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Communication, Correlation and Symmetry in Bargaining

by Karl Wärneryd


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Communication, Correlation and Symmetry in Bargaining

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Communication, correlation, and symmetry in bargaining

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This paper considers two-player normal form games where each player can send a payoff-irrelevant message prior to play. Let \( G \) be the game without communication, and \( G^*(M) \) the extended game with message set \( M \). Any convex combination of Nash outcomes in \( G \) can be approximated in a subgame perfect equilibrium of \( G^*(M) \) for some \( M \). Furthermore, every symmetric game has a symmetric subgame perfect communication equilibrium that is undominated in a limit sense as the message set is enlarged.

1. Introduction

It is often informally suggested that if the players of a game can communicate, then the appropriate equilibrium notion should be not Nash equilibrium, but the larger class of correlated equilibria. [See references as early as Luce and Raiffa (1957) or as late as van Damme (1987).] The assumption is commonplace in discussions of bargaining.

The idea is that if agents can talk, they could reach a self-enforcing agreement to let their actions be \textit{jointly} conditioned on the outcome of a stochastic trial, rather than independently as assumed by the Nash construction. The latter possibility, as formalized by Aumann (1987), expands the set of possible outcomes to include, in particular, the convex hull of Nash outcomes of the underlying game. In some cases, outcomes outside the convex hull are possible.

Aumann's notion of correlated equilibrium is not built on an explicit model of verbal communication, however. It is a much more general attempt to derive an equilibrium concept from Bayesian foundations. That is, games are viewed as standard decision-theoretic problems, where each player is assumed to act rationally given his partition of the set of states of the world, where the description of a state of the world includes a specification of the strategies played in it. As noted by Lipman and Srivastava (1990), the definition of correlated equilibrium is silent on the subject of where this information partition comes from. Lipman and Srivastava study the equilibria attainable when agents acquire information through costly computation. The present paper considers correlated equilibria that arise in two-player games when information about the strategies to be played is...
endogenously generated through 'cheap talk', i.e., explicitly modeled payoff-irrelevant pre-play communication.

The cheap talk device was introduced to economics by Crawford and Sobel (1982) [but previously suggested by the philosopher David Lewis (1969)] in the context of games of one-sided incomplete information. Cheap talk is intended as an explicit model of verbal communication. Recently the general idea, with the strong additional assumption that the costless messages have natural meanings in some language shared by the players, has been applied by, e.g., Farrell (1987) (to complete information games), Myerson (1989) and Rabin (1990), Kim and Sobel (1990), Matsui (1991), and Wärneryd (1991) select equilibria in two-player cheap talk games of complete information by applying criteria of evolutionary stability. Wärneryd (1990) does the same for a class of games of incomplete information. The main purpose of these contributions is to argue that only equilibria efficient within the set of equilibria are plausible under communication.

In contrast, one purpose of this paper is to show how the larger set of correlated equilibria can arise in cheap talk games with complete information. It thus provides some support for the intuition that communication and correlated equilibrium are related. The fundamental result, that any convex combination such that the weights are rational numbers of two equilibrium outcomes of the game without communication can be achieved as a subgame perfect equilibrium outcome of the game with communication, is proved in section 2. The method of proof is very simple. It appears indirectly in Kim and Sobel (1990). Apart from the cheap talk literature mentioned earlier, which is concerned not so much with characterizing the set of equilibria under communication as refining it, there are some related contributions. De Groote (1990) proves a similar, but weaker result for games with one-sided communication and moral hazard. For a very general discussion of communication mechanisms and correlated equilibria, see Forges (1986).

The possibility of correlation is most interesting in bargaining-like situations where the parties order the equilibria differently. This paper shows that cheap talk can be effective in situations of partial conflict of interests. However, symmetric games do not necessarily have symmetric undominated communication-correlated equilibria. Section 3, the main contribution of this paper, shows how outcomes which are undominated in the convex hull of Nash outcomes of the underlying game can be implemented as the limit outcome of a sequence of symmetric subgame perfect communication equilibria where the number of available messages tends toward infinity. The 'Battle of the Sexes' is used as an example.

Finally, section 4 remarks on some weaknesses of the approach.

2. Correlation

Let \( G \) be a two-player normal form game with mixed strategy sets \( S_1 \) and \( S_2 \), which contain as degenerate cases the finitely many pure strategies of \( G \). Then \( S := S_1 \times S_2 \) is the set of strategy profiles of \( G \). For \( s \in S \), \( \pi_i(s) \) is the expected payoff of player \( i \). Write \( \pi := (\pi_1, \pi_2) \). Let \( N(G) \subseteq S \) be the set of Nash equilibrium profiles of \( G \).

Now extend \( G \) by allowing the players to simultaneously send one message each from the finite set \( M \) prior to playing \( G \). A message set is uniquely identified by its cardinality, \( |M| \). Call this extended game \( G^*(M) \). A (behavioral) strategy for player \( i \) for \( G^*(M) \) is a probability distribution \( \mu_i \) over \( M \) and a function \( \sigma_i: M \times M \rightarrow S_i \), which yields a probability distribution over strategies of \( G \) conditional on the information sets possibly reached after the communication stage. Write \( \sigma := (\sigma_1, \sigma_2) \). The communication game involves cheap talk in the sense that payoffs \( \pi^*(\sigma) \) of \( G^*(M) \) are assumed to depend only on the actions taken in the \( G \) subgame.
We note that in an equilibrium of $G^*(M)$ the local strategies played at any information set reached with positive probability after communication constitute an equilibrium of $G$. For assume one of the players plans to play a local strategy that is not a best reply to the other player’s local strategy at some information set that is reached with positive probability. Then the first player could increase his expected payoff by changing to a local best reply at that information set, while leaving his probability distribution over messages unaltered.

It follows that no equilibrium of $G^*(M)$ gives the players expected payoffs outside the convex hull of Nash outcomes of $G$. For this to happen, of course, the profile would have to specify the play of local strategies that are not an equilibrium of $G$ at some information set reached with positive probability.

This property is the main feature that sets correlated equilibria under cheap talk apart from the general concept of Aumann (1987). In Aumann’s conception, the possibility of outcomes outside the convex hull of equilibria of the underlying game arises because the players may not be able to observe the state of the world directly, merely the suggested play of a third party or mediator.

We now turn to the first main result, which shows that given a large enough message set, any outcome in the convex hull of equilibria of $G$ may be approximated by a subgame perfect equilibrium of $G^*(M)$.

**Proposition 1.** Let $s^*$ and $s^{**}$ be equilibria of $G$. Then for any rational number $\delta \in (0, 1)$, there exists a message set $M$ such that there is a subgame perfect equilibrium of $G^*(M)$ with expected payoff $\delta \pi(s^*) + (1 - \delta)\pi(s^{**})$.

**Proof.** Here is a constructive proof. Let $k$ and $\bar{m}$ be positive integers such that $k/\bar{m} = \delta$. Pick $M$ such that $|M| = \bar{m}$. Note that there is an infinity of pairs $(k, \bar{m})$ such that $k/\bar{m} = \delta$, and therefore an infinity of message sets that will allow the required equilibrium. Now recursively let

$$\sigma(m_i, m_j) = \begin{cases} s^* & \text{for } i = 1 \text{ and } j = 1, \ldots, k, \\ s^{**} & \text{for } i = 1 \text{ and } j = k + 1, \ldots, \bar{m}, \\ \sigma(m_{i-1}, m_{j'}) & \text{for } i > 1 \text{ and } j = 1, \\ \sigma(m_{i-1}, m_{j-1}) & \text{otherwise}, \end{cases}$$

and let $\mu_1(m) = \mu_2(m) = 1/\bar{m}$ for all $m \in M$. To see that this is an equilibrium of $G^*(M)$, consider the choice of Player 1 given that Player 2 complies with the profile. For any $m_i \in M$ that Player 1 sends with positive probability, we must have that $\sigma(m_i, m_j) \in B_i(\sigma_2(m_i, m_j))$ for $j = 1, \ldots, \bar{m}$, where $B_i(s_2), s_2 \in S_2$, is the best reply set for Player 1 in $G$ against $s_2$. Otherwise Player 1 could increase his expected payoff without altering the probability of sending $m_i$. The proposed profile is rational in this sense. Furthermore, given that this condition must hold, Player 1 is indifferent between probability distributions over $M$, since for any two messages he sends with positive probability, his expected payoff given one message is the same as his expected payoff given the other and equal to $(k/\bar{m})\pi_1(s^*) + ((\bar{m} - k)/\bar{m})\pi_1(s^{**})$. Similar reasoning shows that the proposed scheme involves a best reply of $G^*(M)$ for Player 2. The equilibrium is subgame perfect since every message is sent with positive probability and only equilibria of $G$ are played at each information set after communication. Finally, note that the expected payoff of this equilibrium is equal to $\delta\pi(s^*) + (1 - \delta)\pi(s^{**})$, as required. □

The extension to the case of convex combinations of more than two equilibria is immediate.
3. Symmetry

3.1. An example

In symmetric games it is often thought theoretically desirable not only to find noncooperative solutions that allow coordination, but symmetric ones. If identical agents face identical situations, we do not want a solution to require some arbitrary distinction between them. Farrell (1987) suggests that cheap talk can help. He considers a variant of the simple 'Battle of the Sexes', where two identical firms contemplate entering a market or not, and both would prefer that one enters and the other stays out. This is like a simple bargaining game under complete information. Many economic models have this kind of natural symmetry. However, in symmetric games, the equilibria constructed in the previous section do not necessarily involve symmetric strategies, where symmetry is taken to mean that one player plans to play at the information set \((m', m'')\) what the other plans to play at \((m'', m')\), for all \(m', m'' \in M\), and both have the same distribution over the message set. On the other hand, Farrell's proposal, although symmetric, does not yield an outcome undominated in the convex hull of Nash outcomes. The purpose of this section is to show how cheap talk allows undominated equilibrium outcomes to be implemented by symmetric strategies.

Consider the simplification of Farrell's model given in table 1. This game has three equilibria without communication, two asymmetric ones in pure strategies, \((\text{In}, \text{Out})\) and \((\text{Out}, \text{In})\), and one symmetric in mixed strategies. Let \(Q\) denote the symmetric equilibrium. It gives each player an expected payoff of \(2/3\).

Now extend the game with cheap talk. Let \(|M| = 3\). We clearly have a symmetric subgame perfect equilibrium if \(\mu_1(m) = \mu_2(m) = 1/3\) for all \(m \in M\) and the players' responses at information sets reached with positive probability after communication are as given by table 2. In equilibrium each player gets an expected payoff of \(11/9\), which is an improvement over the symmetric equilibrium in the absence of communication.

Adding two messages, so that \(|M| = 5\), admits the equilibrium given by table 3, with \(\mu_1(m) = \mu_2(m) = 1.5\) for all \(m \in M\). The expected payoff of both players is now \(4/3\). The process can clearly be continued, adding two messages at a time to the message set and arranging the responses at each information set after communication properly. The expected payoff may then be made to

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converge to $1^\frac{1}{2}$, which corresponds to an equal probability of ending up in either of the two asymmetric equilibria of $G$, avoiding realization of the diagonals of $G$ entirely.

### 3.2. Limit undominated communication equilibria

We now generalize the idea of section 3.1.

**Definition 1.** A game is said to be **symmetric** if $S_1 = S_2 = \bar{S}$ and $\pi_1(s', s'') = \pi_2(s'', s')$ for all $s', s'' \in \bar{S}$.

If $s = (s', s'')$, define $\hat{s} = (s'', s')$, the symmetric image of a strategy profile $s$. We note that in a symmetric game, if $s$ is an equilibrium, then so is $\hat{s}$.

**Definition 2.** A game $G$ is said to have a **limit undominated communication equilibrium** (LUCE) with payoff $\pi^{**}$ if there exists a sequence of message sets $M_n$ such that $|M_n| \to \infty$ and subgame perfect equilibria $\sigma_n^*$ of $G^*(M_n)$ such that $\pi^*(\sigma_n^*) \to \pi^{**}$, and $\pi^{**}$ is undominated in the convex hull of equilibrium outcomes of $G$.

Note that, in perhaps a slight abuse of terminology, LUCE does not refer to a particular strategy profile, but the limit of a sequence of strategy profiles.

From Proposition 1 we immediately realize that every game has a LUCE. However, as noted above, Proposition 1 does not guarantee that these are implementable by symmetric strategies in symmetric games.

**Proposition 2.** Every symmetric game has a symmetric LUCE.

**Proof.** To prove this, let $G$ be a symmetric game. Let $s^* \in N(G)$ be such that $\frac{1}{2}(\pi_i(s^*) + \pi_i(\hat{s}^*)) \geq \frac{1}{2}(\pi_i(s) + \pi_i(\hat{s}))$, for all $s \in N(G)$ and $i \in \{1, 2\}$. That is, $s^*$ is an equilibrium such that the arithmetic average of it and its symmetric image's payoffs is undominated among such averages. Such an equilibrium trivially exists. Let $q \in N(G)$ be a symmetric equilibrium. We know that every symmetric game has one [Nash (1950)]. We may have that $q = s^* = \hat{s}^*$. Now let $|M| = \overline{m}$, with $\overline{m} \geq 3$ and $\overline{m}$ an odd number, and let

$$
\sigma(m_i, m_j) = \begin{cases} 
q & \text{for } i = j = 1, \\
\sigma(m_{i-1}, m_{\overline{m}}) & \text{for } i > 1 \text{ and } j = 1, \\
\hat{s}^* & \text{for } i = 1 \text{ and } 1 < j < \overline{m} \text{ and } j \text{ even}, \\
\sigma(m_{i-1}, m_{j-1}) & \text{otherwise,}
\end{cases}
$$
and let $\mu_1(m) = \mu_2(m) = 1/\bar{m}$ for all $m \in M$. Following the same reasoning as in the previous proof, it is easy to see that this is a symmetric subgame perfect equilibrium of $G^*(M)$. Now consider a sequence of such equilibria for $\bar{m}$ odd and approaching infinity. The expected payoff then approaches $\frac{1}{2}\pi(s^*) + \frac{1}{2}\pi(\hat{s}^*)$, which by construction is undominated in the convex hull of Nash outcomes of $G$. □.

4. Remarks

To summarize briefly, correlated equilibria have been shown to be Nash equilibria of a game extended with cheap talk. In particular, communication in this form allows every symmetric game to have a symmetric equilibrium whose payoff approaches a payoff undominated in the convex hull of Nash outcomes as the message set grows large.

The function of the communication round in this model is to generate a space of distinct information sets that actions can be made contingent on. Message combinations thus play the role of a publicly observable stochastic event. It could be noted that a large message set can be replaced by a smaller set and more than one round of communication. The present model is isomorphic to some much models, although not, for instance, to the one proposed by Farrell (1987), where the addition of more rounds does not make payoffs converge to an undominated solution.

An immediate problem with this approach is the use of Nash equilibrium as the solution concept for the communication game. We have no real understanding of what would enable players to coordinate on a Nash equilibrium. In fact, the most compelling argument in favor of Nash equilibrium is itself based on the possibility of communication, which leads to a vicious circularity.

In discussing communication as a feature of games in general, an evolutionary, trial-and-error approach not based on the assumption of coordination seems potentially more convincing.

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