

## How to define UMVU

Drost, F.C.

*Publication date:*  
1988

[Link to publication](#)

*Citation for published version (APA):*

Drost, F. C. (1988). *How to define UMVU*. (Research memorandum / Tilburg University, Department of Economics; Vol. FEW 362). Unknown Publisher.

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

### Take down policy

If you believe that this document breaches copyright, please contact us providing details, and we will remove access to the work immediately and investigate your claim.

HOW TO DEFINE UMVU

F.C. Drost

FEW 362

Tilburg University

November 1988

**abstract.** Consider the estimation of some  $m$ -vector  $\psi(\theta)$ . The usual definition of the UMVU estimator  $\hat{\psi}_0$  requires that  $\hat{\psi}_0$  is more concentrated than any other unbiased estimator. The comparison of covariance matrices, however, is unnecessarily complicated. It is shown that  $\hat{\psi}_0$  is the UMVU estimator iff the components of  $\hat{\psi}_0$  have smallest variances (or even the smallest sum of variances) in the class of unbiased estimators. Hence a componentwise definition of UMVU estimators seems to be more appropriate. A similar result holds true for BLUE's.

**Keywords :** UMVU estimator, BLUE, concentration ellipsoid, mean-squared error.

**AMS 1980 subject classification :** 62F10.

## 1. INTRODUCTION

Suppose  $Y = (Y_1, \dots, Y_n)'$  has distribution  $P_\theta$ , where  $\theta \in \Theta$  is an unknown nuisance parameter. An important case is i.i.d. sampling from some distribution  $F_\theta$ . Another example is a sample from a linear model. In the latter case the  $Y_i$ 's are often neither independent nor identically distributed. In the general setup we consider the estimation of some vector-valued parameter  $\psi(\theta) = (\psi_1(\theta), \dots, \psi_m(\theta))' \in \mathbb{R}^m$ . There are a lot of estimators and we inquire whether there exists some optimal estimator  $\hat{\psi}_0$ . Usually optimality is defined in terms of concentration ellipsoids (cf. Malinvaud (1970)). Let  $G$  be some class of estimators, then  $\hat{\psi}_0 \in G$  is called  $G$ -optimal if the concentration ellipsoid of any estimator  $\hat{\psi} \in G$  covers the concentration ellipsoid of  $\hat{\psi}_0$  or equivalently if, for each  $c \in \mathbb{R}^m$ , the mean-squared error (MSE) of  $c'\hat{\psi}$  is larger than the MSE of  $c'\hat{\psi}_0$ .

DEFINITION 1.1. The estimator  $\hat{\psi}_0 \in G$  is called  $G$ -optimal if, for all  $\theta \in \Theta$  and  $\hat{\psi} \in G$ ,

$$(1.1) \quad E_\theta(\hat{\psi} - \psi(\theta))(\hat{\psi} - \psi(\theta))' - E_\theta(\hat{\psi}_0 - \psi(\theta))(\hat{\psi}_0 - \psi(\theta))' \geq 0. \quad \square$$

The comparison of matrices in (1.1) is rather complicated. A componentwise evaluation of estimators is more simple.

DEFINITION 1.2. The estimator  $\hat{\psi}_0 = (\hat{\psi}_{01}, \dots, \hat{\psi}_{0m})' \in G$  is called componentwise  $G$ -optimal if, for all  $\theta \in \Theta$ ,  $\hat{\psi} = (\hat{\psi}_1, \dots, \hat{\psi}_m)' \in G$  and  $i=1, \dots, m$ ,

$$(1.2) \quad E_\theta(\hat{\psi}_1 - \psi_1(\theta))^2 - E_\theta(\hat{\psi}_{01} - \psi_1(\theta))^2 \geq 0. \quad \square$$

In stead of (1.1) or (1.2) we consider also a criterion based on the sum of MSE's.

DEFINITION 1.3. The estimator  $\hat{\psi}_0 \in G$  is called weakly  $G$ -optimal if, for all  $\theta \in \Theta$  and  $\hat{\psi} \in G$ ,

$$(1.3) \quad E_\theta \|\hat{\psi} - \psi(\theta)\|^2 - E_\theta \|\hat{\psi}_0 - \psi(\theta)\|^2 \geq 0. \quad \square$$

Implicitly the three definitions require that  $E_\theta \|\hat{\psi}_0 - \psi(\theta)\|^2$  is finite for all  $\theta \in \Theta$ . Obviously (1.1)  $\Rightarrow$  (1.2)  $\Rightarrow$  (1.3). It is not difficult to invent classes of estimators such that the reverse implications do not hold. This motivates the different notions of optimality. Commonly the strong

Definition 1.1 is taken as a basis, then optimality remains invariant under linear transformations.

In this paper we investigate whether the three definitions of optimality are very different. In Section 2 some results concerning the different notions of optimality are given. Under general conditions it is shown that the optimal estimator is unique (Proposition 2.1) and that the Definitions 1.1-1.3 are equivalent, i.e. (1.1)  $\Leftrightarrow$  (1.2)  $\Leftrightarrow$  (1.3) (Proposition 2.2 and Theorem 2.3). Although one of the assumptions is rather complicated the main ideas and proofs are simple. In Section 3 we consider the important class of unbiased estimators  $G_u$  of  $\psi(\theta)$ . Usually only Definition 1.1 is given to define the uniformly minimum variance unbiased (UMVU) estimator of  $\psi(\theta)$ . The other definitions are ignored or they are not connected to Definition 1.1. The results of Section 2 lead to our main statement: a weakly  $G_u$ -optimal estimator is also  $G_u$ -optimal. Componentwise UMVU implies UMVU (Theorem 3.1). The proof is based on a generalization of the Gauss-Markov Theorem (cf. Rao (1976)). Similar results hold true for several subclasses of  $G_u$ . E.g. the best linear unbiased estimator (BLUE) can also be characterized componentwise (Corollary 3.2). A complicated comparison of covariance matrices is overdone in these cases and Definition 1.2 (or 1.3) provides a more handsome requirement.

## 2. GENERAL THEORY

If a  $G$ -optimal estimator exists then it is easily verified that all weakly  $G$ -optimal estimators are  $G$ -optimal. This implies the equivalence of Definitions 1.1-1.3. Similarly the existence of a componentwise  $G$ -optimal estimator implies the equivalence of Definitions 1.2 and 1.3. However, the existence of a (componentwise)  $G$ -optimal estimator is not guaranteed and it may be hard to prove that some estimator is (componentwise)  $G$ -optimal. We proceed in a different way to show that the different optimality definitions are equivalent, we derive some general conditions upon the class of estimators  $G$ .

Identify estimators that are equal a.s.  $\{P_\theta : \theta \in \Theta\}$ , let  $G$  be some class of estimators of  $\psi(\theta)$  and suppose  $\hat{\psi}_1$  and  $\hat{\psi}_2$  are elements of  $G$ . Often the weighted average  $\hat{\psi}_\alpha = (1-\alpha)\hat{\psi}_1 + \alpha\hat{\psi}_2$  belongs to  $G$  for all  $0 \leq \alpha \leq 1$ . Under the somewhat more general condition

A1. For each  $\hat{\psi}_1, \hat{\psi}_2 \in G$  there exists  $\alpha \in (0,1)$  such that the estimator  $\hat{\psi}_\alpha = (1-\alpha)\hat{\psi}_1 + \alpha\hat{\psi}_2$  belongs to  $G$ .  $\square$

we show that the  $G$ -best estimator is unique (if it exists).

PROPOSITION 2.1. Assume A1. If  $\hat{\psi}_0 \in G$  is a weakly  $G$ -optimal estimator then any solution of (1.1), (1.2) or (1.3) is equal to  $\hat{\psi}_0$  a.s.  $\{P_\theta : \theta \in \Theta\}$

PROOF.  $G$ -optimality in the sense of (1.1) and (1.2) implies weak  $G$ -optimality; it suffices to prove the proposition for the latter case. Let  $\hat{\psi} \in G$  be any weakly  $G$ -optimal estimator. Apply A1 to  $\hat{\psi}$  and  $\hat{\psi}_0$  and let  $\hat{\psi}_\alpha \in G$  be the corresponding estimator. Then, for all  $\theta \in \Theta$ ,

$$0 \leq E_\theta \|\hat{\psi}_\alpha - \psi(\theta)\|^2 - E_\theta \|\hat{\psi}_0 - \psi(\theta)\|^2 = -\alpha(1-\alpha)E_\theta \|\hat{\psi}_0 - \hat{\psi}\|^2$$

implying  $\hat{\psi}_0 = \hat{\psi}$  a.s.  $\{P_\theta : \theta \in \Theta\}$ .  $\square$

REMARK 2.1. If a componentwise  $G$ -optimal estimator exists condition A1 can be relaxed. Then a componentwise analogon of A1 suffices to prove Proposition 2.1 (use the equivalence of Definitions 1.2 and 1.3).  $\square$

Let  $\hat{\psi}_1$  and  $\hat{\psi}_2$  be elements of  $G$  and define the estimator  $\hat{\psi}$  by replacing one component of  $\hat{\psi}_1$  by the corresponding component of  $\hat{\psi}_2$ . Since estimators are usually derived componentwise it seems natural to require that  $\hat{\psi} \in G$ . This leads to the following assumption:

A2. For each  $\hat{\psi}_1, \hat{\psi}_2 \in G$  and  $\delta_1, \dots, \delta_m \in \{0,1\}$  the estimator  $\hat{\psi}_\delta = ((1-\delta_1)\hat{\psi}_{11} + \delta_1\psi_{21}, \dots, (1-\delta_m)\hat{\psi}_{1m} + \delta_m\psi_{2m})'$  belongs to  $G$ .  $\square$

Condition A2 implies the equivalence of Definitions 1.2 and 1.3.

PROPOSITION 2.2. Assume A2. If  $\hat{\psi}_0 \in G$  is a weakly  $G$ -optimal estimator then  $\hat{\psi}_0$  is componentwise  $G$ -optimal.  $\square$

To prove that the three definitions of optimality are equivalent we need a very strong assumption. We suppose that, for each  $\eta \in \Theta$  and  $\hat{\psi}_1, \hat{\psi}_2 \in G$ , some estimator  $\hat{\psi}_\eta \in G$  exists which is locally more concentrated than  $\hat{\psi}_1$  and  $\hat{\psi}_2$  under  $P_\eta$ . (It is not required that  $\hat{\psi}_\eta$  is better than  $\hat{\psi}_1$  or  $\hat{\psi}_2$  under  $P_\theta$  when  $\theta \neq \eta$  !)

A3. For each  $\eta \in \Theta$ ,  $\hat{\psi}_1, \hat{\psi}_2 \in G$  there exists  $\hat{\psi}_\eta \in G$  such that, for  $\hat{\psi} = \hat{\psi}_1$  and  $\hat{\psi} = \hat{\psi}_2, = \hat{\psi}_2$ ,

$$(2.1) \quad E_\eta(\hat{\psi} - \psi(\eta))(\hat{\psi} - \psi(\eta))' - E_\eta(\hat{\psi}_\eta - \psi(\eta))(\hat{\psi}_\eta - \psi(\eta))' \geq 0. \quad \square$$

Obviously this condition is fulfilled for the class of all estimators (take  $\hat{\psi}_\eta = \psi(\eta)$ ). Under A1 and A3 the Definitions 1.1-1.3 are equivalent.

THEOREM 2.3. Assume A1 and A3. If  $\hat{\psi}_0 \in G$  is a weakly  $G$ -optimal estimator then  $\hat{\psi}_0$  is  $G$ -optimal.

PROOF. Let  $\theta \in \Theta$ ,  $\hat{\psi} \in G$ . Apply A3 to  $\theta$ ,  $\hat{\psi}$  and  $\hat{\psi}_0$  and let  $\hat{\psi}_\theta$  be the corresponding estimator. Similar to the proof of Proposition 2.1 one easily shows that

$$P_\theta(\hat{\psi}_0 = \hat{\psi}_\theta) = 1$$

(using A1 and (2.1)). The proof of the theorem is now immediate:

$$\begin{aligned} & E_\theta(\hat{\psi} - \psi(\theta))(\hat{\psi} - \psi(\theta))' - E_\theta(\hat{\psi}_0 - \psi(\theta))(\hat{\psi}_0 - \psi(\theta))' \\ &= E_\theta(\hat{\psi} - \psi(\theta))(\hat{\psi} - \psi(\theta))' - E_\theta(\hat{\psi}_\theta - \psi(\theta))(\hat{\psi}_\theta - \psi(\theta))' \geq 0. \quad \square \end{aligned}$$

The present results are easily generalized to the case of strictly convex loss functions for the components and to the case of infinite dimensional parameters.

### 3. UNBIASED ESTIMATION

We prefer classes of estimators  $G$  which are as large as possible. Only in trivial cases one can take for  $G$  the class of all estimators; then, however,  $\psi(\theta)$  is essentially known after  $Y$  is observed and this is not a very serious problem. Commonly attention is restricted to the subclass of all unbiased estimators

$$G_u = \{\hat{\psi} : E_{\theta}(\hat{\psi}) = \psi(\theta) \text{ for all } \theta \in \Theta\}$$

or the subclass of all linear unbiased estimators

$$G_{1u} = \{\hat{\psi} \in G_u : \hat{\psi} = AY + b \text{ for some } A \in \mathbb{R}^{m \times n} \text{ and } b \in \mathbb{R}^m\}.$$

Optimality in  $G_u$  and  $G_{1u}$  is usually defined in the sense of (1.1) and the optimal estimator is called UMVU estimator respectively BLUE, cf. Malinvaud (1970), Lehmann (1983), Fabian and Hannan (1984). The existence of an optimal estimator is not guaranteed; moreover, sometimes unbiased estimators do not exist at all.

If the class  $G_u$  is nonempty and if there is a complete sufficient statistic the Rao-Blackwell theorem implies the unique existence of the UMVU estimator and hence the three definitions of optimality are equivalent. We show that the latter statement also holds true in the general case. Proofs are deferred to the Appendix.

**THEOREM 3.1.** *Definitions 1.1-1.3 are equivalent for the class of unbiased estimators and the  $G_u$ -optimal (or UMVU) estimator is unique (if it exists).  $\square$*

**COROLLARY 3.2.** *Definitions 1.1-1.3 are equivalent for the class of linear unbiased estimators and the  $G_{1u}$ -optimal estimator (or BLUE) is unique (if it exists).  $\square$*

**REMARK 3.1.** The corollary is easily extended to other subclasses of  $G_u$ , e.g. to classes of unbiased estimators based on order statistics, U-statistics, V-statistics or statistics polynomial in  $Y$ .  $\square$

The conclusion of Theorem 3.1 and Corollary 3.2 is that for the classes  $G_u$  and  $G_{1u}$  it suffices to check optimality componentwise. This seems to be a more handsome definition than the classical one.



## 4. APPENDIX

PROOF of Theorem 3.1. Assumptions A1 and A2 are trivially fulfilled for  $G_u$ . It remains to show that Theorem 2.3 applies to  $G_u$ . Suppose  $\hat{\psi}_0$  is weakly  $G_u$ -optimal, fix  $\psi^* \in \psi(\Theta)$  and define  $\beta(\theta)$  by

$$(4.1) \quad \psi(\theta) - \psi^* = W\beta(\theta), \quad \theta \in \Theta,$$

where the columns of  $W$  are a basis of the space spanned by  $\psi(\theta) - \psi^*$ ,  $\theta \in \Theta$ . If  $W$  has  $k$  (say) columns, then  $\beta(\Theta)$  contains a basis in  $\mathbb{R}^k$ ,  $0 \in \beta(\Theta)$ .

Let  $\eta \in \Theta$ ,  $\hat{\psi}_1, \hat{\psi}_2 \in G_u$ . Replace the components of  $\hat{\psi}_1$  and  $\hat{\psi}_2$  with infinite variances under  $P_\eta$  by the corresponding components of  $\hat{\psi}_0$ . These new estimators,  $\hat{\psi}_{1\eta}$  and  $\hat{\psi}_{2\eta}$ , have finite covariance matrices under  $P_\eta$  and belong to  $G_u$ . Under  $P_\eta$  they are more concentrated than  $\hat{\psi}_1$  and  $\hat{\psi}_2$  (in the sense of (1.1)). It suffices to prove (2.1) for  $\hat{\psi} = \hat{\psi}_{1\eta}$  and  $\psi = \hat{\psi}_{2\eta}$ . Define  $X = (W' W)'$ ,  $Z = (\hat{\psi}'_{1\eta} \hat{\psi}'_{2\eta})'$  and  $\zeta^* = (\psi^* \psi^*)'$ , then

$$Z - \zeta^* = X\beta(\theta) + \varepsilon \quad \text{with } E_\theta(\varepsilon) = 0 \quad \text{for all } \theta \in \Theta.$$

Observe that  $\Sigma_\eta = \text{var}_\eta(\varepsilon) = \text{var}_\eta(Z)$  is finite. Let  $Q_\eta = [Q_{\eta 1} \quad Q_{\eta 2}]$  be an orthogonal matrix such that

$$\Sigma_\eta = Q_{\eta 1} \Lambda_\eta Q_{\eta 1}',$$

where  $\Lambda_\eta$  is a nonsingular diagonal matrix. Put

$$\Sigma_\eta^- = Q_{\eta 1} \Lambda_\eta^{-1} Q_{\eta 1}',$$

$$P_\eta = I - X' Q_{\eta 2} (Q_{\eta 2}' X X' Q_{\eta 2})^{-1} Q_{\eta 2}' X$$

and note that

$$(4.2) \quad Q_{\eta 1} Q_{\eta 1}' X P_\eta = (Q_{\eta 1} Q_{\eta 1}' + Q_{\eta 2} Q_{\eta 2}') X P_\eta = X P_\eta.$$

Define a generalized least-squares type estimator  $\hat{\psi}_\eta$  (cf. also Rao (1976)) by

$$\begin{aligned} \hat{\psi}_\eta &= W P_\eta (P_\eta' X' \Sigma_\eta^- X P_\eta)^{-1} P_\eta' X' \Sigma_\eta^- \{ I - X (X' Q_{\eta 2} Q_{\eta 2}' X)^{-1} X' Q_{\eta 2} Q_{\eta 2}' \} (Z - \zeta^*) \\ &\quad + W (X' Q_{\eta 2} Q_{\eta 2}' X)^{-1} X' Q_{\eta 2} Q_{\eta 2}' (Z - \zeta^*) + \psi^*. \end{aligned}$$

Put  $I^* = [I \quad 0]$  and observe that  $\hat{\psi}_\eta$  is an unbiased estimator of  $\psi(\theta)$  (use (4.2) and  $V(V'V)^-V'V = V$ ):

$$\begin{aligned}
E_{\theta}(\hat{\psi}_{\eta}) &= I^* Q_{\eta 1} Q'_{\eta 1} X P_{\eta} (P'_{\eta} X' \Sigma_{\eta}^{-1} X P_{\eta})^{-1} P'_{\eta} X' \Sigma_{\eta}^{-1} \{I - X(X' Q_{\eta 2} Q'_{\eta 2} X)^{-1} X' Q_{\eta 2} Q'_{\eta 2}\} X \beta(\theta) \\
&\quad + I^* X(X' Q_{\eta 2} Q'_{\eta 2} X)^{-1} X' Q_{\eta 2} Q'_{\eta 2} X \beta(\theta) + \psi^* \\
&= I^* Q_{\eta 1} Q'_{\eta 1} X P_{\eta} (P'_{\eta} X' \Sigma_{\eta}^{-1} X P_{\eta})^{-1} P'_{\eta} X' \Sigma_{\eta}^{-1} X P_{\eta} \{I - (X' Q_{\eta 2} Q'_{\eta 2} X)^{-1} X' Q_{\eta 2} Q'_{\eta 2} X\} \beta(\theta) \\
&\quad + I^* \{Q_{\eta 1} Q'_{\eta 1} X P_{\eta} + Q_{\eta 2} Q'_{\eta 2} X + Q_{\eta 1} Q'_{\eta 1} X X' Q_{\eta 2} (Q'_{\eta 2} X X' Q_{\eta 2})^{-1} Q'_{\eta 2} X\} \\
&\quad \quad \quad (X' Q_{\eta 2} Q'_{\eta 2} X)^{-1} X' Q_{\eta 2} Q'_{\eta 2} X \beta(\theta) + \psi^* \\
&= I^* \{Q_{\eta 1} Q'_{\eta 1} X P_{\eta} + Q_{\eta 1} Q'_{\eta 1} X X' Q_{\eta 2} (Q'_{\eta 2} X X' Q_{\eta 2})^{-1} Q'_{\eta 2} X + Q_{\eta 2} Q'_{\eta 2} X\} \beta(\theta) + \psi^* \\
&= I^* X \beta(\theta) + \psi^* = \psi(\theta) \quad \text{for all } \theta \in \Theta.
\end{aligned}$$

Let  $\hat{\psi}_{Ab} = A(Z - \zeta^*) + b \in G_u$ , then  $E_{\theta}(\hat{\psi}_{Ab}) = AX\beta(\theta) + b = \psi(\theta) = W\beta(\theta) + \psi^*$  for all  $\theta \in \Theta$ . Hence (using (4.1))

$$AX = W \text{ and } b = \psi^*.$$

This relation and (4.2) imply that (2.1) holds for all  $\hat{\psi}_{Ab} \in G_u$ :

$$\begin{aligned}
\text{var}_{\eta}(\hat{\psi}_{Ab}) - \text{var}_{\eta}(\hat{\psi}_{\eta}) &= A \Sigma_{\eta} A' - W P_{\eta} (P'_{\eta} X' \Sigma_{\eta}^{-1} X P_{\eta})^{-1} P'_{\eta} W' \\
&= \{A - W P_{\eta} (P'_{\eta} X' \Sigma_{\eta}^{-1} X P_{\eta})^{-1} P'_{\eta} X' \Sigma_{\eta}^{-1}\} \Sigma_{\eta} \{A - W P_{\eta} (P'_{\eta} X' \Sigma_{\eta}^{-1} X P_{\eta})^{-1} P'_{\eta} X' \Sigma_{\eta}^{-1}\}' \geq 0.
\end{aligned}$$

Since  $\hat{\psi}_{1\eta}$  and  $\hat{\psi}_{2\eta}$  are of type  $\hat{\psi}_{Ab}$  the proof is complete for  $G_u$ .  $\square$

PROOF of Corollary 3.2. The proof for  $G_{1u}$  is almost identical to the proof of Theorem 3.1. Note that if  $\hat{\psi}_0$ ,  $\hat{\psi}_1$  and  $\hat{\psi}_2$  are linear in  $Y$  then  $\hat{\psi}_{1\eta}$ ,  $\hat{\psi}_{2\eta}$ ,  $\hat{\psi}_{\eta}$  and  $\hat{\psi}_{Ab}$  are also linear in  $Y$ .  $\square$

REMARK 4.1. The main ideas of the proof are conserved if the covariance matrix  $\Sigma_{\eta}$  is supposed to be nonsingular. Then the terms involving  $Q_{\eta 2}$  do not exist and generalized inverses can be replaced by inverses. Note, however, that singular matrices are inevitable in  $G_u$  and  $G_{1u}$ .  $\square$

## REFERENCES

- FABIAN, V. and HANNAN, J. (1984), *Introduction to Probability and Mathematical Statistics*, Wiley, New York.
- LEHMANN, E.L. (1983), *Theory of Point Estimation*, Wiley, New York.
- MALINVAUD, E. (1970), *Statistical Methods of Econometrics*, North-Holland Publishing Company, Amsterdam.
- RAO, C.R. (1976), Estimation of parameters in a linear model, *Ann. Stat.* 4, 1023-1035. Correction: *Ann. Stat.* 7, 696.6.

## IN 1987 REEDS VERSCHENEN

- 242 Gerard van den Berg  
Nonstationarity in job search theory
- 243 Annie Cuyt, Brigitte Verdonk  
Block-tridiagonal linear systems and branched continued fractions
- 244 J.C. de Vos, W. Vervaat  
Local Times of Bernoulli Walk
- 245 Arie Kapteyn, Peter Kooreman, Rob Willemse  
Some methodological issues in the implementation  
of subjective poverty definitions
- 246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel  
Sampling for Quality Inspection and Correction: AOQL Performance  
Criteria
- 247 D.B.J. Schouten  
Algemene theorie van de internationale conjuncturele en structurele  
afhankelijkheden
- 248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence  
On  $(v,k,\lambda)$  graphs and designs with trivial automorphism group
- 249 Peter M. Kort  
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic  
Investment Policy
- 250 R.H.J.M. Gradus  
Preliminary version  
The reaction of the firm on governmental policy: a game-theoretical  
approach
- 251 J.G. de Gooijer, R.M.J. Heuts  
Higher order moments of bilinear time series processes with symmetrically  
distributed errors
- 252 P.H. Stevers, P.A.M. Versteijne  
Evaluatie van marketing-activiteiten
- 253 H.P.A. Mulders, A.J. van Reeken  
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen
- 254 P. Kooreman, A. Kapteyn  
On the identifiability of household production functions with joint  
products: A comment
- 255 B. van Riel  
Was er een profit-squeeze in de Nederlandse industrie?
- 256 R.P. Gilles  
Economies with coalitional structures and core-like equilibrium concepts

- 257 P.H.M. Ruys, G. van der Laan  
Computation of an industrial equilibrium
- 258 W.H. Haemers, A.E. Brouwer  
Association schemes
- 259 G.J.M. van den Boom  
Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining
- 260 A.W.A. Boot, A.V. Thakor, G.F. Udell  
Competition, Risk Neutrality and Loan Commitments
- 261 A.W.A. Boot, A.V. Thakor, G.F. Udell  
Collateral and Borrower Risk
- 262 A. Kapteyn, I. Wolttiez  
Preference Interdependence and Habit Formation in Family Labor Supply
- 263 B. Bettonvil  
A formal description of discrete event dynamic systems including perturbation analysis
- 264 Sylvester C.W. Eijffinger  
A monthly model for the monetary policy in the Netherlands
- 265 F. van der Ploeg, A.J. de Zeeuw  
Conflict over arms accumulation in market and command economies
- 266 F. van der Ploeg, A.J. de Zeeuw  
Perfect equilibrium in a model of competitive arms accumulation
- 267 Aart de Zeeuw  
Inflation and reputation: comment
- 268 A.J. de Zeeuw, F. van der Ploeg  
Difference games and policy evaluation: a conceptual framework
- 269 Frederick van der Ploeg  
Rationing in open economy and dynamic macroeconomics: a survey
- 270 G. van der Laan and A.J.J. Talman  
Computing economic equilibria by variable dimension algorithms: state of the art
- 271 C.A.J.M. Dirven and A.J.J. Talman  
A simplicial algorithm for finding equilibria in economies with linear production technologies
- 272 Th.E. Nijman and F.C. Palm  
Consistent estimation of regression models with incompletely observed exogenous variables
- 273 Th.E. Nijman and F.C. Palm  
Predictive accuracy gain from disaggregate sampling in arima - models

- 274 Raymond H.J.M. Gradus  
The net present value of governmental policy: a possible way to find the Stackelberg solutions
- 275 Jack P.C. Kleijnen  
A DSS for production planning: a case study including simulation and optimization
- 276 A.M.H. Gerards  
A short proof of Tutte's characterization of totally unimodular matrices
- 277 Th. van de Klundert and F. van der Ploeg  
Wage rigidity and capital mobility in an optimizing model of a small open economy
- 278 Peter M. Kort  
The net present value in dynamic models of the firm
- 279 Th. van de Klundert  
A Macroeconomic Two-Country Model with Price-Discriminating Monopolists
- 280 Arnoud Boot and Anjan V. Thakor  
Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing
- 281 Arnoud Boot and Anjan V. Thakor  
Appendix: "Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing"
- 282 Arnoud Boot, Anjan V. Thakor and Gregory F. Udell  
Credible commitments, contract enforcement problems and banks: intermediation as credibility assurance
- 283 Eduard Ponds  
Wage bargaining and business cycles a Goodwin-Nash model
- 284 Prof.Dr. hab. Stefan Mynarski  
The mechanism of restoring equilibrium and stability in polish market
- 285 P. Meulendijks  
An exercise in welfare economics (II)
- 286 S. Jørgensen, P.M. Kort, G.J.C.Th. van Schijndel  
Optimal investment, financing and dividends: a Stackelberg differential game
- 287 E. Nijssen, W. Reijnders  
Privatisering en commercialisering; een oriëntatie ten aanzien van verzelfstandiging
- 288 C.B. Mulder  
Inefficiency of automatically linking unemployment benefits to private sector wage rates

- 289 M.H.C. Paardekooper  
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues
- 290 Pieter H.M. Ruys  
Industries with private and public enterprises
- 291 J.J.A. Moors & J.C. van Houwelingen  
Estimation of linear models with inequality restrictions
- 292 Arthur van Soest, Peter Kooreman  
Vakantiebestemming en -bestedingen
- 293 Rob Alessie, Raymond Gradus, Bertrand Melenberg  
The problem of not observing small expenditures in a consumer expenditure survey
- 294 F. Boekema, L. Oerlemans, A.J. Hendriks  
Kansrijkheid en economische potentie: Top-down en bottom-up analyses
- 295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber  
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note
- 296 Arthur van Soest, Peter Kooreman  
Estimation of the indirect translog demand system with binding non-negativity constraints

## IN 1988 REEDS VERSCHENEN

- 297 Bert Bettonvil  
Factor screening by sequential bifurcation
- 298 Robert P. Gilles  
On perfect competition in an economy with a coalitional structure
- 299 Willem Selen, Ruud M. Heuts  
Capacitated Lot-Size Production Planning in Process Industry
- 300 J. Kriens, J.Th. van Lieshout  
Notes on the Markowitz portfolio selection method
- 301 Bert Bettonvil, Jack P.C. Kleijnen  
Measurement scales and resolution IV designs: a note
- 302 Theo Nijman, Marno Verbeek  
Estimation of time dependent parameters in linear models  
using cross sections, panels or both
- 303 Raymond H.J.M. Gradus  
A differential game between government and firms: a non-cooperative  
approach
- 304 Leo W.G. Strijbosch, Ronald J.M.M. Does  
Comparison of bias-reducing methods for estimating the parameter in  
dilution series
- 305 Drs. W.J. Reijnders, Drs. W.F. Verstappen  
Strategische bespiegelingen betreffende het Nederlandse kwaliteits-  
concept
- 306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg  
Regression sampling in statistical auditing
- 307 Isolde Woittiez, Arie Kapteyn  
A Model of Job Choice, Labour Supply and Wages
- 308 Jack P.C. Kleijnen  
Simulation and optimization in production planning: A case study
- 309 Robert P. Gilles and Pieter H.M. Ruys  
Relational constraints in coalition formation
- 310 Drs. H. Leo Theuns  
Determinanten van de vraag naar vakantiereizen: een verkenning van  
materiële en immateriële factoren
- 311 Peter M. Kort  
Dynamic Firm Behaviour within an Uncertain Environment
- 312 J.P.C. Blanc  
A numerical approach to cyclic-service queueing models



- 313 Drs. N.J. de Beer, Drs. A.M. van Nunen, Drs. M.O. Nijkamp  
Does Morkmon Matter?
- 314 Th. van de Klundert  
Wage differentials and employment in a two-sector model with a dual  
labour market
- 315 Aart de Zeeuw, Fons Groot, Cees Withagen  
On Credible Optimal Tax Rate Policies
- 316 Christian B. Mulder  
Wage moderating effects of corporatism  
Decentralized versus centralized wage setting in a union, firm,  
government context
- 317 Jörg Glombowski, Michael Krüger  
A short-period Goodwin growth cycle
- 318 Theo Nijman, Marno Verbeek, Arthur van Soest  
The optimal design of rotating panels in a simple analysis of  
variance model
- 319 Drs. S.V. Hannema, Drs. P.A.M. Versteijne  
De toepassing en toekomst van public private partnership's bij de  
grote en middelgrote Nederlandse gemeenten
- 320 Th. van de Klundert  
Wage Rigidity, Capital Accumulation and Unemployment in a Small Open  
Economy
- 321 M.H.C. Paardekooper  
An upper and a lower bound for the distance of a manifold to a nearby  
point
- 322 Th. ten Raa, F. van der Ploeg  
A statistical approach to the problem of negatives in input-output  
analysis
- 323 P. Kooreman  
Household Labor Force Participation as a Cooperative Game; an Empiri-  
cal Model
- 324 A.B.T.M. van Schaik  
Persistent Unemployment and Long Run Growth
- 325 Dr. F.W.M. Boekema, Drs. L.A.G. Oerlemans  
De lokale produktiestructuur doorgelicht.  
Bedrijfstakingverkenningen ten behoeve van regionaal-economisch onder-  
zoek
- 326 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel  
Sampling for quality inspection and correction: AOQL performance  
criteria

- 327 Theo E. Nijman, Mark F.J. Steel  
Exclusion restrictions in instrumental variables equations
- 328 B.B. van der Genugten  
Estimation in linear regression under the presence of heteroskedasticity of a completely unknown form
- 329 Raymond H.J.M. Gradus  
The employment policy of government: to create jobs or to let them create?
- 330 Hans Kremers, Dolf Talman  
Solving the nonlinear complementarity problem with lower and upper bounds
- 331 Antoon van den Elzen  
Interpretation and generalization of the Lemke-Howson algorithm
- 332 Jack P.C. Kleijnen  
Analyzing simulation experiments with common random numbers, part II: Rao's approach
- 333 Jacek Osiewalski  
Posterior and Predictive Densities for Nonlinear Regression.  
A Partly Linear Model Case
- 334 A.H. van den Elzen, A.J.J. Talman  
A procedure for finding Nash equilibria in bi-matrix games
- 335 Arthur van Soest  
Minimum wage rates and unemployment in The Netherlands
- 336 Arthur van Soest, Peter Kooreman, Arie Kapteyn  
Coherent specification of demand systems with corner solutions and endogenous regimes
- 337 Dr. F.W.M. Boekema, Drs. L.A.G. Oerlemans  
De lokale produktiestructuur doorgelicht II. Bedrijfstakverkenningen ten behoeve van regionaal-economisch onderzoek. De zeescheepsnieuwbouwindustrie
- 338 Gerard J. van den Berg  
Search behaviour, transitions to nonparticipation and the duration of unemployment
- 339 W.J.H. Groenendaal and J.W.A. Vingerhoets  
The new cocoa-agreement analysed
- 340 Drs. F.G. van den Heuvel, Drs. M.P.H. de Vor  
Kwantificering van ombuigen en bezuinigen op collectieve uitgaven 1977-1990
- 341 Pieter J.F.G. Meulendijks  
An exercise in welfare economics (III)

- 342 W.J. Selen and R.M. Heuts  
A modified priority index for Günther's lot-sizing heuristic under capacitated single stage production
- 343 Linda J. Mittermaier, Willem J. Selen, Jeri B. Waggoner, Wallace R. Wood  
Accounting estimates as cost inputs to logistics models
- 344 Remy L. de Jong, Rashid I. Al Layla, Willem J. Selen  
Alternative water management scenarios for Saudi Arabia
- 345 W.J. Selen and R.M. Heuts  
Capacitated Single Stage Production Planning with Storage Constraints and Sequence-Dependent Setup Times
- 346 Peter Kort  
The Flexible Accelerator Mechanism in a Financial Adjustment Cost Model
- 347 W.J. Reijnders en W.F. Verstappen  
De toenemende importantie van het verticale marketing systeem
- 348 P.C. van Batenburg en J. Kriens  
E.O.Q.L. - A revised and improved version of A.O.Q.L.
- 349 Drs. W.P.C. van den Nieuwenhof  
Multinationalisatie en coördinatie  
De internationale strategie van Nederlandse ondernemingen nader beschouwd
- 350 K.A. Bubshait, W.J. Selen  
Estimation of the relationship between project attributes and the implementation of engineering management tools
- 351 M.P. Tummers, I. Woittiez  
A simultaneous wage and labour supply model with hours restrictions
- 352 Marco Versteijne  
Measuring the effectiveness of advertising in a positioning context with multi dimensional scaling techniques
- 353 Dr. F. Boekema, Drs. L. Oerlemans  
Innovatie en stedelijke economische ontwikkeling
- 354 J.M. Schumacher  
Discrete events: perspectives from system theory
- 355 F.C. Bussemaker, W.H. Haemers, R. Mathon and H.A. Wilbrink  
A (49,16,3,6) strongly regular graph does not exist
- 356 Drs. J.C. Caanen  
Tien jaar inflatieneutrale belastingheffing door middel van vermogensaftrek en voorraadaftek: een kwantitatieve benadering

- 357 R.M. Heuts, M. Bronckers  
A modified coordinated reorder procedure under aggregate investment  
and service constraints using optimal policy surfaces
- 358 B.B. van der Genugten  
Linear time-invariant filters of infinite order for non-stationary  
processes
- 359 J.C. Engwerda  
LQ-problem: the discrete-time time-varying case
- 360 Shan-Hwei Nienhuys-Cheng  
Constraints in binary semantical networks
- 361 A.B.T.M. van Schaik  
Interregional Propagation of Inflationary Shocks