrace{\frac{d\hat{n}}{db}}_{>0} +$$

$$+\underbrace{f\,(n_s)\,\bar{s}\,(x\,(n_s) - b)}_{>0}\frac{dn_s}{db}$$

$$= \underbrace{1 - \bar{s}\,[1 - F\,(n_s)] + v'\,(b)\,\bar{s}\int_{\hat{n}}^{n_1} nf\,(n)\,dn.}_{>0}$$

Hence, we find that $\frac{dn_s}{db} > 0$.

By definition, $T\,(n_z) + b = 0$ (because $n_z = \hat{n}$ is the least efficient type that can work without subsidy). We therefore have

$$\frac{dT\,(n_z)}{db} = -1.$$

Finally, using similar arguments, one can derive that for $n \in \langle n_s, n_z \rangle$

$$-\frac{dT\,(n)}{db} = \frac{dx\,(n_z)}{db} = \frac{dx\,(n_z)}{dn_z}\frac{dn_z}{db} > 0,$$

since $\frac{dx(n_z)}{dn_z} > 0$ (from $v'(x(n_z)) = 1/n_z$).

Proof of proposition 4

We use the following lemma.

Lemma 9 *For given $g < g^*(0)\,, f\,(.)\,, n_0, n_1$ and $\gamma = 0$, a benchmark value $\bar{b} > 0$ exists such that*

for $b < \bar{b}$ we have a low welfare economy;

for $b > \bar{b}$ the government budget cannot be financed;

for $b = \bar{b}$ we have a high welfare economy with $n_s = n_0$.

In other words, without search costs (i.e. $\gamma = 0$), $\underline{b} = \bar{b}$ in proposition 2.

Proof. In the high welfare economy, everyone works who does not collect a search subsidy: $n_z = \hat{n}$ where $v'(x(\hat{n})) = v'(b) = \frac{1}{\hat{n}}$. The equality $x(\hat{n}) = b$ follows from the proof of lemma 8 with $\gamma = 0$. Note that this implies that $n_s = n_0$. Now write the government budget constraint as

$$\int_{\hat{n}(b)}^{n_1} f(n)\,\bar{s}\,(n(v(x(n)) - v(b)) - x(n))\,dn + \int_{n_0}^{\hat{n}(b)} f(n)\,\bar{s}\,(-x(\hat{n}))\,dn = (1-\bar{s})\,b + g.$$

The left-hand side is decreasing in b because $\hat{n}'(b) > 0$. Furthermore, the right-hand side is increasing in b. The assumption $g < g^*(0)$ implies that the left-hand side exceeds the right-hand side for $b = 0$. Furthermore, if b satisfies $v'(b) = \frac{1}{n_1}$, the left-hand side is negative. We thus have a unique point $\bar{b} > 0$ where the equality above holds, namely in a high welfare economy with $n_s = n_0$. If we increase b above $\bar{b}$ we can no longer afford the welfare benefits and the economy goes bankrupt. If $b < \bar{b}$, we are in a low welfare economy. ■

Now return to the claim made in the proposition. To prove the result, we have to show that a high welfare economy is impossible. We prove this by contradiction. Suppose that an optimal b produces a high welfare economy. In that case, the following equalities hold

$$\begin{aligned}
\hat{b} &= \bar{b}, \\
n_s &= n_0, \\
x(\hat{n}) &= b, \\
\lambda_E &= \frac{1}{\hat{n}}, \\
\hat{n} &= \frac{1}{v'(b)},
\end{aligned}$$

where $\bar{b}$ is the benchmark value defined in the lemma above. The other equations follow from the features of the high welfare economy.

Employing the results from proposition 2 (i.e. $v'(x(n)) = 1/n$ for $n > n_z$ and $\lambda_E = 1/n_z$)

and the expression for $\eta(n)$ in Eq. (21), we can write (6) as

$$v'(b) = \lambda_E + \frac{v'(b)\bar{s}\int_{n_z}^{n_1} f(n)\left[\frac{n}{n_z} - 1\right]dn}{\left[F(n_s) + (1-\bar{s})(1-F(n_s))\right]}. \tag{25}$$

Substituting $\lambda_E = v'(b)$ in this equation, we arrive at

$$v'(b) = \frac{1-\bar{s}+\bar{s}F(\hat{n})}{(1-\bar{s})\frac{1}{v'(b)} + \bar{s}\int_{\hat{n}}^{n_1} f(n)\frac{1}{v'(x(n))}dn}.$$

Because $v'(x(n)) < v'(b)$ for $n > \hat{n}$, we have a contradiction, thus ruling out a high welfare economy.

To prove $T(n)+b < 0$ for $n \in [n_0, n_z]$, first recall from proposition 2 that consumption $x(n)$ for these types is determined by $v'(x(n)) = \frac{1}{n_z}$. Second, Eq. (6) with $n_z < n_1$ and $\eta(n) > 0$ for $n > n_z$ implies that $v'(b) > \lambda_E = \frac{1}{n_z}$. Hence, we find that $T(n)+b = -x(n)+b < 0$ for $n \in [n_0, n_z]$. Q.E.D.