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A CONVERGENT PRICE ADJUSTMENT PROCESS *

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In this paper we describe a price adjustment mechanism which leads to an economic equilibrium, starting from an arbitrarily chosen initial price vector. The process is global and universal.

1. Introduction

The aim of this paper is to present a meaningful price adjustment process, which converges both globally and universally. This problem goes back to Walras, who was concerned with the problem of finding a price adjustment process leading from an initial price system to an equilibrium price system. However, the classical Walras' tâtonnement process may fail to converge to a vector of equilibrium prices, even when the set of initial price systems is restricted. In fact, neither global nor local convergence can be guaranteed. So, the mechanism is not effective in the sense that, from any initial price system in any given standard pure exchange economy, the process always yields a path which converges to a system of equilibrium prices [see Saari and Simon (1978)]. Counterexamples where the prices can spiral forever have been constructed by Scarf (1960). Sonnenschein (1972) proved that any continuous function satisfying Walras' law can be realized as the excess demand function for some pure exchange economy. So, we need a process that converges universally and globally, i.e., a process that converges for any continuous function satisfying Walras' law and from any initial starting price vector.

We know that the classical Walras' tâtonnement process does not converge universally, while for Smale's global Newton process [see Smale (1976)], convergency may not hold for an arbitrarily chosen starting point. In Saari and Simon (1978), it is shown that an effective mechanism needs information about both the excess demand at any price and the value of the gradients of all except one of its component functions. Saari (1985) shows that an effective iterative mechanism which depends upon information obtained solely from the excess demand function and its derivatives does not exist.

In this paper we present a price adjustment process that converges globally and universally. The process is governed by both the value of the excess demand and the location of the corresponding

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price vector with respect to the initial price vector. The paper contains three sections including this introduction. In section 2 we describe the process, while in section 3 we discuss the economic justification of the process.

2. The adjustment process

In this section we consider economies with $n+1$ commodities. For such economies we design a price adjustment mechanism on the n-dimensional unit price simplex $S^n = \{ p \in R^{n+1} | \Sigma_j p_j = 1,\ p_j \geq 0,\ j = 1,\ldots,\ n+1\}$. We assume that the total excess demand function belongs to the class of continuously differentiable functions satisfying

(a) for all $p \in S^n$, $p^{\mathrm{T}} z(p) = 0$ (Walras' law),
(b) $z_j(p) \geq 0$ if $p_j = 0$ (non-negative excess demand if $p_j = 0$).

To design a universal and global adjustment process to find an equilibrium price vector p^* with $z(p^*) = 0$, we define a collection of primal and a collection of dual sets on the unit price simplex S^n. The primal sets are induced by the initial price vector and define the location of the prices with respect to this initial point. The dual sets are induced by the excess demand function and define subsets of prices by revealing the sign pattern of the corresponding excess demand.

To describe the process, let Z be the set of all sign vectors in R^{n+1} having at least one component equal to $+1$ and one component equal to -1. Further, for $s \in Z$, we define $I(s) = \{ i \in I_{n+1} \mid s_i = 0\}$, where $I_{n+1} = \{1, 2,\ldots,\ n+1\}$, and $k(s)$ the number of elements in $I(s)$. Given an arbitrarily chosen initial strictly positive price vector v, we define for each $s \in Z$ the primal $(k(s)+1)$-dimensional set $P(s)$ by

$$P(s) = \left\{ p \in S^n \middle| p_j/v_j = \min_h p_h/v_h \text{ if } s_j = -1 \text{ and } p_j/v_j = \max_h p_h/v_h \text{ if } s_j = 1 \right\}.$$

So, $P(s)$ is the set of prices p in S^n such that the ratio between p_j and v_j is minimal if s_j is negative and the ratio is maximal if s_j is positive. When $s_j = 0$ the price ratio of commodity j lies between the maximum and the minimum of the ratios. Observe that the number of different sign vectors s in Z for which $k(s) = 0$ is equal to $2^{n+1} - 2$, implying that there are $2^{n+1} - 2$ one-dimensional subsets $P(s)$, called rays. We will see that the initial price vector v is left along one of these rays. For $n = 2$, the sets $P(s)$, $s \in Z$ are drawn in fig. 1. Observe that the collection of primal sets cover S^n.

We now define the dual subsets of S^n. These sets are induced by the sign pattern of the excess demand function z. Formally, for each $s \in Z$ we define the dual set $D(s)$ by

$$D(s) = \mathrm{Cl}(\{ p \in S^n \mid \text{sign } z(p) = s \}),$$

with sign $a_j = 0$ if $a_j = 0$ and where $\mathrm{Cl}(A)$ denotes the closure of the set A. We are now ready to describe the process. It starts by leaving the initial price vector along the ray $P(s^0)$ with $s^0 = \text{sign } z(v)$, assuming that $z_j(v)$ is not equal to zero, for all j. The process continues along this ray until for one of the commodities, for instance, i, the excess demand becomes equal to zero. Then s_i becomes equal to zero and the process continues in the corresponding region $P(s)$ by tracing a path of points p in $P(s)$ for which sign $z(p) = s$. In general, for varying s in Z, a path of prices p in $PD(s)$ is followed, where $PD(s) =$

$$\begin{aligned} P(s) \cap D(s) = \{ p \in S^n | & p_j/v_j = \min_h p_h/v_h \text{ and } z_j(p) \leq 0 \text{ if } s_j = -1, \\ & \min_h p_h/v_h \leq p_j/v_j \leq \max_h p_h/v_h \text{ and } z_j(p) = 0 \text{ if } s_j = 0, \\ & p_j/v_j = \max_h p_h/v_h \text{ and } z_j(p) \geq 0 \text{ if } s_j = +1 \}. \end{aligned}$$

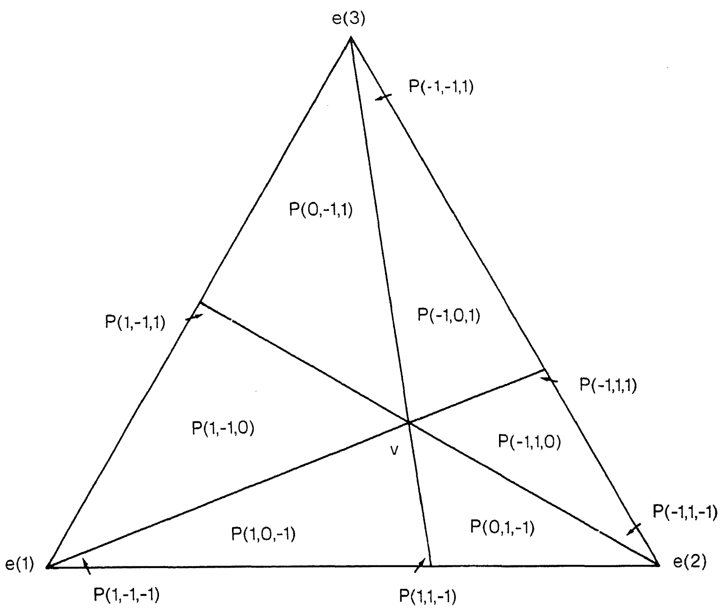

Fig. 1.

The process traces such a path of prices in $PD(s)$ until the path reaches either the boundary of $P(s)$ or the boundary of $D(s)$. In the first case we get an equality in the price ratios for some i in the set of indices $\{j \mid s_j = 0\}$, i.e., for a commodity with zero excess demand either p_i/v_i becomes equal to

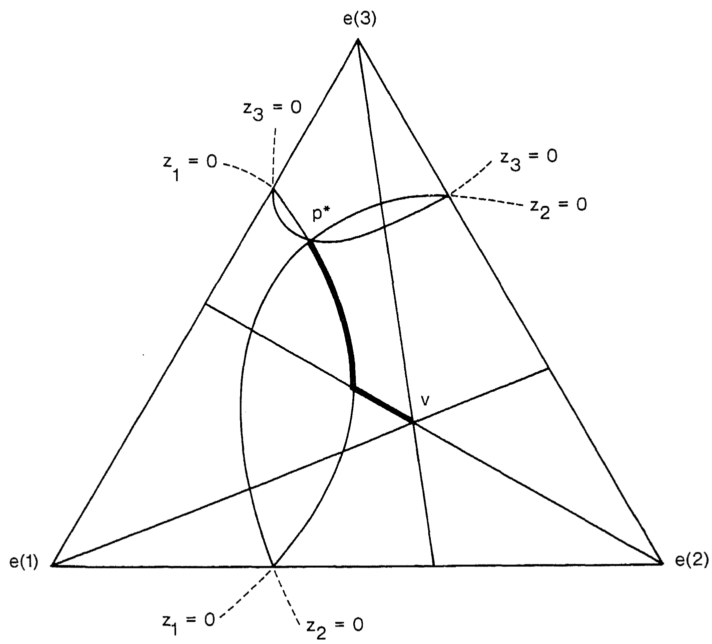

Fig. 2.

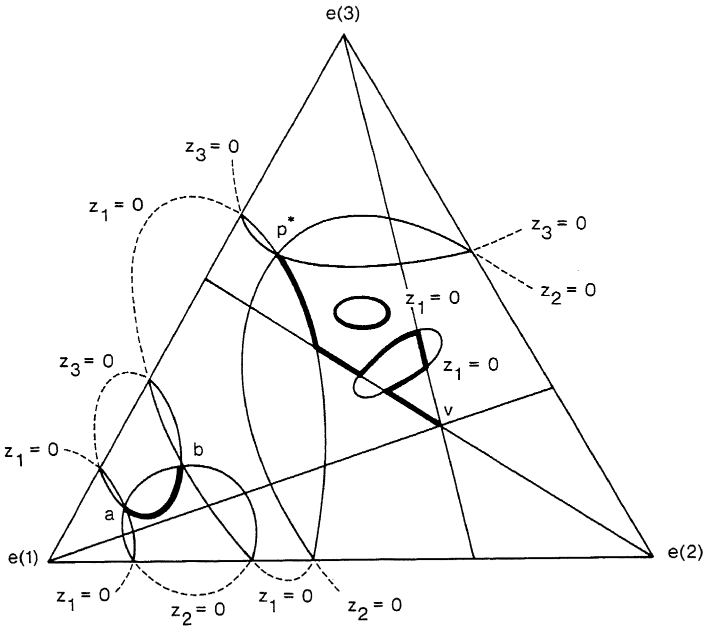

Fig. 3.

$\max_h p_h/v_h$ or p_i/v_i becomes equal to $\min_h p_h/v_h$. Then the process continues in $P(s')$ with $s'_j = s_j$ for all j except i and s'_i equal to $+1$ and -1 respectively. In case the boundary of $D(s)$ is reached, i.e., $z_i(p)$ becomes equal to zero for some i with $s_i \in \{-1, +1\}$, then the process continues in $P(s')$ with $s'_i = 0$ and $s'_j = s_j$ for all other components of s. In this way the sets $PD(s)$ can be linked together and the union $PD = \cup_s PD(s)$ over all sign vectors s in Z contains a curve leading from the initial price system v to an equilibrium price system p^*. This is illustrated in the figs. 2 and 3. In these figures the curves along which $z_i = 0$ are drawn for $i = 1, 2, 3$. In fig. 2, PD contains just one curve. In fig. 3 PD is a collection of three one-dimensional manifolds, namely, a curve from v to an equilibrium price system p^*, a curve connecting two equilibrium price vectors and a loop along which $z_1 = 0$ in $P(0, -1, +1)$. The formal proof that PD contains a curve leading from v to an equilibrium price vector p^* is given in Van der Laan and Talman (1985). Therefore, it is necessary that z is a continuously differentiable function satisfying some regularity conditions.

3. The economics of the process

In the previous section we described an adjustment process on the unit price simplex S^n leading from an arbitrarily chosen starting point v to an equilibrium price system p^*. The process converges for any v and for any continuously differentiable excess demand function z, i.e., the process is global and universal. This is the most appealing feature of the adjustment mechanism. It also shows the attractiveness of the process above, both the classical tâtonnement process and Smale's (1976) global Newton method. The process is also economically meaningful. To define the path of the process we introduced primal sets $P(s)$ and dual sets $D(s)$. The primal sets contain information about the location of a price p with respect to v. The dual sets contain information about the corresponding excess demand at price p. So, the path can be traced by the auctioneer by keeping in mind the starting price vector v and by observing the reaction of the people in the market as reflected by the excess demand. The behaviour of the auctioneer is governed by the total excess demand expressed by the individual agents. Initially, the auctioneer decreases all prices of the commodities with negative excess demand and increases the prices of all commodities with positive excess demand in such a way that the ratio between any two prices with either positive or negative excess demand is kept constant. Prices are adapted in this way until one of the markets attains equilibrium. Then the auctioneer adjusts the prices in order to keep the excess demand of this commodity equal to zero. In general, the auctioneer keeps, with respect to their initial values at v, the relative prices of the commodities with positive (negative) excess demand maximal (minimal) and allows the relative prices of the commodities with zero excess demand to vary between these two bounds. As soon as one of the markets with positive (negative) excess demand attains equilibrium the corresponding price is decreased (increased) away from the relative upper (lower) bound and the auctioneer adjusts this price simultaneously with the other prices of the commodities with zero excess demand in order to keep these markets in equilibrium. On the other hand, if one of the prices of the commodities with zero excess demand reaches the relative upper (lower) bound, then this market is no longer kept in equilibrium but the corresponding price is kept equal to the current relative upper (lower) bound. In this way the auctioneer traces a path of prices leading to an equilibrium price system.

We want to conclude this paper by considering the necessary information to follow the path of prices. Suppose that for some s, a path of prices is traced in $P(s)$. Along this path $z_j(p) = 0$ for all j with $s_j = 0$. So, the prices p_j with j in the set $I(s)$ solve the differential equation

$$\mathrm{d}z^I(p)/\mathrm{d}t = -\mu z^I(p),$$

with $z^I(p)$ the $k(s)$-dimensional vector containing the elements of $z(p)$ in $I(s)$, under the restriction that p belongs to $P(s)$. For the adjustment mechanism induced by this process the auctioneer needs information about $z^I(p)$ and the corresponding gradients. Moreover the auctioneer has to keep in mind the starting price vector. It is also possible to follow the path discretely by a simple zero point algorithm. Then a piecewise linear path of prices in $P(s)$ is followed by generating a sequence of adjacent $(k(s)+1)$-dimensional simplices, needing the excess demand at each vertex. A new vertex is completely determined by the vertices of the current simplex, their corresponding excess demands and the initial point v. This confirms the observation of Saari (1985) that a convergent adjustment procedure should depend on the values of the prices. This paper shows that not only the values of the current prices are needed, but that also the value of the initial price system might play a very important role.

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