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Published in:
Game theory and applications

Publication date:
1990

Citation for published version (APA):
Tijs, S. H. (1990). Big boss games, clan games and information market games. In T. Ichiishi, A. Neyman, & Y. Tauman (Eds.), *Game theory and applications* (pp. 410-412). Academic Press.

BIG BOSS GAMES, CLAN GAMES AND INFORMATION MARKET GAMES

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EXTENDED ABSTRACT

A trend in cooperative game theory is to consider subclasses of games in characteristic function form and to study the behaviour of solution concepts just on such a subclass. Well-studied are e.g. the cone of convex games (Cf. Shapley (1971), the cone of 1-convex games (Cf. Driessen and Tijs (1983)) and of k-convex games (Cf. Driessen (1985)). Other well-known classes are those consisting of simple games, airport games, flow games, minimum spanning tree games, sequencing games etc.

Recently the three classes of games, mentioned in the title, were introduced in [4],[5] and [3]. We will give here the definitions, indicate some economic situations giving rise to such games and discuss some properties of these classes and of the solution concepts.

First recall that a *game* (in characteristic function form) is an ordered pair $<N,v>$, where $N := \{1,2,\ldots,n\}$ is the set of players and $v : 2^N \to \mathbb{R}$ is a real valued map with $v(\phi) = 0$ defined on the family 2^N of subsets (coalitions) of N. The characteristic function v assigns to each coalition $S \in 2^N$ the *worth* v(S), where v(S) can be seen as the amount of (say) money, which S can gain if the members in S cooperate. In the following a role is played by the *marginal vector* $M(v) = (M_1(v), M_2(v),\ldots,M_n(v))$ of the game $<N,v>$, where $M_i(v) := v(N) - v(N\setminus\{i\})$ for each $i \in N$.

A game $<N,v>$ is called a *big boss game* (with player 1 as big boss) iff the following three conditions hold:

(B.1) $v \geq 0,\ M(v) \geq 0$ *(Non-negativity condition)*

(B.2) $v(S) = 0$ if $1 \notin S$ *(Big boss property)*

(B.3) $v(N) - v(N\setminus T) \geq \sum_{i\in T} M_i(v)$ if $1 \notin T$ *(Union property)*

Condition (B.1) expresses the fact that in such a game the worth of each coalition is non-negative and also the marginal contribution of each player to the grand coalition N.
Condition (B.2) states that there is no gain by cooperation if the big boss is not included in the coalition, and (B.3) tells that for a union (i.e. a coalition without the big boss) the marginal contribution to the grand coalition is at least as large as the sum of the marginal contributions of each of its members. It turns out (cf.[4]) that there are many economic situations, which give rise to big boss games. We mention

(i) one seller-many buyers situations of certain type
(ii) landlord-workers games with a concave production function
(iii) bankruptcy games with one big claimant.

Also information market games as introduced in [3] turn out to be big boss games, where the big boss is the initially informed trader. Such games correspond to information markets where one trader (player 1) is informed about a new product and where for each $T \subset N$ there is a market M_T into which only the members of T have entrance and where the possible reward for the new product in that market is $r_T \geq 0$. The characteristic function of the corresponding information market game is given by $v(S) = 0$ if $1 \notin S$ and

$$v(S) = \Sigma_{T:T \cap S \neq \phi} r_T \quad \text{if } 1 \in S$$

In the following theorem we collect some results of big boss games (and consequently of information market games)

<u>Theorem 1</u>. Let <N,v> be a big boss game. Then

(i) The core of <N,v> is a paralellotope, consisting of those vectors $x \in \mathbb{R}^n$ with $\Sigma_{i \in N} x_i = v(N)$ and $0 \leq x_i \leq M_i(v)$ for all $i \in \{2,3,\ldots,n\}$.
(ii) The τ-value $\tau(v)$ and the nucleolus $n(v)$ of <N,v> coincide both with the center

$$(V(N) - \frac{1}{2}\Sigma_{i=2}^{n} M_i(v), \frac{1}{2}M_2(v), \ldots, \frac{1}{2}M_n(v))$$

of the core, which is also the unique point in the kernel
(iii) The bargaining set $M_1^{(i)}$ (for the grand coalition) coincides with the core and is also a subsolution à la Roth.

(iv) For the Shapley value $\Phi(v)$ we have $\Phi_1(v) \le \tau_1(v)$.
(v) $\Phi(v) = n(v) = \tau(v)$ iff $\langle N,v\rangle$ is convex.

A game $\langle N,v\rangle$ is a *clan game* (cf.[5]) with clan C, where $\phi \neq C \subset N$, iff the following conditions hold

(C.1) $v \ge 0$, $M(v) \ge 0$ *(Non-negativity property)*
(C.2) $v(S) = 0$ if $C \not\subset S$ *(Clan property)*
(C.3) $v(N) - v(N\backslash T) \ge \sum_{i \in T} M_i(v)$ if $C \subset N\backslash T$ *(Union property)*

Note that a big boss game is a clan game with clan $C = \{1\}$. Note further that clan games with fixed clan C form a polyhedral cone. Examples of clan games are (i) unanimity games, (ii) bankruptcy games where the set of claimants N can be divided into two parts C and N\C, where the members of C each claim at least the whole remaining capital E and where the sum of the claims of the members in N\C is at most the capital E, (iii) landlord, machine owner and workers games, (iv) information market games arising from a situation where each member of a group of players C has an essential part of the information needed to develop a new product. A characterisation of clan games in terms of the core is given in the next theorem and a summary of some properties in theorem 3.

Theorem 2. Let $\langle N,v\rangle$ be a game with $v \ge 0$ and let C be a non-empty subset of N. Then $\langle N,v\rangle$ is a clan game with clan C iff $v(N)e^i \in$ core (v) for each $i \in C$ and there is an $x \in core(v)$ such that $x_i = M_i(v)$ for all $i \in N\backslash C$.

Theorem 3. Let $\langle N,v\rangle$ be a clan game with clan C and $|C| \ge 2$. Then

(i) Core $(v) = \{x \in \mathbb{R}^N | \sum_{i=1}^n x_i = v(N),\ 0 \le x_i \le M_i(v)$ for all $i \in N\backslash C\}$
(ii) $\tau(v)$ is a multiple of $M(v)$
(iii) For the nucleolus $n(v)$ we have

$$n_i(v) = \begin{cases} \beta & \text{if } i \in C \\ \min\{\beta, \frac{1}{2} M_i(v)\} & \text{if } i \in N\backslash C \end{cases}$$

where β is such that $\sum_{i=1}^n n_i(v) = v(N)$.

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