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CO-MOVEMENT, CAPITAL AND CONTRACTS: 'NORMAL' CYCLES THROUGH CREATIVE DESTRUCTION

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Co–Movement, Capital and Contracts: 'Normal' Cycles Through Creative Destruction

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Abstract

We develop a unified theory of endogenous business cycles in which expansions are neoclassical growth periods driven by productivity improvements and capital accumulation, while downturns are the result of Keynesian contractions in aggregate demand below potential output. Recessions allow skilled labor to be reallocated to growth–promoting activities which fuel subsequent expansions. However, rigidities in production and contractual limitations, inherent to the process of creative destruction, leave capital severely underutilized. A key feature of our equilibrium is the endogenous emergence of long–term supply contracts between capitalist owners and producers.

Key Words: Long–term contracting, investment irreversibility, putty–clay technology, asset–specificity, Endogenous cycles and growth
JEL: E0, E3, O3, O4

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"*The recurring periods of prosperity of the cyclical movement are the form progress takes in capitalistic society.*" (Joseph Schumpeter, 1927)

1 Introduction

A defining feature of the *normal* business cycle is the observed co–movement across diverse sectors of the economy amongst both outputs and inputs to production.[1] Indeed, sectoral co–movement in output was one of the key characteristics discussed in the work of Burns and Mitchell (1946), and forms the basis for the NBER's business cycle dating methodology.[2] Recently, Christiano and Fitzgerald (1998) have documented the striking degree of co–movement in labor hours across highly disaggregated sectors of the US economy. This sectoral co–movement has long been recognized, at least informally, by macroeconomists. For example, Schumpeter (1927) argued that the key to understanding business cycles was to understand why entrepreneurial activity would be clustered over time. Similarly, Keynes (1936) emphasized the role of co–movement in investment behavior — encapsulated in his notion of "animal spirits" — as a key causal factor in the business cycle.

Despite these observations, mainstream macroeconomic analysis has, in recent decades, tended to put aside questions regarding the *causes* of co–movement and, instead, has largely focussed on the propagation mechanisms implied by various aggregate shocks. In particular, the real business cycle literature emphasizes the response of the economy to aggregate technology shocks in driving short–run fluctuations. However, while *technology* shocks may be important at the firm level, it is not obvious why they would be important for economy–wide aggregate output fluctuations. As Lucas (1981) reasons, positive technology shocks for some firms would surely be offset by negative shocks for others. Even if one were to accept the existence of aggregate technology shocks as a driving force in the co–movement of *outputs* across sectors, standard RBC models still have trouble explaining the observed co–movement in *inputs* across sectors. In particular, as has been pointed out recently (Christiano and Fitzgerald, 1998, Siu, 1999), standard RBC models imply that hours worked in the consumption goods sector are inherently *counter–cyclical*.[3]

[1] In using the expression "normal business cycle" we are borrowing from Schumpeter (1927, p. 288), who distinguishes such cycles from seasonal fluctuations, "long waves" and secular trends.

[2] "... a recession is a period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy." (see http://www.NBER.org/cycles.html).

[3] Several papers, including Benhabib, Rogerson and Wright (1991) and Boldrin, Christiano and Fisher (2001),

There are, however, good reasons why economic activity might be correlated across different industries, even in the absence of aggregate shocks. In the presence of imperfect competition, for example, the implementation of a productivity improvement by one firm may increase the demand for anothers' products by raising aggregate demand. In a dynamic general equilibrium setting, Shleifer (1986) shows that this may induce innovators, who anticipate short-lived profits, to implement simultaneously, thereby generating self–enforcing booms in activity. Unfortunately, as a vehicle for understanding actual business cycles, Shleifer's model suffers from a number of serious limitations. Firstly, the multiplicity of equilibria that arise yield rather imprecise predictions for the cyclical process. Secondly, the temporary nature of profits relies on the ad hoc assumption of drastic, but costless imitation. Thirdly, his cycle features only booms and slowdowns, but no downturns. Finally, Shleifer's theory depends critically on the impossibility of any type of storage (including physical capital).[4]

In Francois and Lloyd-Ellis (2003), we demonstrate how a similar endogenous cyclical process can arise through a process of "creative destruction" familiar from Schumpeterian growth models. Like imitation, potential obsolescence limits the longevity of profits and provides incentives to cluster implementation. We show that when productive resources are needed to generate new innovations, allowing for the possibility of storage does *not* rule out cycles, and in fact yields a *unique* cyclical equilibrium.[5] Moreover, because this costly innovation tends to be clustered just before a boom, it causes a *downturn* in aggregate output (even if the measure of GDP includes this investment). Although promising, our earlier model allowed investment only in *intangible* assets (i.e. "ideas"). This limits its applicability because fluctuations in *tangible* capital formation (and its utilization) are, obviously, a key characteristic of the business cycle.[6]

The omission of physical capital from our earlier paper was not innocuous. If consumption–smoothing households have access to fully reversible and continuously variable physical capital, they would "eat" it in anticipation of a boom, thereby undermining the existence of cycles where delay plays an important role. As Matsuyama (1999) notes, this is true more generally of work

develop extended RBC frameworks in which hours worked in the consumption sector need not be counter-cyclical. As will be seen this problem need not arise in our framework either. We discuss this issue in Section 7.

[4] If they could, innovators would choose to produce when costs are low (i.e. before the boom), store the output and then sell it when demand is high (i.e. in the boom). Such a pattern of production would undermine the existence of cycles.

[5] Shleifer (1986) assumes that innovations arise exogenously. When innovation is endogenous, growth is intimately related to the business cycle.

[6] A related limitation of that model is that for realistic long–run growth rates, the existence of the cyclical equilibrium requires unrealistically high values of the elasticity of intertemporal substitution of 3 or above.

featuring agglomeration with implementation delay (Shleifer 1986, Deneckere and Judd 1992, and Gale 1996). The results of such models are not robust to allowing the accumulation of (fully reversible) physical capital. Matsuyama's (1999, 2001) resolution of this impasse is to develop a model of endogenous cycles with physical capital, but without implementation delays. His model features increasing returns in production (as in Romer, 1990), exogenous imitation (as in Shleifer, 1986), and a fixed entry cost for new firms. This results in a growth process in which the economy fluctuates between periods of high capital accumulation coupled with low total factor productivity (TFP) growth, and periods of high TFP growth but little accumulation of capital. While this model is useful as a tool for understanding longer term phenomena (e.g. East Asian post war growth and the US productivity slowdown), it has a number of shortcomings when applied to business cycle frequency fluctuations. In particular, sectoral contractions in output — a key aggregate feature of normal business cycles — are absent from this process. Similarly, consumption never actually falls in absolute terms and factors of production are always fully utilized.[7]

Here we consider an alternative approach that allows for physical capital, but preserves the implementation delays emphasized by previous authors. Specifically, we develop a model of innovation–driven endogenous growth in which capital is irreversible, putty-clay and lumpy. Our emphasis on the rigidities of installed capital is consistent with our focus on normal business cycles rather than secular trends and, as we will demonstrate, generates dynamics that are qualitatively consistent with key features of the data at this frequency. Moreover, there is considerable direct evidence that many types of physical investment are not reversible and feature putty-clay characteristics (see Ramey and Shapiro, 2001, Kasahara, 2002). Doms and Dunne (1993) have also documented the considerable "lumpiness" of plant level investments, while Cabellero and Engel (1998) have demonstrated the high skewness and kurtosis observed in aggregate investment data.[8] Moreover, the variation in "shiftwork" over the business cycle (see Bresnahan and Ramey 1994, Hamermesh 1989 and Mayshar and Solon, 1993) is consistent with the putty–clay assumption, since it implies that capital is being used less intensively during recessions than is

[7]There is a large literature on endogenous cycles and growth. However, most previous work has been restricted to single sector settings, which also seems more consistent with the analysis of general purpose technologies and longer term, secular trends. See Aghion and Howitt (1998, ch. 8) and Lloyd-Ellis and Francois (2003) for more extensive discussions of this literature.

[8]A related literature emphasizes the role of plant level investment irreversibilities. However, recent work (see Veracierto, 2002 and Thomas 2002,) in the RBC tradition has found that the effects of such irreversibilities at the aggregate level are virtually non–existent.

optimal ex ante.

We characterize an endogenous stationary cyclical equilibrium that features dramatic shifts in economy–wide investment behavior through the cycle. Expansions are "neoclassical", supply–side phenomena which directly raise both *potential* output, through the delayed implementation of productivity improvements, and *actual* output through increased labor effort and subsequent capital accumulation. Recessions are "Keynesian" demand–side contractions during which actual output falls below its potential and some capital resources are left under–utilized. These reductions in aggregate demand are not autonomous, however. Rather they are an equilibrium response to the anticipated future expansion, as effort shifts into long–run growth promoting activities, and out of current production.[9]

In addition to an endogenous treatment of growth, our model *endogenously* generates the following qualitative behavior of key aggregates and prices over the business cycle:

• Implementation of innovations is strongly pro–cyclical, so that total factor productivity rises during booms, but remains constant during downturns.

• Labour productivity is strongly pro–cyclical.

• Wages rise during booms and expansions, but do not fall during contractions.

• Investment and consumption are strongly pro–cyclical, but investment is more volatile. Investment is strongly correlated with output and sales growth.

• Labor and capital inputs into consumption and investment sectors are both pro–cyclical.

• Capacity utilization is strongly pro–cyclical.

• Profits are strongly pro–cyclical.

• Term spread is flat during expansions and steep midway through contractions.

• Marginal Q is strongly pro–cyclical, but because the stock market anticipates the boom, the cyclicality of Tobin's average Q tends to be more complex.

A crucial feature of the equilibrium growth path that we study is the endogenous emergence of optimal long term supply contracts between capital owners (e.g. banks) and entrepreneurs. Contractual agreements negotiated between the two parties are necessarily limited because the process of creative destruction implies that an entrepreneur's productivity advantage terminates when replaced by superior innovators in their sectors.[10] The parties can, however, enter into long

[9] This has the flavor of the so–called "paradox of thrift": current savings are chanelled into investments whose return will not be realized until the long run (when, according to Keynes (1936), "we are all dead").

[10] The growth implications of incomplete contracting have been explored recently in a number of papers: Martimort and Verdier (2000) explore the growth implications of cooperative non-productive behavior between owners

term binding arrangements that specify both the quantity and price of capital to be transacted, conditional upon the relationship continuing. As we shall see, the existence of contracts between intermediate producers and capital suppliers, also precipitates the emergence of contracts between final goods producers and their intermediate suppliers.

We defer discussion of previous literature on growth and cycles until after the paper's main results are presented. The paper proceeds as follows: Section 2 sets out the framework, Section 3 characterizes the cycling steady state, Section 4 presents the necessary conditions for existence of the steady state and Section 5 provides numerical examples of cycling economies, and comparative statics. Section 6 deals with the model's dynamics and Section 7 concludes with a discussion of the main implications. **All proofs are in the Appendix.**

2 The Model

2.1 Assumptions

There is no aggregate uncertainty. Time is continuous and indexed by $t \geq 0$. We consider a closed economy with no government sector. The representative household has isoelastic preferences

$$U(t) = \int_t^\infty e^{-\rho(\tau-t)} \frac{C(\tau)^{1-\sigma} - 1}{1-\sigma} d\tau \tag{1}$$

where ρ denotes the rate of time preference and σ represents the inverse of the elasticity of intertemporal substitution. The household maximizes (1) subject to the intertemporal budget constraint

$$\int_t^\infty e^{-[R(\tau)-R(t)]} C(\tau) d\tau \leq S(t) + \int_t^\infty e^{-[R(\tau)-R(t)]} w(\tau) d\tau \tag{2}$$

where $w(t)$ denotes wage income, $S(t)$ denotes the household's stock of assets (firm shares and capital) at time t and $R(t)$ denotes the discount factor from time zero to t. The population is normalized to unity and each household is endowed with one unit of labor hours, which it supplies inelastically.

Final output can be used for the production of consumption, $C(t)$, investment, $\dot{K}(t)$, or can be stored at an arbitrarily small flow cost of $\nu > 0$ per unit time. It is produced by competitive firms according to a Cobb–Douglas production function utilizing a continuum of intermediates,

and workers. Francois and Roberts (2002) show that the productivity slowdown, and a number of accompanying labor market changes can be explained in an endogenous growth framework featuring such incomplete contracting. Acemoglu, Aghion and Zilibotti (2002,2003) explore the implications for such incompleteness for LDC development.

x_i, indexed by $i \in [0,1]$:

$$C(t) + \dot{K}(t) \leq Y(t) = \exp\left(\int_0^1 \ln x_i(t)di\right). \tag{3}$$

For simplicity we also assume that there is no physical depreciation.

Output of intermediate i depends upon the state of technology in sector i, $A_i(t)$, utilized capital, $K_i^u(t)$, which cannot exceed the stock of installed capital, $K_i(t)$, and labor hours, $L_i(t)$, according to the following putty–clay technology:

$$x_i^s(t) = \begin{cases} [K_i^u(t)]^\alpha [A_i(t)L_i(t)]^{1-\alpha} & \text{where } K_i^u(t) = K_i(t) \\ \kappa_i(z)^\alpha A_i(t)^{1-\alpha} L_i(t) & \text{where } K_i^u(t) = \kappa_i(z) L_i(t) < K_i(t) \end{cases} \tag{4}$$

where $\kappa_i(z)$ is the capital–labor ratio chosen at $z < t$, the time at which the last increment to capital was installed. The unit measure of labor hours is perfectly mobile across sectors and inelastically supplied by households in aggregate. However, the amount of this supply that is used in production of intermediates potentially varies due to its opportunity cost in an alternative activity, specified shortly. Installed capital, $K_i(t)$, is sector–specific and is owned by "capitalists" who rent it to entrepreneurs at the rate $q_i(t)$. Installed capital is non-divisible so that any part of it that is not utilized cannot be used elsewhere.[11] Once installed, sector–specific capital is irreversible, $\dot{K}_i \geq 0$, as well as putty-clay. We assume that intermediates are completely used up in production, but can be produced and stored for use at a later date. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later at the flow storage cost ν.

An implication of this structure is that during an expansion, when new capital is being built, firms can choose (K, L) combinations along the Cobb–Douglas production isoquants (curved in Figure 1) and choose an optimal capital–labor ratio that reflects relative factor prices. However during a contraction, if labor is removed and the firm produces below capacity, production must utilize factors along a ray from the chosen point on the full-capacity isoquant, which reflects the same factor ratios. In such a situation, the installed capital is used less intensively in proportion to the labor hours allocated to production. One interpretation of this is that there are fewer shifts.

This production set up implies that if firms were to reduce output below capacity, one of the following three outcomes must occur:

[11] For similar examples of putty-clay capital, see Johansen (1959), Gilchrist and Williams (2000) and Kasahara (2003). Evidence of underutilization is widespread: for example, Bils and Cho (1994) and Basu (1996).

(1) the entire stock of capital is rented to another (presumably more productive) entrepreneur, if one exists;

(2) capital is fully utilized, but some output is stored to be sold later, or

(3) the installed capital remains in place but is under–utilized, $K_i^u(t) < K_i(t)$.

Along the cyclical growth path that we will study here, only outcome (3) turns out to be consistent with equilibrium. However, we will discuss in some detail the conditions that rule out (1) and (2).

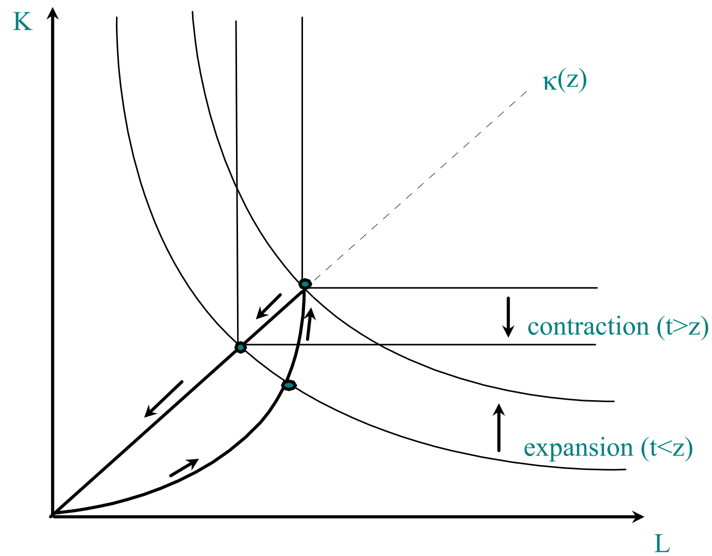

Figure 1: Implications of Irreversibility and Putty–Clay Technology

2.1.1 Innovation

Competitive entrepreneurs in each sector attempt to find ongoing marginal improvements in productivity by allocating labor effort to innovation rather than production.[12] They finance their activities by selling equity shares to households. The probability of an entrepreneurial success in instant t is $\delta H_i(t)$, where δ is a parameter, and H_i is the labor effort allocated to innovation in sector i. At any point in time, entrepreneurs decide whether or not to allocate labor effort to innovation, and if they do so, how much. The aggregate labor hours allocated to innovation is given by $H(t) = \int_0^1 H_i(t)dt$.

[12] All of the labor considered here is skilled and capable of substituting between the two activities. We discuss the implications of allowing unskilled labor in the concluding section.

New ideas and innovations dominate old ones by a factor e^{γ}. Successful entrepreneurs must choose whether or not to implement their innovation immediately or delay implementation until a later date.[13] Once they implement, the knowledge associated with their improvement becomes publicly available, and can be built upon by rival entrepreneurs. However, prior to implementation, the knowledge is privately held by the entrepreneur.[14] We let the indicator function $Z_i(t)$ take on the value 1 if there exists a successful innovation in sector i which has not yet been implemented, and 0 otherwise. The set of instants in which entrepreneurial successes are implemented in sector i is denoted by Ω_i. We let $V_i^I(t)$ denote the expected present value of profits from implementing a success at time t, and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable time in future.

2.1.2 Contracts

The nature of innovation is such that entrepreneurs cannot simply "sell" their ideas to capitalists, but must be involved in its implementation themselves. We assume that entrepreneurs do not have the wealth required to purchase the capital stock needed to implement, and hence must borrow from the capitalists. In effect, there is a separation of ownership and control with respect to the capital stock of the firm, which may necessitate the writing of a long–term capital supply contract. The effective user cost of capital is the outcome of such a contractual relationship between the entrepreneur and the capitalists in each sector. Incumbent capital owners are limited in the extent of their monopoly pricing by the threat of "replacement" capital being built in their sector.[15]

Capital Supply Contracts: Intermediate producers and capital owners in every sector i can contract over a future binding utilized capital level, $K_i^u(\tau)$, and a price for each unit of utilized capital, $q_i(\tau)$, for all τ up to a chosen contract termination date, T_i^K. Thus a contract signed at time t is a tuple $\left\{\{K_i^u(\tau), q_i(\tau)\}_{\tau\in\left[t,T_i^K\right]},\ T_i^K\right\}$.[16] Since the productive advantage of an inter-

[13] As in Francois and Lloyd–Ellis (2003) we adopt a broad interpretation of innovation. Recently, Comin (2002) has estimated that the contribution of measured R&D to productivity growth in the US is less that 1/2 of 1%. As he notes, a larger contribution is likely to come from unpatented managerial and organizational innovations.

[14] Even for the case of intellectual property, Cohen, Nelson and Walsh (2000) show that firms make extensive use of secrecy in protecting productivity improvements. Secrecy likely plays a more prominent role for entrepreneurial innovations, which are the key here.

[15] In order to maintain competition in capital supply it will be assumed that, in the event of a competing capital stock being built, ties in tended prices are always broken in favour of the entrant. Due to storage costs, entry of replacement capital will imply scrapping of the pre-existing stock.

[16] Identical results obtain if instead of specifying the time-varying utilization rate of capital, $K_i^u(\tau)$, contracts can only be written over $K(\tau)$.

mediate producer lasts only until a superior technology is implemented in that sector, contracts allow the termination of agreements before T_i^K if shutting down production. Otherwise, the parties can break contracts only by mutual agreement.

Although supply contracts with particular entrepreneurs only last until their ideas become obsolete, capitalist owners can retain their incumbency permanently (subject to competition from other capitalists). The present value of the capitalist's net income in sector i under the utilization–price sequence $\{K_i^u(\tau), q_i(\tau)\}_{\tau=t}^{\infty}$ is therefore:

$$V_i^K(t) = \int_t^{\infty} e^{-[R(\tau)-R(t)]} \left[q_i(\tau) K_i^u(\tau) - \dot{K}_i(\tau)\right] d\tau. \tag{5}$$

Intermediate Supply Contracts: Final goods producers are also able to contract intermediate good deliveries from each of the intermediate producing sectors, i. Such contracts written at t involve a similar tuple: $\left\{\{x_i(\tau), p_i(\tau)\}_{\tau\in\left[t,T_i^X\right]},\ T_i^X\right\}$ where the unit price is $p_i(t)$ and the contract termination date is T_i^X. Contracts can be altered under the same conditions as in capital contracts.[17] The value of final goods producers is denoted $V^Y(t)$.

2.2 Definition of Equilibrium

Given initial state variables[18] $\{A_i(0),\ Z_i(0),\ K_i(0)\}_{i=0}^1$, an equilibrium for this economy is:

(1) a sequence of capital supply contracts $\left\{\widehat{T}_{iv}^K, \left\{\widehat{K}_i^u(t), \widehat{q}_i(t)\right\}_{t\in\left[\widehat{T}_{iv-1}^K,\widehat{T}_{iv}^K\right]}\right\}_{v\in I}$,

(2) a sequence of intermediate supply contracts $\left\{\widehat{T}_{iv}^X, \{\widehat{x}_i(t), \widehat{p}_i(t)\}_{t\in\left[\widehat{T}_{iv-1}^X,\widehat{T}_{iv}^X\right]}\right\}_{v\in I}$,

(3) sequences $\left\{\widehat{K}_i(t),\ \widehat{L}_i(t),\ \widehat{H}_i(t),\ \widehat{A}_i(t),\ \widehat{Z}_i(t),\ \widehat{V}_i^I(t),\ \widehat{V}_i^D(t),\ \widehat{V}_i^K(t)\right\}_{t\in[0,\infty)}$ for each intermediate sector i, and

(4) economy wide sequences $\left\{\widehat{Y}(t),\ \widehat{R}(t),\ \widehat{w}(t),\ \widehat{V}^Y(t),\ \widehat{C}(t),\ \widehat{S}(t)\right\}_{t\in[0,\infty)}$

which satisfy the following conditions:

• Households allocate consumption over time to maximize (1) subject (2). The first–order conditions of the household's optimization require that

$$\widehat{C}(t)^{\sigma} = \widehat{C}(\tau)^{\sigma} e^{\widehat{R}(t)-\widehat{R}(\tau)-\rho(t-\tau)} \qquad \forall\ t,\tau, \tag{6}$$

and that the transversality condition holds

$$\lim_{\tau\to\infty} e^{-\widehat{R}(\tau)}\widehat{S}(\tau) = 0 \tag{7}$$

[17] Though conceptually feasible, contracts written over the supply of labor and final output are redundant in the equilibria we study and will not be considered further.

[18] Without loss of generality, we assume no stored output at time 0.

• Labor markets clear:

$$\int_0^1 \widehat{L}_i(t)di + \widehat{H}(t) = 1 \tag{8}$$

• Arbitrage trading in financial markets implies that for all assets that are held in strictly positive amounts by households, the rate of return between time t and time s must equal $\frac{\widehat{R}(s)-\widehat{R}(t)}{s-t}$.

• Free entry into innovation — entrepreneurs select the sector in which they innovate so as to maximize the expected present value of the innovation, and

$$\delta \max[\widehat{V}_i^D(t), \widehat{V}_i^I(t)] \leq \widehat{w}(t), \quad \widehat{H}_i(t) \geq 0 \qquad \text{with at least one equality.} \tag{9}$$

• At instants where there is implementation, entrepreneurs with innovations must prefer to implement rather that delay until a later date

$$\widehat{V}_i^I(t) \geq \widehat{V}_i^D(t) \quad \forall\, t \in \widehat{\Omega}_i. \tag{10}$$

• At instants where there is no implementation, either there must be no innovations available to implement, or entrepreneurs with innovations must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } \widehat{Z}_i(t) &= 0, \qquad (11)\\ \text{or if } \widehat{Z}_i(t) &= 1,\ \widehat{V}_i^I(t) \leq \widehat{V}_i^D(t) \quad \forall\, t \notin \widehat{\Omega}_i. \end{aligned}$$

• For all capital supply contracts written at date t, the equilibrium contract is such that no other contract dominates for the capitalist and all existing entrepreneurs indexed by technologies $A_i(\tau) \leq A_i(t)$ in sector i:

$$\max\left[\widehat{V}_i^I(t), \widehat{V}_i^D(t)\right] + \widehat{V}_i^K(t) \geq \max\left[V_i^I(t), V_i^D(t)\right] + V_i^K(t) \tag{12}$$

$$\forall \text{ mutually determined } V_i^J \neq \widehat{V}_i^J, \quad J = I, D, K. \tag{13}$$

• For all intermediate supply contracts written at date t, the equilibrium contract is such that no other contract dominates for the final goods producer and all existing entrepreneurs in sector i:

$$\max\left[\widehat{V}_i^I(t), \widehat{V}_i^D(t)\right] + \widehat{V}^Y(t) \geq \max\left[V_i^I(t), V_i^D(t)\right] + V^Y(t), \tag{14}$$

$$\forall \text{ mutually determined } V_i^J \neq \widehat{V}_i^J, \quad J = I, D, K, \tag{15}$$

where $V^Y(t)$ holds contracts with other intermediate suppliers fixed.

• Free entry into final output production: $\widehat{V}^Y(t) \leq 0$

• Free entry of replacement capital: $\widehat{V}_i^K(t) \leq \widehat{K}_i(t)$

3 The Acyclical Balanced Growth Path

In this section, we briefly consider the existence of an equilibrium growth path along which the utilized capital of firms grows monotonically, entrepreneurship is continuous and implementation is never delayed. We derive the equilibrium without utilizing long–term contracts, so that all transactions occur in spot markets, since it will be seen that allowing for them does not affect the equilibrium. While the acyclical growth path is not our main focus, it is useful for understanding our later results.

Consumption satisfies the familiar differential equation:

$$\frac{\dot{C}\,(t)}{C\,(t)} = \frac{r\,(t) - \rho}{\sigma}, \tag{16}$$

where $r\,(t) = \dot{R}\,(t)$, denotes the continuous time interest rate. In the absence of uncertainty or adjustment costs, and as long as utilized capital is anticipated to grow, capitalists never acquire more capital than is needed for production, so that

$$K_i^u(t) = K_i(t). \tag{17}$$

Within each sector, i, the existence of potential capital entrants implies that capital owners cannot earn excess returns on marginal units. Hence:

Lemma 1 *: As long as new capital is being built, free–entry into capital markets implies that*

$$q_i\,(t) = q\,(t) = r\,(t) \qquad \forall\ i. \tag{18}$$

Final goods producers choose intermediates to maximize profits, taking their prices as given. The derived demand for intermediate i is

$$x_i^d(t) = \frac{Y(t)}{p_i(t)}. \tag{19}$$

The unit elasticity of demand for intermediates implies that limit pricing, which drives out the previous incumbent, is optimal:

Lemma 2 *: The limit price is given by*

$$p_i(t) = \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(t)}. \tag{20}$$

where $\mu = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

The resulting instantaneous profit earned in each sector is given by

$$\pi(t) = (1 - e^{-(1-\alpha)\gamma})Y(t). \tag{21}$$

Aggregate final output can be expressed as

$$Y(t) = [K(t)]^{\alpha} \left[\overline{A}(t)L(t)\right]^{1-\alpha}, \tag{22}$$

where $\overline{A}(t) = \exp\left(\int_0^1 \ln A_i(t)di\right)$. Along the acyclical steady–state growth path, a constant fraction of the labor force is allocated to entrepreneurship. The standard solution method yields the following steady state implication:

Proposition 1 *: If*

$$\frac{(1 - e^{-(1-\alpha)\gamma})\gamma(1-\sigma)}{1 - \alpha e^{-(1-\alpha)\gamma}} < \frac{\rho}{\delta} \tag{23}$$

then there exists an acyclical equilibrium with a constant growth rate given by

$$g^a = \max\left[\frac{[\delta(1 - e^{-(1-\alpha)\gamma}) - \rho(1-\alpha)e^{-(1-\alpha)\gamma}]\gamma}{1 - \alpha e^{-(1-\alpha)\gamma} - \gamma(1-\sigma)(1-\alpha)e^{-(1-\alpha)\gamma}}, 0\right]. \tag{24}$$

Along this equilibrium growth path, the inequality in (23) implies that $r(t) > g^a(t)$ at every moment. Along a balanced growth path, this condition must hold for the transversality condition to be satisfied and hence for utility to be bounded. However, this condition also ensures both that no output is stored, and that the implementation of any innovation is never delayed (see Francois and Lloyd–Ellis, 2003, for further elaboration). Allowing long–term supply contracts would only undermine the existence of this equilibrium growth path if contracting for non-spot market prices could make both parties to a contract better off. However, since all quantities are chosen optimally in the spot market in this equilibrium, such contracts would necessarily involve one side being made worse off.

4 The Posited Cyclical Growth Path

In this section, we begin by informally positing a cyclical equilibrium growth path in which, due to the rigid nature of capital, under–utilization may occur during downturns. We then posit the equilibrium behavior for capitalists and entrepreneurs over the cycle and detail the implications for contracting, consumption and aggregate entrepreneurship. In Section 5, we derive more formally the implications of this behavior over each phase of the cycle, and Section 6 then demonstrates the consistency of the posited behavior of entrepreneurs and capitalists in an equilibrium steady state and derives the conditions for existence.

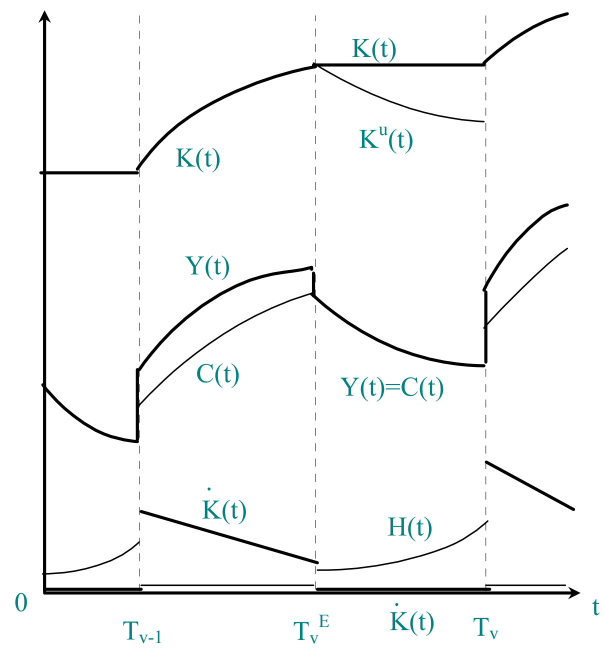

Figure 2: Evolution of Key Aggregates over the Cycle

Figures 2 and 3 depict the movement of key variables during the cycle. Cycles are indexed by the subscript v, and feature a consistently recurring pattern through their phases. The vth cycle features three distinct phases:

• The **expansion** is triggered by a productivity boom at time T_{v-1} and continues through subsequent capital accumulation, leading to continued growth in output, consumption and wages. Over this expansion phase the interest rate falls and investment, though positive, declines as the capital stock rises. Also, since labor's productivity in manufacturing intermediates is relatively high, no labor is allocated to entrepreneurship. Through time, continued capital accumulation lowers returns to further investment, rendering entrepreneurship relatively more attractive. Eventually innovation and reorganization re-commence, drawing labor hours from production. At this point, the return on investment in physical assets drops to zero, and investment ceases temporarily.

• The **contraction** starts with a collapse in fixed capital formation at time T_v^E as investment shifts towards longer-term focused activities. Intermediate producers experience a reduction in aggre-

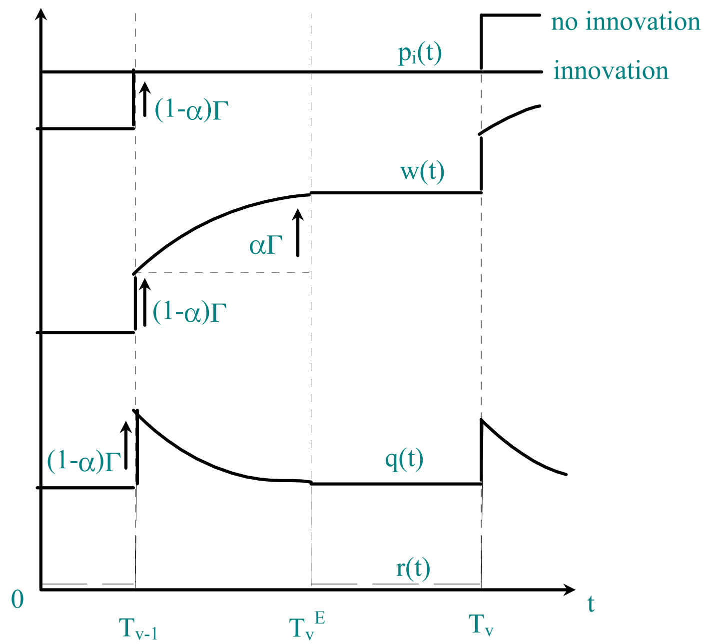

Figure 3: Evolution of Prices over Cycle

gate demand and then optimally re-allocate resources to relatively labor intensive entrepreneurial reorganization in order to raise productivity for the forthcoming boom. Due to irreversibility and the putty-clay nature of installed capital, labor's departure from production implies that capital cannot be fully utilized. Through the downturn, capital utilization falls and is traded at a constant price. Innovation and reorganization continue to increase throughout this phase so that the economy continues to contract through declining consumption expenditure.

• The **boom** occurs at an endogenously determined date, T_v, when the value of implementing stored innovations first exceeds the value of delaying their implementation. At that point, all entrepreneurs implement their successful innovation, starting the upswing once again. During the boom the returns to production rise above those of innovation and re–organization, drawing skilled labor out of entrepreneurship and into production. Returns to capital also rise with the new more productive technologies, so that capital accumulation recommences and the cycle begins again.

4.1 Contracts

Because competition from replacement capitalists weakens during a recession, contracts arise endogenously as capitalists compete to offer guaranteed prices to capital users in advance of the downturn. At this point, we anticipate the form these contracts will take and derive their implications. In Section 6 we shall verify that these contracts are constrained optimal given these implications. Contracts are written during the expansion of each cycle and terminate just before the boom of the next. Contracts between intermediate and final goods producers specify a sequence of quantities and prices for intermediates through the cycle. These contracted prices reflect the marginal costs of the main competitor: the previous incumbent holding the next best technology. The prices and quantities agreed to in the intermediate goods contract take the same form as the spot market values along the acyclical growth path:

$$p_i^c(t) = \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(T_{v-1})} \quad \forall t \in [T_{v-1}, T_v). \tag{25}$$

$$x_i^c(t) = \frac{Y(t)}{p_i^c(t)} \quad \forall t \in [T_{v-1}, T_v). \tag{26}$$

Through the upturn, contracts between capitalists and entrepreneurs specify steady expansion in each sector's capital stock, with capital being traded at a declining market–clearing price. During a contraction, the contracts specify a fixed rental price for capital and a declining utilization rate. The equilibrium contracts are written before T_{v-1}^E over rental rate and utilized capital $\{q_i^c(t), K_i^c(t), T_v\}$ and take the following form:

$$q_i^c(t) = \begin{cases} \alpha e^{-(1-\alpha)\gamma} \overline{A}^{1-\alpha}(T_{v-1}) K(t)^{\alpha-1} & \forall \ t \in \left[T_{v-1}, T_v^E\right] \\ \alpha e^{-(1-\alpha)\gamma} \overline{A}^{1-\alpha}(T_{v-1}) K(T_v^E)^{\alpha-1} & \forall \ t \in (T_v^E, T_v) \end{cases} \tag{27}$$

$$K_i^c(t) = \begin{cases} K(t) & \forall \ t \in \left[T_{v-1}, T_v^E\right] \\ \lambda(t) K\left(T_v^E\right) & \forall \ t \in (T_v^E, T_v), \end{cases} \tag{28}$$

where $\lambda(t) < 1$ denotes the utilization rate. Note that the posited contracts are symmetric across sectors. In fact, as we will see, a contract specifying a constant price through the downturn is not necessary. What will be required is that the price sequence is such that its average equals a unique value. However, a constant price with this property is an equilibrium, and if there is any arbitrarily small cost to price adjustment it is unique.

4.2 Consumption

Over intervals during which the discount factor does not jump, consumption is allocated as described by (16). However along the cyclical growth path, the discount rate jumps at the boom, so that consumption exhibits a discontinuity during implementation periods. We therefore characterize the optimal evolution of consumption from the beginning of one cycle to the beginning of the next by the difference equation

$$\sigma \ln \frac{C_0(T_v)}{C_0(T_{v-1})} = R(T_v) - R(T_{v-1}) - \rho\,(T_v - T_{v-1})\,. \tag{29}$$

where the 0 subscript is used to denote values of variables the instant after the implementation boom. Note that a sufficient condition for the boundedness of the consumer's optimization problem is that $\ln \frac{C_0(T_v)}{C_0(T_{v-1})} < R(T_v) - R(T_{v-1})$ for all v, or that

$$\frac{(1-\sigma)}{T_v - T_{v-1}} \ln \frac{C_0(T_v)}{C_0(T_{v-1})} < \rho \quad \forall\, v. \tag{30}$$

In our analysis below, it is convenient to define the discount factor that will be used to discount from some time t during the cycle to the beginning of the next cycle. This discount factor is given by

$$\beta(t) = R(T_v) - R(t) = R(T_v) - R(T_{v-1}) - \int_{T_{v-1}}^{t} r(s)ds. \tag{31}$$

4.3 Innovation

Let $P_i(s)$ denote the probability that, since time T_v, no entrepreneurial success has been made in sector i by time s. It follows that the probability of there being no entrepreneurial success by time T_{v+1} conditional on there having been none by time t, is given by $P_i(T_{v+1})/P_i(t)$. Hence, the value of an incumbent firm in a sector where no entrepreneurial success has occurred by time t during the vth cycle can be expressed as

$$V_i^I(t) = \int_t^{T_{v+1}} e^{-\int_t^{\tau} r(s)ds} \pi_i(\tau)d\tau + \frac{P_i(T_{v+1})}{P_i(t)} e^{-\beta(t)} V_{0,i}^I(T_{v+1}). \tag{32}$$

The first term here represents the discounted profit stream that accrues to the entrepreneur with certainty during the current cycle, and the second term is the expected discounted value of being an incumbent thereafter.

Lemma 3 *In a cyclical equilibrium, successful entrepreneurs can credibly signal a success immediately and all entrepreneurship in their sector will stop until the next round of implementation.*

In the cyclical equilibrium, entrepreneurs' conjectures ensure no more entrepreneurship in a sector once a signal of success has been received, until after the next implementation. The expected value of an entrepreneurial success occurring at some time $t \in \left(T_v^E, T_v\right)$ but whose implementation is delayed until time T_v is thus:

$$V_i^D(t) = e^{-\beta(t)} V_{0,i}^I(T_v). \tag{33}$$

Since no implementation occurs during the cycle, the entrepreneur is assured of incumbency until at least T_{v+1}. Incumbency beyond that time depends on the probability that there has not been another entrepreneurial success in that sector up until then.[19] The symmetry of sectors implies that entrepreneurial effort is allocated evenly over all sectors that have not yet experienced a success within the cycle. This clearly depends on some sectors not having already received an entrepreneurial innovation, an equilibrium condition that will be imposed subsequently (see Section 6). Thus the probability of not being displaced at the next implementation is

$$P_i(T_v) = \exp\left(-\int_{T_v^E}^{T_v} \widetilde{H}_i(\tau)d\tau\right), \tag{34}$$

where $\widetilde{H}_i(\tau)$ denotes the quantity of labor that would be allocated to entrepreneurship if no entrepreneurial success had occurred prior to time τ in sector i. The amount of entrepreneurship varies over the cycle, but at the beginning of each cycle all industries are symmetric with respect to this probability: $P_i(T_v) = P(T_v)\ \forall i$.

5 The Three Phases of the Cycle

5.1 The (Neoclassical) Expansion

We denote the improvement in aggregate productivity during implementation period T_v (and, hence, the growth in the average unit cost) by $e^{(1-\alpha)\Gamma_v}$, where

$$\Gamma_v = \ln\left[\overline{A}_v / \overline{A}_{v-1}\right], \tag{35}$$

[19] A signal of further entrepreneurial success submitted by an incumbent is not credible in equilibrium because incumbents have incentive to lie to protect their profit stream. No such incentive exists for entrants since, without a success, profits are zero.

and $\overline{A}_v = \exp\left(\int_0^1 \ln A_i(T_v)di\right)$. Following an implementation boom, the state of technology in use remains unchanged for the rest of the cycle. An implication of the Cobb–Douglas structure is that, through competition, the unit factor price index simply reflects this level of technology.

Lemma 4 *: The input price index for $t \in [T_{v-1}, T_v^E]$ is constant and uniquely determined by the level of technology*

$$q(t)^\alpha w(t)^{1-\alpha} = \mu e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}. \tag{36}$$

As a result of the boom, wages rise rapidly. Since the next implementation boom is some time away, the present value of engaging in entrepreneurship falls below the wage, $\delta V^D(t) < w(t)$. During this phase, no labor is allocated to entrepreneurship and no new innovations come on line. However, final output grows in response to capital accumulation financed from household savings. In equilibrium the Euler equation and aggregate resource constraint imply dynamics that are almost identical to those of the Ramsey model:[20]

Proposition 2 *During the expansion, capital and consumption evolve according to:*

$$\frac{\dot{C}(t)}{C(t)} = \frac{\alpha e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}K(t)^{\alpha-1} - \rho}{\sigma} \tag{37}$$

$$\dot{K}(t) = \overline{A}_{v-1}^{1-\alpha}K(t)^\alpha - C(t). \tag{38}$$

Since all capital is utilized, Lemma 1 applies so that

$$r(t) = q(t) = \alpha e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}K(t)^{\alpha-1}. \tag{39}$$

Thus, as capital accumulates, the interest rate declines. Since technology is unchanging, Lemma 4 implies the wage must be rising

$$\frac{\dot{w}(t)}{w(t)} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{\dot{q}(t)}{q(t)} = \alpha\frac{\dot{K}(t)}{K(t)} > 0 \tag{40}$$

During the expansion, the expected value of entrepreneurship, $\delta V^D(t)$, is necessarily growing at the rate of interest, but continues to be dominated by the wage in production. After enough capital has been accumulated, however, $\delta V^D(t)$ eventually equals $w(t)$. At this point, if all workers were to remain in production, returns to entrepreneurship would strictly dominate those in production. As a result labor hours are re–allocated from production and into innovation, which triggers the contractionary phase.

[20]Note that, unlike the Ramsey model, the rate of return on savings is not equal to the marginal product of capital, but rather is a fraction $e^{-(1-\alpha)\gamma}$ of it. This reflects the entrepreneurial share of this marginal product accruing as a monopoly rent.

5.2 The (Keynesian) Contraction

Because of the putty–clay nature of capital, as labor starts to be withdrawn from production the capital–labor ratio cannot be adjusted from $\kappa(T_v^E)$ and output must contract. Since technology is also fixed during this phase, the wage must be constant:

Lemma 5 *: The wage for $t \in [T_v^E, T_v]$ is constant and determined by the level of technology and the capital–labor ratio chosen at the last peak, $\kappa(T_v^E)$:*

$$w(t) = \bar{w}_v = (1-\alpha)e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}\kappa(T_v^E)^{\alpha}. \tag{41}$$

Since there is free entry into entrepreneurship, $w(t) = \delta V^D(t)$, so that the value of entrepreneurship, $\delta V^D(t)$, is also constant. Since the time until implementation for a successful entrepreneur is falling and there is no stream of profits, because implementation is delayed, the instantaneous interest rate necessarily equals zero. If it were not, entrepreneurial activity would be delayed to the instant before the boom. Therefore:

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{w}(t)}{w(t)} = 0. \tag{42}$$

Note that this zero interest rate is consistent with the fact there is now excess (under–utilized) capital in the economy. Since marginal returns to capital in this phase are zero, physical investment ceases and the only investment is that in innovation, undertaken by entrepreneurs.

Lemma 6 *: At T_v^E, investment in physical capital falls discretely to zero and entrepreneurship jumps discretely to $H_0(T_v^E) > 0$.*

A switch like this across types of investment is also a feature of the models of Matsuyama (1999, 2001) and Walde (2002). However, here factor intensity differences between entrepreneurship and investment lead to a crash in output followed by continued decline through the recession. Although investment falls discretely at $t = T_v^E$, consumption must be constant across the transition between phases because the discount factor does not change discretely. With putty–clay technology, the decline in output due to the fall in investment demand is proportional to the fraction of labor hours withdrawn from production. It follows that the fraction of labor hours allocated to entrepreneurship at the start of the downturn, $H_0(T_v^E)$, which we denote as H_v from now on, equals the rate of investment at the peak of the expansion:

$$H_v = \frac{\dot{K}\left(T_v^E\right)}{Y\left(T_v^E\right)} = 1 - \frac{C\left(T_v^E\right)}{\overline{A}_{v-1}^{1-\alpha} K(T_v^E)^\alpha}. \tag{43}$$

Although consumption cannot fall discretely at T_v^E, the zero interest rate implies that consumption must be declining after T_v^E,[21]

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\sigma} = -\frac{\rho}{\sigma}, \tag{44}$$

as resources flow out of production and into entrepreneurship.

Since $Y\left(t\right) = C\left(t\right)$, the growth rate in the hours allocated to production is also given by (44) and so aggregate entrepreneurship at time t is given by

$$H(t) = 1 - (1 - H_v)\, e^{-\frac{\rho}{\sigma}[t - T_v^E]}. \tag{45}$$

Note that the putty–clay nature of capital implies that as labor leaves current production, capital utilization falls in the same proportion. It follows that the capital utilization rate specified in the equilibrium contract (28) is given by

$$\lambda(t) = (1 - H_0(T_v^E)) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}. \tag{46}$$

During the downturn, in the absence of a capital contract, entrepreneurs would be vulnerable to an increasing rental price through the downturn. To see why, observe that, in order to forestall entry by a competing capitalist, the incumbent capitalist is constrained to offer a price–quantity sequence which satisfies

$$V^K\left(K(t), t\right) = \int_t^{T_v} e^{-[R(\tau) - R(t)]} \left[q\left(\tau\right) K^u(\tau) - \dot{K}(\tau)\right] d\tau + e^{-\beta(T_v)} V^K\left(K(t), T_v\right) \le K(t), \tag{47}$$

where $V^K\left(K(t), \tau\right)$ denotes the value of the installed capital at time τ. During the downturn $r\left(t\right) = 0$ and $\dot{K}(\tau) = 0$, so that for $t \in \left[T_v^E, T_v\right]$, the condition becomes:

$$\int_t^{T_v} q\left(\tau\right) \lambda(\tau) K(T_v^E) dt + e^{-\beta(T_v)} V^K\left(K(T_v^E), T_v\right) \le K(T_v^E). \tag{48}$$

[21] Although $r = 0$, strict preference for zero storage results from the arbitrarily small storage costs.

However competition from potential replacement capitalists at the beginning of the next cycle ensures that $V^K\left(K(T_v^E), T_v\right) = K(T_v^E)$. Dividing by $K(T_v^E)$ and re–arranging, using (46), yields a necessary restriction to forestall entry during the downturn:

$$\int_t^{T_v} q\left(\tau\right)\left(1-H_v\right)e^{-\frac{\rho}{\sigma}\left(\tau-T_v^E\right)}d\tau \leq 1-e^{-\beta(T_v)}. \tag{49}$$

The right hand side of this expression is constant throughout the downturn, but the left–hand side would be decreasing through the downturn if $q\left(t\right)$ were constant. It follows that, in the absence of a contracted price, the capitalist could raise q through the downturn and still satisfy (49).

The main implication of this is that, without a contract written before T_v^E delineating the price charged by the capitalist for the remainder of the cycle, entrepreneurs will face an increasing rental rate for capital through the downturn. Given the potential for such price gouging, entrepreneurs will demand the writing of such contracts before T_v^E, when the cost of replacement capital is low.

The first thing to note about any such contract is that it must satisfy the capital feasibility constraint above, which will bind at $t = T_v^E$:

Lemma 7 *Any capital supply contract* $\{q_i^c(\tau), K_i^u(\tau)\}$ *signed at some date* $t \in [T_{v-1}, T_v^E)$ *must satisfy:*

$$\int_{T_v^E}^{T_v} q^c\left(\tau\right)\left(1-H_v\right)e^{-\frac{\rho}{\sigma}\left(\tau-T_v^E\right)}d\tau = 1-e^{-\beta(T_v)} \tag{50}$$

There are a number of price sequences $q^c(t)$ that could satisfy this condition, however the average level of prices through $t \in [T_{v-1}, T_v^E)$ is unique. Let this average in the vth cycle be

$$\overline{q}_v \equiv \frac{\int_{T_v^E}^{T_v} q^c\left(\tau\right)\left(1-H_v\right)e^{-\frac{\rho}{\sigma}\left(\tau-T_v^E\right)}d\tau}{\int_{T_v^E}^{T_v}\left(1-H_v\right)e^{-\frac{\rho}{\sigma}\left(\tau-T_v^E\right)}d\tau}. \tag{51}$$

Using 50, and integrating the denominator through the downturn, Δ_v^E, this implies:

$$\overline{q}_v = \frac{1-e^{-\beta(T_v)}}{\left(1-H_v\right)\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)}. \tag{52}$$

A further feature of such contracts is that they must induce a cost minimizing capital/labor ratio, in order again to forestall entry by competing capital providers. The standard marginal condition applies at every instant through the upturn. With putty/clay capital and zero discounting through the downturn, it is possible to treat the whole of the contractionary phase as if it were a single production period. Consequently, a condition analogous to the standard marginal condition applies to the optimal capital/labor ratio through the downturn,

$$\frac{(1-\alpha)\, K\left(T_v^E\right)}{\alpha L\left(T_v^E\right)} = \frac{\overline{w}_v}{\overline{q}_v}.$$

Since, $L\left(T_v^E\right) = 1$, it follows that:

Proposition 3 *For a capital–supply contract to be efficient through the downturn it is necessary that capital is installed only up to the point at which the marginal return to capital is equal to its average rental price:*

$$q(T_v^E) = \alpha e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} K(T_v^E)^{\alpha-1} = \overline{q}. \tag{53}$$

Equating (52) and (53), substituting for $1 - H_v$ using (43), it follows that the capital–consumption ratio at the height of the expansion can be expressed as:

$$\frac{K\left(T_v^E\right)}{C\left(T_v^E\right)} = \frac{\alpha e^{-(1-\alpha)\gamma}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)}{1-e^{-(1-\alpha)\Gamma_v}}. \tag{54}$$

Note that $K\left(T_v^E\right)$ is also the effective capital stock at the beginning of the next boom since there is no depreciation and no capital is accumulated through the recession.

5.3 The Boom

Productivity growth at the boom is given by $\Gamma_v = (1 - P(T_v))\gamma$, where $P(T_v)$ is defined by (34). Substituting in the allocation of labor to entrepreneurship through the downturn given by (45) and letting

$$\Delta_v^E = T_v - T_v^E, \tag{55}$$

yields the following implication.

Proposition 4 *: In an equilibrium where there is positive entrepreneurship only over the interval* $(T_v^E, T_v]$*, the growth in productivity during the succeeding boom is given by*

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma(1 - H_v)\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right). \tag{56}$$

For an entrepreneur who is holding an innovation, $V^I(t)$ is the value of implementing immediately. During the boom, for entrepreneurs to prefer to implement immediately, it must be the case that

$$V_0^I(T_v) > V_0^D(T_v), \tag{57}$$

recalling that 0 subscripts denote values immediately after implementation. Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:

$$\delta V^I(T_v) = \delta V^D(T_v) = w\left(T_v\right). \tag{58}$$

From (57), the return to entrepreneurship at the boom is the value of immediate (rather than delayed) incumbency. It follows that free entry into entrepreneurship at the boom requires that

$$\delta V_0^I(T_v) \leq w_0\left(T_v\right). \tag{59}$$

The opportunity cost of financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation has occurred. Just prior to the boom, this is given by the capital gains in sectors where no entrepreneurial successes have occurred;

$$\beta(T_v) = \log\left(\frac{V_0^I(T_v)}{V^I(T_v)}\right). \tag{60}$$

Note that since the short–term interest rate is zero over this phase, $\beta(t) = \beta(T_v)$, $\forall\ t \in (T_v^E, T_v)$. Combined with (58) and (59) it follows that asset market clearing at the boom requires

$$\beta(T_v) \leq \log\left(\frac{w_0\left(T_v\right)}{w(T_v)}\right) = (1-\alpha)\Gamma_v. \tag{61}$$

Free entry into entrepreneurship ensures that $\beta(T_v) > (1-\alpha)\Gamma_v$ cannot obtain in equilibrium.

Provided that $\beta(t) > 0$, households will never choose to store final output from within a cycle to the beginning of the next either because it is dominated by the long–run rate of return on claims to future profits. However, unlike final output, the return on stored intermediate output in sectors with no entrepreneurial successes is strictly positive, because of the increase in its price that occurs as a result of the boom. Even though there is a risk that the intermediate becomes obsolete at the boom, if the anticipated price increase is sufficiently large, households may choose to purchase claims to intermediate output rather than claims to firm profits.

If innovative activities are to be financed at time t, it cannot be the case that households are strictly better off buying claims to stored intermediate goods. In sectors with no entrepreneurial success, incumbent firms could sell such claims, use them to finance greater current production and then store the good to sell at the beginning of the next boom when the price is higher. In this case, since the cost of production is the same whether the good is stored or not, the rate of return on claims to stored intermediates in sector i is $\log p_{i,v+1} / p_{i,v} = (1-\alpha)\Gamma_v$.

It follows that the long run rate of return on claims to firm profits an instant prior to the boom must satisfy

$$\beta(T_v) \geq (1-\alpha)\Gamma_v. \tag{62}$$

Free–entry into arbitrage ensures that $\beta(T_v) < (1-\alpha)\Gamma_v$ cannot obtain in equilibrium. Because there is a risk of obsolescence, this condition implies that at any time prior to the boom the expected rate of return on claims to stored intermediates is strictly less than $\beta(t)$.

Combining (61) and (62) yields the following implication of market clearing during the boom for the long–run growth path:

Proposition 5 *Asset market clearing at the boom requires that*

$$\beta(T_v) = (1-\alpha)\Gamma_v. \tag{63}$$

Asset market–clearing thus yields a unique relationship between the discount applied over the boom, and productivity growth.[22]

The growth in output at the boom exceeds the growth in productivity for two reasons: first labor is re–allocated back into production, and second the previously unutilized capital is now being used productively. Since just before the boom, both inputs are a fraction $(1-H_v)e^{-\frac{\rho}{\sigma}\Delta_v^E}$ of their peak levels, output growth through the boom is given by

$$\begin{aligned} \Delta \ln Y(T_v) &= (1-\alpha)\Gamma_v + (1-\alpha)\Delta \ln L + \alpha \Delta \ln K^u \\ &= (1-\alpha)\Gamma_v + \frac{\rho}{\sigma}\Delta_v^E - \ln(1-H_v) \end{aligned} \tag{64}$$

It follows directly from Proposition 5 that growth in output exceeds the discount factor across the boom. Since profits are proportional to output, this explains why firms are willing to delay implementation during the downturn.

[22] Shleifer's (1986) model featured multiple expectations–driven steady state cycles. Such multiplicity cannot occur here because, unlike Shleifer, the possibility of storage that we allow forces a tight relationship between Γ_v and Δ_v^E as depicted in Proposition 4. Since Γ_v, Δ_v^E pairs must satisfy this restriction as well, in general, multiple solutions cannot be found. This however does not rule out cycles of a qualitatively different nature to those analyzed here.

The boom in output can be decomposed into a boom in consumption and investment. From the Euler equation, we can compute consumption growth across the boom:

$$\Delta \ln C(T_v) = \frac{(1-\alpha)}{\sigma}\Gamma_v. \tag{65}$$

Notice that whether the growth in consumption exceeds the growth in productivity at the boom, depends on the value of σ. In particular, if $\sigma < 1$, consumption growth must exceed aggregate productivity growth. Finally, since in the instant prior to the boom $C(T_v) = Y(T_v)$, it follows that the investment rate at the boom jumps to

$$\frac{\dot{K}_0(T_v)}{Y_0(T_v)} = \left(1 - (1-H_v)e^{\left(\frac{1-\sigma}{\sigma}\right)(1-\alpha)\Gamma_v - \frac{\rho}{\sigma}\Delta_v^E}\right) \tag{66}$$

6 Optimal Behavior During the Cycle

Given the dynamics implied above, in this section we derive conditions which must be satisfied in order for the posited behavior of capitalists and entrepreneurs to be optimal.

6.1 Optimal Entrepreneurship and Implementation

Equilibrium entrepreneurial behavior imposes the following requirements on our hypothesized cycle:

• Successful entrepreneurs at time $t = T_v$ must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) > V_0^D(T_v). \tag{E1}$$

• Entrepreneurs who successfully innovate during the downturn must prefer to wait until the beginning of the next cycle rather than implement earlier and sell at the limit price:

$$V^I(t) < V^D(t) \qquad \forall\, t \in (T_v^E, T_v) \tag{E2}$$

• No entrepreneur wants to innovate during the slowdown of the cycle. Since in this phase of the cycle $\delta V^D(t) < w(t)$, this condition requires that

$$\delta V^I(t) < w(t) \qquad \forall\, t \in (0, T_v^E) \tag{E3}$$

The conditions on the value functions above take as given that entrepreneurs do not produce in excess of current demand and store their output until the boom. Provided that the incumbent entrepreneur does not terminate the capital supply contract, (63) ensures that storage across the boom is not optimal. However, since in the posited equilibrium the capital stock is being under–utilized, it is possible that just before the boom a rival entrepreneur who has successfully innovated may be able to "buy out" the contract and utilize all the capital, meeting the current demand for output and storing the remainder until the boom.

This rival would not benefit from taking over the capital contract of the incumbent under identical terms. From (63), producing output and storing it until the boom is not optimal if he must pay a constant amount $\bar{q}$ for capital. Moreover, under (E2) implementation and sale before the boom is not optimal. However, the rival may be willing to take-over the use rights if able to pay $\overline{q}$ for the amount $K^c(t)$ as in the incumbent's contract, and utilize extra units of idle capital at some price $\widetilde{q} < \overline{q}$. Clearly any $\widetilde{q} > 0$ for the excess units would be amenable to the capitalist. The most the rival will be willing to pay per period for the current capital is $\overline{q}K(T_v^E)$, since $e^{-\beta(T_v)}q(T_v) = \overline{q}$. To buy out the contract, the rival must compensate the incumbent for the loss of profits sustained for the remainder of the cycle and must offer the capitalist at least the payment he is currently receiving, $\overline{q}(1-H_v)e^{-\frac{\rho}{\sigma}(t-T_v^E)}K(T_v^E)$ per period. It follows that such a contract buy–out will not a be mutually acceptable at time t if

$$\int_t^{T_v} \pi(\tau)d\tau + \int_t^{T_v} (1-H_v)e^{-\frac{\rho}{\sigma}(\tau-T_v^E)}d\tau \geq \int_t^{T_v} \overline{q}K(T_v^E)d\tau. \tag{67}$$

The following proposition provides a sufficient condition for this to hold throughout the downturn:

Proposition 6 *: If*

$$(1-(1-\alpha)e^{-(1-\alpha)\gamma})(1-H_v)e^{-\frac{\rho}{\sigma}\Delta_v^E} > \alpha e^{-(1-\alpha)\gamma} \tag{E4}$$

then entrepreneurs who successfully innovate during the downturn prefer to wait until the beginning of the next cycle rather than displace the incumbent, produce now and store until the boom.

In effect, condition $(E4)$ explains how it is possible for there to be under–utilized capital during a recession even though there exist rivals who could potentially use the capital stock more profitably. The reason is that the capital stock is "lumpy", so that the rival cannot use a part of it while the incumbent continues to produce. For this reason the rival must compensate the incumbent for his profit loss and this "endogenous" fixed cost is too large for entry to be profitable under recessionary demand conditions. Entry does not become profitable until the boom. There, demand is high and entry costs low because the previous incumbent's profits do not need to be compensated as they have already been destroyed by the implementation of a superior production process.

Note finally that in constructing the equilibrium above we have implicitly imposed the requirement that the downturn is not long enough that all sectors innovate. Thus the following condition must be satisfied with strict inequality:

$$P(T_v) > 0. \tag{E5}$$

Taken together conditions (E1) through (E5) are restrictions on entrepreneurial behavior that must be satisfied for the cyclical growth path we have posited to be an equilibrium. However, we must first check that under these conditions, the contracts we have specified are indeed undominated.

6.2 The Optimality of Contracts

6.2.1 The Intermediate Good Supply Contract

The need for an intermediate contract arises because the lumpiness and sector specificity of installed capital implies that only one intermediate producer can use the sector's capital. Though it is possible to commission the building of a new capital stock, if guaranteed a sufficiently high rental rate, the rental rate so required increases through the cycle, so that the threat of entry provides progressively weaker restraint through time. By negotiating a contract before the downturn, bids from the previous intermediate producer force limit pricing by the current incumbent at a relatively low marginal cost, since at this time the cost of building replacement capital is relatively low. Thus the intermediate goods contract guards the final goods producers against the increasing monopoly power of the incumbent through the downturn by pinning the producer down to a price/quantity pair while the previous incumbent's threat of entry is greatest.

Lemma 8 *The contracted price sequence for intermediate i, $p_i^c(t)$, and quantity $x_i^c(t)$, for $t \in [T_{v-1}, T_v)$, satisfying (25) and (26) is optimal, given a sequence of input prices $w(t), q(t)$ for $t \in [T_{v-1}, T_v)$ faced by the previous incumbent.*

The input price for labor, $w(t)$ is determined in the per period spot market so it remains now to determine the sequence of capital prices.

6.2.2 The Capital Supply Contract

The aim of capital supply contracts is to forestall hold-up by the capitalist, but the contract's "reach" is limited on the entrepreneur's side. Unlike capital which is infinitely lived, entrepreneurs lose their productive advantage when displaced by superior producers, so that they cannot make unconditional promises to purchase capital into the indefinite future. All contracts are thus contingent upon the entrepreneur's continuing production. We show now that the earlier posited contract comprising (27) and (28) is an optimal response to the posited behavior of other agents over the cycle:

Proposition 7 *Provided (E1)–(E5) hold then, in each sector* i, *at the boom of every cycle* (T_v, $v = 1...\infty$), *an equilibrium contract for the capitalist and leading entrepreneur is a sequence of prices* $q^c(t)$ *and capital* $K^c(t)$ *for all* $t \in [T_v, T_{v+1})$ *that satisfies (27) and (28).*

Note that the cyclical equilibrium is supported by the limitations on contracting that we have imposed. The critical, and we think realistic, assumption is that only future prices and quantities can be contracted ex ante. Allowing for a richer set of contracting possibilities would overturn this result. The sort of environments required would need to allow that, in addition to a time varying price q and quantity K for capital, it would be possible to condition transfers between the parties on other actions that they or other parties take. For example if the new incumbent entrepreneur (who arrives probabilistically in the downturn) could somehow be party to the contract at time T_v, then full utilization of the capital through the downturn could also be contracted ex ante. Such a rich contracting environment, however, seems to require unrealistically complex and difficult to observe details to be enforceable between the parties. Thus endogenous underutilization, which corresponds to that observed in actual business cycles, arises here due to seemingly natural limitations in contracting.

7 The Stationary Cyclical Growth Path

Here we characterize the stationary cyclical growth path implied by Propositions 2 to 5. To allow a stationary representation, we normalize all aggregate by dividing by $\bar{A}_{v-1}$ and denote the result with lower case variables.

First recall from Proposition 2, that the dynamics of the economy during the expansion are analagous to those in the Ramsey model without technological change. Let $c_v = c(T_v^E)$ and $k_v = k(T_v^E)$ denote the normalized values of consumption and capital at the peak of the vth expansion. Given initial values $c_0(T_{v-1})$ and $k_0(T_{v-1})$, and an expansion length Δ_v^X, it is possible to summarize the expansion as follows:

$$c_v = f(c_0(T_{v-1}), k_0(T_{v-1}), \Delta_v^X) \tag{68}$$

$$k_v = g(c_0(T_{v-1}), k_0(T_{v-1}), \Delta_v^X), \tag{69}$$

where $f(\cdot)$ and $g(\cdot)$ are well–defined functions. Since capital accumulation stops in the recession, and $\overline{A}$ rises by $e^{\Gamma_{v-1}}$, it follows that $k_0 = e^{-\Gamma_{v-1}} k_{v-1}$. From (44), consumption declines by a factor $e^{-\frac{\rho}{\sigma}\Delta_{v-1}^E}$ in the recession. When combined with its increase at the boom, from (65), this yields $c_0 = e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1}-\frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}$. Substituting for c_0 and k_0 then yields

$$c_v = f(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1}-\frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}, e^{-\Gamma_{v-1}} k_{v-1}, \Delta_v^X) \tag{70}$$

$$k_v = g(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1}-\frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}, e^{-\Gamma_{v-1}} k_{v-1}, \Delta_v^X). \tag{71}$$

Substituting for $1 - H_v$ in Proposition 4 using (43), we can express the size of the boom as

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma \frac{c_v}{k_v^\alpha} \left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma} \right), \tag{72}$$

Propositions 5 and 3 yield

$$\alpha e^{-(1-\alpha)\gamma} \frac{c_v}{k_v} = \frac{1 - e^{-(1-\alpha)\Gamma_v}}{\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma} \right)}. \tag{73}$$

Finally, asset market clearing over the boom (conditions (58) to (61)) imply:

$$\delta v_0^I(c_v, c_{v-1}, k_v, k_{v-1}, \Gamma_v, \Gamma_{v-1}, \Delta_v^E, \Delta_v^X) = \frac{w_0\left(T_{v-1}\right)}{\bar{A}_{v-1}} = (1-\alpha)e^{-(1-\alpha)\gamma} e^{-\alpha\Gamma_{v-1}} k_{v-1}^\alpha, \tag{74}$$

where $v_0^I = V_0^I(T_{v-1})/\bar{A}_{v-1}$ is explicitly derived in the appendix.

In the stationary cycle $\Gamma_v = \Gamma$, $k_v = k$, $c_v = c$, $\Delta_v^E = \Delta^E$ and $\Delta_v^X = \Delta^X$ for all v. Imposing these on the equations above, yields a system of five equations in the five unknowns:

$$\Gamma = \delta\gamma\Delta^E - \delta\gamma\frac{c}{k^\alpha}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right) \tag{75}$$

$$\alpha e^{-(1-\alpha)\gamma}k^{\alpha-1}\frac{c}{k^\alpha} = \frac{1-e^{-(1-\alpha)\Gamma}}{\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)} \tag{76}$$

$$c = f(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma-\frac{\rho}{\sigma}\Delta^E}c, e^{-\Gamma}k, \Delta^X) \tag{77}$$

$$k = g(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma-\frac{\rho}{\sigma}\Delta^E}c, e^{-\Gamma}k, \Delta^X) \tag{78}$$

$$\delta v_0^I(c,k,\Gamma,\Delta^E,\Delta^X) = (1-\alpha)e^{-(1-\alpha)\gamma}e^{-\alpha\Gamma}k^\alpha. \tag{79}$$

We demonstrate existence of the stationary cycle, and analyze this system numerically in the next section. However, the dynamics of the model can be understood heuristically from the phase diagram in Figure 4. Here the process of capital accumulation in the expansionary phase, $t \in \left(T_{v-1}, T_v^E\right)$, within a cycle, when in steady state, is depicted.

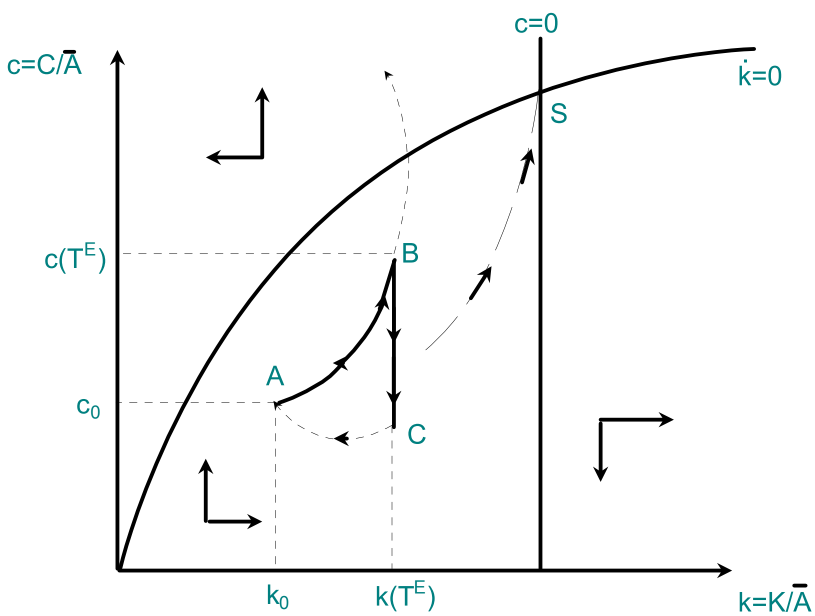

Figure 4: Phase Diagram

The economy does not evolve along the standard stable trajectory of the Ramsey model terminating at the steady state, S. Instead, the evolution of the cycling economy during the expansion is depicted by the path between A and B in the figure. Capital is accumulated starting at the point k_0 corresponding to point A in the diagram, according to (37) and (38). The point

k_0 denotes the inherited capital stock at the boom. Accumulation ends at $k\left(T^E\right)$, at which point investment stops until the next cycle. Note that if allowed to continue along such a path the economy would eventually violate transversality, but capital accumulation stops and consumption declines so that the economy evolves from B to C through the downturn. During this phase, the dynamics of the economy are no longer dictated by the Ramsey phase diagram. When this phase ends, implementation of stored productivity improvements occurs at the next boom, and $\bar{A}$ increases, so that k fall discretely. If $\sigma < 1$, consumption grows by more than productivity at the boom, so that c rises discretely. The boom is therefore depicted by the dotted arrow back to point A. At this point, investment in the expansionary phase recommences for the next cycle. The connection between the two phases of the cycle arises due to the allocation of resources to entrepreneurship. This allocation of resources will be reflected in the size of the increment to $\bar{A}$, Γ.

7.1 Existence of the Stationary Cycle

To demonstrate the existence of the stationary cycle, we numerically solve the model for various combinations of parameters and check the existence conditions (E1)–(E5). We choose parameters to fall within reasonable bounds of known values, and present a baseline case given in Table 1:

Table 1: Baseline Parameters

Parameter	Value
α	0.22
γ	0.13546
σ	0.79
ρ	0.02
δ	1.39

The parameters α and γ were chosen so as to obtain a labor share of 0.7, a capital share of 0.2 and a profit share of 0.1. These values correspond approximately to those estimated by Atkeson and Kehoe (2002). The value of γ corresponds to a markup rate of around 15%. The intertemporal elasticity of substitution $\frac{1}{\sigma}$ is slightly high, but we solve for various values below, including $\sigma = 1$. Given $\sigma = 0.79$, we calibrated δ and ρ so as to match a long–run annual growth rate of 2.2% and an average risk–free real interest rate of 3.8%, values which correspond to annual data for the post–war US. The baseline case above yields a cycle length of a little less than 4 years, $H_v = .2044$,

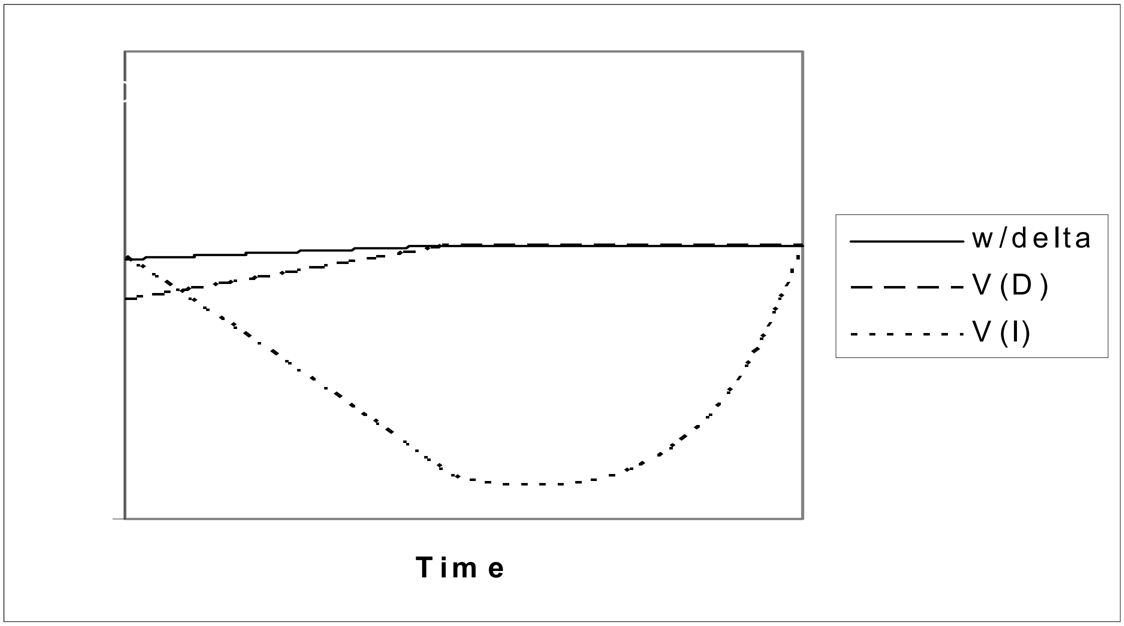

Figure 5: Existence

and $k_v = 7.668$. In this, and all simulations we have computed, steady state values are unique.[23]

Numerically simulated value functions are plotted for the baseline case in Figure 5. The figure allows direct verification of conditions E1, E2, and E3.[24]

The plot starts with implementation at a boom, when $V^I > V^D$ and w/δ. For a discrete interval, V^I remains above V^D implying that, in the event of entrepreneurial success, implementation would dominate delay. However, over this part of the phase, the relative value of labor in production, w/δ, exceeds returns to entrepreneurship, so that no entrepreneurial successes are available to implement. Throughout this expansionary phase, investment occurs so that the wage continues to rise. At the same time, V^D is also rising because the time until implementation of entrepreneurial successes falls. Note that this increase in V^D is simply a function of discounting, i.e., the ensuing boom at which stored successes will be implemented is drawing closer. Throughout this phase V^I declines as the duration of guaranteed positive profits falls.

The end of the expansion corresponds to the commencement of entrepreneurship, i.e., when the increasing value of a delayed entrepreneurial success eventually meets the opportunity cost

[23] Francois and Lloyd-Ellis (2003) explicitly establish uniqueness of the stationary cycle when capital accumulation is not allowed. It seems likely that the introduction of capital would not lead to an additional stationary cycle here, but we have not been able to establish this analytically.

[24] Condition E4 and E5 are not depicted but can be directly checked.

of labor in production. This corresponds to the point at which $w/\delta = V^D$. Since V^D rose due to discounting during the expansion, during the contraction, when capital is no longer accumulated and interest rates are zero, V^D remains constant. This implies that, by arbitrage, the wage must also be constant, until the contraction ends. Through the downturn V^I continues to fall, but must eventually rise again as the probability of remaining the incumbent at the boom, given that an entrepreneurial success has not arrived in one's sector, increases. This increase in V^I is the force that will eventually trigger the next boom that ends the recession. It occurs when V^I just exceeds V^D and entrepreneurs implement stored entrepreneurial successes, leading to an increase in productivity, a jump in demand, movement of labor back to production, and full capacity utilization.

Table 2 lists the numerical implications for growth, cycle length and terminal values of capital stocks for various combinations of parameter values, including the baseline case. A first thing to note is the extreme sensitivity of cycle length, $\Delta = \Delta^X + \Delta^E$, to changes in parameters. In contrast, the long–run growth rate is much less sensitive to changes in parameters than along the acyclical growth path. Generally, increases in parameters that directly raise the impact of entrepreneurship, δ and γ, increase the growth rate, as in the acyclical steady state. Changes in σ and ρ also have effects similar to those present in the acyclical steady state. Additionally, however, changes in these parameters alter cycle length in ways which counterveil, and sometimes overshadow, the direct effects. For example, increasing σ, lowering inter-temporal substitutability, generally induces lower growth in the acyclical steady state because consumers are less willing to delay consumption to the future. A similar effect is present here. However, as the table shows, this increase also raises cycle length and amplitude, inducing more entrepreneurship and a larger boom. The net effect, as the table shows, is an increase in growth rate for this configuration. A similar sequence of effects is present for increases in ρ. Increasing the capital share, α, increases capital accumulation and lowers interest rates, but because this also induces a shorter cycle length, the net effect is a fall in growth rate.

Values of σ closer to 1 do not satisfy our existence conditions given the values of other parameters assumed in the baseline case. However, if we allow δ to rise somewhat, higher values of σ are consistent with the cycle (see the last two rows of Table 2). Intuitively, with higher entrepreneurial productivity, both the size of booms and the average growth rate tend to be higher in equilibrium. As a result, households are willing to delay consumption enough even for low elasticities of intertemporal substitution. As can be seen, the long–run growth rate in such cases tends to be higher and the cycles shorter.

Table 2: Comparative Stationary Cycles

Parameters					$H\left(T_v^E\right)$	$k(T_v^E)$	g	Δ
σ	δ	ρ	α	γ				
.79	1.39	.02	.22	.13546	0.2044	7.668	2.2	3.92
.78					0.2141	7.577	2.185	1.69
.80					0.1963	7.743	2.213	5.87
	1.38				0.1995	7.778	2.191	5.06
	1.40				0.2099	7.553	2.207	2.65
		.0197			0.2154	7.565	2.183	1.46
		.0203			0.1949	7.74	2.213	6.05
			.213		0.1942	7.332	2.213	5.61
			.227		0.2151	8.008	2.185	2.08
				.13446	0.1977	7.843	2.177	5.29
				.13646	0.2115	7.491	2.221	2.47
.9	1.593				0.2008	6.574	2.541	5.29
1	1.861				0.2065	5.407	2.967	4.12

7.2 Qualitative Behavior of Key Variables

7.2.1 Investment and Consumption

Investment is strongly pro-cyclical here. Contemporaneous with the productivity boom, investment jumps discretely to its highest point in the cycle. It remains positive throughout the expansionary phase and then declines sharply as the economy enters recession. Note importantly that the driving force for investment here is the marginal product of capital in production: investment is zero in the recession when interest rates are zero, and jumps to its highest point at the start of the boom, when interest rates are at their highest level.[25] Consumption is also strongly procyclical, but evolves more smoothly than investment At the boom, consumption jumps discretely, and continues to increase throughout the expansionary phase. Consumption falls smoothly in the recession and continues to decline throughout the downturn until the next boom.

The model also generates pro-cyclical allocation of labor to production of consumables and investment goods, as has been reported (e.g. Christiano and Fisher 1995). Although the investment and consumption good sectors are not distinguished, per se, in the model, the allocation of labor to consumption good production can be inferred from equation (65). As long as $\sigma < 1$

[25] Output and sales' growth closely follow investment, whereas investment has almost no relationship, at business cycle frequencies, with the user cost of capital; see Hassett and Hubbard (1996).

consumption growth exceeds productivity growth so that the allocation of labor and capital to consumption must have risen at the boom. The reason labor in both consumption and investment good production can rise is because of the endogenous shutting down of entrepreneurship at the boom. This mechanism is similar to that generated by introducing "homework" in Benhabib, Rogerson and Wright (1991).

7.2.2 Productivity

Even though underlying productivity improvements arise in a decentralized and time-varying manner across sectors, the incentives for strategic delay identified here lead to simultaneous implementation at the boom. Consequently total factor productivity (TFP) rises uniformly there. Output jumps discretely at the boom also, in part, due to the reallocation of additional resources to production then. The effects of the productivity boom persist in further output increases, as capital is then accumulated through the expansion. Note that it is not optimal to accumulate capital before the boom because its short term returns there are zero. It is this delayed accumulation of capital in response to the productivity boom which leads to a prolonged expansion in output. This is consistent with the evidence that expansions tend to start with relatively aggressive growth, and are then followed by milder increases thereafter (see Dahl and Gonzalez–Rivera, 2003).

In the expansion, all labor is used in production and capital is fully utilized. In the contraction, labor is reallocated to entrepreneurial activity, capital utilization falls, and output declines.[26] If utilized capital and labor were correctly measured this would imply that measured productivity should remain constant through the recession. As already discussed, capacity utilization is well known to fall in recessions. However, even if capital utilization is correctly measured in US data, but labor allocated to innovation is not fully measured, then it will appear that labor is being hoarded (see Fay and Medoff 1985).[27] If this occurs, measured productivity would fall, which is consistent with the evidence (see for example Fernald and Basu 1999). Even if entrepreneurship is being correctly measured, but the fall in capital utilization is not, measured productivity falls.

[26] The fall in output does not result from failure to measure the productive contribution of entrepreneurs alone. Even if this were fully measured, which is unlikely to be the case, output would still fall since during the contraction. The reallocation of labor from production, where workers are paid less than their marginal value (due to profits), to entrepreneurship, which if measured correctly implies workers being paid marginal value, will lead to a decline in measured output.

[27] Entrepreneurship is, at best, likely to be only partially measured in the data, since much of it involves activities that will raise long-term firm profits but have little directly recorded output value contemporaneously.

Through the expansion, total factor productivity is constant, and labor productivity rises.

7.2.3 Returns to Factors

In US data, corporate profits increase mildly through upturns but show clear and marked falls in contractions. This is again consistent with the model. As productivity is unchanged through the cycle, equation (21) shows profit to be proportional to demand, so that the pattern of profits will be consistent with this pro–cyclical pattern. Wages rise rapidly in the boom due to the increase in labor productivity. The wage then continues to increase throughout the expansionary phase as capital is accumulated, but is constant through the contraction. This is because the marginal product of labor in production is constant. Competition does not put downward pressure on wages because labor in this phase enters into its alternative activity, entrepreneurship or reorganization. The wage then here is best interpreted as the skilled wage, since it is assumed that all labor is equally able to work in entrepreneurship. The introduction of unskilled labor, which has no role in these tasks, would see a similar increase in wages (and full employment) through the economy's upturn and then a decline (in either wages, and/or employment if labor market frictions are modeled) during the recession.

7.2.4 The Term Spread

In US data the spread between interest rates on a ten year treasury note and a three month treasury bill tends to be large in recessions (i.e. the long term interest rate exceeds the short term). In expansions smaller and seems to be a good predictor of recessions. In particular, relatively low values of the term spread, high short term interest rates relative to long, suggest a higher probability of recession. The cycle analyzed here exhibits a low value of the yield curve through the expansion, and a high value in the recession. The highest value of the yield curve is at the start of the recession. Towards the end of the recession it tracks down as the three month rate starts to include the increased discount over the boom. This implies, particular for short cycles in the model, a good fit with the data. Estrella and Mishkin (1996) argue that the yield curve is a superior predictor over other leading indicators at leads from 2 to 4 quarters. Similarly, at the start of the expansion the value of the yield curve is at its lowest point, thus again providing a leading indication of the imminent contraction to follow the expansionary phase.

7.2.5 Tobin's Q

The market value of a marginal unit of capital to its replacement cost is pro–cyclical in our cycle. It is equal to one during an expansion and zero during a contraction when capital is under–utilized. However, the aggregate behavior of Tobin's (average) Q, defined as the ratio of the value of firms to the book value of their capital stock, is somewhat more complicated. In our model Tobin's Q is given by

$$Q(t) = \frac{V^K(t) + \Pi(t)}{K(t)}, \tag{80}$$

where $\Pi(t)$ denotes the stock market value of the intangible capital tied up in firms, and recall that $V^K(t)$ is the market value of their physical capital.

During an expansion $V^K(t) = K(t)$ and, the value of intangible capital with the value of incumbent firms: $\Pi(t) = V^I(t)$. It follows that

$$Q(t) = 1 + \frac{V^I(t)}{K(t)} \qquad \forall t \in (T_{v-1}, T_v^E). \tag{81}$$

Since $V^I(t)$ declines and $K(t)$ grows during the expansion, $Q(t)$ must decline.

In the downturn, the value of the physical capital stock declines below the capital stock, so that

$$V^K(t) = \left[\bar{q}(1 - H_v)\int_t^{T_v} \lambda(\tau)d\tau + e^{-\beta(T_v)}\right] K(T_v^E) < K(T_v^E). \tag{82}$$

Also some sectors experience innovations, so there exist terminal firms who are certain to be made obsolete at the next round of innovation. At any point in time the measure of sectors in which no innovation has occurred is $P(t)$, therefore the total value of firms on the stockmarket is given by

$$\Pi(t) = (1 - P(t))[V^T(t) + V^D(t)] + P(t)V^I(t), \tag{83}$$

where $V^T(t)$ denotes the value of "terminal" firms who are certain to be made obsolete during the next wave of implementation. The value of these firms can be written as

$$V^T(t) = V^I(t) - \frac{P(T_v)}{P(t)} V^D(t). \tag{84}$$

Substituting into (83) yields

$$\hat{\Pi}(t) = V^I(t) + (1 - P(t))\left[1 - \frac{P(T_v)}{P(t)} V^D(t)\right]. \tag{85}$$

Through the downturn, the value of intangible capital initial falls and then rises again as the economy approaches the next boom.[28] Immediately prior to the boom $P(t) = P(T_v)$, so that again $\Pi(T_v) = V^I(T_v)$. The value of Q during the downturn is thus given by

$$Q(t) = \bar{q}(1 - H_v) \int_t^{T_v} \lambda(\tau) d\tau + e^{-\beta(T_v)} + \frac{\hat{\Pi}(t)}{K(T_v^E)} \qquad \forall\ t \in [T_v^E, T_v) \tag{86}$$

During the contraction, then, $Q(t)$ initially declines as $K(t)$ remains unchanged and the the decline in $V^k(t)$ dominates. However, eventually the growth in the value of intangible capital, $\hat{\Pi}(t)$, starts to dominate as we approach the boom, so that $Q(t)$ rises in anticipation. At the boom, since the book value of capital remains unchanged, but the market value of both physical and capital growth by a factor $e^{(1-\alpha)\Gamma_v}$, Tobin's Q rises rapidly.

The qualitative behavior of Tobin's Q in our model thus accords quite well with its aggregate counterpart in US data. As illustrated by Caballero (1999, Figure 2.1), Tobin's Q tends to reach a peak prior to the peak of expansions and then reaches a minimum midway through NBER–dated recessions. The most rapid periods of growth in Tobin's Q therefore start to occur before the end of recessions and continue through the subsequent boom just as they do in our stationary cycle.

8 Concluding Remarks

The basic mechanism underlying the model discussed in this paper captures a simple and compelling reason for cross-sectoral co–movement: entrepreneurs delay innovative activity (or reorganization) until demand conditions are slack, and delay implementing productivity improvements until the point at which demand conditions are favorable and the costs of acquiring the necessary capital are sufficiently low — this is when other entrepreneurs are doing the same. Since ensuing prices must also satisfy consumers' optimization and asset market clearing when there is the possibility of storage and arbitrage trading, this does not lead to the possibility of multiple self-fulfilling cycles. Moreover, these conditions do not rule out cycles because the re–allocation of resources inherent to the process of endogenous growth ensures that profits grow more than the discount rate across the boom.

The model generates movements in aggregates over the cycle which are qualitatively similar in many respects to those observed in US data. It should be reiterated that these results arise in a framework where both the economy's cyclical behavior and its growth path are fully endogenized.

[28] This cyclical anticipation of future profits implicit in aggregate stock prices accords well with the findings of Hall (2001).

Moreover, the framework we explore has remarkably few degrees of freedom; the model is fully specified by five exogenous parameters: two summarizing household preferences, two underlying the productivity of entrepreneurship, and one pinning down factor shares in production. We do not claim that the current framework is capable of providing a quantitative account of the business cycle. However, in future work we will build on this parsimonious structure to explore a number of key extensions:

• Aggregate uncertainty and stochastic cycle lengths — The length and other characteristics of actual business cycles, vary from cycle to cycle and look rather different from the deterministic stationary equilibrium cycle described here. Introducing some degree of aggregate uncertainty would help to address this. However, in order to develop such an extension we need to develop a deeper understanding of the local transitional dynamics of the model. It turns out that these dynamics are not as complex as one might expect at first blush. The reason is that the path back to the stationary cycle (at least locally) involves the accumulation of only one factor: either physical capital or intangible. Although a full analysis of these local dynamics is beyond the scope of the current paper, we believe it is feasible.

• Unemployment — A natural way to introduce unemployment into the model is to allow for unskilled labour which cannot be used in entrepreneurship and is not directly substitutable with skilled labor in production. With putty–clay production, the marginal value of this unskilled labor falls to zero during the downturn and some fraction of unskilled workers would become unemployed (just like physical capital). In a competitive labor market, this would drive unskilled wages down to their reservation level. However, in the presence of labor market imperfections, such as efficiency wages and search frictions, the dynamics of unemployment and wages interact with the process of creative destruction in a more complex manner. In further work we explore these dynamics more fully.

• Government policy — The framework developed here (as well as its extensions) provide a natural framework for thinking about counter–cyclical policy. First, the question arises as to whether removing or reducing cycles is a valid policy objective at all. In Francois and Lloyd–Ellis (2003) we showed that switching from the cyclical equilibrium to a corresponding acyclical one would raise long–run growth but lower welfare. Similar results are likely to carry over the stationary cycle in the current model. A second issue is that of how to implement a counter–cyclical policy. The recession here is Keynesian in that it is associated with deficient demand,

and the government could intervene, for example, by raising demand for goods and services and taxing savings. However, such a policy would effectively channel resources away from innovative activities and may dampen growth. On the other hand the anticipation of higher demand during a downturn might stimulate innovation, so the overall effect is unclear.

9 Appendix

Proof of Lemma 1: Differentiating (5) with respect to time yields

$$\dot{V}_i^K(t) = r(t) V_i^K(t) - q_i(t) K_i(t) + \dot{K}_i(t) = \dot{K}_i(t). \tag{87}$$

Since $V_i^k(t) = K_i(t)$, (18) follows.■

Proof of Lemma 2: Given factor prices $q(t)$ and $w(t)$, entrepreneurs choose the combination of capital and labor that minimizes the cost of producing $x_i(t)$:

$$K_i(t) = \frac{x_i(t)}{A_i^{1-\alpha}(t)} \left[\left(\frac{\alpha}{1-\alpha}\right)\frac{w(t)}{q(t)}\right]^{1-\alpha} \quad \text{and} \quad L_i(t) = \frac{x_i(t)}{A_i(t)^{1-\alpha}} \left[\left(\frac{1-\alpha}{\alpha}\right)\frac{q(t)}{w(t)}\right]^{\alpha} \tag{88}$$

The resulting unit cost is:

$$\frac{w(t)}{A_i(t)^{1-\alpha}} \left[\left(\frac{1-\alpha}{\alpha}\right)\frac{q(t)}{w(t)}\right]^{\alpha} + \frac{q(t)}{A_i^{1-\alpha}(t)} \left[\left(\frac{\alpha}{1-\alpha}\right)\frac{w(t)}{q(t)}\right]^{1-\alpha} = \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu A_i^{1-\alpha}(t)}. \tag{89}$$

Since the productivity of the most productive rival is $e^{-\gamma} A_i(t)$, the limit price is given by (20).■

Proof of Proposition 1: Using (19) and (20) to substitute for x_i and p_i into (88) yields $K_i = K$, and $L_i = L = 1$ for all i with q and w given by:

$$q(t) = \frac{\alpha e^{-(1-\alpha)\gamma} Y(t)}{K(t)} \tag{90}$$

$$w(t) = (1-\alpha) e^{-(1-\alpha)\gamma} Y(t). \tag{91}$$

Since $q(t) = r(t) > 0$, accumulating capital dominates storage, so that:

$$\dot{K}(t) = Y(t) - C(t), \tag{92}$$

Since all successes are implemented immediately, the aggregate rate of productivity growth is

$$g(t) = \delta\gamma H(t) \tag{93}$$

No–arbitrage implies that

$$r(t) + \delta H(t) = \frac{\pi(t)}{V^I(t)} + \frac{\dot{V}^I(t)}{V^I(t)} \tag{94}$$

Since, innovation occurs in every period, free entry into entrepreneurship implies that

$$\delta V^I(t) = w(t). \tag{95}$$

Along the balanced growth path, all aggregates grow at the rate g. From the Euler equation it follows that

$$r(t) = \rho + \sigma g. \tag{96}$$

Differenting (91) and (95) w.r.t. to time, using these to substitute for $\frac{\dot{V}^I(t)}{V^I(t)}$ in (94), and using (96) to substitute for $r(t)$ and (21) to sustitute for $\pi(t)$, we get

$$\rho + \sigma g + \frac{g}{\gamma} = \frac{\delta(1 - e^{-(1-\alpha)\gamma})}{(1-\alpha)e^{-(1-\alpha)\gamma}} + g. \tag{97}$$

Solving for g yields (24).■

Proof of Lemma 3 We show: (1) that if a signal of success from a potential entrepreneur is credible, other entrepreneurs stop innovation in that sector; (2) given (1) entrepreneurs have no incentive to falsely claim success.

Part (1): If entrepreneur i's signal of success is credible then all other entrepreneurs believe that i has a productivity advantage which is e^{γ} times better than the existing incumbent. If continuing to innovate in that sector, another entrepreneur will, with positive probability, also develop a productive advantage of e^{γ}. Such an innovation yields expected profit of 0, since, in developing their improvement, they do not observe the non-implemented improvements of others, so that both firms Bertrand compete with the same technology. Returns to attempting innovation in another sector where there has been no signal of success, or from simply working in production, $w(t) > 0$, are thus strictly higher.

Part (2): If success signals are credible, entrepreneurs know that upon success, further innovation in their sector will cease from Part (1) by their sending of a costless signal. They are thus indifferent between falsely signalling success when it has not arrived, and sending no signal. Thus, there exists a signalling equilibrium in which only successful entrepreneurs send a signal of success.■

Proof of Lemma 4: From the production function we have

$$\ln Y(t) = \int_0^1 \ln \frac{Y(t)}{p_i(t)} di \tag{98}$$

Substituting for $p_i(t)$ using (20) yields

$$0 = \int_0^1 \ln \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(T_{v-1})} di \tag{99}$$

which re-arranges to (36).■

Proof of Lemma 6: Suppose instead that there exists an intermediate phase in which neither capital is accumulated nor entrepreneurship occurs. Consider the first instant of that phase. Since in the instant prior to that capital was being accumulated, the marginal return to investment in physical capital must exceed ρ. Since the marginal product of capital cannot jump downwards discretely at full capital utilization, there are only two possibilities: either (1) $r(T_v^E) = \rho$ at the start of the intermediate phase or (2) $r(T_v^E) > \rho$ at the start of the intermediate phase. Situation (2) can be ruled out directly since, by assumption, in the intermediate phase there is no entrepreneurship, and so it must be the case that $r > \rho$ and investment will occur. Situation (1) occurs if the marginal return to capital converges continuously to $r = \rho$ along the neoclassical accumulation phase. But this corresponds exactly with the path of accumulation along the stable trajectory of the Ramsey model which does not converge in finite time — this would then imply an infinite length to the capital accumulation phase.■

Proof of Proposition 3: Given the constant wage w_v and the average rental rate on capital through the downturn, $\bar{q}$, the efficient contract will be constructed so as to solve the following cost–minimization problem through the downturn:

$$\min_{\tilde{K}_i, \tilde{L}_i} \; w_v \tilde{L}_i + \bar{q}\tilde{K}_i \quad \text{s.t.} \quad \tilde{x}_i \leq \tilde{K}_i^{\alpha}(A_i \tilde{L}_i)^{1-\alpha} \tag{100}$$

where $\tilde{L}_i = \int_{T_v^E}^{T_v} L_i(t)dt$, $\tilde{K}_i = \int_{T_v^E}^{T_v} K_i^u(t)dt$ and $\tilde{x}_i = \int_{T_v^E}^{T_v} x_i(t)dt$. This temporal aggregation is possible for two reasons: (1) the interest rate is zero through the downturn and intermediate prices are constant, so that the value of a unit of output is time–independent, and (2) as labour is withdrawn from production, the capital labour ratio is constant. Thus the entire downturn can be treated as if it were a single production period. The necessary condition from the problem is simple that

$$\frac{\tilde{K}_i}{\tilde{L}_i} = \frac{\alpha}{1-\alpha}\frac{w_v}{\bar{q}}. \tag{101}$$

At the peak of the expansion, the optimal capital labour ratio is

$$\kappa_i(T_v^E) = \frac{\alpha}{1-\alpha}\frac{w_v}{q(T_v^E)}. \tag{102}$$

Since the capital labour ratio through the downturn must equal that at the peak and the wage is constant through the downturn, it follows that the efficient contract must satisfy $\bar{q} = q(T_v^E)$.■

Proof of Proposition 4: From (35), long–run productivity growth is given by

$$\Gamma_v = (1 - P(T_v))\gamma \tag{103}$$

Integrating (45) over the downturn and substituting for $H(\cdot)$ using (45) yields

$$1 - P(T_v) = \delta \int_{T_v^E}^{T_v} \left(1 - (1 - H_v)e^{-\frac{\rho}{\sigma}[t - T_v^E]}\right) d\tau. \tag{104}$$

Substitution into (103) and integrating gives (56).■

Proof of Lemma 8: $p_i(t) = \frac{q(t)^\alpha w(t) 1-\alpha}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(t)}$ is the marginal cost of the previous incumbent given the sequence $w(t), q(t)$. Due to the unit elasticity of final producer demand, intermediate producing entrepreneurs wish to set price as high as possible. Thus, contracting a lower price at any instant is not optimal for the leader in i. Offering a $p_i^c(t) > p_i(t)$ in any instant would lead to a bid by the previous incumbent that would be both feasible and preferred by the final good producer. Thus $p_i^c(t)$ is the profit maximizing price and $x_i^c(t) = \frac{Y(t)}{p_i^c(t)}$for all $t \in [T_v, T_{v+1})$.■

Proof of Lemma 7: If $V_i^k(t) > K_i(t)$ it is feasible for the leading producer to write an alternative $\{q_i(t), K_i(\tau)\}$ with the builder of a new capital stock in sector i which would lead to new capital being constructed and which would be preferred by the producer. A preferred sequence for the leading producer would be one in which prices were no higher than the contracted sequence above, but which had a strictly lower price in at least one instant. This is feasible if $V_i^k(t) > K_i(t)$. Finally, no new capitalist would enter offering a sequence $V_i^k(t) < K_i(t)$, so that any equilibrium price sequence must at least satisfy (50).■

Proof of Proposition 6: Condition (67) can be expressed as

$$\begin{aligned}
\int_t^{T_v} \pi(\tau) d\tau &\geq \int_t^{T_v} \overline{q} K(T_v^E) \left(1 - (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau \\
(1 - e^{-(1-\alpha)\gamma}) Y(T_v^E) \int_t^{T_v} (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \overline{q} K(T_v^E) \int_t^{T_v} \left(1 - (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau
\end{aligned}$$

Since $\overline{q} K(T_v^E) = \alpha e^{-(1-\alpha)\gamma} Y(T_v^E)$, this can be expressed as

$$\begin{aligned}
(1 - e^{-(1-\alpha)\gamma}) \int_t^{T_v} (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \alpha e^{-(1-\alpha)\gamma} \int_t^{T_v} \left(1 - (1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau \\
(1 - (1 - \alpha)e^{-(1-\alpha)\gamma})(1 - H_v) \int_t^{T_v} e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \alpha e^{-(1-\alpha)\gamma} (T_v - t)
\end{aligned}$$

Since this holds with equality at $t = T_v$, a sufficient condition is that the left hand side declines more rapidly with t than the right hand side. That is

$$(1-(1-\alpha)e^{-(1-\alpha)\gamma})(1-H_v)e^{-\frac{\rho}{\sigma}(t-T_v^E)} > \alpha e^{-(1-\alpha)\gamma} \tag{105}$$

This will hold for all t if holds for $t = T_v$. Hence, condition $(E4)$ follows.■

Proof of Proposition 7: The proof proceeds by demonstrating that there does not exist another mutually improving price/quantity sequence that could be written at time T_v. Note first that in considering alternative contracts we can restrict attention to those that also satisfy Lemma 7. Since contracts not satisfying this will always be dominated by alternatives that do so for at least one of the parties. Since $q^c(t) = r(t)$ for $t \leq T_v^E$ and $K^c(t)$ satisfies (37), there is no potential for mutually beneficial changes in the terms of the contract through the upturn, as capital is efficiently utilized then. At the peak of boom $K^c\left(T_v^E\right) = K\left(T_v^E\right)$ and thereafter, through the downturn, the level of installed capital remains at $K\left(T_v^E\right)$, but since proportion $1-(1-H_0(T_v^E))e^{-\frac{\rho}{\sigma}(\tau-T_v^E)}$ remains idle every instant, utilization is not efficient, which indicates a possibly improving contract. We show that no such contract exists.

If the entrepreneur does not store intermediate, there is no demand for the idle capital because $K^c(t)$ meets production needs. Even free use of excess capital to produce and sell more intermediate output at any instant τ during the downturn would be rejected due to the unit elasticity of demand. That is, since total revenue remains fixed at $p_i(t)x_i^d(t) = Y(t)$, producing more simply incurs extra labor costs with unchanged revenues.

Now consider alternative sequences under which the entrepreneur may store some output, and which increase capital utilization. Entrepreneurs will never choose to store output beyond the boom when a new incumbent has arrived. If competing with a new incumbent, pricing at the new incumbent's marginal cost requires - $p_i(t) = \frac{r(T_v)^\alpha w(T_v)1-\alpha}{\mu e^{-(1-\alpha)\gamma}A_i^{1-\alpha}(t+1)}$ which is less than the price if selling before the boom - $p_i(T_v) = \frac{r\left(T_{v+1}^E\right)^\alpha w\left(T_{v+1}^E\right)1-\alpha}{\mu e^{-(1-\alpha)\gamma}A_i^{1-\alpha}(t-1)}$ since $r\left(T_{v+1}^E\right)^\alpha w\left(T_{v+1}^E\right)e^\gamma = r\left(T_v\right)^\alpha w\left(T_v\right)$, but $A_i^{1-\alpha}(t+1) = e^{2\gamma}A_i^{1-\alpha}(t-1)$. Thus, terminal entrepreneurs strictly prefer to sell stored output before the boom. Proposition 5 ensures the same holds for an entrepreneur who expects to remain the incumbent at the boom.

Capitalists may be willing to modify the contract to allow storage if they receive a positive rent on the proportion $1-(1-H_0(T_v^E))e^{-\frac{\rho}{\sigma}(\tau-T_v^E)}$ which lies idle in the equilibrium contract. We show now that capitalists would only accept a rent of $\overline{q}$, on the extra units. To see why, suppose that the capitalist were to accept a modified contract which maintained the price $\overline{q}$ for units $K^c(t)$ but allowed a lower price $\widetilde{q}$ for utilization of the idle units. Given such a contract the entrepreneur would optimally choose to meet contracted production at each t by utilizing $K^c(t)$ and utilize the remaining $\widetilde{K} = 1 - (1 - H_0(T_v^E))e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}$ to produce and store output. The entrepreneur will then optimally choose to shut down at time τ given by

$$\int_{\tau}^{T_{v+1}} K^c(t)^{\alpha} A_i(t) L_i^*(t)^{1-\alpha} = \int_{T_v^E}^{\tau} \widetilde{K}^{\alpha} A_i(t) L_i^*(t)^{1-\alpha}, \tag{106}$$

where $L_i^*(t)$ is the optimal amount of labor to use at time t. Recall that shutting down production is always an available option for the entrepreneur. The RHS of (106) is the stored output produced using the extra capital up to τ and the LHS is the demand for output under the contract with the final goods producer from τ on. By utilizing the extra units of capital at $\widetilde{q} < \overline{q}$ the entrepreneur simply replaces future demand for capital at price $\overline{q}$ with stored output produced using capital priced at $\widetilde{q}$ and thus lowers costs. Note also that a similar path of storage and shut down would be chosen for a non-stationary price on the extra units, $\widetilde{q}(t)$, provided that, for some instants at least, $\widetilde{q}(t) < \overline{q}$.

Now consider returns to the capitalist from such a deviation. There are two cases to consider depending on whether there has been an entrepreneurial success in sector i or not. If there has not been a success up to time $t > \tau$, which occurs with probability $P_i(t)$, then demand for capital at $t = 0$. As already established, the entrepreneur shuts down at τ in (106) under this deviation, so that his demand is zero. Moreover the previous incumbent cannot sell output to the final goods sector since the current incumbent has contracted sale to the final goods sector, under (25) (26). Thus the previous incumbent (and all earlier) have zero demand for capital for $t < T_v$. However, with probability $(1 - P_i(t))$ there has been a success in sector i at time t. In that case a single entrepreneur has some use for the capital before the boom. But note that it is not essential for him to access capital then, since he does not have an intermediate delivery contract yet, and along the equilibrium path will wait for implementation until the boom. Since such an individual is the only producer with demand for the capital, she is able to drive the capitalist down to marginal cost, which implies a price $q(t) \to 0$ over the interval $t \in [\tau, T_{v+1}]$, where τ solves (106), under this deviation. Since this yields strictly lower returns for the capitalist than (27) (28) a deviation allowing intermediate storage will not be accepted by the capitalist.

It thus follows that any alternative contract which allows utilization of capital in the downturn above $K^c(t)$ cannot be mutually improving. Since $K^c(t)$ is the maximal contracted level of capital possible, and any alternative rental rate necessarily makes one party worse off, the contract $(q^c(t), K^c(t))$ is not dominated by an alternative contract that could be written at T_v.

There is finally the possibility of remaining uncontracted, which would be preferred by the capitalist. But, since this would allow for a strictly increasing unit price for capital through the downturn, in accordance with (49), this would be rejected by the entrepreneur. If the sector's capital owner refused to contract, the entrepreneur would contract capital to be built with another producer at terms $(q^c(t), K^c(t))$. ■

Definition of v_0^I :

$$v_0^I = V_0^I(T_{v-1})/\bar{A}_{v-1} \tag{107}$$

$$= \int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} \frac{\pi(\tau)}{\bar{A}_{v-1}} d\tau + e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds} \int_{T_v^E}^{T_v} \frac{\pi(\tau)}{\bar{A}_{v-1}} d\tau + P(T_v) \frac{V_0^I(T_v)}{\bar{A}_v}. \tag{108}$$

$$= (1 - e^{-(1-\alpha)\gamma}) \left[\int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} y(\tau) d\tau + e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds} \int_{T_v^E}^{T_v} y(\tau) d\tau \right] + \left(1 - \frac{\Gamma_v}{\gamma}\right) \frac{w_0(T_v)}{\delta \bar{A}_v} \tag{109}$$

$$= (1 - e^{-(1-\alpha)\gamma}) e^{-\alpha \Gamma_{v-1}} k_{v-1}^{\alpha} \int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} \left(\frac{k(\tau)}{k_0(T_{v-1})} \right)^{\alpha} d\tau$$
$$+ e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds} \left\{ (1 - e^{-(1-\alpha)\gamma}) c_v \delta \left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma} \right) \right.$$
$$\left. + \left[1 - \delta \Delta_v^E + \delta \frac{c_v}{k_v^{\alpha}} \left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma} \right) \right] (1-\alpha) e^{-(1-\alpha)\gamma} k_v^{\alpha} \right\}. \tag{110}$$

10 References

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