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Abstract

In this paper, we analyze a lobby game, modelled as an all-pay auction in which interest groups submit bids in order to obtain a political prize. The bids are restricted to be below a cap imposed by the government. For both an incomplete and a complete information setting we show the following results. While ex post a lower cap may lead to higher lobbying expenditures, ex ante a lower cap always implies lower expected total lobbying expenditures. The incompletely informed government maximizes social welfare by implementing a cap equal to zero.

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1 Introduction

Lobbying has become an established practice in modern democracies. Its role in society is an intriguing phenomenon, and it has received a lot of attention from game theorists. Tullock (1980) views lobbying as an all-pay auction, in which interest groups submit "bids" in order to win a political prize. The literature that follows Tullock focuses mainly on the social costs of lobbying, which are associated with the fact that the money spent on lobbying cannot be used for other economic activities. Therefore, this branch of the literature devotes much attention to the calculation of total lobbying expenditures by the interest groups (Baye et al., 1993, 1996; Amann and Leiniger, 1995, 1996; Krishna and Morgan, 1997). Another stream of work focuses on the social benefits of lobbying, which arise when interest groups have the opportunity to separate themselves choosing bids that are contingent on policy relevant private information. This stream of work views lobbying as a signaling game, in which interest groups submit informative signals to the government (Potters and Van Winden, 1992; Lohmann, 1993; Lagerlöf, 1997).

In this study, we combine the two views of the literature on lobbying by making a trade-off between social costs and social benefits of lobbying. We do so, taking Tullock's all-pay auction model, and investigating the effect of a cap on the amount of money interest groups are allowed to spend on lobbying. We assume that the cap is chosen by the government with the target of maximizing social welfare. In deciding the optimal cap, the government needs to make a trade-off between the informational benefits lobbying provides, and the social costs. The trade-off turns out to be non-trivial, as both total lobbying expenditures and informational benefits are higher with a higher cap.

We will focus on the following two questions. "What effect would a cap on lobbying expenditures have on their total?" and "Should there be legislation to introduce such a cap in order to increase social welfare?" While the latter question is not answered yet by the economic literature, the former one is addressed in Che and Gale (1998). Their findings challenge the intuitively appealing expectation that a cap on lobbying expenditures decreases their total. They show that a cap "may have the perverse effect of increasing aggregate expenditures and lowering total surplus".

Before answering these questions, we need to emphasize the importance of distinction between the *ex ante* and *ex post* effect of a cap. The distinction is important as it allows us to model the legislative role of the government. New legislation, once introduced, regulates all lobbying activities for a long period of time. As a result, when taking a legislative initiative, the government cannot predict the exact effect of a proposed cap. It is therefore appropriate to model the government's decision on a cap as an *ex ante* choice, i.e., a decision made before the government learns the realizations of the interest groups' values. In contrast, the "perverse effect" described by Che and Gale holds *ex post*, i.e., after the interest groups' values are realized.

Depending on the situation, the interest groups may be or may not be better informed than the government about the characteristics of other interest groups. In this paper, we will investigate the effect of a cap in two different settings. In an incomplete information setting, we assume that each interest group is privately informed about its own value for the prize. The government and the other interest groups only know the distribution function this value is drawn from. In a complete information setting, we assume, following Che and Gale, that the interest groups commonly know each others' values. However, the government is only aware of the value distribution function.

Our contribution is threefold. First, in the case of incompletely informed interest groups, we derive equilibrium bidding in the case that the interest groups are confronted with a cap. Second, we show that the ex ante expected lobbying expenditures decrease by imposing a cap. Thus, legislators need not be overly concerned about the "perverse effect" of a cap, in contrast to what the result of Che and Gale suggests. Third, we point out that the government should optimally ban lobbying by imposing a prohibitive cap. Although a high cap generates information benefits by allowing the government to choose the socially optimal action more often, we show that these benefits do not outweigh the expected social costs.

Two other papers are closely related to ours. McAfee and McMillan (1992) show that weak cartels optimally let all cartel members submit zero bids in the first-price sealed-bid auction. The proof of this result follows the same logic as the proof of the optimality of a prohibitive cap in the incomplete information setting. Also, for the incomplete information setting, Gavious et al. (2001) simultaneously and independently develop alternative proofs for the results on equilibrium bidding and ex ante total lobbying expenditures in the all-pay auction with caps.

We proceed as follows. In Section 2, we outline the structure of our model. In Section 3, we derive the results about the effect of a cap in the incomplete information setting. In Section 4, we show that these results hold in the complete information setting. In Section 5 we conclude with some critical remarks on the results, and with an indication for some directions for further research.

2 The model

Consider the following lobby game. There is a government, G, which owns a political prize,[1] and n interest groups, numbered $1, ..., n$. Let

$$N \equiv \{1, ..., n\}$$

denote the set of all interest groups. We will use i and k to represent typical interest groups in N. Interest groups participate in the all-pay auction, in which they submit **bids**[2] in order to obtain the prize. We will let b_i denote the bid submitted by interest group i. G restricts b_i to be contained in the interval $[0, c]$, where c denotes a **cap**. The interest group that submits the highest bid wins, but each interest group has to pay its bid. In case of ties, the winner is chosen among the interest groups with the highest bid with equal probabilities.

Each interest group i learns its private value v_i of the prize. The v_i's are drawn, independently from each other, from a distribution function F. F

[1] The prize could for instance be a license to operate in a certain market, a building contract, or the right to organize an important event.

[2] We use the terminology from the literature on all-pay auctions and refer to the amount paid by an interest groups as its bid. Direct bribes, writing research reports, or hiring lobbyists are instances of bids. We use the term **total lobbying expeditures** for the sum of all bids.

has support on the interval $[0, 1]$, and has a continuous density function f with $f(v_i) > 0$, for every $v_i \in [0, 1]$. We consider two information structures. In the incomplete information setting, each interest group only knows its own value, and not the values of the other interest groups. In the complete information setting, the values of all interest groups are commonly known among the interest groups. In both settings, G is incompletely informed, and only knows F.

We assume that interest groups are risk neutral expected utility maximizers. Let $u_i(k, v_i, b_i)$ be the utility of interest group i when its value is v_i, its bid is b_i and interest group k wins the prize. Then, interest group i's utility is given by

$$u_i(k, v_i, b_i) \equiv \begin{cases} v_i - b_i & \text{if } k = i \\ -b_i & \text{otherwise.} \end{cases} \tag{1}$$

G chooses c that maximizes ex ante social welfare among the interest groups. Let $SW(k, v_1, ..., v_n, b_1, ..., b_n)$ denote ex post social welfare given that interest group k wins, given the values $v_1, ..., v_n$, and the bids $b_1, ..., b_n$. Ex post social welfare is defined as the sum of interest groups' utilities, so that

$$SW(k, v_1, ..., v_n, b_1, ..., b_n) \equiv \sum_{i=1}^{n} u_i(k, v_i, b_i) = v_k - \sum_{i=1}^{n} b_i. \tag{2}$$

Ex ante social welfare is the expectation of ex post social welfare over the values and the played strategies. We assume that interest groups play a Bayesian Nash equilibrium.

3 Incomplete information

Consider the incomplete information setting. Before we establish our main results, we derive two useful lemmas and a corollary. Define the differentiable functions $C : [0,1] \to \Re$ and $D : [0,1] \to \Re$ with

$$C(y) \equiv \int_0^y [zf(z) + F(z) - 1]F(z)^{n-1}dz + \frac{y}{n} * [1 - F(y)^n]$$

and

$$D(y) \equiv \frac{y}{n} * \frac{1 - F(y)^n}{1 - F(y)} - \int_0^y F(z)^{n-1}dz$$

for all $y \in [0,1]$.

Lemma 1 C is strictly increasing.

Proof. See the Appendix. ■

Lemma 2 D is strictly increasing.

Proof. See the Appendix. ■

Corollary 3 If $c \leq 1 - \int_0^1 F(z)^{n-1}dz$, then there is a unique ξ for which $D(\xi) = c$.

Proof. See the Appendix. ■

Let $v^*(c)$ be the unique solution to $D(v^*(c)) = c$ if $c \leq 1 - \int_0^1 F(z)^{n-1}dz$, and let $v^*(c) = 1$ otherwise. Proposition 4 shows that in equilibrium, the strategy of interest groups with a value below the threshold value $v^*(c)$ is not affected by the cap. Interest groups with a value above $v^*(c)$ submit a bid equal to c. This equilibrium is derived using an indirect approach based on the Revenue Equivalence Theorem (Myerson, 1981), which states that an interest group's interim utility (i.e., its utility as a function of its private value) is entirely determined by the function that assigns a probability that the interest group wins the prize given each possible realization of its value (provided that the utility of an interest groups is zero when is has the lowest possible value). As the bid function (3) determines this probability function, the interim utility for each interest group is fixed. In order to prove that (3) is an equilibrium, we show that the interim utility of each interest group is compatible with (3).

Proposition 4 Consider the lobby game with incomplete information. Let

$$B(v_i, c) = \begin{cases} \int_0^{v_i} [F(v_i)^{n-1} - F(z)^{n-1}] dz & \text{if } v_i \in [0, v^*(c)] \\ c & \text{if } v_i \in (v^*(c), 1], \end{cases} \tag{3}$$

where $v^*(c)$ follows uniquely from $D(v^*(c)) = c$ if $c \leq 1 - \int_0^1 F(z)^{n-1} dz$, and $v^*(c) = 1$ otherwise. Then B constitutes a symmetric Nash equilibrium of the lobby game.[3]

Proof. By Corollary 3, $v^*(c)$ is uniquely determined if $c \leq 1 - \int_0^1 F(z)^{n-1} dz$. Myerson (1981) shows that in equilibrium, the interim utility $\pi_i(v_i)$ of interest group i when having value v_i is given by

$$\pi_i(v_i) = \pi_i(0) + \int_0^{v_i} Q_i(w_i) dw_i, \text{ for all } v_i \in [0, 1] \text{ and } i \in N, \tag{4}$$

[3] For an alternative proof, derived simultaneously and independently, see Gavious et al. (2001). The result can also be derived indirectly from Laffont and Robert (1996).

where $Q_i(w_i)$ is the conditional probability that interest group i wins the prize, given that it has value w_i.

The proposed bid function implies that

$$Q_i(p, w_i) = F(w_i)^{n-1} \text{ if } w_i \in [0, v^*(c)], \tag{5}$$

as $B(w_i, c)$ is strictly increasing in w_i for all $w_i \in [0, v^*(c)]$, and

$$Q_i(w_i) = \bar{Q} = \frac{1 - F(v^*(c))^n}{n(1 - F(v^*(c)))} \text{ if } w_i \in (v^*(c), 1]. \tag{6}$$

The last expression follows from the ex ante probability (i.e., before the interest groups know their value) that a given interest group wins, which is given by

$$\frac{1}{n} = (1 - F(v^*(c)))\bar{Q} + \int_0^{v^*(c)} F(v_i)^{n-1} dF(v_i) = (1 - F(v^*(c)))\bar{Q} + \frac{1}{n} F(v^*(c))^n.$$

It remains to be checked if B is compatible with (4). As $\pi_i(0) = 0$, with (5) and (6), (4) can be rewritten as

$$\pi_i(v_i) = \int_0^{v_i} F(w_i)^{n-1} dw_i, \text{ if } v_i \in [0, v^*(c)], \text{ and} \tag{7}$$

$$\pi_i(v_i) = \int_0^{v^*(c)} F(w_i)^{n-1} dw_i + \int_{v^*(c)}^{v_i} \bar{Q} dw_i, \text{ if } v_i \in (v^*(c), 1] \tag{8}$$

for all $i \in N$. Moreover, the expected utility of interest group i can be expressed as follows

$$\pi_i(v_i) = F(v_i)^{n-1} v_i - b(v_i, c) \text{ if } v_i \in [0, v^*(c)], \text{ and} \tag{9}$$

$$\pi_i(v_i) = \bar{Q} v_i - b(v_i, c) \text{ if } v_i \in (v^*(c), 1], \tag{10}$$

where $b(v_i, c)$ is the bid made by an interest group with value v_i when the cap equals c. It is readily verified that the proposed bid function B is a solution to (7)-(10). Therefore, B constitutes a Bayesian Nash equilibrium. ■

Proposition 4 implies that imposing a lower cap can ex post lead to higher lobbying expenditures. This can be seen as follows. It is readily verified that the equilibrium bid function makes a jump upwards at threshold value $v^*(c)$. Now, take $v_1, ..., v_n$ and c such that $v_2, ..., v_n < v^*(c)$, and $v_1 = v^*(c)$. As v^* is the inverse function of D, by Lemma 2, v^* is strictly increasing in c. Therefore, when c is marginally decreased, interest group 1 will change its bid to c, which is higher than its original bid, whereas the bids of the other bidders remain unchanged, so that total lobbying expenditures increase.

Thus, there are two opposing effects of a decrease in c. On the one hand, it lowers the bids of interest groups with high values, which on the other hand induces interest groups with intermediate values to increase their bid to c so as to pool with the high types and to increase their probability of winning the prize. Depending on the specific values, the second effect sometimes dominates the first.

Proposition 5 shows that the "ex post" result does not hold "ex ante". Let ex ante expected total lobbying expenditures be the expected sum of interest groups' equilibrium bids, where the expectation is taken over the values of the interest groups. The proof follows by calculating the sum of the equilibrium bids given by Proposition 4 as a function of c, and by showing that the resulting function is strictly increasing in c.

Proposition 5 Consider the lobby game with incomplete information. Suppose that c is strictly decreased. Then ex ante expected total lobbying expenditures are strictly decreased as well.

Proof. Let $\tilde{L}^a(c)$ denote the expected ex ante total lobbying expenditures as a function of c. Then

$$\begin{aligned}
\frac{1}{n}\tilde{L}^a(c) &= \frac{1}{n}\sum_{i=1}^{n}\int_0^1 B(v_i,c)f(v_i)dv_i \\
&= \int_0^{v^*(c)}\left[zF(z)^{n-1}-\int_0^z F(y)^{n-1}dy\right]f(z)dz+[1-F(v^*(c))]c \\
&= \int_0^{v^*(c)} zF(z)^{n-1}f(z)dz - F(v^*(c))\int_0^{v^*(c)} F(z)^{n-1}dz+\int_0^{v^*(c)} F(z)^n dz \\
&\quad +\frac{v^*(c)}{n}*[1-F(v^*(c))^n]-[1-F(v^*(c))]\int_0^{v^*(c)} F(z)^{n-1}dz \\
&= \int_0^{v^*(c)} [zf(z)+F(z)-1]F(z)^{n-1}dz+\frac{v^*(c)}{n}*[1-F(v^*(c))^n] \\
&= C(v^*(c)).
\end{aligned}$$

Now, as v^* is the inverse function of D, by Lemma 2, v^* is strictly increasing in c. Then, by Lemma 1, $\tilde{L}^a(c)$ is strictly increasing in c. ■

Proposition 5 implies that if total lobbying expenditures were the only part of social welfare, then a lower cap would always be preferred to a higher one. However, social welfare as defined in (2) is also an increasing function of the winner's private value. As $v^*(c)$ is strictly increasing in c, a lower cap leads to more bidders pooling at the cap, so that the probability that the winner is the interest group with the highest value decreases. Therefore, a lower cap implies less informational benefits.

The trade-off between social costs and social benefits is non-trivial. In order to make the trade-off, we make the simplifying assumption that $\frac{1-F}{f}$ is a strictly decreasing function, which is the case for several standard distributions such as the uniform distribution. Suppose that G is not restricted in letting the interest groups play the all-pay auction, but that it has a much broader class of **feasible mechanisms** to choose from.

We start by defining a **mechanism**. In a mechanism, interest groups are asked to simultaneously and independently choose an action. Interest group i chooses an action $a_i \in A_i$, where A_i is the set of actions for interest group i. The mechanism has the following outcome functions

$$\widehat{p} : A_1 \times ... \times A_n \rightarrow \Re^n,$$

and

$$\widehat{x} : A_1 \times ... \times A_n \rightarrow \Re^n.$$

If $\mathsf{a} = (a_1, ..., a_n)$, then $\widehat{p}_i(\mathsf{a})$ is interpreted as the probability that interest group i gets the prize and $\widehat{x}_i(\mathsf{a})$ is the expected expenditures for interest group i. Interest group i's utility when a is played is, consistently with (1), given by

$$\hat{U}_i(\mathsf{a}) = v_i \widehat{p}_i(\mathsf{a}) - \widehat{x}_i(\mathsf{a}).$$

Let a strategy be a function $\widehat{b}_i : [0,1] \rightarrow A_i$ such that $\widehat{b}_i(v_i)$ is the action interest group i plays when it has value v_i. A feasible mechanism is a mechanism including strategies, which have the following properties: (1) each interest group expects nonnegative utility, and (2) the strategies form a

Bayesian Nash equilibrium of the mechanism. A socially optimal auction is a feasible mechanism that maximizes ex ante social welfare.

By the Revelation Principle (Myerson, 1981), we may assume, without loss of generality, that G only considers feasible direct revelation mechanisms, which is are feasible mechanisms in which each interest group is asked to announce its value, in which it has an incentive to participate (individual rationality) and in which it has an incentive to announce its value honestly (incentive compatibility). Let (p, x) be a feasible direct revelation mechanism, with

$$p : V \to [0, 1]^n$$

having

$$\sum_j p_j(v) \leq 1,$$

and

$$x : V \to \Re^n.$$

We interpret $p_i(v)$ as the probability that interest group i wins, and $x_i(v)$ as the expected payment by i when $v \equiv (v_1, ..., v_n)$ is announced.

Let

$$Q_i(p, v_i) \equiv E_{v_{-i}}\{p_i(v)\}$$

be the conditional probability that interest group i wins given its value v_i, and

$$U_i(p, x, v_i) \equiv v_i Q_i(p, v_i) - E_{v_{-i}}\{x_i(v)\}$$

be interest group i's interim utility from (p, x), with $v_{-i} \equiv (v_1, ..., v_{i-1}, v_{i+1}, ..., v_n)$. Myerson (1981) shows that individual rationality and incentive compatibility are equivalent to

$$
\begin{aligned}
\text{if } w_i &\leq v_i \text{ then } Q_i(p, w_i) \leq Q_i(p, v_i), \forall w_i, v_i, i, && (11)\\
U_i(p, x, v_i) &= U_i(p, x, 0) + \int_0^{v_i} Q_i(p, y_i) dy_i, \forall v_i, i, \text{ and} && (12)\\
U_i(p, x, 0) &\geq 0, \forall i. && (13)
\end{aligned}
$$

Ex ante social welfare from (p, x) is given by

$$
\tilde{S}(p, x) = \sum_{i=1}^{n} \int_0^1 U_i(p, x, v_i) f(v_i) dv_i.
$$

Then,

$$
\begin{aligned}
\tilde{S}(p, x) &= \sum_{i=1}^{n} \int_0^1 \left(U_i(p, x, 0) + \int_0^{v_i} Q_i(p, y_i) dy_i \right) f(v_i) dv_i \\
&= \sum_{i=1}^{n} U_i(p, x, 0) + \int_0^1 \frac{(1 - F(v_i))}{f(v_i)} Q_i(p, v_i) f(v_i) dv_i \\
&\leq \sum_{i=1}^{n} U_i(p, x, 0) + \int_0^1 (1 - F(v_i)) dv_i * \int_0^1 Q_i(p, v_i) f(v_i) dv_i \\
&= \sum_{i=1}^{n} U_i(p, x, 0) + E\{v_i\} \int_0^1 Q_i(p, v_i) f(v_i) dv_i \\
&= E\{v_i\}. && (14)
\end{aligned}
$$

The first equality follows from (12), and the second by integration by parts. The first inequality follows from a theorem from Statistics which tells that the expectation of a product is less or equal than the product of the expectations in case the first term of the product is strictly decreasing, and the second term is increasing (McAfee and McMillan, 1992). Here, $\frac{1-F}{f}$ is strictly decreasing (by assumption), and Q_i is increasing v_i (by (11)). The other manipulations are straightforward.

Consider a feasible direct revelation mechanism $(\tilde{p}, \tilde{x})$ with

$$\begin{aligned} \tilde{p}_i(v) &= \frac{1}{n}, \text{ and} \\ \tilde{x}_i(v) &= 0, \end{aligned}$$

for all i. Basically, $(\tilde{p}, \tilde{x})$ is a lottery in which each interest group has the same probability of winning. The expected social welfare among the interest groups is then expected value generated by the lottery, so that

$$\tilde{S}(\tilde{p}, \tilde{x}) = E\{v_i\}.$$

With (14) it follows that $(\tilde{p}, \tilde{x})$ is a socially optimal mechanism, as

$$\tilde{S}(\tilde{p}, \tilde{x}) \geq \tilde{S}(p, x)$$

for all feasible direct revelation mechanisms (p, x). $(\tilde{p}, \tilde{x})$ is straightforwardly implemented with $c = 0$.

Proposition 6 Consider the lobby game with incomplete information. If $\frac{1-F}{f}$ is strictly decreasing, then $c = 0$ maximizes ex ante social welfare.

An intuition behind Proposition 6 is the following. Observe in the second line of the chain (14) that player i, if winning the object, adds $\frac{1-F(v_i)}{f(v_i)}$ to social welfare. As, by assumption, $\frac{1-F}{f}$ is a strictly decreasing function, G prefers a low type of interest group i to win more often than a high type. However, (11) requires the probability for interest group i to win the object to be (weakly) increasing in v_i. Hence, the best G can do is make the probability that a low type wins equal to the probability that a high type wins. G can do this optimally by implementing a cap equal to zero.

4 Complete information

Consider the complete information setting with two interest groups. For completeness, we first report the finding by Che and Gale (1998) showing that ex post lobbying expenditure may increase as a result of a decrease in c. Let $v_h \equiv \max\{v_1, v_2\}$ and $v_l \equiv \min\{v_1, v_2\}$ and let $L^p(c, v_h, v_l)$ be the ex post expected total lobbying expenditures by both interest groups, given the cap c, v_h and v_l. We speak of *expected* total lobbying expenditures as in equilibrium, interest groups play mixed strategies (Che and Gale, 1998).

Proposition 7 *Consider the lobby game with complete information. Let $n = 2$. Then generically, there is a unique Nash equilibrium,[4] in which $L^p(c, v_h, v_l)$ is given by*

$$L^p(c, v_h, v_l) = \begin{cases} v_l(v_h + v_l)/2v_h & \text{if } c > v_l/2 \\ 2c & \text{if } c < v_l/2. \end{cases} \tag{15}$$

If $c \in \left(\frac{v_l(v_h+v_l)}{4v_l}, \frac{v_l}{2}\right)$, then $L^p(c, v_h, v_l) > L^p(\infty, v_h, v_l)$

Proof. See Che and Gale (1998). ■

Note that for a non-zero mass of realizations of c, v_h, and v_l, $L^p(c, v_h, v_l) > L^p(\infty, v_h, v_l)$, which implies that there is a substantial set of cases in which

[4] For the zero mass event $c = v_l/2$, there a continuum of equilibria, which results in total lobbying expenditures in the interval $[v_l(v_h + v_l)/2v_h, 2c]$. See Che and Gale (1998).

a decrease in c results in an increase in total lobbying expenditures. The intuition behind this result is that a decrease in the cap limits the interest group with the highest value, so that the interest group with the lowest value is willing to bid more aggressively, which in certain cases leads to an increase in total lobbying expenditures.

Assume that each interest group draws its value from a uniform distribution on the interval $[0,1]$. We calculate ex ante expected total lobbying expenditures taking the expectation of (15) with respect to v_l and v_h. Proposition 8 shows that ex ante total lobbying expenditures are always increasing in the cap.

Proposition 8 Consider the lobby game with complete information. If $n = 2$ and $v_i \sim U[0,1]$, then the ex ante expected total lobbying expenditures are strictly increasing in c for all $c \in [0, \frac{1}{2}]$.

Proof. Let $L^a(c)$ denote the ex ante expected total lobbying expenditures as a function of c. Then,

$$\begin{aligned} L^a(c) &\equiv 2\int_0^1 \int_{v_l}^1 L^p(c, v_h, v_l) dv_h dv_l \\ &= 2\int_0^{2c} \int_{v_l}^1 \frac{v_l(v_h + v_l)}{2v_h} dv_h dv_l + 2c(1-2c)^2. \end{aligned}$$

The expression is multiplied by 2 as the role of the interest group with the higher and the lower value is interchanged with probability $\frac{1}{2}$. Taking the first derivative of L^{a} w.r.t. c yields

$$\begin{aligned}\frac{\partial L^{a}(c)}{\partial c} &= 4\int_{2c}^{1}\frac{2c(v_{h}+2c)}{2v_{h}}dv_{h}+2-16c+24c^{2} \\ &= 2-12c+16c^{2}-8c^{2}\log(2c).\end{aligned}$$

As $\log(z)<z-1$ for all $z\in(0,1)$, it holds for all $c\in(0,\frac{1}{2})$ that

$$\frac{\partial L^{a}(c)}{\partial c}>2-12c+16c^{2}-8c^{2}(2c-1)=2(1-2c)^{3}>0. \tag{16}$$

Therefore, as $L^{a}(c)$ is a continuous function of c, $L^{a}(c)$ is strictly increasing in c. ■

Proposition 9 shows that the $c=0$ result of the incomplete information setting has its parallel in the complete information setting. This result follows from Che and Gale (1998), who show that for $c>\frac{1}{2}v_{l}$, expected utility for the bidder with the highest value is $v_{h}-v_{l}$, and expected utility for the bidder with the lowest value equals 0. Hence, in this case, social welfare equals $v_{h}-v_{l}$. If $c<\frac{1}{2}v_{l}$, both bidders bid c, so that ex post social welfare is given by $\frac{1}{2}(v_{l}+v_{h})-2c$. Taking the expectation of ex post social welfare with respect to v_{h} and v_{l}, ex ante social welfare is determined. Straightforward calculations reveal that ex ante social welfare is maximized at $c=0$.

Proposition 9 Consider the lobby game with complete information. If $n=2$ and $v_{i}\sim U[0,1]$, then $c=0$ maximizes ex ante social welfare.

Proof. If $v_{l}>2c$, both interest groups bid c, so that ex post social welfare is given by $\frac{v_{1}+v_{2}}{2}-2c$, and if $v_{l}<2c$, expected utility for the interest groups is $v_{h}-v_{l}$ and 0 respectively for the high and the low value interest group (Che and Gale, 1998). Let $S(c)$ denote ex ante social welfare as a function of the imposed cap c. Then

$$S(c) = \int_{2c}^{1}\int_{2c}^{1}\left[\frac{v_1+v_2}{2}-2c\right]dv_1dv_2 + 2\int_{0}^{2c}\int_{v_2}^{1}(v_1-v_2)dv_1dv_2.$$

The first term of the RHS refers to the case that $v_l > 2c$. The second term of the RHS applies to $v_l < 2c$. Calculating the integrals we find

$$S(c) = \frac{1}{2} - c + 2c^2 - \frac{4}{3}c^3.$$

The first order derivative of S is then given by

$$\begin{aligned}\frac{dS(c)}{dc} &= -(1-2c)^2 \\ &\leq 0\end{aligned}$$

so that $S(c)$ is maximized at $c = 0$. ■

5 Concluding remarks

Our results encourage governments to introduce caps on lobbying. We have found for both the incomplete and the complete information setting that although introducing caps on lobbying may ex post lead to an increase in total lobbying expenditures, this effect is reversed for ex ante expected total lobbying expenditures. Moreover, making the trade-off between social costs and social benefits of lobbying, we have shown that it is optimal for a benevolent government to completely ban lobbying.

This conclusion, however, relies heavily on at least three debatable assumptions. By far the strongest, and therefore most serious assumption, is that interest groups play a Bayesian Nash equilibrium. This assumption is probably not valid in many real-life cases of political lobbying, as often, interest groups cannot be viewed as a single entity, but are poorly organized lobbies instead that suffer seriously from free-riding problems. Second, our results are built on the assumption of a benevolent government which maximizes social welfare, which at first sight seems to be strong as well. However, also a self-interested government may rationally aim at maximal social welfare, so that its probability of being re-elected is maximized. Finally, we have limited the action space of the government to the choice of a cap on lobbying expenditures. We implicitly assume that the government is not able to implement other, probably more efficient mechanisms such as auctions, for instance because the constitution precludes this.[5]

There are several interesting directions for future research. For instance, the analysis was simplified by the assumption of independence (the interest groups' values are drawn independently) and symmetry (the values are drawn from the same distribution). The assumption of independence is not valid when there are external factors which influence the interest groups' values equally. For instance, the value for a license to operate in a certain market depends on consumer's demand, which effects the values for the different interest groups in the same direction. In this respect, extensions to models with affiliated values, interdependent values, or multidimensional signals may provide additional insights. In Onderstal (2002), the model with incomplete information is extended to allow for interest group specific distribution

[5] See Moore (1992) and Palfrey (1992) for a survey of the literature on the implementation of efficient mechanisms in environments with complete and incomplete information respectively.

functions. Onderstal shows that a cap of zero is still optimal, provided that interest groups with low ex ante values (i.e., expected values) are not allowed to participate in the lobby game.

6 Appendix

Proof of Lemma 1. The first and the second order derivatives of C have the following properties.

$$C'(y) = \frac{(n-1)F(y)^n - nF(y)^{n-1} + 1}{n}$$

for all $y \in [0,1]$, so that

$$C'(1) = 0.$$

$$C''(y) = (n-1)f(y)F(y)^{n-2}(F(y)-1) < 0$$

for all $y \in [0,1)$. It immediately follows that $C'(y) > 0$ for all $y < 1$.

Proof of Lemma 2. We deduce for all $y \in [0,1)$,

$$\begin{aligned}
D'(y) &= \frac{[1-F(y)][1-F(y)^n - yf(y)nF(y)^{n-1}] + f(y)y[1-F(y)^n]}{n[1-F(y)]^2} - F(y)^{n-1} \\
&= \frac{1-F(y)+yf(y)}{[1-F(y)]^2} * \frac{(n-1)F(y)^n - nF(y)^{n-1} + 1}{n} \\
&= \frac{1-F(y)+yf(y)}{[1-F(y)]^2} * C'(y) \\
&> 0,
\end{aligned}$$

where the inequality follows from Lemma 1. Therefore, D is strictly increasing.

Proof of Corollary 3. As $D(0) = 0$, $\lim_{y\uparrow 1} D(y) = 1 - \int_0^1 F(z)^{n-1}dz$, and D differentiable and strictly increasing (by Lemma 2), ξ is uniquely determined.

7 References

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