How to define UMVU

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Publication date:
1988

Citation for published version (APA):
Drost, F. C. (1988). *How to define UMVU*. (Research memorandum / Tilburg University, Department of Economics; Vol. FEW 362). Unknown Publisher.

HOW TO DEFINE UMVU

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FEW 362

Tilburg University

November 1988

abstract. Consider the estimation of some m-vector $\psi(\theta)$. The usual definition of the UMVU estimator $\hat{\psi}_0$ requires that $\hat{\psi}_0$ is more concentrated than any other unbiased estimator. The comparison of covariance matrices, however, is unnecessarily complicated. It is shown that $\hat{\psi}_0$ is the UMVU estimator iff the components of $\hat{\psi}_0$ have smallest variances (or even the smallest sum of variances) in the class of unbiased estimators. Hence a componentwise definition of UMVU estimators seems to be more appropriate. A similar result holds true for BLUE's.

Keywords : UMVU estimator, BLUE, concentration ellipsoid, mean-squared error.

AMS 1980 subject classification : 62F10.

1. INTRODUCTION

Suppose $Y = (Y_1,\dots,Y_n)'$ has distribution P_θ, where $\theta \in \Theta$ is an unknown nuisance parameter. An important case is i.i.d. sampling from some distribution F_θ. Another example is a sample from a linear model. In the latter case the Y_i's are often neither independent nor identically distributed. In the general setup we consider the estimation of some vector-valued parameter $\psi(\theta) = (\psi_1(\theta),\dots,\psi_m(\theta))' \in \mathbb{R}^m$. There are a lot of estimators and we inquire whether there exists some optimal estimator $\hat{\psi}_0$. Usually optimality is defined in terms of concentration ellipsoids (cf. Malinvaud (1970)). Let G be some class of estimators, then $\hat{\psi}_0 \in G$ is called G-optimal if the concentration ellipsoid of any estimator $\hat{\psi} \in G$ covers the concentration ellipsoid of $\hat{\psi}_0$ or equivalently if, for each $c \in \mathbb{R}^m$, the mean-squared error (MSE) of $c'\hat{\psi}$ is larger than the MSE of $c'\hat{\psi}_0$.

DEFINITION 1.1. *The estimator $\hat{\psi}_0 \in G$ is called G-optimal if, for all $\theta \in \Theta$ and $\hat{\psi} \in G$,*

$$(1.1)\quad E_\theta(\hat{\psi} - \psi(\theta))(\hat{\psi} - \psi(\theta))' - E_\theta(\hat{\psi}_0 - \psi(\theta))(\hat{\psi}_0 - \psi(\theta))' \geq 0. \quad \square$$

The comparison of matrices in (1.1) is rather complicated. A componentwise evaluation of estimators is more simple.

DEFINITION 1.2. *The estimator $\hat{\psi}_0 = (\hat{\psi}_{01},\dots,\hat{\psi}_{0m})' \in G$ is called componentwise G-optimal if, for all $\theta \in \Theta$, $\hat{\psi} = (\hat{\psi}_1,\dots,\hat{\psi}_m)' \in G$ and* $i=1,\dots,m$,

$$(1.2)\quad E_\theta(\hat{\psi}_i - \psi_i(\theta))^2 - E_\theta(\hat{\psi}_{0i} - \psi_i(\theta))^2 \geq 0. \quad \square$$

In stead of (1.1) or (1.2) we consider also a criterion based on the sum of MSE's.

DEFINITION 1.3. *The estimator $\hat{\psi}_0 \in G$ is called weakly G-optimal if, for all $\theta \in \Theta$ and $\hat{\psi} \in G$,*

$$(1.3)\quad E_\theta\|\hat{\psi} - \psi(\theta)\|^2 - E_\theta\|\hat{\psi}_0 - \psi(\theta)\|^2 \geq 0. \quad \square$$

Implicitly the three definitions require that $E_\theta\|\hat{\psi}_0 - \psi(\theta)\|^2$ is finite for all $\theta \in \Theta$. Obviously (1.1) $\Rightarrow$ (1.2) $\Rightarrow$ (1.3). It is not difficult to invent classes of estimators such that the reverse implications do not hold. This motivates the different notions of optimality. Commonly the strong

Definition 1.1 is taken as a basis, then optimality remains invariant under linear transformations.

In this paper we investigate whether the three definitions of optimality are very different. In Section 2 some results concerning the different notions of optimality are given. Under general conditions it is shown that the optimal estimator is unique (Proposition 2.1) and that the Definitions 1.1-1.3 are equivalent, i.e. (1.1) $\Leftrightarrow$ (1.2) $\Leftrightarrow$ (1.3) (Proposition 2.2 and Theorem 2.3). Although one of the assumptions is rather complicated the main ideas and proofs are simple. In Section 3 we consider the important class of unbiased estimators G_u of $\psi(\theta)$. Usually only Definition 1.1 is given to define the uniformly minimum variance unbiased (UMVU) estimator of $\psi(\theta)$. The other definitions are ignored or they are not connected to Definition 1.1. The results of Section 2 lead to our main statement: a weakly G_u-optimal estimator is also G_u-optimal. Componentwise UMVU implies UMVU (Theorem 3.1). The proof is based on a generalization of the Gauss-Markov Theorem (cf. Rao (1976)). Similar results hold true for several subclasses of G_u. E.g. the best linear unbiased estimator (BLUE) can also be characterized componentwise (Corollary 3.2). A complicated comparison of covariance matrices is overdone in these cases and Definition 1.2 (or 1.3) provides a more handsome requirement.

2. GENERAL THEORY

If a G-optimal estimator exists then it is easily verified that all weakly G-optimal estimators are G-optimal. This implies the equivalence of Definitions 1.1-1.3. Similarly the existence of a componentwise G-optimal estimator implies the equivalence of Definitions 1.2 and 1.3. However, the existence of a (componentwise) G-optimal estimator is not guaranteed and it may be hard to prove that some estimator is (componentwise) G-optimal. We proceed in a different way to show that the different optimality definitions are equivalent, we derive some general conditions upon the class of estimators G.

Identify estimators that are equal a.s. $\{P_\theta : \theta \in \Theta\}$, let G be some class of estimators of $\psi(\theta)$ and suppose $\hat{\psi}_1$ and $\hat{\psi}_2$ are elements of G. Often the weighted average $\hat{\psi}_\alpha = (1-\alpha)\hat{\psi}_1 + \alpha\hat{\psi}_2$ belongs to G for all $0 \le \alpha \le 1$. Under the somewhat more general condition

A1. For each $\hat{\psi}_1$, $\hat{\psi}_2 \in G$ there exists $\alpha \in (0,1)$ such that the estimator $\hat{\psi}_\alpha = (1-\alpha)\hat{\psi}_1 + \alpha\hat{\psi}_2$ belongs to G. □

we show that the G-best estimator is unique (if it exists).

PROPOSITION 2.1. *Assume* A1. *If* $\hat{\psi}_0 \in G$ *is a weakly G-optimal estimator then any solution of* (1.1), (1.2) *or* (1.3) *is equal to* $\hat{\psi}_0$ a.s. $\{P_\theta: \theta \in \Theta\}$

PROOF. G-optimality in the sense of (1.1) and (1.2) implies weak G-optimality; it suffices to prove the proposition for the latter case. Let $\hat{\psi} \in G$ be any weakly G-optimal estimator. Apply A1 to $\hat{\psi}$ and $\hat{\psi}_0$ and let $\hat{\psi}_\alpha \in G$ be the corresponding estimator. Then, for all $\theta \in \Theta$,

$$0 \le E_\theta \|\hat{\psi}_\alpha - \psi(\theta)\|^2 - E_\theta \|\hat{\psi}_0 - \psi(\theta)\|^2 = -\alpha(1-\alpha) E_\theta \|\hat{\psi}_0 - \hat{\psi}\|^2$$

implying $\hat{\psi}_0 = \hat{\psi}$ a.s. $\{P_\theta : \theta \in \Theta\}$. □

REMARK 2.1. If a componentwise G-optimal estimator exists condition A1 can be relaxed. Then a componentwise analogon of A1 suffices to prove Proposition 2.1 (use the equivalence of Definitions 1.2 and 1.3). □

Let $\hat{\psi}_1$ and $\hat{\psi}_2$ be elements of G and define the estimator $\hat{\psi}$ by replacing one component of $\hat{\psi}_1$ by the corresponding component of $\hat{\psi}_2$. Since estimators are usually derived componentwise it seems natural to require that $\hat{\psi} \in G$. This leads to the following assumption:

A2. For each $\hat{\psi}_1$, $\hat{\psi}_2 \in G$ and $\delta_1, \ldots, \delta_m \in \{0,1\}$ the estimator $\hat{\psi}_\delta = ((1-\delta_1)\hat{\psi}_{11} + \delta_1\hat{\psi}_{21}, \ldots, (1-\delta_m)\hat{\psi}_{1m} + \delta_m\hat{\psi}_{2m})'$ belongs to G. □

Condition A2 implies the equivalence of Definitions 1.2 and 1.3.

PROPOSITION 2.2. *Assume* A2. *If* $\hat{\psi}_0 \in G$ *is a weakly G-optimal estimator then* $\hat{\psi}_0$ *is componentwise G-optimal.* □

To prove that the three definitions of optimality are equivalent we need a very strong assumption. We suppose that, for each $\eta \in \Theta$ and $\hat{\psi}_1$, $\hat{\psi}_2 \in G$, some estimator $\hat{\psi}_\eta \in G$ exists which is locally more concentrated than $\hat{\psi}_1$ and $\hat{\psi}_2$ under P_η. (It is not required that $\hat{\psi}_\eta$ is better than $\hat{\psi}_1$ or $\hat{\psi}_2$ under P_θ when $\theta \neq \eta$!)

A3. For each $\eta \in \Theta$, $\hat{\psi}_1$, $\hat{\psi}_2 \in G$ there exists $\hat{\psi}_\eta \in G$ such that, for $\hat{\psi} = \hat{\psi}_1$ and $\hat{\psi} = \hat{\psi}_2$, $= \hat{\psi}_2$,

(2.1) $E_\eta(\hat{\psi} - \psi(\eta))(\hat{\psi} - \psi(\eta))' - E_\eta(\hat{\psi}_\eta - \psi(\eta))(\hat{\psi}_\eta - \psi(\eta))' \geq 0.$ □

Obviously this condition is fulfilled for the class of all estimators (take $\hat{\psi}_\eta = \psi(\eta)$). Under A1 and A3 the Definitions 1.1-1.3 are equivalent.

THEOREM 2.3. *Assume* A1 *and* A3. *If* $\hat{\psi}_0 \in G$ *is a weakly G-optimal estimator then* $\hat{\psi}_0$ *is G-optimal.*

PROOF. Let $\theta \in \Theta$, $\hat{\psi} \in G$. Apply A3 to θ, $\hat{\psi}$ and $\hat{\psi}_0$ and let $\hat{\psi}_\theta$ be the corresponding estimator. Similar to the proof of Proposition 2.1 one easily shows that

$$P_\theta(\hat{\psi}_0 = \hat{\psi}_\theta) = 1$$

(using A1 and (2.1)). The proof of the theorem is now immediate:

$$E_\theta(\hat{\psi} - \psi(\theta))(\hat{\psi} - \psi(\theta))' - E_\theta(\hat{\psi}_0 - \psi(\theta))(\hat{\psi}_0 - \psi(\theta))'$$

$$= E_\theta(\hat{\psi} - \psi(\theta))(\hat{\psi} - \psi(\theta))' - E_\theta(\hat{\psi}_\theta - \psi(\theta))(\hat{\psi}_\theta - \psi(\theta))' \geq 0. \quad \square$$

The present results are easily generalized to the case of strictly convex loss functions for the components and to the case of infinite dimensional parameters.

3. UNBIASED ESTIMATION

We prefer classes of estimators G which are as large as possible. Only in trivial cases one can take for G the class of all estimators; then, however, $\psi(\theta)$ is essentially known after Y is observed and this is not a very serious problem. Commonly attention is restricted to the subclass of all unbiased estimators

$$G_u = \{\hat{\psi} : E_\theta(\hat{\psi}) = \psi(\theta) \text{ for all } \theta \in \Theta\}$$

or the subclass of all linear unbiased estimators

$$G_{lu} = \{\hat{\psi} \in G_u : \hat{\psi} = AY + b \text{ for some } A \in \mathbb{R}^{m\times n} \text{ and } b \in \mathbb{R}^m\}.$$

Optimality in G_u and G_{lu} is usually defined in the sense of (1.1) and the optimal estimator is called UMVU estimator respectively BLUE, cf. Malinvaud (1970), Lehmann (1983), Fabian and Hannan (1984). The existence of an optimal estimator is not guaranteed; moreover, sometimes unbiased estimators do not exist at all.

If the class G_u is nonempty and if there is a complete sufficient statistic the Rao-Blackwell theorem implies the unique existence of the UMVU estimator and hence the three definitions of optimality are equivalent. We show that the latter statement also holds true in the general case. Proofs are deferred to the Appendix.

THEOREM 3.1. Definitions 1.1-1.3 *are equivalent for the class of unbiased estimators and the* G_u*-optimal (or* UMVU*) estimator is unique (if it exists).* □

COROLLARY 3.2. Definitions 1.1-1.3 *are equivalent for the class of linear unbiased estimators and the* G_{lu}*-optimal estimator (or* BLUE*) is unique (if it exists).* □

REMARK 3.1. The corollary is easily extended to other subclasses of G_u, e.g. to classes of unbiased estimators based on order statistics, U-statistics, V-statistics or statistics polynomial in Y. □

The conclusion of Theorem 3.1 and Corollary 3.2 is that for the classes G_u and G_{lu} it suffices to check optimality componentwise. This seems to be a more handsome definition than the classical one.

4. APPENDIX

PROOF of Theorem 3.1. Assumptions A1 and A2 are trivially fulfilled for G_u. It remains to show that Theorem 2.3 applies to G_u. Suppose $\hat{\psi}_0$ is weakly G_u-optimal, fix $\psi^* \in \psi(\Theta)$ and define $\beta(\theta)$ by

$$(4.1)\quad \psi(\theta) - \psi^* = W\beta(\theta), \quad \theta \in \Theta,$$

where the columns of W are a basis of the space spanned by $\psi(\theta) - \psi^*$, $\theta \in \Theta$. If W has k (say) columns, then $\beta(\Theta)$ contains a basis in $\mathbb{R}^k$, $0 \in \beta(\Theta)$.

Let $\eta \in \Theta$, $\hat{\psi}_1$, $\hat{\psi}_2 \in G_u$. Replace the components of $\hat{\psi}_1$ and $\hat{\psi}_2$ with infinite variances under P_η by the corresponding components of $\hat{\psi}_0$. These new estimators, $\hat{\psi}_{1\eta}$ and $\hat{\psi}_{2\eta}$, have finite covariance matrices under P_η and belong to G_u. Under P_η they are more concentrated than $\hat{\psi}_1$ and $\hat{\psi}_2$ (in the sense of (1.1)). It suffices to prove (2.1) for $\hat{\psi} = \hat{\psi}_{1\eta}$ and $\hat{\psi} = \hat{\psi}_{2\eta}$. Define $X = (W' \ W')'$, $Z = (\hat{\psi}'_{1\eta} \ \hat{\psi}'_{2\eta})'$ and $\zeta^* = (\psi^{*\prime} \ \psi^{*\prime})'$, then

$$Z - \zeta^* = X\beta(\theta) + \varepsilon \quad \text{with } E_\theta(\varepsilon) = 0 \quad \text{for all } \theta \in \Theta.$$

Observe that $\Sigma_\eta = \mathrm{var}_\eta(\varepsilon) = \mathrm{var}_\eta(Z)$ is finite. Let $Q_\eta = [Q_{\eta 1} \ Q_{\eta 2}]$ be an orthogonal matrix such that

$$\Sigma_\eta = Q_{\eta 1}\Lambda_\eta Q'_{\eta 1},$$

where Λ_η is a nonsingular diagonal matrix. Put

$$\Sigma^-_\eta = Q_{\eta 1}\Lambda^{-1}_\eta Q'_{\eta 1},$$

$$P_\eta = I - X'Q_{\eta 2}(Q'_{\eta 2}XX'Q_{\eta 2})^- Q'_{\eta 2}X$$

and note that

$$(4.2)\quad Q_{\eta 1}Q'_{\eta 1}XP_\eta = (Q_{\eta 1}Q'_{\eta 1} + Q_{\eta 2}Q'_{\eta 2})XP_\eta = XP_\eta.$$

Define a generalized least-squares type estimator $\hat{\psi}_\eta$ (cf. also Rao (1976)) by

$$\hat{\psi}_\eta = WP_\eta(P'_\eta X'\Sigma^-_\eta XP_\eta)^- P'_\eta X'\Sigma^-_\eta\{I - X(X'Q_{\eta 2}Q'_{\eta 2}X)^- X'Q_{\eta 2}Q'_{\eta 2}\}(Z - \zeta^*)$$
$$+ W(X'Q_{\eta 2}Q'_{\eta 2}X)^- X'Q_{\eta 2}Q'_{\eta 2}(Z - \zeta^*) + \psi^*.$$

Put $I^* = [I \ 0]$ and observe that $\hat{\psi}_\eta$ is an unbiased estimator of $\psi(\theta)$ (use (4.2) and $V(V'V)^-V'V = V$):

$$E_\theta(\hat{\psi}_\eta) = I^* Q_{\eta 1}Q'_{\eta 1}XP_\eta(P'_\eta X'\Sigma^-_\eta XP_\eta)^- P'_\eta X'\Sigma^-_\eta\{I - X(X'Q_{\eta 2}Q'_{\eta 2}X)^- X'Q_{\eta 2}Q'_{\eta 2}\}X\beta(\theta)$$
$$+ I^* X(X'Q_{\eta 2}Q'_{\eta 2}X)^- X'Q_{\eta 2}Q'_{\eta 2}X\beta(\theta) + \psi^*$$
$$= I^* Q_{\eta 1}Q'_{\eta 1}XP_\eta(P'_\eta X'\Sigma^-_\eta XP_\eta)^- P'_\eta X'\Sigma^-_\eta XP_\eta\{I-(X'Q_{\eta 2}Q'_{\eta 2}X)^- X'Q_{\eta 2}Q'_{\eta 2}X\}\beta(\theta)$$
$$+ I^*\{Q_{\eta 1}Q'_{\eta 1}XP_\eta + Q_{\eta 2}Q'_{\eta 2}X + Q_{\eta 1}Q'_{\eta 1}XX'Q_{\eta 2}(Q'_{\eta 2}XX'Q_{\eta 2})^- Q'_{\eta 2}X\}$$
$$(X'Q_{\eta 2}Q'_{\eta 2}X)^- X'Q_{\eta 2}Q'_{\eta 2}X\beta(\theta) + \psi^*$$
$$= I^*\{Q_{\eta 1}Q'_{\eta 1}XP_\eta + Q_{\eta 1}Q'_{\eta 1}XX'Q_{\eta 2}(Q'_{\eta 2}XX'Q_{\eta 2})^- Q'_{\eta 2}X + Q_{\eta 2}Q'_{\eta 2}X\}\beta(\theta) + \psi^*$$
$$= I^* X\beta(\theta) + \psi^* = \psi(\theta) \quad \text{for all } \theta \in \Theta.$$

Let $\hat{\psi}_{Ab} = A(Z - \zeta^*) + b \in G_u$, then $E_\theta(\hat{\psi}_{Ab}) = AX\beta(\theta) + b = \psi(\theta) = W\beta(\theta) + \psi^*$ for all $\theta \in \Theta$. Hence (using (4.1))

$$AX = W \text{ and } b = \psi^*.$$

This relation and (4.2) imply that (2.1) holds for all $\hat{\psi}_{Ab} \in G_u$:

$$\text{var}_\eta(\hat{\psi}_{Ab}) - \text{var}_\eta(\hat{\psi}_\eta) = A\Sigma_\eta A' - WP_\eta(P'_\eta X'\Sigma^-_\eta XP_\eta)^- P'_\eta W'$$
$$= \{A - WP_\eta(P'_\eta X'\Sigma^-_\eta XP_\eta)^- P'_\eta X'\Sigma^-_\eta\}\Sigma_\eta\{A - WP_\eta(P'_\eta X'\Sigma^-_\eta XP_\eta)^- P'_\eta X'\Sigma^-_\eta\}' \geq 0.$$

Since $\hat{\psi}_{1\eta}$ and $\hat{\psi}_{2\eta}$ are of type $\hat{\psi}_{Ab}$ the proof is complete for G_u. □

PROOF of Corollary 3.2. The proof for G_{1u} is almost identical to the proof of Theorem 3.1. Note that if $\hat{\psi}_0$, $\hat{\psi}_1$ and $\hat{\psi}_2$ are linear in Y then $\hat{\psi}_{1\eta}$, $\hat{\psi}_{2\eta}$, $\hat{\psi}_\eta$ and $\hat{\psi}_{Ab}$ are also linear in Y. □

REMARK 4.1. The main ideas of the proof are conserved if the covariance matrix Σ_η is supposed to be nonsingular. Then the terms involving $Q_{\eta 2}$ do not exist and generalized inverses can be replaced by inverses. Note, however, that singular matrices are inevitable in G_u and G_{1u}. □

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