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## **Law enforcement and the black market exchange rate**

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This paper examines how a system of multiple exchange rates can give rise to unreported exports and imports and a black market for foreign exchange. Risk aversion rather than an assumed smuggling or detection technology generally guarantees the coexistence of legal and illegal trading. Evidence for Brazil suggests that enforcement of the exchange rate and trade regime is an important fundamental of the black market exchange rate that has so far invariably been left out of empirical work.

Several authors have analyzed how international trade in violation of exchange or trade restrictions can give rise to black markets for foreign exchange. After the early graphical analysis of Sheikh (1976), de Macedo (1982), Dornbusch (1983), and Nowak (1984) have modeled black markets for foreign exchange in the tradition of asset market models or exchange rate determination. Greenwood and Kimbrough (1987) examine illegal international trade in a two period model that allows for explicit maximizing behavior on the part of international traders. Pitt (1984) considers a smuggling technology where legal and illegal trade coexist as legal trade serves to cloak illegal trade. Bhandari and Végh (1990) consider dual exchange markets with incomplete market segmentation. Essential in these models, and in de Macedo (1987), who extends a flow model of demand for and supply of illegal foreign exchange to a full portfolio model, is the treatment of the illegal trading technology. All papers implicitly or explicitly assume convex cost functions or probabilities of detection that increase in the share of illegal trade to obtain the coexistence of legal and illegal trading channels. If costs were linear or the probabilities of detection constant, then presumably there would be only legal or illegal trading.

In this paper risk aversion on the part of traders is shown to be sufficient for traders generally to combine legal with illegal transactions. The probability of detection is assumed to be exogenous to the trading decision and is treated as a policy variable, as is the penalty imposed on detected illegal exporters and importers. Risk aversion motivates trading in the black market for foreign exchange, as lucky and unlucky black marketeers have a need to readjust their financial portfolios after any penalties have been imposed. Risk aversion as a rationale for the coexistence of legal and illegal trading thus provides a basis for

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distinguishing between illegal international trade and trading on the black market for foreign exchange.

The *sine qua non* of a black market for foreign exchange is restrictive exchange rate policy. In this paper, restrictive exchange rate policy takes the form of multiple exchange rates for imports and exports, or alternatively of different present and future exchange rates. The exchange rate wedge between the rates at which private agents buy and sell foreign exchange can be interpreted as a tax on reported international transactions. As a result, the analysis of exchange restrictions violations is largely parallel to models of tax evasion in the domestic economy such as Allingham and Sandmo (1972).<sup>1</sup> The model analyzes the impact of a change in the exchange rate wedge on the relative magnitudes of legal and illegal trade and on the black market exchange rate for the cases where the change is expected and unexpected, *i.e.*, before and after the initial legal and illegal export decision is made.

Law enforcement, as the word implies, has two sides: the letter of the law, and the degree of its enforcement. While legal statutes and the penalties they prescribe generally evolve only slowly over time, there may be considerable fluctuation in the zeal with which the authorities apply them. Thus an indicator of law enforcement is the number of people charged with violations of importation laws. For Brazil we show empirically that there has been large variation in the number of people charged with import violations, and that these numbers are negatively correlated to changes in the black market exchange rate premium.

The remainder of this paper is organized as follows. Section I describes the basic model, and it derives the sources of demand and supply in the black market for foreign exchange. Section II examines when legal and illegal channels of international trade exist and coexist. Section III examines the black market equilibrium, and establishes some comparative statics results on the black market exchange rate and relative volumes of legal and illegal trading. Section IV presents some elementary evidence on the importance of law enforcement to black market exchange rate determination for the case of Brazil. Section V concludes the paper.

## I. The model

After a basic outline of the model, the sources of demand for and supply of foreign exchange in the black market are described in turn.

### *I.A. Basic outline of the model*

Domestic residents receive an endowment  $E$  of an exportable good  $X$  that is either consumed or exported. In addition to the exportable good, residents consume an importable good  $I$ .  $X$  and  $I$  can be thought of as two different traded goods in a flow model, but also as present and future goods in a two-period model. The international prices of  $X$  and  $I$  in terms of foreign exchange,  $F$ , are one. Foreign exchange and domestic money,  $M$ , only serve as stores of value. Thus the private sector is only interested in acquiring these financial assets by way of exporting to be able to finance later imports. Domestic residents are assumed to start out without any financial assets.

The exchange rate system requires exporters to surrender all foreign exchange earnings to the central bank in return for domestic money at the export exchange

rate  $e_x$ . On the import side, traders can convert domestic currency into foreign exchange at the import exchange rate  $e_i$  without any convertibility limit.<sup>2</sup> The import exchange rate is no less than the export exchange rate, or  $e_i \geq e_x$ . For  $e_i = e_x$ , the exchange rate system is neutral and the authorities only serve as a clearing house for foreign exchange. However, if  $e_i > e_x$ , the system of multiple exchange rates is equivalent to a tariff of import tax  $\tau$  equal to  $1 - e_x/e_i$ . In this instance, exchange controls have positive revenue implications, which may explain why they exist in the first place.

For the implicit exchange tax to be meaningful, it has to be enforced. Enforcement policy and the punishment of violators can take various forms, and any formalization is likely to be incomplete. Here we assume that enforcement takes the form of penalties that are proportional to the volumes of the illegal trading transactions. A look at enforcement policy in Brazil in Section IV reveals that proportional fines indeed are the usual means of punishment in that country. A further question is when in the cycle of illegal trading transactions enforcement takes place. Exchange restriction violations entail the acquisition, subsequent holding or trading, and ultimate disposal of illegal foreign exchange. Each of these steps is illegal, and enforcement in principle could occur at each of the three stages. The modeling choice has nontrivial consequences for the path of the black market exchange rate, as illegal foreign exchange is more valuable after than before any round of inspection and enforcement. In this paper we focus on enforcement at the time when illegal international trade is transacted, which implies foreign exchange can be held or exchanged for domestic money with impunity. Illegal foreign exchange trading may be harder to police than illegal international goods trade, as financial assets are small in volume and easy to conceal.

Enforcement is conducted by a border police that checks exporters and importers with probabilities  $\pi_x$  and  $\pi_i$ , respectively. The two probabilities can in principle be different from each other, and again a reason may be that a country's exports and imports differ in physical volumes. When a violation is detected, the police confiscates shares  $\Gamma_x$  and  $\Gamma_i$  of the illegal foreign exchange involved in the unauthorized export and import transaction, respectively. The private sector views  $\pi_x$ ,  $\pi_i$ ,  $\Gamma_x$ , and  $\Gamma_i$  as given, but these enforcement parameters are choice variables of the government.<sup>3</sup>

Regardless of any implied illegality, the representative agent sets out to maximize his expected utility using legal and illegal trading channels. There are a number of ways in which we can sequence the agent's trading and consumption decisions. If we were interested in a rather short-term flow model of illegal trade, we could assume that the agent postpones all consumption till after all trading has taken place and all uncertainty regarding enforcement has been resolved. However, as most illegal assets are probably held for a while, more relevant may be the scenario, examined here, where the agent first makes a consumption and exporting decision, then at an intermediate stage chooses how much foreign exchange to buy or sell in the black market, and finally consumes the proceeds of his legal and illegal assets.

The sequence of events is thus as follows. Exporters first decide how much to consume, and export legally or illegally ('stage 1'). Simultaneously with the exports, export enforcement takes place. After the export stage, exporters hold domestic money and the illegal foreign exchange. Exporters then meet with

prospective importers at a black market for foreign exchange, where the black market exchange rate is determined ('stage 2'). At the conclusion of black market trading, importers use all their financial assets to import ('stage 3'). During importation, import enforcement takes place. After the import stage, importers consume their importable ('stage 4').

The central link in the sequence is the black market for foreign exchange at stage 2 that brings together the supply of foreign exchange stemming from the underinvoicing of exports at stage 1, and the demand for illegal foreign exchange to finance unauthorized imports at stage 3. After the resolution of the enforcement uncertainty at stage 1, there are two types of agents at stage 2: lucky and unlucky traders depending on whether their illegal exports were detected and partially confiscated. The demand for or supply of foreign exchange of any individual trader at stage 2 is a stochastic variable. By the law of large numbers, however, total supply and demand for foreign exchange, and thus the black market exchange rate, are deterministic for a given enforcement policy.

The agent's preferences can be described by the following separable, iso-elastic utility specification

$$\langle 1 \rangle \quad U(C_x, C_i) = C_x^\delta + C_i^\delta \quad 0 < \delta \leq 1,$$

where  $C_x$  and  $C_i$  are the levels of consumption of the exportable and importable goods, respectively.

We now turn to the determination of the demand for and supply of illegal foreign exchange.

#### *I.B. Demand for illegal foreign exchange*

Let an agent have domestic money holdings  $M$  and illegal foreign exchange holdings  $F$  at the closing of the black market of stage 2. As illegal imports are detected with probability  $\pi_i$ , his importable consumption at stage 4 will be

$$\langle 2 \rangle \quad C_i^1 = \frac{M}{e_i} + (1 - \Gamma_i)F \quad \text{with probability } \pi_i,$$

and

$$\langle 3 \rangle \quad C_i^2 = \frac{M}{e_i} + F \quad \text{with probability } 1 - \pi_i.$$

It should be remembered that the international price of the importable in terms of foreign exchange is one.

As stated before, agents differ in their wealth at stage 2, as some have been caught by the border police and others have not. The iso-elastic utility index has the important property, as discussed in Samuelson (1969), that the optimal composition of a portfolio of a safe and a risky asset is independent of total wealth. This means that at the conclusion of illegal exchange trading at stage 2 all agents will hold domestic money and illegal foreign exchange in equal proportion, although they come to the black market with unequal portfolios depending on whether their illegal exports were detected at stage 1. Thus, relative demand for domestic money and foreign exchange depends on the implied rates of return on these two assets, but not on individual wealth.

An agent, with arbitrary nominal wealth in terms of domestic currency  $W_n$  at stage 2, now has to determine his optimal share, denoted  $s$ , of his wealth that he wishes to hold in the form of illegal foreign exchange. The share  $s$ , once determined, implies the following holdings of domestic money and illegal foreign exchange

$$\langle 4 \rangle \quad M = (1-s)W_n,$$

$$\langle 5 \rangle \quad F = s \frac{W_n}{B},$$

where  $B$  is the black market exchange rate.

The resident chooses the share  $s$  so as to maximize his expected utility of importable consumption  $V = EU(C_i)$ , which yields the following problem

$$\langle 6 \rangle \quad V = EU(C_i) = \max_s \left( \frac{W_n}{e_i} \right)^\delta \left\{ \left[ 1-s+s(1-\Gamma_i) \frac{e_i}{B} \right]^\delta \pi_i + \left[ 1-s+s \frac{e_i}{B} \right]^\delta (1-\pi_i) \right\}.$$

The first-order condition to problem  $\langle 6 \rangle$  with respect to  $s$  is given by

$$\langle 7 \rangle \quad \left[ \frac{(1-\Gamma_i)e_i}{B} - 1 \right] \left[ 1-s+s(1-\Gamma_i) \frac{e_i}{B} \right]^{\delta-1} \pi_i + \left[ \frac{e_i}{B} - 1 \right] \left[ 1-s+s \frac{e_i}{B} \right]^{\delta-1} (1-\pi_i) = 0.$$

It can be checked that the optimal share  $s$  decreases with  $\Gamma_i$ ,  $\pi_i$  and  $B$ , but increases with  $e_i$ . It is also evident from  $\langle 7 \rangle$  that the optimal share  $s$  is independent of  $W_n$ , which allows us to write expected utility  $V = EU(C_i)$  as

$$\langle 8 \rangle \quad V = EU(C_i) = \beta \left( \frac{W_n}{e_i} \right)^\delta$$

where

$$\langle 9 \rangle \quad \beta = \beta(\Gamma_i, \pi_i, e_i, B) = \max_s \left\{ \left[ 1-s+s(1-\Gamma_i) \frac{e_i}{B} \right]^\delta \pi_i + \left[ 1-s+s \frac{e_i}{B} \right]^\delta (1-\pi_i) \right\}.$$

Combining  $\langle 4 \rangle$  and  $\langle 5 \rangle$ , we can write the demand  $D$  for illegal foreign exchange relative to domestic money as a function of the share  $s$  and the black market exchange rate.

$$\langle 10 \rangle \quad D = \frac{F}{M} = \frac{s}{(1-s)B}.$$

From the dependence of  $s$  on  $\Gamma_i$ ,  $\pi_i$ ,  $B$ , and  $e_i$ , it follows that the demand for illegal foreign exchange relative to domestic money can be written as  $D = D(\Gamma_i, \pi_i, B, e_i)$ , where

$$\frac{\delta D}{\delta \Gamma_i} < 0, \quad \frac{\delta D}{\delta \pi_i} < 0, \quad \frac{\delta D}{\delta B} < 0, \quad \frac{\delta D}{\delta e_i} > 0.$$

## I.C. Supply of illegal foreign exchange

The supply of illegal foreign exchange is provided by unreported and unconfiscated export earnings. The exporter's legal and illegal export is decided by the black market exchange rate at stage 2. This subsection derives the supply of illegal foreign exchange, relative to domestic money, as a function of the black market exchange rate and the export enforcement parameters.

If the agent exports quantities  $X_1$  and  $X_i$  legally and illegally at stage 1, then at stage 2 his nominal wealth  $W_n$  in terms of claims on future imported goods can have two values depending on whether his illegal exports are detected at stage 1. In particular, his wealth  $W_n$  can take on the following two values

$$\langle 11 \rangle \quad W_n^1 = e_i(1-\tau) \left[ X_1 + (1-\Gamma_x) \frac{B}{e_x} X_i \right] \quad \text{with probability } \pi_x$$

and

$$\langle 12 \rangle \quad W_n^2 = e_i(1-\tau) \left[ X_1 + \frac{B}{e_x} X_i \right] \quad \text{with probability } 1-\pi_x.$$

Equations  $\langle 11 \rangle$  and  $\langle 12 \rangle$  reflect that the agent can transform 1 unit of exportable with certainty into domestic money worth  $1-\tau$  units of importable by exporting through legal channels. At the same time, the agent can export 1 unit of exportable to obtain an asset that yields  $(1-\tau)(1-\Gamma_x)B/e_x$  units of importable with probability  $\pi_x$  and  $(1-\tau)B/e_x$  units of importable with probability  $1-\pi_x$ .

At stage 1 the agent has to solve the following problem

$$\langle 13 \rangle \quad \max_{C_x, X_1, X_i} C_x^\delta + \beta V \left( \frac{W_n}{e_i} \right),$$

where  $V$  and  $\beta$  are defined in  $\langle 8 \rangle$  and  $\langle 9 \rangle$ . Consumption and total exports cannot exceed the exportable endowment  $E$  according to the resource constraint

$$\langle 14 \rangle \quad C_x + X_1 + X_i \leq E.$$

Using the value function  $V$  from  $\langle 8 \rangle$ , we can write the exporter's problem  $\langle 13 \rangle$  as

$$\langle 15 \rangle \quad \max_{C_x, X, r} (C_x)^\delta + \beta X^\delta \left\{ \left[ (1-\tau) \left( 1-r+r(1-\Gamma_x) \frac{B}{e_x} \right) \right]^\delta \pi_x + \left[ (1-\tau) \left( 1-r+r \frac{B}{e_x} \right) \right]^\delta (1-\pi_x) \right\},$$

where now  $X$  is total exports, *i.e.*,  $X = X_1 + X_i$ , and  $r$  is the share of exports not to be reported to the authorities.

The first-order condition to problem  $\langle 15 \rangle$  with respect to  $r$  is

$$\langle 16 \rangle \quad \left( (1-\Gamma_x) \frac{B}{e_x} - 1 \right) \left[ 1-r+r(1-\Gamma_x) \frac{B}{e_x} \right]^{\delta-1} \pi_x + \left( \frac{B}{e_x} - 1 \right) \left( 1-r+r \frac{B}{e_x} \right)^{\delta-1} (1-\pi_x) = 0.$$

From <16> it is clear that  $r$  does not depend on  $C_x$ ,  $X$ , or  $\beta$ . As shown above, the parameters  $\Gamma_i$ ,  $\pi_i$ , and  $e_i$  influence the value of  $\beta$  and thus the overall attractiveness of exportation. However, from <16> we see that these parameters do not affect the relative export decision independently or other than through  $B$ . Thus the black market exchange rate reflects all information about import enforcement and the value of official import exchange rate. Some inspection goes to show that  $r$  depends negatively on  $\pi_x$ ,  $\Gamma_x$  and  $e_x$ , but positively on  $B$ .

As a share  $\pi_x \Gamma_x$  of underinvoiced export earnings is confiscated, the ratio of illegal foreign exchange holdings to domestic money at stage 2 equals  $(1 - \pi_x \Gamma_x) X_i / (X_1 e_x)$  by the law of large number. The deterministic supply  $S$  of illegal foreign exchange, relative to the supply of domestic money, in terms of the share  $r$  is given by

$$\langle 17 \rangle \quad S = \frac{F}{M} = \frac{r(1 - \Gamma_x \pi_x)}{(1 - r)e_x},$$

which implies  $S = S(\Gamma_x, \pi_x, B, e_x)$ , where

$$\frac{\delta S}{\delta \Gamma_x} < 0, \quad \frac{\delta S}{\delta \pi_x} < 0, \quad \frac{\delta S}{\delta B} > 0, \quad \frac{\delta S}{\delta e_x} < 0.$$

This completes the derivation of demand and supply in the black market for foreign exchange. In what follows, we examine when in fact legal and illegal trading occur, and how the equilibrium depends on the exogenous variables and in particular on enforcement policy.

## II. Existence of legal and illegal trading

Not for all values of the enforcement parameters will legal and illegal trading channels coexist. Clearly, if penalties inflicted on apprehended black marketeers are prohibitively severe, there will be no black market at all. Conversely, if enforcement of international trade is relatively mild, there will be no legal trade. There will be unreported exports if at  $X_i = 0$  the relative gain of underinvoicing, given by  $B/e_x$ , is sufficiently high for the prospective illegal exporter to accept the risk of confiscation. As  $X_i = 0$  implies  $r = 0$  in equation <16> this means there will be unreported exports only if

$$\langle 18 \rangle \quad \frac{B}{e_x} \geq \frac{1}{1 - \Gamma_x \pi_x}.$$

There similarly will be illegal imports if at  $s = 0$  in <7> the agent can benefit from increasing  $s$ . Thus there will be illegal imports if

$$\langle 19 \rangle \quad \frac{B}{e_i} \leq 1 - \Gamma_i \pi_i.$$

There will only be a black market for foreign exchange if there are illegal exports as well as imports. From <18> and <19> this means there will be a black market for foreign exchange if

$$\langle 20 \rangle \quad 1 - \tau < (1 - \Gamma_x \pi_x)(1 - \Gamma_i \pi_i),$$

where  $\tau = 1 - e_x/e_i$  again is the tax rate implicit in the system of exchange controls.



If <20> holds, legal and illegal trading generally coexist unless traders are sufficiently risk neutral, *i.e.*,  $\delta$  is close enough to 1, in which case there is only illegal trade. Equations <18> and <19> also imply that if there is a black market, then  $e_x \leq B \leq e_i$ .<sup>4</sup> Thus the black market premium with respect to the export rate is not negative, while the premium *vis-à-vis* the import rate is not positive.

### III. Equilibrium and comparative statics

Now we are in a position to examine the equilibrium in the black market economy. The relative demand relationship of equation <10> is represented as the *DD* schedule in Figure 1. This schedule reflects that for  $B > e_i(1 - \Gamma_i \pi_i)$ , there is no demand for illegal foreign exchange. At the same time, relative demand for illegal foreign exchange decreases with  $B$ , and it approaches infinity for some positive value of the black market exchange rate, denoted  $B_1$ .

Similarly, the relationship between the supply of illegal foreign exchange at stage 2, relative to domestic money, and the black market exchange rate of equation <17> is represented as the *SS* schedule in Figure 1. The schedule also reflects the fact that there are no illegal exports if  $B < e_x[1 - \Gamma_x \pi_x]^{-1}$ . Further, relative supply increases with  $B$ , and approaches infinity as  $B$  approaches some finite value, denoted  $B_2$ . The equilibrium is indicated by the intersection of the *SS* and *DD* curves, and  $B^*$  and  $(F/M)^*$  are the equilibrium black market exchange rate and illegal foreign exchange relative to money holdings.

We can now check how the equilibrium depends on the exchange rate policy and enforcement policy variables  $e_x$ ,  $e_i$ ,  $\Gamma_x$ ,  $\pi_x$ ,  $\Gamma_i$ , and  $\pi_i$ . Starting with exchange

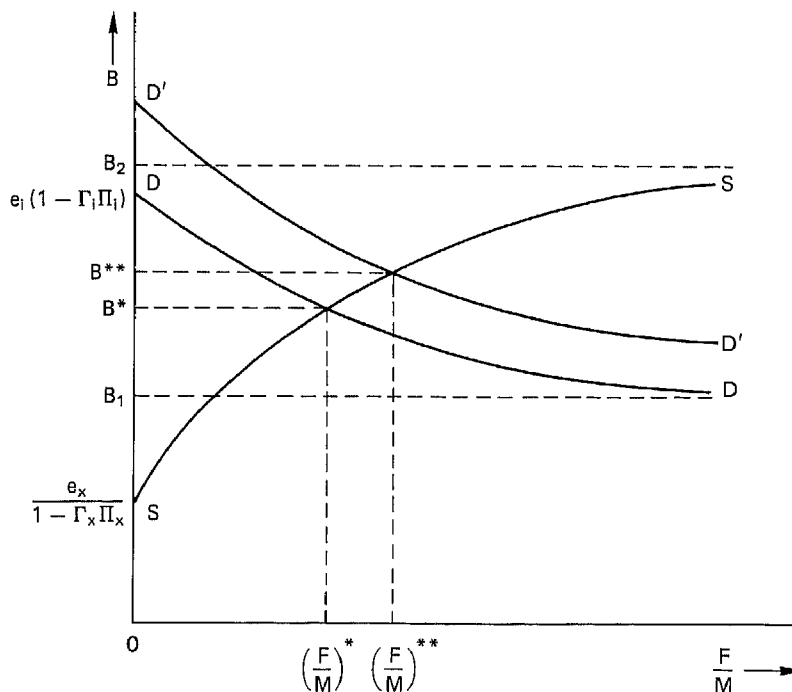


FIGURE 1. The equilibrium in the black market for foreign exchange.

rate policy, if the government were to announce an increase in  $e_i$  before the private sector makes its stage 1 decisions, then the  $DD$  curve in Figure 1 would shift out towards  $D'D'$ . The anticipated devaluation of the official import exchange rate thus depreciates the black market exchange rate from  $B^*$  to  $B^{**}$ , and it increases the equilibrium financial assets ratio  $F/M$  from  $(F/M)^*$  to  $(F/M)^{**}$ . In similar fashion, we can vary  $e_x$ ,  $\Gamma_x$ ,  $\pi_x$ ,  $\Gamma_i$ , and  $\pi_i$ . Again for changes that are fully anticipated at stage 1, we can show that the black market exchange rate varies with the exchange rate and enforcement parameters as follows

$$\langle 21 \rangle \quad B = B(e_i, e_x, \Gamma_i, \Gamma_x, \pi_i, \pi_x),$$

where

$$\frac{\delta B}{\delta e_i} > 0, \quad \frac{\delta B}{\delta e_x} > 0, \quad \frac{\delta B}{\delta \Gamma_i} < 0, \quad \frac{\delta B}{\delta \Gamma_x} > 0, \quad \frac{\delta B}{\delta \pi_i} < 0, \quad \frac{\delta B}{\delta \pi_x} > 0.$$

The financial assets ratio  $\rho = F/M$  and the policy variables are related as follows

$$\langle 22 \rangle \quad \frac{F}{M} = \rho(e_i, e_x, \Gamma_i, \Gamma_x, \pi_i, \pi_x),$$

where

$$\frac{\delta \rho}{\delta e_i} > 0, \quad \frac{\delta \rho}{\delta e_x} < 0, \quad \frac{\delta \rho}{\delta \Gamma_i} < 0, \quad \frac{\delta \rho}{\delta \Gamma_x} < 0, \quad \frac{\delta \rho}{\delta \pi_i} < 0, \quad \frac{\delta \rho}{\delta \pi_x} < 0.$$

Alternatively we can assume that news about changes in exchange rate policy or import enforcement policy reaches the private sector at stage 2, after stage 1 consumption and export decisions have been made. In this instance, the relative supply of illegal foreign exchange to money is predetermined. The effects of changes in  $e_i$ ,  $\pi_i$ , and  $\Gamma_i$  now have effects on the black market exchange rate of equal sign, but larger magnitude.

We can also consider how the policy parameters affect the ratio of illegal to legal exports,  $X_i/X_1$ . It is easily seen that the signs of changes in this ratio with respect to the policy parameters are as for  $\rho$  in  $\langle 22 \rangle$ . It is less straightforward to see how the policy parameters affect absolute trade flows. To know how absolute trade flows are affected one needs to know how relative exports as well as total exports change. For the class of iso-elastic utility functions, the relationship between absolute trade flows and the policy parameters is generally ambiguous.

Finally, let us examine the volume of illegal foreign exchange trading. For the assumption of iso-elastic utility, we know that agents have equal shares of money and illegal foreign assets in their portfolios after stage 2. The volume of foreign exchange trading, then, depends on the discrepancies in the portfolios of the two classes of consumers as well as on the black market exchange rate. Unlucky traders, of which there is a share  $\pi_x$ , own  $e_x X_1$  units of domestic currency, and  $(1 - \Gamma_x) X_i$  units of illegal foreign exchange. Lucky exporters, of which there is a share  $1 - \pi_x$ , on the other hand, own  $e_x X_1$  units of domestic currency, and  $X_i$  units of illegal foreign exchange. Using the fact that all traders will hold foreign exchange and domestic money in equal proportion, we can derive the volume of

black market trading as follows

$$\langle 23 \rangle \quad \pi_x X_i \left[ \frac{(e_x X_1 + (1 - \Gamma_x) X_i B)(1 - \Gamma_x \pi_x)}{e_x X_1 + B X_i (1 - \Gamma_x \pi_x)} - (1 - \Gamma_x) \right].$$

Equation <23> indicates that black market trading volume is zero for a zero enforcement strength, *i.e.*,  $\pi_x = 0$ , as well as for prohibitively severe enforcement parameters which give rise to  $X_i = 0$ . In both instances, all traders are alike at stage 2, and there is no rationale for foreign exchange trading. Interestingly, for low values of  $\pi_x$ , and in particular for the value of  $\pi_x$  given by <18> where illegal exports just come into play, the volume of black market trading increases with  $\pi_x$ . For large values of  $\pi_x$ , of course, illegal international trade and the volume of illegal foreign exchange trading are both reduced to zero.

#### IV. Empirical evidence

This section presents some evidence on the relationship between enforcement and the black market premium for the case of Brazil.<sup>5</sup> The section starts with a description of the penalties imposed on illegal traders in Brazil. This evidence confirms that proportional fines are the common means of punishment. The section then shows that law enforcement, as measured by the number of people charged with import violations, appears an important determinant of the black market exchange rate.

Brazilian exchange control regulations are contained in Law 4231.<sup>6</sup> The law first stipulates that exchange operations must be performed by duly authorized banks, brokerages, and other exchange institutions. These institutions are responsible for the correct identification of their clients, and they can be subjected to a fine of up to three times the value of the exchange operation if found negligent in confirming a client's identity. However, if the client is solely responsible for a false statement of identity only he is liable to a fine of 100 per cent of the value of the exchange operation. The exchange institutions are fined as well for incorrectly classifying a client's data on the official exchange form, and in case of recurrent violations the Bank of Brazil will revoke the exchange institution's operating license.

Brazilian residents are required to declare property or earnings in either cash or securities which they possess abroad to the Bank of Brazil. Any declared earnings or possessions are then to be deposited with the Bank of Brazil or the Ministry of Finance into special accounts. Failure to comply with these requirements is met with confiscation of either cash or securities. On the import side, it is unlawful to import without a license or to overprice or underprice goods as stated on the import invoice. Violators of these regulations are again subject to a 100 per cent penalty in case of unlicensed importation, and they receive a penalty equal to the difference between stated and actual value of imports in case of incorrectly invoiced prices.

As set out in the introduction, law enforcement varies probably more than the law itself, and thus the number of people charged over time is an indicator of law enforcement changes. Figure 2 plots the year-end black market premium, and the number of people charged with importation violations in Brazil for the years 1971 to 1983.<sup>7</sup> The violations' statistic for the year 1974 is missing and has

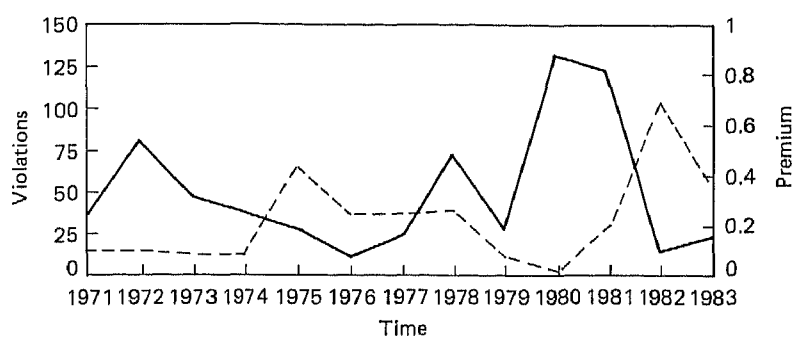


FIGURE 2. Import violations and the black market premium in Brazil 1971-83.

— = number of violations  
 ---- = black market premium

been interpolated in the figure. There appears to be a negative correlation, which is consistent with changes in the probability  $\pi_i$  in our theoretical model.

A simple regression of the black market premium on the number of violations, the official exchange rate, the first difference of the official exchange (as a proxy of the expected devaluation), and the money supply<sup>8</sup> can be motivated by the consumer's problem at stage 2, as reflected in the first-order condition <7>. The results of this regression, and of a regression that excludes the money variable, are reported in Table 1. The violations variable has negative sign, and is significant at the 5 per cent level in both regressions. Given the DW statistics, the hypothesis of no serial correlation is not rejected. Clearly, these results are only indicative given the limited available data. However, the results do suggest that enforcement, which has been ignored in all previous empirical studies, is an important fundamental of the black market exchange rate.<sup>9</sup>

## V. Conclusion

This paper has analyzed illegal international trade and the resulting black market for foreign exchange in an economy characterized by multiple exchange rates. Multiple rates are equivalent to a tax on authorized international trade, and black market activities are efforts to evade this tax. Risk aversion can explain the coexistence of legal and illegal trading channels, and provides a rationale for trading on the black market for foreign exchange. On the black market, exporters meet to readjust their financial portfolios after uncertainty regarding detection and confiscation at the export round has been lifted. The model allows us to examine how changes in levels of the export and import exchange rates as well as enforcement policy affect the black market exchange rate and the relative volumes of legal and illegal trading. Some necessarily cursory evidence for the case of Brazil suggests that exchange restrictions' enforcement is indeed an important fundamental of the black market exchange rate.

TABLE 1. The black market premium and enforcement in Brazil.

C	0.507 (1.19)	0.417 (2.17)
LVIO	-0.130 (2.58)	-0.128 (2.76)
LE	0.177 (1.02)	0.139 (2.00)
LRATE	-0.526 (1.50)	-0.494 (1.63)
LM	-0.032 (0.24)	
DW	2.85	2.76
$R^2$	0.61	0.60
$\bar{R}^2$	0.35	0.43
N	11	11

*Notes:*

LVIO = log of the number of people charged with import violations.

LE = log of the official exchange rate.

LRATE = log difference of the official exchange rate.

LM = log of monetary aggregate.

The dependent variable is the log of the ratio of the black market and official exchange rates. All variables apart from the violations variable are year-end values. The time period is 1972-83, excluding 1974. Parentheses provide absolute  $t$ -statistics.

The violations variable is from *Anuario Estatístico do Brasil*, various issues. The official exchange rate and money are from lines *ae* and 34 of the IMF's *International Financial Statistics*. The black market exchange rate is from *Pick's Currency Yearbook*, various issues.

**Notes**

1. Martin and Panagariya (1984) examine a crime-theoretic model of smuggling without considering a black market for foreign exchange.
2. The analysis thus abstracts from rationing of official foreign exchange for imports, which frequently is an independent reason for the existence of a black market for foreign exchange.
3. The detection probability can more generally be made a function of resources spent by violators to confound the authorities, as in a study of domestic tax evasion in Usher (1986).
4. If there were rationing of official foreign exchange for imports, then  $B \leq e_t$  would not necessarily hold.
5. Brazil did not maintain a multiple exchange rate system during the period from 1971 to 1983 considered in this paper. However, the foreign exchange proceeds of coffee exports were subject to a contribution quota set by the Brazilian Coffee Institute from time to time in terms of foreign exchange. The contribution quota is adjusted whenever the exchange rate is changed in order to ensure that exporters' returns in cruzeiros, using a minimum export price in US dollars, is constant. See the IMF's *Annual Report on Exchange Restrictions*

- for 1972, p. 74. This means that a devaluation of the unified exchange rate *de facto* acts as a tax on international trade.
6. All information in this paragraph and the next is from *Guidelines to Brazil's Foreign Investment Law*, pp. 91–97, where exact references to the Brazilian code can also be found.
  7. For data sources, see the notes to Table 1.
  8. The monetary aggregate is a properly predetermined variable at stage 2 of the illegal trading sequence.
  9. References to previous empirical work on black market exchange rates are as follows. Culbertson (1975) tests black market exchange rate equations that stress relative prices as a measure of the equilibrium exchange rate and the foreign reserve position. Dornbusch *et al.* (1983) estimate a black market premium equation for Brazil that investigates seasonal influences on the black market exchange rate. Olgun (1984) fits a macro model including a black market exchange rate equation in relative prices and the official exchange rate using Turkish data. Kamin (1986) examines a multicountry set of 108 devaluations and by taking averages establishes the stylized paths of the black market exchange rate and the premium before and after devaluations. Some early contributions on illegal trade and smuggling are reprinted in a volume edited by Bhagwati. Among these, Bhagwati *et al.* (1974) provide statistics on export underinvoicing, as indicated by trade data discrepancies, and the black market premium for a set of 27 countries. Finally, McDonald (1985) carries out formal tests relating export underinvoicing to the black market premium and export taxes.

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