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"The museum pass game and its value" revisited

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Abstract:

This paper is a reaction on Ginsburgh and Zang (2003). It reconsiders the problem where a group of museums offer a pass such that the owner can visit these museums an unlimited number of times during a fixed period of time. The problem addressed is how to share the total joint income of this pass system among the museums. Ginsburgh and Zang propose to use the Shapley value of an associated cooperative game. Arguments are provided to model this problem within the framework of bankruptcy problems.

Keywords: Museum pass problem, bankruptcy problem.

1 The Ginsburgh and Zang's model

Ginsburgh and Zang (2003) stated the following problem:" Museum passes which give visitors (tourists as well as residents) unlimited access to participating museums during a limited period of one to several days (perhaps a whole year for residents) have become very common in many European or North-American cities, regions, or even entire countries. The problem is how to share the net income from the sale of passes among the participating museums."

Ginsburgh and Zang consider the following cooperative game (N, v) to analyze this allocation problem. The set of museums that participate in the museum pass system is denoted by N. The price of the museum pass is denoted by π. Let M be the set of customers that buy a museum pass. For each $j \in M$, let $K_j \subset N$ denote the set of museums that customer j visits (at least once). The value of a coalition $S \subset N$ is then defined as $v(S) = \pi \cdot |\{j \in M : K_j \subset S\}|$, i.e., the game is based on the sum of $|M|$ unanimity games: $v = \pi \sum_{j \in M} u_{K_j}$.

The Shapley value of this game is proposed as a way to share the joint income, $|M|\pi$, from the pass system. We now provide two examples which illustrate serious drawbacks of Ginsburgh and Zang's proposal.

Example 1.1. Suppose that we have two museums, 1 and 2, with a museum pass. The regular prices of a ticket for museum 1 and 2 are 10 and 5 Euro, respectively. The price of the museum pass is 12 Euro. Assume that each person who buys the museum card goes once to each museum and that the total number of museum passes sold is 100. Using Ginsburgh and Zang's solution, each museum would obtain 600 Euro from the museum pass system. In this case museum 2 obtains more than it would have obtained if all visitors had bought regular tickets. □

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Example 1.2. Suppose again that we have two museums, 1 and 2, with a museum pass. The regular price of a ticket for both museum 1 and 2 is 10 Euro while the price of the museum pass is 15 Euro. Assume that each person who buys the museum card goes once to museum 1 and twice to museum 2 and that the total number of museum passes sold is 100. Using Ginsburgh and Zang's solution, each museum would obtain 750 Euro from the museum pass system. In this case both museums obtain the same amount, although museum 1 and 2 would have earned 1000 and 2000 Euro respectively if all visits had been on the basis of regular tickets. □

These examples clearly show that Ginsburgh and Zang's solution is not always suitable. In particular Ginsburgh and Zang's solution does not take into account possible asymmetries with respect to both the prices of the regular tickets of the museums and the number of times that a visitor (with a pass) goes to one museum. Moreover, Ginsburgh and Zang's model also has an informational drawback: it requires to know the exact list of museums that each particular pass holder visits.

2 A bankruptcy approach to museum pass problems

In this section we model the museum pass problem as addressed in Ginsburgh and Zang (2003) as a bankruptcy problem.

A *museum pass problem* is denoted by (N, π, p) where N is the (finite) set of museums that are involved in a museum pass system, π is the price of the museum pass and $p = (p_i)_{i \in N} \in \mathbb{R}^N$ is the vector of regular prices of the museums. Let (μ, m) be a realization of the museum pass system, i.e, μ is the number of museum passes sold and $m = (m_i)_{i \in N} \in \mathbb{R}^N$ measures the number of visits[1] that have been made to the individual museums using a museum pass. Since it is reasonable to assume that $\mu\pi \leq \sum_{i \in N} m_i p_i$ (all customers want to "profit" from their passes) we can define the associated bankruptcy problem (E, c) as follows. The estate E is the total revenues that are obtained from the sale of the museum passes, i.e., $E = \mu\pi$, and the vector of claims represents the total amount that each museum would have obtained if all visits had been made with a regular ticket, $c \in \mathbb{R}^N$ with $c_i = m_i p_i$ for $i \in N$.

There exist various types of allocations rules for bankruptcy problems (for a survey see Thomson (2003)). We will consider the following classic rules: the random arrival rule, the constrained equal award rule, the constrained equal loss rule and the Talmud rule. Besides, we introduce a modification of the proportional rule that takes into account minimal rights of the museums. The *proportional rule with minimal rights* assigns to each museum its minimal right plus the amount that it gets when the proportional rule is applied to the remainder (the original estate minus the sum of all minimal rights) with a new vector of claims which is the difference between the initial vector of claims and the vector of minimal rights. The *minimal right* of a museum $i \in N$ is the

[1]If a particular visitor with a museum pass visits a museum twice, this counts as two visits.

amount that remains if all other museums' claims have been fully satisfied (or 0 if the estate is not high enough to satisfy these claims).

Next we reconsider Example 1.1 and Example 1.2 using the random arrival rule and the proportional rule with minimal rights.

Example 2.1. Consider the museum pass problem introduced in Example 1.1. For the associated bankruptcy problem (E, c) we find that $E = 1200$ and $c = (1000, 500)$. Then the allocation proposed by the random arrival rule and the proportional rule with minimal rights coincide and equals $(850, 350)$. This outcome clearly takes into account the asymmetry in regular prices between the two museums. □

Example 2.2. Consider the museum problem introduced in Example 1.2. For the associated bankruptcy problem (E, c) it holds that $E = 1500$ and $c = (1000, 2000)$. Then the allocation proposed by the random arrival rule is $(500, 1000)$. For the proportional rule with minimal rights this is $(400, 1100)$. Both outcomes reflect the fact that there is a clear difference in the visiting attitude of museum pass holders although the museums are symmetric on the basis of prices. □

In the examples above we have just considered two rules: the random arrival rule and the proportional rule with minimal rights. Not without reason: the following example illustrates context-specific drawbacks of the constrained equal award rule, the constrained equal loss rule and the Talmud rule.

Example 2.3. Let (N, π, p) be a museum pass problem with $N = \{1, 2, 3, 4, 5, 6\}$, $\pi = 15$ and $p = (10, 10, 8, 5, 3, 3)$. Consider a realization (μ, m) with $\mu = 10$ and $m = (16, 11, 7, 10, 7, 5)$[2]. The associated bankruptcy problem has an estate $E = 150$ and a vector of claims $c = (160, 110, 56, 50, 21, 15)$. Table 1 provides possible allocations to this problem on the basis of several bankruptcy rules.

Rules \ Museums	1	2	3	4	5	6
Random Arrival	53.4667	42.8667	20.8000	18.8000	8.0500	5.7500
Prop. with minimal rights	58.2524	40.0485	20.3884	18.2039	7.6456	5.4612
Constrained Equal Loss	100	50	0	0	0	0
Constrained Equal Award	28.5	28.5	28.5	28.5	21.0	15.0
Talmud	39.5	39.5	28.0	25.0	10.5	7.5

Table 1: Several bankruptcy rules applied to a museum pass problem.

□

[2] Note that we, contrary to Ginsburgh and Zang, do not need data about how many times a specific visitor goes to a particular museum.

The general principle behind the constrained equal loss rule is to divide the losses of each museum (with respect to their claims) as equal as possible. In Example 2.3 we see that museums 3, 4, 5 and 6 do not get anything whereas they have visitors using the museum pass. For this reason it seems that the constrained equal loss rule is not a good solution in this specific context.

The general principle behind the constrained equal award rule is that it tries to divide the estate as equal as possible (taking the claims as upper bounds). In Example 2.3 we see that museums 5 and 6 get their claims whereas museums 1, 2, 3 and 4 get an equal amount. Assuming that the claims somehow reflect the "quality" or "attraction" of a museum[3] then the museums with higher claims are essential to make the pass system a success. For this reason the proposed equality within the "top" of museums seems an undesirable aspect and we do not consider the constrained equal award rule as a good choice to solve the museum pass problem.

The Talmud rule for this particular case boils down to the constrained equal award rule for the bankruptcy problem with the same estate but where all claims are halved. Obviously this implies that we have the same kind of reasons to discard the Talmud rule as the constrained equal award rule itself.

3 Conclusions

This paper provides a bankruptcy model for the museum pass problem introduced by Ginsburgh and Zang (2003). It is illustrated that some classical bankruptcy rules are more appropriate to solve museum pass problems than others. Contrary to the rule proposed by Ginsburgh and Zang (2003), the strength of these rules is that they take into account asymmetries with respect to prices in regular tickets and the number of visits to museums. We want to stress, however, that this paper does not provide a final general answer: it just provides a more adequate framework to analyze museum pass problems. A specific solution will typically depend on the particular features of the type of instance at hand (large or small asymmetries, informational aspects, etc.).

References

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[3]High claims can reflect that either the the number of visits is high or the price in the regular tickets is high.