LABOR TAX REFORM AND EQUILIBRIUM UNEMPLOYMENT: 
A SEARCH AND MATCHING APPROACH

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Labor Tax Reform and Equilibrium Unemployment: A Search and Matching Approach

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Abstract

The paper studies simple strategies of labor tax reform in a search and matching model of the labor market featuring endogenous labor supply. Changing the composition of the tax wedge—that is, reducing a payroll tax and increasing a progressive wage tax such that the marginal tax wedge remains unaffected—increases employment, reduces the equilibrium unemployment rate, and increases public revenue as long as workers do not have all the bargaining power in wage negotiations. A strategy of replacing employment taxes by payroll taxes increases employment and reduces the equilibrium unemployment rate, while the effect on public revenue is ambiguous.

JEL codes: J3, J680.

Keywords: equilibrium unemployment, employment taxes, invariance of incidence proposition, payroll taxes, search and matching model, labor tax reform, and wage tax.

1 Introduction

Labor tax reform continues to be a key item on the policy agenda of many European countries. The high tax wedges1 of these countries are often seen as one of the main culprits of their high equilibrium unemployment rates. A piecemeal reduction in the marginal tax wedge is therefore

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1The nominal tax wedge is typically defined as the difference between the gross labor costs that the firm has to pay and the after-tax wage income of workers, which equals the sum of marginal income taxes, social security contributions paid by employees and social security contributions paid by employers. In this view, compulsory social security contributions are considered to be more akin to a tax than a social insurance premium from which individuals derive personal rights. The real tax wedge corrects the nominal tax wedge for consumption taxes, such as value-added tax and excises, but is not considered in this paper.
a common measure found in European tax reform programs aimed at stimulating employment. In particular, policy makers have recommended cuts in payroll taxes to reduce unemployment (for example, OECD (1994)). Moreover, some observers claim that the composition of the tax wedge matters to employment and real wages. They argue that shifting the composition of the tax wedge from payroll (or employers’) taxes to wage (or employees’) taxes could increase employment. Indeed, in view of governments’ revenue needs, several OECD countries have taken this approach in reforming their direct tax system. For example, Hungary cut its employers’ social insurance contribution rates by 11 percentage points during 1994-1999, while increasing its effective personal income tax rates from 24 percent to 28 percent so as to keep the tax wedge constant. Several other Central and Eastern European transition countries—featuring high rates of unemployment and overstretched public finances—still have to decide on their structural reform strategies. A pivotal policy question is therefore whether a shift in the composition of the tax wedge would be of help in alleviating the unemployment and budgetary problems of these countries.

As a matter of general principle, it is well known that in perfectly competitive labor markets the invariance of incidence proposition holds. This proposition says that it does not matter to gross wages, net wages, and employment whether (statutory) labor taxes are levied on workers or firms. It is not immediately evident, however, whether payroll and wage taxes are equivalent in an imperfectly competitive labor market. For example, unions are unlikely to accept a wage reduction to offset an increase in payroll taxes, but may agree to refrain from wage increases if wage taxes are increased. Empirical studies employing wage-bargaining models—for example, Lockwood and Manning (1993) and Holm, Honkapohja and Koskela (1994)—suggest that payroll and wage taxes have quite different effects on wage formation. Various econometric studies find that a shift from payroll taxes to wage taxes decreases firms’ wage costs (see Symons and Robertson (1990)).

Few formal studies have analyzed the invariance of incidence proposition in imperfectly competitive labor markets, where households and firms may have different degrees of bargaining power over wages and thus differ in their ability to shift labor taxes. Koskela and Schöb (1999) employ a “right-to-manage” trade union model and find that a revenue-neutral sub-

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2Empirical studies on the employment effects of payroll taxes generally find small negative effects (see Hamermesh (1993)). Summers (1989) argues that the effect may even be negligible if workers value the social security benefits—that is, health care services, unemployment protection and pension allowances—enough so that they are willing to accept a lower wage in combination with these benefits.

3See Ligthart (1999) for further details on the Hungarian tax reform.

4This raises the question why, in practice, social security contributions are divided between employers and employees. Musgrave and Musgrave (1987) argue that the legislative intent may have been to share the tax burden equally, although this does not say anything about the economic incidence. Other factors may play a role as well such as liability to tax evasion and tax administration concerns.

5Wage taxes are defined to include all taxes levied on employees (for example, personal income taxes and employees’ social security contributions) whereas payroll taxes refer to all labor taxes levied on employers.

6In this setup, firms bargain with unions over the wage subject to the restriction that employment is on
stitution of wage taxes for payroll taxes has no effect on employment. Picard and Toulemonde (2001) obtain a similar result in an efficiency wage model. So far, little attention has been paid to labor tax incidence issues in search-theoretic models of the labor market. A notable exception is Boone and Bovenberg (2002), who focus on the optimal design of labor taxation in a search framework, however, and Pissarides (1998), who provides a numerical analysis of the employment effects of changes in the wage tax. None of these authors, however, have focused on the allocation and welfare effects of the kind of labor tax reform described below.

The first purpose of the paper is to analyze—employing a search and matching model of the labor market—whether a coordinated wage-payroll tax reform, which reduces payroll taxes and increases wage taxes so as to keep the marginal tax wedge constant, would affect employment. Rather than focusing on a revenue-neutral tax reform, as is common in the literature so far (for example, Koskela and Schöb (1999)), our reform considers a strategy of keeping the marginal tax wedge constant. This simple and practicable strategy is shown to yield efficiency gains—in terms of a smaller search externality to job-seeking workers, yielding higher employment—without jeopardizing public revenue if the wage tax structure is progressive. Indeed, public revenue is shown to increase. Moreover, such a reform strategy is to yield welfare increases even if there are pre-existing fiscal distortions.

The second aim of the paper is to study whether the form of taxation—that is, specific versus ad valorem taxes—matters to its qualitative effect on wages and employment. In perfectly competitive markets the distinction is immaterial, but this is not necessarily true for imperfect labor markets. Recently, Pisauro (1991) and Rasmussen (1998) have addressed this issue in efficiency wage models. Pisauro (1991) shows that an employment tax (that is, a specific or head tax on the number of employees) increases real wages whereas an ad valorem payroll tax depresses real wages. Rasmussen (1998) extends Pisauro’s analysis to the long run and shows that a balanced budget substitution of employment taxes for payroll taxes leads to higher employment. Our paper studies a coordinated payroll-employment tax reform, which involves reducing the employment tax while increasing the payroll tax so as to keep the wage costs of the firm constant at a given wage rate. Such a strategy increases aggregate employment and reduces the equilibrium unemployment rate, but has an ambiguous effect on public revenue.

Our approach embeds Pissarides’ (1990) search-theoretic framework in a simple intertemporal macroeconomic model of a small open economy with endogenous labor supply. The labor demand curve. See also Heijdra and van der Ploeg (2002, Chapter 8).

In contrast to revenue-neutral restructuring strategies, the marginal tax wedge can readily be calculated on the back of an envelope. Furthermore, our strategy allows us to focus on the composition of the tax wedge and thus abstract from the effects of a change in the level of the marginal tax wedge.

The few tax reform studies dealing with imperfectly competitive labor markets typically do not pay attention to welfare effects.

Shi and Wen (1999) also consider endogenous labor supply in an intertemporal search and matching model of unemployment to study the allocation and welfare effects of labor and capital income taxes.
framework features flows in the labor market from the creation of new jobs and the (exoge-
nous) destruction of existing jobs. It is assumed that job-seeking workers and vacancy-offering
firms match in a stochastic fashion. This modeling setup gives rise to a search externality that
is represented by agents’ contact probabilities as a function of labor market tightness. By
endogenizing the household’s labor supply decision—which is exogenous in Pissarides’ model
(1990)—and allowing for pre-existing labor taxes, we are able to study tax incidence issues.
In particular, a linear tax scheme with an income threshold is considered to study the effects
of tax progressivity on employment. This builds on the work of Koskela and Vilmunen (1996)
and Pissarides (1998), who have found that progressive taxes can be good for employment
in a partial equilibrium bargaining model. Our results show that progressive taxes are also
beneficial to employment in a search and matching model with endogenous labor supply.
Specifically, we show that the degree of progressivity matters to the allocation effects of a
coordinated wage-payroll tax reform.

The paper is organized as follows. Section 2 develops the basic search-theoretic model,
which is extended for endogenous labor supply. Section 3 studies the two types of labor tax
reform experiments. Section 4 concludes.

2 A Simple Search and Matching Model

2.1 Firms

The economy consists of a large number of risk-neutral firms producing a homogeneous good
under perfect competition.\footnote{For convenience, the number of firms is fixed and normalized to unity.}
The representative firm employs $L(t)$ units of labor to produce output, $Y(t)$, according to a simple linear production function:

$$ Y(t) = \alpha_0 L(t), \quad \alpha_0 > 0, $$

where $\Omega_0$ is a (constant) productivity index. The price of final output is normalized to unity.
Physical capital is abstracted from for reasons of simplicity.\footnote{Abstracting from capital keeps the analysis analytically tractable. The model cannot, however, allow for a change in the factor intensity in response to a change in the tax mix. See Section 4 for a further discussion.} The firm’s labor stock adjusts sluggishly because it has to post vacancies, $V(t)$, and find workers to fill them. New employment is defined as the difference between job creation, $q(t)V(t)$, and job destruction, $sL(t)$:

$$ \dot{L}(t) \equiv q(t)V(t) - sL(t), $$

where $q(t)$ represents the probability of the firm finding an unemployed worker with whom it concludes a deal (and thus $1/q(t)$ is the expected duration of a vacancy) and $s$ is the
exogenous job destruction rate.\footnote{The assumption of a constant job break-up rate is common in the literature and not too unrealistic for purposes of comparative statics, but less suitable if one wants to study short-run dynamics. The present paper focuses on the former, however. See Mortensen and Pissarides (2003) for a model that endogenizes the job break-up rate.} A dot above a variable denotes a time derivative (for example, $\dot{L}(t) = dL(t)/dt$).

The firm’s objective function is given by:

\[\dot{A}_P(t) \equiv \int_t^{\infty} \left[ Y(z) - \gamma_0 V(z) - w_P(z)L(z) - \tau_Z L(z) \right] e^{-r(z-t)} dz, \quad (3)\]

where $A_P(t)$ is the value of the firm, $\gamma_0$ is the (constant) flow cost per vacancy, $w_P(t) \equiv w(t)(1 + \tau_E)$ denotes the producer wage, $w(t)$ is the before-tax wage, $\tau_E$ is a payroll (or \textit{ad valorem}) tax, and $\tau_Z$ is an employment (or specific) tax on the total number of hours worked. The rate of interest ($r$) is given, reflecting that the domestic economy is small in world capital markets.

The firm is assumed to have perfect foresight. It chooses time paths of employment and vacancies in order to maximize (3) subject to the production function (1) and the employment accumulation constraint (2). Optimal firm behavior is characterized by:

\[\lambda_E(t)q(t) = \gamma_0, \quad (4)\]

\[A_P(t) = \lambda_E(t)L(t), \quad (5)\]

\[\lambda_E(t) = \int_t^{\infty} \left[ \alpha_0 - \tau_Z - w_P(z) \right] e^{-(r+s)(z-t)} dz, \quad (6)\]

where $\lambda_E(t)$ is the value of an additional worker in the planning period, which is equal to the present discounted value of the surplus the firm earns on that job (see equation (6)), using $r + s$ as the effective discount rate.\footnote{Equation (5) follows from integrating forward the first-order condition for the state variable, $L(t)$, and imposing the terminal condition $\lim_{z \to \infty} \lambda_E(z)L(z)e^{-r(z-t)}$.} According to (4) the firm chooses its vacancies such that the value of an additional worker is equalized to the expected recruitment costs (that is, $\gamma_0/q(t)$). Equation (5) says that the value of the firm should equal the replacement value of its labor force.

\subsection*{2.2 Households}

The representative household comprises infinitely many members caring only about the lifetime utility enjoyed by the household unit at time $t$. Lifetime utility of the representative household, denoted by $\Lambda(t)$, takes a simple logarithmic form:

\[\Lambda(t) \equiv \int_t^{\infty} \log X(z)e^{-\rho(z-t)} dz, \quad \rho > 0, \quad (7)\]

where $\rho$ denotes the pure rate of time preference and $X(t)$ is \textit{full consumption}, which is defined as the sum of the value of goods consumption and the opportunity cost of leisure consumption.
Following Greenwood, Huffman and Hercowitz (1988), full consumption is specified in such a way that the wealth effect on the household’s labor force participation decision is eliminated:

\[ X \cdot [C(t), L_P(t)] \equiv C(t) - \left( \frac{\sigma}{\sigma + 1} \right) L_P(t)^{\sigma+1}/\sigma, \quad \sigma \geq 0, \]  

(8)

where \( C(t) \) denotes private consumption, \( L_P(t) \equiv U(t) + L(t) \) denotes labor force participation, \( U(t) \) is time spent searching while unemployed, and \( \sigma \) is the intratemporal labor supply elasticity. Labor supply is exogenous if \( \sigma = 0 \). The household has a time endowment of unity so that leisure is defined as \( M(t) \equiv 1 - U(t) - L(t) \). Each employed worker pays a wage tax \( (\tau_L) \) on its wage income above a certain threshold \( a \geq 0 \). After-tax wage income amounts to \( (w_N(t) + \tau_L a) L(t) \), where \( w_N(t) = w(t)(1 - \tau_L) \) denotes the after-tax “marginal” wage and \( w_A = w_N(t) + \tau_L a \) is the consumer (or after-tax “average”) wage. Unemployed workers receive an unemployment benefit denoted by \( b \), which is constant and untaxed. The household’s flow budget identity is then given by:

\[ \dot{A}(t) \equiv r A(t) + [w_N(t) + \tau_L a] L(t) + bU(t) + T(t) - C(t), \]

(9)

where \( A(t) \) is the stock of real tangible assets and \( T(t) \) are lump-sum transfers or taxes (if \( T(t) < 0 \)). The household’s wealth portfolio, \( A(t) \), consists of shares in domestic firms, \( A_P(t) \), and net foreign assets, \( A_F(t) \).

At each instant of time, \( f(t) U(t) \) unemployed household members find a job, but some employed members, \( sL(t) \), lose their job owing to idiosyncratic shocks. Then, employment evolves according to:

\[ \dot{L}(t) \equiv f(t) U(t) - sL(t), \]

(10)

where \( f(t) \) denotes the worker-finding rate for the firm (to be determined in Section 2.4).

The optimization problem of the household can be solved in two stages. The first stage involves solving for the optimal time profile of full consumption by maximizing (7) subject to the household’s budget constraint (9) and the transversality conditions:

\[ \lim_{z \to -\infty} \eta_A(z) A(z) e^{-\rho(z-t)} = \lim_{z \to -\infty} \eta_L(z) L(z) e^{-\rho(z-t)} = 0, \]

(11)

where \( \eta_A(t) \) and \( \eta_L(t) \) are the co-state variables associated with the state variables \( A(t) \) and \( L(t) \). The optimization problem yields:

\[ \frac{\dot{X}(t)}{X(t)} = r - \rho, \quad t \in [0, \infty), \]

(12)

\[ X(t) = \rho [A(t) + H(t)], \quad t \in [0, \infty), \]

(13)

Footnotes:

\footnote{In the background the following linear tax function is employed: \( \Upsilon(t) \equiv -a + \tau_L w(t) \), where \( a \geq 0 \) is a tax allowance. If \( a = 0 \) the tax system is proportional, otherwise it is progressive. See Section 2.3 for a further discussion.}

\footnote{In Section 2.5, it is shown that a meaningful steady state requires that \( r = \rho \), thereby yielding a flat profile for full consumption.}
and after-tax human wealth is defined as:

\[ H(t) \equiv \int_t^{\infty} \left[ (w_N(z) + a_t L(z)) + b U(z) - \left( \frac{\sigma}{1 + \sigma} \right) w_R(z)^{1+\sigma} - T(z) \right] e^{-r(z-t)} dz, \quad (14) \]

where \( w_R(t) \) is the household’s reservation wage. Equation (12) shows that full consumption growth is equal to the difference between the rate of interest and pure rate of time preference. The household consumes a constant proportion of its total wealth, comprising financial wealth and after-tax human wealth (see (13)).

In the second step, full consumption is optimally allocated over its components:

\[ U(t) + L(t) = w_R^*(t), \quad w_R(t) \equiv b + f(t) \lambda_L(t), \quad (15) \]

\[ \lambda_L(t) = \int_t^{\infty} [w_N(z) - (b - \tau_L a)] \exp \left[ -\int_z^{\infty} (s + f(\nu) + r) d\nu \right] dz, \quad (16) \]

where \( \lambda_L(t) \equiv \eta_L(t)/\eta_A(t) \) is the pecuniary value of an additional job and \( w_R(t) \) is the worker’s reservation wage. Labor market participation is a positive function of the reservation wage (equation (15)), which is defined as the sum of the unemployment benefit and expected value of a job. Equation (16) shows that the value of an additional job is the present discounted value of the “dividend” earned on the job, consisting of the excess of net labor income over the household’s reservation wage.

2.3 Government Budget and Tax Progression

Lump-sum transfers to households, \( T(t) \), and outlays on unemployment benefits, \( b U(t) \), are covered by revenue from a payroll tax, a wage tax and an employment tax:

\[ T(t) + b U(t) = [\tau_E w(t) + \tau_L (w(t) - a)] L(t) + \tau_Z L(t). \quad (17) \]

In line with general practice, wage taxes are taken to be progressive, whereas social security contributions are assumed to be proportional.

A useful measure of the degree of tax progression is the coefficient of average tax progression:\(^\footnote{To keep matters simple, the employment tax is not included in the definition of the tax wedge and is thus not incorporated in the measure of average tax progression either.}\)

\[ \Psi(\tau_E, \tau_L, a, t) \equiv \tau^M - \tau^A(t) = \frac{\tau_L a}{w(t)(1 + \tau_E)}, \quad (18) \]

where the marginal tax wedge, \( \tau^M \), and average tax wedge, \( \tau^A \), are defined as follows:

\[ \tau^M \equiv \frac{\tau_L + \tau_E}{1 + \tau_E}, \quad \tau^A(t) \equiv \tau^M - \frac{\tau_L a}{w_P(t)}, \quad (19) \]

Both tax wedges are expressed as a percentage of the producer wage as is common in the literature. The tax system is progressive (that is, the average tax burden rises with the wage)
if $\Psi(.) > 0$, which is satisfied for $a > 0$, whereas the tax system is proportional for $a = 0$. An increase in one of the wage tax parameters ($\tau_L$ or $a$) at a given wage rate yields an increase in average tax progression, whereas an increase in the payroll tax reduces average tax progression ceteris paribus. Tax progression can also change owing to a change in the wage at a given tax rate. This is analyzed further in Section 3.

2.4 Job Matching and Wage Bargaining

Firms with vacancies and job-seeking workers are matched in a stochastic fashion. The matching function, $Z(t) = U(t)^{\varepsilon}V(t)^{1-\varepsilon}$, $0 \leq \varepsilon \leq 1$, describes the number of labor contracts, $Z(t)$, that are concluded given the number of job seekers and vacancies (Pissarides, 1990). From the matching function, the job-finding rate of the worker can be derived:

$$f[\theta(t)] \equiv \frac{Z(t)}{U(t)} = \theta(t)^{1-\varepsilon}, \quad f' > 0 > f''$$

where $f[\theta(t)]$ measures the Poisson probability that a vacancy will be filled in a specific time interval $dt$, $\theta(t) \equiv V(t)/U(t)$ is the relative number of traders in the market (an indicator of labor market tightness) and $f(\theta(t)) = \theta(t)q(\theta(t))$, so that $q' > 0 > q''$, where $q(\theta(t)) \equiv Z(t)/V(t)$. Defining $\varepsilon$ as the (absolute value of the) elasticity of the $q(\theta(t))$ function ($0 < \varepsilon \equiv -\theta q'(\theta)/q(\theta) < 1$) it can be determined that $1 - \varepsilon$ is the elasticity of the $f(\theta(t))$ function.

When a firm with a vacancy and a job-seeking worker meet, a local monopoly rent is created by the encounter, which is equal to $\lambda_i E + \lambda_i L$, where the superscript $i$ indicates a particular firm-worker pairing. Upon separation this rent is lost. The division of the rent between the worker and the firm is determined by bargaining over the wage rate. The generalized Nash bargaining objective can then be written as:

$$w_i(t) = \arg \max \left[ \left( \lambda_i E(t) \right)^{\chi} \left( \lambda_i L(t) \right)^{1-\chi} \right], \quad 0 \leq \chi \leq 1,$$

where $\chi$ and $1 - \chi$ represent the bargaining power of the worker and the firm, respectively. Since all firms are identical they all choose the same wage. The wage resulting from the bargaining process can be written as follows:

$$w(t) = \chi \left( \frac{\alpha_0 + \gamma_0 \theta(t) - \tau Z}{1 + \tau_E} \right) + (1 - \chi) \left( \frac{b - \tau_L a}{1 - \tau_L} \right),$$

where it is assumed that $w(t)(1 - \tau_L) + \tau_L a > b$ so that it pays off to work. The negotiated wage is a weighted average of the firm’s surplus—consisting of the marginal product of labor and the foregone search costs ($\gamma_0 \theta(t)$) adjusted for payroll and employment taxes—and the tax-adjusted unemployment benefit (that is, $\frac{b - a + \tau_L a}{1 - \tau_L} > 0$ because $b > a$). The marginal tax rates depress wages because they reduce the surplus of the match, but also because firms and workers save on tax payments by agreeing to keep wages low. If employers have all the bargaining power, that is, $\chi = 0$, the negotiated wage is constant over time. In that case, employers’ taxes and employment taxes do not affect wage setting.
2.5 Steady State and Stability

In steady state, total employment is not changing, although there are still labor market flows taking place. From (10) an expression for the equilibrium unemployment rate (that is, \( u(t) \equiv U(t)/L_P(t) \)) can be obtained, that is, \( u^* = s/(s+f(\theta^*)) \), where asterisks denote steady-state values. Also, in steady state, the rate of interest equals the pure rate of time preference \((r = \rho)\), so that there is no transition in full consumption (that is, \( \dot{X}(t) = 0 \)). Without anticipation effects of policy changes, there are no transitional dynamics in \( \theta, f, q, \lambda_L, \lambda_E \) and \( w \) either. This is summarized in Proposition 1.

**Proposition 1** Without anticipation effects, there are no transitional dynamics in \( \theta, q, f, \lambda_E, \lambda_L, w_R \), and \( w \). These variables jump to their new steady-state values following an unanticipated and permanent shock to any of the model parameters \((s, r, \Omega_0, \gamma_0, \varepsilon, \rho)\) or policy variables \((a, b, \tau_L, \tau_E, \tau_Z)\).

**Proof.** See Appendix, Section A.1.

The model can be graphically summarized by two schedules in the \((w, \theta)\) space (see Panel (a) of Figure 1). The wage setting (WS) schedule (see (A.5) in the Appendix, Section A.2) is upward sloping. At higher rates of labor market tightness the worker gets a larger share of the pure rents associated with a labor contract (via a higher wage). The vacancy creation (VC) curve (see (A.4) in the Appendix) is downward sloping and convex toward the origin, reflecting diminishing returns in vacancy creation. Intuitively, if there are more vacancies, unemployment is lower because unemployed households find it easier to locate a job. The long-run equilibrium is represented by \( E_0 \). Panel (b) of Figure 1 shows the Beveridge Curve (BC), which is downward sloping and convex to the origin in the \((V, U)\) space. The ray through the origin represents the equilibrium unemployment-vacancy ratio (the indicator of labor market tightness (LMT)). Equilibrium unemployment and vacancies are at the intersection of the LMT and BC curves.

3 Tax Reform Analysis

This section develops the two labor tax reform strategies set out in the Introduction. It is assumed that all changes in the policy variables are permanent and unanticipated. The government balances its budget at each point in time via lump-sum transfers/taxes. The formal derivations can be found in the Appendix.

3.1 Substituting Wage Taxes for Payroll Taxes

Consider a labor tax reform which involves simultaneously cutting a payroll tax by \( d\tau_E < 0 \) and increasing a wage tax by \( \phi d\tau_L > 0 \), where the coefficient of initial taxes is denoted by
\[ \phi \equiv \frac{1 + \tau_E}{1 - \tau_L} > 1 \text{ for } \tau_L, \tau_E > 0. \]

Due to the positive initial wage and payroll taxes, the objective of keeping the marginal tax wedge constant does not yield an equiproportionate absolute change in tax rates. For convenience, the initial employment tax has been set to zero. This reform leaves the marginal tax wedge entirely unaffected, while the average tax wedge declines, and the degree of average tax rate progression increases. In terms of Panel (a) of Figure 1, the WS curve rotates to the left and the VC curve shifts to the right, pushing up the wage rate. The new steady state (E₁) lies to the north east of the old equilibrium (E₀) if the wage tax is progressive. In that case, the shift in the VC curve dominates the shift in the WS curve. Proposition 2 shows that such a reform has desirable effects on economic efficiency—it yields a smaller search externality to workers and a higher level of employment in the new steady state—for a progressive wage tax. The result is extremely sharp as it is as close to practicability as one could hope for.¹⁷

**Proposition 2** A cut in the payroll tax coordinated with a rise in the wage tax that keeps the marginal tax wedge unchanged strictly increases employment, labor market tightness, the before-tax wage rate, and average tax progression iff: (i) the wage tax schedule is progressive; and (ii) workers do not have all the bargaining power in wage negotiations.

**Proof.** See Appendix, Section A.3.1.

¹⁷Alternatively, we could have studied a revenue-neutral wage-payroll tax reform. Such a reform is less closely in line with a simple and practicable strategy as discussed in this paper, because it is not straightforward for policy makers to assess whether a particular tax change would yield a revenue-neutral outcome. Below, however, it is shown that under the assumption of a proportional wage tax schedule the two tax reform strategies are identical in their economic effects.
The proposed reform yields a lower rate of unemployment without increasing the marginal
tax wedge if two conditions are satisfied. First, workers should not have all the bargaining
power in wage negotiations (that is, \(0 < \chi < 1\)), otherwise they could demand a wage
increase large enough to fully compensate for the wage tax rise, making vacancy creation more
expensive to firms. Second, the wage tax structure should be progressive (that is, \(a > 0\)).\(^{18}\)

This somewhat counterintuitive result can be explained by the intratemporal income effect
in labor supply; households will work harder to compensate for the income loss associated
with the higher rate of tax progression. Producer wages are sure to fall, thereby increasing
the value of an additional job to the firm. Consumer wages rise, despite the increase in the
wage tax, inducing households to supply more hours of labor. Clearly, the presence of a tax
allowance moderates the increase in before-tax wages because the benefit of wage increases is
reduced owing to increasing tax progression. In sum, under a progressive wage tax, the reform
increases employment via two channels: (i) increased labor force participation, particularly if
the labor supply elasticity is large, workers have a lot of bargaining power, and the tax exempt
threshold is large; and (ii) a reduction in the unemployment rate, owing to firms opening up
more vacancies.\(^{19}\)

If the Hosios (1990) condition—that is, the worker’s relative bargaining power equals the
elasticity of the matching function—holds and initial labor taxes and the unemployment rate
are zero, introducing a progressive wage tax does not yield any first-order welfare effect. In
that case, the decentralized market outcome coincides with the socially efficient solution.
With pre-existing labor taxes, a coordinated wage-payroll tax yields a welfare improvement
if the sufficient conditions set out in Proposition 3 are met:

**Proposition 3** Substituting a progressive wage tax for a linear payroll tax yields an increase
in welfare if the following sufficient conditions hold: (i) \(\chi \geq \varepsilon\) so that workers have a sufficient
amount of bargaining power; and (ii) initial unemployment benefits are not too large (if labor
supply is very elastic). If labor supply is exogenous and the Hosios (1990) condition is met,
welfare unambiguously rises.

**Proof.** See Appendix, Section A.4.

What is the effect of the reform on public revenue? Without tax base effects, a negative
revenue effect would materialize. Intuitively, the tax rate on a relatively large tax base (that
is, the payroll base) is reduced while the tax rate on a relatively small base (that is, the
wage income base) is increased.\(^{20}\) The reform, however, increases the tax bases—via higher

\(^{18}\)It is easy to show that Proposition 2 would not hold if households enjoy a tax credit rather than an
individual tax allowance.

\(^{19}\)Thus, with inelastic labor supply (that is, \(\sigma = 0\)) the labor tax reform would still increase employment
through the unemployment channel.

\(^{20}\)Note that, in practice, this is not so clear cut. Personal income taxes may include certain employment
benefits (for example, a company car) and exclude certain expenses (for example, interest payments on a
wages and employment—thereby more than offsetting the negative tax rate effect. If labor supply is sufficiently inelastic aggregate unemployment falls too (see equation (A.17) in the Appendix), causing a fall in unemployment outlays. Accordingly, net lump-sum transfers—that is, gross public revenue minus unemployment outlays—rise by more than gross public revenue. Proposition 4 summarizes the revenue effects.

**Proposition 4** Cutting a proportional payroll tax while increasing a progressive wage tax so as to keep the marginal tax wedge constant increases public revenue.

**Proof.** See Appendix, Section A.3.1.

Propositions 2-4 provide for a surprisingly simple way of reaping the benefits from labor tax reform without jeopardizing the government’s revenue position. All that is needed is a reallocation of the statutory burden from the proportional payroll tax to the progressive wage tax while keeping the marginal tax wedge constant. The employment and revenue dividends, however, depend crucially on the progressive nature of the wage tax system. The insight that progressive taxes can be good for employment in imperfectly competitive labor markets is not new. What this study contributes is that this conclusion carries over to policy relevant labor tax reform experiments in search and matching models. Moreover, the previous studies have not looked at the revenue and welfare implications of tax reform experiments.

If the wage tax does not provide for an initial allowance, thus yielding a proportional tax system, then:

**Corollary 1** Cutting a proportional payroll tax while increasing a proportional wage tax so as to keep the marginal tax constant leaves employment, unemployment and public revenue unaffected.

The revenue effect is zero because with proportional taxes the wage and payroll tax bases are equal. In addition, agents’ wage setting power is irrelevant to the macroeconomic outcome. Accordingly, the distribution of the statutory burden of payroll and wage taxes does not matter, just like in a setting of a perfectly competitive labor market. It is then immediately apparent that fully replacing the payroll tax by a proportional wage tax would not affect employment and public revenue either. This equivalence result is closely related to Koskela and Schöb’s (1999) result obtained in a “right-to-manage” trade union model with exogenous labor supply, which thus carries through in a search and matching environment with endogenous labor supply.

---

mortgage) from taxable income, whereas payroll taxes are levied upon salary with few such adjustments.

21See, for example, Koskela and Vilmunen (1996) for an application to trade union models. Pissarides (1998) reaches a similar conclusion employing a numerical study of a wage tax cut in a search and matching model.

22Picard and Toulemonde (2001) show—in a perfectly competitive labor market—that a revenue-neutral restructuring of payroll and wage taxes does not affect employment if and only if both producer and consumer wages remain constant.
3.2 Substituting Payroll Taxes for Employment Taxes

The second tax reform experiment consists of cutting the employment tax while increasing the payroll tax so as to keep the firm’s cost of labor constant at a given wage rate. Under perfect competition, this would be a trivial issue because it does not matter to employment whether labor taxes are defined in specific form (employment tax) or \textit{ad valorem} form (payroll tax). In imperfectly competitive labor markets, it is generally relevant in what form labor taxes are levied. Indeed, Pisauro (1991) finds in a moral-hazard efficiency wage model that employment and payroll taxes both depress employment, whereas the payroll tax reduces after-tax wages, but the employment tax increases the after-tax wage. Replacing employment taxes by payroll taxes is then expected to yield a positive effect on employment.\footnote{Pisauro (1991) did not look into the issue of labor tax reform, however.} Below, we show that this line of reasoning is relevant to search and matching models too.

Our reform experiment involves keeping fixed \( \pi \equiv \bar{w}(1 + \tau_E) + \tau_Z \), where \( \bar{w} \) is the given (before-tax) wage rate, so that \( d\tau_E = -d\tau_Z/\bar{w} > 0 \), with \( d\tau_Z < 0 \). Accordingly, the rise in the payroll tax is smaller than the absolute value of the employment tax change. At a given wage rate, the firm’s employment decision would not be affected by such a reform, but allowing for a wage change could give rise to efficiency gains in the new steady state.

Employment and payroll taxes have qualitatively the same comparative static effects on labor market tightness and wages—both variables fall upon a tax increase as long as firms do not have all the bargaining power—but differ in their quantitative effects. The fall in labor market tightness is larger under an employment tax than under a payroll tax. In terms of Panel (a) of Figure 2, the reform leaves unaffected the VC curve, but rotates down the WS curve, thereby moving the equilibrium from \( E_0 \) to \( E_1 \). Consequently, the wage rate falls and

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{Substituting Payroll Taxes for Employment Taxes}
\end{figure}
labor market tightness rises, yielding a fall in the unemployment rate (as is shown in Panel (b) of Figure 2). Due to the lower wage rate households will supply less hours of labor. In addition, the reform depresses firm’s labor costs, inducing firms to open up more vacancies and hire more labor from the pool of unemployed households. Accordingly, aggregate employment increases. Proposition 5 summarizes these positive efficiency effects.

Proposition 5 Combining a reduction in the employment tax with an increase in the payroll tax while holding constant the firm’s cost of labor at a given wage increases employment and labor market tightness and reduces the equilibrium unemployment rate if and only if none of the players has full bargaining power.

Proof. See Appendix, Section A.3.2.

What is the effect of the coordinated payroll-employment tax reform on public revenue? The employment tax has a smaller base than the payroll tax so that without tax base effects a smaller increase in the payroll tax is needed to raise the same amount of revenue. Indeed, the reform is designed in such a way that the tax rate effect is zero. Matters are quite different if tax bases change as well. The fall in the wage rate depresses the wage tax base, but the rise in employment increases the employment tax base. As a result, the effect on public revenue is ambiguous. Proposition 6 summarizes the result.

Proposition 6 Cutting the employment tax and raising the payroll tax while holding constant the firm’s cost of labor at a given wage rate has an ambiguous effect on public revenue.

Proof. See Appendix, Section A.3.2.

4 Concluding Remarks

Many European countries face high equilibrium unemployment rates at a time when their public finances are already overstretched. Therefore, policy makers have shifted their attention away from resource-intensive public subsidies to resource-conserving reforms of the labor tax structure as ways to address the unemployment problem. The formal theory on labor tax reform offers surprisingly little guidance on what kind of reform policy makers should embark on to reduce equilibrium unemployment without putting the government’s revenue position at risk. In this context, the paper has developed a simple search and matching model of the labor market, which allows for endogenous labor supply and various labor taxes on the employers’ and employees’ side. With the aid of the model, two simple and practicable labor tax reform strategies are studied.

It is shown that a strategy of cutting a payroll tax and increasing a progressive wage tax so as to keep the marginal tax wedge constant, yields a “double dividend” if workers have less
than complete bargaining power. It reduces the equilibrium unemployment rate and increases public revenue. Aggregate unemployment is reduced too if labor supply is sufficiently inelastic so that the increase in aggregate employment dominates the labor market participation effect. Welfare increases even in the presence of fiscal distortions and endogenous labor supply as long as the rate of unemployment benefits is not too high. If wage taxes are proportional, however, the reform does not affect employment, public revenue and welfare.

The second strategy involves a piecemeal reduction in the employment tax accompanied by a rise in the payroll tax so as to keep the firm’s labor costs constant at a given wage rate. This reform yields a reduction in the wage rate, which reduces the firm’s labor costs, thereby increasing aggregate employment. Public revenue rises if the additional revenue generated by the positive employment effect exceeds the drop in revenue owing to a fall in the wage tax base.

The analysis has abstracted, it should be emphasized, from physical capital so that the personal income tax applies to labor income only. Allowing for physical capital would broaden the income tax base. Because of such differences in the income tax base, a shift from payroll taxes to income taxation would involve an increase in the overall rate of capital taxation, thereby favoring labor-intensive production methods. In that case, a shift in the composition of the tax wedge would have real effects even if the labor income tax is proportional. In addition, the analysis assumed constant unemployment benefits. The equivalence between proportional payroll taxes and wage taxes is expected to break down if unemployment benefits are dependent on wage or social security contribution rates. Further limitations of the analysis stem from ignoring compliance and tax administration issues. Social security contributions are withheld at source and are therefore more difficult to evade than personal income taxes, which is self assessed in most industrialized countries. A shift from payroll taxes to personal income taxes may then reinforce the positive employment effects though.

Mathematical Appendix

This Appendix presents the formal analysis underlying the results described in the main text. First, it sets out the approach taken to solve the model. Then, it studies the comparative statics of the labor tax reform scenarios. Finally, the welfare effects of the reforms are discussed.

A.1 Steady State and Stability (Proposition 1)

In order to study the out-of-steady-state dynamics of the model, an equation is derived for the key dynamic variable (that is, $\theta(t)$). By substituting the time derivative of (6) in the time
derivative of (4), the following expression is obtained

\[
\varepsilon \left( \frac{\dot{\theta}(t)}{\theta(t)} \right) = s + r - \left( \frac{\alpha_0 - \tau_Z - w(t)(1 + \tau_E)}{\gamma_0} \right) \theta(t)^{-\varepsilon},
\]

(A.1)

where use is made of \( q(t) = \theta(t)^{-\varepsilon} \) (where \( -\dot{q}(t)/q(t) = \varepsilon \dot{\theta}(t)/\theta(t) \)). By using (22) the wage rate from (A.1) can be eliminated so that a nonlinear differential equation in \( \theta(t) \) is obtained:

\[
\varepsilon \left( \frac{\dot{\theta}(t)}{\theta(t)} \right) = s + r - \left( \alpha_0 - \tau_Z - \varepsilon \frac{\gamma_0}{\alpha_0 - \tau_Z - w(t)(1 + \tau_E)} \right) \theta(t)^{-\varepsilon}.
\]

(A.2)

Differentiating (A.2) with respect to \( \theta(t) \) and evaluating the resulting expression around the steady state yields

\[
\varepsilon \left( \frac{\partial \dot{\theta}(t)}{\partial \theta(t)} \right) = \theta(t)^{1-\varepsilon} \left[ \chi + \varepsilon \left( \frac{\alpha_0 - \tau_Z - w(t)(1 + \tau_E)}{\gamma_0 \theta(t)} \right) \right] > 0.
\]

(A.3)

Accordingly, equation (A.3) is an unstable differential equation in \( \theta(t) \) and the only economically sensible solution is that \( \theta(t) \) is equal to its constant steady-state value \( \theta^* \). Following an unanticipated and permanent shock to any of the exogenous variables, \( \theta(t) \) immediately jumps to its new steady-state value. Accordingly, without anticipation effects, there are no transitional dynamics in \( \theta, f, g, \lambda_E, \lambda_L \) and \( w \).

A.2 Loglinearized System

In steady state,\(^{24}\) the model can be reduced to two equations and two unknowns (that is, \( w(t) \) and \( \theta(t) \)), which define the graphical apparatus. The vacancy creation (VC) schedule and the wage setting (WS) schedule are given by:

\[
\alpha_0 - w(t)(1 + \tau_E) - \tau_Z = (r + s) \gamma_0 \theta(t)^{\varepsilon},
\]

(A.4)

\[
w(t) = \chi \left( \frac{\alpha_0 + \gamma_0 \theta(t) - \tau_Z}{1 + \tau_E} \right) + (1 - \chi) \left( \frac{b - \tau_L a}{1 - \tau_L} \right),
\]

(A.5)

where the vacancy creation curve follows from substituting (6) into (4) and noting (20).

The model is non-linear and therefore can be log-linearized around the initial steady state (SS). A relative change of a variable is denoted by a tilde (\( \tilde{\cdot} \)), for example \( \tilde{\theta} \equiv \delta \theta/\theta \) (except for \( \tilde{\tau}_E \equiv d \tau_E/(1 + \tau_E), \tilde{\tau}_L \equiv d \tau_L/(1 - \tau_L) \), and \( \tilde{\tau}_Z \equiv d \tau_Z/w_P \)). This yields the following system of equations:

\[
\begin{bmatrix}
\frac{\varepsilon \gamma_0 (r+s)}{q} & (1 + \tau_E)w \\
\chi \gamma_0 \theta & -(1 + \tau_E)w
\end{bmatrix}
\begin{bmatrix}
\tilde{\theta}(t) \\
\tilde{w}(t)
\end{bmatrix}
= 
\begin{bmatrix}
\delta \theta(t) \\
\delta w(t)
\end{bmatrix},
\]

(A.6)

\(^{24}\)To save on notation, the asterisks identifying steady state values have been dropped.
where the change in the policy variables is given by:

\[
\begin{bmatrix}
\delta \theta(t) \\
\delta \omega(t)
\end{bmatrix} = \begin{bmatrix}
0 \\
-(1 - \chi)\phi(b - a)
\end{bmatrix} \tilde{\tau}_L + \begin{bmatrix}
-w(1 + \tau_E) \\
\chi w(1 + \tau_E)
\end{bmatrix} \tilde{\tau}_Z
\]

\[
+ \begin{bmatrix}
-w(1 + \tau_E) \\
\omega(1 + \tau_E) - (1 - \chi)\phi(b - \tau_La)
\end{bmatrix} \tilde{\tau}_E,
\]  

(A.7)

where \( \phi \equiv (1 + \tau_E)/(1 - \tau_L) > 1 \). This allows us to solve the model for \( \tilde{\theta}(t) \) and \( \tilde{\omega}(t) \), which, on their turn, determine the equilibrium unemployment rate and vacancy rate.

### A.3 Allocation Effects

#### A.3.1 Coordinated Wage-Payroll Tax Reform (Propositions 2 and 4)

Perturbing the marginal tax wedge, \( \tau^M \) (see (19)) and setting \( d\tau^M = 0 \) yields \( \phi d\tau_L = -d\tau_E > 0 \), where \( d\tau_E > 0 \). By substituting this expression in equations (A.6)-(A.7) and imposing \( d\tau_Z = \tau_Z = 0 \), the change in labor market tightness and the market wage is obtained:

\[
\frac{d\theta}{d\tau_L} \tau_{L,E} = \frac{a\phi(1 - \chi)f}{\gamma_0(\chi f + \varepsilon(r + s))} \geq 0,
\]  

(A.8)

\[
\frac{d\omega}{d\tau_L} \tau_{L,E} = \frac{w(\chi f + \varepsilon(r + s)) - (1 - \chi)\varepsilon(r + s)a}{(1 - \tau_L)(\chi f + \varepsilon(r + s))} \left( \frac{\chi(\alpha_0 + \gamma_0\theta)}{(1 + \tau_E)(1 - \tau_L)} + \frac{(1 - \chi)[\chi f(b - \tau_La) + \varepsilon(r + s)(b - a)]}{(1 - \tau_L)^2(\chi f + \varepsilon(r + s))} \right) > 0,
\]  

(A.9)

where use is made of (22) in going from the first to the second line of (A.9). It is assumed that \( b > a > \tau_La \) so that the second term on the second line of (A.9) is positive. Labor market tightness rises as long as \( a > 0 \), \( \chi \neq 1 \) and search costs \( \gamma_0 \) are bounded.

Perturbing (18) yields the increase in average tax progression:

\[
\frac{d\Psi}{d\tau_L} \tau_{L,E} = \frac{a}{wp} \left[ 1 - \frac{\tau_L}{1 - \tau_L} \epsilon^P_{\tau_L} \right] > 0, \quad a \neq 0,
\]  

(A.10)

where the elasticity of the producer wage with respect to the tax change \( \epsilon^P_{\tau_L} \) is given by:

\[
\epsilon^P_{\tau_L} \equiv \left( \frac{dw}{d\tau_L} \right) \frac{1 - \tau_L}{wp} = -\frac{\varepsilon(r + s)(1 - \chi)(a/w)}{\chi f + \varepsilon(r + s)} < 0,
\]  

(A.11)

\[
\epsilon_{\tau_L} \equiv \left( \frac{dw}{d\tau_L} \right) \frac{1 - \tau_L}{w} = 1 + \epsilon^P_{\tau_L},
\]  

(A.12)

where (A.12) represents the tax elasticity of the wage rate. Average tax progression increases particularly if the initial wage tax, the elasticity of the producer wage and the initial tax
allowance are large. The consumer wage and producer wage change as follows:

\[
\begin{align*}
\left(\frac{dw_A}{d\tau_L}\right)_{\tau_L,\tau_E} & = \frac{[\chi f + \chi\varepsilon(r + s)]a}{\chi f + \varepsilon(r + s)} > 0, \\
\left(\frac{dw_P}{d\tau_L}\right)_{\tau_L,\tau_E} & = -\frac{\varepsilon(r + s)(1 - \chi)(1 + \tau_E)a}{[\chi f + \varepsilon(r + s)](1 - \tau_L)} < 0,
\end{align*}
\]

(A.13)

so that the consumer wage increases whereas the producer wage falls.

Differentiating the expression for the steady-state unemployment rate, \(u = s/(s + f)\) (from (10) for \(\dot{L} = 0\), yields the change in the unemployment rate with respect to the wage tax:

\[
\left(\frac{du}{d\tau_L}\right)_{\tau_L,\tau_E} = -u \left(\frac{f'(\theta)}{s + f}\right) \left(\frac{d\theta}{d\tau_L}\right)_{\tau_L,\tau_E} \leq 0, \quad f'(\theta) > 0,
\]

(A.15)

where the unemployment rate is defined in the usual fashion as the proportion of job seekers in the labor force, that is, \(u = U/L_P\). The change in employment follows from differentiating

\[
\frac{dL}{d\tau_L} = \left(\frac{1 - u}{w}\right) \left(\frac{\chi f}{1 - \chi} + u \left(\frac{f'}{s + f}\right)\right) \left(\frac{d\theta}{d\tau_L}\right)_{\tau_L,\tau_E} \geq 0.
\]

(A.16)

The first term on the right-hand side of (A.16) is the positive labor force participation effect— which is increasing in the labor supply elasticity—and the second term of (A.16) is the unemployment rate effect. If labor supply is exogenous (that is, \(\sigma = 0\)),

if labor supply is exogenous then \(U + L = 1\) so that changes in aggregate unemployment and the unemployment rate mirror changes in aggregate employment (that is, \(du = dU = -dL\)).
A.3.2 Coordinated Payroll-Employment Tax Reform (Propositions 5-6)

The firm’s labor cost can be defined by $\pi \equiv \bar{w}(1 + \tau_E) + \tau_Z$. Keeping the firms labor cost constant at a given wage yields $d\tau_E = -d\tau_Z/w > 0$, where $d\tau_Z < 0$. By substituting this expression in equations (A.6)-(A.7) and solving the system of equations, we arrive at:

\[
\left( \frac{d\theta}{d\tau_E} \right)_{\tau_E, \tau_Z} = \frac{\chi(1 - \chi)\left[a_0 + \gamma_\theta \tau - \phi(b - \tau_L a)\right]}{\gamma_\theta \left[\chi f + \varepsilon(r + s)\right]} L > 0, \quad (A.20)
\]

\[
\left( \frac{dw}{d\tau_E} \right)_{\tau_E, \tau_Z} = -\frac{\chi(1 - \chi)\varepsilon(r + s)\left[a_0 + \gamma_\theta \tau - \phi(b - \tau_L a)\right]}{\left[\chi f + \varepsilon(r + s)\right]} L < 0, \quad (A.21)
\]

where use has been made of (22). It is assumed that $w(1 - \tau_L) + \tau_L a > b$ from which it follows that $a_0 + \gamma_\theta \tau - \phi(b - \tau_L a) > 0$. Note that labor market tightness is unaffected by the tax reform if workers have all the bargaining power (that is, $\chi = 1$) or firms have all the bargaining power (that is, $\chi = 0$). The producer wage and firm’s labor cost change as follows:

\[
\left( \frac{dwP}{d\tau_E} \right)_{\tau_E, \tau_Z} = w \left[ 1 + \frac{(1 + \tau_E)}{w} \left( \frac{dw}{d\tau_E} \right)_{\tau_E, \tau_Z} \right] \quad (A.22)
\]

\[
\left( \frac{d\pi}{d\tau_E} \right)_{\tau_E, \tau_Z} = (1 + \tau_E) \left( \frac{dw}{d\tau_E} \right)_{\tau_E, \tau_Z} < 0. \quad (A.23)
\]

If firms have a lot of bargaining power the first term of (A.22) is negative and dominates the second term if, in addition, $a$ and $b$ are small. In that case, the producer wage falls. The effect on the firm’s labor costs is negative, which results from (A.21). Obviously, the consumer wage falls too. The decline in the unemployment rate and rise in aggregate employment follows

\footnote{Here it is assumed that $0 < \chi < 1$.}
from expressions similar to (A.15)-(A.16):

\[
\left( \frac{du}{d\tau} \right)_{\tau E,\tau Z} = -u \left( \frac{f'(\theta)}{s + f} \right) \left( \frac{d\theta}{d\tau} \right)_{\tau E,\tau Z} \leq 0, \quad f'(\theta) > 0, \tag{A.24}
\]

\[
\left( \frac{dL}{d\tau} \right)_{\tau E,\tau Z} = w_R^\sigma \left( 1 - u \right) \left( \frac{\chi}{1 - \chi} \right) \frac{\gamma_0}{\phi w_R} + u \left( \frac{f'}{s + f} \right) \left( \frac{d\theta}{d\tau} \right)_{\tau E,\tau Z} \geq 0. \tag{A.25}
\]

Public revenue changes according to:

\[
\left( \frac{dR}{d\tau} \right)_{\tau E,\tau Z} = w \sigma \left[ \frac{\left( 1 - u \right) \sigma \left( \chi \right)}{1 - \chi} \frac{\gamma_0}{\phi w_R} + u \left( \frac{f'}{s + f} \right) \right] \left( \frac{d\theta}{d\tau} \right)_{\tau E,\tau Z}, \tag{A.26}
\]

where the first term represents the tax rate effect, which drops out. The second and third term are of opposite sign. Public revenue rises if the positive employment effect (third term) dominates the negative wage base effect (second term).

A.4 Welfare Effects (Proposition 3)

Lifetime utility is given by:

\[
\Lambda(t) = \int_t^\infty \log X(z) e^{-\rho(z-t)} dz = \log \frac{X(t)}{\rho} = \rho \Lambda(t) = \log \rho + \log \left[ A(t) + H(t) \right], \tag{A.27}
\]

where use is made of \( r = \rho \). Maximizing welfare is equivalent to maximizing total wealth, which is defined as \( \Omega(t) = A(t) + H(t) \). Total wealth can be rewritten by using (3), (14), and (17):

\[
\Omega(t) = A_F(t) + \int_t^\infty \left[ \alpha_0 L(z) - \gamma_0 V(z) - \left( \frac{\sigma}{1 + \sigma} \right) w_R^{1+\sigma} \right] e^{-\rho(z-t)} dz, \tag{A.28}
\]

where it should be borne in mind that variables like \( f(\theta) \) and \( w_R \) are at their steady-state values. The term in square brackets on the right-hand side of (A.28) can be rewritten as:

\[
\left[ . \right] = \left( \alpha_0 + \gamma_0 \theta \right) L(z) - \gamma_0 \theta w_R^{1+\sigma} - \left( \frac{\sigma}{1 + \sigma} \right) w_R^{1+\sigma}. \tag{A.29}
\]

By using (10) we can write \( \dot{L}(t) + (s + f)L(t) = f w_R^\sigma \). Solving this first-order differential equation yields:

\[
L(z) = \left[ 1 - e^{-(s+f)(t-z)} \right] \left( \frac{\dot{f}}{s + f} \right) w_R^\sigma + e^{-(s+f)(t-z)} L(t), \tag{A.30}
\]

20
where \( L(t) \) is the initial stock of employment. By using (A.29) and (A.30) we can rewrite (A.28) as:

\[
\Omega(t) = A_F(t) + \left( \frac{1}{\rho} \right) \left[ \left( \frac{\alpha_0 + \gamma_0 \theta}{s + f + \rho} \right) [f + \rho L(t)] - \gamma_0 \theta w_R^\sigma - \frac{\sigma w_R^{1+\sigma}}{1+\sigma} \right],
\]

(A.31)

\[
w_R = b + \left( \frac{\chi}{1-\chi} \right) \left( \frac{1}{\phi} \right) \gamma_0 \theta.
\]

(A.32)

Equation (A.31) expresses \( \Omega(t) \) in terms of the predetermined variables (that is, \( A_F(t) \) and \( L(t) \)) and labor market tightness (\( \theta \)) for given values of the policy variables.

### A.4.1 Exogenous Labor Supply

We first look at a special case of the model for which labor supply is exogenous (that is, \( \sigma = 0 \)). Equation (A.31) reduces to:

\[
\Omega(t) = A_F(t) + \left( \frac{1}{\rho} \right) \left[ \left( \frac{\alpha_0 + \gamma_0 \theta}{s + f + \rho} \right) [f + \rho L(t)] - \gamma_0 \theta \right].
\]

(A.33)

By differentiating (A.33) with respect to \( \theta \) we obtain:

\[
\rho \left( \frac{d\Omega(t)}{d\theta} \right) = \left( 1 - \frac{f + \rho L(t)}{s + f + \rho} \right) \left[ \left( \frac{\alpha_0 + \gamma_0 \theta}{s + f + \rho} \right) \frac{df}{d\theta} - \gamma_0 \right]
\]

\[
= \left( \frac{s}{s + f} \right) \left( \frac{q}{s + f + \rho} \right) \left( \alpha_0 + \gamma_0 \theta \right) (1 - \varepsilon) - \frac{\gamma_0}{q} (s + f + \rho),
\]

(A.34)

where \( 1 - \varepsilon \equiv \theta f^*(\theta)/f(\theta), f(\theta) \equiv \theta q(\theta), \) and we have assumed that the economy is initially in the steady state (so that \( L(t) = L^* = f/(s + f), \)) where asterisks denote equilibrium values.

The term in square brackets on the right-hand side of (A.34) is simplified as follows:

\[
[.] = \alpha_0 (1 - \varepsilon) - \left( \frac{\gamma_0}{q} \right) (s + \varepsilon f + \rho)
\]

\[
= \alpha_0 (\chi - \varepsilon) \left( \frac{s + f + \rho}{s + \chi f + \rho} \right) + (1 - \chi) \tau^P \left( \frac{s + \varepsilon f + \rho}{s + \chi f + \rho} \right),
\]

(A.35)

where the composite pre-existing tax rate, \( \tau^P \), is given by:

\[
\tau^P \equiv \left[ \tau_Z + (1 + \tau_E) \left( \frac{b - \tau_L a}{1 - \tau_L} \right) \right] > 0,
\]

(A.36)

where we have used \( \lambda_E = \gamma_0/q = (1 - \chi)(\alpha_0 - \tau_Z - \phi(b - \tau_L a))/(s + f + \rho) \) (from (4), (6) and (22)) to eliminate \( \gamma_0/q \) in going from the first to the second line of (A.35). By substituting (A.35) into (A.34) and noting that \( d\Lambda/d\tau_L = (1/X^*)(d\Omega/d\theta)(d\theta/d\tau_L) \), we arrive at:

\[
\rho \left( \frac{d\Lambda(t)}{d\tau_L} \right) = \left( \frac{q U^*}{s + \chi f + \rho} \right) \Phi \left( \frac{\Phi}{X^*} \right) \left( \frac{d\theta}{d\tau_L} \right),
\]

(A.37)

where

\[
\Phi \equiv (\chi - \varepsilon) \alpha_0 + (1 - \chi) \tau^P \left( \frac{s + \varepsilon f + \rho}{s + f + \rho} \right),
\]

(A.38)
so that the sign of the welfare effect is fully determined by $\Phi$. If the Hosios (1990) condition holds, that is, $\chi = \varepsilon$, and there are no pre-existing fiscal distortions (that is, $\tau_Z = b = a = 0$) then a labor tax change does not have a first-order welfare effect. In that case, the decentralized market outcome coincides with the socially optimal outcome. If the Hosios (1990) condition is met, but pre-existing labor taxes and unemployment benefits are positive, a coordinated wage-payroll tax reform (with $a > 0$) raises welfare.

**A.4.2 Endogenous Labor Supply**

For endogenous labour supply (that is, $\sigma > 0$) we need to recognize the endogeneity of the reservation wage, $w_R$. By differentiating (A.31) with respect to $\theta$ and simplifying we obtain:

$$\rho \left( \frac{d\Omega(t)}{d\theta} \right) = \sigma \Upsilon + \left( \frac{q \Omega^*}{s + \chi f + \rho} \right) \Phi, \quad (A.39)$$

where $\Upsilon$ is given by:

$$\Upsilon = w_R^{\sigma-1} \left( \frac{dw_R}{d\theta} \right) \left[ f \left( \frac{\alpha_0 + \gamma_0 \theta}{s + f + \rho} \right) - \gamma_0 \theta - w_R \right]$$

$$= w_R^{\sigma-1} \left( \frac{\gamma_0}{\phi} \right) \left( \frac{\chi}{1 - \chi} \right) \left( \frac{f}{s + f + \rho} \right) \left[ \alpha_0 \omega \chi \tau^M + (1 - \tau^M \omega \chi) \tau^P \right] - b, \quad (A.40)$$

where $\Phi$ is defined in (A.38), $\tau^M$ is given by (19) and $\omega$ is:

$$0 < \omega \equiv \frac{s + f + \rho}{s + \chi f + \rho} < 1. \quad (A.41)$$

Note that we have made use of $L^* = f w_R^\sigma / (s + f)$, $U^* = s w_R^\sigma / (s + f)$ and (A.32) in deriving (A.39). Thus, we arrive at:

$$\rho \left( \frac{d\Omega(t)}{d\tau_L} \right) = \left[ \sigma \Upsilon + \left( \frac{q \Omega^*}{s + \chi f + \rho} \right) \Phi \right] \frac{1}{X^*} \left( \frac{d\theta}{d\tau_L} \right). \quad (A.42)$$

Sufficient conditions for the coordinated wage-payroll tax reform to yield an increase in welfare are that: (i) $\chi \geq \varepsilon$ so that workers have a sufficient amount of bargaining power; (ii) initial unemployment benefits are not too large or labor supply is not too elastic; and (iii) wage taxes are progressive.

**References**


