Cash and Tournament Poker

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Cash and tournament poker: games of skill?

Report commissioned by W.M.C. van den Berg, examining magistrate in charge of criminal matters in the district of Amsterdam.

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Authors' note

The original 2009 Dutch version of this report also contained a CD-ROM with 15 appendices to facilitate its practical use at the time. Most of these appendices are listed in the references but are unfortunately only available in Dutch. In any case they are not essential for understanding the main line of argument of the report. In order not to deviate too much from the original version however we have opted to maintain all references to the appendices in the current English version.

1. Introduction

The direct reason for this report was the request by Mr. W.M.C. van den Berg, examining magistrate in charge of criminal matters in the district of Amsterdam, to answer the following question as part of the BKB case (Blaas, Kurver and Blaas):

Why, in your view, should poker be treated as a game of skill, rather than as a game of chance?

First we provide a brief outline of our expertise. Ben van der Genugten is professor of Probability and Statistics and Peter Borm is professor of Mathematics and Game Theory. We both work at the Department of Econometrics and Operations Research of Tilburg University. Since 1990 we have acted as expert witnesses in several court cases concerning the distinction between games of skill and games of chance in the Netherlands and Austria. Several projects were performed jointly with Marcel Das, professor of Econometrics and Data Collection and director of CentERdata, a research institute affiliated with Tilburg University. Professor Das has reviewed a draft version of the current report. We have also studied this topic intensively at a more theoretical level, through scientific publications as well as through the supervision of PhD and Master students.

The current report builds on the conceptual framework of the general method of relative skill as introduced in Van der Genugten & Borm (1994a), and explicitly draws on the simplifications and refinements that this method has undergone over the past 15 years through both practical experience and theoretical deliberations. In essence it has not changed, however; the method provides a tool for the objective and consistent classification of games in terms of relative skill.

The classification method of relative skill has been developed and elaborated to serve Dutch law as set forth in Article 1 of the Dutch Gaming Act: a game of chance is understood as

"an opportunity to compete for prizes or premiums, if the indication of the winners is subject to any form of chance, on which the participants generally cannot exercise any predominant influence."

In this description, the term *opportunity* stands for game in the widest sense of the word. Games in which money does not play a role are not relevant to the law on games of chance. We will therefore consistently assume that prizes and premiums are awarded in the form of money, and that the sums of money depend on the results of the game.
The legal definition of a game of chance also alludes to *chance elements*. Apparently, to qualify as a game of chance, the game must contain some chance element that designates one possibility among a series of possibilities as the winning one, while this element generally cannot be influenced by the participants in any meaningful way. Such an element is usually referred to as a chance element or an uncertainty element, but often simply as "chance". A recurrent theme in relevant verdicts is moreover the idea that a game is more skillful if the players can improve their ability through study or through frequent practice. The game is then less likely to be qualified as a game of chance. We therefore designate a game as a *game of skill* if it is not a game of chance. Every game involving money is therefore either a game of chance or a game of skill.

Our general method regarding the level of relative skill of a game produces a number between 0 and 1, whereby the value of 0 corresponds with a pure game of chance (skill plays *no role whatsoever*) and the value of 1 denotes a pure game of skill (in which chance plays *no role whatsoever*). This specifically allows for a comparison and classification of games in terms of skill level, and hence to determine the minimally required skill level (the *skill threshold level*), which will determine the game's classification as a game of skill.

In this report we choose to present our method and earlier conclusions in a fairly compact manner. For a more extensive and detailed description and analysis, in several places we refer to the appendixes on the enclosed CD-ROM containing integral versions of some of our earlier reports and publications. In the text we therefore concentrate specifically on the main lines of argument leading up to our earlier conclusions that a regular cash poker variant like Texas Hold'em should be considered a game of skill.

This report furthermore focuses on tournament poker, as this is the most frequently occurring form of poker in the BKB case. It is not at all obvious to us that any tournament version of a game of skill will automatically qualify as a game of skill as well. Accordingly, we have conducted a new and specific analysis for a tournament model that is suitable both for the specific tournament involved in the BKB case, and for the tournaments held regularly at Holland Casino. Given the time restrictions under which this report was produced, it is a non-complex tournament model that should be viewed as a reasonable, initial approach, which can be elaborated further through additional research. For this reason we can only draw conclusions from this analysis with due caution. In paragraph 5 we again keep to the main lines of argument. One of the appendixes on the CD-ROM contains a detailed overview of the analyses and computer simulations of tournament poker, performed specifically for this report.

Regarding poker variants, as far as we know only the fixed-limit cash poker variant has been assessed under the Gaming Act. This is the first time that tournament poker is under assessment. One typical feature of a tournament is that the prizes are awarded on the basis of an ultimate ranking of participants, which is determined by the game results (generally in the form of tokens or points) achieved during the rounds that make up the tournament. In this respect tournaments can well be compared to the so-called management games, which have been legally assessed previously with a view to the Gaming Act. In this study we were therefore able to benefit from our previous experience with management games. It moreover turns out that tournament poker and management games are comparable not only regarding the issue at hand, but also regarding the essence of our conclusions.

The structure of the remainder of this report is as follows. Paragraph 2 presents our main conclusions. Paragraph 3 outlines existing Dutch case law regarding the Gaming Act, with particular attention for the state of affairs concerning poker. The situation concerning management games is also considered explicitly. Paragraph 4 describes the method of relative skill for single-player and multiple-player games, with reference to the scientific and practical validation of the method. The paragraph furthermore discusses on a general level how the relative skill of a cash game compares to the relative skill of this same game in a tournament context. Paragraph 5 sketches the considerations based on previous research to qualify common multiple-player cash variants of poker, such as Texas Hold'em, as a game of skill. Here we furthermore present a separate, quantitative analysis of tournament poker.

2. Conclusions

Our conclusions issue from the method of relative skill. This method in principle enables a classification of a large number of practical games in terms of their relative skill levels. However, it does not answer the question exactly where to draw the skill threshold level. We have argued previously that, with a view to consistency in case law, this threshold should be located between 0.1 and 0.3.

The classification of a game as one of chance or one of skill cannot automatically be made to apply to tournament versions of the same game.
In analyzing the relative skill of tournaments, the number of participants in relation to the number of tournament rounds and the prize structure co-determine the outcome.

Cash poker variants of Texas Hold'em should be classified as a game of skill, as their level of relative skill exceeds the 0.3 threshold.

Our analysis of the Texas Hold'em tournament in the BKB case, explicitly taking into account the number of participants, the number of tournament rounds and the prize structure, does not give reason to classify this tournament version as a game of chance. The same conclusion applies to Texas Hold'em tournaments with around 50 participants, as organized for example by Holland Casino.

3. Dutch case law

For a detailed overview of the drafting of the Gaming Act, and our commentary on the rulings and verdicts regarding specific games until approximately the year 2000, please see Appendix 1 on the CD-ROM.

3.1. The state of affairs regarding cash poker

Fixed-limit Texas Hold'em, along with other cash variants such as Five card draw poker, Omaha Hold'em and Seven card stud poker, has been the subject of court cases. The first time that a multiple-player game was made subject to the Gaming Act occurred in the district court of Amsterdam, on 7 May 1996. The judge exonerated the organizers for a lack of evidence. As expert witnesses in this trial, we presented our assessment system based on relative skill and recommended that the poker variants be classified as games of skill. We emphatically pointed out the wide range of strategic aspects involved in such multiple-player games, particularly where it concerns games that involve incomplete information, as in this instance. This advice was followed by the court: "It is after all conceivable that starting players quickly develop a certain measure of skill with which, combined with other variables such as the strategy of other players, they can develop a personal strategy to such an extent that it cannot be ruled out that they can generally exercise predominant influence over the role of chance". The court also adopted the view that not only chance elements such as the cards one is dealt are essential to the poker variants, but also and especially the application of randomized strategies by the players: "The in itself correct observation by the public prosecutor that in the poker variants offered by the accused, a maximum of 41 to 47 (of the 52) cards may be unknown at the end of the fifth round (just before the showdown), does not contradict the foregoing. It does not force the conclusion that the participants cannot in general exercise predominant influence over the chances of winning."

This line of argument stands in stark contrast to that followed by the Court of Appeal and the Supreme Court (in which we were no longer directly involved). The Supreme Court followed the argumentation of the Court of Appeal. The Court of Appeal based its opinion on the argument that the game rules directly determine that the impact of chance in (these variants of) the game of poker is such that the players cannot have any influence. The game could only be considered a game of skill if it can be demonstrated that this impact can be "overcome" through the use of some form of probabilistic calculations. With this argument, the Court of Appeal ruled that the impact of chance cannot be overcome. Here, the court followed the advice of an expert witness who stated that the average player is not prepared to develop his skill as he only plays for the purpose of relaxation. This is not a sound argument. What exactly does "overcome" mean? To overcome what exactly? The argument presumes a type of player who simply performs a random lottery over all his possible actions at any possible decision moment. Yet no player will do so, also not in card games like blackjack and bridge: given the structure of the game he will always pursue some form of strategy with considerably better game results than by using a random lottery. In that sense alone he or she more or less overcomes the impact of pure chance. To overcome should be replaced by "to do significantly better than a beginner". Here, the beginner is definitely another person than the above described (non-existent) player using random lotteries all the time. The Court of Appeal relies on the – to put it cautiously: -- non-verifiable judgment of an expert witness. We hold a different view. In practice, poker players frequently play with a variety of other players. Our observation is that they always strive to achieve a good result. This is quite typical, incidentally, for many multiple-player card games, regardless of whether cash or game points are at stake. Finally, the Court's general deliberations are wholly

directed at poker games, without considering the implications of any comparative application of these deliberations to other card games such as bridge. In fact, it would result in the erroneous conclusion that bridge is a game of chance rather than a game of skill.

The district court's argumentation regarding the wealth of strategic aspects of poker variants is annulled by subsequent imprecise deliberations by higher judges, who were possibly also insufficiently cognizant of the difference in game characteristics between single-player games and multiple-player games.

3.2. Management games

A recent ruling in proceedings concerns the so-called 'management games' of *Competitie manager* and *Grand Prix manager GPM* (2 February 2005, no.105364, included as Appendix 2 on the CD-ROM). These management games are operated via the internet and are the subject of a detailed analysis in Van der Genugten, Borm & Dreef (2004), included as Appendix 3 on the CD-ROM, and in Van der Genugten, Borm & Dreef (2005), Appendix 4 on the CD-ROM.

A typical feature of management games is that the final prize structure is determined by the final ranking of all participants as determined by the game results (in points) achieved over the course of the game rounds. In terms of structure, these games are thus comparable to tournaments. Since in this case, the court basically adopted the arguments and conclusions of our reports in full, we shall briefly discuss a few relevant details with regard to GPM here.

The goal of GPM is to compile a Formula 1 team in terms of car parts and staffing that will achieve the best performance in a simulated season of Formula 1 competitions. It was possible to quantify the role of chance in GPM on the basis of extensive data collection combined with statistical techniques. In this way, the relative skill level was determined for various GPM variants that only differ with respect to the actual prize structure. Our first report concluded that, given a fairly horizontal or gradually increasing prize structure (in which the prizes are not only awarded to a small number of highly placed players in the final ranking), the relative skill level of GPM can be set at around 0.3, so that the final verdict on the game is: game of skill. For less gradual prize structures (as occurred in actual practice), the relative skill level came to a maximum of 0.1, so that the final verdict on the game is: game of chance. Following the recommendations in our second report, the prize structure of GPM was modified to create a more gradual structure, putting the relative skill level at round 0.3 and assuring its legal classification as a game of skill.

How can these conclusions be explained in qualitative terms? The scores in GPM vary widely, but there is a relatively large group of players who achieve scores that fall only slightly short of those achieved by advanced players in top ranking positions. The small difference in scores amply remains within the margin generated through chance. So if the prize structure is restricted to the players in the top ranking segment, then chance plays an important role in the awarding of prizes. For if the game is played repeatedly, then this relatively large group of players will often wind up as winners, while an advanced player will not. This implies that the advanced players achieve a low game result on average, with a relatively low learning effect as a result. In case of a more gradual or horizontal prize structure, the results achieved by an advanced player will vary across repeated plays, but distributed evenly at a high level. The learning effect will thus be greater in case of a gradual prize structure.

3.3. Review and skill threshold level

If we combine the data regarding management games with previous legal rulings on explicit games of chance and skill slot machines (cf. Van der Genugten, 1997a), then the designation of the *skill threshold level* would reasonably be located between 0.1 and 0.3. Games with a level of relative skill above 0.3 should in any case be classified as game of skill, and games with a relative skill level below 0.1 as games of chance. This method is in any case consistent with court rulings so far. The only exception to this is the Supreme Court ruling following the Court of Appeal with regard to a number of fixed-limit cash poker variants. It would have been preferable if they had followed the well-argued judgment of the district court.

4. Method of relative skill

The method of relative skill has been described extensively and meticulously in the general publications by Hilbers, Hendrickx, Borm & Van der Genugten (2008), Dreef, Borm & Van der Genugten (2004a, 2004b), Van der Genugten, Das & Borm (2001), Borm & Van der Genugten (2001), Borm & Van der Genugten (1998), Van der Genugten (1997a), Van der Genugten & Borm (1996a,1996b,1996c), and Van der Genugten & Borm (1994a).

Our method has been developed particularly with a view to studying so-called *strategy games* with monetary rewards. Strategy games are about mental dexterity: the ability to make sensible decisions systematically, which boils down to choosing a *comprehensive game plan* or *strategy*. A strategy thus does not correspond to a general game attitude or approach, but it provides a detailed specification of actions to be taken at any conceivable decision point in the game.

Strategy games can be classified according to a number of characteristic *game features.* A detailed classification is provided in Van der Genugten and Borm (2005), as a contribution to a book written for a legally versed audience. This contribution is included in full as Appendix 5 on the CD-ROM. Game features are: the presence or absence of chance elements, the degree of complexity (e.g. frequent or few decision moments), complete or incomplete information, equal or unequal information among different players, and of course the number of players. As in more advanced card games such as bridge, poker games typically involve multiple players, they contain chance elements, they are highly complex, the information is incomplete, and there is an information disparity among the players. Perhaps needless to say, but when referring to poker games we mean multiple-player games such as Texas Hold'em, Seven card stud poker or Five card draw poker, and not single-player games such as Caribbean stud poker or American poker.

In Paragraph 4.1 we describe in general terms the method of relative skill for strategy games with chance elements. In Paragraph 4.2 we discuss the elaboration of this method for *single-player games*. The main characteristic of a single-player game is that, although more players may be engaged in playing the same game, any one player's game result depends only on his or her own chosen strategy, and not on any actions taken by other participants. Roulette, Golden Ten and blackjack are typical examples. In *multiple-player games* such as Texas Hold'em and bridge, the result achieved by each player typically depends *partly* on the decisions taken by other players. This makes such games

intrinsically much more complex. This interaction between players must be incorporated in the method of relative skill for multiple-players games in an adequate and consistent manner. Paragraph 4.3 explains how this is done. Paragraph 4.4 offers a practical and theoretical validation of this method, drawing on relevant literature. Paragraph 4.5 offers a more general discussion of the relationship between the relative skill of a strategy game and tournament versions of the same game.

4.1. General

Classification as a “game of chance” or a “game of skill’ depends on the relative skill of a game, which is determined by weighing two effects on the *game result:*

- The *learning effect* (*LE*), due to the skill elements involved in the game,
- The *random effect* (*RE*), attributable to the chance elements involved in the game.

By “game result” we mean, in this context, the (probabilistic) expected gain: that is, the average gain over an, in principle, infinite number of repeated game plays. This concretely concerns an amount of money that is itself no longer dependent on chance. For games in which the stake amount is *not* an intrinsic component of the strategy or game plan but more of a random choice in advance (as in Roulette), we standardize the expected gains by dividing by the expected stake amount. Obviously, the classification as game of chance or game of skill does not depend on the exact value for the game result or on whether the game result is positive or negative for a certain strategy. We have opted for the term learning effect since skill is achieved through study or experience. The random effect can then be associated with any further improvement in terms of game result that could be achieved, if the effects of the random factors were known in advance. The latter is of course a *fictional* situation. The random effect offers a very elucidating means of measuring the variation in game results, solely attributable to the chance elements involved in the game.

The relative skill of a game is expressed as a number, say *S* (*“Skill”*), which is large (maximum of 1) if the learning effect is dominant, and is small (minimum of 0) if the random effect dominates. A simple formula to express this, incorporating the terms of the underlying learning effect and random effect, is as follows:

$$S = LE / (LE + RE).$$

Thus, no learning effect (*LE* =0) yields *S* = 0 and no random effect (*RE* = 0) yields *S* = 1. Every game will always have a relative skill level of between 0 and 1, so that all games can be ranked in terms of their relative skill.

4.2. Single-player games

The concepts of *LE* and *RE* have been operationalized for single-player games by distinguishing three types of players:

1) the *beginner*, who plays in naïve manner, knows and understands the rules of the game but lacks experience in actual play, with game result *R*(0),
2) the *advanced* player, who has mastered every aspect of the game, with game result *R*(*m*), and

3) the *fictional* (advanced) player, an advanced player in the theoretical situation that he knows the outcome of all chance elements during play (but cannot influence these), with game result *R(f)*.

A specific player can learn the game and can become increasingly proficient at it, until he reaches the level of an advanced player. That is why the learning effect equals:

$$LE = R(m) - R(0).$$

The extra result that the fictional player can attain over the result of the advanced players is solely attributable to the fictional knowledge of the outcome of the random factors. That is why the random effect equals:

$$RE = R(f) - R(m).$$

For many casino games (such as Roulette and Golden Ten), the fictional player's game result can simply be equated with the maximum result possible per game. For games with a small chance of a big win, only the fictional player will attain a high game result, so that the random effect *RE* is large and the relative skill, therefore, is small.

We wish to point out that the precise specification of the beginner's strategy is a freely definable parameter in our model. The skill level analysis can therefore be performed with any specification of the beginner's strategy. This means, specifically, that the fairly vague discussion about possible skill elements in a game can be reduced to the more concrete question of what population of beginners should function as reference point in determining the learning effect and the random effect.
When applying relative skill to specific games, it is necessary in each instance to adequately characterize the three types of players. Experience shows that the application of the method of relative skill to specific games requires a tailor-made analysis of the learning effect and random effect, based on the particular features of the game. Also in the event of analytical restrictions, the conceptual framework outlined above in any case provides a way of consistently comparing the learning effect and random effect.

To get a better feeling for the method of relative skill for single-player games we refer to Appendixes 6 and 7 on the CD-ROM, in which the skill level analysis is illustrated with reference to the well-known casino game blackjack and the less known but simpler game Spiel 21.

4.3 Multiple-player games

In multiple-player games, the different *player roles* are generally asymmetrical. For poker, think for instance of each player's specific position at the table. We shall therefore start by examining a single player in a specific player role, in the midst of several beginners as opponents.
We will portray this player as three different types: as a beginner, as an advanced player, and as a fictional player, but each time playing against a group of beginners. This yields a specific game result for each type. Both the advanced and the fictional type are presumed to be aware of the beginner's strategy for each player role. They will apply this knowledge to determine their own strategy.
We will then consider all the player roles for each of these three types of player, enabling us to calculate the average game result across the player roles for each type of player: $R(0)$ for the beginner type, $R(m)$ for the advanced type, and $R(f)$ for the fictional type. The resulting figures serve as input for the learning effect LE and the random effect RE, with which the relative skill S can be determined.

It is a typical feature of card games that players do not know each other's cards in many phases of the game. In that sense they have incomplete and different information. This also applies to many other multiple-player games. It can therefore be to one's benefit to vary the decisions one takes in a certain game situation. This can formally be portrayed as a randomized choice between certain possible actions with individually chosen probabilities. This *internal* decision-making mechanism, not perceptible to other players, we describe as *randomized*. A strategy containing such randomized decision-making mechanisms is accordingly referred to as a randomized strategy. For a closer examination of the role and meaning of randomization, we refer to the discussion on bluffing and sandbagging in the specific context of multiple-player poker variants in Paragraph 5. Players that randomize in this internal way add their own chance elements to the game. In our analysis we therefore assume that the fictional player also knows the outcome of these internal lottery mechanisms.

To determine the skill level, the point of departure is again the beginner's strategy. This strategy must be established for every player role and may in principle be randomized. This specification can again be considered as a freely definable parameter in our model. The analytical restrictions for concrete multiple-player games are much larger than for single-player games. In any case, the methodical approach again offers a useful conceptual framework for the consistent weighing of the learning effect and random effect.

For a more detailed conceptual discussion on relative skill in multiple-player games we refer to Hilbers, Hendrickx, Borm & Van der Genugten (2008), included on the CD-ROM as Appendix 8.

4.4. Theoretical and practical validation

First developed in Van der Genugten and Borm (1994a) and subsequently refined over the next 15 years, the method has amply proved its scientific value and practical applicability as an objective and consistent means of classifying games based on their relative skill.

Scientific output

Relative skill is the subject of the dissertations “Skill and strategy in games” (Dreef, 2005) and “Golden ten and related trajectory games” (De Vos, 1997), which were both successfully defended at Tilburg University. The method is furthermore addressed in two recent graduation theses, by Maaten (2009) and by Hilbers (2007). Scientific reports and publications relating to the method of relative skill are: Van der Genugten (2008, 2003, 1997c, 1993), Hilbers, Hendrickx, Borm & Van der Genugten (2008), Dreef & Borm (2006), Dreef, Borm & Van der Genugten (2004a, 2004b, 2003), Borm & Van der Genugten (2001, 1998), Van der Genugten, Das & Borm (2001) and Van der Genugten & Borm (1996c).

Practical applications

The method of relative skill has been applied to a large number of actual games, on the request of the public prosecutor, defense lawyers and/or the examining magistrate, as part of judicial proceedings: *Blackjack:* Van der Genugten & Das (1999b), *Fruitcard, Carribean Studpoker en American poker 2*: Van der Genugten & Das (1999a, 1999c, 1998), *ROTA-roulette*: Van der Genugten, De Vos & Das (1999), *Dromus-24*: Van der Genugten & De Vos (1998), *Jokeren, Jahtzee en Eurobsgame*: Van der Genugten (1997b), *Concard Aces*: Van der Genugten (1996), *Piramidesystemen*: Van der Genugten & Borm (1994b), *Spot the balll*: Van der Genugten & Brekelmans (2006), *Random Flashback*: Van der Genugten & Das (2005), *Managementspelen*: Van der Genugten, Borm & Dreef (2005, 2004), and, finally, *Poker*: Borm & Van der Genugten (2005) and Van der Genugten, Borm & Grossmann (1997).

4.5. Tournaments

So far, tournaments have not formally been the subject of Dutch cases of law. There has been a case of Swedish case of law in which a tournament version of Texas Hold'em was classified as a game of skill. We quote the key formulation: “…in this version of poker it is the players’ actual aptitude and analytical capabilities that are the winning point and not the actual luck factor”. For further information we refer to Appendix 9 on the CD-ROM.

Without performing an explicit skill level analysis, in this paragraph we wish to already discuss a few general aspects that play a role in the classification of tournaments. Virtually every game can be organized as a tournament, with the actual game functioning as separate game rounds. There are no grounds, however, for equating “game of chance (or of skill) game round” with “game of chance (or of skill) tournament”. The classification of a tournament very much depends on its design.

Take, as an example, a game in which each player’s goal is simply to win. The winning probability of the advanced player is greater than the winning probability of a beginner. The qualification as a game of chance or of skill also depends on the winning probability of the fictional player. These probabilities determine the relative skill S of a single play of the game. Now suppose this game were to be played in a tournament form, in which all players consistently participate in each game round, and all the game rounds are independent of each other. The tournament winner is the player who wins the largest number of game rounds, and only this person is awarded a prize. In this tournament form, the (tournament) winning probability of the advanced player increases in tandem with the number of game rounds. The game may continue to be a game of chance if the number of game rounds is limited. However, if the tournament consists of a large number of game rounds, then the tournament prize ultimately will be won by the advanced player,

and the tournament will be (almost purely) a game of skill. This always applies, regardless of how small the difference between the winning probabilities per tournament round of the advanced player and the beginner.

Let us now adapt the rules of the tournament such that, after a fixed number of rounds, only those players that have won a certain number of the game rounds may progress to the next round. This introduces the possibility that an advanced player will be eliminated after a certain number of game rounds, and thus does not win a prize. This person's probability of winning the tournament is therefore limited, even if the tournament consists of a very large number of game rounds. This design of the tournament means that the level of relative skill remains limited with an increasing number of game rounds.

Finally, we return to the original tournament design (no eliminations), but we introduce a prize structure that corresponds strictly with the number of game rounds won. Compared to 'only the tournament winner gets the prize', for the same number of tournament rounds the relative skill of this new type of tournament will be higher. After all, the advanced player will receive a prize more often than the beginner, even if he does not end up as the tournament winner.

For a game in tournament form, the number of game rounds and the prize structure thus play an important role. In fact we already encountered this phenomenon in Paragraph 3.2, with respect to management games.

In practice, game rounds are usually not independent of each other. Maximizing the result per game and achieving the highest possible final ranking in a tournament version of the same game are different goals, then. This implies, in particular, that good or advanced strategic behavior in a game and in a tournament version of the same game can be an intrinsically different matter, and hence that a game and its tournament version can differ in terms of relative skill. A straightforward example of this is provided in Maaten (2009), showing that the relative skill levels of a two-player cash poker game and a tournament version (aimed at winning more tokens than the opposite player and a fixed sum for the one who wins most tokens) can differ considerably.

5. Poker

5.1. Cash poker: general

Among the more common multiple-player cash poker variants such as Seven card stud, Five card draw poker and Texas Hold'em, the latter game is most popular in the Netherlands. Each game has different betting structures: fixed-limit, pot-limit or no-limit.

In earlier, general publications such as Van der Genugten (2008, Appendix 10 on the CD-ROM) and Van der Genugten & Borm (2005, Appendix 5 on the CD-ROM), it was shown how skill in poker variants is closely tied to practical skills regarding the calculation of probabilities, which play a role in evaluating the quality of one's own cards in combination with information from the flop, or in evaluating the quality of other players' cards. This calculation of probabilities is described in the many books written about cash poker, discussing the various rules of thumb to determine one's expected gains. The quality of these calculations can be appraised through computer simulations. The Pokerstove program, included as Appendix 11 on the CD-ROM, is a good example.

Besides skills in terms of evaluating one's options through calculations of probabilities, more psychological techniques such as bluffing (bidding on a bad hand, instead of folding) and sandbagging (not bidding on a good hand, but going along or checking) form an essential part of good game strategy. Both techniques aim to make the best possible use of the fact that all of the players, without exception, are in a situation of incomplete information (nobody knows another player's cards); this is something that inexperienced poker (but also bridge) players tend to realize insufficiently. The best way to apply these techniques can, in theory, be calculated mathematically. Naturally, players should not always bluff or use a sandbagging type of strategy, as this would neutralize the uncertainty factor. This leads to randomization, in quite a natural manner: a good bluffing or sandbagging strategy uses an internal randomization mechanism. As noted above, the odds to apply here (how often should I bluff with this type of hand?) can be exactly calculated in principle, but players can also develop a good feel for this simply through frequent play. In that sense they learn to play the game, and skill is acquired. There is an extensive literature on this subject. There is even special software to help develop such strategies through self-study; see for instance the computer program "Turbo Texas Hold'em for Windows", as described in Wilson (2005).

To characterize a beginner's strategy, it seems reasonable to assume that a beginner does not apply the techniques described in various poker handbooks. After all, studying and mastering these techniques requires a considerable effort, which the beginner has yet to make. An advanced player can of course be assumed to have made this effort. In accordance with this line of reasoning, the typical ingredients of a beginner's strategy would be: to stay in the game for too long with relatively poor cards (no or wrong estimation of probabilities), and not (successfully) applying the techniques of bluffing and sandbagging (too little game experience). The general import of these ingredients has been confirmed to some extent by the results of the Texas Hold'em experiment, as described in Maaten, Borm, Van der Genugten & Hendrickx (2008), included as Appendix 12 on the CD-ROM. This experiment clearly demonstrated that many beginners in any case wish to stay in the game until the flop, that they do not use sandbagging techniques, and that they attempt to bluff sporadically, without careful timing. In our skill level analysis concerning cash poker, as reported in various publications, these ingredients will always, in one way or another, form the foundation for the choices made in a beginner's strategy, which, as argued before, serves as reference point for the determination of relative skill.

We conclude this paragraph with a general comment on the possible difference in relative skill between fixed-limit, pot-limit and no-limit variants of multiple-player poker. The results by Hilbers (2007) and Hilbers, Hendrickx, Borm & Van der Genugten (2009) concerning the variation in the height of the possible fixed-limit bidding from low to high, indicate that the difference in relative skill between the three variants is not significant. Restricting the skill level analysis to fixed-limit variants thus appears justified.

5.2. Cash poker: a game of skill

The report entitled “Poker: ein Geschicklichkeitsspiel!” (Borm & Van der Genugten, 2005, Appendix 13 on the CD-ROM) extensively argues that, with a view to consistent jurisprudence, common multiple-player cash poker variants such as Texas Hold’em should be classified as games of skill, on the basis of a relative skill level analysis. This conclusion was based on both an exact quantitative analysis of relatively simple stylized poker variants that did however incorporate key skill aspects of real poker variants, and on a more qualitative analysis based on a comparison of so-called skill indicators.

More recent studies into complex poker variants that more closely approximate the real poker game, by means of straight poker in Hilbers (2007) and Hilbers, Hendrickx, Borm & Van der Genugten (2009), once again confirm this conclusion. Depending on the exact specifications of the game (number of players, bet heights, number of bidding rounds, the rake, choice of beginner’s strategy), the relative skill level varies consistently between 0.3 and 0.5. A further confirmation that the skill level is at this level is provided by the simulation results regarding Texas Hold’em, with two players and two phases (“pre-flop” directly followed by “river”); see Appendix 14 on the CD-ROM.

In realistic poker variants, there is at least one additional source that demonstrates skill, compared to straight poker or the just-indicated two-phase poker. The point is that the ultimate composition of a player’s hand of five cards is built up in a larger number of phases, possibly with open own cards or shared (community) cards. Generally speaking, in a complex game with various phases an advanced player will have more scope to obtain information – information which is freely available to the fictional player. In a global sense we could thus say that, in a more complex game, the difference in information between an advanced and a fictional player plays a comparatively smaller role, which means that the relative skill level is greater.

5.3. A skill analysis of tournament poker

The study into the relative skill level of multiple-player cash poker reveals that, depending on the precise specifications, this game has a skill level ranging between 0.3 and 0.5.

Tournament poker differs from cash poker in the following essential manner. Players purchase tokens, which are the only stake throughout the tournament, and the ultimate number of tokens won determines players’ final ranking. Payouts are subsequently awarded on the basis of this ranking only. The relative skill of tournament poker therefore comes to depend in part on the prize structure, and as argued in paragraph 4, this skill level can differ from that of the underlying cash poker game.

To obtain insight into the relative skill level of tournament poker, we assume a stylized “*base form*” of a single game round. If such a single game round is only played for cash, then this game corresponds with cash poker. On account of the limited time available to us, the chosen base form of a single play round is a somewhat crude model, but this could be refined in further research.

Tournament poker consists of a series of such stylized game rounds, with tokens at stake. The tokens determine the final ranking, which then translates into cash payouts in accordance with the prize structure. Given our choice for this base form, we studied a number of base tournament variants which explicitly incorporate the characteristic aspects that play an effective role in practice. The step taken in our model from a game round to a tournament is thus an accurate reflection of actual practice. The model

particularly takes into account the number of players, the anticipated duration of the tournament, the number of game rounds, the changes of table, the variation in the number of players per table due to possible quitters, the initial amount of tokens and the stake increases across the game rounds. The only aspect not factored in is the dependency between game rounds, generated by a game strategy that stretches across the entire tournament. To incorporate this dependency would make the analysis hugely complex and time-consuming.

The so-called BKB base tournament models the BKB poker tournament with 45 participants. Unfortunately, the information available to us did not indicate the exact prize structure. However, it appears from several witness accounts that this structure does not differ significantly from the prize structure used in comparable poker tournaments in Holland Casino, and thus we have used this structure in our analysis. We additionally performed a specific analysis with regard to the so-called HC base tournaments that model the poker tournaments as organized by Holland Casino, based on the Master Classics of Poker. In our analysis we specifically vary the number of players within a realistic duration of play. A more detailed description of the simplified form of a single game round and the corresponding tournaments is offered in the second part of Appendix 15 (poker tournaments). We shall restrict our discussion to the main points here.

An important quantity in the analysis concerns the winning percentage (Win%) of the bet that an advanced player will achieve during a base game round with a single beginner as opponent. In our simplified model, this winning percentage should correspond with the winning percentage that an advanced player would achieve at a real cash poker table with a single beginner as opponent. This winning percentage may possibly be derived as well from the real winning percentage achieved by an advanced player at a cash poker table with multiple beginners. Ideally, an accurate estimate of the winning percentage Win% should be derived from empirical material. However, given the nature of this quantity – an advanced player among beginners – such material is not immediately available. The only usable data available derives from the Poker Experiment as documented in Maaten, Borm, Van der Genugten & Hendrickx (2008, Appendix 12). However, given the statistical limitations of the experiment on account of the small number of observations, the analysis contained in the first part of Appendix 15 (bids and gains) only indicates a lower limit for the winning percentage Win%. This percentage will in any case be higher than 5%, and will probably be considerably higher, but we wish to be cautious about drawing any clear-cut conclusions on the basis of this limited experiment. If we take the following rule of thumb (see Meinert (2007), page 270, a standard reference book on poker): “A skilled, solid player places approximately two big bets per hour. So if you play 3 / 6 Euro fixed-limit poker, the hourly wage comes to … around 12 Euros” -- and through some mathematical operations translate this to correspond with our base model, then this means a percentage of 3.5%. It should be noted that the quoted rule of thumb assumes a regular, real-life poker table; so instead of having one “skilled, solid player” facing only beginners, there will be other experienced or even advanced players at the table. The 3.5% derived from the rule of thumb thus signifies a lower limit for the winning percentage Win%. This percentage will in fact be considerably higher than 3.5%, and the percentage of 5% determined through the poker experiment would appear to be a cautious lower limit, based on the quoted rule of thumb.

In our analysis we therefore only consider winning percentages starting from 5%. Further, the relative skill level (S-cash) of the corresponding cash poker should be located between 0.3 and 0.5, to remain in conformity with our previous findings for cash poker variants. For all base forms that meet the conditions above, we subsequently calculate the relative skill level of the associated base tournament in four different ways: S-1, S-2,

S-3 and S-4. The difference between them is due to the different choices with regard to the prize structure. We have opted to do so in order to make visible the effect of the prize structure on the relative skill.
We will now explain the differences between the four calculation methods.

S-1: the level of relative skill that is based on a prize structure in which only the final tournament winner receives a payout.

S-2: the level of relative skill that is based on a gradual prize structure in which all players receive a payout proportional to their final ranking. With 45 participants, this leads to the structure [45 44 43 … 3 2 1 0 … 0].

S-3: the level of relative skill that is based on a prize structure in which all players at the final table receive a payout proportional to their final ranking while the other players do not receive any payout. With 9 participants at the final table, this leads to the structure [9 8 7 6 5 4 3 2 1 0 … 0].

S-4: the level of relative skill that is based on the prize structure as used by HC and BKB. The 6 best players at the final table in the final ranking receive payouts in a less gradual way (than with respect to S-3) to their final ranking while the other players do not receive any payout. This leads to the structure [37 23 15 11 8 6 0 … 0].

For the classification of the BKB and HC tournaments, the relative skill S-4 is essential given the fact that the associated prize structure most resembles the prize structure that is actually used in these tournaments.

The relative skill levels S-1, S-2 and S-3 not only serve for comparison but also to verify the qualitative arguments with respect to the form of the prize structures, as indicated in paragraph 3.2 with respect to management games. The relative skill level S-1 corresponds with the most extreme prize structure imaginable: only the tournament winner wins a prize. On the opposite side of the spectrum, S-2 corresponds with a very gradual prize structure that is fully proportionate to a player's final position in the final ranking. We shall see that, also for base tournaments, the relative skill level S-1 (corresponding with an extreme prize structure) turns out significantly lower that skill level S-2 with a gradual prize structure. Relative skill levels S-3 and S-4 correspond with prize structures that might be considered intermediate forms, with the prize structure that corresponds with S-3 having a more gradual structure than the structure for S-4.

To give an idea of our simulation results for the BKB base tournament, a representative amount of data is summarized below in Table 1. The full version is given in the second part of Appendix 15 (poker tournaments).

Table 1. Relative skill levels of poker tournaments, depending on the winning percentage (Win%) and the prize structure.

Win%	S-cash	S-1	S-2	S-3	S-4
5	0.50	0.35	0.51	0.44	0.40
5	0.45	0.30	0.44	0.38	0.35
5	0.42	0.26	0.41	0.35	0.31
5	0.38	0.23	0.39	0.32	0.28
5	0.36	0.19	0.34	0.27	0.24
5	0.33	0.17	0.32	0.25	0.22
5	0.31	0.15	0.31	0.24	0.20

10	0.50	0.28	0.47	0.39	0.34
10	0.45	0.24	0.43	0.35	0.30
10	0.43	0.22	0.41	0.32	0.27
10	0.37	0.17	0.36	0.27	0.22
10	0.35	0.15	0.34	0.25	0.20
10	0.32	0.14	0.32	0.23	0.19
10	0.30	0.13	0.31	0.23	0.18
13	0.50	0.28	0.48	0.39	0.34
13	0.46	0.25	0.45	0.36	0.31
13	0.43	0.23	0.43	0.34	0.28
13	0.38	0.20	0.40	0.30	0.25
13	0.36	0.19	0.39	0.29	0.24
13	0.32	0.18	0.37	0.28	0.23
13	0.30	0.17	0.36	0.27	0.22
15	0.50	0.30	0.53	0.43	0.37
15	0.45	0.27	0.50	0.40	0.33
15	0.43	0.26	0.48	0.38	0.32
15	0.39	0.24	0.46	0.36	0.30
15	0.35	0.23	0.44	0.34	0.29
15	0.33	0.23	0.44	0.34	0.28
15	0.31	0.22	0.43	0.34	0.28
17	0.50	0.33	0.53	0.45	0.39
17	0.46	0.31	0.51	0.43	0.37
17	0.44	0.30	0.50	0.41	0.36
17	0.39	0.29	0.49	0.40	0.35
17	0.35	0.28	0.48	0.39	0.34
17	0.33	0.28	0.47	0.39	0.34
17	0.30	0.28	0.47	0.39	0.33
20	0.50	0.40	0.52	0.46	0.43
20	0.45	0.38	0.50	0.44	0.41
20	0.43	0.37	0.48	0.42	0.39
20	0.40	0.36	0.47	0.41	0.38
20	0.35	0.35	0.45	0.40	0.37
20	0.33	0.35	0.45	0.40	0.37
20	0.30	0.35	0.45	0.39	0.37

For illustration purposes, we shall examine the following row in this table:

Win%	S-cash	S-1	S-2	S-3	S-4
15	0.39	0.24	0.46	0.36	0.30

This row indicates that, when the fixed winning percentage equals 15% and the fixed relative skill level S-cash = 0.39 in the base form of the cash poker game, the relative skill level S-4 of the corresponding BKB base tournament can be determined as 0.30. A more gradual prize structure for the final table results in a higher relative skill level of S-3 = 0.36, and a fully gradual prize structure results in an even higher skill level of S-2 = 0.46. However, with a payout for only the winner, the relative skill level would result in a lower figure: S-1 = 0.24.
To analyze the influence of the winning percentages, in Table 1 we consider 6 different winning percentages: 5%, 10%, 13%, 15%, 17% and 20%. For each of these percentages we consider seven values for the relative skill level S-cash for the base form of cash poker, which adequately cover the potential range from 0.3 and 0.5.

From the last column in Table 1, listing the relative skill level S-4, we may conclude that our analysis does not support the classification of the BKB poker tournament as a game of chance. The relative skill S-4 is mostly above 0.2 for all winning percentages, and mostly above 0.3 for a winning percentage of 15% or higher. Remember also that, in

choosing the winning percentage, we assumed a real minimum level of 5%. More theoretical considerations such as the analysis for two-phase Texas Hold'em in Appendix 14 suggest that Win% may well exceed 20%, so that it seems entirely justified to classify this a game of skill. However, exercising due caution with respect to our rough modeling, we prefer to restrict our conclusion to saying that there is "no reason for classification as a game of chance."

If we compare the level of relative skill associated with the different prize structures, we see that the BKB base tournament, in which only the tournament winner is awarded a payout with a winning percentage of up to approximately 15%, mostly results in skill levels (see column beneath S-1) of above 0.1 and below 0.3, so that the skill classification remains unclear. However, with a winning percentage of 20% the skill levels would amply exceed 0.3 for this prize structure as well. If the fully gradual prize structure that corresponds with the relative skill levels in the column beneath S-2 were to be applied, then the classification as a game of skill is clear. Mainly on account of the low number of participants of 45, this would also apply in case of the more gradual prize structure at the final table, corresponding with the relative skill levels as listed in the column beneath S-3.

The above implies that, for the BKB poker tournament to be classified even more clearly as a game of skill, it would be advisable to adopt a somewhat more gradual prize structure than currently applied.

An interesting ancillary conclusion from the above results is that the statement "tournament poker involves more skill than cash poker", or "cash poker involves more skill than tournament poker", both prove to be incorrect. With a winning percentage of 17% and a skill level for cash poker of 0.39, the relative skill level S-4 of the corresponding tournament equals 0.35, whereas, with a relative skill level for cash poker of 0.30, the relative skill level of the corresponding tournament equals 0.33.

The analysis with respect to the HC base tournaments yields a picture comparable to that for BKB base tournaments, also regarding the important effect of the prize structure on the level of relative skill. On the basis of our findings, there is no reason to classify an HC poker tournament with 50 participants as a game of chance. Our analysis does show very clearly, however, that the number of participants in a tournament in relation to the duration of the tournament plays a crucial role in the skill classification. Already with 100 participants, the prize structures that only award prizes to players on the final table result in relative skill levels (S-3 and S-4) that predominantly fall below 0.3. With 250 or 500 participants, this effect is only amplified. Where such variants are concerned, classification as "game of chance" would seem more appropriate.

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