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Structural remedies in merger regulation in a Cournot framework*

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Abstract

To prevent possible abuse of market power in the future an antitrust agency can force merging firms to divest some of their assets. The divested assets can be sold through an auction either to existing competitors or a new entrant. An analysis of a Cournot market with fixed capital (assets) is conducted. Divesture of assets extends the range of parameters when a merger satisfies consumer surplus standard and should be approved. The rationalization of production between the merging firms and the viability of a new entrant play a crucial role in determining whether to approve a merger. If the agency takes more active stance toward the selection of a purchaser of the assets, then it could lead to a favorable outcome for consumers and merging firms.

Keywords: merger regulation, structural remedies, efficiency, auction
JEL classification: D43, K21, L51

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1 Introduction

There is an extensive literature on mergers for different market structures and types of competition[1]. It has been shown that if there are no cost reductions due to a merger, firms find it profitable to exercise their market power through a price increase, which decreases consumer surplus.

At the same time an antitrust agency can choose between behavioral and structural remedies to restore effective competition in relevant markets[2]. Behavioral remedies set constraints on the merged firms' future behavior such as engagements by the merging parties not to abuse certain assets, compulsory licensing, access to intellectual property. However, in the case of behavioral remedies the prime difficulty is in overseeing the implementation of the remedies at the post-merger phase. Structural remedies modify the allocation of property rights and create new firms through entire or partial divesture of assets. In the EU the competition agency prefers to use structural remedies, because it is easy to implement and once implemented there is no need to monitor behavior of merging firms afterwards[3].

In this paper I analyze structural remedies in merger regulation. Although there are discussions by antitrust practitioners, academics, and lawyers, to my knowledge the idea of structural remedies as a way to protect consumers is not formally analyzed in the literature. The model presented in this paper captures and allows a theoretical analysis of all main issues that are at stake when the agency makes a merger approval decision.

This paper is an extension of Shapiro and Farrell (1990), Perry and Porter (1985), McAfee and Williams (1992) models. Those papers conduct an equilibrium analysis of a Cournot market before and after a merger with a focus

[1] Salant et al. (1983), Perry and Porter (1985), Deneckere and Davidson (1985), Zang and Kamien (1990), Shapiro and Farrell (1990), Horn and Persson (2001).

[2] "The vast majority of cases raising competition concerns were solved through viable remedies offered by the notifying parties in due time." Monti M., the EU Competition Commissioner, Paris, January 2002.

[3] See Monti (2002), Motta et al. (2002), Motta (2004)

on profitability and welfare changes. A common feature in those models is the existence of a fixed capital (assets) in an industry. The amount of fixed capital in possession of a firm determines firms' production costs. In this paper I modify their analysis by allowing partial divesture of assets.

In order to prevent possible abuse of market power in the future the agency can force merging firms to divest some of their assets[4]. The divested assets can be sold either to existing competitors or stimulate a new entrant into the market by selling him the divested assets. One of the viable mechanisms to sell divested assets is an auction. In the paper we analyze certain auction outcomes and possible ways to enhance consumer and total welfare. According to the EU merger regulation[5] merging firms suggest a purchaser, which must be approved by the agency. Therefore, the agency has veto power over the choice of a purchaser of divested assets.

It is assumed that an antitrust agency applies a consumer surplus standard; it wants to approve mergers that decrease prices, while rejecting the ones that increase prices[6]. However, mergers could lead to significant synergies (efficiencies). If synergies between merging firms are substantial, then price might decrease after a merger, i.e. cost reduction outweighs market power effect[7]. This issue plays an important role in merger regulation after both the EU and US agencies allowed for an efficiency defense in merger approvals.

The structure of the paper is as follows. In Section 2 I describe the basic model. Then I analyze a symmetric cost structure case, and proceed with a non-symmetric case. Then results are discussed, followed by a conclusion.

[4]We can think about landing slots at airports that airlines possess, licences, radio or mobile phone frequencies, electricity generation facilities, etc. The agency can ask to divest certain number of them as a structural remedy.

[5]The EU Merger Regulation and Competition Commission "Best practice guidelines for divesture commitments" published on
http://europa.eu.int/comm/competition/mergers/legislation.

[6]For a discussion about consumer vs. total surplus approaches in merger regulation, see Motta (2004), Neven and Roller (2000), Shapiro and Farrell (2001), Besanko and Spulder (1993).

[7]See Williamson (1968), Werden (1996), Shapiro and Farrell (1990), Roller et al. (2000), Besanko and Spulder (1993).

2 Model

Let us consider market inverse demand function $P = a - bX$, where the total output $X = \sum_{i=1}^{n} x_i$ and x_i is firm's i output. Each firm maximizes its profit: $max_{x_i}(a - bX)x_i - C(x_i, s_i)$, where s_i is firm's i assets. The idea of the fixed capital s in the industry is captured through a form of the cost function, $C(x_i, s_i) = \frac{d_i}{s_i}x_i$, the more capital a firm possesses, the lower marginal costs of production. Parameter d_i describes firm's i production technology. Therefore, the market is characterized by constant marginal costs of production $C_{x_i} = \frac{d_i}{s_i}$. This is a simplified form of Perry and Porter (1985), McAfee and Williams (1992) cost function $C(x, s) = sg + dx + \frac{e}{2s}x^2$ and, consequently, marginal costs $C_x = d + \frac{e}{s}x$ and Shapiro and Farrell (1990) cost function $C(x, s) = wx^{\frac{1}{b}}s^{-\frac{a}{b}}$ and, consequently, marginal costs $C_x = \frac{w}{b}x^{\frac{1}{b}-1}s^{-\frac{a}{b}}$, where s is amount of fixed capital, and a, b, e, d, g, w are constants. The constant marginal costs function allows to capture the main feature of their functions, the inverse relation between assets and marginal costs, but it simplifies derivations significantly. Although many results in the paper would be valid for any convex cost function with respect to a fixed capital ($C_s < 0,\ C_{ss} > 0$).

The equilibrium for a Cournot type of competition with constant marginal costs is the following. Each firm maximizes its profit:
$max_{x_i}(a - bX)x_i - \frac{d_i}{s_i}x_i$ for $i = 1, ..., n$. There is a restriction on parameters: $a - \frac{d_i}{s_i} > 0$. A firm with too little capital, that $a - \frac{d_i}{s_i} < 0$, would always prefer not to produce. Given demand and costs functions, the equilibrium output and price before the merger are derived from the system of n first-order conditions (FOCs): $a - 2bx_i - b\sum_{j=1, j\neq i}^{n} x_j = \frac{d_i}{s_i}$.

Hence, $x_i = \frac{1}{(n+1)b}(a - n\frac{d_i}{s_i} + \sum_{j\neq i}^{n}\frac{d_j}{s_j})$ is the output by the i-th firm[8] and

therefore the total market output is $X_{total} = \frac{1}{(n+1)b}(na - \Sigma_{i=1}^{n}\frac{d_i}{s_i})$.

Hence, equilibrium price is $P = \frac{1}{n+1}(a + \Sigma_{i=1}^{n}\frac{d_i}{s_i})$.

[8]Further restrictions on exogenous parameters are: $x_i = \frac{1}{(n+1)b}(a - n\frac{d_i}{s_i} + \sum_{j\neq i}^{n}\frac{d_j}{s_j}) > 0$

Profit of the i-th firm is $\Pi_i = \frac{1}{(n+1)^2 b}[a - n\frac{d_i}{s_i} + \sum_{j=1, j\neq i}^{n} \frac{d_j}{s_j}]^2$.

3 Symmetric case

First, we derive a pre-merger equilibrium and then compare it with cases when there is no agency's intervention, when the agency can force divesture and the divested assets go either to an existing competitor or to a new entrant. The objective of the agency is to protect consumers from a price increase due to the merger. Having veto power over a purchaser of the divested assets, the agency approves a merger only if the price after the merger will not increase.

Consider first a symmetric case with n=3, i.e. each firm possesses an equal amount of fixed capital s, i.e. $s_1 = s_2 = s_3 = s$, and the same production technology, i.e. $d_1 = d_2 = d_3 = d$, to get a feeling how the system behaves.

3.1 Pre-merger case

The pre-merger equilibrium output of each firm is $x_1 = x_2 = x_3 = \frac{1}{4b}(a - \frac{d}{s})$, and $X_{total}^{Before} = \frac{3}{4b}(a - \frac{d}{s})$ and, consequently, the equilibrium price before the merger is $P^{Before} = \frac{1}{4}(a + 3\frac{d}{s})$.
Equilibrium profit of each firm is $\Pi_i^{Before} = \frac{1}{16b}(a - \frac{d}{s})^2$.

3.2 Merger to duopoly without agency's intervention

When any two firms in the market merge the number of firms decreases and the market becomes more concentrated. However, the merged firm becomes twice bigger than its competitor and possesses $(2s)$ assets, which decreases its marginal costs of production. The equilibrium output and price after the merger is derived from the system of two FOCs for the merged firm (M) and the firm-outsider (o):

$$\begin{cases} a - 2bx_M - bx_o = \frac{d}{2s} \\ a - 2bx_o - bx_M = \frac{d}{s} \end{cases} \quad \Leftrightarrow \quad \begin{cases} x_M = \frac{a}{3b} \\ x_o = \frac{a}{3b} - \frac{d}{2bs} \end{cases}$$

Therefore, total output is $X_{total}^{After} = \frac{1}{3b}(2a - \frac{3}{2}\frac{d}{s})$.

Consequently, the equilibrium price is $P^A = a - b\frac{1}{3b}(2a - \frac{3}{2}\frac{d}{s}) = \frac{a}{3} + \frac{d}{2s}$

***Remark 1*:** Price after the merger will not increase, $P^A \leq P^B$, if $\frac{a}{3} \leq \frac{d}{s}$.
Proof: Compare : $P^A = \frac{a}{3} + \frac{d}{2s} \leq \frac{1}{4}(a + 3\frac{d}{s}) = P^B \quad \Leftrightarrow \quad \frac{a}{3} \leq \frac{d}{s}$

Marginal costs of production of the merged firm could decrease substantially because the firm possesses more fixed capital. Lower marginal costs for the merged firm could outweigh market power effect and be sufficient not to increase market price. This result copies the one, which was shown by Shapiro and Farrell (1990)[9].

Profit of the merged firm is $\Pi_M = \frac{1}{9b}[a - 2\frac{d}{2s} + \frac{d}{s}]^2 = \frac{a^2}{9b}$.
Profit of the outsider firm is $\Pi_o = \frac{1}{9b}[a - 2\frac{d}{s} + \frac{d}{2s}]^2 = \frac{1}{9b}[a - \frac{3}{2}\frac{d}{s}]^2$.

The merger is possible only if it is profitable for the merging firms itself, i.e. if a joint profit after the merger is higher than the sum of profits before the merger when they are separate firms. However, this condition usually holds for a wide range of parameters[10].

3.3 Merger to duopoly with divesture to an existing competitor

The agency can use structural remedies to 'correct' new market situation and keep prices at least unchanged after the merger. The agency can ask the merged firm to divest some of obtained assets. A volume of divested

[9]Shapiro and Farrell (1990) showed that a merger raises price if and only if a markup of the would-be merging firms is less than the sum of the pre-merger markups at its constituent firms, where a merged firm produces just as much as its constituent firms together did before the merger.

[10] $\Pi_M^A > \Pi_1^B + \Pi_2^B \quad \Leftrightarrow \quad \frac{a^2}{9b} > 2\frac{1}{16b}(a - \frac{d}{s})^2 \quad \Leftrightarrow \quad \frac{a}{3}(3 - \sqrt{8}) < \frac{d}{s}$. Given $\frac{a}{3}(3 - \sqrt{8}) < \frac{a}{3}$, it is always profitable for the merging firms to merge, if the price will not increase (Remark 1: $\frac{a}{3} \leq \frac{d}{s}$).

assets is denoted by Δ with $\Delta \in [0, s_j]$, where s_j is amount of fixed assets which belongs to the acquired firm. There is an upper limit on the amount the agency can ask the merged firm to divest because otherwise the merger makes no sense: asking more the agency would leave the acquiring firm with less assets than before the merger. Therefore, if two firms merge then they control $(2s)$ assets. After the agency asks them to divest (Δ) assets, then $(2s - \Delta)$ assets remain in their possession. If divested assets go to an existing competitor, then the outsider to the merger possesses $(s + \Delta)$ assets. Equilibrium output and price are derived from the system of two FOCs for the merged firm (M) and the firm-outsider(o):

$$\begin{cases} a - 2bx_M - bx_o = \frac{d}{2s-\Delta} \\ a - 2bx_o - bx_M = \frac{d}{s+\Delta} \end{cases} \quad \Leftrightarrow \quad \begin{cases} x_M = \frac{1}{3b}[a - \frac{2d}{2s-\Delta} + \frac{d}{s+\Delta}] \\ x_o = \frac{1}{3b}[a + \frac{d}{2s-\Delta} - \frac{2d}{s+\Delta}] \end{cases}$$

Therefore, total output is $X^{A1}_{total}(\Delta) = \frac{1}{3b}[2a - \frac{d}{(2s-\Delta)} - \frac{d}{(s+\Delta)}]$.

Consequently, the equilibrium price is $P^{A1}(\Delta) = \frac{1}{3}[a + \frac{d}{(2s-\Delta)} + \frac{d}{(s+\Delta)}]$.

Profit of the merged firm is $\Pi^{A1}_M = \frac{1}{9b}[a - 2\frac{d}{2s-\Delta} + \frac{d}{s+\Delta}]^2$ and profit of the outsider is $\Pi^{A1}_o = \frac{1}{9b}[a - 2\frac{d}{s+\Delta} + \frac{d}{2s-\Delta}]^2$. The merging firms will proceed with the merger only if it is profitable for them: $\Pi^{A1}_M > \Pi^B_1 + \Pi^B_2$. However, like in the previous section, it is possible to show that this profitability constraint is not binding for parameters when the price not increases after the merger.

If we compare this situation with divesture with the one without (i.e. when the agency doesn't intervene), then the following proposition holds.

Proposition 1: Given pre-merger symmetric cost structure ($s_1 = s_2 = s_3$ and $d_1 = d_2 = d_3$), any divesture $\Delta \in (0; s)$ leads to lower prices than without it.

Proof: P^{A1} will never exceed P^A: $\frac{1}{3}[a + \frac{d}{(2s-\Delta)} + \frac{d}{(s+\Delta)}] \leq \frac{a}{3} + \frac{d}{2s} \Leftrightarrow$
$\frac{1}{2s-\Delta} + \frac{1}{s+\Delta} \leq \frac{3}{2s} \quad \Leftrightarrow \quad 2s^2 \leq (2s - \Delta)(s + \Delta)$, because $\Delta(s - \Delta) \geq 0$.

The divesture leads to more symmetric market. From the proposition above it is seen that no matter how large the decrease in marginal costs due to more fixed capital for the merged firm it is always better to divest some

assets and restore the symmetry: take assets from the bigger firm and give it to the smaller one. This results come from the convexity of the cost function, ($C_x = \frac{d}{s}$): marginal costs are inversely related to the amount of fixed capital a firm possesses. Given firms' identical cost structure and a Cournot type of competition the maximum output and the lowest price are achieved when all firms at the market possess equal amount of fixed capital, i.e. in case of duopoly the best result is achieved when ($\frac{s}{2}$) assets are divested and both firms possess ($\frac{3}{2}s$).

Therefore we can distinguish three effects that affect equilibrium price in the market: number of players, marginal costs, and symmetry of the market[11]. Number of players in the market and degree of symmetry negatively effects the price, while marginal costs positively effect the price.

From Section 3.2 we know that if $\frac{a}{3} \leq \frac{d}{s}$, then we do not need any divesture to make prices not to increase, i.e. $\Delta = 0$. Now let's check if $\frac{a}{3} > \frac{d}{s}$, then how many assets the agency should ask the merging firms to divest in order to keep prices at the pre-merger level, i.e. $P^{A1} \leq P^B$:

$\frac{1}{3}[a + \frac{d}{(2s-\Delta)} + \frac{d}{(s+\Delta)}] \leq \frac{1}{4}(a + 3\frac{d}{s}) \qquad \Leftrightarrow$

$-\Delta^2 + s\Delta + 2s^2\frac{3d-as}{9d-as} \geq 0$

The solutions to this quadratic expression is the following:

$$\Delta^{Required} = \frac{-s \pm s\sqrt{1+8\frac{3d-as}{9d-as}}}{-2} = \frac{-s \pm s\sqrt{\frac{33d-9as}{9d-as}}}{-2},$$

The expression under the square root should be positive but less than one, $0 \leq \frac{33d-9as}{9d-as} \leq 1$, to make divesture less than s assets (otherwise the agency asks to divest more than the acquiring firm obtained from the merger). If $(9d - as) < 0$, then $(3d - as) < 0$ and $(33d - 9as) < 0$ as well, and it would be impossible to have the square root expression to be less than 1. Given $(9d - as) > 0$, the price will stay unchanged after the merger if :

[11] One of the measurements of symmetry of the market is the Herfindahl-Hirschman index: the higher HHI, the least symmetric a market.

$\Delta_1^{Req} = \frac{-s+s\sqrt{\frac{33d-9as}{9d-as}}}{-2}$ and $\Delta_2^{Req} = \frac{-s-s\sqrt{\frac{33d-9as}{9d-as}}}{-2}$,

and the following inequalities hold $0 \leq \frac{33d-9as}{9d-as} \leq 1 \quad \Leftrightarrow \quad \frac{9a}{33} \leq \frac{d}{s} \leq \frac{a}{3}$

There are two solutions to the quadratic equation, but the one with least divesture, Δ_1^{Req}, should be a focus of the analysis. Both solutions lead to the same result (keep the price unchanged) and the agency should choose the one with minimum possible intervention into the market.

The results show that a possibility of divesture of assets from the merging firms extends the range of parameters to satisfy consumer surplus standard from $\frac{a}{3} \leq \frac{d}{s}$ (case of a merger without divesture) to $\frac{9a}{33} \leq \frac{d}{s}$. Therefore, on the interval $\frac{9a}{33} \leq \frac{d}{s} \leq \frac{a}{3}$ there is such divesture Δ between 0 and $\frac{s}{2}$ that would keep the price after the merger unchanged. If $\frac{9}{33}a = \frac{d}{s}$, then $\Delta^{Req} = \frac{s}{2}$, i.e. the merged firm should divest exactly half of what it obtained through the merger and, therefore, two firms in the market would possess equal amount of fixed capital (the fact that we discussed above).

Since the agency's prime objective is to protect consumers from unjust price increases after the merger due to increased market power and the agency doesn't pay attention to firms' profits (or absence of it), the agency should choose such divesture that keeps price unchanged rather than the one that minimizes price[12].

3.3.1 Efficiencies

At the same time there could be efficiency gains due to merger specific synergies between merging firms' assets. In our setting it is equivalent to the decrease in parameter d (from d to αd, where $\alpha \in [0, 1]$) for the merged firm. If divested assets (Δ) go to a competitor, then the merged firm possesses $(2s - \Delta)$, while the competitor has $(s + \Delta)$ assets. At the same time it is assumed that the merging firms are able to achieve efficiencies even if some

[12]The merger with required divesture should be profitable for the merging firms as we discussed before.

of its combined assets are divested. Equilibrium output and price are derived from the system of two FOCs for the merged firm (M) and the outsider (o):

$$\begin{cases} a - 2bx_M - bx_o = \frac{\alpha d}{2s-\Delta} \\ a - 2bx_o - bx_M = \frac{d}{s+\Delta} \end{cases} \quad \Leftrightarrow \quad \begin{cases} x_M = \frac{1}{3b}[a - \frac{2\alpha d}{2s-\Delta} + \frac{d}{s+\Delta}] \\ x_o = \frac{1}{3b}[a + \frac{\alpha d}{2s-\Delta} - \frac{2d}{s+\Delta}] \end{cases}$$

Therefore, total output is $X^{A1}_{total}(\alpha) = \frac{1}{3b}[2a - \frac{\alpha d}{(2s-\Delta)} - \frac{d}{(s+\Delta)}]$.

Consequently, the equilibrium price will be $P^{A1}(\alpha) = \frac{1}{3}[a + \frac{\alpha d}{(2s-\Delta)} + \frac{d}{(s+\Delta)}]$.

Proposition 2: Price after the merger will not increase, $P^{A1}(\alpha) \leq P^B$, if $\alpha \leq (2s - \Delta)[\frac{1}{4}(\frac{9}{s} - \frac{a}{d}) - \frac{1}{s+\Delta}]$.

Function $\Delta(\alpha)$ could be plotted for given values of parameters a and d (see Graph 1). The line captures a trade-off between efficiency gains due to the merger and divested assets the agency asks the merged firm to sell-off to the competitor in order to keep prices unchanged after the merger[13]. This trade-off plays an important role in merger regulation after both the EU and USA agencies recently allowed for the efficiency defense in merger approvals. It illustrates that the amount of divested assets depends on verified efficiencies merging firms are able to bring in front of the agency.

The inclusion of exogenously given "synergy parameter" α implies that a cost function is not convex in s anymore. It allows for a shift in marginal costs function. Although the problem with such types of changes in parameters is that they are exogenously given.

3.4 Merger with divesture to a new entrant

The agency can enforce divesture of (Δ) assets to a new entrant into the market. Therefore, the merging firms possess $(2s - \Delta)$ assets, old competitor has (s) assets, while their new competitor has (Δ) assets. Equilibrium output

[13] As we discussed above in a Cournot type of competition the maximum output and the lowest price are achieved when market structure is symmetric, i.e. firms have equal marginal costs. In the presence of efficiencies α it means that this best outcome is achieved when $\frac{\alpha d}{2s-\Delta} = \frac{d}{s+\Delta} \quad \Leftrightarrow \quad \Delta = s\frac{2-\alpha}{1+\alpha}$

and price are derived from the system of three FOCs for the merging firms (M), a firm-outsider (o), and a new entrant to the market (N):

$$\begin{cases} a-2bx_M-bx_o-bx_N=\frac{d}{2s-\Delta} \\ a-2bx_o-bx_M-bx_N=\frac{d}{s} \\ a-2bx_N-bx_M-bx_o=\frac{d}{\Delta} \end{cases} \Leftrightarrow \begin{cases} x_M=\frac{1}{4b}[a+\frac{d}{\Delta}-\frac{3d}{2s-\Delta}+\frac{d}{s})] \\ x_o=\frac{1}{4b}[a+\frac{d}{\Delta}+\frac{d}{2s-\Delta}-\frac{3d}{s})] \\ x_N=\frac{1}{4b}[a-\frac{3d}{\Delta}+\frac{d}{2s-\Delta}+\frac{d}{s})] \end{cases}$$

Therefore, total output is $X^{A2}_{total}=\frac{1}{4b}[3a-\frac{d}{\Delta}-\frac{d}{2s-\Delta}-\frac{d}{s})]$.

Consequently, the equilibrium price is $P^{A2}=\frac{1}{4}[a+\frac{d}{(2s-\Delta)}+\frac{d}{s}+\frac{d}{\Delta}]$

Proposition 3: If the pre-merger market is characterized by symmetric cost structure ($s_1=s_2=s_3$ and $d_1=d_2=d_3=d$) and a new entrant possesses the same technology as all other firms ($d_N=d$), then there is no such divesture Δ to a new entrant that would decrease price after the merger.

Proof: Price after the merger with a divesture to a new entrant, P^{A2}, is always greater than price before the merger, P^B:

$P^{A2}=\frac{1}{4}[a+\frac{d}{(2s-\Delta)}+\frac{d}{s}+\frac{d}{\Delta}]\geq\frac{1}{4}[a+3\frac{d}{s}]=P^B$

$\Leftrightarrow \quad \frac{1}{(2s-\Delta)}+\frac{1}{\Delta}\geq\frac{2}{s} \quad \Leftrightarrow \quad s^2\geq(2s-\Delta)\Delta \quad \Leftrightarrow \quad (s-\Delta)^2\geq 0.$

The price will never decrease due to the form of the marginal costs function ($\frac{d}{s}$). Under the symmetric cost structure and a Cournot type of competition the maximum output and lowest price are achieved when all firms possess equal amount of fixed capital (which is the case for the pre-merger situation). After the merger the number of players stays the same (one firm is eliminated through the merger but a new one is formed) but cost structure becomes non-symmetric[14].

Remark 2: It is possible to show that this proposition is valid for any convex cost function with respect to fixed capital ($C_s<0,\ C_{ss}>0$).

From Sections 3.3 and 3.4 it is seen that if the pre-merger market is characterized by symmetric cost structure, then the agency would never approve

[14]There is an increase in market concentration. The market becomes less symmetric and the Herfindahl index goes up: the merged firm is bigger than Firm 1 before the merger ($2s-\Delta>s$) and a new entrant is smaller than Firm 2 ($\Delta<s$).

a merger with a divesture of assets to a new entrant. Therefore, there is no need to auction the divested assets because an existing competitor is the only potential purchaser of the assets that could be approved by the agency. However, if a market is characterized by a non-symmetric cost structure the auction can lead to different outcomes.

4 Non-symmetric case

First let's consider a three-firm industry with the identical technology parameters for the marginal costs function $d = d_1 = d_2 = d_3$ but not equal amount of fixed capital $s_1,\ s_2,\ s_3$, i.e. each firm faces identical marginal costs function but possesses different amount of fixed capital.

Proposition 4: If firms in the market differs only in amount of fixed capital they possess then a merger between any two firms with divesture to a new entrant leads to a price increase.
Proof: This proposition follows from the convexity of the costs function. Assume the price after the merger is lower than before the merger:
$P^{Before} = \frac{1}{4}[a + \frac{d}{s_1} + \frac{d}{s_2} + \frac{d}{s_3}] > \frac{1}{4}[a + \frac{d}{(s_1+s_2-\Delta)} + \frac{d}{s_3} + \frac{d}{\Delta}] = P^{NonSym}$
$\frac{d}{s_1}+\frac{d}{s_2} > \frac{d}{s_1+s_2-\Delta}+\frac{d}{\Delta} \Leftrightarrow \frac{1}{s_1}+\frac{1}{s_2} > \frac{1}{s_1+s_2-\Delta}+\frac{1}{\Delta} \Leftrightarrow s_1 s_2 < (s_1+s_2)\Delta-\Delta^2$
Inequality holds if $\Delta \in (s_2, s_1)$ assuming $s_2 < s_1$, which requires to divest more than was acquired. It is impossible.

***Remark 3*:** It is possible to show that this proposition is valid for any convex cost function with respect to fixed capital.

Conclusion: Similarly to the symmetric case the agency will never approve divesture of assets to a new entrant. For certain values of parameters it can approve a sale of the divested assets to the existing competitor, or approve the merger without any divesture, or reject the merger at all. Therefore, this non-symmetric case does not provide us with any new insights on the divesture problem.

Consider a three-firm industry with equal amount of assets across firms $s_1 = s_2 = s_3 = s$ but with different technology parameters $d_1,\ d_2,\ d_3$. Let's assume that firm 1 and 2 are merging and the merged firm would produce at marginal costs which are the lowest among the two merging firms[15]. Without loss of generality we can assume that $d_1 < d_2$.

Then there are 2 cases:
a) A merger between two firms with divesture to the existing competitor. Hence, the equilibrium price in the non-symmetric case with the divesture to an existing competitor (P^{EC}) is $P^{EC} = \frac{1}{3}[a + \frac{d_1}{2s-\Delta} + \frac{d_3}{s+\Delta}]$.

b) A merger between two firms with divesture to a new entrant. The marginal costs of production for a new entrant are characterized by a parameter d_N. This parameter determines viability of a new entrant, which is a prime concern for the EU Competition Commission while deciding on the divesture of assets[16]. Hence, the equilibrium price in the non-symmetric case with the divesture to a new entrant (P^{NE}) is $P^{NE} = \frac{1}{4}[a + \frac{d_1}{2s-\Delta} + \frac{d_3}{s} + \frac{d_N}{\Delta}]$.

In merger approval decisions the agency and the merging firms negotiate required amount of divested assets Δ and then can decide to auction it. In principle an auction seems a viable mechanism to sell divested assets and often parties in interest opt for it[17]. At the auction either an existing competitor or a new entrant purchases the divested assets. A winner of the auction determines a market structure and, consequently, prices. The agency approves a purchaser only if the price will not increase. At the same time a merger and, consequently, a divesture are possible only if the merger is beneficial for the merging firms itself (firm 1 and 2), i.e. if the merging firms expect higher joint profit after the merger than the sum of profits before the merger when they are separate firms: $\Pi_1^{Before}(s) + \Pi_2^{Before}(s) < \Pi_{Merged}^{After}(2s-\Delta)$. It is worth noticing that divesture is a plausible instrument only if exogenous parameters are such that without any agency's intervention

[15] In the literature it is known as a rationalization of production, i.e. a shift of output to the facility with lower marginal cost (see Shapiro and Farrell 1990).

[16] Exogenous parameters should satisfy a condition: $x_N = \frac{1}{4b}(a - 3\frac{d_N}{\Delta} + \frac{1}{2-\Delta} + d_3) > 0$

[17] An alternative to the auction is a direct sale of assets.

price will increase after the merger (like in the Remark 1, Section 3.2), i.e. only if $P^{Before} < P^{After}(without\ divesture)$.[18]

The analysis proceeds in the following way. First, given exogenous parameters and values of Δ between 0 and s we check who has a higher expected profit from the purchase of the divested assets: an existing competitor or a new entrant. Then, given the amount of divested assets and the winner at the auction we check whether the merger is profitable for the merging firms. Finally, we answer a question what would happened to the price. The agency approves the divesture only if it will not increase the price.

In Section 4.1-4.3 we formally introduce the conditions mentioned above and then (Section 4.4) proceed with the analysis of possible auction outcomes.

4.1 Incentives for an existing competitor and a new entrant to purchase the divested assets

The existing competitor (firm 3) compares profits when it purchases divested assets (Δ) and then operates in duopoly market with the situation when it stays away from the purchase while a new entrant buys the assets. Profit of the existing competitor if it purchases divested assets and operates in duopoly market is $\Pi_3^{buys} = \frac{1}{(3)^2 b}[a - 2\frac{d_3}{s+\Delta} + \frac{d_1}{2s-\Delta}]^2$, while profit of the existing competitor if it stays away from the purchase while a new entrant buys the assets is $\Pi_3^{away} = \frac{1}{(4)^2 b}[a - 3\frac{d_3}{s} + \frac{d_1}{2s-\Delta} + \frac{d_N}{\Delta}]^2$. If a new entrant purchases divested assets then its profit is $\Pi_N = \frac{1}{(4)^2 b}[a - 3\frac{d_N}{\Delta} + \frac{d_1}{2s-\Delta} + \frac{d_3}{s}]^2$. Therefore, if the inequality $\Pi_3^{buys} - \Pi_3^{away} > \Pi_N$ holds, then the existing competitor bids a higher price than a new entrant, because its expected profit is higher. Here and later on in the paper without loss of generality we can assume that $s = 1$ and $d_1 = 1$ to simplify the further calculations. With this the above condition becomes:

$$\frac{16}{9}[a - 2\frac{d_3}{1+\Delta} + \frac{1}{2-\Delta}]^2 - [a - 3d_3 + \frac{1}{2-\Delta} + \frac{d_N}{\Delta}]^2 > [a - 3\frac{d_N}{\Delta} + \frac{1}{2-\Delta} + d_3]^2$$

[18] Further in the paper we will consider only such values of exogenous parameters, i.e. $P^{Before} = \frac{1}{4}[a + \frac{d_1}{s} + \frac{d_2}{s} + \frac{d_3}{s}] < \frac{1}{3}[a + \frac{d_1}{2s} + \frac{d_3}{s}] = P^{After}(without\ divesture)$.

There are 4 exogenous parameters in the model ($a,\ d_2,\ d_3,\ d_N$), therefore it is difficult to derive explicitly conditions for the inequality to hold. However, given certain values of exogenous parameters, it is seen (Graph 0) that by changing values of Δ the divested assets could be purchased either by a new entrant or by the existing competitor.

4.2 Incentives for the merging firms

The merging firms (firm 1 and 2) proceed with the merger if expected joint profit after the merger is higher than the sum of profits before the merger when they are separate firms: $\Pi_1^{Before}(s) + \Pi_2^{Before}(s) < \Pi_{Merger}^{After}(2s - \Delta)$. Depending on who wins the auction the condition above becomes:

a) if the existing competitor purchases the assets (given $s = 1$ and $d_1 = 1$)

$\frac{1}{16b}[a - 3 + d_2 + d_3]^2 + \frac{1}{16b}[a + 1 - 3d_2 + d_3]^2 < \frac{1}{9b}[a - 2\frac{1}{2-\Delta} + \frac{d_3}{1+\Delta}]^2$

b) if a new entrant purchases the assets

$\frac{1}{16b}[a - 3 + d_2 + d_3]^2 + \frac{1}{16b}[a + 1 - 3d_2 + d_3]^2 < \frac{1}{16b}[a - 3\frac{1}{2-\Delta} + \frac{d_N}{\Delta} + d_3]^2$

4.3 Price change after a divesture

a) If the divested assets go to the existing competitor, then the price will not increase after the merger if

$P^{Before} = \frac{1}{4}[a + \frac{d_1}{s} + \frac{d_2}{s} + \frac{d_3}{s}] \geq \frac{1}{3}[a + \frac{d_1}{2s-\Delta} + \frac{d_3}{s+\Delta}] = P^{EC}$

Given the assumptions $s = 1$ and $d_1 = 1$, we obtain the following inequality: $3(1 + d_2 + d_3) \geq a + 4\frac{1}{2-\Delta} + 4\frac{d_3}{1+\Delta}$

b) If the divested assets go to a new entrant then the price will not increase after the merger if

$P^{Before} = \frac{1}{4}[a + \frac{d_1}{s} + \frac{d_2}{s} + \frac{d_3}{s}] \geq \frac{1}{4}[a + \frac{d_1}{2s-\Delta} + \frac{d_3}{s} + \frac{d_N}{\Delta}] = P^{NE}$

Given the assumptions $s = 1$ and $d_1 = 1$, we obtain: $1 + d_2 \geq \frac{1}{2-\Delta} + \frac{d_N}{\Delta}$

4.4 Possible divesture auction outcomes

In this section we investigate some outcomes of the auction of divested assets and possible ways to enhance consumer welfare. It is difficult to solve analytically the system of all inequalities above because there are 6 exogenous parameters $a,\ d_1,\ d_2,\ d_3,\ d_N,\ s$. We know that $d_1 < d_2$ and without loss of generality we can assume that $s = 1$ and $d_1 = 1$, and conditions $a > \frac{d_i}{s_i}$ and $x_i = (a - n\frac{d_i}{s_i} + \sum_{j \neq i}^{n} \frac{d_j}{s_j}) > 0$ must hold. However, we can conduct a numerical analysis and look at some possible outcomes of structural remedies.

The crucial parameters in the model are firms' marginal costs. The table below reflects technology parameter d_i of marginal costs of each firm relatively to all others. In columns there is a ranking of two merging firms parameters (d_1 and d_2) relatively to the outsider to the merger (d_3). In rows there is a ranking of a new entrant's parameter (d_N) relatively to the three pre-merger firms[19].

	Merging firms d_1 and d_2 wrt d_3		
New entrant d_N	Two lowest	Two highest	Lowest and Highest
Lowest	Case 1	Case 2	Case 3
b/w first and second	Case 4	Case 5	Case 6
b/w second and third	Case 7	Case 8	Case 9
Highest	Case 10	Case 11	Case 12

A numerical analysis is conducted by using MATLAB software (see a sample of the code in the Appendix). Without loss of generality it is assumed that $a = 4$ (a is a parameter of the inverse demand function). We check for different types of equilibria for $\Delta \in (0; 1)$ (grid is 100) and parameters $d_2, d_3, d_N \in (0; 4)$ (grid is 100). Given inequalities in sections 4.1-4.3 and conditions in footnotes 8 and 17, the following results are obtained.

[19]There are 4 parameters (d_1, d_2, d_3, d_N) and $(4!) = 24$ combinations when the order matters. Assuming $d_1 < d_2$ there are only 12 possible cases left.

If Δ is small and/or d_N is high then the condition $x_N = \frac{1}{4b}(a - 3\frac{d_N}{\Delta} + \frac{1}{2-\Delta} + d_3) > 0$ does not hold, i.e. a new entrant cannot have a positive output level. The viability of a new entrant that is captured by the ratio $\frac{d_N}{\Delta}$ is crucial for the firm's competitiveness. It is seen that the higher d_3, marginal costs of the existing competitor, and the more efficient a new entrant relatively to the merging firms, which efficiency is normalized to one, the more likely the condition to hold. If the condition does not hold, then the existing competitor would always win the auction whenever it is profitable for him[20] and for the merging firms (see section 4.2), otherwise the merger would not happened. Depending on the exogenous parameters the price will increase or decrease, and hence the merger be rejected or approved by the agency, respectively.

There are parameters when a new entrant wins the auction, the price decreases, and it is profitable for the merging firms to proceed with the merger and divesture. From the numerical analysis we can say that such situation can emerge in Cases 1, 2, 3 (most frequently it appears in Case 1, for example: $a = 4,\ d_1 = 1,\ d_2 = 1.33,\ d_3 = 1.77,\ d_N = 0.44,\ \Delta = 0.36$). In all these cases a new entrant is the most efficient firm in the market (d_N is the lowest among all firms). Although in many industries it is difficult to imagine that a firm, which is a newcomer to a market, could possess the most advanced technology but there are cases where it could be true. There is a tough competition to operate flights from the Heathrow Airport in London. Landing slots are the necessary assets to do business. Incumbents (British Airways, United, and others) do not allow other airlines to buy or lease landing slots at the airport in order to keep competitors away from a lucrative Trans-Atlantic flights business. However, it is possible that a low-cost carrier enters the market[21] by buying divested assets and it is the most efficient player (efficient enough to reduce the price in the market). Probably a new efficient entrant needs not much assets to start profitable business: if a new entrant is efficient, d_N is small, then divesture Δ could be small. At the same time 'little' divesture keeps the merger profitable for the merging firms. In this case the results of the auction is beneficial for all parties involved:

[20]Probability to have it profitable for Firm 3 increases with d_3 and decreases with d_2: $\Pi_3^{After} = \frac{1}{9b}(a - 2\frac{d_3}{1+\Delta} + \frac{1}{2-\Delta})^2 > \frac{1}{16b}(a + 1 + d_2 - 3d_3)^2 = \Pi_3^{Before}$.

[21]Here the definition of the market is flights from Heathrow.

consumers, merging firms, a new entrant, and the existing competitor (it is more profitable for Firm 3 to stay away from the purchase of assets at the auction).

Another situation is when a new entrant wins the auction and the price decreases, but this new market structure is unprofitable for the merging firms. It is possible in Cases 1, 2, 3, and 5 (most frequently it appears in Case 1, for example: $a = 4,\ d_1 = 1,\ d_2 = 1.2,\ d_3 = 1.3,\ d_N = 0.5, \Delta \in [0.80; 1.00]$). In this situation we can use the same example of the Heathrow airport as above with the only difference that the merging firms would not allow the divesture to a new entrant to happen. Appearance of a new more efficient competitor will decrease their future profits. Therefore the merging firms prefer to abandon the merger.

It is possible that if a new entrant wins the auction, the price would have decreased and it would have been profitable for the merging firms, however the existing competitor bids higher price for the assets and a new market structure either leads to higher prices or makes the merger unprofitable for the merging firms.
The situation when it leads to higher prices can emerge in Cases 1, 2, 3, 5, and 8 (most frequently it appears in Case 2, for example: $a = 4, d_1 = 1, d_2 = 1.2,\ d_3 = 0.9,\ d_N = 0.8,\ \Delta = 0.6$), i.e. when either a new entrant is the most efficient firm or the merging firms are the least efficient firms in the pre-merger market (Firm 1 and 2 have the highest marginal costs). The existing competitor does not allow the merger to happen simply by overbidding a new entrant and causing the price to increase. As a result the agency rejects the purchaser of divested assets and the merger, the merging firms have to abandon the merger and forgo expected profits in the future, and consumers have to stay with the pre-merger price, which could have decreased due to the merger.
The situation when divesture to the existing competitor makes the merger unprofitable can emerge in Cases 1, 3, 4, 6, 8, 9 (most frequently it appears in Case 4, for example: $a = 4, d_1 = 1, d_2 = 1.47,\ d_3 = 1.68,\ d_N = 1.26,\ \Delta = 0.79$). As a result the merging firms have to abandon the merger and forgo expected profits in the future, and consumers have to stay with the pre-

merger price, which could have decreased due to the merger with divesture to a new entrant.
In both cases the agency (and consumers) would be better off if a new entrant wins the auction. Therefore, by excluding the existing competitor (incumbent) from the auction the agency can enhance consumer and merging firms welfare. Furthermore, this policy is easy to implement.

Whenever a new entrant has the highest marginal costs among all firms (Case 10, 11, 12) and the assets are divested to him, the price will always increase and the agency will reject the divesture and, consequently, the merger. The intuition is that given the number of firms in the market stays the same (three) and convexity of the marginal costs function it is unreasonable to divest assets from 'efficient' to the least efficient firm because in the model the market price depends on the sum of marginal costs across all firms.

Under wide range of parameters the existing competitor wins the auction, the price decreases, and such market structure is profitable for the merging firms to proceed with the merger. However, the price could also increase in some cases or a new market structure could be unprofitable for the merging firms. These results are similar to the ones discussed in the symmetric case.

5 Conclusion

The presented model introduces a simple theoretical framework to analyze structural remedies in merger regulation. It captures all main issues that are at stake in merger approval decisions: efficiency defense and consumer welfare, amount of divesture and auction design, viability of a new entrant and rationalization of output between merging firms.

Under the current merger guidelines the merging firms can sell divested assets through an auction, while a purchaser of the assets must be approved by the agency. Evidently the merging firms choose a purchaser, which is the most profitable for them: based on revenue from the auction and future profit from a new market structure. The agency only checks whether the price will

decrease or increase after the purchase and, respectively, approves or rejects a purchaser and the merger.

The analysis of the symmetric case shows that divesture allows to extend the range of parameters when a merger should be approved. While the nonsymmetric case shows the importance of the rationalization of production between the merging firms and the viability of a new entrant.

From the results of the numerical analysis we can suggest to make the agency more active in the selection of a potential purchaser of divested assets. The agency can write down in the merger guidelines that first they want to look for a new entrant (a viable one) and only if the one is not found then to consider existing competitors. For some parameters the agency is better-off to exclude the existing competitor (incumbent) from the auction. As it was shown in the paper it could lead to a more favorable outcome for consumers and merging firms.

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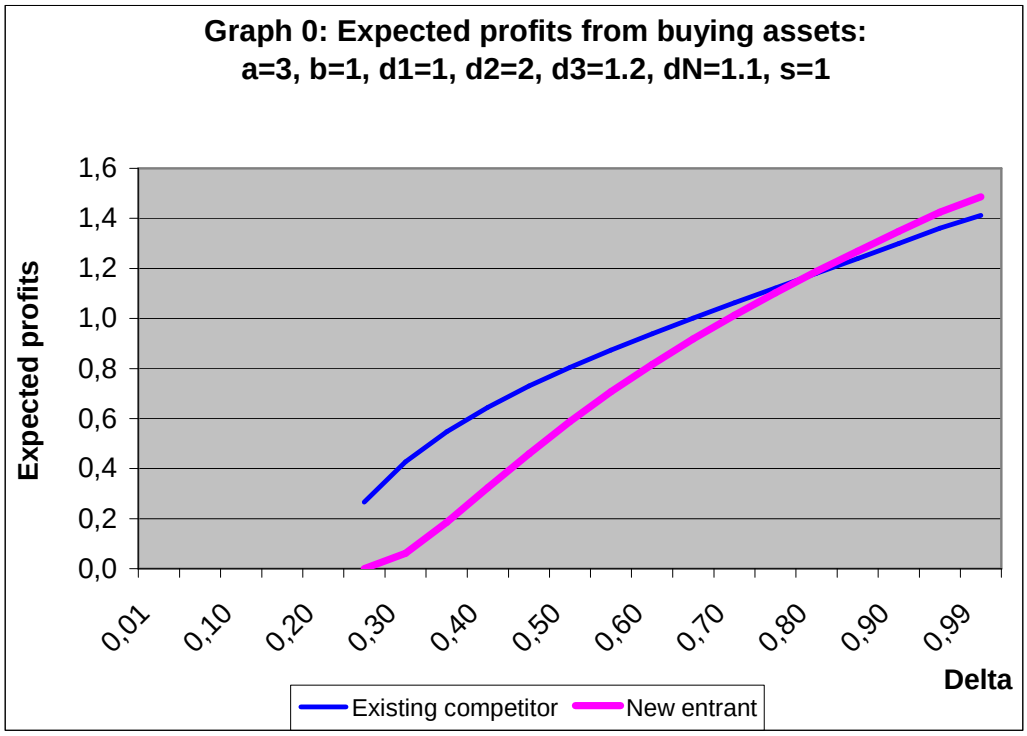
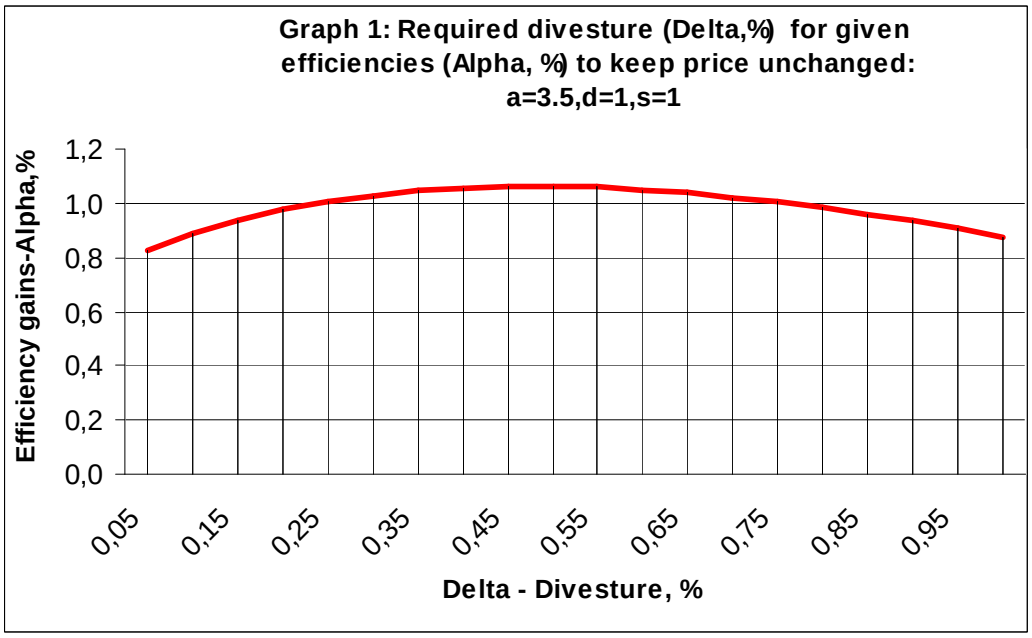

Graph 0: Expected profits from buying assets:
a=3, b=1, d1=1, d2=2, d3=1.2, dN=1.1, s=1
Expected profits
1,6
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0,0
0,01
0,10
0,20
0,30
0,40
0,50
0,60
0,70
0,80
0,90
0,99
Delta
Existing competitor
New entrant


Graph 1: Required divesture (Delta,%) for given
efficiencies (Alpha, %) to keep price unchanged:
a=3.5,d=1,s=1
Efficiency gains-Alpha,%
1,2
1,0
0,8
0,6
0,4
0,2
0,0
0,05
0,15
0,25
0,35
0,45
0,55
0,65
0,75
0,85
0,95
Delta - Divesture, %

MATLAB code for the situation when an existing competitor wins the auction but the price will increase. However, if a new entrant would have won the auction, the price would decrease and it would be profitable for the merging firms

```
clear; tic;
a=4;
N=100;
K=100;
delta=linspace(.0001,1,N);
d2=linspace(.0001,4,K);
d3=linspace(.0001,4,K);
dN=linspace(.0001,4,K);

X1=0;  X2=0; X3=0; X4=0; X5=0; X6=0;
X7=0;  X8=0; X9=0; X10=0; X11=0; X12=0;

for i1=1:N
  for i2=1:K
   for i3=1:K
    for i4=1:K

          deltai=delta(i1);
          d2i=d2(i2);
          d3i=d3(i3);
          dNi=dN(i4);

 I1=(a-1-3*d2i+d3i)>0;                          % without divesture the price will increase

 I211=(a-3+d2i+d3i)>0;                          % positive output for Firm 1 before
 I212=(a-3/(2-deltai)+d3i+dNi/deltai)>0;        % positive output for Firm 1 after with new entrant
 I213=(a-3/(2-deltai)+d3i/(1+deltai))>0;        % positive output for Firm 1 after with existing competitor
 I22=(a+1-3*d2i+d3i)>0;                         % positive output for Firm 2 before
 I231=(a+1+d2i-3*d3i)>0;                        % positive output for Firm 3 before
 I232=(a-3*d3i+1/(2-deltai)+dNi/deltai)>0;      % positive output for Firm 3 after with new entrant
 I233=(a-3*d3i/(1+deltai)+1/(2-deltai))>0;      % positive output for Firm 3 after with existing competitor
 I2N=(a-3*dNi/deltai+1/(2-deltai)+d3i)>0;       % positive output for the new entrant with Delta assets

 % Existing competitor bids higher price at an auction
 I3=((16/9)*(a-2*d3i/(1+deltai)+1/(2-deltai))^2-(a-3*d3i+1/(2-deltai)+dNi/deltai)^2-
(a-3*dNi/deltai+1/(2-deltai)+d3i)^2)>0;

 % it is profitable for the merging firms 1 and 2 if a new entrant buys assets
 I41=((a-3/(2-deltai)+d3i+dNi/deltai)^2-(a-3+d2i+d3i)^2-(a+1-3*d2i+d3i)^2)>0;

 % it is profitable for the merging firms 1 and 2 if an existing competitor (EC) buys assets
 I42=((16/9)*(a-2/(2-deltai)+d3i/(1+deltai))^2-(a-3+d2i+d3i)^2-(a+1-3*d2i+d3i)^2)>0;

 % price NOT increases if a new entrant (N) buys assets
 I51=(1+d2i-1/(2-deltai)-dNi/deltai)>0;

 % price INCREASES if an existing competitor (EC) buys assets
 I52=(a-3+4/(2-deltai)-3*d2i-3*d3i+4*d3i/(1+deltai))>0;


 I=I1*I211*I212*I213*I22*I231*I232*I233*I2N*I3*I41*I42*I51*I52;
 % if all conditions hold then it is 1, if at least one doesn't then 0
```

```
   Case1=(dNi<1)&(1<d2i)&(d2i<d3i);          % All possible ordered combinations d1,d2,d3,dN
   Case2=(dNi<d3i)&(d3i<1)&(1<d2i);          % given that d2>d1=1
   Case3=(dNi<1)&(1<d3i)&(d3i<d2i);
   Case4=(1<dNi)&(dNi<d2i)&(d2i<d3i);        % If the conditions are satisfied then 1, otherwise 0
   Case5=(d3i<dNi)&(dNi<1)&(1<d2i);
   Case6=(1<dNi)&(dNi<d3i)&(d3i<d2i);
   Case7=(1<d2i)&(d2i<dNi)&(dNi<d3i);
   Case8=(d3i<1)&(1<dNi)&(dNi<d2i);
   Case9=(1<d3i)&(d3i<dNi)&(dNi<d2i);
   Case10=(1<d2i)&(d2i<d3i)&(d3i<dNi);
   Case11=(d3i<1)&(1<d2i)&(d2i<dNi);
   Case12=(1<d3i)&(d3i<d2i)&(d2i<dNi);

if Case1*I==1
   X1=X1+1;              % count number of situations that fall in Case 1 and satisfy inequalities in I
elseif Case2*I==1
   X2=X2+1;
elseif Case3*I==1
   X3=X3+1;
elseif Case4*I==1
   X4=X4+1;
elseif Case5*I==1
   X5=X5+1;
elseif Case6*I==1
   X6=X6+1;
elseif Case7*I==1
   X7=X7+1;
elseif Case8*I==1
   X8=X8+1;
elseif Case9*I==1
   X9=X9+1;
elseif Case10*I==1
   X10=X10+1;
elseif Case11*I==1
   X11=X11+1;
elseif Case12*I==1
   X12=X12+1;
end

end
end
end
end
toc;

X1      % Display total number of situations that satisfied Case 1 and all specified inequalities
X2      % matters whether it is 0 or not
X3
X4
X5
X6
X7
X8
X9
X10
X11
X12
```