

## Tilburg University

### Oskar Morgenstern

van Damme, E.E.C.

*Publication date:*  
2004

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication in Tilburg University Research Portal](#)

*Citation for published version (APA):*  
van Damme, E. E. C. (2004). *Oskar Morgenstern*. (CentER Discussion Paper; Vol. 2004-42). Vakgroep CentER.

#### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

#### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

CentER



# Discussion Paper

No. 2004-42

**OSKAR MORGENSTERN**

By E.E.C. van Damme

April 2004

ISSN 0924-7815

# OSKAR MORGENSTERN<sup>§</sup>

Eric van Damme<sup>\*</sup>

CentER and TILEC, Tilburg University

January 2004

---

<sup>§</sup> Prepared for the “Encyclopedia of Social Measurement”, edited by Kimberly Kempf-Leonard, to be published by Elsevier.

<sup>\*</sup> Prof.dr. E.E.C. van Damme, Tilburg University, CentER for Economic Research and Tilburg Law and Economics Center (TILEC), P.O. Box 90153, 5000 LE Tilburg, The Netherlands. Phone +31-13-4663045, Fax +31-13-4663266, e-mail: [Eric.vanDamme@TilburgUniversity.nl](mailto:Eric.vanDamme@TilburgUniversity.nl), <http://center.uvt.nl/staff/vdamme/>.

**JEL: B21, B31, C70**

## **GLOSSARY**

**cooperative game:** a game model that assumes that players can make coalitions and agree on side payments outside of the formal rules

**game theory:** a mathematical theory to model and analyse conflicts arising from economic, social and political situations

**Nash equilibrium:** solution concept for non-cooperative games: a strategy profile is a Nash equilibrium if no player can profit by unilaterally deviating from the solution

**non-cooperative game:** a game model which is such that the rules of the game fully describe the situation; all possibilities for coalition formation or side payments are explicitly included in the rules

**stable set:** solution concept for cooperative games introduced by von Neumann and Morgenstern; a stable set satisfies both internal stability (no element in the set dominates another element in the set) as well as external stability (every element outside is dominated by an element from inside)

**von Neumann Morgenstern utility theory:** an axiomatic theory of individual decision making under uncertainty; if an individual's choice behavior satisfies certain assumptions, he will have a numerical utility function and he will evaluate each lottery (random outcome) by its expected utility

Together with John von Neumann, Oskar Morgenstern wrote *The Theory of Games and Economic Behavior*, one of the great social science books of the 20<sup>th</sup> century. The book argues strongly that traditional models and methods are unsuited to develop a general theory of rational behavior, and it proposes an alternative based on game theory. Besides expounding the theory of zero-sum two person games that had earlier been developed by von Neumann, the book contains at least three seminal contributions (the theory of expected utility, the concept of a cooperative game, and the stable set solution concept) and it provides the first applications of game theory to economics and political science. Morgenstern also worked and wrote on economic growth, on predictability of stock prices, on the accuracy of economic observations, and on methodological issues. In this entry, we describe Morgenstern's work and influence, focusing on his work in game theory. For a selection of Morgenstern's writings and a bibliography, I refer to Schotter (1976). For an overview of game theory and for definitions of the technical terms used in this article, the reader is suggested to consult Aumann (1987) or Aumann and Hart (1992-2002).

## **I. Early Work: Impossibility of Perfect Foresight**

Oskar Morgenstern was born in Görlitz (Germany) on January 24, 1902. When he was fourteen, the family moved to Vienna, where, in 1925, Oskar received his doctorate for a thesis on marginal productivity, which enabled him to describe himself as a product of the Austrian School of Economics (Morgenstern, 1976). In the same year, he was awarded a Rockefeller Fellowship, which allowed him to further study in London, Paris and Rome and at Harvard and Columbia. In 1928 he returned to the University of Vienna, where he defended his habilitation thesis *Wirtschaftsprognosen* and where he was appointed as Privatdozent in 1929. In this second thesis, a study of the theory and applications of economic forecasting, Morgenstern points out two problems in making predictions in the social sciences: the predictions may influence the predicted events, and an individual makes predictions about the behavior of other individuals, behavior that in turn is guided by their predictions about his own behavior. This raises the issue of whether a (good) prediction is possible at all, and by means of an example, Morgenstern suggests that the answer is no:

“Sherlock Holmes, pursued by his opponent, Moriarty, leaves for Dover. The train stops at a station on the way, and he alights there rather than traveling on to Dover. He has seen Moriarty at the railway station, recognizes that he is very clever, and expects that Moriarty

will take a special faster train in order to catch him at Dover. Holmes' anticipation turns out to be correct. But what if Moriarty had been still more clever, had estimated Holmes' mental abilities better and had foreseen his actions accordingly? Then obviously he would have traveled to the intermediate station. Holmes, again, would have had to calculate that, and he himself would have decided to go on to Dover. Whereupon Moriarty would have "reacted" differently. Because of so much thinking they might not have been able to act at all or the intellectually weaker of the two would have surrendered to the other in the Victoria Station, since the whole flight would have become unnecessary. Examples of this kind can be drawn from everywhere." (Morgenstern, 1928, p. 98)

In the same year as Morgenstern published his *Wirtschaftsprognose*, John von Neumann published *Zur Theorie der Gesellschaftsspiele*. Morgenstern's example can be viewed as a 2-player game in which each of the players, Holmes and Moriarty, has two strategies: to take a slow train that stops at the intermediate station and get out there, or to take a fast train to Dover. Let us assume that Holmes escapes for sure if they take different types of trains, that he is caught otherwise, and that Holmes (respectively Moriarty) wants to maximize (respectively minimize) the probability of escape. Obviously, no player has a *deterministic* optimal strategy and it is not possible to make a *determinate* prediction of the outcome: if the prediction is that Holmes will take the slow train, Holmes will realize that Moriarty will know this, inducing Holmes to violate the prediction, and similar for every other possibility. In modern game theoretic language: this is a game without pure strategy equilibrium. In his paper, von Neumann, however, showed that equilibrium can be obtained if we allow for randomized decisions: a player that bases his decision on the toss of a coin, cannot be outsmarted by his opponent and will be happy with his choice; we can confidently predict that Holmes will escape with probability 1/2. The main result in von Neumann's paper shows that mixed strategies not only solve the prediction problem in this example, but in 2-person constant sum games in general.

In 1931, Morgenstern succeeded von Hayek as director of the Austrian Institute for Business Cycle Research, but he continued to be bothered by problems of prediction and foresight. He also became editor of the *Zeitschrift für Nationalökonomie*, in which he published his paper *Vollkommene Voraussicht und Wirtschaftliches Gleichgewicht* that returns to issues from the 1929 thesis and that argues that the theory of general economic equilibrium crucially depends on this paradoxical assumption of perfect foresight and, hence, is

unsatisfactory. After a presentation of this paper in Karl Menger's Vienna Colloquium, the mathematician Eduard Čech pointed out to Morgenstern that his work was related with that of von Neumann and he urged him to read the latter's paper. Morgenstern's many duties (in addition to being director of the mentioned institute, he was an advisor to the Austrian National Bank, an advisor to the Ministry of Commerce and a member of the Committee of Statistical Experts of the League of Nations) apparently prevented him from doing so.

In January 1938, with support of the Carnegie Endowment for International Peace, Morgenstern left for the United States. In March that year, the Nazis took over in Austria and Morgenstern was dismissed from his Vienna position. Finding himself unable to return, he accepted a three-year appointment in political economy at Princeton University, in the hope that he would get to know von Neumann and be able to work with him.

## **II. The Collaboration with von Neumann**

In Morgenstern (1976) the impression is given that, even though von Neumann did most of the work, and certainly was responsible for the mathematical parts, the writing was a truly cooperative effort: "We wrote virtually everything together and in the manuscript there are sometimes long passages written by one or the other and also passages in which the handwriting changes two or three times on the same page". Morgenstern's diaries, however, allow one to conclude that his account is somewhat misleading: von Neumann did the bulk of the work; see Rellstab's chapter in Weintraub (1992). Morgenstern's direct contributions are mainly in Chapter 1, in his insistence on the measurement of utility (see below), and in the economic examples, but the indirect contribution should not be underestimated: he succeeded in getting, and keeping von Neumann interested in economic problems and inducing him to spend his great intellectual powers on these, and thereby advancing social science. As Morgenstern (1963, p.25) himself wrote: "Scientific activity consists largely in asking the right kind of question. There comes a point where economists and mathematicians must get together to do precisely this in order to advance our knowledge." In his chapter in Weintraub's book, Schotter writes: "Without Oskar Morgenstern we would not have the theory of games as we know it today. (...) Game theory would probably not have been introduced into the social sciences until many years later. (...) I can think of no other economists at the time who could have walked into a room with John von Neumann and walked out later with a 600-page book on the theory of games complete with economic

examples.” By combining their respective comparative advantages, von Neumann as a creative innovator, and Morgenstern as a visionary entrepreneur and arbitrageur between two fields, the two constructed a unique product.

When Morgenstern met with von Neumann in 1938, the latter had again seriously taken up his interest in game theory. The first intensive discussions between the two took place in October 1939 when Morgenstern was starting to write down the material on “maxims of behavior” that would eventually be published in Chapter 1 of the *Theory* (from now on: TGEB) and continued till the summer of 1940, when von Neumann gave three lectures on games to the department of mathematics at Washington University (Seattle). After his return to Princeton, von Neumann’s cooperation with Morgenstern intensified: he informed Morgenstern about the progress that he had made and wrote two papers, while Morgenstern supplied von Neumann with criticism about established economic thinking and with an agenda of things to do. In the summer of 1941, von Neumann proposed that they write a paper together and this induced Morgenstern to work on Von Neumann’s manuscripts, the explicit purpose being to write an introduction to them. The period June 1941-Christmas 1942, during which the book was written, probably was the intellectually most stimulating and most productive period of Morgenstern’s life. On July 30, 1941, he writes in his diary “This is the book I have been dreaming of for years”.

### **III. Axioms and Measurable Utility**

From the outset, the authors of TGEB make clear that their work is solidly based on methodological individualism: “In the course of the development of economics, it has been found, and it is now well-nigh universally agreed, that an approach to this vast problem is gained by the analysis of the behavior of the individuals which constitute the economic community” (TGEB, p. 10). Of course, this approach requires an adequate representation of the motives of the individuals. At that time, the standard approach in economics was to represent an individual’s preferences by a set of indifference curves, which corresponds to an ordinal ranking of alternatives. Even though this standard approach is based on the strong assumption that preferences are complete (the individual is able to compare all possible prospects), it has only limited applicability, since it does not allow for uncertainty, and it is also somewhat clumsy to work with. Von Neumann and Morgenstern argue that the theory should be able to deal with uncertainty, and that, if one is willing to assume completeness of



preferences as well as certain other “natural” properties, then one actually obtains a much simpler, numerical representation. Specifically, in the 2<sup>nd</sup> edition of TGEB, von Neumann and Morgenstern (from now on: vNM) propose axioms that imply that utility is measurable, unique up to a positive affine transformation, and linear in the probabilities that represent the uncertainty. TGEB provides one of the first applications of the axiomatic method in economics and it shows that this method, which had already been successfully used in mathematics, is equally promising in this domain. Morgenstern (1963, p. 23-25) provides a brief description of the axiomatic method in general; Nash’s work on bargaining and that of Shapley on value and on measuring political power are excellent examples illustrating how fruitful the approach can be in dealing with economic problems.

It is useful to illustrate the axiomatic approach pioneered by von Neumann and Morgenstern by explicitly deriving their utility function. Assume an individual can choose from a (finite) set of possible outcomes  $X$ . The traditional approach assumes that  $i$  has a complete preference ordering over  $X$ , that is, for each two alternatives  $x, y \in X$ , he can say whether he (weakly) prefers  $x$  to  $y$  (written  $x \check{S} y$ ) or  $y$  to  $x$  ( $y \check{S} x$ ), the relation is reflexive ( $x \check{S} x$  for all  $x$ ) and transitive (if  $x \check{S} y$  and  $y \check{S} z$  then  $x \check{S} z$ ). Clearly, associated with  $\check{S}$ , there is a strict preference relation  $\check{S}^{\text{TM}}$  ( $x \check{S}^{\text{TM}} y$  if  $x \check{S} y$  but not  $y \check{S} x$ ) and an indifference relation  $\sim$  ( $x \sim y$  if  $x \check{S} y$  and  $y \check{S} x$ ). Since  $X$  is finite it follows that there is a best element  $b$  in the set ( $b \check{S} x$  for all  $x$ ) as well as a worst element  $w$  ( $x \check{S} w$  for all  $x$ ).

vNM now assume that the individual can also compare uncertain prospects yielding outcomes in  $X$ , as long as these are associated with known, objective probabilities. Formally, if  $P$  denotes the set of all lotteries on  $X$ , i.e. probability distributions with outcomes in  $X$ , we assume that the preference relation  $\check{S}$  can be extended to a complete, reflexive, transitive relation on  $P$ . Now take  $x \in X$ , assume  $b \check{S}^{\text{TM}} x \check{S}^{\text{TM}} w$ , and write  $+ \lambda; w, b, = (1-\lambda).w + \lambda.b$  for the lottery that yields  $b$  with probability  $\lambda$  and  $w$  with the complementary probability  $1-\lambda$ . It is natural to assume that  $w \sim +0; w, b, , b \sim +1; w, b, ,$  and that the preference relation is continuous, i.e. there exists some  $\lambda \in (0,1)$  such that  $x \sim + \lambda; w, b, .$  It is equally natural that, if  $\lambda > \mu$ , then  $+ \lambda; w, b, \check{S}^{\text{TM}} + \mu; w, b, ,$  and in this case there exists a unique number, let us denote it by  $u(x)$ , such that  $x \sim +u(x); w, b, .$  Note that  $u(w) = 0$  and  $u(b) = 1$ ; we have obtained that the utility of any outcome  $x \in X$  is measurable:  $u(x)$  is simply the probability that one has to put on the good outcome to make the individual indifferent between  $x$  and the lottery.

What can be said about the utility of a lottery? Can  $u(\cdot)$  be extended from  $X$  to  $P$ , just as we have extended  $\succsim$ ? With an additional assumption, the answer is affirmative. Assume that the individual can also compare compound lotteries, that is, objects of the type  $\alpha p + (1-\alpha)r$ , where  $p, r \in P$ . The final axiom that we need is independence:  $p \succsim q$  if and only if  $\alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$ , for all  $\alpha \in [0,1]$  and  $r \in P$ . Again this appears to be natural: each of the compound lotteries yields  $r$  with the same probability, hence, my preference should not be guided by that event; in the alternative event, the lottery coincides with  $p$  or  $q$ , hence, my preferences are determined by these lotteries. This assumption implies that the individual is indifferent between a compound lottery and the simple lottery that induces the same distribution on  $X$ . Consequently, if  $p \in P$  is the lottery that yields outcome  $x$  with probability  $p(x)$ , we have  $p \sim \alpha u(p) + (1-\alpha)w$ , where  $u(p) = \sum_{x \in X} p(x)u(x)$ . We have derived our main result: if preferences  $\succsim$  on  $P$  are complete, transitive, continuous and satisfy the independence axiom, then there exists a function  $u$  on  $X$  such that  $p \succsim q$  if and only if

$$\sum_{x \in X} p(x)u(x) \geq \sum_{x \in X} q(x)u(x) \quad (1)$$

Preferences can thus be represented by a numerical utility function, and one lottery is preferred to another if and only if it yields higher expected utility. Furthermore, the utility function  $u(\cdot)$  is determined up to a positive affine transformation: since  $x \sim \alpha u(x) + (1-\alpha)w$ , we must have  $v(x) = \alpha(u(x)v(b) + (1-\alpha)v(w)) + (1-\alpha)v(w)$ , or  $v(x) = v(w) + \alpha(u(x)(v(b)-v(w)))$  for any alternative utility function  $v$  that satisfies (1).

This approach has been extended to the case where the probabilities are subjective. The axiomatic approach makes clear what are the “critical” assumptions underlying expected utility theory, as such, it allows one to attack the theory on its weakest points, and offers room for alternative development and improvement. Even though the above axioms appear natural, actual human behavior violates them in systematic ways; see Nobel Foundation (2002). This should not lead us to belittle the contribution of von Neumann and Morgenstern, as Morgenstern (1972) has written: “Each problem solved usually suggests new ones which could not have been stated without a given problem first having been solved”.

#### IV. Games and Solutions

vNM are quick to point out that a participant in a social exchange economy faces a much different problem from that discussed above: “He too tries to obtain an optimum result. But in order to achieve this, he must enter into relations of exchange with others. If two or more persons exchange goods with each other, then the result for each one will depend in general not merely upon his own actions but on those of others as well.” (TGEB p. 11) Obviously, as actions depend on expectations, we are back in the setting of the Holmes-Moriarty example. vNM point out that this conceptual problem is neglected in traditional economics and that this kind of problem is nowhere dealt with in classical mathematics. They then propose an alternative game theoretic solution.

The traditional economic method abstracts away from strategic interaction by focusing on the case where the number of players is large. While one may indeed hope that the influence of every particular participant is negligible in that case, vNM are right to argue that this had not been formally proved and that such a proof requires explicit consideration of the finite player case. As they also argue, the traditional approach is all the more unsatisfactory since it makes several assumptions (such as the existence of a price system, the fact that individuals know prices and take them as given, and the absence of coalitions of traders that could wield power), that should preferably should be derived from more primitive ones.

An example may make this clear. Suppose that there is one seller who owns one indivisible unit and who values this at  $a$ . Also suppose that there are two potential buyers and that each of these values the unit at  $b$  with  $b > a$ . As trading generates a surplus, the natural questions are: who will trade and how will the surplus be divided? The traditional approach assumes the existence of prices and searches for a price that clears the market. As is clear, the unique price where supply is equal to demand is  $p = b$ , hence, the standard prediction is that the short side of the market appropriates the entire surplus. There is no prediction about who will actually trade, but, since, in equilibrium, each buyer is indifferent between trading or not, this really is irrelevant. However, where do the prices come from? Why focus on a market institution? Why is it justified to assume that the buyers do not form a cartel and negotiate with the seller as a team?

vNM want to address these more basic questions. They clearly state their goal “We wish to find the mathematically complete principles which define “rational behavior” for the participants in a social economy, and to derive from them the general characteristics of that behavior” (TGEB, p. 31). Furthermore, these rules have to deal with all possible situations that may arise: “if the superiority of “rational behavior” over any other kind is to be established, then its description must include rules of conduct for all conceivable situations – including those where “the others” behaved irrationally, in the sense of the standards which the theory will set for them” (TGEB, p. 32).

### **A. Non-cooperative Games**

In defining what should be understood as a solution, it is important, using terminology that was introduced by John Nash, to distinguish between cooperative and non-cooperative games. In many parlor games, the rules offer a complete description: it is clear what is allowed and what is not. Such games are said to be non-cooperative: it is not possible to move outside of the game. Note that the game being non-cooperative does not mean that coalitions cannot be formed: it simply means that all such possibilities are included in the rules. For such a game, a solution is “a set of rules for each participant which tell him how to behave in every situation which may conceivably arise” (TGEB, p. 31).

Given Morgenstern’s interest in predictability, it is remarkable that Chapter 1 of TGEB circumvents the question of what constitutes “rational behavior” in a non-cooperative game. For the special zero-sum two-person case, the issue is, however, discussed on pp. 147-148. There it is argued that, if a complete and absolutely convincing theory of rational behavior exists, then each player should still be willing to play according to this theory even if he knows that other players know that he will play that way. vNM use this argument to justify the minmax solution that was originally advocated, and proved to exist in Von Neumann (1928), that is, they recommend that each player chooses a strategy that guarantees the highest possible payoff under the assumption that his strategy has been found out by the opponent. In later work, John Nash turned around this argument and reasoned that, when a convincing theory exists, each player himself should not be able to profit from knowing what the others will do: each player’s plan should be a best response against the plans of the others. A strategy profile satisfying this condition is known as a Nash equilibrium of the game, hence, the vNM-argument seems to naturally lead to the Nash equilibrium concept, by now the most

important solution concept for non-cooperative games. The Holmes-Moriarty example shows that a Nash equilibrium in pure strategies need not exist, but Nash proved that there is always at least one equilibrium in mixed strategies. He also showed that his concept coincides with that of von Neumann for two-person zero-sum games. Since Nash's concept seems the natural solution to the problem of "perfect foresight" that occupied Morgenstern since the start of his career, it is remarkable that this concept is not discussed in TGE, neither to reject it, nor to endorse it. Perhaps, vNM rejected this concept as it does not have good "defensive" properties (an equilibrium strategy need not do well when the others don't play according to the solution), but since they did not discuss this, although they mention Nash's work in the preface to the 3<sup>rd</sup> edition of TGE, we don't know. In fact, it is quite remarkable that also in his later work Morgenstern never discussed the Nash concept.

## **B. Cooperative Games**

As vNM are, obviously, mainly interested in situations with more than two players, in which coalitions play an important role, the bulk of TGE is devoted to cooperative games. What is even more important, as is stressed on p. 35 and in Chapter V of TGE, if there are more than two players, they will typically have incentives to agree coalitions and to make side payments *outside of the formal rules* of the game: one must expect that a player will be willing to compensate another in order to secure the latter's cooperation. Consequently, even if the rules of the game do not explicitly allow for these possibilities, the formal analysis has to take them into account, and this induces vNM to introduce a different game form, the so called characteristic function, which describes for each coalition  $S$  of players that might form, the total value  $v(S)$  that these players can divide among themselves. Note that in moving to this representation, vNM assume that utility is freely transferable between the players. For cooperative games, the question that TGE seeks to answer is: which coalition  $S$  will form and how its members will divide the value  $v(S)$ ?

It should be immediately clear that one cannot expect the theory to give a unique answer to the first question. For example, in the 3-person symmetric game in which each 2-person coalition has value 1 and all other coalitions have value 0, one may expect two players to agree on an equal split ( $\frac{1}{2}, \frac{1}{2}$ ), but one cannot say which players this will be. As vNM stress, the competition between coalitions will determine which one will form and how it will divide the surplus. A second example may make this clear. Suppose we have a 3-player game with

$v(12) = 3$ ,  $v(13) = 2$ ,  $v(23) = 1$  and  $v(S) = 0$  otherwise. Clearly, the coalition  $\{1,2\}$  is most attractive, but how should it divide the spoils? Each of the players in this coalition will look at what he can get with the outside player, while the outside player 3 will clearly investigate what bribe he has to offer to each of them in order to induce one of them to form a coalition with him. Consequently, one can state something about the division in one coalition only if one simultaneously looks at the other coalitions. In this case, if  $x = (x_1, x_2, x_3)$  satisfies  $x_1 + x_2 = 3$ ,  $x_1 + x_3 = 2$  and  $x_2 + x_3 = 1$ , and each coalition  $\{i, j\}$  expects  $v(ij)$  to be divided according to  $x$ , then, in each coalition, each inside player will be satisfied as he cannot get more by switching, hence,  $x$  determines a stable division for each possible coalition that might form. Obviously, the unique  $x$  that satisfies these conditions is  $x = (2,1,0)$ . Even if the coalition  $\{1,2\}$  is eventually formed, the payoff division  $(2,1)$  associated with this coalition derives its stability only from consideration of the other possible coalitions. The solution consists of a triple  $\{ \{1,2\}, 2, 1, \{1,3\}, 2, 0, \{2,3\}, 1, 0 \}$ ; it is the system that is stable, not the single imputations from which it is composed.

Accordingly, vNM come to the conclusion that “A solution should be a system of imputations possessing in its entirety some kind of balance and stability the nature of which we shall try to determine” (TGEB, p. 36). The stability notion that they propose makes use of the concept of dominance. An imputation  $x$  is said to dominate another imputation  $y$  ( $x \succ y$ ) if there exists a coalition  $S$  of individuals each of whom strictly prefers  $x$  to  $y$  and such that  $S$  is able to enforce  $x$ . Note that this dominance relation typically will not be transitive: in the one-seller two-buyers example that we discussed above, let  $x_i$  be the imputation resulting from a trade at price  $x$  between  $S$  and buyer  $B_i$ , then, if  $b > x > y > z > a$ , we have  $x_1 \succ z_2$  (through the coalition  $\{S, B_1\}$ ) and  $x_2 \succ y_1$ , but there is no dominance relation between  $x_2$  and  $z_2$ . As a result, a single imputation that dominates all others, in general, need not exist.

When can a set  $X$  of imputations be considered stable? A necessary requirement is that  $X$  should be free of inner contradictions, no element in  $X$  should dominate another element of  $X$ . Secondly, vNM insist that any alternative imputation (i.e. one that is not in  $X$ ) can be discredited by referring to some element that is in  $X$ : if  $y \notin X$ , then  $x \succ y$  for some  $x \in X$ . A set  $X$  that satisfies both internal and external stability is called a stable set. vNM argue that such sets correspond to stable standards of behavior for a society: “once they are generally accepted, they overrule anything else and no part of them can be overruled within the limits of

the accepted standards. This is clearly how things are in actual social organizations” (TGEB, p. 42).

The one-seller two-buyers example may be used to illustrate this concept. The characteristic function is given by  $v(12) = v(13) = v(123) = 1$  and  $v(S) = 0$  otherwise. The competitive solution allocates all surplus to the seller, hence, it produces the imputation  $(1,0,0)$ , and the reader may verify that this imputation is undominated by any other one. However, as a singleton set, this imputation is not stable: for example, it does not dominate the imputation  $(\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$  that results if the two buyers decide to collude, bargain with the seller as a team, and insist that the seller gives up half of the pie. Stable sets have to allow for such collusion. It turns out that many stable sets exist. Suppose that the buyers agree that, whatever they get out, they will split this in proportion  $(\alpha, 1-\alpha)$ , and assume, with vNM, that in the bilateral bargaining game between the seller and the team any distribution of the surplus is possible. Then one will predict an outcome in the set  $X(\alpha) = \{(1-x, x\alpha, x(1-x)) : 0 \leq x \leq 1\}$  and any such set is stable. We may conclude, as Morgenstern had conjectured all along, that other forms of social organization, different from unbridled competition, will be stable.

The fact that a game may have multiple stable sets does not bother vNM: “this has a simple and not unreasonable meaning, namely that given the same physical background different “established orders of society” or “accepted standards of behavior” can be built” (TGEB, p. 42). On the other hand, they write “There can be, of course, no concessions as regards existence.” In 1969, William Lucas constructed an example of a game without a stable set. This counter-example, as well as the others that have been constructed, is, however, contrived and does not correspond to an actual economic, political or social reality. Given Morgenstern’s emphasis on “realistic modelling”, it is, hence, unlikely that he would have been bothered by these examples. While, in “realistic models” stable sets have been found to exist, stable set theory has nevertheless only found relatively few applications, mainly as a consequence of the fact that the concept is difficult to work with.

## V. Other Contributions

In the preface of his selection of the writings of Oskar Morgenstern, Schotter writes: “This book contains a selection of writings of one social scientist whose entire scholarly life has

been devoted to both building up as well as tearing down the basic traditions of economics. As a builder, he has helped to introduce a radically new way of looking at economic problems, along with hastening the introduction of more powerful mathematical techniques. As a critic, he has constantly attacked economics for refusing to address itself to the empirically given problem, and has asked his profession to confront head-on the enormous complexity of the social world rather than hide behind methodological tools borrowed from other disciplines.” This characterization is accurate and in line with that of other observers. For example, in his chapter in Weintraub (1992), Leonard writes that Morgenstern has taken a “position as an arbiter of ideas, an intermediary between theorists in disparate fields, and one ultimately most capable of giving his energy to penetrating criticism rather than alternative theoretical construction”. If one looks at Morgenstern’s later work one can hardly come to a different conclusion; see Morgenstern (1972).

Schotter’s volume is divided into six sections: game theory and utility theory, linear economic systems, economic theory, economic statistics, methodology, and history of economic theory. Above we have extensively described Morgenstern’s early work on the first topic, and after the death of von Neumann in 1957, one cannot see new developments in his work in this area. For sure, there are papers written until the 1970s, but these elaborate on ideas that were already present in earlier work. A similar remark applies to Morgenstern’s work on methodology. Morgenstern’s main work in economics since the mid 1950s has been on the von Neumann model of the expanding economy. This work successfully relaxed several assumptions in the seminal von Neumann growth model from 1937. In econometrics and statistics, especially noteworthy is Morgenstern’s collaboration with Clive Granger, Nobel prize winner in 2003, on the predictability of stock market prices. This work showed that the short-run movements of prices are well-described by a simple random walk, but that for the longer-run the random walk hypothesis performs less well, and that there was a surprisingly small connection between prices and quantities, thus casting doubt on the standard competitive model of price formation. Since then, a lot of work has been done in this area; we refer to Nobel Foundation (2003) for further details.

Morgenstern not only was an arbiter between different fields of science, he also intermediated between science and the community of business and politics. He was a consultant to the Rand Corporation, the Atomic Energy Commission and the White House and he was active in business as director of the consulting firm Mathematica. This firm was



responsible for a large scale project for the military on games with incomplete information; it gave an important impulse to game theoretic research, while Morgenstern took pride in the fact that Mathematica was a profitable enterprise delivering high quality work. Morgenstern also employed his entrepreneurial talents elsewhere; he was instrumental, in 1954, in the founding of *Naval Research Logistics Quarterly* and in 1971 he founded the *International Journal of Game Theory*, the first field journal for game theory. In 1963, together with Paul Lazarsfeld, he was one of the founders of the Institute of Advanced Studies in Vienna.

As early as 1957, Morgenstern received an Honorary Doctorate from the University of Mannheim. He got a second such Doctorate, from the University of Basel, in 1960. Later he also received an honorary doctorate from the University of Vienna. In 1970, Morgenstern left Princeton for New York University. He became an honorary member of the American Economic Association and of the American Academy of Sciences in 1976. Oskar Morgenstern died on July 26, 1977, regretting that he had not received the Bank of Sweden Prize in Economic Sciences in Honor of Alfred Nobel.

## BIBLIOGRAPHY

Aumann, R. J., (1987). Game Theory. In *The New Palgrave Dictionary of Economics* (J. Eatwell, M. Milgate, P. Newman, eds.) 2, 460-482.

Aumann, R.J. and S. Hart, eds. (1992-2002) *Handbook of Game Theory with Economic Applications*, volume 1 (1992), volume 2 (1994), volume 3 (2002), North Holland Publ. Comp., Amsterdam.

Morgenstern, O. (1928). *Wirtschaftsprognose, eine Untersuchung ihrer Voraussetzungen und Möglichkeiten*, Julius Springer Verlag, Vienna.

Morgenstern, O. (1935). 'Vollkommene Voraussicht und wirtschaftliches Gleichgewicht'. *Zeitschrift für Nationalökonomie* 6(3), 337-357. (English translation by Frank Knight reprinted in Schotter, 1976).

Morgenstern, O. (1963). Limits to the Uses of Mathematics in Economics. *Mathematics and the Social Sciences*, a symposium sponsored by the American Academy of Political and Social Sciences, (J. Charlesworth (ed.), 12-29. (Reprinted in Schotter, 1976.)

Morgenstern, O. (1972). Thirteen Critical Points in Contemporary Economic Theory: An Interpretation. *Journal of Economic Literature* 10(4), 1163-1189. (Reprinted in Schotter, 1976.)

Morgenstern, O. (1976). The Collaboration between Oskar Morgenstern and John von Neumann on the Theory of Games. *Journal of Economic Literature* 14(3), 805-816.

Neumann, J. von (1928). Zur Theorie der Gesellschaftspiele. *Mathematische Annalen* 100, 295-320.

Neumann, J. von, and Morgenstern, O. (1944). *Theory of Games and Economic Behavior*. Princeton University Press, Princeton, NJ. (Page numbers refer to Third, expanded, edition printed in 1953).

Nobel Foundation (2002). The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2002 – Advanced Information

<http://www.nobel.se/economics/laureates/2002/adv.html>

Nobel Foundation (2003). The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2003 – Advanced Information

<http://www.nobel.se/economics/laureates/2003/adv.html>

Schotter, A. (1976). *The Selected Economic Writings of Oskar Morgenstern*. New York University Press.

Weintraub, E. (1992). *Toward a History of Game Theory*. Duke University Press.