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COGNITIVE PROCEDURES AND HYPERBOLIC DISCOUNTING

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Cognitive Procedures and Hyperbolic Discounting[†]

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Abstract

"Hyperbolic discount functions are characterized by a relatively high discount rate over short horizons and a relatively low discount rate over long horizons" (Laibson 1997). We suggest two cognitive procedures where individuals perceive future utility as decreasing at a decreasing rate as a function of time. Such a perception is similar to hyperbolic discounting. The first procedure shows that individuals hyperbolically discount marginal utility from money when they follow a cognitive procedure in which they believe that their wealth might increase or decrease in each future period under the constraint of a perceived small probability that wealth will decrease below its current level. The second procedure shows that individuals hyperbolically discount expected utility from consumption when they believe that they will rationalize their actions and thus alter their utility function over time. The difference in how perceived utility changes over the short and long horizon generates the hyperbolic discounting phenomenon. We find that greater tendencies toward rationalization and greater volatility in consumption increase the hyperbolic discounting phenomenon. Although hyperbolic discounting is usually regarded as impulsive and irrational, Azfar (1999) and this author suggest that hyperbolic discounting may be rational in some cases.

JEL classification: D90, D91

Keywords: Hyperbolic discounting, present-biased preferences, cognitive procedure, cognitive dissonance, rationalization, inconsistency of preferences, marginal utility.

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1 Introduction

A consistent psychological finding on individual time preferences is that discount functions are hyperbolic, suggesting that people are impatient at present, but claim to be patient in the future. Phelps and Pollak (1968) introduced hyperbolic discount functions into economic theory in the context of consumption and savings across generations. Laibson (1994) utilized hyperbolic discount functions in the context of intertemporal one-person decisions to study consumption and saving patterns[1]. This paper illustrates two cognitive procedures that would lead people to act as if they were hyperbolic discounters. One procedure leads to hyperbolic discounting of money and the other leads to hyperbolic discounting of consumption.

Two papers have offered explanations for hyperbolic discounting. Azfar (1999) showed that hyperbolic discounting could occur when agents were uncertain about their discount rate or hazard rate, as well as the probability of not receiving payment. Azfar explains the intuition:

> The apparent discount rate at time t depends on the true discount rate and the expected hazard rate at t. The hazard rate at date t is the weighted average of the initial hazard rates, where the weights are the probability of survival till t. These weights decline more rapidly for higher hazard rates, and thus as t rises, the hazard rate converges to the lower end of the initial distribution. (p. 247)

[1] Experimental evidence was first offered by Thaler (1981), who used hypothetical questions and found that the discount rate declines sharply with the length of time to be waited. Benzion, Rapoport, and Yagil (1989) replicated these findings for undergraduate and graduate students of economics and finance at two Israeli universities. Kirby and Herrnstein (1995) demonstrated the same results using both monetary and non-monetary real rewards. Kirby (1997) showed, using real rewards, that subjects are hyperbolic discounters when subjects were induced to convey their true value using second bid auctions.

Rubinstein (2000) claims that the experimental results which hyperbolic discounting is based upon are better explained by a decision-making procedure that is based on similarity relations than by hyperbolic discounting.

Impulsiveness might be seen as a negative and patience as a positive attribute. Kirby (1997) stated that "[a]fter all, behavior consistent with normative discounting should be our goal, even if it is not our norm" (p. 68). Kirby and Herrnstein (1995) wrote: "One may stably prefer a smaller benefit to a larger one that is more remote in time. But if preference reverses simply because of changes in one's temporal vantage point . . . then the violation of stationarity meets the ordinary criterion of impulsiveness" (p. 83). Contrary to these statements, Azfar (1999) and this author suggest that the hyperbolic discounting phenomenon might be rational. On the other hand, hyperbolic discounting may not need an explanation: People may simply be impulsive.

Experiments in hyperbolic discounting generally ask subjects whether they prefer a small, earlier *monetary* reward to a larger, later *monetary* reward. The first cognitive procedure presented here describes a situation where the perceived *marginal* utility from money is hyperbolic. The utility was marginal because the participants in the experiments were asked about their preferences over a sum of money that was marginal to their total wealth. The second cognitive procedure describes a situation where the perceived utility from a non-monetary reward — i.e., consumption of a particular good — is hyperbolic.

A crucial assumption for the procedure leading to hyperbolic discounting from money is that individuals *perceive* the probability that their wealth will decrease below its current level as relatively small. For simplicity, in the formal model we assume that probability to be zero. This assumption can be justified by loss-aversion and status-quo bias [see Rabin (1998) and Kahneman, Knetsch and Thaler (1992) for a detailed analysis of these phenomena]. The individual is extremely averse to his wealth decreasing below its current level. He can actually make sure that his wealth will not deteriorate by insuring himself against a loss in wealth. Alternatively, he might tell himself that if, in the future, his financial situation will be close to the current level, he will work hard to try to ensure that it does not deteriorate further. We believe that such an assumption is reasonable because the individual himself is the one who assigns the probabilities to what might happen to his wealth. A second assumption is that the individual understands that future wealth can fluctuate between periods due to life's changing circumstances: for example, stock market fluctuations, real price fluctuations, unexpected expenditures or windfalls.

Both assumptions imply that an individual perceives his future wealth as a random walk under the constraint that wealth will not fall below current level. Marginal utility is hyperbolically discounted when the individual perceives future added utility as decreasing at a decreasing rate over time. Hyperbolic marginal utility from money is thereby generated: In the near future, wealth can only increase, causing expected marginal utility from money to decrease. The probability that wealth will decrease in the far future causes the expected marginal utility from money to decrease by a lesser amount in the far than in the near future. This possible difference in the decrease of expected marginal utility in the far future generates hyperbolic marginal utility[2].

The second cognitive procedure leads to hyperbolic discounting from consumption of some particular good. We assume that there is an optimal level of

[2] This procedure and the procedure that follow are alternative explanations to the one provided by Azfar (1999). The explanations do not conflict. When both prevail, marginal utility will be more intensely hyperbolic.

consumption; hence, excess consumption is harmful. An individual's optimal consumption likewise follows a random walk. The individual knows this and contemplates the expected utility of future consumption from his perspective in the present. In the near future, an agent's utility has a positive probability to decrease but in the far future his utility might return to the present level. The difference in how expected utility decreases in the near and far future induces hyperbolic discounting. The assumption that consumption follows a random walk can be justified by assuming that: (1) Actual consumption is volatile around optimal consumption[3] and (2) If actual consumption does not equal optimal consumption, cognitive dissonance occurs; the individual then rationalizes his preferences in such a way that optimal consumption in the following period equals actual consumption in the present period.

The fact that people rationalize and convince themselves that their actions are good is captured by Leon Festinger's (1957) theory of cognitive dissonance. The theory states that humans have a powerful motive to maintain cognitive consistency[4]. Cognitive consistency is a state of mind in which one's beliefs, attitudes, and behavior are all compatible with each other (Abelson et al., 1968). Cognitive dissonance theory predicts

[3] This volatility can arise when there are costs for being rigid.

[4] There is much evidence for the validity of the cognitive dissonance theory. According to Gilad et al. (1987) more than 900 studies were conducted up to 1982. Two seminal experiments are those of Festinger and Carlsmith (1959), who showed that when people lie in situations where dissonance is aroused, individuals tend to believe their own lies, and of Freedman (1965), who showed that when children are mildly threatened not to act in a certain manner, they produce more behavior change than when they are severely threatened. The reason for this response is that a mild threat does not produce sufficient deterrence, causing behavior to be rationalized by a change in attitudes. An experiment that relates to consumption was conducted by Middlestaedt (1969), in which student volunteers were asked to rank 9 swimsuits. When given a choice between their third- and fourth-ranked choice or between their third- and fifth-ranked choice, the choice between the third and the fourth choice produced more dissonance than did a choice between the third and fifth choice because it required rejecting a more desirable alternative (that is, the fourth versus the fifth choice). The choice's rejection thus demanded some cognitive explanation. Afterwards, all volunteers were given the choice between their second and third choices. Those in the high dissonance group were more apt to choose their third choice over their second. The experiment showed that

that under certain circumstances, people will change their attitudes to be congruent with their behavior. A person maintaining two conflicting cognitions will experience cognitive dissonance, an unpleasant state of tension akin to hunger or thirst. He will then wish to change one of the cognitions in such a way that both will no longer conflict. When attitudes regarding a behavior dictate that the behavior should be different than the behavior already performed, a person might suffer from cognitive dissonance and alter his attitudes in such a way that they will coincide with his behavior. One implication of cognitive dissonance for economic theory is that an agent who does not act according to his utility function might alter his utility function to accord with his behavior. A theory of endogenous formation of preferences can thus be derived from cognitive dissonance theory. After reviewing the endogenous preferences literature, Bowles (1998) claimed that:

> "Markets and other economic institutions do more than allocate goods and services: they also influence the evolution of values, tastes, and personalities. Economists have long assumed otherwise; the axiom of exogenous preferences is as old as liberal political philosophy itself…[yet] most economists have not asked how we come to want and value the things we do" (p. 75).

In the second cognitive procedure, we assume that an individual's preferences vary over time because individuals may experience cognitive dissonance when their actions differ from their beliefs. Preferences in each period are thus derived from realizations of behavior in the previous period[5]. Individuals know that in the future their utility function

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consumers who experience cognitive dissonance after buying a product enhance the utility of the product bought.

[5] This paper follows several other economic papers incorporating the psychological phenomenon of cognitive dissonance into economic theory. This stream in the literature was initiated by Hirschman (1965), who used cognitive dissonance to describe attitude change regarding modernization. Akerlof and Dickens (1982) considered a formal model of the implications of cognitive dissonance on workers in hazardous industries. Specifically, such workers would suffer from cognitive dissonance and convince themselves that

might change; however, they evaluate future consumption based on current preferences. Peleg and Yaari (1973) noted that:

> "The whole question of preferences that change over time is, at the outset, rather troublesome…But changing of tastes…is a real phenomenon and we feel that it is worthy of examination, even at the cost of a certain amount of methodological deficiency."

One might argue that if individuals understand that their tastes are changing, they should evaluate future utility based on future preferences. We assume this not to be true. A person may acknowledge the fact that in the future he might become overweight and rationalize his actions. But in the present, he does not enjoy the fact that he might become overweight in the future. This assumption is paradoxical. On the one hand, we assume that people are sophisticated; they are aware of a possible cognitive dissonance reaction which will cause a change their behavior because of a change in preferences. On the other hand people evaluate future utility from the perspective of the present, not according to future utility.

the industry where they were working was safe. If safety equipment were to become available in the future, they would not purchase it. A Pareto inferior outcome would result whenever safety equipment was inexpensive relative to the cost of an accident. Dickens (1986) showed that increasing punishment could increase the crime rate. When punishment is mild, potential criminals who decided not to commit a crime rationalized their action by convincing themselves that there was no benefit from committing the crime. In the future, they would not commit a crime even when there was no punishment. When punishment is severe, no rationalization takes place because potential criminals have sufficient justification — the punishment — for not committing the crime. Gilad et al. (1987) described a model where cognitive dissonance generates an information filter that blocks the flow of information when a discrepancy appears between new and old information. Hence, not all information reached the decision-maker. Nagler's (1993) model on deceptive advertising assumed that individuals find it difficult acknowledging that they were deceived as a result of cognitive dissonance. Firms have an incentive to deceive because individuals might repeat purchasing from a deceiving firm due to this response. Rabin (1994) showed that increasing cognitive dissonance that arises from immoral behavior could increase that behavior. He assumed that conformists enjoy immoral behavior yet find it difficult to justify such behavior; hence, they suffer from cognitive dissonance because their immoral behavior differs from their beliefs. An increase in the level of cognitive dissonance causes immoral behavior to decrease but also causes the person to justify such behavior. Because everyone justifies such behavior, the behavior becomes more acceptable in society, ultimately causing individuals to increase that immoral behavior. Montgomery (1994) assumed that social norms require husbands to provide a minimum level of family support and that failure to do so generates cognitive dissonance. He showed that an increase in social norms towards greater family support might decrease the level of family support provided by low-income men.

This paper aims to elucidate the discount rate generated by the two cognitive procedures. Thus, we will assume that there is no initial discount rate per se. If such a discount rate did exist, the overall discount rate would be a function of the real discount rate and the discount rate generated by the cognitive procedures.

2 Hyperbolic Discounting from Monetary Rewards

The hyperbolic discounting theory states that an individual will prefer an SER (smaller earlier reward) to an LLR (larger later reward) in the near future, but an LLR to an SER in the far future. We demonstrate this phenomenon with a three-period example and then with an infinite horizon model.

2.1 A Three-Period Example

Consider a three-period example where period 0 stands for the present, period 1 for the near future, and period 2 for the far future. Assume the following preferences: In period 0, the individual's marginal utility from money is 10. His marginal utility diminishes by 1 for each $1 he gains[6]. The individual perceives future wealth as fluctuating between periods, but not falling below current wealth. Economic fluctuations cause volatility in wealth. Loss aversion and status quo bias can cause aversion to thinking that wealth might decrease below its current level.

[6] It follows that if the individual gains $11, his marginal utility will be negative, which is clearly unrealistic. Nevertheless, we model a linear diminishing marginal utility because we wish to show that a linear diminishing marginal utility turns hyperbolic and to focus on this point. Also, note that the number 10 is arbitrary. Had we assumed an arbitrarily large amount, the individual would have to gain a large amount for marginal utility to become negative.

The individual's perception about future wealth is specified by the following probabilities: In the first period, he might gain $1 with probability m. If he does not gain $1 now, he might gain it with the same probability in the next period. However, if he gains $1 in the first period, in the next period he might gain an additional $1 with probability a, lose $1 with probability c, and neither gain or lose with probability b, where $a ? b ? c ? 1$. Table 1 below describes the probabilities for enjoying various utilities from an additional $1 in the two subsequent time periods as perceived from period 0. For example, the probability that additional utility will be 9 in period 1 is m, and the probability that additional utility will be 8 in period 2 is ma. The bottom row summarizes the individual's VNM expected utility from an additional $1. For example, the expected utility in period 1 is $10 ? m$. Since $10 ? 10 ? m ? 10 ? m^2 ? 11m ? m(8a ? 9b ? 10c)$, expected utility from an additional $1 diminishes over time.

Marginal utility is hyperbolically discounted when the individual perceives future additional utility as decreasing at a decreasing rate as a function of time. Expected marginal utility from an additional $1 in period i is defined as Ed_i. The difference between expected additional utility in period i-1 and period i is defined as $?i$, thus $?i ? Ed_{i?1} ? Ed_i$. In order to show that the discounting is hyperbolic, we must show that the difference between expected additional utility in period i-1 and i decreases as i increases, that is, $?1 ? ?2 ? 0$[7].

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[7] The discount rate between the short and long horizon is represented by $?1$. It is usually referred to as $?$ in the hyperbolic discounting literature.

Table 1: The columns indicate the probabilities of enjoying various utilities from an additional $1 in each period. The bottom row indicates the expected additional utility from an additional $1 in each period.

Additional utility from $1	Period 0	Period 1	Period 2
10	1	$1 ? m$	$(1 ? m)^2 ? mc$
9	0	m	$(1 ? m)m ? mb$
8	0	0	ma
Expected additional utility from $1	10	$10 ? m$	$10 ? m^2 ? 11m$ $? m(8a ? 9b ? 10c)$

<u>RESULT 1</u>: *The expected marginal utility from money is hyperbolic when* $a ? m ? c ? 1 ? a$.

<u>Proof:</u> Substitute the probabilities from Table 1, $?1 ? m$ and $?2 ? m(1 ? a ? m ? c)$. The expected marginal utility from money is hyperbolic when $?1 ? ?2$ *and* $?1$ are positive and $?2$ is not negative. $?1$ is always positive. $?2$ is not negative when $m ? c ? 1 ? a$, while $?1 ? ?2$ is positive when $a ? m ? c$. QED

The intuition behind the condition $m ? c ? 1 ? a$ is that a must be large enough so that expected additional utility will not increase in period 2. The intuition behind $a ? m ? c$ is that a must be small enough so that expected additional utility will not decrease much, causing the decrease in expected additional utility between periods 1 and 2 to be smaller than between periods 0 and 1.

In order to demonstrate the inconsistency of preferences provided by the experimental results on hyperbolic discounting, we need to show that an individual prefers an SER to an LLR in the near future, but the converse in the far future. Recall that Table 1 shows the probabilities for enjoying various utilities from an additional $1 as a result of different levels of wealth. Table 2 shows the utilities from an addition of a bit more than $1 ($1.10) under the same conditions as in Table 1[8]. The only difference between the tables is that in Table 2, we increased additional utility by 0.5 for each level of wealth.

Table 2: The columns indicate the probabilities of enjoying various utilities from an additional $1.10 in each period. The bottom row indicates expected utility from the additional $1.10 in each period.

Additional utility from $1.10	Period 0	Period 1	Period 2
10.5	1	$1 ? m$	$(1 ? m)^2 ? mc$
9.5	0	m	$(1 ? m)m ? mb$
8.5	0	0	ma
Expected additional utility from $1.10	10.5	$10.5 ? m$	$10.5 ? m^2 ? 11m$ $? m(8.5a ? 9.5b ? 10.5c)$

[8] The values of $1 for the SER and $1.10 for the LLR are arbitrary. The results depend on the utilities attached to these values. These utilities are subjective and can, thus, differ across individuals.

RESULT 2: *Inconsistency of preferences will prevail (the individual will prefer the SER in the near future, but the LLR in the far future) when the probability of gaining the first $1 is sufficiently high,* $m ? 0.5$*, and the probability of losing $1 is sufficiently high,* $c ? a ? 0.5 ? \sqrt{2}$.

Proof: We need to show that $Ed(\$1)_0 ? Ed(\$1.1)_1$ and $Ed(\$1.1)_2 ? Ed(\$1)_1$. Substituting expected additional utilities from Tables 1 and 2, the first condition holds when $m ? 0.5$, and the second condition holds when $c ? a ? 0.5 ? \sqrt{2}$.   QED

The intuition behind this result is that the probability of gaining $1 in the first period (m) must be sufficiently high to decrease the expected additional utility of the LLR in period 1. In that case, the individual will prefer the SER in period 0 to the LLR in period 1, which is the near-future tradeoff. The probability of losing $1 in period 2, (c), must be sufficiently high so that the expected additional utility in period 2 will not be too small. Thus, the individual will prefer the LLR in period 2 to the SER in period 1, which is the far-future tradeoff.

2.2 Infinite Horizon Case

We now extend our example to a multi-period environment. We assume that the marginal utility from money in period t, d_t, decreases linearly as a function of wealth in period t, w_t[9]. Specifically,

$$d_t \; ? \; A \; ? \; w_t \qquad (1)$$

when A is a constant parameter.

Without loss of generality, we assume that the agent contemplates future utility from the perspective of period 0. Let $pr_0(w_t \; ? \; w)$ be the probability in period 0 that $w_t \; ? \; w$. We assume that $pr_0(w_t \; ? \; w)$ is derived by the following procedure: An individual is endowed with wealth in period 0. He can win, lose, or neither win nor lose $1 in each period under the constraint that when his wealth is equal to his wealth in period 0, he cannot lose $1; thus, he has a lower boundary for the amount of wealth he might hold. The expected marginal utility in period t as seen from the perspective of period 0 is:

$$E_0 d_t \; ? \; \underset{w?0}{\overset{?}{?}} (A \; ? \; w_t) pr_0(w_t \; ? \; w) \qquad (2)$$

In order to demonstrate that the expected marginal utility is hyperbolic, we now turn to a numerical analysis and compute equation (2) for each period using a Mathematica 3 computer program[10].

Figure 1 depicts the individual's expected marginal utility from money 400 periods into the future in a situation where A=15 and the probability to gain the first $1 is

[9] See footnote 6 for a note on linear decreasing utility from money.
[10] The computer programs are available from the author by request.

0.5. Whenever the individual's wealth is more than his initial wealth, he can win $1 with probability 0.25 or lose $1 with probability 0.25. Figure 1 shows that in such a situation, the expected marginal utility from money will hyperbolically decrease in time.

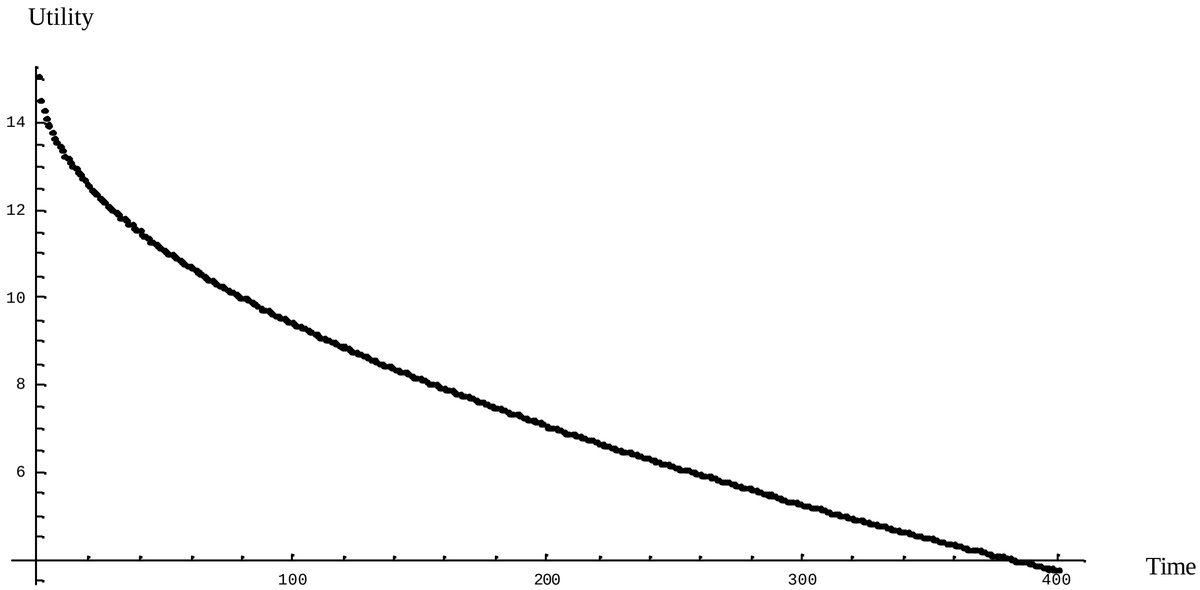

Figure 1. Expected utility from an additional $1 as a function of time. The probability of winning $1 when the individual’s wealth is equal to his wealth in period 0 is m=0.5, the probability of winning or losing $1 when the individual’s wealth is greater than his wealth in period 0 is a=c=0.25, and the probability of neither winning nor losing $1 when the individual has some initial wealth is b=0.5.

Figure 2 shows that individual's choices are inconsistent when the dynamic described in this section occurs. For each time period t, the curve marked LLR indicates expected utility from a larger sum of money to be received in period t+1, while the curve marked SER indicates expected utility from a smaller sum of money to be received in period t. The expected utility from a large reward is greater than the expected utility from a small reward in *any specific time period*[11]. But when the larger sum of money is given

[11] The levels of additional utilities in the program are arbitrary.

later than the smaller sum, the agent prefers the SER in the near future, and the LLR in the far future. This result is congruent with the experimental evidence.

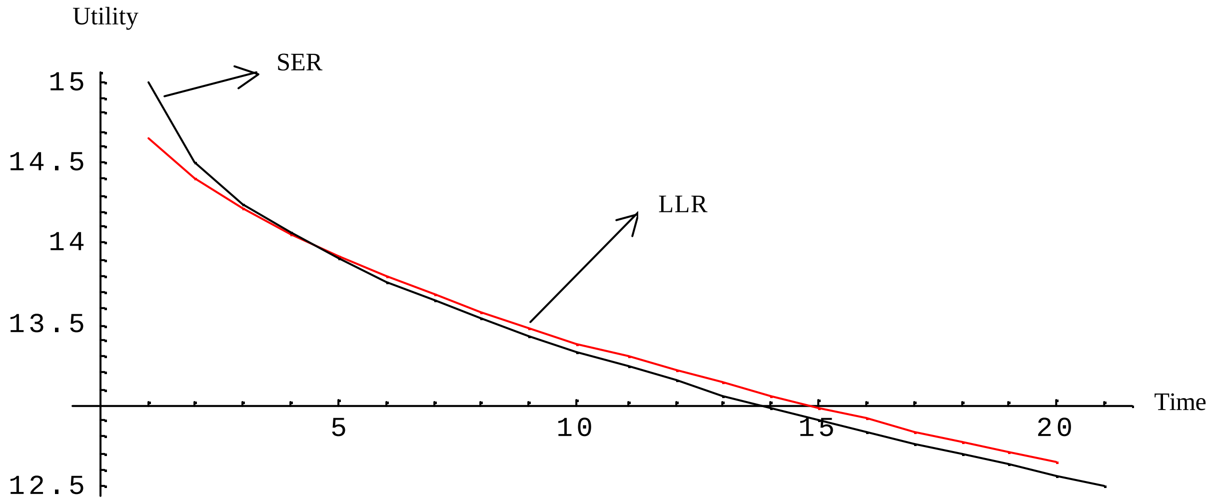

Figure 2. Expected utility from an additional $1.00 in period t (SER) and expected utility from an additional $1.10 in period t+1 (LLR). The probabilities for winning or loosing $1 in each period are the same as in Figure 1.

3 Hyperbolic Discounting from Non-Monetary Rewards

This section describes the second procedure where an individual perceives future expected utility as decreasing at a decreasing rate as a function of time and displays inconsistency of preferences. Consider an agent who contemplates his instantaneous utility from consumption of product x for each future time period from the perspective of the present. At every period there is an optimal consumption level, $x_t^{?}$. The utility function is:

$$u_t(x_t)\ ?\ A\ ?\left|x_t^{?}\ ?\ x_t\right| \qquad (3)$$

where x_t denotes actual consumption and the constant parameter is denoted by A.

Thus we assume that excessive consumption is harmful.

We assume that actual consumption is volatile and distributed around optimal consumption. For simplicity we assume that actual consumption, x_t, is distributed discretely around optimal consumption and is equal to:

$$x_t \; ? \; \begin{cases} x_t^? & with\ probability \quad r \\ x_t^? \; ? \; 1 & with\ probability \quad p \\ x_t^? \; ? \; 1 & with\ probability \quad q \end{cases} \qquad (4)$$

where r+p+q=1.

In each period, the agent chooses $x_t^?$ in order to maximize (3). We endogenize preferences by assuming that when $x_t \; ? \; x_t^?$, cognitive dissonance arises and the agent convinces himself that his actual consumption is his optimal consumption[12]. That is, the next period's utility function is:

$$u_{t?1}(x_{t?1}) \; ? \; A \; ? \left| x_{t?1}^? \; ? \; x_{t?1} \right| \qquad (5)$$

where $x_{t?1}^? \; ? \; x_t$

The agent at period t understands that his preferences might alter at period t+1. In period t+1, the process repeats itself[13]. Thus, from period t's perspective, the probability that optimal consumption will diminish in period t+2 by two units is q^2, the probability

[12] Cognitive dissonance might arise because individual behavior (actual consumption) deviates from individual preferences (optimal consumption). Cognitive dissonance theory [Festinger, 1957] claims that in such a situation, the individual will experience an unpleasant physiological response and adjust one of the cognitions in such a way that the cognitions do not conflict. It is difficult to change what one has already done, so we assume that the agent will change his preferences in such a way that they will reflect behavior. Hence, optimal consumption follows a random walk.

[13] In the program that generates the results we assumed that $x_t^?$ is large and thus the agent does not reach a level at which he cannot reduce consumption.

that it will increase by two units is p^2 and the probability that it will decrease by one unit is 2qr. The probability that it will increase by one unit is 2pr and the probability that it will not change is $r^2 ? 2pq$. In each period, the agent can alter his utility function with the same probabilities. In period t, the individual knows that in the future he might experience cognitive dissonance, which might alter his utility function and his behavior. But viewed from the present, future behavior might not be optimal in terms of the present utility function. The utility at period t from consuming x_z at any period $z ? t$ is:

$$u_t(x_z) ? A ? \left|x_t^? ? x_z\right| \qquad (6)$$

Without loss of generality, we will assume that the agent makes his decisions at the beginning of period 1, when period 1's optimal consumption is known but actual consumption is unknown. The probability at period 1 of consuming x in period t is $?_t(x_1^?, x)$. The agent begins by optimizing to consume $x_1^?$ but understands that he might consume much more or much less in the future because he is aware that he might rationalize his actions. The VNM expected utility function for consumption, x, at period t from the perspective of period 1 is:

$$u_t ? \overset{?}{\underset{x?0}{?}} ?_t(x_1^?, x) ?A ? \left|x_1^? ? x\right| ? \qquad (7)$$

We also assume that in each period, agents decide their optimal consumption according to (3) but contemplate their future expected utility according to (7). Thus, for purposes of understanding how individuals evaluate future utility, we must investigate u_t as a function of time.

3.1 A Three-Period Example

In this section we demonstrate the hyperbolic discounting phenomenon in a simple three-period model. Period 0 represents the present, period 1 the near future and period 2 the far future. Consider an individual having the utility function (7) when A=10. The individual consumes $x_0 ? x_0^? ? x_1^?$ in period 0 and receives utility of 10. Table 3 below shows probabilities for various utilities from consumption in the present and in the next two periods that are seen from the perspective of period 0. Expected utilities for the following two periods as seen from this perspective (i.e., period 0) are summarized in the table's bottom row.

Table 3: The columns indicate the probabilities of having various utilities in each period. The bottom row indicates expected utility in each period. The variables p, q and r are probabilities; p+q+r=1.

Utility	Period 0	Period 1	Period 2
10	1	r	$r^2 ? 2pq$
9 8	0 0	$p ? q$ 0	$2r(p ? q)$ $p^2 ? q^2$
Expected utility	10	$10r ? 9(p ? q)$	$10r^2 ? 20pq ? 18r(p ? q)$ $? 8(p^2 ? q^2)$

We define Eu_i as expected utility at period i from the perspective of period 0. The discount of expected utility between periods is defined by $?i ? Eu_{i?1} ? Eu_i$. Thus $?1 ? p ? q$ and $?2 ? p ? q ? 4pq$. The short-horizon discount is represented by $?1$ while the long-horizon discount is represented by $?2$.

<u>RESULT 3</u>: *In the three-period model described above, individuals perceive their future expected utility as hyperbolically discounted.*

<u>Proof:</u> In order to show that the utility function is hyperbolic, we show that it is decreasing between periods ($?1 ? 0$ and $?2 ? 0$) at a decreasing rate ($?1 ? ?2 ? 0$). Thus, the individual discounts highly his expected utility function over the short relative to the long horizon. $?1 ? p ? q ? 0$ when either p or q are positive. We need to show that $?2 ? p ? q ? 4pq ? 0$. Adding and subtracting $4p^2$ yields $4p^2 ? 4(p ? q)p ? (p ? q) ? 0$, which holds for every $0 ? p ? q ? 1$. When p and q are positive, $?1 ? ?2 ? 4pq$ is positive. QED

A necessary condition for expected utility to be hyperbolic is that both p and q are positive. This means that if the individual rationalizes his behavior only for more or only for less consumption, the expected utility function will diminish at a constant rate. In order for the expected utility function to diminish hyperbolically, the individual must rationalize decisions regarding both behaviors, that is, for more as well as for less consumption.

The extent to which expected utility is hyperbolic is measured by $?1 ? ?2$. A large $?1 ? ?2$ means that expected utility declines quickly in the first period relative to the second period. Any positive $?1 ? ?2$ means that the expected utility function is hyperbolic when this function always decreasing over time.

RESULT 4: i) *The expected utility function is more hyperbolic when p and/or q are larger. ii) Subject to* $p ? q ? const.$*, the more equal p and q are, the more hyperbolic the expected utility function.*

Proof: i) $?1 ? ?2 ? 4pq$ increases in both p and q. ii) Subject to $p ? q ? const.$, maximum $?1 ? ?2$ is reached when $p ? q$. Note that $?1 ? ?2 ? 4pq ? 4[p(p ? q) ? p^2]$ and $p ? q ? 2p ? (p ? q)$ represent the size of the inequality between p and q.

$\frac{d4[p(p ? q) ? p^2]}{d[2p ? (p ? q)]} ? 2(q ? p) ? 0$ when $p ? q$. The larger the p-q, the lower the $?1 ? ?2$. The same argument holds when $q ? p$. QED

The intuition behind the first part of Result 4 is that an increase in p or q means that there is more volatility in consumption. The hyperbolic discounting results from the probability of deviation from optimal consumption and a probability that consumption will return to optimal. An increase in p or q increases both phenomena. The intuition behind the second part of Result 4 is that p>q indicates a high probability to deviate toward greater consumption than optimal but low probability to return to optimal consumption. Expected utility consequently decreases faster and is less hyperbolic. Because the model is symmetric, the same result will arise when q>p.

An increase in p and/or q represents an increase in volatility of consumption. But it can also be interpreted as an increase in the likelihood of cognitive dissonance. We assume that once actual consumption differs from optimal consumption, a reaction in the form of cognitive dissonance always occurs, but this is not necessarily always the case. Cognitive dissonance can occur with positive probability. An increase in that probability will increase p and q. Thus, a greater tendency toward cognitive dissonance induces an increase in the hyperbolic discounting phenomenon.

Hyperbolic discounting causes dynamic inconsistency of choice in the sense that individuals prefer a smaller earlier reward over a larger later reward in the near future but the converse in the far future. Table 4 depicts a commodity that has a higher utility for the individual than the commodity in Table 3.

Table 4: The utility in this table is higher than the utility in Table 3. The columns indicate the probability of having various utilities in each period. The bottom row of the table indicates expected utility in each period. The variables p, q and r are probabilities; p+q+r=1.

Utility	Period 0	Period 1	Period 2
10.5	1	r	$r^2 ? 2pq$
9.5	0	$p ? q$	$2r(p ? q)$
8.5	0	0	$p^2 ? q^2$
Expected utility	10.5	$10.5r ? 9.5(p ? q)$	$10.5r^2 ? 21pq ? 19r(p ? q)$ $? 8.5(p^2 ? q^2)$

RESULT 5: *Inconsistency of preferences will prevail when* $4pq ? p ? q ? 0.5 ? 0$ *and* $p ? q ? 0.5$ *for the utilities presented in Tables 3 and 4.*

Proof: The individual prefers the smaller reward in period 0 to the larger reward in period 1 when $10 ? 10.5r ? 9.5(p ? q)$, which holds when $p ? q ? 0.5$, and prefers the larger reward in period 2 to the smaller reward in period 1 when $10.5r^2 ? 21pq ? 19r(p ? q) ? 8.5(p^2 ? q^2) ? 10r ? 9(p ? q)$, which holds when $4pq ? p ? q ? 0.5 ? 0$. There exists p and q such that both conditions hold. QED

3.2 Infinite Horizon Case

We now extend our example to a multi-period environment. The results are demonstrated using computer programs written in Mathematica 3[14]. We will show the hyperbolic discounting phenomenon, inconsistency of preferences as well as investigate how a different p, q, and r influence the expected utility function (7).

Figure 1 depicts a hyperbolic expected utility function from the perspective of period 1 for 500 periods into the future. The probability that actual consumption will equal optimal consumption in each period is r=0.5. The probability that consumption level will decrease or increase by one is p=q=0.25. The computer program follows the model described in section 3 when the utility from consumption in the first period is set at A=15.

[14] The computer programs are available from the author by request.

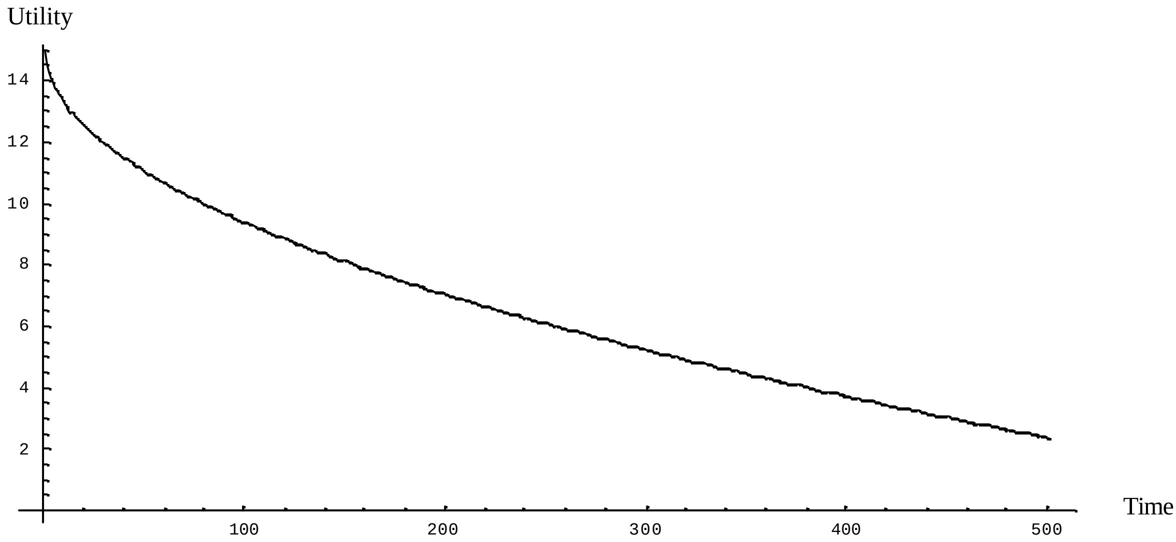

Figure 3: Expected utility as a function of time. The probability that utility will not change in each period is equal to r=0.5. The probability that utility will increase is equal to p=0.25 and the probability that utility will decrease is equal to q=0.25. Utility from consumption during the first period equals A=15.

Hyperbolic discounting causes dynamic inconsistency of choice. Figure 4 shows how expected utility from two different commodities can change over time. One curve depicts expected utility from a larger reward at period t+1 whereas the other curve depicts expected utility from a smaller reward at period t. Individuals prefer the smaller, earlier reward over the larger, later reward in the near future and the converse in the far future.

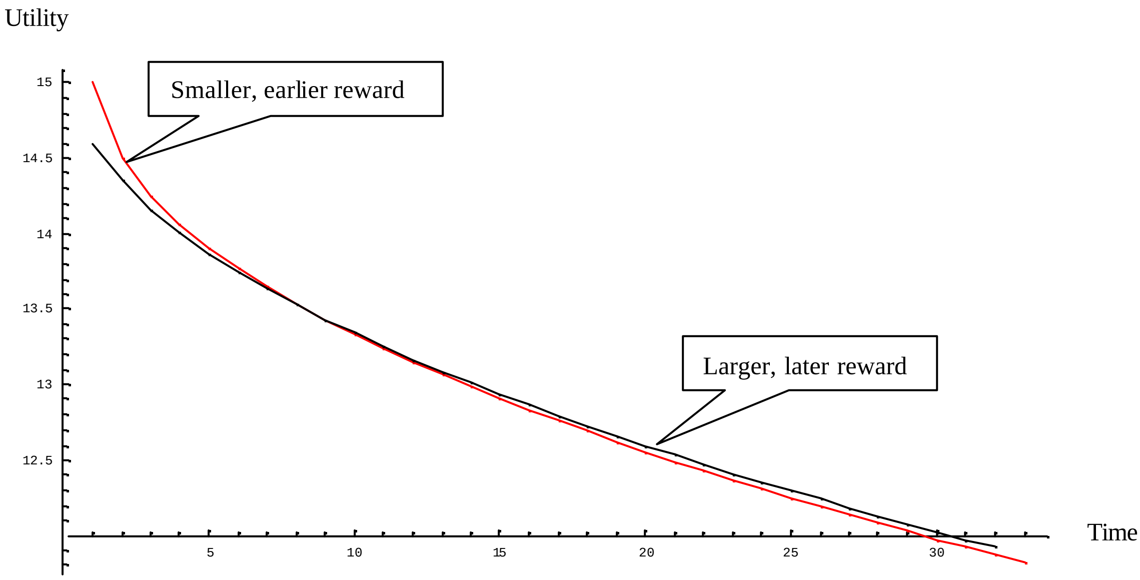

Figure 4: Hyperbolic discounting causes inconsistency of preferences. In the near future, the individual prefers a smaller, earlier reward at period t to a larger, later reward at period t+1 whereas in the far future the individual prefers the larger later reward to a smaller, earlier reward. A=15 for the smaller, earlier reward and A=15.1 for the larger, later reward. For both rewards, r=0.5 and p=q=0.25.

Some individuals are more prone than others to deviate from their optimal consumption level. A smaller r means that individuals are more prone to this deviation either because of greater volatility in each period's consumption or because of a greater tendency toward cognitive dissonance. Figure 5 depicts expected utility functions for two individuals, one is more prone to change his consumption, represented by r=0.1, than the other, represented by r=0.9. It is difficult to compare the two functions in terms of degree of hyperbolic discounting because the functions decrease at a different rate in all time periods. However, we view the individual who is more prone to changing his consumption as having a more hyperbolic function because his expected utility decreases by a relatively large amount in the first periods.

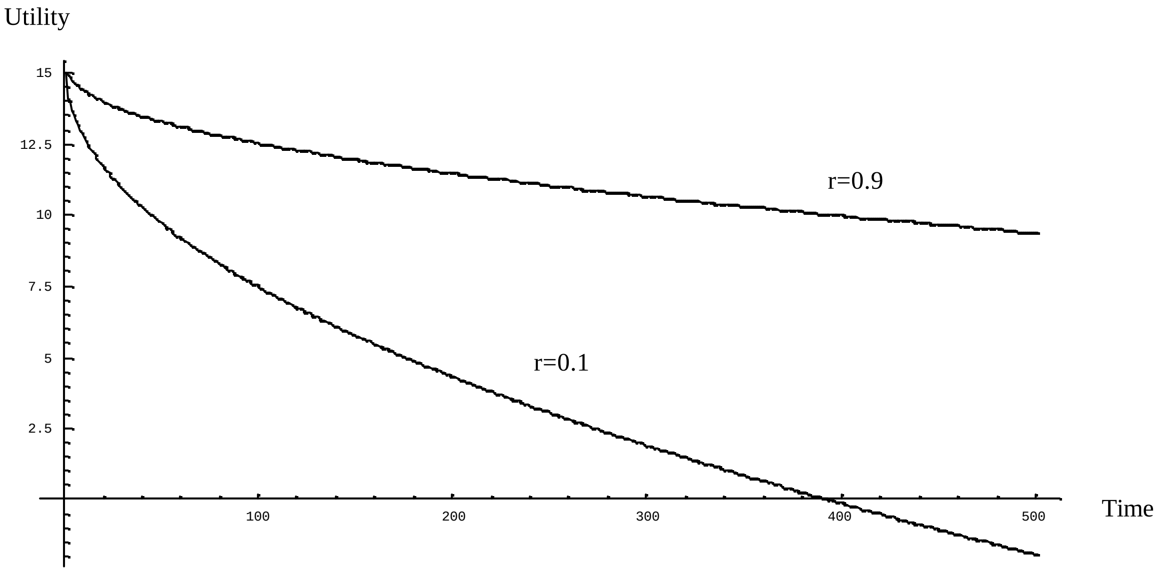

Figure 5: Expected utilities as a function of time when consumption volatility differs or when susceptibility to cognitive dissonance differs. High consumption volatility and high susceptibility to cognitive dissonance are represented by r=0.1 whereas low consumption volatility and low susceptibility to cognitive dissonance are represented by r=0.9. In both cases, p=q and p+r+q=1.

Some individuals are prone to consume more than they originally planned while others are prone to consume less than they originally planned. Figures 6 shows the influence of such asymmetric behavior, which causes the expected utility function to be less hyperbolic.

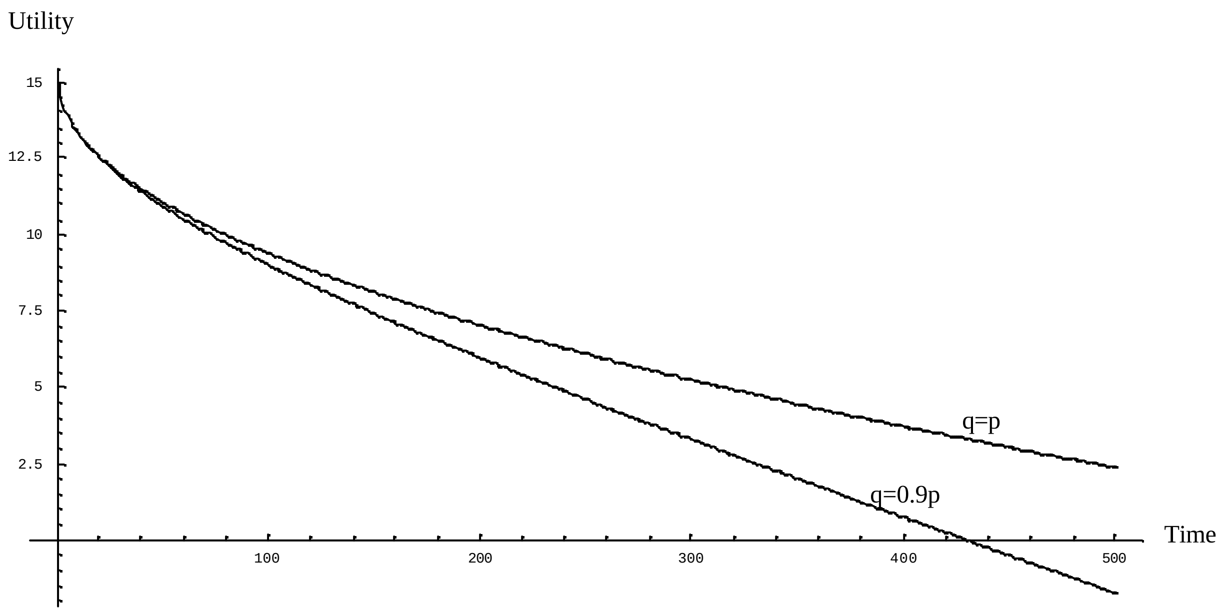

Figure 6: Expected utilities as a function of time. The tendency to consume more than optimal consumption is represented by q=0.9p and the tendency to consume symmetrically around optimal consumption is represented by p=q. In both cases r=0.5, p+r+q=1.

We find that although the rate of decrease in the expected utility function always decreases at a decreasing rate on average, the rate of decrease does not always decrease between every period. This phenomenon can be explained by an example where the probability that consumption will not change is r=0. During the first period, the individual consumes optimally. During the second period, the individual will not consume optimally from the perspective of the first period. He will either consume more or less than optimal. During the third period, his consumption will deviate once more. He will either consume optimally from the perspective of the first period or deviate even further from optimal when compared to the second period. Expected utility from consumption will equal his expected utility during the second period. Thus, expected utility in the third period will not diminish. During the fourth period, his consumption may be even further from optimal and his expected utility will diminish even further. During odd periods, expected utility will not diminish. During even periods expected utility decreases at a decreasing rate. Figure 7 depicts such a situation.

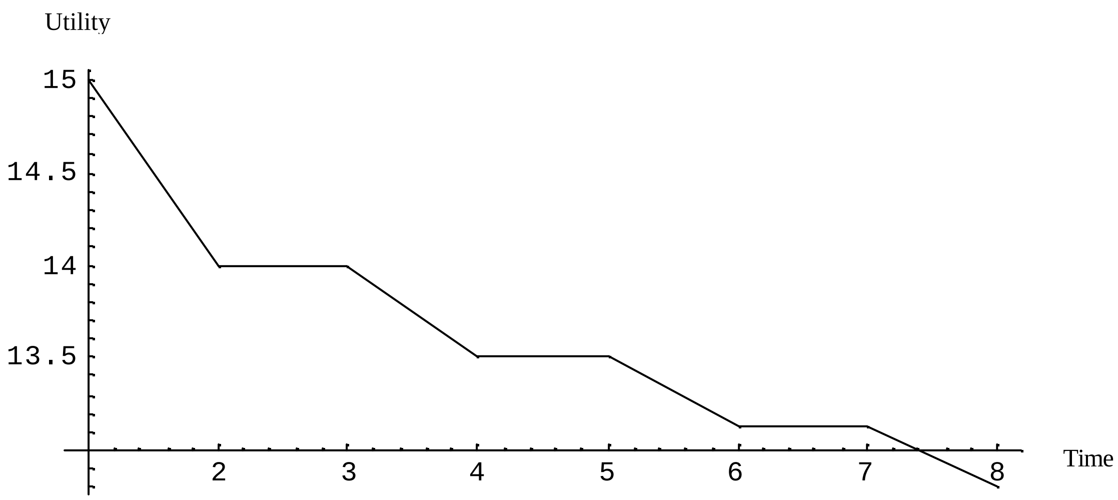

Figure 7: Expected utility from consumption, 8 periods into the future. The probability that utility will not change is r=0, p=q and p+r+q=1.

The cyclical phenomenon might suggest that the three-period model in section 3.1 does not demonstrate hyperbolic discounting because the second period is effectively a cycle. However, the cyclical phenomenon is salient only when r is small. The three-period model also holds when r is not small. Whenever there are cycles, discounting will be hyperbolic, on average, over time.

The cyclical phenomenon of diminishing expected utility decreases when the probability for consumption to remain consistent increases. Figure 8 depicts this when r=0.15.

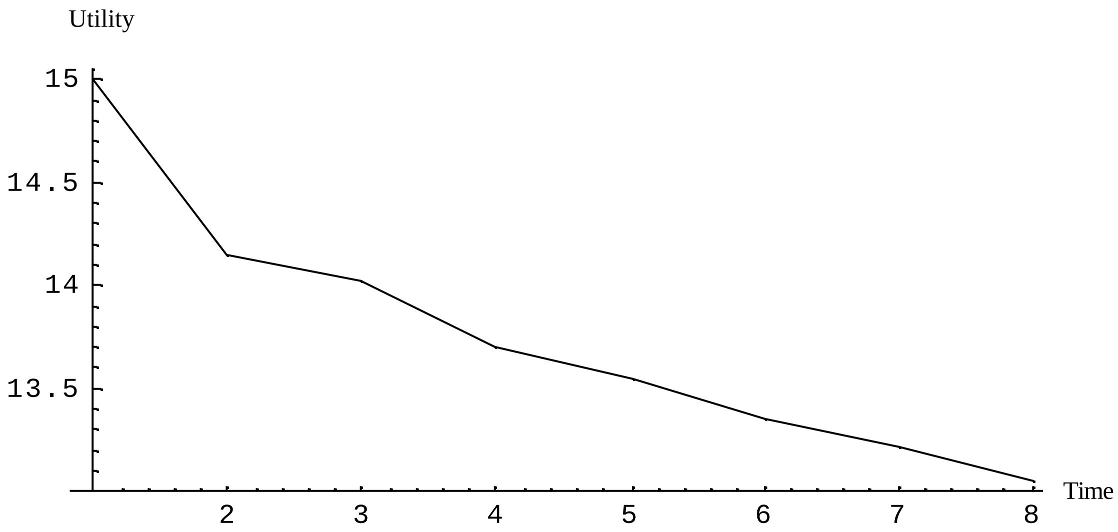

Figure 8: Expected utility from consumption, 8 periods into the future. The probability that utility will remain consistent is r=0.15, p=q and p+r+q=1.

4 Concluding Remarks

Hyperbolic discounting agents perceive future utility as decreasing at a decreasing rate as a function of time. Cognitive procedures might cause people to perceive future utility in the same way. We provide two such procedures. The first procedure shows that hyperbolic discount function for money can be derived when individuals, in each period, assume that their wealth might change under the constraint that the probability that their wealth will decrease below the current level is small. People contemplate how much money they are likely to have in the future and evaluate their expected marginal utility from money accordingly. When marginal utility as a function of wealth is itself hyperbolic, utility will be hyperbolic over time if, in each period, there is a positive probability of gaining money. This situation does not require a positive probability of losing money in any period. In such circumstances, a positive probability of losing money and returning to previous levels of marginal utility enhances the hyperbolic discounting phenomenon. The framework provided here shows how hyperbolic discounting can occur even when the marginal utility as a function of wealth is not hyperbolic.

The second part of the paper offers a cognitive procedure that leads to hyperbolic discounting phenomenon with respect to consumption of a particular good. When people know that they will rationalize their consumption behavior in the future but evaluate future consumption on the basis of current standards, expected utility function will be hyperbolically discounted given a unique maximum for the utility function. The phenomenon will be stronger when individuals are more prone to rationalizing their consumption, when rationalization is symmetric for more and less consumption and when consumption is more volatile.

Seemingly impulsive behavior is therefore shown to be rational under set circumstances. Dynamic inconsistency is an outcome of the hyperbolic discounting phenomenon. Again, individuals are only seemingly inconsistent. They alter their future choices because they know that their preferences might change. We do not claim that the cognitive procedures presented here are the only procedures that lead to hyperbolic discounting; rather, we only argue that they can lead to hyperbolic discounting under the specific assumptions indicated.

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