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Determination of optimal penalties for antitrust violations in a dynamic setting.[*]

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Abstract

We analyze a differential game describing the interactions between a firm that might be violating competition law and the antitrust authority. The objective of the authority is to minimize social costs (loss in consumer surplus) induced by an increase in prices above marginal costs. It turns out that the penalty schemes which are used now in EU and US legislation appear not to be as efficient as desired from the point of view of minimization of consumer loss from price-fixing activities of the firm. In particular, we prove that full compliance behavior is not sustainable as a Nash Equilibrium in Markovian strategies over the whole planning period, and, moreover, that it will never arise as the long-run steady-state equilibrium of the model. We also investigate the question which penalty system enables us to completely deter cartel formation in a dynamic setting. We found that this socially desirable outcome can be achieved in case the penalty is an increasing function of the degree of offence and is negatively related to the probability of law enforcement.

JEL-Classification: L41, K21, C73 .

Keywords: Antitrust Policy, Antitrust Law, Dynamic games

1 Introduction.

In this paper we incorporate specific features of antitrust law enforcement, which are in practice now in the US and the European Union, into a dynamic framework of utility maximization with two players having conflicting objectives. In the particular case of violations of antitrust law, those two players are the firm of regulated monopoly type, which rises prices above marginal costs level or the firm, which participates in cartel agreements, and the Antitrust Authority, whose aim is to prevent price-fixing or cartel formation in the industry.

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According to the US sentencing guidelines for organizations (2001) and the guidelines on the method of setting fines imposed for violations of competition law in Europe (1998), the penalty schemes for antitrust violations are based mainly on the gravity of the violations, which is determined on the basis of the turnover involved in the infringement. To be more precise, in the European regulation the penalty imposed depends on the gravity and duration of the infringement in a linear manner. The level of offence is measured by the turnover involved in the infringement, which is defined as the total sales of the product involved over the whole period of existence of the cartel. In the US sentencing guidelines for organizations the system of fine imposition for antitrust violations is different. There we observe that the penalty schedule for the base fine is represented by a convex increasing function of the level of offence.

In order to investigate the efficiency of the current penalty schemes we incorporate these two features of penalty systems for antitrust law violations into a dynamic model of intertemporal utility maximization by modelling penalty schedule in the stylized form as a linear or quadratic functions of the degree of price-fixing and time. Similar to Feichtinger (1983) the set up of the problem leads to a differential game. The authorities attempt to minimize the social loss caused by price-fixing, whereas the firm wants to maximize the profit gained from price-fixing.

It is found that the stylized form of the existing penalty schemes would not succeed, in the sense that it cannot provide complete deterrence. Therefore, we try to find a more efficient functional form of penalty schedule for violations of antitrust law. Finally, we suggest a new penalty system which is most efficient from the point of view of complete deterrence of cartel formation in dynamic settings.

We relate our analysis to the general literature on crime and punishment, starting with Becker (1968). In his seminal paper, Becker (1968) studied the problem of how many resources and how much punishment should be used to enforce different kinds of legislation. The decision instruments are the expenditures on police and courts influencing the probability that the offender is convicted, and the type and size of punishment for those convicted. The goal was to find those expenditures and punishments that minimize the total social loss. This loss is the sum of damages from offences, costs of apprehension and conviction, and costs of carrying out the punishment imposed.

The main contribution of Becker's work was to demonstrate that the best policies to combat illegal behavior were part of an optimal allocation of resources. Becker (1968) investigates this problem using a static economic approach to crime and punishment. He derives that in a static environment the optimal fine should be a multiple of the social cost of the crime and inversely related to the probability of detection. So, since an increase in the probability of control causes an increase in the costs of detection, the least costly policy for the antitrust authority would be to decrease the probability of control and increase the fine itself. But in this case legal limitations concerning the upper bound of fine can exist. And this poses the problem. Later, in Leung (1991), Feichtinger (1983), Fent et al. (1999) and (2003) dynamic (intertemporal) trade-offs between the damages generated from the offences and the costs of the control instruments were studied. More precisely, there papers try to determine a mix of policy variables, like prevention, treatment and law enforcement, which minimize the discounted stream of total social loss.

Now we give a more detailed review of the papers related to the problem, addressed in the current paper. Leung (1991) introduces a dynamic model of optimal punishment, where the optimal fine is calculated as a solution to an optimal control problem. This model considerably improves the efficiency and cost effectiveness if compared to the static mechanism of Becker (1968). It was found that the optimal fine is negatively related to the offender's returns from engaging in some criminal activity and positively related to the social cost of the crime. Moreover, the author finds that the fine which would block the crime can actually be less than the harm induced by the infringement, which contradicts the result of Becker. Leung argues that Becker's approach will not generate the optimal outcome, i.e. the outcome which maximizes welfare, in a dynamic environment. In fact, according to Leung (1991) it would cause overcomplience because the multiple fine imposes too heavy a penalty on the offender.

A considerably different approach was suggested in Fent et al. (1999) and (2003). They investigated optimal law enforcement strategies in case punishment is modelled as a function depending not only on the intensity of crime (offence rate) but also on the offenders prior criminal record. This idea was adopted in Fent et al. (1999) in an optimal control model with the aim to discover the optimal intertemporal strategy of a profit maximizing offender under a given, static punishment policy in the model with only one agent. In Fent et al.(2003) the framework described above was extended to an intertemporal approach of utility maximization, considering two players with conflicting objectives. The authorities attempt to minimize the social loss caused by criminal offences, whereas the offending individual wants to maximize the profit gained from offending. This leads to a differential game, which makes it possible to study competitive interactions in a dynamic framework. The criminal record takes the role of a state variable. A high record increases the punishment an offender expects in case of being convicted.

Modelling intertemporal trade-offs requires application of tools like dynamic programming, optimal control theory and, if there is strategic interaction between players, differential games. All the papers mentioned above investigate the problem of optimal dynamic law enforcement and minimization of social loss from crime by modelling the interactions between the offender, who commits the crime, and the authority, whose aim is to prevent the crime. In this paper we suggest a similar approach. We analyze a differential game between the offender and the authority, whose aim is to prevent the crime, to study the situation of violation of antitrust law by the firm, which performs price-fixing activities or participates in a cartel.

Technically our analysis will be close to the paper by Feichtinger (1983), in which he studies violations of criminal law by means of a differential game solution to a model of competition between a thief and the police. We extend his framework by allowing for the penalty for violation to vary over time. Moreover, we introduce the fine as a function of the current degree of offence and probability of law enforcement at each instant of time.

The paper is organized as follows. In section 2 we set up the model describing the intertemporal game played between a firm engaged in price-fixing and the antitrust authority, and recall the modified static microeconomic model of price-fixing. In section 3 the differential game will be solved and we show that it is impossible to have complete deterrence under current European and US systems of penalties for antitrust violations. In section 4 a new penalty scheme, which gives the desired outcome with no collusion, will be suggested. Section 5 provides a summary of our results and outlines possible extensions and generalizations of the model. Finally, in appendixes we provide proofs of the main results of the paper.

2 Description of the problem.

A model is designed to determine optimal penalty schemes for antitrust violations and cartel deterrence in the framework of differential games. There are two types of agents. First, there are the firms, which can perform illegal activities, such as price-fixing and cartel formation or violations of the price limits imposed by the authority on the regulated monopoly. They obtain strictly positive gains from price-fixing in each period, that the cartel was present in the market. Second, we have the antitrust authority, which can inspect those firms, and, in case violation is detected, punish them by imposing a fine $s(t)$, where t reflects the time index.. The interactions of the agents are modelled as a continuous time problem with planning horizon $(0, T)$, where $T < \infty$.

The aim of the firm is to dynamically maximize its total expected gain from increase of price above competitive level over time by choosing $q(t)$. We will call this variable the degree of illegal activities with respect to price-fixing (analogous to the "pilfering rate" in the model of competition between thief and police in Feichtinger (1983)). This variable will be described in more detail in the subsection 2.1, which deals with the microeconomic model underlying the problem of fighting price-fixing agreements.

The antitrust authority is modelled as a second decision maker. It also has one instrument, which is the "rate of law enforcement" (or probability of control by the antitrust authority) $p(t)$. The aim of the antitrust authority is to maximize welfare. This implies that the rents from collusion for the firms need to be reduced. So the aim of the antitrust authority is to prevent cartel formation at the lowest possible costs.

The profit of the firm in each period or the rent from collusion per period above the competitive profit ($\pi^c = 0$) is $\pi(t) > 0$.

Moreover, in order to be able to set up the model and determine the objective functionals of both players, we first describe the static microeconomic model of price-fixing and define our terms.

2.1 Static microeconomic model of price-fixing.

Let us consider an industry with M symmetric firms engaged in a price fixing agreement. Assume that they can agree and increase prices from $P^c = c$ to $P > c$ each, where c is the marginal cost in the industry. Since firms are symmetric, each of them has equal weight in the coalition and consequently total cartel profits will be divided equally among them.[1] Hence, the whole market for the product (in which the price-fixing agreement has been achieved) will be divided equally among M firms, so each firm operates in a specific market in which the inverse demand function equals $P(Q) = 1 - Q$. They are identical in all submarkets. Under these assumptions we can simplify the setting by considering not the whole cartel (group of violators) but only one firm, and apply similar sanctions to all the members of cartel.[2]

Let P^m be the monopoly price in the industry under consideration, and $P = 1 - Q$ is the

[1] We also assume that there is no strategic interaction between the firms in the coalition in the sense that we abstract from the possibility of self-reporting or any other non-cooperative behavior of the firms towards each other.

[2] Of course, in these settings the incentives of the firms to betray the cartel can not be taken into account and the possibility to influence the internal stability of the cartel is not feasible. But this is the topic for another paper.

inverse demand for a particular firm. In order to be able to represent consumer surplus and extra profits from price fixing for the firm (π) in terms of the degree of collusion, we specify the variable q, which denotes the degree of price-fixing. Let $q = \frac{P-c}{P^m-c}$,where P is the price level agreed by the firms. Then it holds that $q \in [0, 1]$ and extra profits from price fixing for this particular firm will be determined according to the following formula:

$$\pi = q(\frac{(1-c)}{(P^m-c)} - q)(P^m - c)^2.$$

Let $(P^m - c)^2 = A$. With linear demand $P = 1 - Q$ we observe that $P^m = \frac{1+c}{2}$, so that $\frac{1-c}{P^m-c} = 2$ and, consequently, it holds that $A = \frac{(1-c)^2}{4} = \Pi^m$ (monopoly profit in this particular market).

The producer surplus, consumer surplus and net loss in consumer surplus are represented in Figure 1.

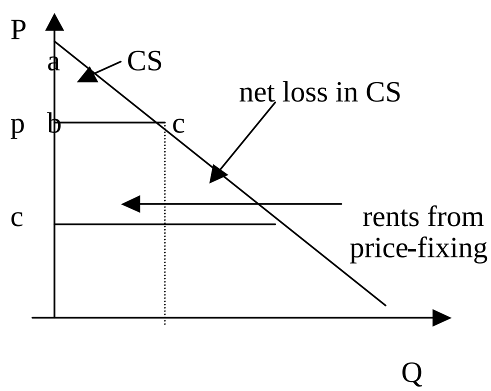

Figure 1: Representation of producer and consumer surpluses and net loss of consumer surplus in the price-quantity diagram.

The Producer Surplus equals

$$PS(q) = \pi(q) = \Pi^m q(2 - q) \ ,$$

the Net Loss of Consumer Surplus is the area of the right triangle

$$NLCS = \frac{1}{2}\Pi^m q^2 \ ,$$

while the Consumer Surplus is determined by the area of triangle *abc*:

$$CS(q) = \frac{1}{2}\Pi^m (2 - q)^2 \ .$$

Under the assumption that Π^m is equal to $\frac{1}{4}$ (or $c = 0$), these three functions are presented in Figure 2.

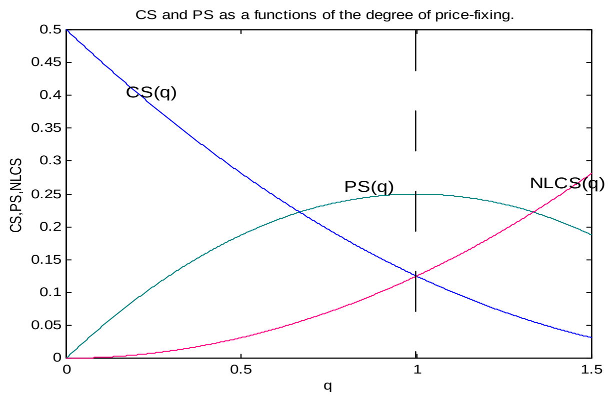

Figure 2: Consumer surplus, producer surplus and net loss of consumer surplus as continuous differentiable functions of the degree of price-fixing.

The consumer surplus is lower the higher the degree of collusion. The loss in consumer surplus is higher the higher the degree of collusion, while the rents from cartel for the firm are higher the higher the degree of collusion.

It should be mentioned that in the literature two main objectives of the authority are considered. First, the authority aims to maximize total welfare, i.e. the sum of consumer and producer surpluses. Second, the authority's aim could be to maximize consumer surplus and at the same time minimize the rents from collusion for the firm. The second approach can be justified by the fact that the rents obtained through illegal activities are lost for society in most of the cases. So they should not be included in the regulator's maximization function.

Let us consider the first problem in a static setting. The antitrust authority is aimed to maximize $(CS + PS)$ i.e. to maximize $\{\frac{1}{2}\Pi^m(2-q)^2 + \Pi^m q(2-q)\}$ s.t. $q \in [0,1]$. So, given $q \in [0,1]$ the total welfare is maximized when $q = 0$. Note, that this is equivalent to the minimization of $NLCS$.

Let us consider now the second problem in a static setting. The antitrust authority is aimed to maximize CS and at the same time minimize the rents from collusion, i.e. keeping $PS = 0$ equal to competitive profit. In other words, the sum of net loss of consumer surplus and producer surplus will be minimized. This means that the problem can be rewritten to minimize$\{\Pi^m q(2-q) + \frac{1}{2}\Pi^m q^2\}$. This is equivalent to minimize $\{(2q - \frac{1}{2}q^2)\Pi^m\}$, which is equal to the minimization of the total loss from price-fixing for society. Consequently, in the settings where antitrust authority cares only about CS the social welfare will be maximized when there is no collusion.

So we can conclude that in the static setting the two problems described above are equivalent in the sense that antitrust authority should not allow for any collusion irrespective of whether it cares about total welfare of the society or only about the consumer surplus. The same holds in a dynamic setting. The aim of the antitrust authority will thus be to achieve $q = 0$ in all the periods of the planning horizon in a dynamic setting as well.

2.2 Description of the dynamic game.

To investigate the interactions between the firm and the antitrust authority we develop a differential game. We consider a firm (player 2) playing against the antitrust authority (player 1). The probability that the firm gets caught at time t , $F(t)$, is influenced by the degree of collusion of the firm, $q(t)$, as well as the law enforcement rate of the antitrust authority $p(t)$, in the following manner:

$$\dot{F}(t) = p(t)q(t)[1 - F(t)] \tag{1}$$

Note that $\Phi(t) = \dot{F}(t)[1 - F(t)]^{-1}$ is the hazard rate of the process leading to conviction of the firm. $\Phi(t)$ is the conditional probability of getting caught at time t provided that the firm has not yet been caught. (1) says that the hazard rate Φ increases linearly with increasing activities of the firm and antitrust authority, and $F(0) = 0$ is the initial condition.

As usual two types of variables appear in the model: a state variable $F(t)$ (the probability distribution function of the time until the detection of the violation of the firm) and control instruments $q(t)$ (degree of collusion of the firm) and $p(t)$ (law enforcement rate of antitrust authority). Note that the state constraint $0 \leq F(t) \leq 1$ is satisfied automatically. The idea to use $F(t)$ as a state variable is based on Kamien and Schwartz (1971). Assume also that a once convicted firm is not able to collude any more until time T (so if punishment is harsh enough, the firm needs a lot of time to recover). Parameter r denotes discount rate.

The objective function for the antitrust authority is given by:

$$\max \int_0^T e^{-rt}[-(NLCS(t) + C(p(t)))[1 - F(t)] + s(t)\dot{F}(t)]dt - e^{-rT}C_1(T)[1 - F(T)] \tag{2}$$

The term $C(p(t))$ reflects the costs for the antitrust authority of performing the checking activities (such as the number of inspections, salaries for auditors, etc.). The analysis of the game will be conducted for the case when costs of law enforcement are quadratic, i.e. $C(p) = Np^2(t)$[3]. The instantaneous consumer surplus, $CS(t)$, is negatively related to $q(t)$ (the higher the q, the higher the degree of collusion, the less competition in the market, thus the higher the

[3] However, the results obtained in the paper hold for costs of law enforcement being any increasing convex function of p. Solution of the game for linear case $C(p) = Np$ is available from the author upon request.

price). The term $NLCS(t)$ reflects the loss in instantaneous consumer surplus due to a price increase by the firm. $NLCS(t)$ increases when $q(t)$ increases. The term $s(t)\dot{F}(t)$ reflects the expected revenue for the authority at time t if the cartel is discovered at this particular instant of time. $C_1(T)$ is the terminal value (disutility) assessed by the antitrust authority if the firm is not yet caught at time T. [4] Note also that we assume that no additional costs arise after the firm has been caught. This is a reasonable assumption in the context of violations of antitrust law, since it is assumed that only monetary fine can be imposed and this , on the contrary to imprisonment, is costless for the authority.

The objective function for the firm is given by

$$J^2(q(t)) = \max \int_0^T e^{-rt}[PS(t)[1-F(t)] + PS^{comp}F(t) - s(t)\dot{F}(t)]dt + e^{-rT}C_2(T)[1-F(T)] \quad (3)$$

Here the term $PS(t)$ reflects the instantaneous rents from collusion, while $-s(t)\dot{F}(t)$ denotes the expected punishment for the firm at time t, i.e. the fine times the probability of being caught. $s(t)$ is the instantaneous penalty at the moment the firm is caught. For further analysis we assume it is a function of both control variables and time. Note that the higher the degree of collusion, $q(t)$, the higher the probability to be caught for the firm, and, consequently, the higher the expected punishment. The term $PS^{comp}F(t)$ reflects the profits of the firm during the period after the conviction, when there is no price-fixing. Consequently, the expression PS^{comp} is assumed to be zero. Finally, $C_2(T)$ is the terminal value (utility) of the firm being not yet convicted in cartel formation at time T.

The corresponding differential game with two players, one state variable $F(t)$, and two control variables, $q(t)$ and $p(t)$, is represented by the expressions (1)-(3). The state space is $F(t) \in [0,1]$, and the set of feasible controls is $p(t) \in [0,1]$ for player 1 and $q(t) \in [0,1]$ for player 2.

The major difference with earlier papers on crime control (Feichtinger (1983)) is that we introduce $s(t)$ being the penalty imposed on the firm as a function of the degree of offence, which

[4] From the underlying static microeconomic model of price-fixing (section 2.1) we derive that $CS^{\max} = 2\Pi^m$. Taking this into account the maximization of (2) is equivalent to a maximization problem of the following form:

$$\max \int_0^T e^{-rt}[(PS(t) + CS(t) - C(p(t)))[1-F(t)] + CS^{\max}F(t) + s(t)\dot{F}(t)]dt - e^{-rt}C_1(T)[1-F(T)]$$

where the term $PS(t)$ reflects the instantaneous producer surplus, the term CS(t) reflects the instanteneous consumer surplus, and the term $CS^{\max}F(t)$ reflects the expected instantaneous consumer surplus in periods after the conviction.

can vary over time. Moreover, the penalty could be a function of both the degree of offence and the rate of law enforcement by the antitrust authority. This case will be considered in later sections of the paper. Another extension compared to Feichtinger (1983) is that we determine explicitly the instantaneous utilities for the antitrust authority and the firm in the case of price-fixing, as functions of the degree of offence on the basis of the underling microeconomic model.

An economically reasonable assumption would be to set salvage values to be nonnegative, i.e. $C_1(T) \geq 0, C_2(T) \geq 0$. Moreover, further, in order to simplify the calculations, we assume zero discount rate[5] $(r = 0)$.

We also assume that players make their choices simultaneously and that they present the solutions to their control problems by Markovian strategies or open-loop Nash Equilibrium strategies (for a reference see ,e.g., Dockner et al. (2000)).

Definition 1 *The tuple* (ϕ, ψ) *of functions* $\phi, \psi : F \times [0, T) \longmapsto R^{m^i}$, *is called a Markovian Nash Equilibrium if, for each* $i \in \{1, 2\}$, *an optimal control path* $u^i(t)$ *of the control problem exists and is given by the Markovian Strategy* $u^1(t) = \phi(F(t), t)$ *and* $u^2(t) = \psi(F(t), t)$.

Definition 2 *The tuple* (ϕ, ψ) *of functions* $\phi, \psi : [0, T) \longmapsto R^{m^i}$, *is called an open-loop Nash Equilibrium if, for each* $i \in \{1, 2\}$, *an optimal control path* $u^i(t)$ *of the control problem exists and is given by the open-loop Strategy* $u^1(t) = \phi(t)$ *and* $u^2(t) = \psi(t)$.

In the solution of the game described above we will search for the open-loop Nash Equilibria of the differential game. It can be shown that for this particular game the set of Markovian (closed-loop) Nash Equilibria will coincide with the set of open-loop Nash Equilibria. The proof will be provided in Appendix 2.

3 Analysis of the current EU and US penalty schemes.

3.1 Stylized EU penalty scheme.

In this section we consider a penalty scheme, which resembles the current European or Dutch systems[6]. We model the main feature of these systems, namely that the base penalty must be

[5] However, the main results of the paper would not change if we relax this assumption. Except for the results of Appendix 3 and section 6.1.4. of Appendix 1, where we were not able to find closed form solution for the dynamics of control variables in case $r > 0$.

[6] Guidelines on the method of setting fines imposed for violations of competition law in Europe can be found in PbEG 1998, while guidelines for the setting of fines in the Netherlands are described in Section 57(1) of

proportional to the gravity of infringement or to the turnover involved in the undertaking and should not depend on the rate of law enforcement. It should be mentioned that the functional form described in equation (4) below does not capture all the properties of the penalty schemes, which are determined in the current Guidelines for the setting of fines (such as bigger duration of the offence or leading role in the infringement would increase the penalty). That is why we call this scheme "Stylized EU penalty scheme". Consequently, the penalty in this case is modeled as a linear increasing function of the degree of offence, $q(t)$:

$$s(q(t)) = K\Pi^m q(t), \tag{4}$$

where K is a positive constant and Π^m is the instantaneous monopoly profit to the firm[7].

We first find an open-loop Nash equilibrium of the game described above. In Appendix 2 we show that in this differential game candidates for open-loop Nash optimality are also candidates for closed-loop Nash optimality. To find Nash Equilibria in open-loop strategies we first find a tuple (ϕ, ψ) where $\phi : [0, T] \longmapsto [0, 1]$ and $\psi : [0, T] \longmapsto [0, 1]$ are the fixed strategies for antitrust authority and firm respectively. ϕ corresponds to the control variable $p(t)$, and ψ corresponds to the control variable $q(t)$.

As the static analysis in section 2.1 suggests, it is reasonable to assume concavity of the terms $-NLCS(t) - C(p(t))$ and $PS(t)$. This allows to obtain the expressions for an interior solution of differential game (1)-(3)[8].

The solution of the problem of the firm gives us the following expression being the reaction function of the firm in each period[9]:

$$q^*(t) = \frac{2\Pi^m + \mu(t)\phi(t)}{2\Pi^m + 2\Pi^m K\phi(t)} = B, \tag{5}$$

$$q^*(t) = \begin{cases} \frac{\frac{0 \text{ if } B \leq 0}{B \text{ if } 0 < B \leq 1}}{1 \text{ if } B > 1} \end{cases}. \tag{6}$$

In (5) $\mu(t)$ is the shadow price (costate variable) of the state variable $F(t)$ for the firm.

According to (5) the optimal degree of price-fixing for the firm decreases with decreasing shadow price $\mu(t)$. Moreover the higher the penalty at the instant the firm is caught, the lower

Competition Act.

[7]The multiplier $K\pi^m$ is derived from the static optimization problem for the firm. The firm decides on the level of offence given the rate of law enforcement, p, and the functional form of the penalty scheme, which is linear. And the aim of the antitrust authority is to achieve zero price-fixing outcome.

[8]However, in general all the results of the model will hold also for arbitrary concave objective functions.

[9]For a complete derivation of this result see appendix 1.

the optimal rate of price-fixing. The influence of the maximal gains from price-fixing on the optimal degree of price-fixing is determined by taking the derivative of expression (5) with respect to Π^m and, based on the proof in Appendix 1, we set $\mu(t) \leq 0$ for $t \in [0, T]$[10]. We get $(\frac{2\Pi^m+\mu(t)\phi(t)}{2\Pi^m+2\Pi^m K\phi(t)})'_{\Pi^m} \geq 0$. So the optimal degree of price-fixing by the firm will increase when the maximal gains from collusion increase. This behavior makes economic sense.

The solution of the problem of the antitrust authority gives us the following expression being the reaction function of the antitrust authority in each period[11]:

$$p^*(t) = \frac{(K\Pi^m\psi(t) - \lambda(t))\psi(t)}{2N} = D, \tag{7}$$

$$p^*(t) = \begin{cases} \frac{0 \text{ if } D \leq 0}{D \text{ if } 0 < D \leq 1} \\ 1 \text{ if } D > 1 \end{cases}. \tag{8}$$

Where $\lambda(t)$ is the shadow price for the antitrust authority.

The intuition behind the formula (7) is as follows. Since the antitrust authority aims to minimize the total loss, the adjoint variable $\lambda(t)$ measures the shadow costs of one additional unit of probability $F(t)$ imputed by the authority. Thus, $-\lambda(t)$ is the shadow price by which the state variable F is assessed by the authority. From (7) we see that a decrease in $\lambda(t)$ results in an increase of the rate of law enforcement p. The increase in the absolute value of the penalty $K\Pi^m$ also will cause an increase in the rate of law enforcement, since it becomes more profitable for the antitrust authority to discover more violations. At the same time it holds that the higher the marginal costs of law enforcement N, the lower p.

3.2 Determination of the Nash Equilibrium.

Based on the expressions (7) and (5) we can prove the following proposition.

Proposition 3 *The outcome with no collusion $q(t) = 0$ for all $t \in [0, T]$ can not arise as equilibrium strategy of the firm , when the penalty schedule has the form $s(q(t)) = K\Pi^m q(t)$, where K is a positive real number, which determines the steepness of the penalty scheme, and the costs of law enforcement are a quadratic function of $p(t)$.*

Proof : (see Appendix 1 (section 6.1.3.)).

[10] For verification see appendix 1.

[11] For a complete derivation of this result see appendix 1.

It should be also mentioned, that the unique steady state of this problem is given by $q^* = 0$ and $p^* = 0$[12]. But considering the phase diagram of this problem in the (p,q)- plane we conclude that this solution is not stable[13].This fact provides an additional argument in favor of rejection of the linear penalty scheme which does not depend on any other variables of the model, except for the degree of price-fixing.

The general result of the analysis of the differential game conducted in this subsection points out the weaknesses of the penalty scheme that is linear in the degree of offence, which is described in the European sentencing guidelines for violations of antitrust law. One of the possible improvements would be to change the functional form of the penalty. For example, in US sentencing guidelines it resembles a convex increasing function of the degree of offence. We treat this case in the subsection below.

3.3 Stylized US penalty scheme.

In this section we consider a differential game where the penalty schedule is a convex function of the degree of the offence. This schedule is given by the following expression:

$$s(q(t)) = K\Pi^m q^2(t). \tag{9}$$

This resembles the current US system of penalties for violations of antitrust law, where the base penalty imposed by court for the firm convicted in price-fixing will be determined as a convex increasing function of the degree of offence, assigned to this particular violation, which is based on the amount of turnover involved in undertaking[14]. Again, this system does not exactly capture all the features of the penalties determined in US guidelines manual (such as dependence on the duration of offence or the role in the infringement). That is why, as in the previous subsection, we call this scheme ”Stylized US penalty scheme”. For the convex penalty scheme Proposition 4 can be obtained. We refrain from presenting its proof, since it is similar to the linear case[15].

[12] For the sake of completeness we also give here the exact definition of the stationary Markovian Nash equilibria. Let us consider game with infinite planning horizon, e.g. $T = \infty$. Assume also that there are two players, and strategy space for each of them has dimension one. *Now The stationary Markovian Nash equilibrium (or steady state) is the tuple* (ϕ, ψ) *of time independent functions* $\phi, \psi : F \longmapsto U^i$ *for* $i \in \{1, 2\}$. Where F is the state space and U^1 and U^2 are the strategy spaces for players 1 and 2 respectively. Moreover, U^1, $U^2 \in \mathbf{R}$.

[13] See the part on the investigation of stability in Appendix 1.

[14] US guidelines manual.

[15] The proof of the proposition 4 and investigation of the stability of the system in the long run are available from the author upon request. Also here it holds that unique steady state given by p=q=0, is not stable.

Proposition 4 *The outcome with no collusion $q(t) = 0$ for all $t \in [0, T]$ can not arise as equilibrium strategy of the firm in the model with finite horizon, when the penalty schedule is convex i.e. $s(q(t)) = K\Pi^m q^2(t)$, where K is a fixed positive real number, and the costs of law enforcement are a quadratic function of $p(t)$.*

The deterrence with convex penalty system works better than the deterrence with a linear penalty scheme for more grave offences, since when q is sufficiently high, it can be shown that for any given probability of law enforcement it gives a lower degree of price-fixing by the firm and, consequently, a lower damage for society. Moreover, this result once again gives support to the argument in favor of deterrence focused not only on cartel benefits but also on the harm to the consumers caused by price-fixing. Recall that the net losses in consumer surplus were proportional to the squared degree of offence[16].

The main implication of the model discussed in this section is the result that in the framework of a differential game between the firm and the antitrust authority the penalty schemes which are used now in the EU and US legislation appear not to be as efficient as desired from the point of view of minimization of consumer loss from price-fixing activities of the firm. The result is that, given this framework, zero collusion (full compliance) behavior is not sustainable as a Nash Equilibrium in Markovian strategies for all periods of the time horizon, and, moreover, this equilibrium will never arise as a long run steady state equilibrium of the model. The reason for this is that the current penalty schemes do not allow the fine to be high enough to outweigh the accumulated expected gains from price-fixing for colluding firms. Another reason could be that fines for antitrust violations do not depend in any way on the probability of law enforcement, which should be an important determinant of the efficiency of penalty schemes as has been mentioned in Becker (1968) and Leung (1991). In the next section we pursue this road.

[16] Section 2: *Net Loss of CS* $= \frac{1}{2}\Pi^m q^2$

4 A penalty schedule that does prevent collusion.

4.1 Solution of the game.

Here the aim is to find an open-loop Nash equilibrium, which is also a Markovian Nash Equilibrium of the game described above, when the penalty schedule is determined as follows [17]

$$s(q(t), p(t)) = K\Pi^m q(t) + \frac{G}{p(t)} \quad with \quad s(0,0) = 0, \tag{10}$$

where G is a positive constant.

The foundation for the penalty schedule determined by expression (10) is based on the following considerations. Looking at the FOC for the firm (5) in the case when the penalty is linear, given $\mu(t) \leq 0$ for all t, we can get $q(t) = 0$ for all t if and only if there is some additional strictly negative term in the numerator of the expression (5). By adding the term $\frac{G}{p(t)}$ into the penalty function we assure the appearance of this additional term in the expression for the reaction function of the firm. Note that this result has a lot in common with the well known result of Becker (1968).

Searching for the open-loop Nash Equilibria of the game we start by solving the optimal control problem of the firm. If the antitrust authority chooses to play $p(t) = \phi(t)$ then the firm's problem is described by

$Max \int_0^T e^{-rt}[\Pi^m q(t)(2-q(t))[1-F(t)] - (K\Pi^m q(t) + \frac{G}{\phi(t)})\phi(t)q(t)(1-F(t))]dt + e^{-rT}C_2(T)[1-F(T)]$

s.t. $\dot{F}(t) = \phi(t)q(t)[1-F(t)]$

The Hamiltonian of this problem equals

$H(q, F, \mu) = \Pi^m q(t)(2-q(t))[1-F(t)] - (K\Pi^m q(t) + \frac{G}{\phi(t)})\phi(t)q(t)(1-F(t)) + \mu(t)\phi(t)q(t)(1-F(t))$, where $\mu(t)$ is the costate variable of the problem of the firm.

Solving for $q(t)$ and $\mu(t)$ we get:

$\dot{\mu}(t) = \Pi^m q(t)(2-q(t)) - s(t)\phi(t)q(t) + \mu(t)\phi(t)q(t)$

$$q^*(t) = \frac{2\Pi^m + \mu(t)\phi(t) - G}{2\Pi^m + 2\Pi^m K\phi(t)} = B, \tag{11}$$

$$q^*(t) = \begin{cases} \frac{0 \text{ if } B \leq 0}{B \text{ if } 0 < B \leq 1} \\ 1 \text{ if } B > 1 \end{cases}. \tag{12}$$

According to (11) the optimal degree of price-fixing for the firm decreases with decreasing shadow price $\mu(t)$. Moreover, the higher the penalty at the instant the firm is caught, the lower

[17] For verification see Appendix 2.

the optimal rate of price-fixing. The influence of the maximal gains from price-fixing on the optimal degree of price-fixing is determined by taking the derivative of expression (11) with respect to Π^m and taking into account $\mu(t) \leq 0$ for $t \in [0,T]$[18]. We get $(\frac{2\Pi^m+\mu(t)\phi(t)-G}{2\Pi^m+2\Pi^m K\phi(t)})'_{\Pi^m} \geq 0$. So the optimal degree of price-fixing by the firm will increase when the monopoly profits from collusion increase. The size of the fixed fine G negatively influences the degree of price fixing.

Now we move to the solution of the optimal control problem of antitrust authority. If firm chooses to play $q(t) = \psi(t)$ then the regulator's problem can be written as

$$Min \int_0^T e^{-rt}[(NLCS(t)+C(p(t)))[1-F(t)]-(K\Pi^m\psi(t)+\frac{G}{p(t)})\dot{F}(t)]dt+e^{-rT}C_1(T)[1-F(T)]$$

s.t. $\dot{F}(t) = p(t)\psi(t)[1-F(t)]$

The Hamiltonian of this problem equals

$H(p,F,\lambda) = (\Pi^m\frac{1}{2}\psi^2(t)+Np^2(t))[1-F(t)]-(K\Pi^m\psi(t)+\frac{G}{p(t)})p(t)\psi(t)(1-F(t))+\lambda(t)\psi(t)p(t)(1-F(t))$, where $\lambda(t)$ is a costate variable of the problem of player 1.

Solving for the optimal $p(t)$ and $\lambda(t)$, and taking into account that the control region for p is constrained by the $[0,1]-$ interval, we get:

$\dot{\lambda}(t) = \Pi^m\frac{1}{2}\psi^2(t)) + Np^2(t) - s(t)p(t)\psi(t) + \lambda(t)\psi(t)p(t),$

$$p^*(t) = \frac{(K\Pi^m\psi(t)-\lambda(t))\psi(t)}{2N} = D, \tag{13}$$

$$p^*(t) = \begin{cases} \frac{0 \text{ if } D\leq 0}{D \text{ if } 0<D\leq 1} \\ 1 \text{ if } D>1 \end{cases}. \tag{14}$$

The intuition behind this result is exactly the same as in section 3.2.

Taking into account the assumptions on the terminal values $C_1(T) \geq 0, C_2(T) \geq 0$ we conclude that the transversality conditions will be as follows:

$$\lambda(T) = -C_1(T) \leq 0 \ \ and \ \ \mu(T) = -C_2(T) \leq 0. \tag{15}$$

4.2 Determination of the Nash Equilibrium.

Let us investigate the stability of the system and the properties of the last period solution. By doing this we are able to establish that, under certain conditions on the parameters of the model, an equilibrium with zero degree of collusion in all periods can be sustained as an open-loop or Markovian equilibrium of the game.

From (11)-(14) it can be concluded that the system of equations describing the solution of the differential game in terms of reaction functions in the final period of the game, given that

[18] For verification see proof of Proposition 5.

the solution is interior (due to the concavity of the Hamiltonians), has the following form:

$$p^*(T) = \frac{(K\Pi^m q(T) - \lambda(T))q(T)}{2N} \tag{16}$$

$$q^*(T) = \frac{2\Pi^m + \mu(T)p(T) - G}{2\Pi^m + 2\Pi^m K p(T)} \tag{17}$$

Studying the reaction functions of both players at each instant of time, we can conclude that the following proposition holds:

Proposition 5 *If the penalty schedule has the form* $s(q(t), p(t)) = K\Pi^m q(t) + \frac{G}{p(t)}$ *with* $s(0,0) = 0$ *where* K *is any positive real number and* $G \geq 2\Pi^m$*, then the unique equilibrium has* $q(t) = 0$ *for all* $t \in [0, T]$*.*

Proof:

From expression (16) it is obtained that $p^*(T) = 0$ if and only if $q(T) = 0$,since expression $K\Pi^m q(T) - \lambda(T)$ can not be equal to zero due to the transversality condition (15). This can be situated on an optimal path for the strategy of player 2, given by expression (17) if and only if $G \geq 2\Pi^m$. Secondly, given that $\mu(T) \leq 0$, the best response function $q(p)$ for player 2 is the constant function passing through the point $(0, 0)$, so $q^*(p) = 0$ for any $p \in [0, 1]$.

In Figure 3 we sketch the period T reaction functions of the firm and antitrust authority.

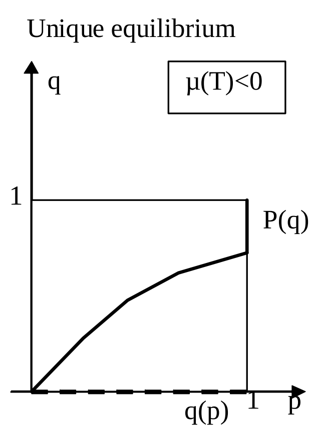

Figure 3: Determination of the Nash Equilibrium in the model when the penalty schedule is given by the function $s(q(t), p(t)) = K\Pi^m q(t) + \frac{G}{p(t)}$ for parameter values $K = 2, \Pi^m = 1, N = 1$ and taking $\lambda = -1$.

We can conclude that $q^*(T) = 0$ can be sustained as an open-loop or Markovian[19] Nash equilibrium in the last period of the game only if $G \geq 2\Pi^m$, i.e. the fixed penalty is high enough to make the reaction curve of the firm a horizontal line, passing through the point $q = 0$.

[19] For the proof of the fact that for this particular game the set of open loop equilibria coincides with the set of Markovian equilibria see Appendix 2.

In order to find equilibrium values of $p^*(t)$ *and* $q^*(t)$ in each period we can draw both reaction functions in a (p, q) diagram at each instant of time. To find analytical expressions for the Nash Equilibria of the game in terms of open-loop strategies for both players, we have to find Nash Equilibria in each period t and compose the optimal path starting from the last period.

The problem here is that expression (10) does not define the penalty in case $p(t) = 0$. The penalty and, consequently, the objective functions become indeterminate when $p(t) = 0$. To overcome this problem we introduce the notion of $\varepsilon -$ *equilibrium* (or *almost equilibrium*).

Definition 6 *An $\varepsilon -$equilibrium of any strategic-form game is a combination of randomized strategies such that no player could expect to gain more than ε by switching to any of his feasible strategies, instead of following the randomized strategy specified for him.*[20]

Obviously, in the equilibrium point with $p^*(t) = 0$ and $q^*(t) = 0$ for all $t \in [0, T]$ values of objective functionals do not exist. Hence, as a candidate for $\varepsilon -$*equilibrium* we consider point $q^*(t) = 0$ and $p^*(t) = \sigma > 0$ for all $t \in [0, T]$, where $\sigma \to 0+$.

Now we can define the $\varepsilon -$*equilibrium* of the game by $q^*(t) = 0$ and $p^*(t) \to 0+$ for all $t \in [0, T]$ and use this equilibrium in further analysis.

In order to show that $p^*(t) \to 0+$ and $q^*(t) = 0$ for all $t \in [0, T)$ can be sustained as an open-loop or Markovian Nash equilibrium of this game, we need to verify that this solution satisfies the necessary conditions for optimality. Obviously, they are satisfied. Assuming $F(t) \neq 1$ we can rewrite differentiated Hamiltonians as follows[21]:

$$\frac{\partial H(p,F,\lambda)}{\partial p} = 2Np(t) - K\Pi q^2(t) + \lambda(t)q(t)\big|_{(p=0,q=0)} = 0$$

$$\frac{\partial H(q,F,\mu)}{\partial q} = 2\Pi^m - 2\Pi^m q(t) + \mu(t)p(t) - 2\Pi^m Kq(t)p(t) - G|_{(p=0,q=0)} = 0 \text{ iff } G = 2\Pi^m$$

Next, we prove that $q(t) = 0$, $p(t) \to 0+$ for all $t \in [0, T]$ is a unique equilibrium. The fact that $\mu(t) \leq 0$ for all $t \in [0, T]$, ensures that $q(t) = 0, p(t) \to 0+$ for all t is a unique solution.

Firstly, $\mu(T) > 0$ can not hold, since according to the transversality condition we have $\mu(T) \leq 0$. Hence, the equilibrium with $q(T) = 0, p(T) \to 0+$ is a unique equilibrium in period T given $G \geq 2\Pi^m$.

We can show that the equilibrium with $q(t) = 0, p(t) \to 0+$ will be also unique for all $t \in [0, T)$. In the problem under consideration the necessary condition for uniqueness of the equilibrium $q(t) = 0, p(t) \to 0+$ for all t is the condition $\mu(t) \leq 0$ for any $t \in [0, T]$. Taking into

[20] For the reference see Myerson (2002)

[21] Note that in case $F(t) = 1$ equalities $\frac{\partial H(p,F,\lambda)}{\partial p} = 0$ and $\frac{\partial H(q,F,\mu)}{\partial q} = 0$ are satisfied for any values of p and q.

account the transversality condition $\mu(T) \leq 0$ above we now show that $\mu(t) \leq 0$ for $t \in [0, T)$. Assume that there is an arbitrary $t' \in [0, T)$ such that $\mu(t') > 0$. Then from the optimality condition we obtain $\mu = \frac{-2\Pi^m + 2\Pi^m q + 2K\Pi^m qp + G}{p}$[22].

And from the costate equation for $\mu(t)$ we obtain that

$\dot{\mu}(t') = 2\Pi^m q(t') - \Pi^m q(t')^2 - (K\Pi^m q(t') + \frac{G}{p(t')})p(t')q(t') + (\frac{-2\Pi^m + 2\Pi^m q(t') + 2K\Pi^m q(t')p(t') + G}{p(t')})p(t')q(t')$
$= q^2(t')\Pi^m \left(1 + Kp(t')\right) \geq 0.$

Hence , a non-positive terminal value given by $\mu(T) = -C_2(T)$ could never be reached. Thus,

$$\mu(t) \leq 0 \quad for \quad t \in [0, T)$$

Hence we can conclude that with $G \geq 2\Pi^m$ the outcome with no collusion $q(t) = 0$ for all $t \in [0, T]$ can arise as an open-loop or Markovian Nash Equilibrium solution of the game and this equilibrium is unique[23].

End of the proof of the existence and uniqueness of the Nash Equilibrium.

To summarize the analysis we stress that this proposition considers the settings, where we model the interactions between the firm and antitrust authority as a differential game. In this game the antitrust authority imposes a penalty of the form $S(q(t), p(t)) = K\Pi^m q(t) + \frac{2\Pi^m}{p(t)}$ at the moment that the cartel is discovered and zero penalty if it checks and does not discover any violation. One important feature of this schedule is that when the cartel is discovered the penalty imposed on the firm must be at least greater than twice the instantaneous monopoly profits from price-fixing in the industry under consideration. It turns out that this penalty scheme is more efficient than the current EU or US penalty schemes, in the sense that this policy leads to the complete deterrence outcome. In particular, the regulator can achieve the outcome with no price-fixing in all the periods of the planning horizon at the lowest possible costs.

Finally consider the infinite horizon problem and let us investigate the stability of the Nash Equilibrium solution in the long run. Studying the phase diagram[24] we can conclude that the following proposition holds.

[22] For a complete derivation see Appendix 3.

[23] Note that this result also goes through with $r > 0$. The only difference is that $\dot{\mu}(t') = r\mu(t') + q^2(t')\Pi^m \left(1 + Kp(t')\right)$, which is also greater or equal than zero given that $\mu(t') > 0$.

[24] See figure 6 in Appendix 3.

Proposition 7 *The outcome with $q^* = 0$ and $p^* \to 0$ is the unique long run steady state equilibrium of the infinite horizon model, where the penalty is given by the expression $S(q(t), p(t)) = K\Pi^m q(t) + \frac{2\Pi^m}{p(t)}$ and the costs of law enforcement for the antitrust authority are convex.*

This proposition states that the equilibrium $(q^* = 0, p^* \to 0+)$ is also the unique steady state equilibrium of the differential game with penalty schedule given by $S(q,p) = K\Pi^m q + \frac{2\Pi^m}{p}$. Complete proof of this fact will be provided in Appendix 3. Referring to Feichtinger (1983) we define the game under consideration as a state-separable game, i.e. a game which has the property that the state variable is absent in the maximization conditions as well as in the adjoint equations. For such a game the system of differential equations for the Nash-optimal controls can be derived, as will be shown in Appendix 3. Also, the qualitative behavior of the optimal solution can be obtained from a phase diagram analysis in the (p,q)- plane, but a closed-form solution of the system of the differential equations for the Nash equilibrium of the game under consideration still cannot be calculated due to the complicated structure of objective functions[25].

Here we give a phase portrait in the $(p,q)-$ plane of the system of differential equations which describes the long run dynamics of the system in terms of control variables. The domain of the controls is determined by the square $[0,1] \times [0,1]$. We also show in Appendix 3, that the solution $(p^* \to 0,\ q^* = 0)$ is the unique stable steady state equilibrium of the game.

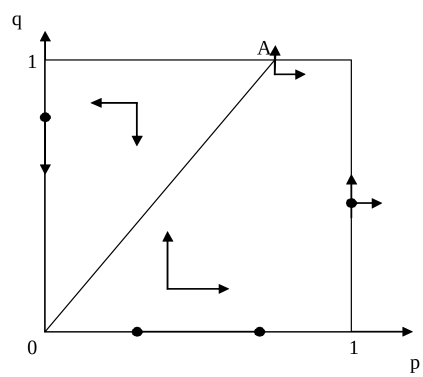

Figure 4: Phase portrait in (p,q)-space for the model where penalty schedule is given by $s(q(t), p(t)) = K\Pi^m q(t) + \frac{G}{p(t)}$ for the set of parameters $K = 2, N = 1, G = 2$, $\Pi^m = 1$.

Considering the dynamics of the system in this domain, we conclude that for certain initial values of control variables, in particular $q > p\sqrt{2}$ (in the example where $K = 2$, $\Pi^m = 1, N = 1, G = 2\Pi^m = 2$) or $q > \sqrt{\frac{2N}{\Pi^m}}p$ (in general case) (or simply for the points in the (p,q)-plane above line OA in the graph above) with arbitrary values of parameters N and Π^m and

[25] See Appendix 3.

$F_x = 2\Pi^m$, the system will always converge to the point $(0, 0)$. Moreover starting in any point with characteristics $q \leq \sqrt{\frac{2N}{\Pi^m} p}$ (below line OA) will bring the system into the point $(1, 1)$, which is clearly suboptimal compared to the solution $(0, 0)$. So we can conclude that $q^* = 0, p^* \rightarrow 0$ is the unique stable steady state solution of the system of differential equations (26), (27). Moreover, this result is not sensitive to the changes of the values of the parameters of the model.

5 Conclusions.

In this paper we analyze dynamic interactions between the antitrust authority and a firm involved in a cartel. We develop a model which can be used to study dynamic optimal enforcement of competition law. We can summarize the results of the paper as follows.

One main result is that the penalty schemes, which are used now in the EU and US legislation, appear not to be as efficient as desired from the point of view of minimization of consumer loss from price-fixing activities of the firm. In particular, we prove the result that zero collusion (full compliance) behavior is not sustainable as a Nash Equilibrium in Markovian strategies. The reason is that the current penalty schemes do not allow the fine to be high enough to outweigh the accumulated expected gains from price-fixing for colluding firms. An additional reason could be that fines for antitrust violations do not depend in any way on the probability of law enforcement, which should be an important determinant of the efficiency of penalty schemes. The latter result was obtained by Becker (1968) and also by Leung (1991).

Furthermore, we determine a penalty system, that is efficient from the point of view of the possibility of complete deterrence of cartel formation in a dynamic setting. We find that there is a possibility to achieve the socially desirable outcome, i.e. the outcome with no price-fixing in all the periods of planning horizon, only with a very specific form of the penalty scheme. The amount of fine should be an increasing function of the degree of offence and it should be negatively related to the probability of law enforcement, which is related to Becker's (1968) result. An interesting implication is that in any case, whatever the degree of offence is, the penalty should be greater than twice the per period maximal gains from price-fixing for the firm. This in some sense confirms the suggestion which has been made in the beginning of the paper that, indeed, the penalty should be related not only to the gains from price-fixing for the firm but also to the loss in consumer surplus due to price-fixing, which is approximately twice the monopoly profits in case of full collusion.

There is a number of possible extensions of the model described in this paper. It seems reasonable to assume that the duration of the game is large. Thus it might be interesting to consider also the case of an infinite time horizon in more details and try to find a more general solution for this setting. In this case the salvage values must be equal to zero and the discount rate must be strictly positive for reasons of convergence of the objective functionals. The introduction of new state variables such as the offender's criminal record or the accumulated gain from cartel formation could give new insights for the determination of optimal penalty schemes for antitrust law violations. New insights may also be gained by looking at heterogeneity among the violating firms and, consequently, different penalty schedules for offences of different gravity and differentiation between industries can also help to improve the deterrence power of the current penalty schemes for violations of competition law.

6 Appendixes.

6.1 Appendix 1. Complete solution of the differential game with linear penalty schedule[26].

6.1.1 Solution of the problem of player 2 (firm).

Let's start by solving the optimal control problem of player 2. If player 1 chooses to play $p(t) = \phi(t)$ then player 2's problem can be written as

$$Max \int_0^T e^{-rt}[\Pi^m q(t)(2-q(t))[1-F(t)] - s(t)\phi(t)q(t)(1-F(t))]dt + C_2(T)[1-F(T)]$$

$$s.t.\ \dot{F}(t) = \phi(t)q(t)[1-F(t)]$$

$F(t) \in [0,1]$ and $F(T)$ is free. This implies a transversality condition of the following form: $\mu(T) = -C_2(T)$, where $\mu(t)$ is the costate variable of the above problem .

$F(0) = 0$ and $q(t) \in [0,1]$

Now $\phi(t)$ is assumed to be fixed functions and $PS(t)$ is determined from the subsection 2.2.

The Hamiltonian of this problem equals

$H(q, F, \mu) = \Pi^m q(t)(2-q(t))[1-F(t)] - s(t)\phi(t)q(t)(1-F(t)) + \mu(t)\phi(t)q(t)(1-F(t))$

Solving for $q(t)$ and $\mu(t)$ we get:

[26] For the sake of completeness we solve this game under assumption that $r \geq 0$. So, that the results stated in section 3.1 (under assumption $r = 0$) will hold automatically.

(i) $\dot{\mu}(t) - r\mu(t) = -\frac{\partial H(q,F,\mu)}{\partial F}$. This implies $\dot{\mu}(t) - r\mu(t) = \Pi^m q(t)(2-q(t)) - s(t)\phi(t)q(t) + \mu(t)\phi(t)q(t)$

(ii) $q^*(t)$ is such that it maximizes $H(q,F,\mu)$ on $q \in [0,1]$.

(iii) $F(T)$ is free, this implies transversality condition of the following form: $\mu(T) = -C_2(T)$

Given $s(q(t)) = K\Pi^m q(t)$ with $K \in (0,\infty)$ i.e. K is any positive real number (ii) implies $\frac{\partial H(q,F,\mu)}{\partial q} = 2\Pi^m - 2\Pi^m q(t) + \mu(t)\phi(t) - 2\Pi^m K q(t)\phi(t)$.

So equation $2\Pi^m - 2\Pi^m q + \mu\phi - 2\Pi^m K q\phi = 0$ gives us $\mu = 2\Pi^m \frac{-1+q+Kq\phi}{\phi}$

Substituting this expression into the costate equation we get:

$\dot{\mu} = r\mu + \Pi^m q(2-q) - \Pi^m K q\phi q + 2\Pi^m(\frac{-1+q+Kq\phi}{\phi})\phi q = r\mu + \Pi^m q^2 + \Pi^m K q^2 \phi$

$\frac{\partial H(q,F,\mu)}{\partial q} = 0$ implies $q^*(t) = \frac{2\Pi^m + \mu(t)\phi(t)}{2\Pi^m + 2\Pi^m K\phi(t)} = B$ and since $q(t) \in [0,1]$ we get $q^*(t) = \begin{cases} 0 \text{ if } B \leq 0 \\ B \text{ if } 0 < B \leq 1 \\ 1 \text{ if } B > 1 \end{cases}$

Proof of $\mu(t) \leq 0$ **for all** $t \in [0,T]$.

The transversality conditions are provided by $\lambda(T) = -C_1(T)$ for player 1 (antitrust-authority) and by $\mu(T) = -C_2(T)$ for player 2 (firm). Given $C_1(T) \geq 0$ and $C_2(T) \geq 0$ we have that $\lambda(T) \leq 0$ and $\mu(T) \leq 0$, where $\lambda(T)$ and $\mu(T)$ are values of the costate variables of the game in the last period.

Taking into account the conditions above we can show that $\mu(t) \leq 0$ for $t \in [0,T)$.

Assume that there is arbitrary $t' \in [0,T)$ such that $\mu(t') > 0$. Then, according to the concavity of $PS(q)$ we obtain from the costate equation for $\mu(t)$ that $\dot{\mu}(t') = \mu(t')r + \Pi^m q(t')^2 + \Pi^m K q(t')^2 p(t') \geq 0$. Hence , a non-positive terminal value given by $\mu(T) = -C_2(T)$ could be never reached. Thus,

$$\mu(t) \leq 0 \quad for \quad t \in [0,T)$$

End of the proof.

Note, since the sign of the expression for costate variable of player 1 $\dot{\lambda}(t) - r\lambda(t) = \frac{1}{2}\Pi^m q^2 - Np^2$ is ambiguous, it holds that in general $\lambda(t)$ has no unique sign.

6.1.2 Solution of the problem of player 1 (antitrust authority).

Now we move to the solution of optimal control problem of player 1. If player 2 chooses to play $q(t) = \psi(t)$ then player 1's problem can be written as

$$Min \int_0^T e^{-rt}(NLCS(t) + Np^2(t))[1 - F(t)] - s(t)\dot{F}(t)]dt + C_1(T)[1 - F(T)]$$

$$s.t.\ \dot{F}(t) = p(t)\psi(t)[1 - F(t)]$$

$F(t) \in [0,1]$ and $F(T)$ is free. This implies a transversality condition of the following form: $\lambda(T) = -C_1(T)$, where $\lambda(t)$ is the costate variable of the above problem.

$F(0) = 0$ and $p(t) \in [0,1]$

Now $\psi(t)$ is assumed to be fixed function.

The Hamiltonian of this problem equals

$H(p,F,\lambda) = (\Pi^m \frac{1}{2}\psi^2(t) + Np^2(t))[1-F(t)] - s(t)p(t)\psi(t)(1-F(t)) + \lambda(t)\psi(t)p(t)(1-F(t))$

Solving for $p(t)$ and $\lambda(t)$ we get:

(i) $\dot{\lambda}(t) - r\lambda(t) = -\frac{\partial H(p,F,\lambda)}{\partial F}$ this implies $\dot{\lambda}(t) - r\lambda(t) = \frac{1}{2}\Pi^m\psi^2(t) + Np^2(t)) - s(t)p(t)\psi(t) + \lambda(t)\psi(t)p(t)$

(ii) $p^*(t)$ is such that it maximizes $H(p,F,\lambda)$ on $p \in [0,1]$.

(iii) $F(T)$ is free, this implies transversality condition of the following form: $\lambda(T) = -C_1(T)$

(ii) and $s(q(t)) = K\Pi^m q(t)$ implies

$\frac{\partial H(p,F,\lambda)}{\partial p} = 2Np(t) - K\Pi^m\psi(t)\psi(t) + \lambda(t)\psi(t) = 0$. This implies $p^*(t) = \frac{(K\Pi^m\psi(t) - \lambda(t))\psi(t)}{2N} = D$ and taking into account limits of the control region for probability of control $p^*(t) = \begin{cases} \frac{\frac{0 \text{ if } D\leq 0}{D \text{ if } 0<D\leq 1}}{1 \text{ if } D>1} \end{cases}$

Solution of the equation $2Np - K\Pi^m\psi\psi + \lambda\psi = 0$ gives $\lambda = \frac{-2Np + K\Pi^m\psi^2}{\psi}$.

Substituting this expression into the costate equation we get:

$\dot{\lambda}(t) - r\lambda(t) = \Pi^m\frac{1}{2}\psi^2 + Np^2 - K\Pi^m\psi p\psi + (-\frac{2Np - K\Pi^m\psi^2}{\psi})\psi p = \frac{1}{2}\Pi^m\psi^2 - Np^2$

6.1.3 Proof of proposition 3.

Consider the value of the control variable of the antitrust authority in the last period of the game given by expression (7). It is clear that $p^*(T) = 0$ if and only if $q(T) = 0$. But this contradicts to the optimal path for the last period strategy of player 2 , which is given by expression (5). This implies that $p^*(T) = 0$ and $q^*(T) = 0$ does not constitute a Nash equilibrium of the game in the last period for arbitrary salvage value $C_2(T)$. Consequently, strategy $q(t) = 0$ for all t can not be sustained as a Nash equilibrium in open-loop or Markovian strategies with $C_2(T) \leq \frac{2\Pi^m}{p(T)}$ and $\mu(t) \leq \frac{2\Pi^m}{p(t)}$ for all $t \in [0,T)$.

We may also notice an interesting argument that follows from the fact that transversality condition implies that $\mu(T) = -C_2(T)$. Then we get $q^*(T) \leq 0$ for any $p(T) \in [0,1]$ if and only if $\mu(T) = -C_2(T) \leq -\frac{2\Pi^m}{p(T)}$. (Note that this result does not hold in general settings with arbitrary terminal value of the firm.) This implies that the reaction function of the firm in the last period can pass through origin only when $C_2(T)$ the terminal utility of the firm being not yet convicted in cartel formation at time T is greater or equal than $\frac{2\Pi^m}{p(T)}$. So the outcome with no collusion in the last period and consequently with no price-fixing in all the preceding periods can arise in equilibrium only under very special circumstances, i.e. when $C_2(T) \geq \frac{2\Pi^m}{p(T)}$ and $\mu(t) \leq -\frac{2\Pi^m}{p(t)}$ for all $t \in [0, T)$, which means the terminal utility of the firm being not yet convicted in cartel formation at time T must be equal exactly the absolute penalty which could be imposed on the firm in the static case in order to block any degree of price-fixing[27] and also the path of the costate variable for the firm should follow exactly $\mu(t) = -\frac{2\Pi^m}{p(t)}$ or lie below. So, here we can observe analogies with the static model.

So, we have that $q^*(t) = 0$ for some $p(t) \in [0,1]$ if and only if $\mu(t) \leq -\frac{2\Pi^m}{p(t)}$ for all $t \in [0, T]$. But in this case the dynamics of the system rules out the result with $p(t) = 0$ for all $t \in [0, T]$. This can be shown as follows.

Consider $-\frac{2\Pi^m}{p(t)} < 0$ for all $t \in [0, T]$ iff $p(t) > 0$. This implies $\dot{\mu}(t) = \Pi^m q(t)^2 + \Pi^m K q(t)^2 p(t) = 0$ for all $t \in [0, T]$. Thus $\dot{p}(t) = 0$ and $p(t) > 0$. This implies $p(T) > 0$. Consequently, the outcome with $q(t) = 0$ for all $t \in [0, T]$ can be sustained in a Nash equilibrium of this game with $\mu(T) = -C_2(T) \leq -\frac{2\Pi^m}{p(T)}$ only when $p(t) > 0$ for some $t \in [0, T]$, which is clearly suboptimal , since there is unnecessary waste of resources compared to the outcome with $q(t) = 0$ and $p(t) = 0$ for all $t \in [0, T]$.

END OF THE PROOF OF PROPOSITION 3.

6.1.4 Investigation of stability of the system when penalty is given by the expression $s(q) = K\Pi^m q$.

From the solution of the problem of the firm (setting $r = 0$) we obtain

$$\mu = \frac{-2\Pi^m + 2\Pi^m q(t) + 2K\Pi^m q(t)p(t)}{p(t)} \tag{18}$$

and $\dot{\mu}(t) = 2\Pi^m q - \Pi^m q^2 - (K\Pi^m q)pq + (\frac{-2\Pi^m + 2\Pi^m q + 2K\Pi^m qp}{p})pq = q^2\Pi^m(1 + Kp)$

From the solution of the problem of player 1 (setting $r = 0$) we have

$$\lambda = \frac{-2Np(t) + K\Pi^m q^2(t)}{q(t)} \tag{19}$$

and $\dot{\lambda}(t) = \frac{1}{2}\Pi^m q^2 + Np^2 - K\Pi^m q^2 p + (\frac{-2Np + K\Pi^m q^2}{q})qp = \frac{1}{2}\Pi^m q^2 - Np^2$

Differentiating (19) and (18) with respect to time and equalizing it to $\dot{\lambda}(t)$ and $\dot{\mu}(t)$ respectively we obtain following system of equations:

[27] Recall section 2.2: $s'(q) = \frac{2\Pi^m}{p}. \implies s(q) = \frac{2\Pi^m}{p}q$,i.e. the penalty imposed on the firm colluding with the degree 1 (earning monopoly profits) must be at least double of monopoly profits.

$$\frac{2p\Pi^m q' + 2K\Pi^m p^2 q' + 2p'\Pi^m - 2p'\Pi^m q}{p^2} = q^2\Pi^m(1+Kp) \tag{20}$$

$$\frac{-2qNp' + K\Pi^m q^2 q' + 2q'Np}{q^2} = \frac{1}{2}\Pi^m q^2 - Np^2 \tag{21}$$

from (21) it follows $p' = \frac{1}{4}\frac{2K\Pi^m q^2 q' + 4q'Np - \Pi^m q^4 + 2Np^2q^2}{qN}$

substituting p' into (20) and solving for $q\prime$ we get $q' = \frac{1}{2}q^2\frac{\Pi^m q^2 - \Pi^m q^3 - 2Np^2 + 4qNp^2 + 2Np^3qK}{2Kp^2qN + K\Pi^m q^2 - K\Pi^m q^3 + 2Np}$

from (20) it follows $q' = \frac{1}{2}\frac{-2p' + 2p'q + q^2p^2 + q^2p^3K}{p(1+Kp)}$,

substituting $q\prime$ into (21) and solving for $p\prime$ we get $p' = \frac{1}{2}q^2p\frac{K^2\Pi^m q^2p^2 + 4Np^2 + 4Np^3K - \Pi^m q^2}{2Kp^2qN + K\Pi^m q^2 - K\Pi^m q^3 + 2Np}$

Solving the system of equations above for $p\prime = 0$ and $q\prime = 0$ we get that solution $p = 0$, $q = 0$ is also a steady state equilibrium of the game described in section 3, i.e. when penalty is given by expression $S(q) = K\Pi^m q$. But after careful analysis of the phase diagram of this system we can conclude that the equilibrium $p = 0$, $q = 0$ is not stable for some policy relevant values of the parameters of the system.

Given parameters are $\Pi^m = 1, N = 2, K = 0.5$ and given the domain of the control variables is $[0,1] \times [0,1]$ we can represent the system dynamics in Figure 5.

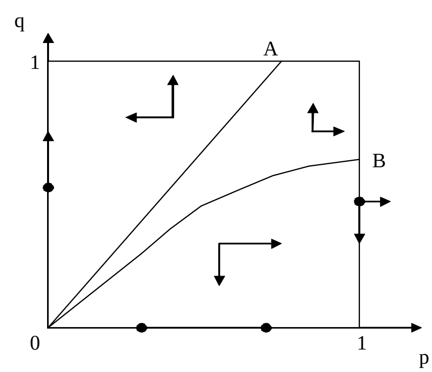

Figure 5: Phase portrait in (p,q)-space for the model with linear penalty schedule and convex costs of law enforcement for the set of parameters $K = 0.5, N = 2$, $\Pi^m = 1$.

Where OA is the locus where variable p changes its dynamics and OB is the locus where variable q changes its dynamics.

By studying the phase diagram we can conclude that solution $p^* = 0$, $q^* = 0$ can not be stable equilibrium, i.e. equilibrium to which system converges in the long run.

6.2 Appendix 2.

In this appendix we show that for the games described in sections 3, 4 and 5 the candidates for open-loop Nash optimality are also candidates for closed-loop Nash optimality and hence the open-loop strategies are also optimal in the set of closed loop strategies.

We have already mentioned, that referring to Feichtinger (1983) we can define the game under consideration as a state-separable game, i.e. the game which has the property of the absence of the state variable from the maximum conditions as well as from the adjoint equations. For such a games the system of differential equations for the Nash-optimal controls can be derived and also the qualitative behavior of the optimal solution can be obtained from a phase diagram analysis in the (p, q)- plane.

According to Feichtinger (1983), in state-separable differential games the candidates for open-loop Nash optimality are also candidates for closed-loop Nash optimality. The strategies are independent of the state variable because neither the Hamiltonian-maximizing conditions nor the adjoint equations depend on state variable F. Thus, the open-loop strategies are also optimal in the set of closed loop strategies. Usually it is shown by verifying the sufficient conditions for closed-loop Nash equilibrium controls as in Leitmann and Stalford (1974).

For the particular game described in section 3 of the paper the procedure of verifying the sufficient conditions will be as follows.

Recall the definition of Markovian Nash Equilibria given in section 2.1. So searching for closed-loop equilibria we assume that the choice of the control variable by each player in the next period will depend on the realization of state variable and also that both players can observe this realization. In that case the optimal strategies of player 1 (antitrust authority) and player2 (firm) must be respectively $p(t) = \phi(F(t), t)$ and $q(t) = \psi(F(t), t)$.

Solving for open-loop Nash equilibria of the game of section 3 we get $q^*(t) = \frac{2\Pi^m + \mu(t)p(t)}{2\Pi^m + 2\Pi^m K p(t)}$ and $p^*(t) = \frac{(K\Pi^m q(t) - \lambda(t))q(t)}{2N}$.

Now we substitute $q^*(t)$ and $p^*(t)$ into $H^2(q, F, \mu)$ and $H^1(p, F, \lambda)$. Then the Maximized Hamiltonians will have the following form:

$H^{2*}(q, F, \mu) = \Pi^m q^*(t)(2 - q^*(t))[1 - F(t)] - s(t)\phi(t)q^*(t)(1 - F(t)) + \mu(t)\phi(t)q^*(t)(1 - F(t))$

$H^{1*}(p, F, \lambda) = (\Pi^m \frac{1}{2}\psi^2(t) + Np^{*2}(t))[1 - F(t)] - s(t)p^*(t)\psi(t)(1 - F(t)) + \lambda(t)\psi(t)p^*(t)(1 - F(t))$

Recall also that in state-separable game described above adjoint equations do not depend on state variable and, consequently, costate variables will not depend on state variable as well.

Taking above considerations into account we can notice that the Maximized Hamiltonian functions of both players are linear (and hence concave) with respect to the state variable. So we can conclude that the candidates characterized by $\frac{\partial H^1(p,F,\lambda)}{\partial p}$ and $\frac{\partial H^2(q,F,\mu)}{\partial q}$ are indeed nondegenerate Markovian Nash Equilibria of the game in section 2.2. Since $q^*(t)$ and $p^*(t)$ do not depend on $F(t)$ this open-loop Nash equilibrium of this game could be also regarded as a Nash Equilibrium of a differential game in which both players have full Markovian information.

The same reasoning holds for the model in section 4.

6.3 Appendix 3. Calculation of steady states in the model where penalty is given by expression $s(q,p) = K\Pi^m q + \frac{2\Pi^m}{p}$.

In this appendix we verify that the equilibrium ($q^* = 0, p^* = 0$) is also unique steady state equilibrium of the differential game with penalty schedule given by $s(q,p) = K\Pi^m q + \frac{2\Pi^m}{p}$. Refiring to Fiechtinger (1983) we define the game under consideration as a state-separable game. For such a games we generally derive the system of differential equations for the Nash-optimal controls. But since objective functions of this game are quite complicated expressions in terms of control variables and co-state variables the stability of the system can not be investigated with the help of general techniques such as evaluation of the trace and determinant of the Jacobian matrix. So, to investigate the qualitative behavior of the optimal solution we use a phase diagram analysis in the (p,q)- plane. Unfortunately, a closed-form solution of the system of the differential equations for the Nash equilibrium of the game under consideration still can not be calculated due to the complicated structure of objective functions.

To simplify the calculations we assume that there is no discounting. Unfortunately, we were not able to obtain closed form expressions even for dynamics of control values in case $r > 0$. However, we can presume that if $r > 0$ the effect of the penalty should be even stronger, since accumulated expected future gain from price-fixing for the firm would be less.

The solution of the problem of player 2 gives

$$\mu = \frac{-2\Pi^m + 2\Pi^m q + 2K\Pi^m qp + G}{p} \tag{22}$$

and $\dot{\mu}(t) = 2\Pi^m q - \Pi^m q^2 - (K\Pi^m q + \frac{G}{p})pq + (\frac{-2\Pi^m + 2\Pi^m q + 2K\Pi^m qp + G}{p})pq = q^2\Pi^m(1 + Kp)$

The solution of the problem of player 1 gives

$$\lambda = \frac{-2Np + K\Pi^m q^2}{q} \tag{23}$$

and $\dot{\lambda}(t) = \frac{1}{2}\Pi^m q^2 + Np^2 - K\Pi^m q^2 p - qG + (\frac{-2Np + K\Pi^m q^2}{q})qp = \frac{1}{2}\Pi^m q^2 - Np^2 - qG$

Differentiating (23) and (22) with respect to time and equalizing it to $\dot{\lambda}(t)$ and $\dot{\mu}(t)$ respectively we obtain following system of equations:

$$\frac{2p\Pi^m q' + 2K\Pi^m p^2 q' + 2p'\Pi^m - 2p'\Pi^m q - p'G}{p^2} = q^2\Pi^m(1 + Kp) \tag{24}$$

$$\frac{-2qNp' + K\Pi^m q^2 q' + 2q'Np}{q^2} = \frac{1}{2}\Pi^m q^2 - Np^2 - qG \tag{25}$$

from (25) it follows

$$q' = \frac{1}{2}q^2 \frac{2(\Pi^m)^2 q^2 - 4\Pi^m Np^2 - 4\Pi^m qG - 2(\Pi^m)^2 q^3 + 8\Pi^m qNp^2 + 3\Pi^m q^2 G + 2GNp^2 + 2qG^2 + 4\Pi^m qNp^3 K}{4K\Pi^m p^2 qN + 2K(\Pi^m)^2 q^2 + 4\Pi^m Np - 2(\Pi^m)^2 q^3 K - GK\Pi^m q^2 - 2GNp} \tag{26}$$

from (24) it follows

$$p' = \Pi^m q^2 p \frac{K^2\Pi^m q^2 p^2 + 4Np^2 + 4Np^3 K - \Pi^m q^2 + 2qG + 2qGpK}{4K\Pi^m p^2 qN + 2K(\Pi^m)^2 q^2 + 4\Pi^m Np - 2(\Pi^m)^2 q^3 K - GK\Pi^m q^2 - 2GNp} \tag{27}$$

In order to be able to conduct more transparent analysis we make assumptions about the parameters of the model. First, we normalize monopoly profits to 1, i.e. $\Pi^m = 1$, then parameter of the penalty scheme $G = 2\Pi^m = 2$. Moreover the costs of law enforcement should be proportional to the amounts of extra gains from price-fixing in every particular industry, since the more the firm has resources the more efficient it will be in hiding the violation and if violation is found the more fears will be the battle in the court, consequently the more resources the antitrust authority has to spend in order to catch and sew the firm. Taking the above considerations into account I assume $N \cong \Pi^m = 1$. Parameter K can be equal to 2 as in static settings or less, this does not influence neither the location of the steady state, no the dynamics of the system around steady state.

This is remarkable, that there is one location of steady state which is not influenced by the values of parameters of the system at all , which is in the point $p = 0, q = 0$. This can be seen immediately from the system (24), (25).

Given parameters values $K = 2, N = 1, F = 2, \Pi^m = 1$

$q' = \frac{1}{4}q^2\frac{4q-q^2+4p^2+4p^3}{2p^2-q^2}$ and $p' = \frac{1}{4}qp\frac{4q^2p^2+4p^2+8p^3-q^2+4q+8qp}{2p^2-q^2}$.

Considering the partial derivatives of the expressions above we can not infer any information about the Jacobian matrix of this system. So no conclusion can be given by evaluating the trace and determinant of the Jacobian matrix. Consequently, another algorithm should be applied.

Constructing the phase diagram of the above system in the (p, q)-plane, we get the Figure 6.

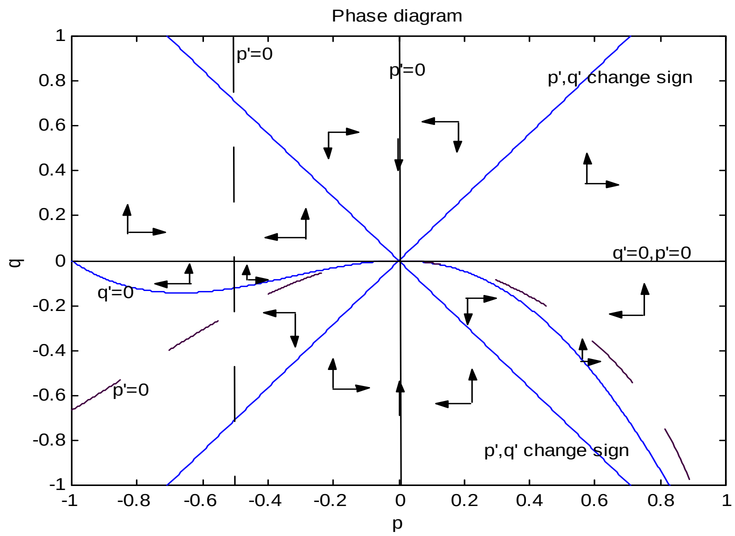

Figure 6: Complete phase portrait in (p,q)-space for the model where penalty schedule is given by $s(q(t),p(t)) = K\Pi^m q(t) + \frac{G}{p(t)}$ for the set of parameters $K = 2, N = 1, F = 2,\ \Pi^m = 1$.

In this diagram the locuses where variable q changes sign are

$q = 0,\ q = 2 + 2\sqrt{(1 + p^2 + p^3)},\ q = 2 - 2\sqrt{(1 + p^2 + p^3)},\ q = \sqrt[2]{2}p,\ q = -\sqrt[2]{2}p.$

And the locuses where variable p changes sign are

$q = 0, p = 0, p = -0.5,\ q = \frac{1}{2p-1}\left(-2 + 2\sqrt{(1 - 2p^3 + p^2)}\right), q = \frac{1}{2p-1}\left(-2 - 2\sqrt{(1 - 2p^3 + p^2)}\right),$
$q = \sqrt[2]{2}p,\ q = -\sqrt[2]{2}p.$

Recall, the domain of the controls is determined as $(p, q) \in [0, 1] \times [0, 1]$. Considering the dynamics of the system in this domain, we conclude that for certain initial values of control variables, in particular $q > \sqrt[2]{2}p$ (in this example) or $q > \sqrt[2]{\frac{2N}{\Pi^m}}p$ (in general case) with arbitrary values of parameters N and Π^m and $G = 2\Pi^m$, the system will always converge to the point $(0, 0)$. Moreover starting in any point with characteristics $q \leq \sqrt[2]{\frac{2N}{\Pi^m}}p$ will bring the system into the point $(1, 1)$, which is clearly suboptimal compared to the solution $(0, 0)$. So we can conclude that $q^* = 0, p^* = 0$ is stable steady state solution of the system of differential equations (26), (27). Moreover this result is not sensitive to the changes of the values of the parameters of the system (26), (27).

Consider the values of objective functionals for both players in points (0,0) and (1,1).

in case $p(t) = 0, q(t) = 0$ for all t$\in [0, T]$ we get

$\dot{F}(t) = p(t)q(t)[1 - F(t)]|_{p=0,q=0} = 0$ and $F(0) = 0 \Longrightarrow F(t) = 0$ for all $t \in [0, T]$

in case $p(t) = 1, q(t) = 1$ for all t$\in [0, T]$ we get

$\dot{F}(t) = p(t)q(t)[1 - F(t)]|_{p=1,q=1} = 1 - F$ and $F(0) = 0 \Longrightarrow F(t) = 1 - e^{-t}$ for all $t \in [0, T]$

Thus, $J_2|_{(0,0)} = C_2(T)$ and $J_2|_{(1,1)} = (K + 1)\Pi^m(e^{-T} - 1) + C_2(T)[e^{-T}]$

$(K+\Pi^m)(e^{-T}-1)+C_2(T)[e^{-T}]<C_2(T)$

Now consider the objective function of the authority:

$J_1|_{(0,0)} = \int\limits_0^T 2\Pi^m dt = 2\Pi^m T$

$J_1|_{(1,1)} = 2\Pi^m T - ((\frac{3}{2}+K)\Pi^m - N)(e^{-T}-1)$

$2\Pi^m T - ((\frac{3}{2}+K)\Pi^m - N)(e^{-T}-1) < 2\Pi^m T$ when $N > (\frac{3}{2}+K)\Pi^m$.

So for non-benevolent regulator the result will depend on the magnitude of costs of control and penalty which can be collected given current penalty scheme. If it is very costly to check, then the outcome with zero price-fixing is preferred.

In terms of minimization of loss of consumer surplus (the regulator is benevolent and does not care about the monetary payoff from the penalty imposed on the firm):

$J_1|_{(0,0)} = \int\limits_0^T \frac{1}{2}\Pi^m q^2(t)[1-F(t)]dt\Big|_{(0,0)} = \textit{total loss in } CS|_{(0,0)} = 0$

$J_1|_{(1,1)} = \int\limits_0^T \frac{1}{2}\Pi^m[e^{-t}]dt = -\frac{1}{2}\Pi^m(e^{-T}-1) = \textit{total loss in } CS|_{(1,1)} > 0$

So we can conclude that the outcome (0,0), which is policy target of antitrust authority, will be also preferred by the firm when the penalty is given by the expression $S(q,p) = K\Pi^m q + \frac{2\Pi^m}{p}$.

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