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Smulders, J.A.; van de Klundert, T.C.M.J.

Published in:
The Monopolistic Competition Revolution in Retrospect

Publication date:
2004

Link to publication

Citation for published version (APA):

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Monopolistic Competition and Economic growth*

SIAK SMULDERS
Tilburg University

THEO VAN DE KLUNDERT
Tilburg University and Groningen University

This version: September 2004

JEL: O41, O31, L16.

Keywords: Dixit-Stiglitz model, monopolistic competition, growth, entry, research and development, trustified capitalism.

Correspondence: Department of Economics, Tilburg University, PO Box 90153, 5000 LE Tilburg, The Netherlands, Phone: +31.13.466.4940, Fax +31.13.466.3042, email: j.a.smulders@kub.nl.

* This paper is a slightly revised version of the chapter with the same title in B. Heijdra and S. Brakman (eds.) “The Monopolistic Competition Revolution in Retrospect”, Cambridge University Press 2004, page 307-331.
Abstract

The Dixit-Stiglitz model of monopolistic competition is an important building block of a number of theories of economic growth. We discuss these theories in a general equilibrium framework with two types of research and development (R&D). First, new product lines can be introduced by incurring a sunk cost. Second, incumbent firms can raise productivity by in-house investment in tacit knowledge. Special cases of the model include a dynamic version of the Dixit-Stiglitz model, the semi-endogenous growth model, the semi-endogenous growth model based on variety expansion, the endogenous growth model based on in-house R&D, and a combination of the latter two. It is shown that the intensity of competition play quite a different role in the various cases distinguished.
1. Introduction

The Dixit-Stiglitz (1977) model of monopolistic competition has been the essential building block for the new generation of growth models that was developed by Romer (1990) and others. It is a well-known property of neoclassical theory that exogenous forces ultimately drive growth. By relying on perfect competition neoclassical growth theory could not model productivity growth and technical change as endogenous variables. R&D efforts lead to increases in productivity, but typically R&D expenditures are a fixed cost, which can be recouped only if firms make profits.

Monopolistic competition can generate profits in the short run if the number of competing firms is not too large. In the Dixit-Stiglitz model entry of new firms will lead to zero profits in the long run. However, the standard assumption that new firms appear out of the blue does not seem realistic. Each new firm in the model of monopolistic competition introduces a new product variant, which may require innovative effort in the first place. The development of a new product requires an up-front R&D expenditure. Entry then takes place as long as the R&D costs do not exceed the net present value of future profits that can be reaped by bringing the new product on the market.

In the Dixit-Stiglitz model the number of firms is finite in the long run. Entry of new firms reduces profits of all firms in the industry. This result is not changed if account is taken of up-front expenditure to develop new products. After the dust of entry is settled we are back in a static economy. What really changes this picture is Romer's idea of intertemporal knowledge spillovers. As the number of products increases the cost of developing new products falls, because innovating firms can build on the knowledge developed by their predecessors. Therefore, the number of products already developed can be associated with the stock of ideas on which researchers can build. It is a public good that creates an intertemporal knowledge spillovers. As a result the cost of R&D decreases and a never ending process of entry and introduction of new products drives economic growth.

Various other models build on Romer's approach and can be classified as "variety expanding models" (cf. Grossman and Helpman, 1991, Chapter 3). In alternative approaches it is not entry but quality improvement and creative destruction that induces growth (cf. Aghion and Howitt 1992, 1998; Grossman and Helpman, 1991, Chapter 4). Knowledge spillovers are also the driving force but competition is often modelled as limit pricing rather than monopolistic pricing. As there appears to be no clear connection with the Dixit-Stiglitz model we will not discuss these other growth models in the present survey.

Instead we would like to focus on a third approach in endogenous growth theory that also starts from the Dixit-Stiglitz approach, but does not rely on the somewhat unrealistic feature of never ending entry. Thompson and Waldo (1994), Smulders and Van de Klundert
Peretto (1996) and Peretto and Smulders (2002) have developed a growth model in which monopolistic competing firms undertake in-house R&D that results in productivity improvements for each firm. As all existing firms expand their production growth is economy-wide. Economic growth is sustained since private research creates a tacit knowledge stock that reduces future R&D costs. In these models the knowledge stock is not a pure public good, as in the endogenous growth theories referred to above, but can be appropriated fully or at least to a large extent by the firms themselves. The models with in-house R&D are representative for the system of "trustified capitalism", which Schumpeter in his later work had in mind as one of regimes of growth and competition (e.g. Soete and Ter Weel, 1999). In the regime of trustified capitalism large firms with unthreatened market positions dominate the economy and undertake their own R&D to reduce cost and improve productivity or product quality.

In the models with in-house R&D entry may be possible in the early stages of development in which the number of firms is small and profits are excessive. Newcomers have to invest in R&D in order to develop new product lines. Under certain conditions it may be profitable to engage in this type of R&D, but after a while entry stops as excessive profits vanish. In this view entry is not essential to explain economic growth. Once entry has stopped incumbents dominate the market and growth is driven by the in-house creation of knowledge. The window of opportunity for new entrants is closed and economic growth in determined by large firms with their own history and future.

This paper aims at discussing the role monopolistic competition plays in several theories of economic growth. We develop a general equilibrium model with in-house R&D as well as R&D that results in the creation of new product lines. It will be shown that different growth models in the growth literature are special cases of our more general model. Throughout the analysis we are concerned with the determinants of equilibrium growth and product variety. Welfare properties of the equilibria are beyond the scope of the present paper.

The paper is organised as follows. In Section 2 we introduce our model of economic growth allowing for two types of R&D. On the demand side account is taken of differentiated goods and a taste for variety. In accordance with the Dixit-Stiglitz model there is monopolistic competition on the markets for goods. Perfect competition prevails in the labour market and in the capital market. Variety expansion models are discussed in Section 3. Special cases consider relate to the dynamic version of the Dixit-Stiglitz models with a finite number of firms, the semi-endogenous growth model of Jones (1995) and the model of endogenous growth of Romer (1990) and Grossman and Helpman (1991, Chapter 3). The model with in-house R&D is analysed in Section 4. To focus on the engine of growth the number of firms is fixed. The latter assumption is relaxed in Section 5, where two types of
innovation – variety expansion and productivity improvement – are introduced. It is shown that depending on the number of firms in the initial state there may be two phases of economic growth. In the first phase both types of R&D generate the same rate of return and variety expansion goes along with productivity improvement. In the second phase entry stops and growth is driven by productivity improvements within existing firms. Conclusions are presented in Section 6.

2. Model

Households

The composite consumption good is defined over a continuum of varieties, a mass \( N \) of which is available in the market:

\[
C = N^{1/(\epsilon - 1)} \left( \int_0^N X_i^{(\epsilon - 1)/\epsilon} di \right)^{\epsilon/(\epsilon - 1)}
\]  

(1)

Following the Dixit-Stiglitz (1977) approach, the elasticities of substitution between different varieties is constant and equal to \( \epsilon > 1 \). We generalize this approach along the lines of Heijdra and van der Ploeg (1996) and Benassy (1996) by disentangling the assumption of product differentiation (measured by \( \epsilon \)) from the taste for variety (measured by \( \nu \); \( \nu = 0 \) implies no taste for variety, \( \nu = 1/(\epsilon - 1) \) brings us back to the canonical Dixit-Stiglitz model).

Intertemporal utility is discounted utility of the composite consumption good:

\[
U = \frac{1}{1 - \rho} \int_0^\infty C(t)^{1-\rho} \exp(-\theta t) dt
\]  

(3)

We assume a constant discount rate \( \theta \). The curvature of the utility function is governed by the constant rate of relative risk aversion \( \rho \). The main advantage is that the elasticity of intertemporal substitution equals a constant \( 1/\rho \), which accompanies the constant elasticity of \( \text{intra} \)temporal substitution (\( \epsilon \)) familiar from the static Dixit-Stiglitz model. As we will see below, the relative size of both elasticities determines the nature of the dynamics in the model.

Utility maximization gives rise to a two-stage maximization problem. First, consumers trade off current consumption against future consumption, taking as given the
relative price of consumption over time, viz. the real interest rate \( r - \hat{p}_c \). This gives rise to the Ramsey rule:

\[
r - \hat{p}_c = \hat{\vartheta} + \varrho \hat{C}
\]

(4)

where hats denote growth rates, and

\[
p_c = N^{-\left(\nu - \left(\frac{1}{\varrho} - 1\right)\right)} \left( \int_0^N p_i^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}
\]

(5)

denotes the consumer price index. Second, consumers allocate per period consumption expenditures over the different varieties, which gives rise to the familiar iso-elastic demand function for good \( i \):

\[
X_i = C(p, p_c) \quad \quad \quad (6)
\]

In the sequel we assume symmetry so that \( X_i \) and \( p_i \) are the same for each variety \( i \). Hence we may write \( C = N^\nu (NX_i) \) for the consumption index and \( p_c = N^{-\nu} p_i \) for the price index. Differentiating with respect to time and substituting the results into the Ramsey equation, we find:

\[
r = \hat{\vartheta} + \varrho (NX_i + \nu \hat{N}) + (\hat{p}_i - \nu \hat{N}) \equiv \hat{r}_c
\]

(7)

We interpret the expression on the right-hand side as the required rate of return in the capital market. The first term is the utility discount rate. The last term in parenthesis is the consumption price index inflation rate. When consumer prices increase, households require higher nominal interest rates. The second term reflects consumption smoothing: household require premium for postponement of consumption; the smaller the elasticity of intertemporal substitution (\( \frac{1}{\varrho} \)), the larger the required premium for a given rate of change in consumption (which equals the first term in parenthesis).\(^1\)

**Firms**

Firms produce with labour only. Firm \( i \) needs \( 1/A_i \) units of labour to produce one unit of its variety:
Variable $A_i$ can be interpreted as the stock of firm-specific knowledge that determines the productivity of the firm. Note that the units of measurement of $X$ are such that $X$ directly enters the utility function, which means that $A$ also reflects the firm’s capability to produce a certain level of product quality per unit of labour input. Hence, any increase in $A$ reflects productivity or quality improvements, which terms we will use interchangeably.

Each firm can expand this stock and thus improve its own productivity over time by allocating research labour to research and development (R&D) activities. The productivity of research labour $L_{Ai}$ depends on the stock of knowledge already accumulated ($A_i$) and on other sources of knowledge, which we will later identify as spillovers from other firms, denoted by $S_i$:

$$\dot{A_i} = S_i A_i^\alpha \cdot L_{Ai}$$  \hspace{1cm} (9)

We do not restrict the sign of $\alpha$. If $\alpha$ is positive, firms use the knowledge they have accumulated in the past as an input in new research and development projects and benefit from experience and learning by doing. If $\alpha$ is negative, further improving productivity becomes harder the more productive the firm already is.

Profits equal revenue minus labour costs:

$$\pi_i = p_i (X, \hat{X}) \cdot X_i - w(L_{Xi} + L_{Ai})$$  \hspace{1cm} (10)

Firms maximize the net present value of profits, taking into account the downward sloping demand for their product. The first order conditions for maximization imply the following pricing and no-arbitrage equation:

$$p_i = \frac{\varepsilon}{\varepsilon - 1} \frac{w}{A_i}$$  \hspace{1cm} (11)

$$\frac{wL_{Xi}}{w/(S_i A_i^{\alpha - 1})} + (\hat{\omega} - \hat{S}_i) \equiv r_i = r$$  \hspace{1cm} (12)

The elasticity of substitution (price elasticity $\varepsilon$) determines the markup over unit labour costs $w/A_i$. The no-arbitrage equation equates costs savings per euro invested in R&D, that is the
rate of return to R&D, \( r_A \), to the cost of capital, that is the required (nominal) rate of return \( r \). The first term on LHS represents marginal labour cost savings in production from investing in knowledge divided by the marginal cost of knowledge accumulation. We see that the larger firm size \( L_X \), or the larger marginal research productivity \( S_A A_i^{\alpha-1} \), the higher returns. The second term on the LHS captures the increases in research costs over time due to factors the firms cannot control (wage cost and spillovers). The larger these cost increases, the more attractive it is to undertake R&D now rather than later, that is, the higher the current return to R&D. The equation only applies in an interior solution with R&D \((L_{Ai} > 0)\). In a corner solution without research, \( r_A < r \) and \( L_{Ai} = 0 \).

Substituting the markup pricing result (11) into the profit function (13), we find:

\[
\pi_i = w \left( \frac{1}{\varepsilon - 1} L_{Xi} - L_{Ai} \right)
\]

(13)

**Market structure: entry**

To model entry as a dynamic process, we assume that potential entrants have to incur a sunk cost to develop a blueprints for a new product line. The introduction of new product lines, or *variety expansion* for short, can be considered as the second type of R&D in the model, alongside in-house R&D directed to quality improvement within existing product lines as described above. The research technology is such that if \( L_N \) units of labour are allocated to variety expansion, a flow of \( S_N L_N \) new firms is created.

\[
\dot{N} = S_N L_N
\]

(14)

Free entry in R&D ensures that the price of a blueprint, \( p_N \), equals the cost of developing a blueprint, \( w / S_N \). The price of a blueprint in turn equals the value of a blueprint for an entrant, \( v_N \). This value must be such that investing money in blueprints gives a return that equals the return in the bonds market, which implies \( \pi / v_N + \hat{v}_N = r \). Substituting \( \pi \), from (10) and (11), and \( v_N = p_N = w / S_N \) we find the following no-arbitrage equation:

\[
\frac{w \left( \frac{1}{\varepsilon - 1} L_{Xi} - L_{Ai} \right)}{w / S_N} + (\hat{w} - \hat{S}_N) \equiv r_N = r
\]

(15)
The LHS represents the return to investing in new product lines, the return to entry \( (r_N) \) for short. The first term represents the profit flow from a new product line relative to the cost to develop the new line. The second term represents the increase in development cost over time. This equation only holds in an interior equilibrium \( (L_N > 0) \). If no entry takes place, we must have \( r_N < r \) and \( L_N = 0 \).

**Labour market equilibrium**

Labour supply is exogenous at level \( L \) and grows at rate \( g_L \). Total labour demand equals demand for production, and for the two types of research. With symmetric firms, this implies the following labour market clearing condition:

\[
L = (L_{\alpha} + L_{\mu})N + L_N
\]

(16)

3. Growth through variety expansion

We first study growth driven by the expansion of product variety. We show how the seminal growth models developed by Romer (1990), Grossman and Helpman (1991, chapter 3), and Jones (1995) can be seen as a dynamic version of the Dixit-Stiglitz model extended for knowledge spillovers that govern the cost of entry.

To focus on entry, we assume that all research effort in the economy is devoted to developing new product lines \( (L_{\alpha} = 0) \). Moreover we assume that the cost of entry endogenously changes over time. The idea is that research on new product lines builds on a stock of public knowledge. Individual research efforts contribute to public knowledge, that is, there are knowledge spillovers from private research to the public knowledge stock. Since research aims at expanding \( N \), we can relate the knowledge stock to \( N \). In particular, research productivity \( S_N \) increases with \( N \):

\[
S_N = \chi N^\phi \quad \text{for } \phi \leq 1
\]

(17)

From labour market clearing condition, we can derive \( \dot{N} = \chi (L/N^{1-\phi}) - \chi (N^\phi L_N) \), which we can rewrite as an expression for how \( L/N^{1-\phi} \) evolves over time:

\[
\left(\frac{L}{N^{1-\phi}}\right) = g_L - (1-\phi)\chi(L/N^{1-\phi}) + (1-\phi)\chi(N^\phi L_N).
\]

(18)
where $g_L$ is the exogenous growth rate of the labour supply.

The capital market is in equilibrium if the return to innovation $r_N$ equals the rate of return required by households $r_c$. Combining the Ramsey equation (7) and the no-arbitrage equation (12), solving for $\hat{N}$, and rewriting using the above labour market condition, $\hat{N} = \chi(L/N^{1-\phi}) - \chi(N^g L_{\chi})$, we find:

$$\left[\frac{N^g L_{\chi}}{N^g L_N}\right] = (N^g L_{\chi})\chi\left[\frac{(1 - v + \phi)}{(\epsilon - 1)^{\phi}}\left[\frac{1}{\rho} + (v + 1 - \phi)\right] - (L/N^{1-\phi})\chi\left[(v + 1 - \phi) - (v - \phi)^{1/\rho}\right]\right] - \frac{1}{\rho} \tilde{\theta}$$

Equations (18) and (19) now form a system of two differential equations in $L/N^{1-\phi}$ and $N^g L_{\chi}$, the first of which is predetermined, while the second can jump. We can therefore easily set up a phase diagram with these two variables on the axes, cf. the three panels in Figure 1. Recall that the relevant parameters are research productivity $\chi$, spillover parameter $\phi$, intratemporal and intertemporal substitution $\epsilon$ and $1/\rho$, taste for variety $v$, discount rate $\tilde{\theta}$, and population growth rate $g_L$. The exact nature of the phase diagram changes with the parameters, as we will discuss below. The adjustment is saddle point stable as long as the first term in brackets in (19) is positive. For $\phi < 1$, the saddle path slopes up (down) if the second term in brackets is positive (negative), that is if the elasticity of intertemporal substitution $(1/\rho)$ is small. Points above the 45-degree line are infeasible since they imply a violation of the labour market constraint (16).

*** insert figure 1 a, b, c Phase diagram of the variety expansion model ***

**Special case i (Dynamic Dixit-Stiglitz model):** $\phi = g_L = 0$

By assuming $\phi = g_L = 0$, we find the simplest dynamic version of the Dixit-Stiglitz model: rather than instantaneous entry, we have time-consuming entry (sunk cost rather than fixed cost). The dynamic model reduces to two differential equations in $N$ and $L_{\chi}$ and is represented in Figure 1a. If the number of firms happens to be large initially, such that $L/N < \tilde{\theta}(\epsilon - 1)/\chi$, the economy is stuck in a stationary equilibrium, in which $N$ is historically determined and $L_{\chi} = L/N$. The market has to be shared by so many firms that the rate of return to entry falls short of the minimum required rate of return $\tilde{\theta}$ and no investment takes place. In contrast, if the initial number of firms is small, such that
entry takes place and the economy approaches a steady state that can be characterized by:

\[ L_{x_i} = (\varepsilon - 1) \frac{\hat{\vartheta}}{\chi}, \quad N = \frac{\chi L}{(\varepsilon - 1)\hat{\vartheta}} \]  

This steady state can be easily related to the static Dixit-Stiglitz model. In the static version with flow fixed cost \( F \) per firm and instantaneous entry, we find \( L_{x_i} = (\varepsilon - 1)F \) and \( N = L/\varepsilon F \) (cf. Neary’s chapter in this book). Hence, in the dynamic model, the term \( \hat{\vartheta}/\chi \) takes over the role of the fixed cost in the static model. Note that this term equals the steady state interest rate \( (\hat{\vartheta}) \) times the set-up cost \( (1/\chi) \), and therefore represents the annualized setup cost. The preference parameters \( \nu \) and \( \rho \) do not affect the steady state solution. The taste-for-variety parameter \( \nu \) measures how the cost of capital changes with a change in product variety (see (7)), the intertemporal substitution parameter \( \rho \) measures how the cost of capital changes with a change in consumption. In the steady state, both consumption and variety are constant so that the two parameters do not play a role.

The dynamics of the model enrich the picture of monopolistic competition. In the static Dixit-Stiglitz model firm size is fixed (which is criticized in Neary’s Chapter). However, in the dynamic model outside the steady state, firm size \( L_{x_i} \) decreases or increases over time along the saddle path for two reasons. First, the number of firms cannot change instantaneously. Second, if total savings in the economy changes, total sales, which is shared among incumbents, changes. A large part of the labour force may be allocated to R&D activities in the short run, thus reducing the amount of labour for the firms that are already in the market. In particular, firm size increases (decreases) with the number of firms if the saddlepath in Figure 1 slopes downward which requires: \((1+\nu)/\nu < 1/\rho\); it slopes downward if the latter inequality is reversed. In the canonical DS formulation (where \( \nu = 1/(\varepsilon - 1) \)) this reduces to \( \varepsilon < (>) 1/\rho \). So what matters is whether the intratemporal elasticity of substitution (the DS variable \( \varepsilon \)) is larger or smaller than the intertemporal elasticity of substitution. Intuitively, if the taste for variety is large (\( \varepsilon \) small) and the intertemporal elasticity is large, and if the economy starts with a small number of firms, it is attractive to postpone consumption and massively invest in more variety, which makes firms small in the short run when variety is still small, but large in the steady state when variety has expanded.

Per capita income equals \( C/L = N^\nu (NL_{x_i})/L \). In the steady state this boils down to \( C/L = N^\nu = (\chi L/(\varepsilon - 1)\hat{\vartheta})^\nu \). Hence we see that if labour supply grows, per capita income grows because of the love-of-variety effect. This previews an important mechanism in the semi-endogenous growth models discussed below.
**Special case ii (Semi-endogenous growth):** \( 0 < \phi < 1, \quad g_L > 0 \)

By allowing for some, but limited, knowledge spillovers and population growth, we find a case in which the steady state is characterized by growth. The reason is that entry goes on forever and thus allows for gains from increasing specialization. This case is a simplified version of Jones’ (1995) model of “semi-endogenous growth” and of Eicher and Turnovsky’s (1999) “non-scale growth model”.

Figure 1b depicts the corresponding phase diagram. The dynamics differ from those in the previous case mainly because of population growth, which prevents a stationary equilibrium to arise. In an economy that starts with a small size relative to the number of firms such that \( L/N^{1-\phi} < \theta(e-1)/\chi \), innovation is not profitable enough, as above. All labour is allocated to production \( (L_N = L/N) \) and population growth makes total production grow (and equilibrium moves along the 45-degree line). Per capita production stays constant in this first stage of economic growth (which captures some aspects of the Malthusian regime). At some moment in time, population growth has resulted in large enough markets to warrant innovation. In this second stage of growth the economy moves along the saddle-path. The growth rate of per capita consumption now gradually increases over time, as is reflected by the vertical distance between the 45-degree line and the saddle-path.

A steady state is characterized by constant \( L/N^{1-\phi} \) and \( N^\phi L_N \), which requires a constant rate of variety expansion \( \dot{N} = g_L/(1-\phi) \) and a constant share of production in total employment \( NL_N/L \). Hence \( \dot{N} + \dot{L}_N = g_L \) and the long-run growth rate of per capita consumption is given by:

\[
g_c - g_L = \nu \dot{N} = \frac{\nu}{1-\phi} g_L \quad (21)
\]

The key difference with the previous case is that per capita growth may be unbounded. This requires \( \nu > 0 \), so that growth is driven by the taste for variety. New firms enter every period, so that consumers can divide their expenditures over an increasing range of product variety, which increases real consumption to the extent that they love variety. Unbounded growth also requires population growth \( (g_L > 0) \). Ongoing entry is threatened by diminishing returns: more firms means smaller firms and thus lower returns to entry so that entry stops. A growing labour force offsets this.

Slightly reinterpreting the model along the lines of Ethier (1986), we may state that an ongoing process of increasing specialization in production drives growth, so as to capture
Adam Smith’s view on growth. One arrives at this view by interpreting the sub-utility function for $C$ as a production function for final goods $C$, in which $X_i$ are intermediate inputs and $N$ is the number of intermediate inputs. Parameter $\nu$ then measures how much producers benefit from using a larger number of specialized inputs, that is, how much they benefit from increased specialization.

**Special case iii (“Endogenous growth”):** $\phi = 1, \quad g_L = 0$

For the case of critically large spillovers, viz. $\phi = 1$, and zero population growth, the model reduces to an “endogenous growth” model in the spirit of Grossman and Helpman (1991, Ch. 3) and Romer 1990.

The phase diagram for this case – depicted in Figure 1c – is one-dimensional since the variable on the horizontal axis $L/N^{1-\phi} = L$ is a constant. As a result $N^\phi L_{X_i} = NL_{X_i}$ immediately jumps to the value for which $N^\phi L_{X_i} = NL_{X_i} = 0$, see (19). Hence, there is no transitional dynamics. From (1), (8), (14), and (16) the growth rate of consumption can be calculated as $g_C = \nu \dot{N} = \nu \chi(L-NL_{X_i})$, which implies $NL_{X_i} = L-(g_C/\nu \chi)$. Substituting this expression and $N^\phi L_{X_i} = NL_{X_i} = 0$ into (19), we find:

$$g_C = \nu \left( \frac{\chi L - (\epsilon - 1) \phi}{\epsilon + (\rho - 1)(\epsilon - 1) \nu} \right)$$  \hspace{1cm} (22)

Different from the previous case is that growth may be unbounded without population growth. More firms means lower profits per firm, but due to strong spillovers ($\phi = 1$), also the cost of entry declines strongly, so that it remains attractive to enter, no matter how large $N$ already is.

The key difference with the previous case is that the long-run growth rate of consumption depends on intertemporal preference parameters. A higher discount rate ($\phi$) or a lower rate of intertemporal substitution ($1/\rho$) implies a lower growth rate. If we would introduce production taxes or research subsidies, these would affect the long-run growth rate, too. The taste-for-variety (or returns-to-specialization) parameter affects growth positively as above.\textsuperscript{6} Lower values of $\epsilon$ imply both higher profits and faster growth\textsuperscript{7} and in this sense competition is bad for growth.

4. Growth through in-house R&D
A less desirable aspect of growth models based on variety expansion is that entry of new firms (or the emergence of new industries) and increased specialization within firms (industries) cannot be disentangled. Although some periods of growth may be characterized by rapid changes in market structure and extended periods of entry of new firms, in most periods growth stems from research within a limited number of established firms. Gort and Klepper (1982) show that over the life cycle of an industry, only in the initial phase entry of new firms is the dominant source of growth. Malerba and Orsenigo (1995) take a sectoral perspective and compare the dominant source of innovation in different sectors. They find that only eleven out of the thirty-three industries patents are predominantly granted to new innovators. This suggests the existence of two different regimes of the innovation process that drives growth, one based on entry and another based on in-house R&D. Schumpeter elaborated on both regimes, labeled “competitive capitalism” and “trustified capitalism” respectively. In the latter, innovation is conducted by large and established firms that engage in monopolistic competition and try to capture some of the market shares of their rivals. Established firms have an advantage over entrants because they can build on experience and tacit firm-specific knowledge. Innovation strengthens their position in the market, since it expands the stock of firm-specific knowledge.

We now model some elements of trustified capitalism by studying growth driven by quality improvement within product lines. We consider the case that is opposite to the one analysed in the previous section: we abstract from entry, and assume that all innovation takes the form of quality improvement within existing firms ($L_N = 0$, $N$ is treated as a parameter). This results in a model similar to Thompson and Waldo (1994) and Smulders and van de Klundert (1995).

As in the previous section, we also allow for knowledge spillovers to capture the idea that research builds on public knowledge. The public knowledge stock rises with research effort. Since research aims at increasing $A$, we can now relate the knowledge stock to average $A$ in the economy, which equals $A_i$ because of our symmetry assumption. In particular, research productivity $S_A$ increases with $A$:

$$S_A = \xi A^\psi$$

Parameter $\psi$ captures intertemporal between-firm knowledge spillovers. We already introduced intertemporal within-firm knowledge spillovers as captured by $\alpha$. We denote total intertemporal spillovers by $\psi + \alpha \equiv \phi$. From this specification of spillovers, the R&D
function (9), the symmetry assumption ($A_i = A$), and the labour market clearing condition (16), we find:

$$\hat{A} = \xi(L/NA^{1-\varphi}) - \xi(L_{Xi}/A^{1-\varphi}) \quad (24)$$

Using this equation, we find the following expression for how $L/NA^{1-\varphi}$ evolves over time:

$$\left(\frac{L}{NA^{1-\varphi}}\right) = g_{L/N} - (1 - \varphi)\chi(L/NA^{1-\varphi}) + (1 - \varphi)\chi(L_{Xi}/A^{1-\varphi}) \quad (25)$$

where $g_{L/N}$ is the growth rate of the per firm labour supply, which we take as a parameter.

The capital market is in equilibrium if the return to innovation $r_A$ equals the rate of return required by households $r_c$. Combining the Ramsey equation and the no-arbitrage equation, solving for $L_{Xi}$, and rewriting using the above labour market condition, we find:

$$\left(\frac{L_{Xi}}{A^{1-\varphi}}\right) = (L_{Xi} / A^{1-\varphi})\xi \left[ (\varphi - \alpha)\frac{1}{\rho} + (2 - \varphi) \right] - (L/NA^{1-\varphi})\xi \left[ (2 - \varphi) - (\alpha + 1 - \varphi)\frac{1}{\rho} \right] - \frac{1}{\rho} \varphi \quad (26)$$

We now have a system of two differential equations in $L/NA^{1-\varphi}$ and $L_{Xi}/A^{1-\varphi}$, so that we can again set up a phase diagram to analyse the dynamics. The adjustment is saddle point stable as long as the first term in brackets of (26) is positive. The saddle path slopes up (down) if the second term in brackets is positive (negative), that is if the elasticity of intertemporal substitution ($1/\rho$) is small. Figure 2a and 2b depict the phase diagrams for two parameter combinations.

*** insert Figure 2 Growth with in-house R&D ***

The phase diagrams for the variety-expansion model and the quality-improvement model are very similar, so that we need not go through all cases. Due to the similar structure, similar conclusions hold with respect to the feasibility of growth in the long run. It can be easily checked that long-run growth is unbounded, either if some exogenous growing factor compensates for the diminishing returns ($g_{L/N} > 0$, see Figure 2a), or if spillovers are
sufficiently large to result in constant returns with respect to the reproducible factor ($\varphi = 1$ and constant returns with respect to $A$ arise, see figure 2b).

In the latter case of endogenous growth ($\varphi = 1, \ g_{L/N} = 0$), the dynamic system boils down to a simple single differential equation:

$$\rho L_{\xi} = \xi [\rho + 1 - \alpha] \cdot L_{\xi} - \xi (\rho - \alpha) \frac{L}{N} - \dot{\vartheta} \tag{27}$$

Since the differential equation is unstable, $L_{\xi}$ immediately jumps to the steady state, which is given by:

$$L_{\xi} = \frac{1}{\rho + 1 - \alpha} \left( \frac{\dot{\vartheta}}{\xi} + (\rho - \alpha) \frac{L}{N} \right) \tag{28}$$

The associated rate of consumption growth equals the rate of productivity growth since with symmetry $C = N^\nu \xi L_{\xi}$ (see (1) and (8)) where $N$ and $L_{\xi}$ are constant:

$$g_c = \dot{\vartheta} = \frac{\xi L/N - \dot{\vartheta}}{\rho + 1 - \alpha} \tag{29}$$

The key property of this expression is that growth depends on intertemporal preference parameters. If we introduced production taxes and research subsidies, growth would be affected by these policy variables. In short, the long-run growth rate is endogenous. Similar to the endogenous growth case of the variety expanding model, growth increases with research productivity ($\xi$) and decreases with $\rho$ and $\vartheta$.

Intertemporal knowledge spillovers show up as a determinant of long-run growth. In particular, the degree to which intertemporal spillovers can be appropriated by firms ($\alpha$) is positively related to growth. The higher $\alpha$, the more the firm’s own research efforts contribute to the reduction in future R&D costs (see (9)), and thus the higher the incentive to invest in R&D. In the variety expansion model, no individual researcher could internalize the contribution of its own research efforts to future research cost reductions, because only the public knowledge stock affects research costs and the own contribution to the public stock is perceived as negligible.

Growth depends positively on the firm size measured by labour available per firm $L/N$. Like in the model of endogenous growth driven by variety expansion, see (22), a larger
firm size implies a larger market in which the results of R&D can be commercialized, so it boosts the return to innovation.

The key parameter from the Dixit-Stiglitz model ($\varepsilon$) does not show up in the expression for the growth rate. There are three reasons for this: symmetry among firms, no creative destruction, and the assumption of fixed number of firms.

Aghion et al. (2001) set up a growth model with a large fixed number (say $N$) of industries, in each of which two firms produce goods that enter consumer preferences with a constant elasticity (say $\varepsilon$) as in the Dixit-Stiglitz approach. Firms set prices as in a Cournot duopoly and choose R&D effort. Since the returns to R&D are stochastic in this model, duopolists may end up with different productivity levels even if they start at the same level. If they start at the same level, R&D is more profitable with a higher degree of substitution $\varepsilon$, since a given quality advantage over the rival firm produce a bigger boost in profits, the more easily consumers substitute for the rival’s output.

Product market competition as measured by $\varepsilon$ does not directly affect innovation of a fixed number of symmetric firms. However, as we have seen in the variety expansion model, once we allow for entry, the number of firms is negatively related to the product market competition parameter $\varepsilon$: lower profit margins require larger firms in equilibrium. Since in the quality improvement model growth is stimulated by larger firm size ($L/N$) we may expect more competition (higher $\varepsilon$) to result in faster growth once we relax the assumption of a fixed number of firms. This is what we turn to in the next section.

5. Growth with variety expansion and in-house R&D

We now combine the two models to explore the interaction between entry and in-house innovation. To simplify we assume the following:

$$g_L = \phi = 0, \quad \varphi = 1, \quad \rho = 1$$

These assumptions imply that, first, there is neither population growth nor intertemporal spillovers in entry, second, there are strong intertemporal spillovers in productivity improvements, and, third, utility is logarithmic. The last assumption only simplifies the expressions for the transitional dynamics. The first two assumptions guarantee that long-run growth is driven by productivity improvements within product lines and that the number of firms is finite. Note that with these assumptions, $\alpha = 1 - \Psi$ and $S_N = \chi$ is a constant. Also note
that now the phase diagrams for the variety expansion and quality improvement model can both be depicted in the $L/N, L_{Xi}$ plane (cf. Figure 1a and 2b).

We first examine which type of innovation is undertaken in equilibrium. Any type of innovation can only be undertaken if its rate of return is at least the market rate $r$. Substituting the labour market clearing condition (16) into the no-arbitrage equations (12) and (15), and normalizing the wage to unity by choice of numeraire ($w = 1$), we may write:

\[ r_N \leq r \iff \chi \left( \frac{1}{\varepsilon - 1} L_{Xi} - L_{Ai} \right) \leq r \iff \frac{1}{\varepsilon} \left( \frac{L - L_N}{N} \right) - \left( \frac{\varepsilon - 1}{\varepsilon \chi} \right) r \leq L_{Ai} \quad (30) \]

\[ r_A \leq r \iff \xi (L_{Xi} - \psi L_{Ai}) \leq r \iff \frac{1}{1 + \psi} \left( \frac{L - L_N}{N} \right) - \left( \frac{1}{\xi (1 + \psi)} \right) r \leq L_{Ai} \quad (31) \]

Figure 3 depicts the two relationships as reaction curves – of entrants and incumbents respectively – in the $L_{Xi}/N, L_{Ai}$ plane, where $r$ and $N$ can be treated as parameters for firms take them as given. Points above an agent’s reaction curve implies that her rate of return falls short of the interest rate $r$ as indicated by the minus sign. Both reaction curves slope negative. The point of intersection represents an equilibrium in which both types of innovation yield a return equal to the market interest rate $r$ and in which the labour market clears.

*** insert Figure 3 reaction curves ***

The figure shows that this equilibrium is stable if the entrants’ reaction curve is steeper than the incumbent’s reaction curve, which requires $\psi > \varepsilon - 1$. If the inequality is reversed, the equilibrium is unstable: slightly increasing (decreasing) the amount of labour in entry increases the return to entry (in-house R&D). Hence, if $\psi < \varepsilon - 1$ in-house R&D and entry never occur simultaneously and equilibrium is driven by either variety expansion or in-house R&D. Since we analysed these situations in the previous sections, we can here restrict attention to the case in which both types of innovation are undertaken simultaneously because they yield the same return in a stable equilibrium. From (30)-(31) we can derive that a necessary conditions for this equilibrium to exist is

\[ \psi > \varepsilon - 1 > \chi/\xi \quad (32) \]

Equating the rates of return in (30) and (31), we find the rate of productivity improvement for an equilibrium with both in-house R&D and variety expansion:
\[ \hat{A} = \xi L_{Ai} = \xi L_{xi} \left( \frac{(\varepsilon - 1) - \chi / \xi}{(\varepsilon - 1)(\psi - \chi / \xi)} \right) \] 

Substituting the second equality and the labour market equilibrium condition (16) into (14), we find the rate of variety expansion:

\[ \hat{N} = \chi \frac{L}{N} - \chi \left( \frac{(\varepsilon - 1) - \chi / \xi + (\varepsilon - 1)(\psi - \chi / \xi)}{(\varepsilon - 1)(\psi - \chi / \xi)} \right) L_{xi} \] 

(34)

Substituting this result into (30) and equating the resulting expression for the rate of return on investment to the required rate of return in (7), we find after setting \( \rho = 1 \):

\[ \hat{L}_{xi} = \left( \frac{\epsilon \chi}{\varepsilon - 1} \right) L_{xi} - \chi \frac{L}{N} - \psi \] 

(35)

We use (34)-(35) to set up the phase diagram in the \( L_{xi}, L/N \) plane, as shown in Figure 4. Under the assumptions made in (32), the \( \frac{\hat{E}}{N} = 0 \) locus has a positive slope that is smaller than 45 degree. Points above the locus imply either \( L_N < 0 \) when \( r_N = r_A \), which must be ruled out, or \( r_N < r_A \) when \( L_N = 0 \), which implies a situation with constant \( N \). The \( \frac{\hat{E}}{N} = 0 \) locus from (34) cuts the \( \hat{L}_{xi} = 0 \) locus derived from (35), which also slopes upward. If the economy starts with a small number of firms, entry takes place and the economy moves along the saddlepath. Production per firm falls over time. In the long run, the number of firms approaches the following value:

\[ N = \frac{\chi L}{\psi - (\varepsilon - 1)} \left( \frac{\psi - (\varepsilon - 1)}{(\varepsilon - 1)(\psi - \chi / \xi) + (\varepsilon - 1) - \chi / \xi} \right) \equiv \bar{N} \] 

(36)

If the economy starts with a large number of firms, \( N > \bar{N} \), the rate of return to entry falls short of that of in-house R&D for any positive amount of labour allocated to variety expansion, as can be easily checked from (30)-(31). Hence, only in-house R&D can be undertaken in equilibrium. We can use the \( \hat{L}_{xi} = 0 \) locus from the model with in-house R&D only, see (28), after setting \( \rho = 1 \). This line cuts the 45-degree line at \( L/N = \psi / \xi \). Points above the 45-degree line must be ruled out since they imply \( L_{xi} > L/N \), which violates the
labour market clearing condition. Hence, for \( \frac{\xi L}{\theta} < N < \bar{N} \), growth is given by (29), and for \( N < \frac{\xi L}{\theta} \), the growth rate is zero.

*** Insert Figure 4. Phase diagram ***

The most natural scenario is that the economy starts with a low number of firms \(( N < \bar{N} )\). Then two stages of growth emerge. In the first stage, entry and in-house R&D occur simultaneously. Once a critical number of firms has entered the market, a second stage of growth starts with in-house R&D only. Note that this growth pattern resembles the product life cycle that is empirically relevant on industry level: in a mature economy, the number of firms is stable, and each of them devotes resources to productivity improvement. The long-run growth rate in this situation can be calculated as:

\[
g_c = \hat{A} = \frac{[(\varepsilon - 1)(\xi/\chi) - 1] \theta}{\psi - (\varepsilon - 1)}
\]

(37)

Competition, as measured by a high value for \( \varepsilon \), boosts long-run growth in this model, as is clear from (37). The reason is already explained above: larger price elasticities imply smaller profit margins for monopolistic firms, so that starting new firms is less attractive. Firms are larger in the mature economy, which makes in-house R&D more profitable and enhances growth. Hence, while growth did not depend on \( \varepsilon \) in the quality improvement model and did depend negatively on \( \varepsilon \) in the variety expansion model, growth depends positively on \( \varepsilon \) once we combine the two models.

Another important contrast to the results in the previous sections, is that the scale of the economy, as measured by the labour force, does not affect the growth rate, but intertemporal preferences do. Hence we have endogenous growth without the so-called scale effect. The scale effect implies that larger economies grow at a faster rate, basically because they can exploit knowledge in a large market. The empirics on postwar growth do not support this model prediction. In the previous sections, the scale effect showed up whenever growth was endogenous (by which we mean that it depends on intertemporal preference parameters and policy variables), see (22) and (29). In the semi-endogenous growth variants above, the scale effect was removed by assuming diminishing returns with respect to knowledge, but that assumption rendered growth independent of intertemporal preference parameters (or policy variables), see (21). In the model that combines quality improvement and variety expansion, an increase in the scale of the economy (larger labour force \( L \)) results – for a given number of firms – in more labour per firm, larger sales per firm, and higher profits and therefore
incentives to both higher levels of in-house R&D and development of new product lines. If indeed more firms enter, the process of variety expansion gradually reduces the market for individual incumbents, thus offsetting the higher incentives to in-house R&D. In the long-run equilibrium with a larger labour force, the number of firms is proportionately larger, but each firm invests at the same rate as in the economy with a smaller labour force.\(^9\)

The rate of growth does not depend on the taste-for-variety parameter \(v\). The number of product varieties in the steady state is constant over time so that the taste for variety does not affect the cost of capital (see our discussion of (20)). If we allow for population growth as well as non-unitary elasticity of intertemporal substitution (\(1/\rho\)), the number of firms continuously grows as in the semi-endogenous growth model (in particular \(\hat{N} = g_L\) so that \(L/N\) is constant) and \(v\) enters the expression for the long-run growth rate:

\[
g_c - g_L = \hat{A} + v\hat{N}_t = \left[\frac{(\epsilon - 1)\xi}{\chi - 1}(\hat{\theta} + \rho g_L) + \left[\psi - (\epsilon - 1)\right]v g_L}{[\psi - (\epsilon - 1)] - (\rho - 1)[(\epsilon - 1)\xi/\chi - 1]} \tag{38}\]

Finally it should be noted that the growth rate depends positively on the discount rate \(\hat{\theta}\), which seems counterintuitive. High discount rates discourage investment, but there are two types of investment now. Investment in variety is reduced (the number of firms \(N\) declines with the discount rate, see (36)), which in turn boosts investment in quality improvement (\(g\)).

6. Conclusions

The Dixit-Stiglitz approach has been the main way of incorporating monopolistic competition in models of economic growth based on research and development. Monopolistic competition is an essential feature of these models since monopoly profits provide both the incentives to undertake R&D and the means to finance R&D.

The degree of monopolistic competition, as measured by (some measure inversely related to) the elasticity of substitution between product varieties (\(\epsilon\)) affects growth in different directions in the various types of growth models. If variety expansion (entry) drives growth (and in models of creative destruction), more competition (higher \(\epsilon\)) is bad for growth as it reduces the profits that can be reaped by the innovators that enter the market with new product lines. If in-house R&D drives growth, which is the case in the “trustified capitalism” regime, an increase in competition is good for growth. A higher elasticity of substitution implies lower profits margins and a smaller number of firms that can survive in the market.
On average firm size will be larger, which boosts innovation since the return to innovation can be captured in a larger share of the market.

The role of product variety also differs in the various models. In the variety expanding growth models, an ongoing process of entry of new firms drives long-run growth. The model captures Adam Smith’s idea that increasing specialization drives productivity gains. However, in these models there is no distinction between the number of firms and the number of specialized consumption goods or intermediary inputs. While the number of goods (or inputs) may indeed capture specialization, the number of firms is an aspect of industrial organization, and ideally the two should be kept separately. In the model with both entry and in-house R&D, entry stops once a critical number of firms is in the market. In-house R&D becomes the engine of growth. This model reflects the industry life-cycle pattern in which entry is followed by consolidation.

This survey has shown that the Dixit-Stiglitz approach has been fruitfully used as an essential building block in growth theory and that a rich set conclusions can be derived about the dynamic implications of monopolistic competition and product variety. It is also clear that further studying the interaction between different types of R&D in monopolistic markets is a promising avenue to study stages of growth and dynamics of market structure. Finally it should be noted that we only dealt with models that explore the elementary driving forces behind growth. Twenty-five years after the publication of the Dixit-Stiglitz model and eleven years after the publication of the first Dixit-Stiglitz-based growth models, we are still witnessing a rapid growth in the literature on how growth is related to capital market imperfections, income inequality, labour markets, social norms, environmental problems and many other issues. It is the Dixit-Stiglitz model that makes it possible to study these important topics.
References


Eicher and Turnovsky (1999), Economic Journal


Notes

1 We will show that \( \sum N X_i = 0 \) in the steady state. Note that then a change in variety has two opposing effects: a consumption smoothing effect (growing variety means growing consumption, so households require higher rate of return) and a price effect (growing variety means declining effective prices so a lower nominal interest rate is accepted). The first dominates if \( \theta > 1 \).

2 In the tradition of the Dixit-Stiglitz model of monopolistic competition, firms ignore the impact of their own actions on total consumption and the price index of consumption, because there is a continuum of firms and the actions of each firm on the total are negligible. In contrast, if the number of firms is not that large and firms act strategically, oligopolistic competition prevails and the mark-up depends not only on the elasticity of substitution (\( \varepsilon \)) but also the number of (symmetric) firms (cf. Yang and Heijdra 1993). Under oligopolistic competition the outcomes depend on the reaction hypothesis introduced with respect to firm behaviour. Different regimes of competition lead to different mark-ups for a given number of firms and substitution parameter. The regime of competition affects growth as is shown in Van de Klundert and Smulders (1997). In the present paper we ignore strategic interactions and oligopoly.

3 This situation can arise endogenously in the full model, for example if \( S_A \) is sufficiently small. Below, we will return to the determination of which type of research is undertaken in equilibrium.

4 In Jones (1995), variety matters for production rather than for preferences, moreover physical capital is required to produce varieties, rather than just labour as in our model.

5 Note that \( NL L/n = \left( N L/n \right) /N \), in which both terms in parentheses are constant.

6 Now even the growth rate of the number of firms (see the term in parentheses) is affected.

$$\text{sign} \left( \frac{dN}{dv} - \text{sign} \left[ (1 - \rho) g_c \right] \right).$$

7 Differentiating we find

$$\text{sign} \left( \frac{dg_c}{d\varepsilon} = \text{sign} \left[ (1 - \rho) \chi L - (\varepsilon \varepsilon + \chi L) \right] \right).$$

Positive growth requires \( \chi L + \varepsilon > \varepsilon \varepsilon \). Bounded utility requires \( \varepsilon \varepsilon + (\rho - 1) g_c > 0 \), which after substitution of \( g_c \) implies \( \varepsilon \varepsilon \varepsilon > (1 - \rho) \chi L \). Hence, if growth is positive and utility is bounded, \( \varepsilon \varepsilon + \chi L > (1 - \rho) \chi L \) and

$$\frac{dg_c}{d\varepsilon} < 0.$$

8 Thompson and Waldo (1994) model research as a stochastic process and ignore within-firm intertemporal spillovers (this case arises in our model if \( \alpha = 0 \) and \( \psi = 1 \)). The key element of trustified capitalism is that “creative accumulation” rather than “creative destruction” drives growth. In this respect, the trustified capitalism model contrasts to the “Schumpeterian” models of Grossman and Helpman (1991 chapter 4) and Aghion and Howitt (1992, 1998), which all rely on creative destruction. What our model of trustified capitalism has in common with these models is that growth is possible without an increase in variety. In these “Schumpeterian” models with creative destruction, new firms replace old firms. There are two reasons not to deal with this class of models in this chapter. First, these models are often structurally isomorph with the variety expansion models (see Grossman and Helpman 1991, p. 98) and yield similar insights. Second, these models rely on perfect substitutability and (Bertrand) limit pricing rather than the Dixit-Stiglitz approach of modeling competition.

9 Young (1998), Dinopoulous and Thompson (1998), and Howitt (1999) have used this mechanism to remove scale effects; Peretto and Smulders (2002) provide the microeconomic foundation for the key assumption that the cost of in-house R&D depends on \( A \) only and not on \( N \).