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PRODUCT LIFE CYCLES AND INVESTMENT: A REAL OPTIONS ANALYSIS

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Product life cycles and investment: A real options analysis

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Abstract

This paper studies a firm's investment decision in production capacity where product demand follows a product life cycle (PLC), implying that demand first grows and then declines. The starting point of the decline phase is uncertain. The investment decision involves deciding about the timing and the size of the investment. We make a distinction between the firm being a product life cycle leader and a product life cycle follower. A PLC-leader will always first experience demand growth before the decline kicks in. In case of a PLC-follower, the firm enters an existing product life cycle, implying that the decline can already start before this firm even has invested.

It turns out that it makes a major difference whether the firm is a PLC-leader or PLC-follower. To benefit from a period of demand growth, the PLC-follower has an incentive to invest early, thus when the current demand level is still low. This forces the firm to limit the investment size. The PLC-leader, on the contrary, always chooses the investment size corresponding to demand growth throughout, where the probability of demand decline simply delays the moment of investment.

Keywords: product life cycle; decision making under uncertainty; real options

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1 Introduction

Product life cycles occur in a number of important product markets, including semiconductors, pharmaceuticals, the energy sector, and telecommunications (Bollen (1999), Lukas et al. (2017), Sendstad et al. (2021)). In general, the implication of a product life cycle for product demand is that initially, when the product is new, consumers learn about its existence and demand has a positive trend. Therefore, demand is expected to increase. Then, after a while, new, better, and more modern products become available due to which the demand trend of the current product will turn to be negative and demand is expected to decline. Beforehand it is not known when this decline phase will start.

In the operations management literature a number of contributions deal with various implications of a product life cycle. Bhattacharya et al. (2006) and Georgiadis et al. (2006) focus on remanufacturing used and unsold products from the previous product generation. Bradley and Guerrero (2008) analyzes the problem resulting from mismatches between product life cycles and parts, like electronic components, used in that particular product. Mehra et al. (2014) studies introducing upgrades of software products becoming obsolete during the downturn of their product life cycle.

The topic of this paper is to study a firm's investment decision when demand of the underlying product faces a product life cycle. In particular, we consider a firm having a capital investment project, where undertaking the investment implies that the firm obtains a production plant. The goods will be sold on the market at which demand is uncertain and follows a product life cycle. The research question is how the future decline phase associated with a product life cycle affects the firm's investment decision. Concerning the latter we apply a modern real options approach in the sense that the investment decision is not only about the timing, as in the literature that mainly started off with the seminal work by Dixit and Pindyck (1994), but also about the size of the investment. Dangl (1999) is one of the first papers that considers such a framework where Huisman and Kort (2015) is the first contribution that combines this approach with a duopoly model. In our setting investment size relates to the production capacity that the firm acquires.

We consider two scenarios. In the first scenario the product life cycle starts with the firm becoming active, which happens when it undertakes the investment. This means that when the firm sells its products, demand first has a positive trend, which turns negative later. In the second scenario the firm has to deal with an existing product life cycle. The main difference with the first scenario is that the decline phase can already start before the firm invests and becomes an active producer. We denote the firm in the first scenario as PLC-leader and in the second scenario as PLC-follower.

The situation of a PLC-leader can arise in connection with a revolutionary innovation (Stibel

(2011), Balter et al. (2022a)). Revolutionary innovations are innovations of products that no one else has thought of before. Then it makes sense that the product life cycle begins at the moment the firm starts the production of such a good with its investment. On the contrary, in case of an evolutionary innovation, where we think of an upgrade of an existing product, it makes sense that with the product launch the firm enters an existing product life cycle, hence the situation of a PLC-follower.

It turns out that it matters a great deal whether the firm is a PLC-leader or follower. In the case of a PLC-follower the fact that demand will decline in the future reduces the firm's investment size, compared to a situation where there is no decline phase. Concerning the timing of an investment taking place before the decline phase has started, there are three contrary effects. First, since the investment size is smaller, the investment is cheaper, which incentivizes the firm to invest earlier compared to the situation where demand does not decline in the future. Second, it is less attractive for the firm to wait with investment. This is because during the waiting phase there is always a probability that the product life cycle starts its decline phase. This also gives an incentive to invest earlier. Third, there is a net present value (NPV) effect: due to the future phase of demand decline the net present value of the investment is lower and therefore the firm will invest later. Overall, either of the effects can dominate so that in general it cannot be said whether the occurrence of a future decline in demand will speed up or delay investment.

For a PLC-leader waiting with investment is more attractive, because where a PLC-follower can already be confronted with a declining demand before it has invested, a PLC-leader is assured of demand that is expected to grow, at least up until the time of investment. The implication for the optimal investment decision of the firm is that, in order to compensate the reduction in the investment's NPV caused by the future demand decline, it simply waits for a higher current demand level to materialize before it undertakes the investment. This enables the firm to keep the investment size at the same level as when there is no future demand decline. So, compared to a situation where demand is expected to grow forever, possible occurrence of the later demand decline phase makes that the firm will delay its investment, where it keeps the size equal.

Although most of the real options literature imposes a constant, and mainly positive, demand trend, there are contributions that deviate from this scenario. We now give a concise overview of the latter and explain how our paper relates to these contributions. Both Driffill et al. (2003) and Guo et al. (2005) adopt a framework considering two regimes with different trend and volatility parameters, which can all the time switch back and forth between the regimes. Driffill et al. (2003) focuses on the timing of one single investment opportunity, whereas Guo et al. (2005) examines capacity choice where the investment behavior is both incremental and lumpy. Contrastingly, we focus on a product life cycle, thus considering just one shift from a regime with positive demand trend to a regime with negative trend, and one investment opportunity of which both the timing and the size needs to be determined.

Sendstad et al. (2021) has in common with our approach that demand trend of an existing product decreases at one moment in time. The cause is the occurrence of a new technology. Differences are that in Sendstad et al. (2021) demand trend always stays positive, while in our case demand trend is negative in the second stage of the product life cycle, and, second, Sendstad et al. (2021) restricts itself to analyzing the PLC-follower case, thus excluding the possibility that the decrease in the demand trend can only occur after the firm invests in the current technology. The fact that they make the model more complex by also considering risk aversion and the options to abandon the current technology and invest in the new technology, prevents that an analytical solution can be obtained, as we have.

Bollen (1999) also looks at a product life cycle with a positive and a negative demand trend. In our model the firm chooses its capacity level when investing and produces up to capacity from that moment on. Bollen's model is more flexible in that capacity can be adjusted along the way and serves just as an upper bound to the quantity produced. Bollen (1999) applies a numerical solution procedure, whereas our solution is analytical.

Lukas et al. (2017) derives the product life cycle from a stochastic version of the Bass (1969) model. As in our model, Lukas et al. (2017) considers a one-time investment where the firm chooses the size and produces up to capacity. Lukas et al. (2017) applies its solution procedure to a case study related to electric vehicle batteries.

Wahab et al. (2022) considers a demand system of two products. Demand of the two products is correlated and in both cases there is first growth and then decline. The paper concentrates on the value of flexibility in producing the two products together versus having fixed capacity systems for each product separately.

In the papers mentioned until now, there is ongoing demand uncertainty. Gutiérrez and Ruiz-Aliseda (2011) has deterministic demand apart from the moment that the decline of the product life cycle starts, which is uncertain. Furthermore, Gutiérrez and Ruiz-Aliseda (2011) focus on investment timing and refrain from modeling capacity choice.

The paper is organized as follows. Section 2 presents the model, whereas Section 3 analyzes the problem of the PLC-leader. The analysis of the PLC-follower is contained in Section 4, and Section 5 concludes.

2 The Model

We consider a framework with a firm that can undertake an irreversible investment to enter a market. The price at time t in this market is given by

$$P(t) = X(t)\left(1 - \eta Q(t)\right), \tag{1}$$

where $Q(t)$ is the market output, $\eta > 0$ is a constant and $X(t)$ is an exogenous shock process, of which the development reflects a product life cycle. The product life cycle implies that, first, demand is expected to increase over time, and afterwards it is expected to decline. This is modeled such that before some, beforehand unknown, time T, $X(t)$ follows a geometric Brownian motion with positive trend $\mu_1 > 0$, and afterwards the trend, denoted by $\mu_2 < 0$, is negative. We express the development of $X(t)$ over time as follows:

$$dX(t) = \begin{cases} \mu_1 X(t)\,dt + \sigma X(t)\,dW(t), & \text{for} \quad t \in [0, T) \\ \mu_2 X(t)\,dt + \sigma X(t)\,dW(t), & \text{for} \quad t \in [T, \infty) \end{cases},$$

in which $dW(t)$ is the increment of a Wiener process, and $\sigma > 0$ is a constant.

During the initial growth phase of demand there is a constant probability λdt, with $\lambda > 0$ being a positive constant, that the decline phase starts. This implies that T is exponentially distributed with parameter λ. This way of modeling the regime switch has been applied in several contributions, see , e.g, Driffill et al. (2003), Guo et al. (2005), and Sendstad et al. (2021).

The inverse demand function (1) being linear in quantity is a frequently used assumption in this literature (see, e.g., Huisman and Kort (2015), Sendstad et al. (2021)), which, unlike iso-elastic demand, shares with many other demand functions the reasonable property that demand elasticity is decreasing in price (Balter et al. (2022b)). The firm is risk neutral and discounts against rate r. To guarantee that it is optimal for the firm to invest in finite time we impose that $r + \lambda > \mu_1$ (Dixit and Pindyck (1994)).

The firm can become active on this market by investing in capacity. A unit of capacity costs δ. This implies that if the firm invests in a plant with capacity K, it incurs investment costs being equal to δK. We impose that the firm produces up to capacity. Arguments backing this assumption can be found in, e.g., Goyal and Netessine (2007). Denoting the timing of investment by t_I, it follows that

$$Q(t) = \begin{cases} 0, & \text{for} \quad t \in [0, t_I) \\ K, & \text{for} \quad t \in [t_I, \infty) \end{cases}.$$

The firm's objective is to maximize the expected discounted cashflow stream over time. Let V

denote the corresponding value of the firm. Then the investment problem that the firm is facing can be formalized as follows:

$$V(X) = \max_{t_I \geq 0, K \geq 0} E\left[\int_{t_I}^{\infty} e^{-rt} K X(t)\left(1-\eta K\right) dt - \delta K e^{-r t_I} | X(0) = X\right]. \tag{2}$$

The value of the firm, and thereby the expectation in equation (2), is conditional on the information that is available at time 0. The level of the geometric Brownian motion at that time is set equal to X.

3 The optimal investment decision of the PLC-leader

This section considers the investment decision of a firm in the situation that the product life cycle starts with this firm becoming active on the product market. This implies that from the moment the investment is undertaken there exists a time period under which product demand is expected to increase. However, during this period, with constant probability λdt the positive demand trend turns negative so that demand is expected to decline from then on.

Let X_L^*, where the subscript L corresponds to PLC-*leader,* be the value of the geometric Brownian motion at which the firm is indifferent between investing and not investing. Hence, if $X(0) < X_L^*$ it is not optimal to invest at the initial point of time. At the moment that X reaches X_L^* for the first time, the firm invests where the corresponding investment size is denoted by $K^*(X_L^*)$. It follows that the optimal investment time t_I equals the first time that the stochastic process X, starting at $X(0)$ at time zero, reaches this level X_L^*. Of course, in the opposite case where $X(0) \geq X_L^*$, it holds that $t_I = 0$ and the investment size equals $K^*(X(0))$.

Assume we are in the stopping region, i.e. $X \geq X_L^*$, so that it is optimal to invest immediately. The optimal investment size $K(X)$ is found by solving

$$\max_{K>0} E\left[\int_{0}^{\infty} e^{-rt} K X(t)\left(1-\eta K\right) dt - \delta K | X(0) = X\right], \tag{3}$$

which gives

$$K^*(X) = \frac{1}{2\eta}\left(1 - \frac{\delta\left(r+\lambda-\mu_1\right)\left(r-\mu_2\right)}{X\left(r+\lambda-\mu_2\right)}\right). \tag{4}$$

In the absence of a product life cycle, thus when the market trend is μ_1 forever and the switch to the negative trend μ_2 does not take place, it follows that if $\lambda = 0$ then $K^*(X) = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu_1)}{X}\right)$, which is larger than the capacity size in (4). We conclude that for a given level of X, indicating the level of demand at the moment of investment, the future decline phase associated with a product life cycle has a negative effect on the acquired capacity size. This is because the decline phase has a negative effect on the investment's NPV, and therefore the firm decides to invest less.

Equation (4) also implies that the optimal capacity is increasing in X. At a higher level of X, it is profitable for the firm to invest in a larger capacity so that the total profit flow increases.

As we said above, in case the initial demand level is such that $X(0) < X_L^*$, the firm waits with investment until demand has grown such that X has reached the level X_L^*, denoted as the investment threshold. Then the firm acquires the production capacity $K^*(X_L^*) = K_L^*$. The next proposition presents the corresponding investment decision.

Proposition 1. *The optimal investment threshold X_L^* and the corresponding optimal capacity level K_L^* are given by*

$$X_L^* = \frac{\beta_{(1)}+1}{\beta_{(1)}-1}\frac{\delta\left(r+\lambda-\mu_1\right)\left(r-\mu_2\right)}{r+\lambda-\mu_2}, \tag{5}$$

$$K_L^* = \frac{1}{\eta\left(\beta_{(1)}+1\right)}, \tag{6}$$

in which

$$\beta_{(1)} = \frac{1}{2}-\frac{\mu_1}{\sigma^2}+\sqrt{\left(\frac{1}{2}-\frac{\mu_1}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} > 1.$$

It is important to notice that the capacity size (6) is independent of the decline phase parameters μ_2 and λ. So, in fact, whatever the probability of the occurrence of the demand decline is, or whatever the value is of the negative trend parameter, it does not influence the investment size. Moreover, comparing the capacity size of expression (6) to the capacity size that is obtained in Huisman and Kort (2015) for the model where demand is expected to grow forever, we conclude that they are exactly the same. It follows that the future decline phase associated with the product life cycle has no effect on the investment size. Note that for given X the investment size $K^*(X)$ is decreasing in the decline parameters. What is going on when the firm invests at the threshold is that when the decline is more severe or is expected to occur earlier, it will reduce the attractiveness of the investment project. This raises the investment threshold, which in turn has a positive effect on the investment size in such a way that it fully compensates the negative effect on the investment size of the decline parameters.

In Huisman and Kort (2015) the following investment threshold is derived for the case where demand grows forever:

$$X_1^* = \frac{\beta_{(1)}+1}{\beta_{(1)}-1}\delta\left(r-\mu_1\right), \tag{7}$$

where the subscript 1 relates to having the trend $\mu = \mu_1$ forever. We obtain that the investment threshold in (5) is larger. This means that the decline phase of the product life cycle has the effect that it delays the investment. The firm waits for a higher demand level to materialize in order to compensate for the reduction in the expected NPV of the investment caused by the future demand decline.

Concerning the effect of the decline-phase parameters λ and μ_2, we obtain that

$$\frac{\partial X_L^*}{\partial \lambda} = \frac{\beta_{(1)}+1}{\beta_{(1)}-1}\frac{\delta\left(\mu_1-\mu_2\right)\left(r-\mu_2\right)}{\left(r+\lambda-\mu_2\right)^2} > 0,$$
$$\frac{\partial X_L^*}{\partial \mu_2} = -\frac{\beta_{(1)}+1}{\beta_{(1)}-1}\frac{\delta\lambda\left(r+\lambda-\mu_1\right)}{\left(r+\lambda-\mu_2\right)^2} < 0.$$

The conclusions are straightforward. First, if the probability that the decline phase starts is larger, which at the same time means that the period of demand increase is expected to be shorter (note that its expected length equals $1/\lambda$), the firm will invest at a larger threshold to compensate for the increased reduction in the expected net present value of the investment. Similarly, if the demand decline is expected to be more influential, i.e. μ_2 is more negative, the investment project is less profitable, so the firm will only invest if the current demand level is higher.

4 The optimal investment decision of the PLC-follower

It is important to realize that in the problem of the PLC-leader demand has a positive trend as long as the firm has not invested yet. This makes waiting with investment attractive, and explains the fact that the future decline phase of the product life cycle does not affect the investment size. In such a case the firm simply waits until demand has grown to a higher level and invests then. However, in the situation of a PLC-follower, to be analyzed in this section, demand can enter its decline phase already before the firm has invested. This makes waiting with investment less attractive and our aim is to establish how this will affect the firm's optimal investment decision.

In principle there are two possibilities regarding the timing of the investment. Like previously, investment takes place at the moment demand has a positive trend, or it can be that the trend has already turned negative at the moment of investment. In the latter case the trend is constant over time from then on and is given by $\mu_2 < 0$. From Huisman and Kort (2015) we get that the firm invests at the moment the demand level is such that the threshold

$$X_2^* = \frac{\beta_{(2)}+1}{\beta_{(2)}-1}\delta\left(r-\mu_2\right), \tag{8}$$

is reached, with

$$\beta_{(2)} = \frac{1}{2}-\frac{\mu_2}{\sigma^2}+\sqrt{\left(\frac{1}{2}-\frac{\mu_2}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} > 1.$$

The corresponding capacity size equals

$$K_2^* = \frac{1}{\eta\left(\beta_{(2)}+1\right)}. \tag{9}$$

It is straightforward to obtain that

$$K_2^* < K_1^* = \frac{1}{\eta\left(\beta_{(1)}+1\right)}. \tag{10}$$

The intuition is that if demand grows more, the firm can sell a given amount against a higher price and then it is more attractive to increase production.

Next, let us turn to the case that the firm invests before the decline phase has started. As we know the investment decision consists of deciding about the timing and the size. Concerning establishing the latter, note that right at the time of the investment the optimization problem is similar to the one of the PLC-leader. In both cases that level of capacity will be chosen that maximizes the investments' expected NPV. This comes down to solving the maximization problem (3), resulting in the capacity level (4), i.e.

$$K^*(X) = \frac{1}{2\eta}\left(1 - \frac{\delta\left(r+\lambda-\mu_1\right)\left(r-\mu_2\right)}{X\left(r+\lambda-\mu_2\right)}\right).$$

The following proposition presents the investment decision X_F^* of the PLC-*follower*, given that the investment is undertaken before the demand decline phase has started.

Proposition 2. *In the situation that the product life cycle has demand trend* $\mu_1 > 0$*, then the optimal investment threshold* X_F^**, is implicitly given by*

$$\frac{\lambda\left(\beta_{(2)}-\beta_\lambda\right)}{\lambda-\beta_{(2)}\left(\mu_1-\mu_2\right)}\frac{\delta}{\eta\left(\beta_{(2)}^2-1\right)}\left(\frac{X_F^*}{X_2^*}\right)^{\beta_{(2)}} + \frac{1}{2\eta}\left(1-\frac{\delta\left(r+\lambda-\mu_1\right)\left(r-\mu_2\right)}{X_F^*\left(r+\lambda-\mu_2\right)}\right)$$
$$\times\left(\frac{1}{2}X_F^*\left(\beta_\lambda-1\right)\left(\frac{\left(r+\lambda-\mu_2\right)}{\left(r+\lambda-\mu_1\right)\left(r-\mu_2\right)}+\frac{\delta}{X_F^*}\right)-\delta\beta_\lambda\right)=0,\ \textit{if}\ X_F^*<X_2^*, \tag{11}$$

and

$$
\begin{aligned}
&\frac{\lambda(1-\beta_\lambda)}{4\eta(r-\mu_2)(r-\mu_1+\lambda)}\left(X_F^*-X_2^*\left(\frac{X_F^*}{X_2^*}\right)^{\beta_\lambda^-}\right)+\frac{\lambda\delta\beta_\lambda}{2\eta(r+\lambda)}\left(1-\left(\frac{X_F^*}{X_2^*}\right)^{\beta_\lambda^-}\right) \\
&-\frac{\lambda\delta^2(r-\mu_2)(1+\beta_\lambda)}{4\eta(r+\mu_1+\lambda-\sigma^2)}\left((X_F^*)^{-1}-(X_2^*)^{-1}\left(\frac{X_F^*}{X_2^*}\right)^{\beta_\lambda^-}\right)+\frac{\lambda\left(\beta_{(2)}-\beta_\lambda\right)}{\lambda-\beta_{(2)}\left(\mu_1-\mu_2\right)}\frac{\delta}{\eta\left(\beta_{(2)}^2-1\right)}(X_2^*)^{\beta_{(2)}}\left(\frac{X_F^*}{X_2^*}\right)^{\beta_\lambda^-} \\
&+\frac{1}{2\eta}\left(1-\frac{\delta\left(r+\lambda-\mu_1\right)\left(r-\mu_2\right)}{X_F^*\left(r+\lambda-\mu_2\right)}\right)\left(\frac{1}{2}X_F^*\left(\beta_\lambda-1\right)\left(\frac{\left(r+\lambda-\mu_2\right)}{\left(r+\lambda-\mu_1\right)\left(r-\mu_2\right)}+\frac{\delta}{X_F^*}\right)-\delta\beta_\lambda\right)=0 \text{ if } X_F^*\geq X_2^*.
\end{aligned} \tag{12}
$$

The constants β_λ and β_λ^- are the positive and negative root of

$$\frac{1}{2}\sigma^2\beta_\lambda(\beta_\lambda-1)+\mu_1\beta_\lambda-(r+\lambda)=0.$$

At the threshold the optimal capacity level to be acquired is given by

$$K_F^*=K^*\left(X_F^*\right)=\frac{1}{2\eta}\left(1-\frac{\delta\left(r+\lambda-\mu_1\right)\left(r-\mu_2\right)}{X_F^*\left(r+\lambda-\mu_2\right)}\right). \tag{13}$$

So unfortunately, an explicit expression for the investment threshold is not available. The reason is that multiple effects play a role regarding the optimal determination of the investment timing. Compared to the case where demand grows forever with demand trend $\mu=\mu_1>0$, it is less attractive for the firm to wait with investment. This is because during the waiting phase there is always a probability that the product life cycle starts its decline phase. This gives an incentive for the investment to take place *sooner* thus at a lower threshold level.

Then there is the NPV effect. Compared to the case of "demand growth forever", the decline phase associated with the product life cycle reduces the NPV of the investment. This makes the investment project less attractive and therefore this gives an incentive for the firm to invest *later*.

Third, there is a so-called quantity effect, resulting from the size of the investment. As we will obtain from Figure 1 later on, the acquired capacity will be in between K_2^* and K_1^*, being the capacity sizes corresponding to investing at the thresholds in case of a trend being equal to μ_2 and μ_1 forever, respectively (see (9) and (10)). So, the investment size of the PLC-follower falls below K_1^*, which also holds for the investment cost. Therefore, the firm can afford to invest at a lower current demand level. Hence, this quantity effect is such that the firm will invest *sooner*. To conclude, compared to the case where demand grows forever with demand trend $\mu=\mu_1>0$, there are two effects pointing to an earlier investment and one effect making that the firm will invest later. The total effect can go both ways, generally speaking.

Since we only have an implicit expression for the investment threshold, we have to resort to a numerical exercise for a further analysis of the investment decision of the PLC-follower. This is done in Figure 1 showing the investment threshold X_F^* and the investment size K_F^* as a function of λ. In fact, the panels in Figure 1 each depict three different curves, namely one, with threshold X_1^* or investment size K_1^*, related to the case where $\mu = \mu_1$ forever, one, with threshold X_2^* or investment size K_2^*, related to the case where $\mu = \mu_2$ forever, and the curve, with threshold X_F^* or investment size K_F^*, corresponding to the PLC-follower's investment decision.

Note that when $\lambda = 0$ the probability that the growth phase of the product life cycle turns into the decline phase is zero. Then the PLC-follower case is similar to the case where $\mu = \mu_1$ forever, which explains why the two corresponding curves intersect at the vertical axis in all panels. In the other extreme case, thus when $\lambda = \infty$, the growth phase immediately turns into the decline phase, so that then the PLC-follower case is similar to the case where $\mu = \mu_2$ forever. This explains why the curves corresponding to the PLC-follower investment decision all converge to the curves related to $\mu = \mu_2$ forever when λ increases.

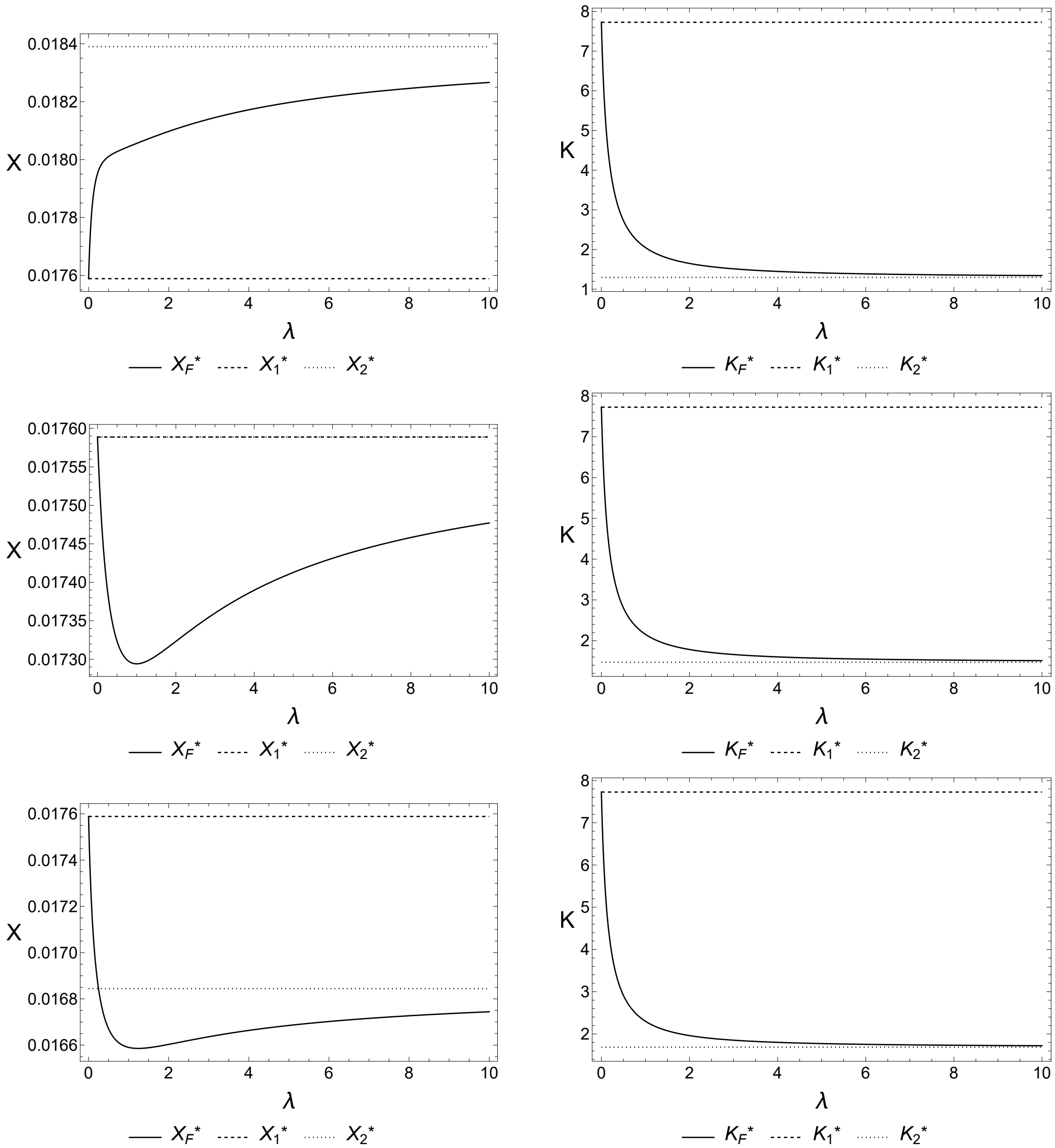
The three cases in Figure 1 differ in the relative position of the investment thresholds X_1^* and X_2^*. In the above two panels we deal with a situation where $X_1^* < X_2^*$, whereas we have the opposite case in the two panels at the bottom. Concerning the two panels in the middle these two thresholds are equal. The reason that we can have these three cases is that, for the problem where the demand trend stays at the same level, the investment threshold is first decreasing and then increasing in the market trend μ. In particular, we obtain from Balter et al. (2022b) that

$$\frac{dX^*}{d\mu} \begin{cases} < 0 & \text{if} \quad \mu < \frac{1}{2}\sigma^2 \\ > 0 & \text{if} \quad \mu > \frac{1}{2}\sigma^2 \end{cases}.$$

Figure 2 illustrates this result for the parameter values of Figure 1. One can check that, indeed, the threshold for $\mu = -0.06$ exceeds the one for $\mu = 0.06$ (upper panels), whereas the latter in turn is larger than the one associated with $\mu = -0.04$ (bottom panels), and the ones for $\mu = 0.06$ and $\mu = -0.05$ are equal (middle panels). In case the investment size is given, the investment threshold X^* would be decreasing in the demand trend μ. The intuition is that when demand is expected to grow more, it is more attractive to invest, and therefore the firm will invest earlier at a *lower* threshold. However, now that the firm can also choose the investment size there is a quantity effect in that when demand is expected to grow more, the firm wants to acquire a larger production capacity. This makes the investment more expensive, and therefore the firm wants to invest at a moment that the demand level is higher, thus when the threshold is *larger.* The two contrary effects cause the nonmonotonicity of the threshold behavior in Figure 2.

To explain Figure 1 it is convenient to start with considering how the investment size depends on λ. This is depicted in the panels on the right side. They all have the same qualitative structure.

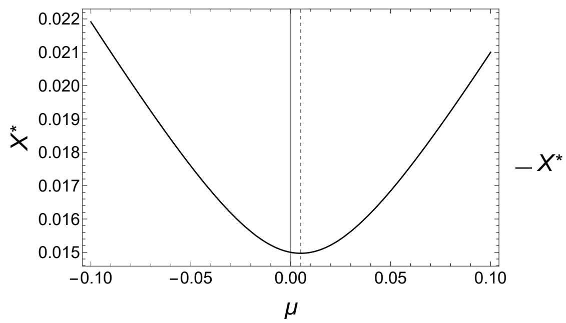
Figure 1: *The investment threshold and investment size corresponding to the PLC-follower's optimal investment decision for three different scenarios. The parameter values are* $r = 0.1, \mu_1 = 0.06, \delta = 0.1, \eta = 0.05, \sigma = 0.1$, *and* $\mu_2 = -0.06$ *for the above two panels,* $\mu_2 = -0.05$ *for the two panels in the middle, and* $\mu_2 = -0.04$ *for the bottom two panels.*



As we said before, $\lambda = 0$ corresponds with demand growth forever with $\mu_1 > 0$, so then the investment size is K_1^*. Furthermore, we also said before that $\lambda = \infty$ in fact means that demand is expected to decline forever with $\mu_2 < 0$, where the investment size is K_2^*. From (10) we obtain that $K_2^* < K_1^*$, as confirmed in Figure 1.

When λ increases it reduces the expected time of the demand growth phase, which equals

Figure 2: *The optimal investment threshold as a function of the demand trend, in case the latter does not change over time. The parameter values are* $r = 0.1, \delta = 0.1, \eta = 0.05$, *and* $\sigma = 0.1$. *The dashed line represents* $\mu = \frac{1}{2}\sigma^2$.



$1/\lambda$. This reduction goes fast for low values of λ whereas it approaches a horizontal asymptote of zero when λ becomes larger. Looking at Figure 1 we see that the same holds for the investment size with the difference that the horizontal asymptote is K_2^*. The reason is that the resulting investment size under the product life cycle is in fact a weighted average of the optimal investment sizes under μ_1 and μ_2, which are K_1^* and K_2^*, respectively. The investment quantity decreases fast when λ is small because the expected time of the demand growth phase reduces fast there, and this then also holds for the weight attached to K_1^*. When λ is larger the expected time of the demand growth phase hardly declines when λ increases marginally, and this then also holds for the investment size.

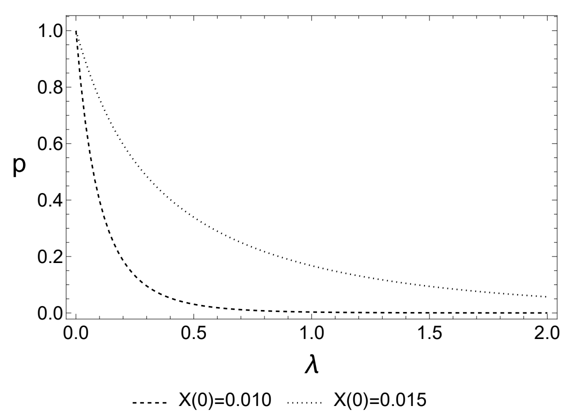
Next, we discuss how the investment threshold depends on λ. Consider first the upper-left panel of Figure 1, where $X_1^* < X_2^*$. This points to the fact that the NPV effect, i.e. investing early at a lower demand threshold when the demand trend is larger, dominates the quantity effect, i.e. investing late at a larger demand threshold when the demand trend is larger, because the firm invests more so that the investment is more expensive. In fact, this holds for every value of λ, because when λ increases the growth phase of the product life cycle is expected to last shorter. This means that the expected NPV of the investment project decreases. This makes that the firm invests later, i.e. the firm waits for a larger demand level to materialize to compensate for the reduction in the NPV, before it undertakes the investment.

Next, consider the lowest-left panel of Figure 1. Here it holds that $X_1^* > X_2^*$, so that the quantity effect dominates. For low levels of λ the investment threshold sharply declines. This is caused by the fact that the investment amount declines sharply too, which makes the investment a lot cheaper. Due to the quantity effect, and also the fact that the firm has an incentive to invest before the decline phase begins, the firm invests sooner. When λ is larger, the investment amount is approaching the horizontal asymptote K_2^*, and thus hardly changes. It follows that there is a very small quantity effect, and therefore, due to the dominating NPV effect, the investment threshold is increasing and converges to X_2^*.

According to the same reasoning the investment threshold first decreases and then increases in the middle-left panel of Figure 1. The difference is that now $X_1^* = X_2^*$ so that the curve representing the investment threshold in the product life cycle case, begins and ends at the same level.

For the PLC-follower the decline phase can basically start anytime, implying that the firm, in case it does not invest at time zero, does not know beforehand whether the investment is undertaken before, thus when (11) and (13) apply, or after the decline phase starts, with decisions (8) and (9). For the parameter constellation of the upper panels of Figure 1, Figure 3 gives the probability that the firm invests before the decline phase.[1] Figure 3 shows that if the initial demand level is higher, the probability that the firm invests during the growth phase is larger. This is because then the investment threshold is reached sooner. Furthermore, if λ increases the probability that demand growth turns into demand decline is larger within a certain amount of time, and, as a consequence, it will be more difficult for the firm to already reach its investment threshold during the growth phase.

Figure 3: *The probability, denoted by p, that the investment is undertaken during the growth phase of the product life cycle. The parameter values are* $r = 0.1, \mu_1 = 0.06, \delta = 0.1, \eta = 0.05, \sigma = 0.1,$ *and* $\mu_2 = -0.06$. *The black dashed curve holds for* $X(0) = 0.010$, *and the black dotted curve results from having* $X(0) = 0.015$.



[1]According to Sarkar (2000), the probability of reaching the critical level X – the probability of investing – within some time period t is given by $P(X,t) = \Phi\left(\frac{\ln(X_0/X)+(\mu-\frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}\right) + \left(\frac{X}{X_0}\right)^{2\frac{\mu}{\sigma^2}-1}\Phi\left(\frac{\ln(X_0/X)-(\mu-\frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}\right)$. Conditional on being before the start of the decline phase, the threshold follows $dX(t) = \mu_1 X(t)dt + \sigma X(t)dW(t), X(0) = X_0$. The probability that the firm has invested while the event of a switch has not taken place yet is given by $p = \int_0^\infty P(X_F^*, t)\lambda e^{-\lambda t}dt$.

5 Conclusions

The development of demand over time of many products behaves according to a product life cycle. In general this means that initially, when consumers have to detect and get used to the product, demand is increasing over time. After a while better products have been introduced that drive the current product out of the market, the current product gets outdated or out of fashion, all of which leads to demand being decreasing over time. In the literature many economic consequences of the product life cycle have been analyzed. The aim of the present paper is to study how the firm should invest in this market knowing that the product's demand is subject to a typical product life cycle development. In our model we take into account that future demand is uncertain, like also the moment in time that the decline-phase starts, and that the investment decision consists of both deciding when to invest and how much to invest.

We look at this investment problem in two different cases. First, we consider a scenario where the product life cycle essentially starts at the moment the firm invests. Such a case will occur if this firm is the market leader and this firm knows for sure that right after the investment it can enjoy a period in which product demand increases over time. We find that occurrence of the future demand decline will delay the firm's investment but does not influence the size. In fact the firm waits for a larger demand level to be realized in order to compensate for the future demand reduction.

The second scenario we consider is where the firm has to deal with an existing product life cycle. The difference is that now the decline phase can start any time, which could then also be before the firm has undertaken the investment. Compared to a situation where demand keeps on growing, in this situation the future demand decline associated with a product life cycle will accomplish that the firm will reduce the investment size. Regarding the optimal time to invest, multiple contrary effects play a role and therefore the future occurrence of a decline in demand may either accelerate or delay investment.

An important extension of the work we present in this paper would be to take into account competition. For instance, with a duopoly model we could consider a situation that the product life cycle starts at the moment the first firm invests. Then this firm is assured of an initial time period in which product demand grows. However, the second investor is in the situation of the second scenario described above where the product life cycle can actually already enter its decline phase before the second firm has invested. This characteristic may influence the incentive to preempt the other firm with investing, and would as such be a relevant addition to the industrial organization and the operations management literature, as well as the theory of preemption games in real options (Thijssen et al. (2012), Huisman and Kort (2015), Riedel and Steg (2017)).

A Appendix

This appendix contains the proofs of the propositions.

A.1 Proof of Proposition 1

Consider the stage in which demand has a positive trend μ_2 but with probability λdt switches to a negative trend μ_1. The expected value of the project at time t in regime one can be expressed as the sum of the operating profit over the interval $(t, t+dt)$ and the continuation value beyond $t+dt$. The Bellman equation for $V^{(1)}(X,K)$ is given by

$$rV^{(1)}(X,K) = P(t)K + \lim_{dt\downarrow 0} \frac{1}{dt}\mathbb{E}[dV^{(1)}(X,K)]. \tag{14}$$

Using Ito's lemma we get

$$\begin{aligned} \mathbb{E}[dV^{(1)}(X,K)] \quad = \quad & \lambda dt\left(-V^{(1)}(X,K)+V^{(2)}(X,K)\right) \\ & +(1-\lambda dt)\left(V_X^{(1)}(X,K)\mu_1 X dt + \frac{1}{2}\sigma^2 X^2 V_{XX}^{(1)}(X,K)dt\right) + o(dt). \end{aligned} \tag{15}$$

Substitution of (15) into (14) gives

$$rV^{(1)}(X,K) = KX(1-\eta K) + V_X^{(1)}(X,K)\mu_1 X + \frac{1}{2}\sigma^2 X^2 V_{XX}^{(1)}(X,K) + \lambda\left(-V^{(1)}(X,K)+V^{(2)}(X,K)\right),$$

where with probability λdt the value switches to the value function belonging to a μ_2-regime which is given by $V^{(2)}(X,K) = \frac{XK(1-\eta K)}{r-\mu_2}$, and follows directly from Huisman and Kort (2015). The associated homogeneous equation (involving the value function terms only) is then

$$rV^{(1)}(X,K) = V_X^{(1)}(X,K)\mu_1 X + \frac{1}{2}\sigma^2 X^2 V_{XX}^{(1)}(X,K) - \lambda V^{(1)}(X,K),$$

with solution $V^{(1)}(X,K) = B_1 X^{\beta_\lambda} + B_2 X^{\beta_\lambda^-}$ where β_λ and β_λ^- are the positive and negative root of $\frac{1}{2}\sigma^2\beta_\lambda(\beta_\lambda - 1) + \mu_1\beta_\lambda - (r+\lambda) = 0$. For a particular solution of the total equation we propose $V^{(1)}(X,K) = aX + b$. The total solution is the sum of the homogeneous solution and particular solution

$$V^{(1)}(X,K) \quad = \quad \frac{K(1-\eta K)X(r-\mu_2+\lambda)}{(r-\mu_1+\lambda)(r-\mu_2)} + B_1 X^{\beta_\lambda} + B_2 X^{\beta_\lambda^-}. \tag{16}$$

The boundary conditions are

$$V^{(1)}(0,K) = 0 \tag{17}$$

$$\lim_{X\to\infty} V^{(1)}(X,K) = wX. \tag{18}$$

Since $\beta_\lambda^- < 0$, $X^{\beta_\lambda^-}$ will go to infinity when X goes to zero. Thus (17) leads to $B_2 = 0$. And (18), which implies $B_1 = 0$, refers to the exclusion of speculative bubbles, i.e., in the limit the value function is linear in X, where w is a constant. Hence, the value function in the μ_1-regime at the moment of investment solves for

$$V^{(1)}(X,K) = \frac{K(1-\eta K)X(r-\mu_2+\lambda)}{(r-\mu_1+\lambda)(r-\mu_2)}.$$

The optimal capacity $K^*(X)$ as given in (4) is obtained by solving

$$\frac{\partial V^{(1)}(X,K)-\delta K}{\partial K} = 0.$$

If the event of a switch in the demand process can only happen after investment, then the waiting region is given by

$$rF^{(1)}(X) = \frac{\partial F^{(1)}(X)}{\partial X}\mu_1 X + \frac{1}{2}\frac{\partial^2 F^{(1)}(X)}{\partial X^2}\sigma^2 X^2,$$

which solves for $F^{(1)}(X) = A_1 X^{\beta_{(1)}}$, where $\beta_{(1)}$ is the positive root of

$$\frac{1}{2}\sigma^2\beta_{(1)}^2 + \left(\mu_1 - \frac{1}{2}\sigma^2\right)\beta_{(1)} - r = 0.$$

To determine the optimal threshold, we employ the value matching and smooth pasting conditions

$$\begin{aligned} V^{(1)}(X_L^*,K)-\delta K &= F^{(1)}(X_L^*) \\ \left.\frac{\partial V^{(1)}(X,K)}{\partial X}\right|_{X=X_L^*} &= \left.\frac{\partial F^{(1)}(X)}{\partial X}\right|_{X=X_L^*}. \end{aligned}$$

Together these imply

$$X_L^*(K) = \frac{\beta_{(1)}}{\beta_{(1)}-1}\frac{\delta(r-\mu_1+\lambda)(r-\mu_2)}{(1-\eta K)(r-\mu_2+\lambda)},$$

and with (4) lead to (5) and (6).

A.2 Proof of Proposition 2

Now that the decline in demand can also occur in the waiting period, $F(X)$ has to be composed as follows. Let $F^{(2)}(X)$ be the option value when the drift of X is μ_2, then

$$rF^{(2)}(X) = \frac{\partial F^{(2)}(X)}{\partial X}\mu_2 X + \frac{1}{2}\frac{\partial^2 F^{(2)}(X)}{\partial X^2}\sigma^2 X^2,$$

which solves for a form of $F^{(2)}(X) = A_2 X^{\beta_{(2)}}$ where $\beta_{(2)}$ is the positive root of

$$\frac{1}{2}\sigma^2\beta_{(2)}(\beta_{(2)}-1)+\mu_2\beta_{(2)}-r=0.$$

After this waiting period, only $V^{(2)}(X,K)$ can happen, as the decline has already been set in motion. The value matching and smooth pasting conditions applied to $V^{(2)}(X,K)$ and $F^{(2)}(X)$ give (8) and (9) similar as in Huisman and Kort (2015) with μ_2 as drift parameter. Recall that if X is smaller than this optimal threshold, it is better to wait, and otherwise it is optimal to invest:

$$F^{(2)}(X) = \begin{cases} A_2 X^{\beta_{(2)}} & \text{if } X < X_2^* \\ V^{(2)}(X, K^*_{(2)}(X)) - \delta K^*_{(2)}(X) & \text{if } X \geq X_2^* \end{cases},$$

where $K^*_{(2)}(X) = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu_2)}{X}\right)$.

In case the switch from growth to decline has not happened yet, the waiting period is characterized by

$$rF^{(1)}(X) = \frac{\partial F^{(1)}(X)}{\partial X}\mu_1 X + \frac{1}{2}\frac{\partial^2 F^{(1)}(X)}{\partial X^2}\sigma^2 X^2 + \lambda\left(-F^{(1)}(X)+F^{(2)}(X)\right). \tag{19}$$

We can now distinguish two cases, $X < X_2^*$ and $X \geq X_2^*$:

If $X \geq X_2^*$ then (19) solves for

$$F^{(c1)}(X) = C_1 X^{\beta_\lambda} + C_2 X^{\beta_\lambda^-} + \frac{\lambda}{4\eta(r-\mu_2)(r-\mu_1+\lambda)}X - \frac{\lambda\delta}{2\eta(r+\lambda)} + \frac{\lambda\delta^2(r-\mu_2)}{4\eta(r+\mu_1+\lambda-\sigma^2)}X^{-1},$$

where β_λ and β_λ^- are the positive and negative root of

$$\frac{1}{2}\sigma^2\beta_\lambda(\beta_\lambda-1)+\mu_1\beta_\lambda-(r+\lambda)=0.$$

Since $X \geq X_2^*$, the boundary condition $F^{(c1)}(0) = 0$ does not hold and thus $C_2 \neq 0$. In the limit the waiting region cannot go to infinity because there is a point in which investment becomes optimal, therefore also $C_1 \neq 0$.

If $X < X_2^*$ then (19) solves for

$$F^{(d1)}(X) = D_1 X^{\beta_\lambda} + \frac{\lambda}{\lambda - \beta_{(2)}\left(\mu_1 - \mu_2\right)} A_2 X^{\beta_{(2)}}.$$

Now $F^{(d1)}(0) = 0$, implies $D_2 = 0$.

For continuity and differentiability, we need, at X_2^*

$$\begin{aligned} F^{(c1)}(X_2^*) &= F^{(d1)}(X_2^*) \\ \frac{\partial F^{(c1)}(X)}{\partial X}\bigg|_{X=X_2^*} &= \frac{\partial F^{(d1)}(X)}{\partial X}\bigg|_{X=X_2^*}. \end{aligned}$$

To solve for the optimal threshold X_F^* we employ the value matching and smooth pasting conditions

$$\begin{aligned} V^{(1)}(X_F^*, K) - \delta K &= F^{(1)}(X_F^*) \\ \frac{\partial V^{(1)}(X,K)}{\partial X}\bigg|_{X=X_F^*} &= \frac{\partial F^{(1)}(X)}{\partial X}\bigg|_{X=X_F^*}, \end{aligned}$$

where

$$F^{(1)}(X) = \begin{cases} F^{(c1)}(X) & \text{if } X \geq X_2^* \\ F^{(d1)}(X) & \text{if } X < X_2^* \end{cases}.$$

Substitution of the value functions into these equations, leads, after some algebra, to expressions (11) and (12).

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