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MODIFICATION OF THE KOJIMA-NISHINO-ARIMA ALGORITHM AND ITS COMPUTATIONAL COMPLEXITY*

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Abstract

A modification of the Kojima-Nishino-Arima algorithm for finding all zeros of a polynomial is presented. This modification yields a polynomial-time result on its computational complexity. The main theorem asserts that the cost of locating all zeros of a polynomial of degree n up to an accuracy of ε grows no faster than $O(\max\{n^4, n^3 \log_2(n/\varepsilon)\})$.

Keywords: zeros of polynomials, piecewise linear homotopy, error estimate, computational complexity

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1 Introduction

In response to a question by Smale [6] about the efficiency of piecewise linear homotopy methods in solving polynomial equations, there have been several results, see [3],[5],[9]. However, these results are all dealing with Kuhn's algorithm for polynomials, see [2]. There are two flaws to be indicated. First, as a zero finding method, Kuhn's algorithm fails to give correct multiplicities ("A counterexample for a theorem of Kuhn", presented by Kuhn at the 17^{th} TIMS/ORSA joint National Meeting, San Francisco, 1984). Second, employing integer labelling, Kuhn's algorithm is not, in the original sense of piecewise linear path-following, an exact PL homotopy method.

On the other hand, Kojima, Nishino and Arima [1] presented a PL homotopy algorithm (KNA algorithm in short) employing vector labelling. Due to the introduction of a perturbation term, it can give all zeros of a polynomial with correct multiplicities. This feature makes the algorithm much more attractive. However, there is not any result on its computational complexity yet. It is well-known that, for a complexity discussion of certain kind, an error estimate of the computation is needed first. In spite of the original asymptotic estimate in [1], the desired estimate should be independent from the coefficients of the polynomial. In this paper we obtain an error estimate which is explicit only in terms of the homotopy parameter and of the degree of the polynomial. In doing so, the perturbation term, which makes the KNA algorithm successful, appears to be the problem. The key step of our work is to modify the perturbation. This modification yields a desired estimate. After that, the main result, that the cost of locating all zeros of a polynomial of degree n up to an accuracy of $\varepsilon > 0$ with correct multiplicities grows no faster than $O(\max\{n^4, n^3 \log_2(n/\varepsilon)\})$, follows.

2 Modification of perturbation

Let $f(z) = z^n + \sum_{k=1}^{n} a_k z^{n-k}$ be a polynomial with complex coefficients. To compute the n roots of $f(z)$ we choose an auxiliary polynomial $h(z) = z^n - r_0^n$, where r_0 is a positive constant. Let the function $H : [0,1] \times \mathbb{C} \to \mathbb{C}$ be defined by

$$H(t,z) = \begin{cases} f(z) + (2t)^{2n}b & \text{if } t \in [0, \frac{1}{2}], \\ (2t-1)h(z) + 2(1-t)f(z) + b & \text{if } t \in [\frac{1}{2}, 1], \end{cases} \tag{2.1}$$

where b is a nonzero complex constant. Then H is a homotopy between the polynomial f on level 0 and h on level 1. Notice that we use $(2t)^{2n}b$ instead of $2tb$ as in [1].

Next, let ρJ_3 be the set of 3-dimensional simplices in $(0,1] \times \mathbb{C}$ as in Todd [7] where ρ is denoting the grid size. Then ρJ_3 is a refining triangulation of $(0,1] \times \mathbb{C}$ such that for each $d = 0, 1, 2, \ldots$, the set $[2^{-(d+1)}, 2^{-d}] \times \mathbb{C}$ is triangulated and the set $\{2^{-d}\} \times \mathbb{C}$ is subdivided into triangles with diameter all equal to $2^{-d}\sqrt{2}\rho$. The set of facets of all simplices in ρJ_3 is denoted by $(\rho J_3)^2$. For $\sigma \in \rho J_3$ or $\sigma \in (\rho J_3)^2$, the number $t(\sigma)$ denotes the depth of σ in $(0,1] \times \mathbb{C}$, i.e., $t(\sigma) = \sup\{t | (t,z) \in \sigma\}$. Let $\Phi : [0,1] \times \mathbb{C} \to \mathbb{C}$ be the piecewise linear approximation of H with respect to the triangulation ρJ_3, i.e., let $(t,z) = \sum_{i=1}^4 \lambda_i (t_i, z^i)$ be a point in the simplex in ρJ_3 with vertices (t_i, z^i) for $i = 1, \ldots, 4$, so that $\sum_{i=1}^4 \lambda_i = 1$ and $\lambda_i \geq 0$ for $i = 1, \ldots, 4$, then

$$\Phi(t,z) = \sum_{i=1}^{4} \lambda_i H(t_i, z^i).$$

Let $J(\varepsilon) = \{(s, s^2, \ldots, s^n)^T \in \mathbb{R}^n | 0 < s < \varepsilon\}$ for any positive number ε. A 2-dimensional simplex τ in $(\rho J_\varepsilon)^2$ with vertices y^1, y^2, y^3 is complete if $J(\varepsilon)$ is a subset of the convex hull of $H(y^1), H(y^2)$, and $H(y^3)$ for some positive ε. A simplex in ρJ_3 is complete if one of its facets is a complete 2-dimensional simplex. Any complete simplex in ρJ_3 has exactly one pair of complete facets.

The following two propositions are straightforward.

Proposition 2.1 *Suppose $\tau \in (\rho J_3)^2$ is complete, then*

$$\bar{\tau} \cap \Phi^{-1}(0) \neq \emptyset,$$

where $\bar{\tau}$ is the closure of τ and $\Phi^{-1}(0) = \{(t,z) \in (0,1] \times \mathbb{C} | \Phi(t,z) = 0\}$ is the zero set of Φ.

Proposition 2.2 *Suppose $\tau \in (\rho J_3)^2$ is complete and none of its vertices is a zero of Φ. Then there is at least a pair of its vertices such that their Φ-images span with respect to the origin an angle not less than $2\pi/3$.*

The next propositions can be found in [1].

Proposition 2.3 *Suppose that* $0 < \rho \leq r_0/(20n)$. *Then there are exactly* n *complete facets in* $\{1\} \times \mathbb{C}$.

To simplify the discussion, in the sequel we consider only the case when $|a_k| \leq 1, k = 1, \ldots, n$. Notice that this is only a technical restriction. Moreover, let $|b| \leq 1, \rho = 1$, and $r_0 = 20n$. Then the condition of Proposition 2.3 is satisfied. The next proposition says that all complete simplices lie in a bounded set.

Proposition 2.4 *All complete facets in* $(\rho J_3)^2$ *lie entirely within a vertical circular cylinder with axis* $(0, 1] \times \{0\}$ *and radius* $R = 40n$.

The propositions above guarantee that there exist n sequences of adjacent simplices in ρJ_3 having common complete facets. Each sequence starts with some complete facet in $\{1\} \times \mathbb{C}$ and consists of an infinite number of simplices. Since all complete facets lie in a bounded set and the number of complete facets in the set $[2^{-d}, 1] \times \mathbb{C}$ for any integer d is finite, each sequence therefore will reach any level $\{2^{-d}\} \times \mathbb{C}, d = 0, 1, \ldots,$ within a finite number of simplices. Each sequence of adjacent complete simplices can be followed by complementarity pivoting. Moreover, the sequences lead to the roots of f on level 0, see [1].

Proposition 2.5 *Complementary pivoting procedures which start from the* n *complete facets in* $\{1\} \times \mathbb{C}$ *give all* n *zeros of the polynomial under consideration with the correct multiplicities.*

3 Error estimate

In this section we discuss how far a complete facet in $(\rho J^3)^2$ lies from one of the roots of f when it lies close to the level 0. The next proposition shows how close the roots of two arbitrary polynomials of degree n, lie at most from each other, see [4] for a proof.

Proposition 3.1 *Suppose* $f(z) = z^n + \sum_{k=1}^n a_k z^{n-k}$ *and* $g(z) = z^n + \sum_{k=1}^n b_k z^{n-k}$ *are two polynomials of degree* n *and let* $\alpha_1, \ldots, \alpha_n$ *and* $\beta_1, \ldots, \beta_n$ *be their zeros, respectively. Let* $\mu = \max\{|\alpha_1|, \ldots, |\alpha_n|, |\beta_1|, \ldots, |\beta_n|\}$ *and* $\theta = (\sum_{k=1}^n |b_k - a_k|\mu^{n-k})^{1/n}$. *Then after appropriate arrangement if necessary, we have that*

$$|\alpha_k - \beta_k| < 2n\theta, \; k = 1, 2, \ldots, n.$$

For any given $t \in (0, \frac{1}{2}]$, let $\xi_1, \ldots, \xi_k$ be the zeros of $H(t, z)$, then Proposition 3.1 yields that after appropriate arrangement

$$|\alpha_k - \xi_k| < w(t), k = 1, 2, \ldots, n,$$

where

$$w(t) = 2n(2t)^2 |b|^{1/n} \leq 2n(2t)^2.$$

Let N be equal to $\lceil \log_2 n \rceil + 2$ where $\lceil r \rceil$ stands for the smallest integer not less than r for any real number r, then for $t \leq 2^{-N}$ we obtain that $2n2t = 4nt \leq 1$, and thus

$$w(t) \leq 2t.$$

The next proposition can be found in [3].

Proposition 3.2 *For any $w \in \mathbb{C}$ with $|w| < 1$ it holds that*

$$|\arg(1 + w)| \leq \frac{1}{2}\pi |w|.$$

Finally, for any $\xi \in \mathbb{C}$ and $r > 0$, let $B(\xi, r)$ denote the open ball in $\mathbb{C}$ with centre ξ and radius r, i.e., $B(\xi, r) = \{z \in \mathbb{C} | |z - \xi| < r\}$, then the next theorem says that any complete facet τ with depth t at most equal to 2^{-N} lies at most a distance of $(5n + 4)t$ from one of the zeros of f.

Theorem 3.1 . *Let τ be a complete facet in $(\rho J^3)^2$ with vertices y^1, y^2, y^3 and let its depth $t(\tau)$ be less than or equal to 2^{-N}. Then*

$$\tau \subset (0, t(\tau)] \times \bigcup_{k=1}^{n} B(\alpha_k, (5n + 4)t(\tau)).$$

Proof. We need only to prove the theorem in the case of $n \geq 2$ and $H(y^1), H(y^2)$ and $H(y^3)$ are all unequal to zero. For $i = 1, 2, 3$, let $y^i = (t_i, z^i)$ and let $\xi_1^i, \ldots, \xi_n^i$ be the zeros of $H(t_i, \cdot)$ such that these zeros are consistent with the zeros $\alpha_1, \ldots, \alpha_n$ of f in the sense of Proposition 3.1. Thus $|\xi_k^i - \alpha_k| \leq 2t_i, i = 1, 2, 3, k = 1, \ldots, n$.

In case $t_1 = t_2 = t_3$, we need only to show that the distance of τ and the set of zeros of $H(t_1, \cdot)$ is not greater than $5nt_1$, implying that $\tau \subset \{t_1\} \times B(\alpha_k, (5n+4)t(\tau))$ for some

$k \in \{1, \dots, n\}$. Suppose that the distance of τ and every ξ_k^1 is greater than $5nt_1$, thus together with Proposition 3.2, for all $i \neq j \in \{1,2,3\}$

$$\begin{aligned}\left|\arg\frac{H(t_1,z_i)}{H(t_1,z_j)}\right| &\leq \sum_{k=1}^{n}\left|\arg\frac{z_i-\xi_k^1}{z_j-\xi_k^1}\right| = \sum_{k=1}^{n}\left|\arg\left(1+\frac{z_i-z_j}{z_j-\xi_k^1}\right)\right| \\ &\leq n\frac{\pi}{2}\frac{\sqrt{2}}{5n} < 2\pi/3,\end{aligned}$$

since $|(z_i - z_j)/(z_j - \xi_k^1)| < \sqrt{2}\, t_1/(5nt_1) < 1$. This contradicts to the result of Proposition 2.2.

In case not $t_1 = t_2 = t_3$, suppose without loss of generality that $t_1 < t_2$ while $t_1 \leq t_3 \leq t_2$. Now $t(\tau)$ is equal to t_2. Since the diameter of the simplex τ projected on $\{t_2\} \times \mathbb{C}$ is at most equal to $\sqrt{2}\, t_2$ we need only to show that the distance of τ and the set of zeros of $H(t_2, \cdot)$ is not greater than $(5n - \sqrt{2})t_2$. Suppose not, then

$$\left|\frac{(z_1 - z_2 + \xi_k^2 - \xi_k^1)}{(z_2 - \xi_k^2)}\right| \leq \frac{\sqrt{2}+4}{5n-\sqrt{2}} < 1$$

and so

$$\begin{aligned}\left|\arg\frac{H(t_1,z_1)}{H(t_2,z_2)}\right| &\leq \sum_{k=1}^{n}\left|\arg\frac{z_1-\xi_k^1}{z_2-\xi_k^2}\right| = \sum_{k=1}^{n}\left|\left(1+\frac{z_1-z_2+\xi_k^2-\xi_k^1}{z_2-\xi_k^2}\right)\right| \\ &\leq n\frac{\pi}{2}\frac{\sqrt{2}+4}{5n-\sqrt{2}} < 2\pi/3.\end{aligned}$$

The argument of $|\arg(H(t_1,z_1)/H(t_3,z_3))| < 2\pi/3$ is exactly the same if $t_3 = t_2$ and the same as above if $t_3 = t_1$. Similarly, $|\arg(H(t_3,z_3)/H(t_2,z_2))| < 2\pi/3$ holds. This again contradicts the result of Proposition 2.2. This completes the proof. □

4 Computational complexity

In this section we discuss how many simplices in the triangulation ρJ_3 will be at most generated before some given accuracy in approximation is reached for computing all zeros of the polynomial f. The calculation of the number of simplices is similar to the one in case of integer labelling.

Proposition 4.1 *Let d be a nonnegative integer and let K be a unit cube in $(0,1] \times \mathbb{C}$. Let $\xi \in \mathbb{C}$ and Z a disk in $\mathbb{C}$ with ξ as center. Denote s_d as the number of 3-dimensional* *simplices in ρJ_3 which meet with $[2^{-(d+1)}, 2^{-d}] \times (K \cap Z)$. Then*

$$s_d \leq 14\ Vol\ (K \cap Z) 2^{2d},$$

where $Vol\ (K \cap Z)$ means the volume of $K \cap Z$ in Euclidean two-space.

Proof. This proposition is just a variant of Proposition 3.1 in [3]. □

The next proposition is a corollary of Theorem 3.1.

Proposition 4.2 *To locate all zeros of f within an accuracy of ε, no computation below level $t = 2^{-D}$ is necessary if ε is small enough, where D is equal to $\log_2 \lceil (5n+4)/\varepsilon \rceil$.*

All the propositions above together with Theorem 3.1 imply the following result.

Theorem 4.1 *For $\varepsilon > 0$ small enough, the number of pivots to compute all zeros of f within an accuracy of ε is at most equal to*

$$14\pi[22000n^4 + n(5n+2)^2(D-N)],$$

where $N = \lceil \log_2 n \rceil + 2$ and $D = \lceil \log_2((5n+4)/\varepsilon) \rceil$.

Proof. By Theorem 3.1 and Proposition 2.4, all complete facets lie in either a cylinder of radius $40n$ between the levels $t = 1$ and $t = 2^{-N}$ with center the origin or in n stepped cylinders centred at the zeros of f when $0 < t \leq 2^{-N}$. With Proposition 4.1 the number of complete simplices between $t = 1$ and $t = 2^{-N}$ is at most equal to

$$\sum_{i=0}^{N} 14\pi(40n)^2 2^{2i} = 14\pi(40n)^2(2^{2N+2} - 1)/3$$

$$\leq 14\pi(4n)^2 2^8 n^2/3 \leq 14\pi(22000n^4).$$

For $d \geq N$, between $t = 2^{-d}$ and $t = 2^{-d+1}$ the number of complete simplices in ρJ_3 is at most equal to

$$n 14\pi(5n+4)^2 2^{-2d} 2^{2d} = 14\pi n(5n+4)^2.$$

Hence, by Proposition 4.2, the total number of simplices encountered in computation up to an accuracy of ε is at most equal to

$$14\pi[22000n^4 + n(5n+4)^2(D-N)].$$

□

The theorem shows that the cost of locating all zeros of f with correct multipliticities and with accuracy equal to ε grows not faster than $O(\max\{n^4, n^3 \log_2(n/\varepsilon)\})$.

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