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CASES IN COOPERATION AND CUTTING THE CAKE

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1 Introduction

Cooperation should be big business, but the main question is: 'How to reach a state of cooperation and how to cut the cake if it comes to cooperation?' Here by cake we mean the cost savings or extra-gains generated by cooperation. The success of cooperative ventures often relies on agreements on how to share the costs and/or benefits generated. Cooperative game theory and sharing problems are closely interrelated in practice. An overview on the game theoretic literature on sharing rules can be found in the work of Tijs and Driessen (1986) and Young (1994). For an introduction in cooperative game theory we refer the reader to the books of Tijs (2003) and Peleg and Sudhölter (2003). Cost/reward allocation is an important practical problem arising from many different real-world situations. There is no single, all purpose solution to the sharing problem. In satisfying the existing need for suitable mechanisms to distribute cost/reward among the agents involved, organizational constraints and goals, environmental aspects as well as the amount of available information should be taken into account. Joint ventures require an allocation mechanism which is efficient, fair, and provides incentives to (different groups of) agents involved to agree upon.

Often sharing situations can be modelled as coalitional games. A coalitional reward game consists of a player set and a characteristic function with assigns to each coalition of players a real number that is to be interpreted as the maximum profit or cost savings that the members in this coalition can realize when they cooperate. To describe a situation where a group of people can decrease their costs by working together a coalitional cost game can be used, where the characteristic function assigns to each coalition its costs. To each coalitional cost game one can associate a related coalitional reward game, which is usually called the corresponding cost savings game. Advantages of modelling sharing situations as coalitional games are that one is forced to structure the underlying situation, and that game theoretic solution concepts which are based on considerations concerning coalitions of players can be applied.

We assume that all players of the game will cooperate to form the grand coalition and cope with the problem of allocating the worth of the grand coalition among them. Such an allocation is called efficient. Further, an allocation for players in a cooperative (reward) game is called stable if each coalition receives at least its value. The need for an efficient and stable allocation makes the core (cf. Gillies (1953)) an attractive solution concept for all groups of) players. Classical solution concepts in cooperative game theory are the Shapley value (cf. Shapley (1953)), the τ-value (cf. Tijs (1981)), and the nucleolus (cf. Schmeidler (1969)). These solution concepts satisfy three different systems of axioms. The Shapley value is the unique solution concept which satisfies the efficiency property, the dummy player property, the symmetry property and the additivity property. The τ-value is the unique solution concept (for semi-balanced games) which satisfies the efficiency property, the minimal right property and the restricted proportionality property. The nucleolus is the unique solution concept which satisfies the efficiency property, the individual rationality property and the consistency property. The choice of a specific solution concept is often based on the properties that are appealing for the situation at stake.

The coalitional games arising from 'phoning in plane' situations have, as we see in Section 4, a specific structure which is relevant for the approached situations and provides strong arguments to choose a specific division of the revenues in such situations. In our opinion 'phoning in planes' situations provide nice examples of how theory and practice may interact.

In general, it is a difficult task to find a suitable model of the practical situation at hand and, moreover, to give a transparent division scheme for the players involved. To stress the difficulties in using cooperative game theory for help in advising, we include here a small but significant part from the contribution of Michael Maschler to the Game Practice I conference volume (2000):

'To sum up, when we make a recommendation on a cooperative-type real issue we are faced with a problem of choosing the right solution concept. To make a good decision we have to examine the foundations of the solution concept to see how they fit reality. The decision, however, may also depend on the coalition function we choose to model the situation. Again, we have to examine the real case in order to make a choice. It is not sufficient to recommend a certain solution on the ground that game theory has defined it. One has to justify why the particular solution is appropriate to the specific issue.'

This paper is mainly intended to show that the use of Cooperative Game Theory for help in advising, or in mechanism design, has seen some notable successes. But, we would like to stress that much work has still to be done in this direction to increase the use of game theory by society. Further, there is still a lot of new ground which can and should be explored by Game Theory. The Game Practice conferences (1998, 2000, 2002, 2004), the ZiF year 'Procedural Approaches to Conflict Resolution', etc. are helpful to bridge the still existing gap between theory and practice.

The outline of this contribution is as follows. In Section 2 we give a list of seminal sharing problems arising from practice. Two cases of cooperation in container transport are treated in Section 3, and two cases of telecom cooperation are extensively discussed in Section 4. Section 5 concludes with lessons which can be learned from the cases.

2 Cooperative game theory with an eye outside

In the past many sharing problems arising from practice were important in the development of cooperative game theory, either as supporters of already known theory or as an inspiration source for new directions. We can mention here some problems:

(i) The Tennessee Valley Problem (cf. Ransmeier (1942) and Driessen (1988)), where some proposed rules were related to the τ-value of the related game. The Tennessee Valley Problem has been faced by the Tennessee Valley Authority when solving the question about (the price of) electricity power in its program. The Authority had to prepare allocations of the costs of the Wilson Dam and of additional reservoir projects which the Authority might implement among five objectives: navigation; flood control; development of power; national defence; fertilizer production.

(ii) The airport fee problem (Littlechild and Thompson (1977)) where the principle 'the users pay and they pay equally' corresponds to the Shapley value of the related airport game. The problem in airport situations is how the cost of a runway should be distributed among users who need runways of different lengths for their airplanes. For airport fee problems where there are restricted benefits we refer to Branzei et al. (2002, 2003) and Tijs and Branzei (2004).

(iii) The bankruptcy problem (O'Neill (1982), Aumann and Maschler (1985)) where the nucleolus of a related game corresponds to proposals for bankruptcy in the Talmud. A bankruptcy problem arises from a situation where an estate has to be divided among several claimants, each of them with a claim on the estate, and the aggregate claim exceeds the available estate.

(iv) The Cornell telephone bill problem (Billera (1978)) where games with a continuum of players played a role. The 'internal telephone billing rates' problem at Cornell University is well known. The question here is: 'How to charge local calls, long-distance calls, etc. to the different departments?' This problem led to the literature of Aumann-Shapley pricing (1974) in multi-commodity cost sharing problems.

(v) Cost sharing problems in irrigation situations (Aadland and Kolpin (1998), Koster et al. (2001)) where weighted Shapley values and weighted constrained egalitarian solutions were interesting. The sample of irrigation ditches used by Aadland and Kolpin (1998) is drawn from Carbon and Stillwater counties of Montana, USA. It consists of a main ditch for common use from which private ditches branch off and transport water to different users' parcels of land. The main ditch begins with the headgate – a device that controls the volume of water delivered from the source stream – and then continues on a sequential path through the land consisting of parcels of each user. Each user is individually responsible for expenses incurred on their private ditches, so the problem is how to share the costs associated with the main ditch.

The last 30 years the interaction between cooperative game theory and Operations Research was also important. It created all kind of Operations Research Games (Curiel (1997), Borm et al. (2001)). We mention a few: linear production games (Owen (1975)), minimum spanning tree games (Bird (1976)), sequencing games (Curiel et al. (1989)), traveling salesman games (Potters et al. (1992)), and holding games (Tijs et al. (2000)). In many of these situations the optimization problem for the grand coalition and the sharing problem could be treated simultaneously, leading to interesting taylor made sharing allocations (produce and pay, construct and charge, switch and share, ...).

3 Cooperation in container transport

In this section we consider two cases. In the first case container transport with trucks from Rotterdam to Germany and the awareness of gains in cooperation (less container movements with almost empty containers) led to cooperation of three firms where one firm played a special role because this firm had developed an (excellent) routing program. The Regiefunctie Informatietechnologie and Telecommunicatie (R2i) of the Gemeentelijk Havenbedrijf Rotterdam contacted the Game Theory Group of Tilburg University. In fact it was the firm owning the routing program who contacted us via R2i, which we call for this reason in the following the 'problem owner'. The three parties alone could not reach an agreement because of the problem of how to compensate the 'problem owner' for the development costs of the routing program. We constructed two cooperative 3-person games v_b and v, where the game v_b should be used the first three years and the game v the later years. The game v_b was more advantageous for the problem owner because it was also meant to compensate the problem owner for the development costs of his routing program. It turned out that the game v_b had a special structure which could be related to the structure of so-called big boss games. A big boss game (cf. Muto et al. (1988)) is a coalitional game where there is one player with veto power – the big boss – and the characteristic function satisfies the monotonicity property and the union property. Such a game has the τ-value as an interesting solution in the barycenter of the core. After our proposal the three firms started working together almost immediately. For more details of this case we refer to van Os (1997).

In the second case only the R2i contacted our game theory group. They had observed that the firms transporting containers over the Rhine from Rotterdam to the Middle Rhine did not work together in container transport, in contrary to the firms active on the Upper and the Lower Rhine. Again we have used cooperative game theory and constructed a cooperative game and proposed a solution. In this case there was no problem owner. It turned out that big extra gains could be made in cooperation but the firms did not react. An extensive description of this case can be found in the work by Nielen (1994).

It is difficult to explain why our advice was followed in the first case, but not in the second case. In our opinion the presence of the problem owner in the first case has been beneficial for incentivating cooperation. In the second case the firms alone were well-doing and this could explain their lack of interest in cooperation. The different attitude towards cooperation in the two cases could also be explained by differences in cultural factors. There was also a difference in the nature of transport – container transport by trucks to Germany for firms in Rotterdam harbour versus transport by ships within Middle Rhine region – which could also influence the willingness to cooperate.

4 Telecom problems and cooperative game theory

In this section we give a description of two situations in telecommunication in the beginning of the nineties of the last century and show how the related sharing problems were tackled by means of cooperative game theory. Our exposition is mainly based on the work of van den Nouweland et al. (1996).

The situation we describe first concerned a public telephone service for passengers in airplanes in which the telephone connections are established by radio communication to a near ground station, from where the connections are provided to the destination subscriber using the existing network. Such a system was called the Terrestrial Flight Telephone System (TFTS). To launch this service, which is economically attractive only if it is sufficiently wide-spread, several national operators had to cooperate by installing the necessary apparatus in airplanes and by placing ground stations that could make it possible to use the service when flying over the countries. A group of European operators decided to cooperate and agreed upon the configuration of ground stations to be placed such that the overlap between stations is minimal and each operator will place (and pay for) the ground stations that are planned to be placed in his country. The problem that they had still to solve was to agree upon how to divide the revenues that will be (potentially) generated through TFTS. Two existing proposals, namely PI, which splits the total revenue proportionally to the investments of the countries, and GR, that lets each country have the revenues which are generated via the ground stations it installs, have been proved to be unappealing for reaching cooperation. To find acceptable divisions of the revenues van den Nouweland et al. (1996) have modelled the TFTS situation as a coalitional game by taking the countries whose national operators and the plane company participate in the cooperation to be the players. The characteristic function is such that for each coalition of players the revenue that its members can jointly realize within the cooperation of the countries is the sum of the revenues that are generated by one and two countries within that coalition. The basic data, i.e. the revenues generated by telephone calls that are made from airplanes of each cooperating country flying over any country, were computed by using real data from Reed Travel Group-ABC International and the Deutsche Bundespost. The coalitional game associated with the TFTS situation is the sum of an additive game and a nonnegative 2-game (i.e. a nonnegative combination of unanimity games based on 2-person coalitions) implying that the game is convex. Since for the class in which the TFTS game is a member the Shapley value, the nucleolus and the τ-value coincide (and occupy a central position in the core of the game), it has been a strong argument to recommend this specific division of the revenues. This proposal has been proved to be very appealing. Indeed, given that it satisfies three axiom systems, one can think that it must be hard to find other divisions that make sense a priori. It has been cooperative game theory helping to gain insight into the sharing problem arising from the TFTS situation.

The second situation analyzed here concerns the rerouting of international telephone calls. The circuits can be used more efficiently if during busy hours the calls are routed via quiet parts of the international network instead of routing them via direct circuits from the originating country to the destination country (see Gibbens et al. (1991)). So, by cooperating and making agreements on the use of transit routes, international carriers are able to use their circuits more efficiently and to reduce the costs of their network. The use of transit routes is, in general, limited to two link paths to prevent that telephone calls bounce back to the originating country, and also that congestion on one link will affect the whole international network. Hence, in order to generate profits through the rerouting of an international telephone call, exactly three international carriers have to cooperate: a carrier from the originating country, a transit carrier, and a carrier in the destination country. The rerouting of international telephone calls situation could be modelled as a coalitional game, where the players are the international carriers, and for each coalition of international carriers the worth is defined to be the sum of cost savings that are obtainable by subcoalitions of size 3 of that coalition. The coalitional game corresponding to the rerouting of international telephone calls is a sum of nonnegative 3-games implying that the game is convex, and, consequently, it has a nonempty core. So, it has been shown via cooperative game theory that it is worthwhile for international carriers to cooperate and reroute their international telephone calls during busy hours. Two important solution concepts, the Shapley value and the τ-value, coincide for the rerouting game and offer an appealing way to divide the gains from cooperation such that not only all individual carriers, but also all coalitions of international carriers are better off than they would be if they did not cooperate with the other carriers.

In both telecommunication situations the contractors have been very thankful to us, but given the 'secret' feature of this research we do not know yet in how far our proposals were actually implemented. Note that we could consider the Dutch Telephone Company as the problem owner. Finally, we would like to confess that during this research there was a lot a communication, but we have had the feeling that many things were not disclosed to us.

5 Concluding remarks

The Tilburg Game Theory Group learned many lessons from their advising practice described in Sections 3 and 4. To know the client and the problem well we took care that a master student worked in the firm full time for some months and a PhD student one day a week during that period. Further there were regularly meetings between members of the firm and the Game Theory Group of Tilburg. The biggest problems were:

(i) to get insight in the cooperative situation;

(ii) to find the good estimate for the characteristic function of the involved cooperative game;

(iii) to find a sharing rule for the cake, which is clear and appealing for the clients.

In the cases of 'phoning in planes' and 'container transport by trucks' there was already a conflict history before we entered. We were able to find new viewpoints on the problems which made the advice successful. We were successful in situations were one of the players (the problem owner) interacted with us. It would be interesting to study all kind of interactions and get some insight in the ‚best' one.

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