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ADJUSTMENT COSTS IN A DYNAMIC MODEL OF THE FIRM

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Summary: In this paper we derive the firm's optimal dynamic investment policy under both financial restrictions and convex adjustment costs. Some characteristic differences will be pointed out between the model concerned and other dynamic models of the firm without adjustment costs, viz.

- investments are a continuous function of time
- capital stock never keeps a stationary value
- adjustment costs have a negative impact on the maximum level and on the growth rate of capital.

Zusammenfassung: In diesem Beitrag will ich den Einfluss einer konvexen Anpassungskostenfunktion auf die optimale Politik eines dynamischen Modells der Unternehmung analysieren. Einige charakteristische Eigenschaften der Lösung dieses Models sind:

- die Investitionen sind eine kontinuierliche Funktion der Zeit
- es gibt keinen stationären Wert des Gesamtkapitals
- Anpassungskosten haben einen negativen Einfluss auf den maximalen Wert des Gesamtkapitals und auf die Wachstumsgeschwindigkeit des Gesamtkapitals

1. Introduction

One of the first dynamic models of the firm is the classical model of Jorgenson (1967). The problem with this model is that the resulting optimal solution dictates an instantaneous adjustment of the stock of capital goods to the level of maximum revenue.

In the literature, two ways in particular have been proposed to avoid this unrealistic immediate adjustment. The first way is the introduction of financing in the dynamic model of the firm. Examples of such models are those of Leland (1972), Ludwig (1978) and van Loon & Verheyen (1984). The second way of getting a smoothed adjustment pattern is the introduction of adjustment costs as another aspect governing the dynamics of the firm. Research into this subject has been conducted by e.g. Gould (1968), Lucas (1967) and Treadway (1969). The article by Söderström (1976) contains a good survey of the theory of adjustment costs.

In this contribution we will analyse the impact of coupling financing and adjustment costs on the optimal policy within a really dynamic model of the firm.

In section 2 we present our dynamic model of the firm and section 3 contains a description and further analysis of the optimal solution.

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Finally, in the appendix we will give the necessary conditions for an optimal solution and we will also mention a method which transforms these necessary conditions into the optimal trajectories of the firm.

2. A dynamic model with financing and adjustment costs

We first assume that the firm behaves as if it maximizes the sum of the capital value of the revenue flow during the planning period and the discounted value of final capital good stock at the end of the planning period. Further, we assume that the firm is operating under decreasing returns to scale. This results in the following expression:

$$\max \int_{T=0}^{z} (S(K) - I(T))e^{-iT}\, dT + K(z)e^{-iz} \qquad (1)$$

in which

I(T) = gross investments
K(T) = total amount of capital goods
T = time
S(K) = earnings, $S(K) > 0, \frac{dS}{dK} > 0, \frac{d^2S}{dK^2} < 0$
i = discount rate
z = planning horizon

The impact of investments on the production structure is described by the, now generally used, formulation of net investments:

$$\dot{K} := \frac{dK}{dT} = I(T) - aK(T) \qquad (2)$$

in which

a = depreciation rate

Disinvestments are not allowed, so:

$$I(T) > 0 \qquad (3)$$

Further, we assume a positive value of capital good stock at T = 0:

$$K(0) = K_0 > 0 \qquad (4)$$

The formulation of the classical model of Jorgenson is now presented in (1) through (4).

The optimal solution of this model dictates an instantaneous adjustment of the stock of capital goods to the level of maximum revenue.

This is presented in figure 2.1

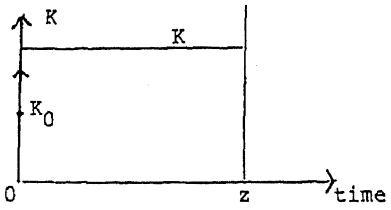

figure 2.1 Optimal trajectory of the capital stock in the model of Jorgenson

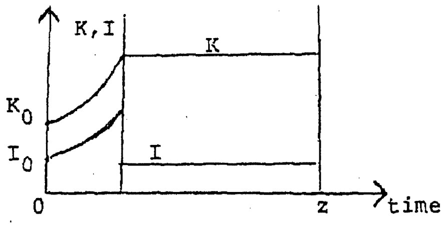

figure 2.2 Optimal trajectory of the capital stock and investments in the financial model

If the stock of capital goods has reached the level of maximum revenue, it holds that the marginal earnings equal marginal cost. Below that level it is worthwile to expand the capital good stock because, due to diminishing returns to scale, marginal earnings exceed marginal cost.
So, from the above we can conclude that in the stationary stage, it holds:

$$\frac{dS}{dK} = i + a \qquad (5)$$

In order to get rid of the unrealistic immediate adjustment at $T = 0$, we introduce financing as another aspect governing the dynamics of the firm.
We assume that earnings after deduction of depreciation are used to issue dividend or to increase the value of equity through retained earnings:

$$\dot{X}(T) := \frac{dX}{dT} = S(K) - aK(T) - D(T) \qquad (6)$$

in which

$D(T)$ = dividend payments by the firm
$X(T)$ = amount of equity

Assuming that the firm will attract only one kind of money capital: equity and has one production factor: capital goods, we get the balance equation:

$$K(T) = X(T) \qquad (7)$$

As far as its dividend policy is concerned, we assume that the firm is exempted from paying dividend, so:

$$D(T) > 0 \qquad (8)$$

The formulation of our financial model is now presented in (1) through (4) and (6) through (8).

The difference between the optimal solution of the Jorgenson model and the financial model is that the growth rate of capital good stock in the second model is limited by the financial structure. But the level of capital good stock of maximum revenue remains the same. This is presented in figure 2.2.

The aim of this paper is to incorporate both financing and a convex cost of adjustment function into the dynamic model of the firm. Therefore, we change the goalfunction (1) and the dynamic equation of equity (6) into the following expressions:

$$\max \int_{T=0}^{z} (S(K) - I(T) - U(I))e^{-it}\, dT + K(z)e^{-iz} \qquad (9)$$

$$\dot{X}(T) := \frac{dX}{dT} = S(K) - aK(T) - U(I) - D(T) \qquad (10)$$

in which

$U(I)$ = adjustment cost function, $U(I) > 0$, $\frac{dU}{dI} < 0$, $\frac{d^2U}{dI^2} < 0$

Now, (2) through (4) and (7) through (10) together form the financial model with convex adjustment costs.

Due to (2), (10) and the deriviative of (7), we can derive:

$$D(T) = S(K) - I(T) - U(I) \qquad (11)$$

The above substitution enables us to get rid of the expressions (7), (8) and (10) by adding:

$$S(K) - I(T) - U(I) > 0 \qquad (12)$$

This model can be solved analytically by using optimal control theory (see Pontryagin et.al. (1962), Kamien & Schwartz (1983)), where the state of the system is described by the amount of capital goods and is controlled by investments. The aim of this control is to reach a maximum value of the objective function.

3. Optimal solution of the financial adjustment cost model

We can obtain the necessary conditions for an optimal solution with the use of the standard maximum principle of Pontryagin. Next, we apply the general solution procedure of van Loon (1983, pp. 115-117) in order to transform the set of necessary conditions into the optimal trajectories of the firm (see appendix).

Each trajectory consists of feasible paths. These paths are characterized by different policies concerning investment expenditures and dividend payments. Our problem has to deal with 3 paths, which we show in tabel 3.1:

path	I	D
1	max	0
2	>0	>0
3	0	max

table 3.1: features of feasible paths.

Comment: when the time index is clear, it will be left out (for instance I in stead of I(T)).

The optimal solution of our problem consists of two master trajectories. The first one is represented by figure 3.1.

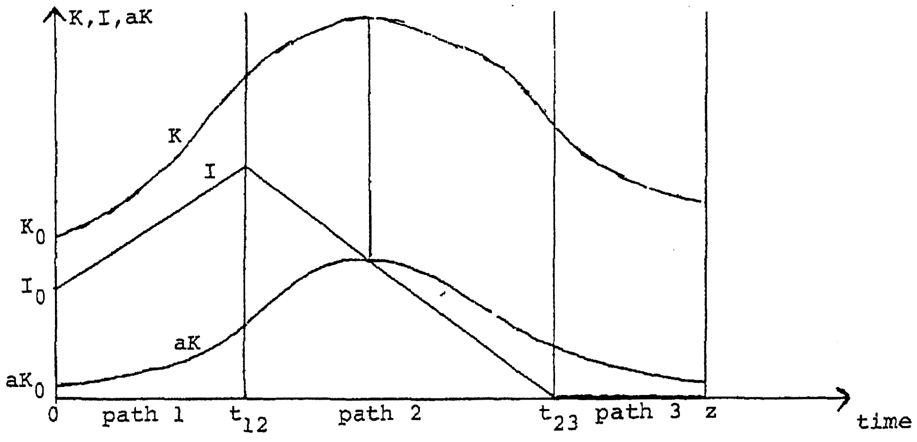

figure 3.1: development of I,K and aK on master trajectory 1.

On path 1, the firm grows at maximum speed. At t_{12} this strategy stops, because the marginal earnings become too low ($\frac{dS}{dK}$ diminishes when K rises) and the adjustment costs become too high ($\frac{dU}{dI}$ rises when I rises). Therefore, on path 2 investments diminish. Capital good stock still increases until the level of I falls below the depreciation level. Then K also diminishes. When investments become zero, path 2 passes into path 3. This happens just before z; then time is too short to defray the adjustment costs.

A noteworthy point is the continuity of I. Large values of I cost a great deal, because the cost of adjustment function is convex. Therefore, it is best for I to develop gradually in time.

It is interesting to compare this trajectory with the optimal solution of the financing model without adjustment costs, which is represented by figure 2.2. It can be proven that, when K has reached its maximum level on master trajectory 1 of the model with adjustment costs, it holds (see Kort 85):

$$\frac{dS}{dK} > (i+a)\ (1+\frac{dU}{dI}) \tag{13}$$

From (5), (13) and the concavity of S(K), we can conclude that adjustment costs have a negative impact on the maximum level of K.

In the model without adjustment costs capital good stock grows faster than in the adjustment cost model. This is because in the first model, only financing has a negative impact on the growth rate, while in the second model the growth rate is also negatively influenced by the adjustment costs.

Further, in the model without adjustment costs it holds that investments are not a continuous function of time.

The second master trajectory of the adjustment cost model is represented by figure 3.2.

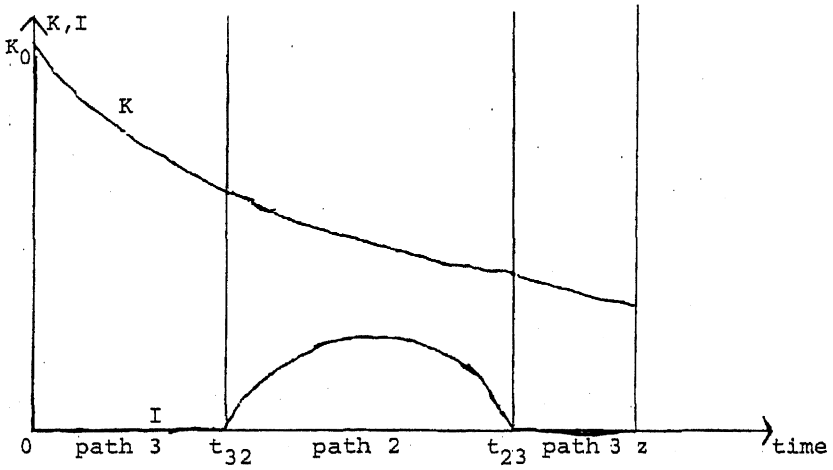

Figure 3.2. development of I and K on master trajectory 2.

This solution will be optimal when K_0 is relatively high and/or the planning period is relatively small.
ξ diminishes because the marginal earnings are too small and/or I is not too high because there is not much time to defray the adjustment costs.

References

Gould J.P., (1968), Adjustment costs in the theory of investment of the firm, Review of Economic Studies, pp. 47-55.

Jorgenson D.W., (1967), The theory of investment behaviour, in: R. Ferber (ed.): Determinants of investment behaviour , National Bureau of Economic Research, (Columbia University Press, New York).

Kamien M.I., & N. Schwartz, (1983), Dynamic optimization, the calculus of variation and optimal control in economics and management, 2nd. reprint (North Holland, New York).

Kort P.M., (1985), Optimal dynamic investment policy under financial restrictions and adjustment costs. Research Memorandum F.E.W., (Tilburg University).

Leland H.E., (1972), The dynamics of a revenue maximizing firm, International Economic Review 113, pp. 376-385.

Loon P.J.J.M. van, (1983), A dynamic theory of the firm: production, finance and investment, Lecture notes in economics and mathematical systems 218, (Springer, Berlin),

Loon P.J.J.M. van & P.A. Verheyen, (1984), Groeifasen bij bedrijven (Growthstages of the firms), Maandblad voor Accountancy en Bedrijfshuishoudkunde 58, 386-394).

Lucas R.E., (1967), Optimal investment policy and the flexible accelerator, International Economic Review, VIII (february 1967), pp. 78-85.

Ludwig Th., (1978), Optimale Expansionspfade der Unternehmung (Optimal growth stages of the firm), (Gabler Verlag, Wiesbaden).

Pontryagin L.S., V.G. Boltyanskü, R.V. Gamkrelidze & E.F. Mischenko, (1962), The mathematical theory of optimal processes (Wiley, New York).

Söderström H.T., (1976), Production and investment under costs of adjustment, a survey. Zeitschrift für Nationalökonomie 36 (1976), pp. 369-388.

Treadway A.B., (1969), On rational entrepreneurial behaviour and the demand for investment, Review of Economic Studies, pp. 227-239.

Appendix Solution of the dynamic adjustment cost model of the firm

We apply the standard maximum principle of Pontryagin to obtain the necessary conditions.
Let the Hamiltonian be:

$$H = (S(K) - I - U(I))e^{-iT} + \psi(I-aK) \tag{14}$$

and the Lagrangian:

$$L = H + \lambda_1(S(K) - I - U(I)) + \lambda_2 I. \tag{15}$$

where

ψ:= adjoint variable or co-state variable which denotes the marginal contribution of capital good stock to the performance level.

λ_j:= dynamic Lagrange multipliers representing the dynamic 'shadow price' or 'opportunity costs' of the j-th restriction.

then it must hold that:

$$\frac{\partial L}{\partial I} = -(1 + \frac{dU}{dI})\ (e^{-iT}+\lambda_1) + \psi + \lambda_2 = 0 \tag{16}$$

$$-\dot{\psi} = \frac{dS}{dK}\ (e^{-iT}+\lambda_1) - a\psi \tag{17}$$

$$\left.\begin{array}{ll} \lambda_1(S(K) - I - U(I)) = 0 & (18) \\ \lambda_2 I = 0 & (19) \end{array}\right\} \text{ complementary constraints}$$

$$\psi(z) = e^{-iz} \quad \text{(transversality constraint)} \tag{20}$$

Next, we can apply the general solution procedure of van Loon to transform these conditions into the optimal trajectories of the firm. These trajectories consist of different paths, which are each of them characterized by the set of active constraints. The properties of these paths are presented by table A.1.

path	λ_1	λ_2
1	+	0
2	0	0
3	0	+
4	+	+

tabe A.1. the different paths

In Kort (1985) we have proved that path 4 is infeasible. In the same article we also couple the remaining three paths into the optimal master trajectories of the firm, which are the following:

master trajectory 1: path 1 - path 2 - path 3
master trajectory 2: path 3 - path 2 - path 3.