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The Combinatorics of Dom de Caen

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Abstract. We give an overview of some of the mathematical results of Dominique de Caen. These include a short proof of König's theorem, results on Turán numbers, biclique partitions, clique coverings, hypergraphical Steiner systems, p-ranks, stellar permutations, the construction of new distance-regular graphs, large sets of equiangular lines, a bound on the probability of a union, and more.

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1. Introduction

On June 19, 2002, Dominique de Caen died. Dom was born on May 11, 1956, in Montreal. He was a native french speaker, but he learned fluent English at a very young age. The story goes that when he was seven years old, his mother came to pick him up from a stay in the hospital. But Dom couldn't leave, he said; he had arranged to teach English to the nurses on the four o'clock shift. There was a teacher in Dom right from the start.

Dom studied mathematics at McGill in Montreal and received a B.Sc. degree there in 1977. He obtained his M.Sc. degree at Queen's University, Kingston in 1979 for his thesis on prime Boolean matrices [T1]. His M.Sc. supervisor, Norm Pullman, later became a friend and colleague of Dom at Queen's. Dom's Ph.D thesis [T2] (from 1982) on Turán's hypergraph problem was supervised by Eric Mendelsohn at the University of Toronto. After a one-year post-doc at Waterloo, Dom obtained a position at Northeastern University in Boston. Since 1985, when he was awarded an NSERC University Research Fellowship, Dom was on faculty at Queen's University, Kingston.

I first met Dom at a workshop on "Graph Symmetry" [8] in Montreal in July 1996. I carried with me a copy of my almost finished thesis, and showed it to Dom. I am pretty sure that he liked my tables with feasible parameter sets for several combinatorial objects that had his interest. Dom enjoyed playing with parameter sets, to try and construct the corresponding object, inspired by "numerology". "If you do operation O to object X, and then do such and so, then you get the same numbers as we want for this construction, so this may work". Dom was very creative and clever in doing this, and probably it is how he found his construction for distance-regular graphs (see Section 9).

During the academic year 1997–1998, I worked as a post-doc with Dom and with David Gregory, a very good friend of Dom and a co-author on 13 publications. I worked with Dom on several topics; we managed to construct nonregular conference graphs (nonbipartite graphs with three eigenvalues, not all integral) [D47], and some interesting association schemes [D48,D49,D52]. With Dom I lost a very special friend. I have very good memories of our cooperation; and also of our competition … at the snooker table. Because of my deep appreciation for Dom, and the things I learned from him, I will give an overview of (some of) the mathematical results he obtained. Since the variety of topics covered by Dom's work is so large, some of them are not covered here (such as "prime Boolean matrices"), or only partially (such as "clique coverings", "biclique partitions", "ranks"). The results mentioned are merely intended to give an impression of the breadth and depth of the combinatorics of Dom de Caen.

2. König's Theorem

Dom's one-page paper [D21] with a short proof of König's theorem is the result of an attempt to extend this theorem to infinite graphs.

König's theorem asserts that if $v = v(G)$ is the size of a largest matching in a bipartite graph G, then there exists a cover of all the edges of G by a set of v vertices.

To prove the theorem, Dom observed that it suffices to prove that every bipartite graph G contains a vertex that meets every matching of size $v(G)$, a so-called universal vertex (use induction). It is then proven (by contradiction) that for every edge in G, at least one of its ends is a universal vertex. This is done by claiming that if for the edge $e = \{x, y\}$ there are matchings M_x and M_y of size v missing x, resp. y, then the subgraph $H = M_x \cup M_y \cup \{e\}$ contains a matching of size $v+1$. To verify this claim, it is noted that H has maximum degree at most two, and hence the component C of H containing e is either a path or even cycle. Thus, if C has t edges, then it contains a $\lceil t/2 \rceil$-matching. Since either M_x or M_y meets C in at most $\lceil t/2 \rceil - 1$ edges, the appropriate matching has $v - \lceil t/2 \rceil + 1$ remaining edges which together with the $\lceil t/2 \rceil$-matching of C give a matching of G of size $v+1$.

3. Turán's Hypergraph Problem

The Turán number $T(n, l, k)$ (for $n \geq l \geq k \geq 1$) is the minimum number of k-subsets from an n-set, such that every l-subset contains at least one of the k-subsets. It is the same as the minimum number of blocks in an $(n-l)$-$(n, n-k, 1)$ covering (take the complements of the k-subsets).

The following result is from [D9].

$$T(n, l, k) \geq \frac{l(n-l+1)}{k(n-k+1)} \cdot \frac{\binom{n}{k}}{\binom{l}{k}}.$$

This gives the best known general lower bound on Turán numbers, for $n \geq k+l$. The latter is not a crucial restriction, since in this area, one is (in general) interested in fixing k and l, and letting n grow. In particular, the number

$$t(l,k) = \lim_{n\to\infty} \frac{T(n,l,k)}{\binom{n}{k}}$$

is of special interest. In his survey [D37] on Turán's hypergraph problem, Dom conjectured that

$$\lim_{k\to\infty} k \cdot t(k+1,k) = \infty,$$

with an offer for a reward of 500 Canadian dollars for a proof or disproof for this conjecture.

It should be noted that the problem of determining $t(l,k)$ for specific l and $k > 2$ is a very hard problem. For $k = 2$, Turán [13] showed that $t(l,2) = (l-1)^{-1}$. He also conjectured that $t(4,3) = \frac{4}{9}$. This problem is equivalent to the question of how many triples one can pick without forming any tetrahedra (all triples from a given quadruple). In [D17], it was shown that $t(4,3) \geq 1 - c = .3786\ldots$, where c is the real root of the polynomial $9x^3 - 33x^2 + 46x - 18$. Currently, the best known lower bound is $t(4,3) \geq \frac{3}{4} - \frac{1}{12}\sqrt{17} = .4064\ldots$, by Chung and Lu [4].

A similar problem is to determine how many triples one can pick without forming any Fano planes (the unique configuration of 7 triples on 7 points such that any two triples intersect in a point). Let $f(n)$ denote the maximum number of such triples from an n-set. It was conjectured by Sós [12] that

$$\lim_{n\to\infty} \frac{f(n)}{\binom{n}{3}} = \frac{3}{4}.$$

Together with Füredi [D50], this long-standing conjecture was proved.

4. Biclique Partitions

The Graham–Pollack theorem [7] states that the complete graph on n vertices cannot be edge-partitioned into fewer than $n-1$ complete bipartite graphs (bicliques). On the other hand, there are many ways to partition the edges of K_n into $n-1$ bicliques.

With Hoffman [D22], it was shown that K_n cannot be edge-decomposed into $n-1$ isomorphic bicliques, except for the case where these bicliques are stars $K_{1,s}$. In these exceptional cases, the biclique decompositions correspond 1-1 with regular tournaments on $2s-1$ vertices.

With Gregory and Pritikin [D32], minimum biclique partitions of the complete multigraph λK_n are studied. It is conjectured that for each λ, the edges of λK_n can be partitioned into $n-1$ bicliques, for n large enough, and this conjecture is proved for infinitely many λ, including all $\lambda \leq 18$. Such biclique partitions with $n-1$ bicliques are related to affine designs and balanced weighing matrices. When all bicliques are isomorphic, there is a relation with balanced bipartite designs, and a bipartite analogue of difference sets.

5. Extremal Clique Coverings in Complementary Graphs

In a joint paper with Erdös, et al. [D14], bounds on the sum, resp. product of the clique covering numbers of two complementary graphs were obtained.

If $cc(G)$ is the least number of cliques needed to cover all edges of G, then

$$\left\lfloor \frac{n^2}{4} \right\rfloor + 2 \leq \max\{cc(G) + cc(\overline{G})\} \leq \frac{n^2}{4}(1 + o(1)),$$

where the maximum is over all graphs on n vertices. The lower bound is attained by the complete bipartite graph with the two parts differing in size by at most one.

Also the conjecture was proven that $\max\{cc(G) \cdot cc(\overline{G})\} \sim n^4/256$, where the maximum is over all graphs on n vertices. The paper also contains similar, but weaker, results on the clique partition numbers.

6. Hypergraphical Steiner Systems

A t-$(\binom{p}{d}, K, \lambda)$ design with points the set of d-tuples from a p-set is called d-hypergraphical if the symmetric group S_p (acting on d-sets) is an automorphism group of the design, or in other words, if the block set is a union of orbits.

With Kreher [D33], it was shown that there are four 3-hypergraphical 3-$(\binom{6}{3}, 4, 1)$ designs. Also their full automorphism groups were determined. Two of the designs, although different under the action of S_6, turned out to be isomorphic.

It was also conjectured that for fixed $d \geq 2$ and λ, there are only finitely many non-trivial d-hypergraphical t-$(\binom{p}{d}, K, \lambda)$ designs (with $t \geq 2$, p, K ranging freely): (A design is trivial if $t \in K$ or for some $k \in K$ all k-subsets of the point set are blocks.) When $t \geq 2$ is also fixed then the above statement is known to be true.

7. *p*-Ranks

In [D26] and [D39], the ranks of tournament matrices were studied. A tournament matrix is a square 0-1 matrix M such that $M + M^T = J - I$ (the all-ones matrix minus the identity matrix). It is shown in [D26] that the rank of an $n \times n$ tournament matrix is at least $\frac{n-1}{2}$ over any field; over a field of characteristic zero it is either $n-1$ or n.

In the one-page paper [D39], it is proven that if $n \equiv p \equiv 3 \pmod 4$, then the rank over a field of characteristic p (the p-rank) cannot equal $\frac{n-1}{2}$. For $n \equiv 1 \pmod 4$, examples with p-rank $\frac{n-1}{2}$ are known. The case $n \equiv 3$, $p \equiv 1 \pmod 4$ remains open.

We remark that also some results about the algebraic multiplicity of zero as an eigenvalue of a tournament matrix were obtained with Gregory, et al. [D29].

With Godsil and Royle [D31], extensions of a bound of Bruen and Ott [2] on the p-rank of the incidence matrix of a partial linear space are obtained. It is shown, for example, that in a finite connected partial linear space with v points, b blocks, and f flags, the p-rank of the incidence matrix B satisfies $\mathrm{rk}_p(B)^2 \geq f - v - b + 1$. Also a more technical result is derived.

In the unpublished paper [D36] with Godsil, the bound of Bruen and Ott [2] on the p-rank of the incidence matrix B of a projective plane (of order n) is slightly improved. It is shown that $(\mathrm{rk}_p(B) - 1)^2 = n^3 + \mathrm{rk}_p(C)$, where C is a certain matrix in the Bose–Mesner algebra of a very natural 5-class association scheme on the antiflags of the plane. It is further proved that $\mathrm{rk}_p(C) \geq \lfloor \frac{n+1}{2} \rfloor$; and if p is odd, then $(\mathrm{rk}_p(C))^2 + \mathrm{rk}_p(C) \geq 2n^3$. The bound of Bruen and Ott follows from the nonnegativity of the rank of C.

8. Stellar Permutations and the Search for Biplanes

One of Dom's goals or wishes was to construct new biplanes. Although he did not succeed in doing this, he came up with an interesting concept [D44] while he was studying the known biplanes.

For a finite set X, a stellar permutation of the set $\binom{X}{2}$ of pairs of X is a permutation such that for each $x \in X$, the image of the star $St(x) = \{\{x, y\} : x \neq y \in X\}$ is a 2-regular graph on $X \setminus \{x\}$.

Dom discovered that stellar permutations arise in some of the known biplanes, and he suggested that it might be possible to construct new biplanes with a so-called stellar representation (see [D44] for details).

Stellar permutations exist if and only if $|X| \geq 4$. One can construct stellar permutations in the following way. Let L be a self-orthogonal Latin square of order $|X| \geq 4$ (these exist if $|X| \neq 6$), i.e., $L(x, x) = x$ for all x and L is orthogonal to its transpose. Then $\phi(\{x, y\}) = \{L(x, y), L(y, x)\}$ is a stellar permutation. Another construction uses skew Room squares, and a recursive method using linear spaces is described.

Although the composition of two stellar permutations need not be stellar, it is of interest to look at groups of stellar permutations (for convenience we call the identity map also stellar). It is shown that the size of such a group can be at most half the size of X. Stellar groups of maximal size ($|X|/2$ for even $|X|$; $(|X| - 1)/2$ for odd $|X|$) can be constructed if $|X|$ is a prime power. For even $|X|$, such maximal stellar groups can be used to construct elation semibiplanes. The problem is raised to construct homology semibiplanes from maximal stellar groups when $|X|$ is odd.

9. Distance-Regular Graphs

A distance-regular graph with intersection array $\{b_0, b_1, \ldots, b_{d-1}; c_1, \ldots, c_d\}$ is a connected graph with diameter d such that for every vertex x and vertex y at distance i from x, the number of neighbours of y at distance $i - 1$ from x equals c_i, and the number of neighbours of y at distance $i + 1$ from x equals b_i, for all i. Such a graph is regular with valency b_0.

One of Dom's great achievements was the construction, with Mathon and Moorhouse [D38], of distance-regular graphs with intersection array $\{2^{2t}-1, 2^{2t}-2, 1; 1, 2, 2^{2t}-1\}$. Such graphs are antipodal covers of the complete graph (for more information on distance-regular graphs, see [1]).

The graphs are easily defined as follows. Let $V = GF(2^{2t-1}) \times GF(2) \times GF(2^{2t-1})$ be the vertex set. Two vertices (a, i, α) and (b, j, β) are adjacent precisely if

$$\alpha + \beta = a^2b + ab^2 + (i+j)(a^3 + b^3).$$

Actually, the graphs are defined in greater generality; and they are related to the Preparata codes; see [D38] for details. The construction also allows for taking quotients. In this way distance-regular graphs with intersection arrays $\{2^{2t}-1, 2^{2t}-2^i, 1; 1, 2^i, 2^{2t}-1\}$ for $i = 1, \ldots, 2t$ arise. Prior to this construction, no distance-regular graphs with these intersection arrays were known for $i < t$.

It is important to note that the distance-regular graphs constructed by Dom are formally dual to the Cameron–Seidel association schemes of linked symmetric designs from Kerdock sets and quadratic forms, as described in the paper by Cameron and Seidel [3]. This paper was a major source of inspiration for Dom, as we shall also see in the next section.

In [D48], 5-class association schemes related to the above distance-regular graphs were constructed, as were their formally dual schemes: by applying vector space duality a fission scheme of a subscheme of the Cameron–Seidel scheme was obtained.

By using highly nonlinear functions (cf. [6]), it might be possible to generalize the construction of the above distance-regular graphs. With Fon-Der-Flaass [D54], Dom did find a generalization of his own graphs, by using Latin squares.

In [D45], the question was raised whether there exist distance-regular antipodal covers of the complete graph with intersection array $\{b^2(b+2)-1, b(b^2-1), 1; 1, b(b+1), b^2(b+2)-1\}$, with b a positive integer. The answer is yes for $b = 1$ or 2, but unknown for other b. From his results on the spectra of complementary graphs in a strongly regular graph, obtained by using Jacobi's identity on determinants, Dom showed that these putative distance-regular graphs cannot arise as the second subconstituent of a strongly regular graph.

In [D40], it is proven that for every $i \times j$ submatrix of zeroes of the incidence matrix of a bipartite distance-regular graph of diameter at most four, we have $i + j \leq v - k + 1$, where v is the number of vertices and k denotes the valency (in other words, the degree of indecomposibility is $k - 1$). From this it follows that the vertex connectivity of a bipartite distance-regular graph of diameter at most four is equal to its valency. It is widely believed that this is true for all distance-regular graphs. We note that Dom's interest in the topic of indecomposibility dates from his Master's thesis on prime Boolean matrices [T1]; see also [D3].

10. Equiangular Lines

A set of lines in Euclidean space is called equiangular if all pairs of lines have the same angle. Finding a large set of equiangular lines in Euclidean space is equivalent to finding a graph whose Seidel matrix S has smallest eigenvalue θ of large multiplicity m. The correspondence follows from the fact that the matrix $I - \frac{1}{\theta}S$ is positive semidefinite of rank $d = n - m$, where n is the number of vertices, and hence it can be respresented as the Gram matrix of n unit vectors in Euclidean d-space, with pairwise inner products $\pm\frac{1}{\theta}$. Thus the lines through these vectors have angles $\arccos(\frac{1}{\theta})$.

In [D51], Dom observed that one of the graphs in the Cameron–Seidel association scheme has smallest Seidel eigenvalue of large multiplicity. Thus one finds a set of $\frac{2}{9}(d+1)^2$ equiangular lines in Euclidean d-space for each $d = 3 \cdot 2^{2t-1} - 1$, for any positive integer t. It should be noted that the size is of the same asymptotic order as the "absolute" upper bound $d(d+1)/2$ for the size of an equiangular set of lines in d-space. Prior to this construction, the largest sets had sizes of order $d\sqrt{d}$.

11. Probability of a union

In [D42], a lower bound is given on the probability of a union of events in terms of the probabilities of the events, and the pairwise intersections of events. More specifically, Dom showed that if $\{A_i\}_{i \in I}$ is a finite family of events in a probability space, then

$$P\Big(\bigcup_{i \in I} A_i\Big) \geq \sum_{i \in I} \frac{P(A_i)^2}{\sum_{j \in I} P(A_i \cap A_j)}$$

(with the convention that $\frac{0}{0} = 0$).

This result was obtained by a clever application of the Cauchy–Schwarz inequality. Dom's inequality was applied by Séguin [11] to obtain a lower bound on the error probability for signals derived from a binary linear code and used on additive white Gaussian noise channels with a maximum-likelihood decoder. For an application in the same field, Kuai et al. [9] obtained a generalization of Dom's bound.

12. Some Other Results

In his first paper [D1], Dom gave counterexamples to the conjectures that every graph has a smallest path (resp. path-cycle) decomposition such that every odd vertex is the endpoint of exactly one path in the decomposition.

With Chee et al. [D23], many new simple t-designs (for $t = 2, \ldots, 5$) were found. Also with Hobart et al. [D35], new t-designs were obtained.

The Erdös–Lovász function $f(n)$ is defined as the smallest k for which there exists a set of k pairwise intersecting n-sets such that the minimum cardinality of a set that intersects every such n-set is n. In a paper with Székely [D34], it is shown that $f(3) = 6$, and there are only two configurations realizing this (one is the Fano plane minus a line), and that $f(4) = 9$, with exactly one corresponding configuration. It is also noted that $12 \leq f(5) \leq 14$.

In [D43], an upper bound on the sum of squares of degrees in a graph was obtained: if G is a graph with n vertices and e edges, and vertex degrees d_i, $i = 1, \dots, n$, then

$$\sum_{i=1}^{n} d_i^2 \leq e\left(\frac{2e}{n-1} + n - 2\right).$$

The inequality follows from the positive semidefiniteness of a certain quadratic form. It inspired the author [5] to obtain a related matrix inequality. Li and Pan [10] found that the only graphs for which equality holds in the above inequality are the stars $K_{1,n-1}$ and the complete graphs K_n. They applied "de Caen's inequality" to obtain an upper bound on the largest Laplacian eigenvalue of a graph.

To conclude this overview, we mention an open problem from [D41] about planar point-line configurations. "Is it true that every configuration of m lines and n points in the plane determines fewer than mn triangles?" Although Dom and Székely [D41] were careful to propose a slightly weaker conjecture (see also below), I am quite sure that Dom believed that the answer to the question is yes.

13. A List of Some Problems and Conjectures

Conjecture 1 [D8]. $t(l, 3) = 4/(l-1)^2$ for every $l \geq 3$. (For odd l this was already conjectured by Turán.)
Conjecture 2 [D32]. For each λ, the edges of λK_n can be partitioned into $n-1$ bicliques, for n large enough.
Conjecture 3 [D33]. For fixed $d \geq 2$ and λ, there are only finitely many non-trivial d-hypergraphical t-$(\binom{p}{d}, K, \lambda)$ designs (with $t \geq 2$, p, K ranging freely).
Conjecture 4 [D37]. $\lim_{k\to\infty} k \cdot t(k+1, k) = \infty$.
Conjecture 5 [D40]. The degree of indecomposibility of a bipartite distance-regular graph is $k-1$, i.e., for every $i \times j$ submatrix of zeroes of its incidence matrix, we have $i + j \leq v - k + 1$, where v is the number of vertices and k denotes the valency.
Conjecture 6 [D41]. For every planar system of n points and m straight lines, the number of triangles determined by this system is $O(mn)$.
Problem 1 [D3]. Which Boolean matrices are (Boolean) products of prime matrices? A Boolean matrix is prime if it is not a permutation matrix, and if whenever it is the product of two matrices, then either of these two is a permutation matrix.
Problem 2 [D14]. Is $\max\{cp(G) + cp(\overline{G})\} \sim 7n^2/25$, where the maximum is over all graphs on n vertices, and $cp(G)$ is the clique partition number of G?
Problem 3 [D16]. For a non-negative integer matrix A, determine the minimum number $b = b(A)$ such that $A = BC$, where B and C are binary matrices, and B has b columns. Also find restrictions on the structure of B and C.
Problem 4 [14]. Let G be a graph embedded in the plane. A facial factor of G is a 2-factor (a spanning 2-regular subgraph) each of whose connected components is the boundary of some face of G. Find non-trivial necessary and sufficient conditions for the existence of a facial factor in a plane graph.

Problem 5 [D33]. Do any of the 3-hypergraphical 3-$(20, 4, 1)$ designs admit a one-point extension to a 4-$(21, 5, 1)$ design?
Problem 6 [D37]. Find all k-uniform hypergraphs (sets of k-subsets) such that every $(k+1)$-subset contains one or $k+1$ edges (such k-subsets).
Problem 7 [D44]. Can one construct a homology semibiplane from a maximal stellar group?
Problem 8 [D45]. Do distance-regular graphs with intersection array $\{b^2(b+2) - 1, b(b^2-1), 1; 1, b(b+1), b^2(b+2)-1\}$, with b a positive integer, exist?
Problem 9 [D47]. Does a graph with three distinct eigenvalues have at most three distinct degrees?
Problem 10 [D53]. Is there a q-analogue of the recurrence relation in [D53] for the rank of the set-inclusion matrices of t-subsets versus k-subsets of a fixed set of size v; i.e., is there some recurrence relation for the rank of the inclusion matrices of t-dimensional subspaces versus k-dimensional subspaces of a v-dimensional space over $GF(q)$?

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