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**A NOTE ON ROBINSON'S  
TEST OF INDEPENDENCE**

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## A Note on Robinson's Test of Independence

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### Abstract

In this paper we reconsider the test of independence proposed by Robinson (1991). Simulations show that the true levels of this test may deviate substantially from the size suggested by the limiting null distribution even if the sample is quite large. This implies that application of this test may lead to unreliable conclusions. We improve the small sample properties by higher order expansions and avoid timeconsuming calculations like density estimates. Therefore we discretize the available data series. Robinson's original test is improved for this case by moment corrections. Moreover, a new test is introduced that is closely related to the second order term of Robinson's test. The limiting null distribution of the new test is well approximated in small samples and the power is generally higher.

KEYWORDS: tests of independence in time-series.

### 1 Introduction

Let  $y_1, \dots, y_T$  be a sample from some ergodic stationary time-series. We want to test the hypothesis  $H$  that these observations are independent and identically distributed drawings from some unknown distribution. The alternative  $K$  is not restricted to special dependency structures. This testing problem is important in many settings, including semi-parametric analysis, where errors are often assumed to be independent. In trying to discriminate between densities  $f$  and  $h$  it is natural to use tests based upon the Kullback-Leibler pseudo distance

$$I(f, h) = \int f(x) \log \frac{f(x)}{h(x)} dx. \quad (1.1)$$

Tests based upon this measure using estimated densities are considered by e.g. Bickel and Rosenblatt (1973) and Rosenblatt (1975). An obvious disadvantage of this approach is of course the need to evaluate an integral and the complex limiting distributions arising if kernel type density estimates of  $f$  and  $h$  are plugged into (1.1). Robinson (1991) avoids the exact computation of the integral in (1.1) by approximating it by a weighted average. To be precise, his test rejects for large values of

$$I_\gamma/\hat{v} = \hat{v}^{-1} [T/2]^{-1/2} \sum_{t=1}^{T-1} c_t(\gamma) \log \left( \frac{\hat{f}(y_t, y_{t+1})}{\hat{h}(y_t)^2} \right), \quad (1.2)$$

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where  $\hat{\nu}$  denotes an estimate of the standard deviation of the limiting distribution (1.3) below,  $c_t(\gamma) = \frac{1+t\gamma}{2\gamma}$  if  $t$  is odd and  $c_t(\gamma) = \frac{1-\gamma}{2\gamma}$  if  $t$  is even. In (1.2)  $\hat{f}$  denotes a kernel estimate of the simultaneous density  $f$  of  $y_t$  and  $y_{t+1}$ ,  $\hat{h}$  of the marginal density  $h$  of  $y_t$ . The positive parameter  $\gamma$  gives different weights to the pairs  $(y_t, y_{t+1})$  when  $t$  is odd or even and is introduced to get a non-degenerate limiting distribution of  $I_\gamma$ . Although evaluation of the integral is avoided, calculation of the test statistic  $I_\gamma/\hat{\nu}$  is still rather cumbersome since it involves density estimation (with the inherent problem of choosing the bandwidth) and the choice of the somewhat artificial constant  $\gamma$ . Robinson (1991) restricts attention to  $\gamma \in (0, 1]$  and shows that, under rather strong conditions (e.g. a nonvanishing density on a compact support is required and thus normality is excluded),

$$I_\gamma \xrightarrow{d_H} N \left( 0, E \left\{ \log \frac{h(y_2)}{h(y_1)} \right\}^2 \right). \quad (1.3)$$

The proof is based on the fact that replacement of the kernel density estimators  $\hat{f}$  and  $\hat{h}$  by the true  $f$  and  $h$  yields an asymptotically negligible term. To get a non-degenerate limiting distribution in (1.3), note that  $y_t$  cannot be uniformly distributed (i.e.  $h$  has to be non-constant). Although one can prove that substitution of estimated densities by true underlying ones does not affect the limiting distribution, it has a large impact on the small sample distribution of the test statistic. Even for large sample sizes the level is quite unsatisfactory (see Table 1 and the discussion in Section 3) and hence power properties are difficult to establish.

It is the purpose of this paper to introduce simple and straightforward modifications of Robinson's test of independence that do not require cumbersome calculations like density estimation and that have nice small sample properties both under the null and the alternative. We restrict attention to categorical data by discretizing the original data set into, say  $r$ , classes. The discretization yields loss of information but, in practice, one can hardly imagine that dependencies are smoothed away. An empirical example conforms this. The discretized version of  $I_\gamma$  suffers from a similar problem as  $I_\gamma$  itself, i.e. the small sample distribution is poorly approximated by its limiting distribution. We suggest some moment corrections to get a more appropriate size. Moreover, we propose a new test statistic closely related to the second order term of Robinson's statistic. This test is simple to calculate and consistent for the same range of alternatives, it does not exclude the uniform distribution under  $H$  and has still power at local alternatives (Robinson's test has limiting power  $\alpha$  at local alternatives). This incidentally shows that the relative efficiency of the discretized version of Robinson's test with respect to our new test is equal to zero. Section 2 presents our theoretical results. In the third section we illustrate our claims with a few simulation results and an empirical example. Proofs are deferred to the Appendix. In the remainder  $[x]$  denotes the largest integer not exceeding  $x$ .

## 2 Categorical data and chi-square tests

From now on we confine attention to (possibly discretized) time-series of categorical data. Let  $A_1, \dots, A_r$  be the  $r$  possible outcomes or labels of  $y_t$ . While Robinson (1991) used kernel density estimators to approximate the true unknown underlying densities, histogram

estimators seem to be more natural in this set-up. This does not invalidate the derivation of the limiting distribution (1.3). As in Robinson (1991) we primarily direct attention to the case involving only the comparison of two succeeding values  $y_t$  and  $y_{t+1}$  ( $t = 1, \dots, T-1$ ). It is more natural to compare more succeeding values if it is at forehand known that  $y_t$  and  $y_{t+1}$  are independent (but longer dependencies may be present); this will be discussed at the end of this section. However, the power of tests based upon comparison of only two succeeding values can be considerable even in this case [cf. Gleser and Moore (1985)]. Tests of dependence over more than two succeeding values might be of interest in series that are potentially seasonal.

To be more precise put  $p_{ij} = P(y_1 \in A_i, y_2 \in A_j)$  and  $p_i = \sum_{j=1}^r p_{ij}$ , the corresponding marginal frequencies ( $i, j = 1, \dots, r$ ) and let

$$\begin{aligned} M_{ij}^* &= \#\{y_{2t} \in A_i, y_{2t+1} \in A_j, t = 1, \dots, [(T-1)/2]\} \text{ and} \\ N_{ij}^* &= \#\{y_{2t-1} \in A_i, y_{2t} \in A_j, t = 1, \dots, [T/2]\}. \end{aligned}$$

Obviously both  $M^* = (M_{ij}^*)$  and  $N^* = (N_{ij}^*)$  have multinomial distributions under  $H$  (but not under  $K$ ). Similarly to (1.3) one easily derives

$$\begin{aligned} R_\gamma &= [T/2]^{-1/2} \sum_{i,j=1}^r \left\{ \frac{1-\gamma}{2\gamma} M_{ij}^* + \frac{1+\gamma}{2\gamma} N_{ij}^* \right\} \log \frac{(M_{ij}^* + N_{ij}^*)/(T-1)}{(\#\{t: y_t \in A_i\}/T)^2} \\ &\rightarrow_{d_H} N \left( 0, \sum_{i,j=1}^r p_i p_j \left( \log \frac{p_i}{p_j} \right)^2 \right). \end{aligned} \quad (2.1)$$

Let  $\hat{v}^2$  be the consistent estimator of the variance in the RHS of (2.1) obtained by plugging in cell-frequencies for the unknown marginal probabilities. The histogram based modification of (1.2) rejects for large values of  $R_\gamma/\hat{v}$ . Under fixed alternatives

$$2\gamma[T/2]^{-1/2} R_\gamma \rightarrow_{d_K} 2 \sum_{i,j=1}^r p_{ij} \log \frac{p_{ij}}{p_i^2}. \quad (2.2)$$

This incidentally shows that the test is consistent against all alternatives where subsequent values are not independent. A more precise analysis shows that the asymptotic local power under contiguous alternatives tends to  $\alpha$ .

Note that in the case where all class probabilities are equal both  $R_\gamma$  and  $\hat{v}$  converge in probability to zero. In that case no asymptotic theory is available for  $R_\gamma/\hat{v}$ . Robinson (1991) found the behavior of  $R_0 = \lim_{\gamma \rightarrow 0} 2\gamma R_\gamma$  too cumbersome. Simulations point out that the approximation to the normal distribution is very poor for  $\gamma \approx 0$ . Even for substantial values of  $\gamma$  the normal approximation is inadequate, see Table 2 below.

The attempt of this paragraph is to propose a simple test that has a better approximation to the asymptotic null distribution, that has more power at local alternatives and that does not have the strange weighting scheme for even and odd pairs of  $(y_t, y_{t+1})$ .

Put  $n = [T/2]$ . To avoid naughty remainder terms in our test statistics replace  $y_T$  by  $y_1$  if  $T$  is odd and add an observation  $y_{T+1} = y_1$  if  $T$  is even. Define

$$\begin{aligned} M_{ij} &= \#\{y_{2t} \in A_i, y_{2t+1} \in A_j, t = 1, \dots, n\} \text{ and} \\ N_{ij} &= \#\{y_{2t-1} \in A_i, y_{2t} \in A_j, t = 1, \dots, n\}. \end{aligned}$$

Let central dots,  $\cdot$ , denote summation over indices and observe  $M_{..} = N_{..} = n$ ,  $M_{\cdot i} = N_{\cdot i}$  and  $M_{i \cdot} = N_{i \cdot}$  ( $i = 1, \dots, r$ ). Put  $\hat{p}_{ij}^{(1)} = N_{ij}/n$  and  $\hat{p}_i^{(0)} = (N_{i \cdot} + N_{\cdot i})/2n$ . A simple test based on the classical likelihood ratio, where the alternative is completely unrestricted, rejects for large values of

$$T_1 = 2n \sum_{i,j=1}^r \hat{p}_{ij}^{(1)} \log \frac{\hat{p}_{ij}^{(1)}}{\hat{p}_i^{(0)} \hat{p}_j^{(0)}}.$$

The behavior of  $T_1$  under  $H$  is well-known:

$$T_1 \xrightarrow{d_H} \chi_{r^2-r}^2.$$

Under fixed alternatives  $T_1/n$  converges to the RHS of (2.2). A disadvantage of  $T_1$  is that knowledge about the probability structure under  $K$  is discarded. In our case the alternative is not completely unrestricted because of the assumed stationarity of the time-series. Hence we know at the forehand that the marginal probabilities are equal. This property is not satisfied by the estimates  $\hat{p}_{ij}^{(1)}$  in  $T_1$  since they do not necessarily satisfy  $\hat{p}_i^{(1)} = \hat{p}_i^{(1)}$ . Optimization of the likelihood under  $K$  seems not to be feasible. A computational more attractive alternative is to use the test statistic

$$T_2 = 4n \sum_{i,j=1}^r \hat{p}_{ij}^{(2)} \log \frac{\hat{p}_{ij}^{(2)}}{\hat{p}_i^{(0)} \hat{p}_j^{(0)}} = 4n \sum_{i,j=1}^r \hat{p}_{ij}^{(2)} \log \frac{\hat{p}_{ij}^{(2)}}{(\hat{p}_i^{(0)})^2}, \quad (2.3)$$

where  $\hat{p}_{ij}^{(2)} = (M_{ij} + N_{ij})/2n$  [for the latter equality in (2.3) use  $M_{\cdot i} = N_{\cdot i}$  and  $M_{i \cdot} = N_{i \cdot}$ ]. Observe that  $\hat{p}_{ij}^{(2)}$  is a consistent estimator of  $p_{ij}$  both under  $H$  and  $K$  and  $\hat{p}_{ij}^{(2)}$  satisfies the condition of equal margins. Apart from a scaling factor of approximately  $2\sqrt{n}$  the test statistic  $T_2$  is close to  $R_0$ . In the Appendix we show that

$$T_2 \xrightarrow{d_H} \chi_{(r-1)^2}^2. \quad (2.4)$$

Under alternatives  $T_2/2n$  converges to the RHS of (2.2). The reduced number of degrees of freedom in the limiting distribution of  $T_2$ , compared to that of  $T_1$ , and the behavior of  $T_1/n$  and  $T_2/2n$  under fixed alternatives suggest that  $T_2$  is better than  $T_1$ . Numerical calculations show that the distribution of  $T_2$  is well approximated by its limiting distribution (even if the sample is rather small), see Section 3.

The test statistic  $T_2$  has several advantages over  $R_\gamma$  for several reasons. In the first place the limiting null approximation is much better. This can be explained by splitting  $R_\gamma$  into two terms

$$\begin{aligned} R_\gamma &= \frac{1}{2} n^{-1/2} \sum_{i,j=1}^r \{N_{ij}^* - M_{ij}^*\} \log \frac{(M_{ij}^* + N_{ij}^*)/(T-1)}{(\#\{t : y_t \in A_i\}/T)^2} \\ &+ \frac{n^{1/2}}{\gamma} \sum_{i,j=1}^r \frac{M_{ij}^* + N_{ij}^*}{2n} \log \frac{(M_{ij}^* + N_{ij}^*)/(T-1)}{(\#\{t : y_t \in A_i\}/T)^2}. \end{aligned} \quad (2.5)$$

The first term on the right yields the limiting distribution (2.1) and the second term vanishes under  $H$ . However, the second term is close to  $\frac{1}{4\gamma} T_2 / \sqrt{n}$  and therefore we expect a

(vanishing) bias of approximately  $\frac{1}{4\gamma}(r-1)^2/\sqrt{n}$ . For small values of  $\gamma$  this expectation is relatively large (even if  $r$  is moderate and  $T$  is large) if compared to the (near zero) variance of the asymptotic distribution and hence this distortion is expected to be important. Expression (2.5) can be used to perform a bias and variance correction of  $R_\gamma$ , as will be explained in Section 3. Secondly, while  $T_2$  still has a limiting chi-square distribution if the marginal probabilities  $p_i$  are equal, the limiting distribution of  $R_\gamma$  is degenerate in this case. Finally, although both tests are consistent against the same range of alternatives, the behavior of  $T_2$  is expected to be much better under local alternatives because of the extra scaling factor of order  $\sqrt{n}$ . In fact,  $T_2$  still has power under contiguous alternatives, while  $R_\gamma$  has limiting power  $\alpha$  in that case. This incidentally shows that the relative efficiency of  $R_\gamma$  with respect to  $T_2$  is zero.

A drawback of all the procedures above is that only two succeeding values  $y_t$  and  $y_{t+1}$  are compared. If the sample is very large it seems to be more natural to consider triples  $(y_t, y_{t+1}, y_{t+2})$  or even longer vectors. Let  $n = \lceil T/3 \rceil$ . Replace or put  $y_{3\lceil T/3 \rceil+1} = y_1$  and  $y_{3\lceil T/3 \rceil+2} = y_2$  and define

$$\begin{aligned} L_{ijk} &= \#\{y_{3t} \in A_i, y_{3t+1} \in A_j, y_{3t+2} \in A_k, t = 1, \dots, n\}, \\ M_{ijk} &= \#\{y_{3t-1} \in A_i, y_{3t} \in A_j, y_{3t+1} \in A_k, t = 1, \dots, n\}, \\ N_{ijk} &= \#\{y_{3t-2} \in A_i, y_{3t-1} \in A_j, y_{3t} \in A_k, t = 1, \dots, n\}. \end{aligned}$$

Note  $L_{i..} = M_{i..} = N_{i..}$  etc. ( $i = 1, \dots, r$ ). Put  $\hat{p}_{ijk}^{(3)} = (L_{ijk} + M_{ijk} + N_{ijk})/3n$  and  $\hat{p}_i^{(0)} = (N_{i..} + N_{.i.} + N_{..i})/3n$ . Then we can prove the following result

$$\begin{aligned} T_3 &= 6n \sum_{i,j,k=1}^r \hat{p}_{ijk}^{(3)} \log \frac{\hat{p}_{ijk}^{(3)}}{\hat{p}_i^{(0)} \hat{p}_j^{(0)} \hat{p}_k^{(0)}} = 6n \sum_{i,j,k=1}^r \hat{p}_{ijk}^{(3)} \log \frac{\hat{p}_{ijk}^{(3)}}{(\hat{p}_i^{(0)})^3} \\ &\rightarrow_{d_H} \chi_{r(r-1)^2}^2 + 2\chi_{(r-1)^2}^2. \end{aligned}$$

As above one easily verifies that  $T_3/3n$  converges to a nonzero constant like (2.2). Generalizing this procedure to even longer vectors, say of length  $m \geq 2$ , we conjecture the following limiting null distribution

$$\begin{aligned} T_m &= 2mn \sum_{i_1, \dots, i_m=1}^r \hat{p}_{i_1, \dots, i_m}^{(m)} \log \{ \hat{p}_{i_1, \dots, i_m}^{(m)} / (\hat{p}_{i_1}^{(0)} \times \dots \times \hat{p}_{i_m}^{(0)}) \} \\ &= 2mn \sum_{i_1, \dots, i_m=1}^r \hat{p}_{i_1, \dots, i_m}^{(m)} \log \{ \hat{p}_{i_1, \dots, i_m}^{(m)} / (\hat{p}_{i_1}^{(0)})^m \} \\ &\rightarrow_{d_H} \sum_{k=1}^{m-1} k \chi_{r^{m-k-1}(r-1)^2}^2. \end{aligned} \tag{2.6}$$

We are not able to give a formal proof of (2.6) for general  $m > 3$  (with  $p$  unknown). However for several  $m > 3$ ,  $r$  and non-specific choices of  $p$  the limiting distribution of  $T_m$  has been exactly calculated under the chosen probability vector  $p$  and the resulting limiting distribution is always the RHS of (2.6). As before,  $T_m/mn$  converges to a non-zero constant like (2.2) under  $K$ .

Of course we can also use the statistics derived above for non-categorical data. Divide  $\mathbf{R}$  into  $r$  disjoint intervals  $A_1, \dots, A_r$  and replace the original time-series by one that only yields the interval labels of the original observations. Proceed as before. This discretization yields information loss and tests derived in this manner are not consistent against all alternatives. However, if the cells  $A_1, \dots, A_r$  are chosen in a suitable manner and if  $r$  is not too small then one can hardly imagine that existing dependencies are smoothed away by discretization. Moreover, one way out is to increase the number of cells in an appropriate manner as the sample size increases [cf. e.g. Morris (1975), Kallenberg et al. (1985) and Inglot et al. (1990)].

### 3 Simulation results and an empirical example

In this section we present some simulation results concerning the various tests introduced in Section 2. First of all we show that the true size of Robinson's original test of independence may be far away from the size suggested by the limiting distribution (1.3). We simulated several truncated normal distributions which satisfy the restrictions of Robinson (1991) and used also the kernels and bandwidths suggested in this paper. Our results are summarized in Table 1. It is clear from this table that Robinson's test of independence is very sensible w.r.t. the behavior of the true underlying density at the borders of the underlying compact support. If the density is small at these borders the test statistic rejects the hypothesis too often. Probably asymptotics do not work even for samples as large as  $T = 1000$  because of the underlying assumption that densities are bounded away from zero. On the other hand if the underlying density is large at the borders of the support the test is very conservative. Probably this is due to the density estimation routine that runs into problems at the boundary of the support.

The considerations above suggest that the rejection probabilities for several exchange rates found by Robinson (1991) are much too high since the distributions of exchange rates are quite smooth. The true size of the test for these type of marginal densities is largely underestimated by the size suggested by the limiting distribution. Hence we also expect that the power properties of this test are rather bad. This conclusion is supported by a small scale simulation study (not reported here). The power of Robinson's test for (truncated) ARMA and GARCH alternatives, like the ones described below, does not exceed the true size very much.

The remainder of this section is devoted to the behavior of the discretized versions of Robinson's test of independence. First we consider the size of the tests by applying them to categorical data. We have chosen six probability vectors under  $H$ :  $\pi = (0.20, 0.20, 0.20, 0.20, 0.20)$ ,  $\pi = (0.15, 0.20, 0.30, 0.20, 0.15)$ ,  $\pi = (0.25, 0.20, 0.10, 0.20, 0.25)$ ,  $\pi = (0.33, 0.33, 0.33)$ ,  $\pi = (0.25, 0.50, 0.25)$  and  $\pi = (0.40, 0.20, 0.40)$ . Note that the first and fourth probability vector imply that both  $R_\gamma$  and  $\hat{v}$  tend to zero in probability and consequently tests based on  $R_\gamma/\hat{v}$  have unknown limiting size (more work has to be done to obtain the limiting distribution in this case). We have chosen sample sizes  $T = 1000$  and  $T = 5000$ . In this way expected cell frequencies are never less than 10 (if  $m = 2$ ) and hence application of asymptotic theory seems reasonable. Equation (2.5) is used to correct  $R_\gamma$  for bias and variance. To get the adapted version of  $R_\gamma$



(column 'adapted') we subtract  $\frac{1}{4\gamma}(r-1)^2/\sqrt{n}$  from  $R_\gamma$  and divide by  $\sqrt{\hat{v}^2 + \frac{2(r-1)^2}{4n}}$  instead of by  $\hat{v}$ . Table 2 indicates that the unadapted  $R_\gamma$  behaves poorly under  $H$ , especially for small  $\gamma$ . (Note that standard errors for the given estimated sizes are approximately 0.001.) The bias and variance correction assures a size approximately equal to the limiting theoretical size if true cell probabilities are unequal. The limiting case  $\gamma = \infty$  behaves rather well. In the case of equal probabilities (for  $R_\gamma$  not motivated by any theory) we still get a large deviation. The estimated size of the test based on  $T_1$  or  $T_2$  resembles the theoretical true value of 0.05.  $T_3$  behaves a little bit worse if  $T = 1000$  and  $r = 5$ , which is due to the fact that the probability of empty cells is relatively large. With less cells or more observations  $T_3$  behaves appropriate.

In order to compare power properties of the tests discussed above, we have simulated two ARMA models and two GARCH models based upon the equations

$$y_t = \rho y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}, \quad \varepsilon_t \sim N(0, \sigma^2),$$

$$y_t = \sqrt{h_t} \varepsilon_t, \quad h_t = \sigma^2 + \beta h_{t-1} + \alpha y_{t-1}^2, \quad \varepsilon_t \sim N(0, 1),$$

with parameters  $(\rho, \theta) = (.08, .02)$  respectively  $(\rho, \theta) = (.02, .08)$  and  $(\alpha, \beta) = (.2, .7)$  and  $(\alpha, \beta) = (.4, .5)$ . The scale parameter  $\sigma$  is chosen such that the unconditional variance of the process is equal to one. Classes are chosen symmetrically around zero. Class-borders are chosen such that, under the assumption of standard normal marginals, the class-probabilities  $\pi$  used before are obtained.

Because of the bad size properties of  $R_\gamma/\hat{v}$ , we only used the adapted version. The results based upon five classes are reported in Table 3. Simulations using only three classes yield a similar picture. As expected it turns out that  $T_2$  has more power than  $R_\gamma$  and  $T_1$ . Recall that comparison of  $T_2$  and  $R_\gamma$  is not fair when class probabilities are equal (because of the bad size properties of the latter). However, even in that case, the power of the  $R_\gamma$  tests hardly deviates from the test based upon  $T_2$  while the  $R_\gamma$  sizes are much larger. Furthermore, while  $R_\infty$  has the best size properties of all  $R_\gamma$  statistics, it has the worst power properties. For  $R_1$  it is just the other way around. This suggests that  $\gamma$  induces a trade-off between the behavior under  $H$  and  $K$ .  $T_2$  and  $T_3$  do not suffer from this drawback. The simulations also suggest that  $T_3$  has more power if longer dependencies are present, for instance in GARCH-processes. Finally  $T_2$  and  $T_3$  behave best if cell-probabilities are almost equal.

We apply the tests above to the same exchange rates as in Robinson (1991), i.e. to logarithmic differences of daily, weekly and monthly observations of the German Mark, Japanese Yen, Swiss Franc and the British Pound exchange rates with respect to the US Dollar (January 2, 1974—June 28, 1985). To make a comparison with the simulation results possible and since the daily and weekly number of observations ( $T = 2997$  respectively  $T = 599$ ) is rather large, we have chosen five classes. In case of monthly observations ( $T = 137$ ) three classes are more appropriate. Class borders are chosen such that, under the assumption of normal marginals, the classes would be equiprobable. We report p-values of the various tests in Table 4. The tests  $T_2$  and  $T_3$  reject independence for all currencies at daily and weekly frequency and have often much smaller p-values than the  $R_\gamma$  statistics. Therefore we prefer  $T_2$  and  $T_3$ . Aggregation of exchange rates decreases the impact of dependency structures, compare e.g. Drost and Nijman (1993). This is supported by p-

values based on monthly observations. Then the conclusion of rejection is less obvious. The Japanese Yen exchange rate series is an exception:  $T_2$  and  $T_3$  clearly reject. This supports the conclusion of several studies that this series seems to be IGARCH, implying long-term dependencies.

#### 4 Conclusions

Simulations indicate that the small sample properties of Robinson's (1991) test of independence may deviate a lot from the asymptotic limiting distribution. We indicate that the problem arises from slowly vanishing higher order terms, caused by the estimation of the unknown densities. In the case of discretized data we are able to compute limiting distributions for these higher order terms and as a consequence suggest a correction. It is even possible to make this higher order term leading and this yields a much more natural test statistic. This test performs much better, both from a theoretical point of view as in simulations. Power properties of the new test dominate Robinson's and the small sample properties are satisfactory. In contrast to the test based on  $R_\gamma$ , the latter holds especially if class probabilities are almost equal. For the general case, not restricted to discretized data and using kernel density estimators, one easily verifies that there is also an influential term of lower order in small and moderate samples. Future research in this direction is recommended.

#### Appendix

PROOF of (2.4): It is convenient to define the normalized probability vector  $\hat{q} = (\hat{q}_{11}, \hat{q}_{12}, \dots, \hat{q}_{rr})^T$ , the  $r \times r$  projection matrix  $P$  and the  $r^2 \times r^2$  matrix  $C$  by their components ( $i, j, k, l = 1, \dots, r$ )

$$\begin{aligned}\hat{q}_{ij} &= \frac{\sqrt{2n} \hat{p}_{ij}^{(2)} - p_{ij}}{\sqrt{p_{ij}}} = \frac{1}{\sqrt{2n}} \sum_{t=1}^{2n} \frac{I\{(y_t, y_{t+1}) \in A_i \times A_j\} - p_{ij}}{\sqrt{p_{ij}}} \\ P_{ij} &= \sqrt{p_i p_j}, \\ C_{(i-1)r+j, (k-1)r+l} &= \delta_{j=k} \sqrt{p_i} + \delta_{l=i} \sqrt{p_j k}.\end{aligned}$$

Let  $\otimes$  denote Kronecker product and observe that the central limit theorem for 1-dependent variables yields

$$\hat{q} \rightarrow_{d_H} N(0, I + C - 3P \otimes P).$$

Note that  $P$  is a projection matrix of rank 1. Define the projection matrix  $M$  as the sum of two projection matrices of rank  $r-1$

$$M = P \otimes (I - P) + (I - P) \otimes P,$$

and observe that, by a Taylor-expansion of  $T_2$ ,

$$T_2 = 4n \sum_{i,j=1}^r \hat{p}_{ij}^{(2)} \log \frac{\hat{p}_{ij}^{(2)}}{p_{ij}} - 8n \sum_{i=1}^r \hat{p}_i^{(0)} \log \frac{\hat{p}_i^{(0)}}{p_i}$$

$$\begin{aligned}
&= 2n \sum_{i,j=1}^r \frac{(\hat{p}_{ij}^{(2)} - p_{ij})^2}{p_{ij}} - 4n \sum_{i=1}^r \frac{(\hat{p}_i^{(0)} - p_i)^2}{p_i} + o_p(1) \\
&= \hat{q}^T (I - M) \hat{q} + o_p(1), \quad n \rightarrow \infty.
\end{aligned}$$

Tedious but straightforward calculations show

$$(I - M)(I + C - 3P \otimes P)(I - M) = I - M - P \otimes P,$$

which is a projection matrix of rank  $r^2 - 2(r-1) - 1 = (r-1)^2$ . This completes the proof.  $\square$   
**REMARK** It is clear from the derivation above that our results easily generalize to other distance measures. Let

$$T_m(\lambda) = 2mnI^\lambda(\hat{p}^{(m)}, \hat{p}^{(0)})$$

be the statistic based upon the directed divergence measure [cf. Rényi (1961)]

$$I^\lambda(q, p) = \frac{1}{\lambda(\lambda+1)} \sum q \left\{ \left( \frac{q}{p} \right)^\lambda - 1 \right\}.$$

(The summation  $\sum$  of a vector denotes the sum of the components. Products, ratios and other operations are defined componentwise.) Define  $I^\lambda$  by continuity when  $\lambda = -1, 0$  and note  $T_m(0) = T_m$ . Moreover, since  $T_m(\lambda) = T_m + o_p(1)$  under  $H$ , we obtain the same limiting null distribution for the whole class  $T_m(\lambda)$ . This class is close to the class of Cressie-Read (1984) statistics.  $T_m(\lambda)/m$  resembles Pearson's  $\chi^2$ -test for  $\lambda = 1$ , the likelihood ratio statistic for  $\lambda = 0$ , Freeman Tukey's statistic for  $\lambda = -\frac{1}{2}$ , the modified likelihood ratio statistic for  $\lambda = -1$  and Neyman's modified  $\chi^2$ -statistic for  $\lambda = -2$ . To compare the behavior of  $T_m(\lambda)$  under alternatives observe that

$$T_m(\lambda)/mn \rightarrow 2I^\lambda((p_{i_1 \dots i_m}), (p_{i_1} \times \dots \times p_{i_m})).$$

Since the limiting null distributions of  $T_m(\lambda)$  are equal, large values of the term on the right are preferred. In case of high peaks of  $p_{i_1 \dots i_m}/(p_{i_1} \times \dots \times p_{i_m})$  high values of  $\lambda$  are preferred and in case of deep dips low values of  $\lambda$ , see also the discussion in Cressie and Read (1984), Moore (1984), Drost et al. (1989) and Kallenberg and Drost (1992). To control the level in small and moderate samples we suggest to choose  $\lambda$  between  $-\frac{1}{2}$  and 1.

#### References

- BICKEL, P.J. and ROSENBLATT, M. (1973), "On some global measures of the deviations of density function estimates", *Annals of Statistics*, 1, 1071-1095.  
 CRESSIE, N. and READ, T.R.C. (1984), "Multinomial goodness-of-fit tests", *Journal of the Royal Statistical Society, Series B*, 46, 440.  
 DROST, F.C., KALLENBERG, W.C.M., MOORE, D.S. and OOSTERHOFF, J. (1989), "Power approximations to multinomial tests of fit", *Journal of the American Statistical Association*, 84, 130-141.  
 DROST, F.C. and NIJMAN, TH. E. (1993), "Temporal aggregation of GARCH processes", *Econometrica*, forthcoming.

- GLESER, L.J. and MOORE, D.S. (1985), "The effect of positive dependence on chi-squared tests for categorical data", *Journal of the Royal Statistical Society, Series B*, **47**, 459-465.
- INGLOT, T., JURLEWICZ, T. and LEDWINA, T. (1990), "Asymptotics for multinomial goodness of fit tests for simple hypothesis", *Theory of Probability and its Applications*, **35**, 771-777.
- KALLENBERG, W.C.M., OOSTERHOFF, J. and SCHRIEVER, B.F. (1985), "The number of classes in chi-Squared goodness-of-fit tests", *Journal of the American Statistical Association*, **80**, 959-968
- KALLENBERG, W.C.M. and DROST, F.C. (1992), "Comparison of tests and local families", To appear.
- MOORE, D.S. (1984), "Measures of lack of fit from tests of chi-squared type", *Journal of Statistical Planning and Inference*, **10**, 151-166.
- MORRIS, C. (1975), "Central limit theorems for multinomial sums", *Annals of Statistics*, **3**, 165-188.
- RÉNYI, A. (1961), "On measures of entropy and information", *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, **1**, 547-561.
- ROBINSON, P.M. (1991), "Consistent nonparametric entropy-based testing", *Review of Economic Studies*, **58**, 437-453.
- ROSENBLATT, M. (1975), "A quadratic measure of deviation of two-dimensional density estimates and a test of independence", *Annals of Statistics*, **3**, 1-14.

support\ $\gamma$	1/8	1/4	1/2	1	$\infty$
$[-1, 1]$	0.000	0.000	0.000	0.000	0.048
$[-1.5, 1.5]$	0.092	0.053	0.036	0.039	0.055
$[-1.75, 1.75]$	0.622	0.359	0.160	0.085	0.053
$[-2, 2]$	0.938	0.710	0.317	0.143	0.055
$[-2.5, 2.5]$	1.000	0.959	0.600	0.240	0.053
$[-3, 3]$	1.000	0.989	0.722	0.308	0.050
$\mathbf{R}^\dagger$	1.000	0.998	0.776	0.333	0.061

Table 1: Estimated true levels of  $I_\gamma/\hat{v}$  for  $\gamma = \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1$  and  $\infty$  using critical values from the normal limiting distribution. The estimates are based upon 1000 replications and each sample consists of 1000 independent drawings from a truncated standard normal distribution. The bandwidth is chosen as  $T^{-\frac{1}{3}}$  in the estimation of  $h$ , the univariate density, and as  $T^{-\frac{1}{4}}$  in the estimation of  $f$ , the bivariate density.

$\dagger$  No asymptotic theory available.

	$T = 1000$ adapted		$T = 5000$ adapted		$T = 1000$ adapted		$T = 5000$ adapted	
	$\pi = (0.20, 0.20, 0.20, 0.20, 0.20)^\dagger$				$\pi = (0.33, 0.33, 0.33)^\dagger$			
$R_{\frac{1}{2}}$	1.000	0.069	1.000	0.067	0.880	0.069	0.880	0.074
$R_{\frac{1}{4}}$	0.991	0.074	0.991	0.071	0.714	0.071	0.716	0.076
$R_{\frac{1}{2}}$	0.895	0.088	0.893	0.086	0.489	0.079	0.494	0.084
$R_1$	0.615	0.112	0.614	0.114	0.305	0.095	0.311	0.101
$R_\infty$	0.146	0.146	0.146	0.146	0.126	0.126	0.132	0.132
$T_1$	0.054	-	0.051	-	0.050	-	0.051	-
$T_2$	0.053	-	0.050	-	0.049	-	0.051	-
$T_3$	0.072	-	0.053	-	0.052	-	0.050	-
	$\pi = (0.25, 0.20, 0.10, 0.20, 0.25)$				$\pi = (0.25, 0.50, 0.25)$			
$R_{\frac{1}{2}}$	0.904	0.064	0.513	0.054	0.209	0.055	0.103	0.052
$R_{\frac{1}{4}}$	0.571	0.058	0.228	0.053	0.111	0.053	0.071	0.050
$R_{\frac{1}{2}}$	0.260	0.056	0.118	0.051	0.077	0.052	0.059	0.049
$R_1$	0.131	0.055	0.078	0.051	0.063	0.051	0.054	0.049
$R_\infty$	0.055	0.055	0.051	0.051	0.052	0.052	0.049	0.040
$T_1$	0.055	-	0.050	-	0.053	-	0.051	-
$T_2$	0.053	-	0.050	-	0.049	-	0.052	-
$T_3$	0.088	-	0.055	-	0.052	-	0.050	-
	$\pi = (0.15, 0.20, 0.30, 0.20, 0.15)$				$\pi = (0.40, 0.20, 0.40)$			
$R_{\frac{1}{2}}$	0.897	0.063	0.504	0.055	0.267	0.057	0.119	0.052
$R_{\frac{1}{4}}$	0.563	0.058	0.223	0.053	0.133	0.053	0.078	0.050
$R_{\frac{1}{2}}$	0.253	0.056	0.116	0.051	0.083	0.052	0.063	0.050
$R_1$	0.127	0.055	0.079	0.052	0.065	0.052	0.056	0.051
$R_\infty$	0.055	0.055	0.051	0.051	0.052	0.052	0.051	0.051
$T_1$	0.054	-	0.050	-	0.052	-	0.050	-
$T_2$	0.053	-	0.052	-	0.051	-	0.050	-
$T_3$	0.081	-	0.056	-	0.053	-	0.050	-

Table 2: Estimated true levels of several test statistics using critical values from asymptotic distributions. The estimates are based upon 40000 replications and for sample sizes  $T = 1000$  and  $T = 5000$ .  $\pi$  denotes the vector of marginal class-probabilities. Level  $\alpha = 0.05$ .

<sup>†</sup> Note that equal cell-probabilities cause a degenerate asymptotic distribution of  $R_\gamma$ . Critical values are, however, still based upon formula (2.1).

	$T = 1000$				$T = 5000$			
	ARMA	ARMA	GARCH	GARCH	ARMA	ARMA	GARCH	GARCH
	$\rho = .08$ $\theta = .02$	$\rho = .02$ $\theta = .08$	$\alpha = .2$ $\beta = .7$	$\alpha = .4$ $\beta = .5$	$\rho = .08$ $\theta = .02$	$\rho = .02$ $\theta = .08$	$\alpha = .2$ $\beta = .7$	$\alpha = .4$ $\beta = .5$
	$\pi = (0.20, 0.20, 0.20, 0.20, 0.20)^\dagger$							
$R_{\frac{1}{8}}$	0.379	0.387	0.700	0.998	0.994	0.996	1.000	1.000
$R_{\frac{1}{4}}$	0.368	0.372	0.651	0.988	0.991	0.992	1.000	1.000
$R_{\frac{1}{2}}$	0.336	0.339	0.532	0.907	0.973	0.974	0.989	1.000
$R_1$	0.284	0.287	0.366	0.638	0.868	0.865	0.857	0.993
$R_\infty$	0.165	0.166	0.108	0.056	0.230	0.232	0.095	0.051
$T_1$	0.162	0.151	0.327	0.948	0.773	0.774	0.992	1.000
$T_2$	0.342	0.348	0.683	1.000	0.993	0.994	1.000	1.000
$T_3$	0.292	0.278	0.705	1.000	0.963	0.966	1.000	1.000
	$\pi = (0.25, 0.20, 0.10, 0.20, 0.25)$							
$R_{\frac{1}{8}}$	0.303	0.296	0.474	0.991	0.879	0.875	0.986	1.000
$R_{\frac{1}{4}}$	0.206	0.204	0.356	0.968	0.580	0.566	0.885	1.000
$R_{\frac{1}{2}}$	0.125	0.122	0.210	0.846	0.263	0.259	0.535	1.000
$R_1$	0.084	0.079	0.123	0.534	0.130	0.130	0.235	0.977
$R_\infty$	0.053	0.051	0.060	0.071	0.052	0.055	0.053	0.074
$T_1$	0.158	0.150	0.225	0.857	0.740	0.733	0.918	1.000
$T_2$	0.332	0.320	0.497	0.994	0.990	0.989	0.999	1.000
$T_3$	0.299	0.296	0.535	0.994	0.950	0.948	1.000	1.000
	$\pi = (0.15, 0.20, 0.30, 0.20, 0.15)$							
$R_{\frac{1}{8}}$	0.340	0.324	0.741	0.998	0.919	0.915	1.000	1.000
$R_{\frac{1}{4}}$	0.239	0.224	0.540	0.972	0.642	0.622	0.982	1.000
$R_{\frac{1}{2}}$	0.142	0.132	0.280	0.775	0.306	0.290	0.736	1.000
$R_1$	0.090	0.087	0.130	0.379	0.143	0.141	0.327	0.911
$R_\infty$	0.059	0.054	0.046	0.033	0.050	0.054	0.044	0.028
$T_1$	0.161	0.160	0.476	0.983	0.794	0.784	0.999	1.000
$T_2$	0.363	0.355	0.831	1.000	0.995	0.995	1.000	1.000
$T_3$	0.317	0.316	0.862	1.000	0.972	0.972	1.000	1.000

Table 3: Estimated true powers several test statistics using critical values from asymptotic distributions. For the  $R_\gamma$ -statistic we only consider the corrected version. The estimates are based upon 10000 replications and sample sizes  $T = 1000$  and  $T = 5000$ . Class-borders are chosen such that, under the assumption of normal marginals,  $\pi$  denotes the vector of class-probabilities. Level  $\alpha = 0.05$ .

<sup>†</sup> Note that equal cell-probabilities cause a degenerate asymptotic distribution of  $R_\gamma$ . Critical values are, however, still based upon formula (2.1). Note: the true size is not  $\alpha$  in this case, see Table 2.

	DM/\$	JY/\$	SF/\$	BP/\$
daily	$r = 5$			
$R_{\frac{1}{2}}$	0.000	0.000	0.000	0.000
$R_{\frac{1}{4}}$	0.000	0.000	0.000	0.000
$R_{\frac{1}{2}}$	0.061	0.000	0.000	0.000
$R_1$	0.262	0.005	0.010	0.000
$R_{\infty}$	0.605	0.431	0.180	0.084
$T_1$	0.001	0.000	0.000	0.000
$T_2$	0.000	0.000	0.000	0.000
$T_3$	0.000	0.000	0.000	0.000
weekly	$r = 5$			
$R_{\frac{1}{2}}$	0.000	0.000	0.000	0.000
$R_{\frac{1}{4}}$	0.000	0.000	0.000	0.000
$R_{\frac{1}{2}}$	0.000	0.001	0.000	0.035
$R_1$	0.000	0.096	0.018	0.388
$R_{\infty}$	0.287	0.671	0.579	0.894
$T_1$	0.011	0.002	0.110	0.227
$T_2$	0.000	0.000	0.000	0.003
$T_3$	0.000	0.000	0.000	0.000
monthly	$r = 3$			
$R_{\frac{1}{2}}$	0.000	0.000	0.000	0.000
$R_{\frac{1}{4}}$	0.000	0.000	0.000	0.000
$R_{\frac{1}{2}}$	0.000	0.000	0.000	0.001
$R_1$	0.000	0.000	0.000	0.108
$R_{\infty}$	0.000	0.348	0.002	0.748
$T_1$	0.140	0.036	0.109	0.413
$T_2$	0.516	0.000	0.551	0.108
$T_3$	0.263	0.000	0.276	0.301

Table 4: Application to standardized logarithmic differences of daily ( $T = 2997$ ), weekly ( $T = 599$ ) and monthly exchange rates ( $T = 137$ ). Class-borders are chosen such that, under the assumption of normal marginals, the classes would be equiprobable. The p-values reported are based on the asymptotic limiting null distribution. For the  $R_\gamma$ -statistics we only used the adapted version.



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