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Discussion paper

Social Security Reform and Population Ageing in a Two-Sector Growth Model

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Abstract

This paper analyses the effects of reducing unfunded social security and population ageing on economic growth and welfare, both for a small open economy and for a closed economy. The economy consists of a service sector and a commodity sector. Productivity growth only occurs in the latter sector and is assumed to depend positively on its size. It is shown that if old agents mainly demand labour intensive services, a decrease of the pay-as-you-go (PAYG) pension scheme reduces long-run growth and thus welfare in a small open economy, whereas current generations are better off. However, reducing social security raises productivity growth in a closed economy, both in the short and long run.

Furthermore, ageing will lead to a lower long-run rate of economic growth in a small open economy, whereas in the short run, the effects depend on the type of ageing and the size of the PAYG-scheme. In a closed economy, the effects of ageing depend on the substitutability of labour and capital.

JEL classification: D91, E60, H55, J14, O41

Keywords: endogenous growth, overlapping generations, pensions, population ageing, privatisation, social security reform

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1 Introduction

Analyses on the sustainability of current social security arrangements are usually based on a comparison of the rates of return of both funded and unfunded schemes. This basically leads to the conclusion that as long as the economy is dynamically efficient, a public pension and health care scheme that is financed on a pay-as-you-go (PAYG)-basis is harmful for future generations, in the sense that they would gain from a switch to more funded schemes. This would even hold more if the population ages, since this deteriorates the internal rate of return of an unfunded scheme. An important aspect in this context that is often neglected is the effect of social security reform and population ageing on economic growth, and thereby on the well-being of current and future generations, which is the focus of this paper.

Conventional analyses on the relation between social security and growth focus on the impact of a PAYG-scheme on savings. As was demonstrated in e.g. Feldstein (1974), Kotlikoff and Summers (1981) and Modigliani (1988), transfers from the young to the old, such as unfunded social security, discourage private savings, resulting in less capital accumulation and a lower output level (see also e.g. Kotlikoff, 1979, and Jones and Manuelli, 1992). However, these analyses do not explicitly model technological progress. If productivity growth is modeled by assuming an externality in production as in Romer (1986) and Lucas (1988), where production is characterised by constant returns to scale at the aggregate level, unfunded social security will decrease the rate of economic growth, see King and Rebelo (1990), Rebelo (1991), Saint-Paul (1992) and Wiedmer (1996).[1] But if economic growth rather results from the accumulation of human capital (as initiated by Uzawa, 1965, and Lucas, 1988), the effect of PAYG-pension schemes on growth may be positive, as is for instance demonstrated in Sala-i-Martin (1996), Kemnitz (2000), Kemnitz and Wigger (2000) and Sánchez-Losada (2000).

As to the effects of ageing, the neo-classical growth model with an exogenous saving rate predicts that a declining population growth would have a positive effect on the growth rate of output per capita, since the number of people that share the capital stock decreases. In Futagami and Nakajima (2001), who apply a model with external effects of the aggregate capital stock on productivity, it is found that the effect of a longer life span on economic growth is unambiguously positive. On the

[1] In that case, a Pareto-improving transition to a more funded scheme is possible, see Belan *et al.* (1998), Corneo and Marquardt (2000) and Gyárfás and Marquardt (2001).

other hand, Pecchenino and Pollard (1997), who consider a similar growth model, show that a longer life span can decrease economic growth, depending on the extent to which all wealth is annuitised. Another channel through which population ageing can affect economic growth is the formation of human capital, as in Kemnitz (2000) and Pecchenino and Utendorf (1999), where ageing has a positive effect on growth.

Not much empirical work has been done on the relation between ageing and growth. Disney (1996, Chapter 8) finds a clear negative relationship between the old-age dependency ratio and the GDP growth rate for 24 OECD countries between 1977 and 1992. Also Lindh and Malmberg (1999) found that the growth pattern of GDP per worker in the OECD countries between 1950 and 1990 are to a large extent explained by age structure changes. In particular, they too find a strong negative correlation between growth and the population share of old-age people.

In this paper, we focus on another mechanism through which social security reform and ageing can affect economic growth. We extend the two-sector model presented in Groezen *et al.* (2002) by including endogenously determined economic growth, and concentrate on the effects of privatisation[2] and population ageing with respect to the sectoral structure of the economy, and on economic growth for different degrees of openness of the economy. Our approach closely follows Baumol (1967), who groups economic activities into two types: "technologically progressive activities in which innovations, capital accumulation, and economies of large scale all make for a cumulative rise in output per man hour and activities which, by their very nature, permit only sporadic increases in productivity." (Baumol, 1967, p. 415-16). As to the first type of activity, "labor is primarily an instrument", whereas the second type consists of "services in which the labor is an end in itself, in which quality is judged directly in terms of amount of labor." So again, we assume two sectors: a labour-intensive service sector and a capital intensive commodity sector. Productivity growth is endogenously determined and mainly stems from technological progress that typically takes place in the commodity (or 'high-tech') sector. As productivity grows, wages paid in this sector also increase. Because labour is assumed to be mobile between both sectors, the wage paid in the services sector will grow equally fast.

The capital-intensive sector produces homogeneous consumption and investment commodities that are demanded by young individuals, whereas elderly derive utility from labour-intensive services. Consequently, a change in the intergenerational

[2]By privatisation we mean a simultaneous reduction of the unfunded part of social security (i.e., a lower PAYG-tax) and an extension of the funded part (i.e., higher private savings).

redistribution and demographic composition of the economy will affect the relative aggregate demand for services and commodities. The structure of the economy, thereby the growth rate and the well-being of successive generations, are thus influenced.

We find that the effects of privatisating social security crucially depend on the openness of the economy. In a small open economy that faces an exogenously given interest rate (that exceeds the rate of economic growth), the short-run effects differ from the long-run consequences. Initially, privatisation immediately decreases the demand for services by the old and thus leads to a higher concentration of labour in the productive sector, which stimulates economic growth. In the long run, however, privatising social security increases the lifetime income of generations born at or just after the time of the decrease in the PAYG-tax. This stimulates their demand for both services and commodities. As services cannot be imported, this implies that more labour will be employed in the services sector and the domestic production of commodities is reduced. Consequently, economic growth is eventually lower than it would have been without privatisation, putting future generations at a disadvantage.

On the other hand, if the economy is closed, the interest rate is endogenously determined by the interaction of savings and investments. In the short run, privatisation reduces the income of the current old and thereby increases the employment in the commodity sector, which implies a higher growth rate. In the long run, the overall demand for both commodities and services increases. Because labour is employed in both sectors, and capital only in one, labour becomes relatively more scarce so that wages increase, and thereby the price of services. This reduces the demand for services, thus stimulating productivity growth in the commodity sector. Furthermore, reducing an unfunded social security scheme increases savings and the capital stock, which attracts employees to the commodity sector as they can earn a higher wage. So also in the long run, growth is stimulated by privatisation in a closed economy.

The second part deals with the effects of ageing, modeled as a permanently higher average lifetime. It turns out that in a small open economy, population ageing leads to a lower long-run rate of economic growth because the increase in the relative number of old implies a lower domestic production of commodities. In the short run, ageing has no effects on the labour share of the productive sector.

In a closed economy, the capital-labour ratio will also be influenced by domestic demographic changes and consequently play an important role. The long-run effects of ageing then crucially depend on the extent to which capital and labour are substitutable in the production process of commodities. If they are substitutes, then ageing will decrease growth, whereas the reverse holds if capital and labour are complements. Again, ageing has no effect in the short run.

The rest of this paper is organised as follows. Section 2 describes the model. In Section 3, we analyse both the long-run and short-run consequences of downsizing the unfunded social security scheme. Section 4 analyses the consequences of increasing longevity. In Section 5 we discuss some extensions and modifications of the model. Section 6 concludes.

2 The model

Production of commodities and services

Two sectors of production are distinguished, both consisting of many suppliers who take prices as given. In the *commodity sector* (labelled Y), homogeneous goods are produced that either serve as consumption or investment good and can be traded internationally. The production process involves the employment of both physical capital and labour according to the following standard neoclassical CRTS production function: $Y_t = F(K_t, A_t L_t^Y)$, where K_t stands for the domestic capital stock, L_t^Y is the number of people employed in the commodity sector and A_t is a productivity parameter reflecting the current state of (technological) knowledge or experience in the economy (all at time t). A_t can therefore be interpreted as the average human capital and $A_t L_t^Y$ as the total human capital employed in the commodity sector. Production per effective unit of labour is described by $f(\kappa_t)$, where $\kappa_t \equiv \frac{K_t}{A_t L_t^Y}$ denotes the effective capital-labour ratio. The elasticity of substitution between capital and labour is thus defined as $\sigma \equiv -\frac{f'(\kappa)(f(\kappa)-f'(\kappa)\kappa)}{f''(\kappa)f(\kappa)\kappa}$. Firms maximise their profits, so the interest and wage rate are given by $r_t = f'(\kappa_t)$ and $w_t = A_t\left(f(\kappa_t) - f'(\kappa_t)\kappa_t\right)$ respectively.

The productivity of an employee in the commodity sector, A_t, grows at the endogenous rate g,[3] which is assumed to depend positively on the number of people

[3]It may well be that the economy also adopts new technologies and increases its knowledge from abroad. In that case, the size of the commodity sector positively affects the *additional* productivity growth of the economy.

employed in the productive sector,[4]

$$A_t = A_{t-1}\left(1 + g(L_t^Y)\right). \tag{1}$$

This is a shortcut for explicitly modeling (labour-intensive) R&D activities or education. Basically, one could assume that part of the employees in the commodity sector devote their time on these activities, so the more people that are active in this sector of the economy, the more knowledge is created and skills are improved, which allows people to produce more final output, i.e., productivity increases (as in e.g. Romer, 1990, Grossman and Helpman, 1991, Mulligan and Sala-i-Martin, 1993).

The *service sector* concerns the provision of labour-intensive services (labelled D) that do not use capital goods, and do not benefit from technological improvements.[5] This sector uses labour only, assuming that one unit of labour translates into one service.[6] Since international trade in services is not possible and labour is assumed to be immobile across borders, the total provision of services equals total labour supply in this sector (L_t^D). The price of services in terms of commodities, p_t, is therefore equal to the wage rate in this sector. As we shall assume labour to be homogeneous and the labour market to be competitive, this wage is equal to the wage paid in the other sector, so $p_t = w_t$.

Households

The economy is inhabited by N individuals who live for two periods, such that in each period, both a young and an old generation are alive.[7] For notational convenience, we set N equal to unity. Apart from age, people are identical. Every young individual faces a probability of ε to grow old, so $1-\varepsilon$ is the fraction of young that dies after one

[4]Note that we assume A to grow instantly with the size of the sector. If learning takes time, however, one would rather want to assume some delay in the accumulation of knowledge. However, assuming that g is a function of L_{t-1}^Y instead of L_t^Y would imply that learning takes about 30 years (i.e., one period in our model), which does not seem to be very realistic.

[5]Examples are nurses, GPs, gardeners, cleaners, housekeepers, hairdressers, butlers, theatre seats and hotel rooms. Note that these are not services like bank services which extensively use high-tech goods like computers.

[6]Another interpretation would be that the quality of services increases with the number of employees.

[7]This size is assumed to be constant. Section 5 investigates the effects of changes in the population growth rate,

period of life.[8] When people grow old, the need for services increases, both because of increased disability (which raises the demand for e.g. care services and housekeeping assistance) and a higher preference for services relative to (high-tech) commodities.[9] To simplify the analysis, we will assume old individuals to demand services only. The expected lifetime utility of a representative agent born at any time t can then be represented by the following function,

$$E_t U(c_t, d_{t+1}^o) = \log c_t + \gamma \varepsilon_{t+1} \log d_{t+1}, \tag{2}$$

where c_t stands for the consumption of commodities when young and d_{t+1} is the number of services enjoyed by the agent when old.[10]

Every young individual supplies one unit of labour inelastically and receives wage. A fraction τ_t of his labour income is taxed by the government to finance a public pension scheme. The young spend their net income on the purchase of commodities, either for consumption or investment purposes (i.e., savings, s_t). Savings are invested in annuities or through an actuarially fair pension fund. As only a fraction ε_{t+1} of young savers born at time t survives to period $t+1$, the assets of those who deceased fall to surviving contemporaries. The total return on the savings is therefore $\frac{r_{t+1}}{\varepsilon_{t+1}}$. Furthermore, the public pension benefit that an old individual at time $t+1$ receives is equal to $\frac{\tau_{t+1} w_{t+1}}{\varepsilon_{t+1}}$. The social security tax is constant (or in case of privatisation, reduced to a lower constant level), so that the benefit level adjusts.[11] Individual consumption possibilities are thus given by the following budget constraints,

$$c_t = w_t(1-\tau_t) - s_t, \tag{3.a}$$

$$p_{t+1} d_{t+1} = \frac{r_{t+1} s_t + \tau_{t+1} w_{t+1}}{\varepsilon_{t+1}}. \tag{3.b}$$

where p_t is the price of services in terms of commodities. Maximising (2) subject to

[8] We can interpret ε as the fraction of the second period of life that people live, i.e., average longevity. An increase of ε is thus referred to as a longer lifespan (one form of population ageing).

[9] Because innovation mainly takes place in the commodity sector, the varieties of high-tech commodities constantly change. As older people have more difficulties learning how to apply these new technologies and/or products, this can also explain why they would have a relatively higher preference for services.

[10] Allowing for consumption of services when young does not qualitatively affect the results of reducing the social security tax and increasing longevity. However, the results can change if ageing is the result of a lower fertility rate. See Subsection 5.2.

[11] In Subsection 5.3 we examine the case of a constant benefit level.

(3.a) and (3.b) results in the following individual demand and saving functions,

$$c_t = \frac{(1-\tau_t)w_t}{1+\gamma\varepsilon_{t+1}} + \frac{\tau_{t+1}w_{t+1}}{(1+\gamma\varepsilon_{t+1})r_{t+1}}, \tag{4}$$

$$d_{t+1} = \frac{\gamma r_{t+1}(1-\tau_t)w_t}{(1+\gamma\varepsilon_{t+1})w_{t+1}} + \frac{\gamma\tau_{t+1}}{1+\gamma\varepsilon_{t+1}}, \tag{5}$$

$$s_t = \frac{\gamma\varepsilon_{t+1}(1-\tau_t)w_t}{1+\gamma\varepsilon_{t+1}} - \frac{\tau_{t+1}w_{t+1}}{(1+\gamma\varepsilon_{t+1})r_{t+1}}. \tag{6}$$

Note that the consumption of commodities is growing over time with the rate of economic growth. But for a given growth rate, the number of services enjoyed is constant, since the (positive) income effect of a rising wage is exactly offset by the (negative) substitution effect of higher prices of services. This is a necessary requirement for an equilibrium to exist, because the supply of labour, and thus the provision of services, is limited.[12,13] Furthermore, for a given interest rate, a higher growth rate decreases the number of services for an old person.

Finally, four assumptions are made throughout the paper (see Appendix for further details).

Assumption I *The growth function g is such that a steady state exists.*

Assumption II *L^Y converges to a steady state, i.e., the equilibrium is asymptotically stable.*

Assumption III *Productivity growth in the commodity sector is bounded by $g' < \frac{(1+\gamma\varepsilon_t)\left(1+g(L_t^Y)\right)^2}{r\gamma\varepsilon_t(1-\tau_t)}$ $\forall t$.*

Assumption IV *The set of parameter values is such that the economy is always dynamically efficient, i.e., $r_t > 1 + g(L_t^Y)$ $\forall t$.*

[12] This is in line with the modelling of the labour/leisure choice in a model with economic growth, where the number of leisure hours cannot grow indefinitely. King *et al.* (1988) and Rebelo (1991) formulate the kind of utility function for which this holds (see also Barro and Sala-i-Martin, 1995, Chapter 9). The log-linear utility function applied here is a special case of this type of utility function.

[13] Instead of putting restrictions on the utility function in order to prevent corner solutions, one could also assume that the services sector experiences exogenous growth equal to $\bar{g}_S$, and growth in the commodity sector is partly exogenous, $\bar{g}_Y$, and partly endogenous, $g(L^Y)$. In that case, it must be assumed that the difference in exogenous growth rates is ultimately compensated by the endogenously determined productivity difference. The share of labour employed in the commodity sector then equals $L^Y = g^{-1}(\bar{g}_S - \bar{g}_Y)$ in the long run, in which case privatisation does not affect the growth rate in the long run.

According to Assumption III, endogenous-growth effects are not very strong. This assumption is necessary in order to assure that (second-order) price effects that are the result of a change in L^Y do not dominate the direct (first-order) effects of a shock.[14] Assumption IV states that the economy is dynamically efficient, both before and after the economy is hit by a shock. So if a reduction of the social security implies a lower level of utility, this is because the economy has become dynamically inefficient.

3 Privatising social security

This section investigates the consequences of reducing the unfunded social security scheme in the economy described above. The first part deals with the case of a small open economy, whereas the second part focuses on a closed economy. The probability to grow old is set equal to unity ($\varepsilon = 1$).

3.1 Privatisation in a small open economy

In a small open economy with perfect capital mobility, the interest rate is determined at the world capital market. If this rate is constant and equal to r, the effective capital-labour ratio is also constant and equal to $\kappa = f^{-1}(r)$. Consequently, wages grow at the rate g. In case of dynamic efficiency ($r > 1+g$), the interest rate exceeds the growth rate and funding yields a higher return than a PAYG-scheme. If the growth rate is also exogenously given, current young and future generations would be better off if the social security tax were reduced. This section examines whether this also holds in the two-sector model as described above.

Four markets can be distinguished. First, because services are not tradable, the total demand has to be met by domestic labour supply, so $L_t^D = d_t$. Labour market clearing implies that the remaining young are employed in the commodity sector: $L_t^Y = 1 - L_t^D$. Together with (6), this implicitly gives the dynamics of the employment share of the commodity sector,

$$L_t^Y = 1 - \frac{rs_{t-1} + \tau_t w_t}{p_t} \tag{7}$$

$$= 1 - \frac{\gamma(1-\tau_{t-1})r}{(1+\gamma)\,(1+g(L_t^Y))} - \frac{\gamma\tau_t}{1+\gamma}. \tag{8}$$

[14]Simulations for the case that $g_t = \bar{g} + \rho L_t^Y$ show that this assumption indeed holds for a wide range of parameter values.

Any excess demand (supply) of commodities will be imported (exported) from (to) abroad, and be equal to the difference between domestic savings and investments.

Note that the current state of the economy, as summarised by (8), is not related to its past. The effects of a change in the social security tax can be traced by linearising (8) around the initial steady state. In case the small open economy is unexpectedly hit by a shock (e.g. a lower value of τ), L^Y will jump to a new value at the time of the shock, and jumps to its new steady-state value after one period, which can easily be seen from the fact that equation (8) only contains one parameter (τ_{t-1}) and no variable (L^Y) that refers to the past.

3.1.1 Long-run consequences of lower transfers

According to conventional wisdom, privatising social security would cause some short-run pain, but also bring substantial long-run gains. However, the long-run consequences of reducing the social security tax are not as straightforward as they seem. In particular, a reduction of the social security tax can change the production structure of the economy, and thereby influence the rate of economic growth. The following proposition holds for the two-sector model presented above.

Proposition 1 *Privatisation will eventually reduce economic growth and thus lead to a lower level of utility in a small open economy.*

Proof The long-run impact of the size of the PAYG-scheme (τ) on the economy can be traced by comparative statics of (8), which yields[15]

$$\frac{dL^Y}{d\tau} = \frac{\gamma(1+g)(r-1-g)}{(1+\gamma)(1+g)^2 - \gamma(1-\tau)rg'} > 0.$$

This expression is positive under assumptions III and IV. ■

In the long run, a reduction of the PAYG-scheme has three effects on the allocation of labour over the two sectors, and thereby on the rate of economic growth. First, if $r > 1 + g$, lifetime income increases when the PAYG-tax decreases, which results in a higher demand for both services and commodities. As savings increase, so will investments abroad. Consequently, net foreign assets increase, which enables the economy to finance increased imports (or decreased exports) of commodities from abroad. Domestic production of commodities will be reduces, which releases labour

[15] Omitting time subscript denotes the (initial) steady-state value of the variable or parameter.

from this sector, and more people will be employed in the services sector. This is necessary because the increased demand for services has to be met by domestic labour. As L^Y decreases, technological progress or knowledge creation slows down. Second, since this implies a lower economic growth, the internal rate of return of the PAYG-system decreases. So for a given social security tax, lifetime income decreases,[16] which leads to a lower demand for services, thus partially offsetting the initial decrease of L^Y. Third, productivity grows at a lower rate, and so do wages and thereby the price of services. Old-age consumption thus becomes cheaper. In other words, the old are confronted with a higher real interest rate, implying a higher demand for services. This also has a negative effect on the employment share of the commodity sector, and accordingly on economic growth.

3.1.2 Short-run consequences of lower transfers

If the social security tax decreases unexpectedly at time $t = 0$, the economy will be in its new steady state as from $t = 1$ on.[17] At the time of the shock, however, the effects differ from its long-run consequences since the old at that time did not foresee the change in the PAYG-benefit, nor the changing rate of economic growth (which reflects the increase of the price of services) when they made their savings decision. As a matter of fact, the short-run effects are the opposite of the long-run effects, as stated in the following proposition.

Proposition 2 *In a small open economy, an unexpected permanent reduction of the social security scheme initially increases economic growth.*

PROOF At time $t = -1$, individual savings are given by $s_{-1} = \frac{\gamma(1-\tau)w_{-1}}{1+\gamma} - \frac{\tau w_{-1}(1+g)}{(1+\gamma)r}$, where τ (g) denotes the social security tax (growth rate) at the time before the shock occurred. Because the PAYG-tax is changed unexpectedly at time $t = 0$, these savings are given, and equation (7) boils down to

$$L_0^Y = 1 - \frac{r\gamma(1-\tau) - \tau(1+g)}{(1+\gamma)\left(1+g(L_0^Y)\right)} - \tau_0.$$

[16]Naturally, this second-order effect cannot dominate the first-order effect if the stability condition applies.

[17]If the privatisation is announced at time $t = 0$ to take place one period later, no changes in L_0^Y will occur: young individuals decide to save more and consume less, so the total demand for commodities is unchanged. Because no shock yet occurs, the elderly do not change their demand for services. In case young individuals also demand services, L_0^Y will increase, as is demonstrated in the Appendix.

The short-run effects on the employment in the commodity sector can be seen from totally differentiating this equation, giving

$$\frac{dL_0^Y}{d\tau_0} = -\frac{(1+\gamma)(1+g)^2}{(1+\gamma)(1+g)^2 - r\gamma(1-\tau)g' + \tau(1+g)g'},$$

which is negative if Assumption III holds. ■

The reason for an increase in the labour share of the commodity sector in the short run is simple: at the time the PAYG-tax is unexpectedly cut down, old individuals will have to economise the entire decrease of their pension benefits on services, so the number of employees in the commodity sector will increase.[18] Furthermore, the wage and thereby the price of services that elderly buy at time $t = 0$ increases, whereas the interest rate does not change. So the retired will use less services, reinforcing the initial rise in L_0^Y.

Figure 1 shows the change in lifetime utility that privatising social security brings about in a small open economy.[19] In this figure, the horizontal axis denotes the period in which a certain generation is born, and the vertical axis gives the change in utility as a fraction of the utility level that the elderly alive at the time of the shock $(t = 0)$ had.

[18] If also young individuals demanded services, they would demand more services and commodities at the time of the shock because their lifetime income increases as long as $r > 1 + g$ holds, which would push L^Y down. However, the young would spread this positive income effect over their entire life, so the increase in the provision of services to the young would be smaller than the reduced demand by the current old, and L^Y increases anyway. Furthermore, the economy will be in its new steady state in the subsequent period, which is characterised by a lower rate of economic (and thereby wage) growth. Hence, the price of the services that the current young would demand in the next period will be lower. This would induces them to substitute current services for future services, thus reinforcing the increase of L^Y at time $t = 0$.

[19] All simulations are based on a productivity growth that is assumed to be a linear function of the number of employees in the productive sector: $g_t = \bar{g} + \rho L_t^Y$, where $\bar{g}$ is chosen such that the equilibrium growth rate is equal to 1 (which corresponds to an annual economywide growth rate of about 2.3%). The cases considered are $\rho \to 0$ and $\rho = 2$ respectively. The production function of commodities is given by $Y_t = \left[0.3K_t^{-0.2} + 0.7(A_t L_t^Y)^{-0.2}\right]^{-5}$, and the utility function is $U_t = \log(c_t^y) + 0.67\log(d_{t+1}^o)$. Furthermore, $r = 3.25$ (which corresponds to an annual interest rate of 4%) and the social security tax is decreased from 20% to 10%.

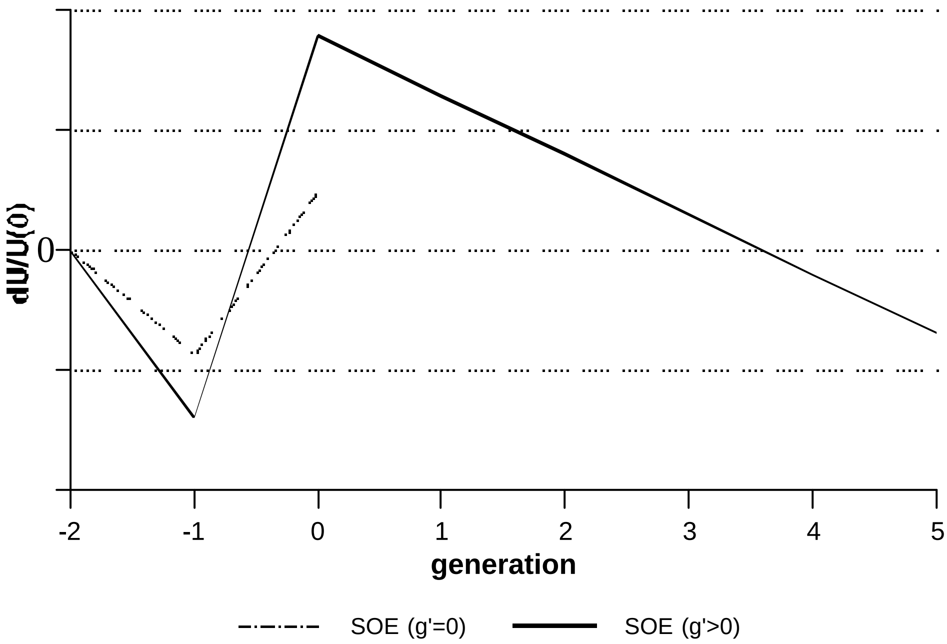

Figure 1 *Effect of privatisation on lifetime utility in a small open economy*

If endogenous-growth effects are absent ($g' = 0$), the elderly at the time of the shock will suffer from a reduction in their pension benefit, whereas the current young and all future generations gain equally much because the wage and interest do not change and accordingly, their lifetime income increases to the same extent.[20]

In case the production of commodities is characterised by endogenous productivity growth ($g' > 0$), the effects are different. At the time the privatisation policy is implemented ($t = 0$), more people will be employed in the commodity sector, which stimulates productivity growth. Hence, wages rise and services become more expensive. This was not foreseen by the current old when they made their savings decision, so they can demand fewer services due to both a lower pension benefit and higher prices, and are therefore worse off. Individuals born at $t = 0$ experience three effects. First, downsizing the social security scheme increases their lifetime income. Second, because L_0^Y increases, the remuneration of their labour duties increases, and third, fewer people will be employed in the commodity sector at time $t = 1$, so the services they consume when old are cheaper. All these elements substantially increase the

[20] Compensating these elderly for this loss would require an amount equal to the present value of the gains of future generations (see Verbon, 1988, and Breyer, 1989).

well-being of the generation that is born at the time of the social security reform. All future generations also benefit from a higher lifetime income that is directly caused by a lower social security tax, as well as from the higher labour productivity they 'inherit' from generation $t = 0$. The number of services they enjoy increases, which has a positive effect on lifetime utility. However, these future generations face a lower value of L^Y, so their own labour productivity grows at a smaller rate. After some time, this effect will become dominant, and the consumption of commodities by the young will be lower than before the reduction of the public pension scheme. How long it takes before individual well-being actually deteriorates depends on both individual preferences and economic factors. First, if agents have a rather strong preference for services (i.e., γ is high), or if people life rather long (ε high), the utility gain of more services is significant, and the utility loss of fewer commodities is small. It then takes quite a long time before the utility loss of fewer commodities, due to a lower growth rate, outweighs the higher utility from more services.[21] Furthermore, if g' takes a high value, the fall in productivity growth is high and the decrease in utility will sooner occur. Third, if the interest rate is relatively low or the initial rate of economic growth high (and the economy is close to the golden rule), a decrease in the social security tax causes a relatively moderate rise in lifetime income. This implies a modest contraction of the commodity sector, and thereby a small decrease of productivity growth, so that utility decreases are delayed.

3.2 Privatisation in a closed economy

A decrease of the unfunded part of social security creates an incentive for individuals to save more for their old age. In a closed economy, this will result in a higher physical capital stock and consequently affect factor rewards. This subsection focuses on the effects of privatisation in such an economy.[22]

As in a small open economy, the labour and service markets both clear if

$$L_t^Y = 1 - d_t. \tag{9}$$

However, no international trade of commodities is possible in a closed economy, so the total domestic demand for commodities, which consists of consumption and in-

[21] This is especially so if services like health care only raise utility when their level is above a certain minimum requirement, in which case a Stone-Geary utility function is most appropriate.

[22] Note that this framework is also applicable to the case of many (small) open economies that simultaneously implement the same privatisation policy.

vestment goods, must equal the aggregate domestic production, i.e., $Y_t = c_t + s_t = w_t(1-\tau_t)$, or

$$L_t^Y = (1-\tau_t)\left(1 - \frac{\kappa_t f'(\kappa_t)}{f(\kappa_t)}\right). \tag{10}$$

Finally, since there is no scope for international lending and borrowing, aggregate savings are entirely invested domestically, and the interest rate is determined endogenously. If capital fully depreciates in one period, this implies that the capital market clears when $s_t = K_{t+1}$ holds, or

$$\frac{s_t}{A_t\left(1+g(L_{t+1}^Y)\right)} = \kappa_{t+1} L_{t+1}^Y. \tag{11}$$

Combining these equations gives a two-dimensional non-linear system in L_t^Y and κ_t. Because the capital stock is determined by savings in the previous period, we get a saddle-point stable system with one forward-looking (or jump) variable, L^Y, and one backward-looking (or state) variable, κ. The effects of a change in the social security tax are analysed by linearising the equilibrium conditions around the initial steady state. We assume this steady state to be saddle-point stable.

3.2.1 Long-run consequences of lower transfers

In the previous section it was shown that for a small open economy characterised by perfect capital mobility, reducing the PAYG-scheme has a negative impact on the rate of economic growth in the long run, because it increases the demand for services and import goods from abroad. Thus, fewer people will be employed in the productive sector and growth effects are smaller. In a closed economy, however, international lending and borrowing is not possible, and the capital stock is entirely determined by domestic savings. As a consequence, the long-run effects of privatising social security on the rate of economic growth are quite different, as stated in the following proposition.

Proposition 3 *Privatisation will eventually stimulate economic growth and thus lead to a higher level of utility in a closed economy. The (effective) capital-labour ratio rises in the long run.*

Proof See Appendix. ■

A lower social security tax increases the lifetime income of individuals. For a given price level and interest rate, this implies a higher demand for both services and commodities. Because capital is only employed in one sector, and labour in both, labour will become relatively more scarce, which pushes wages up. As this implies an increase in the price of services, the relative demand for services will decrease, so the employment share of the commodity sector grows and productivity growth increases. Furthermore, a lower social security tax also increases savings and thereby stimulates the accumulation of physical capital, which is only used in the commodity sector. This increase in the capital stock outweighs the rise in employment in the commodity sector so that the capital-labour ratio rises. This makes labour more productive in the commodity sector and consequently, wages rise and the services sector contracts.

Note that the decrease in the provision of services implies that individuals can enjoy fewer services when they are old, despite the higher lifetime income and increased savings. This is due to the strong decrease in the real rate of interest $\left(\frac{r_t}{p_t}\right)$. The higher lifetime utility that privatisation causes in the long run is therefore solely due to the increased consumption of commodities when young. Consequently, this rise in lifetime utility goes along with a decrease in equity between the old and the young living in the same period: the young profit from the higher savings of the previous generation, while this old generation itself effectively gets poorer.

3.2.2 Short-run consequences of lower transfers

A sudden change in the social security tax at time $t = 0$ initially only affects the economy through a change in the division of labour over the two sectors since the size of the capital stock is determined by the savings of the previous period.[23] In the subsequent periods savings and thus the capital stock gradually move to the new steady-state level, and so does the number of employees in the commodity sector.

Proposition 4 *In a closed economy, an unexpected decline of the social security tax causes more people to be employed in the commodity sector and thus stimulates growth in the short run. Consequently, the (effective) capital-labour ratio decreases.*

PROOF Differentiating the commodity market equation (10) at time $t = 0$ gives $\frac{dL_0^Y}{d\tau} = \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)}\frac{d\kappa_0}{d\tau} - \frac{f(\kappa)-\kappa r}{f(\kappa)}$. Furthermore, differentiating the capital market equation (11) at time $t = 0$, taking savings of the previous period as given, results in $\frac{d\kappa_0}{d\tau} = -\kappa\left(\frac{g'}{1+g} + \frac{1}{L^Y}\right)\frac{dL_0^Y}{d\tau}$. Combining these two expressions and rewriting gives

$$\frac{dL_0^Y}{d\tau}\left[1 + \frac{(1-\tau)(\sigma-1)\kappa^2 f''(\kappa)}{f(\kappa)}\left(\frac{g'}{1+g} + \frac{1}{L^Y}\right)\right] = -\frac{f(\kappa)-\kappa r}{f(\kappa)}.$$

[23] In case of an expected shock, no change in L_0^Y nor k_0 occurs (see Appendix).

The RHS of this expression is always negative, whereas the LHS can numerically be shown to be always positive. Hence, $\frac{dL_0^Y}{d\tau} < 0$. From this it immediately follows that $\frac{d\kappa_0}{d\tau} > 0$. ■

Privatising social security thus immediately increases the growth rate because the purchasing power of the initial elderly is reduced, implying a smaller services sector. On the other hand, κ initially decreases because both A_0 and L_0^Y increase, and the physical capital stock is fixed in the short run. This implies that wages may well fall in the short run if τ is reduced, making services cheaper. This will be so if growth effects are not very substantial and if labour and capital are complements (i.e., if σ is low).

The decrease in τ leads to higher savings, thus to a higher capital-labour ratio and higher wages in the next period. The higher wages raise savings further so that the capital-labour ratio in the subsequent period is again higher. This process of a rising capital stock also implies that more people are attracted to be employed in the commodity sector, so also L^Y steadily rises. Therefore, the number of services enjoyed by elderly individuals decreases, whereas savings gradually rise. That is, although people save more, this does not allow them to consume more services when they have grown old. The reason for this is that the only way to transfer purchasing power to old age is through investment in capital that is only useful for the production of commodities, something the elderly do not want to consume. Instead, they demand more services, which increases the price of services, and together with a lower interest rate offsets the effect of the increase in savings. Figure 2 shows the effects of privatisation on lifetime utility in a closed economy (the effects for a small open economy, as shown in Figure 1, are also displayed).

First consider the case without growth ($g' = 0$). Because at time $t = 0$ the capital-labour ratio decreases, the elderly receive a higher interest on their savings, and wages decrease which makes their services cheaper. This partly offsets the decrease of their public pension benefit, so they are less worse off than they would be in a small open economy. Individuals born at the time of the privatisation ($t = 0$), however, are confronted with a lower wage when they are young, and also save more, resulting in a growing capital stock. This causes the capital-labour ratio to be higher at the time they have become old, so wages and thereby the price of future services will be higher. Overall, these individuals suffer from the social security reform, contrary to their contemporaries in a small open economy.

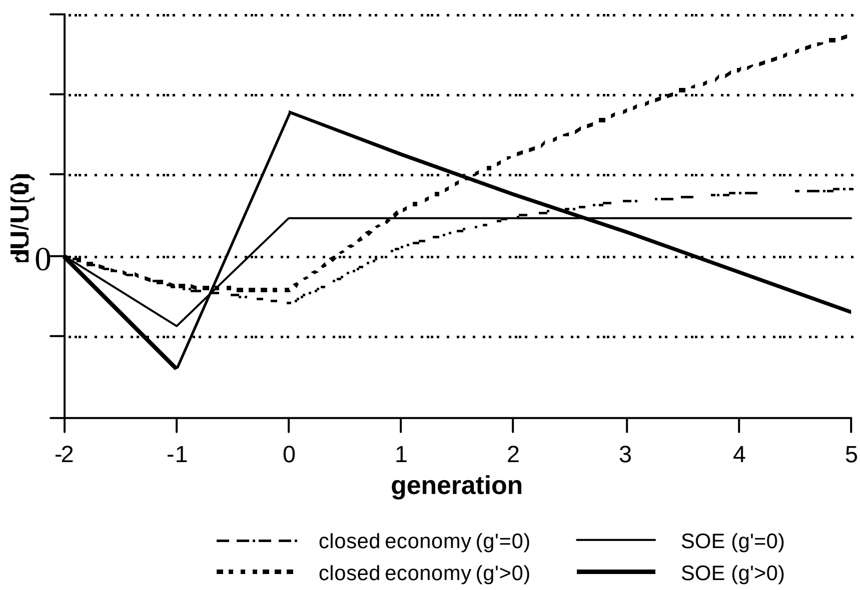

Figure 2 *Effect of privatisation on lifetime utility in a closed economy*

Later generations will experience increasingly higher wages though, and are therefore better off as long as the economy stays dynamically efficient. The consumption of commodities when young increases, but due to a lower PAYG-benefit, a higher wage and a lower interest rate, the number of services receives when old decreases. So the utility gap between the young and old increases. If individuals care a lot about an equal distribution of well-being over their lifetime, or if there is some minimum number of services a person needs when old, they will try to prevent the increase of this utility gap from happening by saving more. However, this will increase the capital-labour ratio further, so wages rise even more and the interest decreases to a greater extent, which only increases the gap further. The economy will thus soon end up in a situation of dynamic inefficiency, and the government can increase welfare by running a PAYG-scheme. Yet in Figure 2, the economy stays dynamically efficient. Because the capital-labour ratio increases, it moves closer to the golden rule level. This does not happen in a small open economy, so future generations gain more from privatisation in a closed economy as compared to a small open economy.

Figure 2 also shows the change in utility in case there is endogenous growth. The higher it is, the more wages rise at time $t = 0$ due to the initial increase of

L^Y in a closed economy. At the same time, however, it makes the capital stock that the elderly at time $t = 0$ own more productive, as the employees apply it more productively, and this raises the interest rate. The difference with the case in which $g' = 0$ is therefore very small. The young at the time of the transition, however, are better off (or less worse off) if g' is higher, and naturally the same holds for all future generations since L^Y increases in the long run after a reduction of the social security tax. Therefore, the welfare effects of privatising social security depend more crucially on the openness of the economy if growth effects in the productive sector are more important. Again, the consumption of commodities when young increases, whereas the number of services enjoyed when old decreases. Endogenous growth adds to this growing utility gap between young and old.

3.3 Conclusions so far

In this section we showed that the welfare effects of reforming social security are not as clear as is often suggested. Because individuals' tastes for tradable commodities and non-tradable services differ with age, a change in the intergenerational redistribution causes a change in the production structure of the economy. Depending on the openness of the economy, this affects international trade and the extent to which the economy specializes in the provision of services. As the accumulation of (technological) knowledge typically takes place in the capital-intensive sector, the rate at which productivity grows subsequently changes and thereby the welfare of future generations.

In particular, in a small open economy with endogenous growth, privatising social security as advocated by Feldstein will lead to a dilemma: lifetime income will initially rise more than with exogenous technological progress, but this rise in wealth will lead to a kind of "Dutch disease": the import of commodities rises and the country specialises more in the production of non-tradable low-tech services, which lowers growth.[24] That is, the larger welfare gains for the current young come at the cost of lower welfare for generations born in the (distant) future. Moreover, endogenous-growth effects increase the welfare loss for the current old caused by the privatisation. So in an open economy, introducing endogenous growth intensifies the difference in

[24] A similar shift of labour from tradables production to non-tradable services and a subsequent productivity loss is found in Smulders and Van de Klundert (2001) who analyse the effects of the initial distribution of financial wealth across countries on long-run growth and cross-country productivity levels.

position between the two "parties" that have to decide on Feldstein's proposal in the political arena, i.e., the current young and the current old. At the same time, however, it may weaken the position of the young, as it deprives them of the argument that privatisation benefits future generations.

In case of a closed economy, the effects are quite different. Increased savings now raise the capital stock and thus increase wages, i.e., the price of services. As a consequence, privatisation does not lead to an increase in the relative importance of the services sector but causes this sector to shrink, thus stimulating growth. As a result, it favours future generations. However, this comes at the cost of a decrease in welfare of both current generations: not only the old but also the young will lose. Endogenous-growth effects reduce the welfare loss for the current young and reinforce the gains for future generations without harming the current elderly. This will not affect decision making in a representative democracy where decisions are made by current generations that do not take the welfare effects of future generations into account. It only increases the costs of myopic decision making in terms of the welfare of future generations.

4 Population ageing

In this section, we analyse the effects of a longer life span on economic growth and welfare. Again we make a distinction between a small open economy and a closed economy. The social security tax is assumed to be constant.

4.1 Ageing in a small open economy

As individuals face a probability ε to live in the second period, ε also reflects the average number of retired, i.e., the dependency ratio. Equilibrium on the services market therefore implies

$$L_t^D = \varepsilon_t d_t. \tag{12}$$

This, together with equilibrium on the labour market and (5) and (6), implicitly gives the dynamics of the employment share of the commodity sector,

$$\begin{aligned} L_t^Y &= 1 - \frac{rs_{t-1} + \tau w_t}{p_t} && (13) \\ &= 1 - \frac{\gamma\varepsilon_t(1-\tau)r}{(1+\gamma\varepsilon_t)\left(1+g(L_t^Y)\right)} - \frac{\gamma\varepsilon_t\tau}{1+\gamma\varepsilon_t}. && (14) \end{aligned}$$

Again, the current state of this small open economy, as implicitly summarised by (14), contains no variable or parameters that refer to the past and is therefore not related to its history.[25] The effects of a change in the rate of population growth can be traced by linearising (14) around the initial steady state, as is described in the Appendix. In case the small open economy is unexpectedly hit by an increase of ε, the variable L^Y will jump to a new value at the time of the shock, and jumps to its new steady-state value one period later.

4.1.1 Long-run consequences of ageing

The long-run effect of a longer life span on the employment level of the commodity sector, and thereby on the rate of economic growth, is negative, as the following proposition asserts.

Proposition 5 *In a small open economy, population ageing will lead to a lower rate of economic growth in the long run.*

PROOF The long-run impact of an increasing life span on the economy can be traced by comparative statics of (15), which yields the following equation,

$$\frac{dL^Y}{d\varepsilon} = -\frac{\gamma(1+g)\left(r(1-\tau)+\tau(1+g)\right)}{(1+\gamma\varepsilon)\left((1+\gamma\varepsilon)(1+g)^2-\gamma\varepsilon r(1-\tau)g'\right)} < 0. \tag{15}$$

Assumption IV implies that the denominator of (15) is positive, so $\frac{dL^Y}{d\varepsilon} < 0$. ■

If people expect to live longer, they will decide to save more when they are young. These increased savings are invested abroad, yielding a return of r. With these returns, the elderly individuals buy services that are provided by their children. Because the retired use more services, the number of young people employed in that sector increases, leaving fewer people in the capital-intensive commodity sector. As a consequence, the growth rate decreases. Furthermore, the decline of domestic commodity production will be compensated by higher imports of these goods from abroad (or lower exports), which is financed by the increased foreign asset returns.

A lower growth rate implies that future services become cheaper, so the real interest rate (in terms of services) increases. As can be seen from (6), savings are not influenced by this. Hence, each retired person can use more services.

[25] If young individuals also demand services, the solution of L_t^Y is a terminal-value problem as L_t^Y is a forward-looking variable. See Subsection 5.2 and the Appendix for more details.

4.1.2 Short-run consequences of ageing

If the life span increases unexpectedly at time $t = 0$, the economy will be in its new steady state as from $t = 1$ on. At the time of the shock, however, the effects differ from the long-run consequences since the old at that time did not foresee the change in the PAYG-benefit, nor the changing rate of economic growth (which reflects the increase of the price of services) when they made their savings decision. In a small open economy, the interest rate and the PAYG-tax do not change, so the elderly still spend the same amount of money on services. Consequently, the allocation of labour over the two sectors does not change either, and growth is not affected by ageing in the short run.

Proposition 6 *Increasing longevity does not affect economic growth in the short run for a small open economy.*

PROOF At time $t = -1$, individual savings are given by s_{-1}. Because ageing occurs unexpectedly at time $t = 0$, these savings are given. Equation (13) then boils down to

$$\begin{aligned} L_0^Y &= 1 - \varepsilon_0 d_0 \\ &= 1 - \frac{r s_{-1}}{w_0} - \tau. \end{aligned}$$

In this equation, ε_0 does not appear, so $\frac{dL_0^Y}{d\varepsilon_0} = 0$. ■

This also holds in case the longer life span is foreseen. Then, people start saving more at time $t = 0$, but this basically implies a shift from the demand of consumption commodities to investment commodities, which does not affect the size of the commodity sector.[26]

Figure 3 shows the consequences of population ageing on the lifetime utility of successive generations.[27] First consider the case without endogenous growth ($g' = 0$).

[26] There will be effects on growth in the short run if agents also demand services when they are young, as described in Subsection 5.2.

If ageing is the result of a lower rate of population growth, growth will decrease if the economy is a net lendor. If the young demand services too, the effects of such a form of ageing depend on the size of the PAYG-scheme. See Subsection 5.1 for details.

[27] Simulations are based on the same production function as applied in Figure 1. Furthermore, $\gamma = 0.9$, $r = 3.25$, $\tau = 0.2$, $\rho = 2$ in case of endogenous growth, and ε increases from $\frac{2}{3}$ to 0.7. In order to compare lifetime utility before and after the shock properly, we took the value of ε constant at unity, so we compare the lifetime utility of individuals who live both periods.

In a small open economy, the factor rewards do not change. As there is no endogenous productivity change either, the lifetime income of individuals remains the same. However, people live longer on average, so the period during which they can spend their lifetime income becomes longer. The consumption of commodities and services thus decreases, and utility decreases likewise (on average).

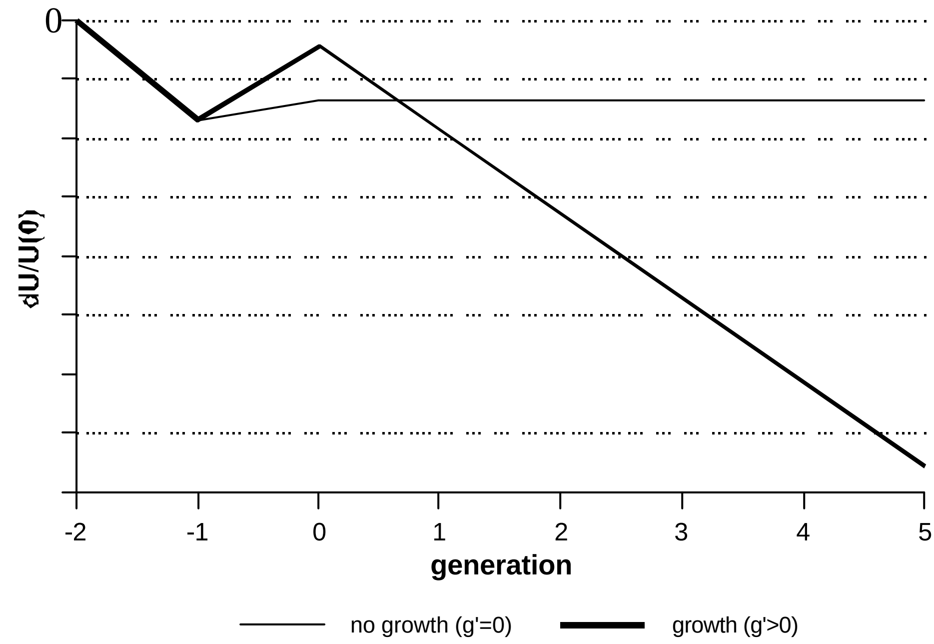

Figure 3 *Effect of increasing longevity on lifetime utility in a small open economy*

With endogenous growth, the effects for the currently old are not different from the case in which $g' = 0$ because the number of employees in the commodity sector does not change in the short run. The wage of the individuals born at time $t = 0$ therefore does not change either. However, less people will be employed in the commodity sector at time $t = 1$, so the wage rate will be lower. Consequently, the services that the generation born at the time of the shock will buy when they are old will be cheaper. This positive price effect alleviates the direct negative effect of a longer life span on utility. In the long run, individuals suffer from the fact that a lower productivity growth reduces the wage they earn when young, enabling them to purchase less commodities.

4.2 Ageing in a closed economy

The labour and services markets both clear if the following condition holds,

$$L_t^Y = 1 - \frac{\gamma \varepsilon_t r_t w_{t-1}(1-\tau)}{(1+\gamma\varepsilon_t)w_t} - \frac{\gamma\varepsilon_t \tau}{1+\gamma\varepsilon_t}. \tag{16}$$

Furthermore, equilibrium on the commodity and capital market is given by (9) and (10) respectively.

4.2.1 Long-run consequences of ageing

The long-run consequences of population ageing on the rate of economic growth depend on the extent to which capital and labour can be substituted in the production process of commodities, as summarised in the following proposition.

Proposition 7 *The long-run rate of economic growth decreases (increases) in a closed economy due to population ageing if the elasticity of substitution between capital and labour is greater (smaller) than unity. Ageing has no effect on growth if this elasticity equals unity. The capital-labour ratio always increases.*

PROOF See Appendix. ■

Increasing longevity gives people an incentive to save more when they are young. This raises the capital stock that is used in the commodity sector, so capital becomes relatively more abundant in that sector. If capital is a good substitute for labour ($\sigma > 1$), this implies that labour too becomes relatively abundant in this sector so that the wage decreases and individuals would earn more in the services sector. A higher number of people will therefore be employed in the services sector (so that wages are equal in both sectors again). However, if capital and labour are complements rather than substitutes ($\sigma < 1$), the productivity of employees in the commodity sector increases, which subsequently attracts labour from the services sector, so growth increases. In case of a Cobb-Douglas production function, capital and labour are not complements nor substitutes, so then ageing causes no change in the division of labour and thereby in the growth rate.[28]

[28] This directly follows from the equilibrium condition for the commodity market, according to which the wage share does not depend on the capital-labour ratio (i.e., $\frac{w_t A_t L_t^Y}{Y_t} = 1 - a$), so that $L_t^Y = (1-\tau)(1-a)$. Allowing for consumption of services by the young does not change this result.

4.2.2 Short-run consequences of ageing

As is the case for the small open economy, increasing longevity does not influence the allocation of labour over the two sectors in the short run, which leads to the following proposition.

Proposition 8 *In a closed economy, increasing longevity does not affect economic growth in the short run, nor the (effective) capital-labour ratio.*

PROOF Differentiating (10) at time $t = 0$ gives $\frac{dL_0^Y}{d\varepsilon} = \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)} \frac{d\kappa_0}{d\varepsilon}$. Furthermore, differentiating the capital market equation (11) at time $t = 0$, taking savings of the previous period as given, results in $\frac{d\kappa_0}{d\varepsilon} = -\kappa \left(\frac{g'}{1+g} + \frac{1}{L^Y} \right) \frac{dL_0^Y}{d\varepsilon}$. Combining these two expressions gives in $\frac{d\kappa_0}{d\varepsilon} = \frac{dL_0^Y}{d\varepsilon} = 0$. ■

Again, elderly individuals cannot react to their longer life span. The savings they invested when young give a lower return r/ε, and the same holds for the social security benefit. Hence, they will be able to use less services and are therefore worse off. The allocation of labour over the two sectors does not change, so the capital-labour ratio also remains the same.

The effects of a longer life span on the lifetime utility of subsequent generations in a closed economy are displayed in Figure 4.[29]

First consider the case of capital and labour being complements in the production process of commodities ($\sigma < 1$). Because at the time of the shock, the labour allocation nor the capital-labour ratio change, the wage and interest rate remain the same. Therefore, the group of retired as a while receives the same return on the savings, but because more people survived, each elderly individual gets a lower return on his savings, so the utility of this generation decreases. The young at that time receive the same wage, but will decide to save a higher fraction of it, so they will consume less commodities. These higher savings translate into a higher capital stock in the next period, which attracts labour to the commodity sector since labour and capital are complements. Hence, L^Y increases, and so does the capital-labour ratio. Consequently, wages rise and services become more expensive, enabling the members of this generation to buy less services. The next generation earns this higher wage

[29] In this figure, we took $\gamma = 0.75$ in order to ensure that the economy is dynamically efficient both before and after the shock. If capital and labour are complements, $\sigma = 0.8$, and in case of substitutability, $\sigma = 1.2$.

when young, so their savings will be higher, resulting in even higher wages the period after that. As this process continues, L^Y rises and productivity grows further, so people will be able to consume more commodities when young. This will eventually compensate the loss of utility that is the result of fewer services during old age.

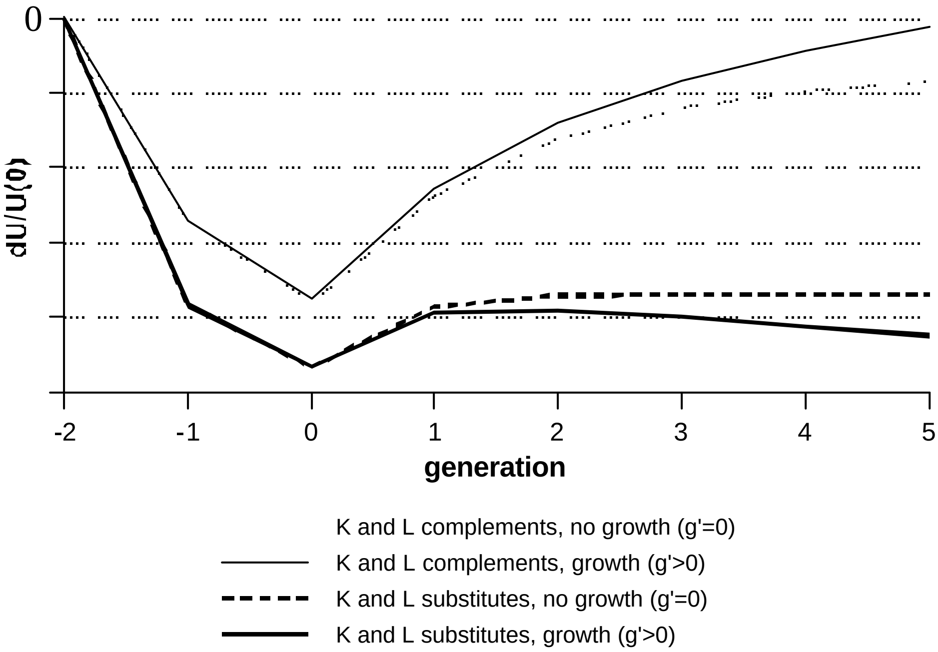

Figure 4 *Effect of increasing life span on lifetime utility in a closed economy*

The short run effects of ageing are (qualitatively) the same if capital and labour are substitutes. In the long run, a higher capital stock implies that labour too becomes relatively abundant in the commodity sector. Therefore, some individuals will move out of that sector and be employed in the services sector, so productivity growth will eventually slow down, and the lifetime utility of future generations decreases due to an increasing life span. The effects of privatisation and population ageing are summarised in Table 1.

Table 1

variable	Privatising SOE	Privatising CLE	Increasing longevity SOE	Increasing longevity CLE ($\sigma < 1$)	Increasing longevity CLE ($\sigma > 1$)
short run					
L_0^Y, g_0	+	+	0	0	0
κ_0	0	-	0	0	0
$p_0 = w_0$	+	+	0	0	0
c_0	+	+	-	-	-
d_0	-	-	-	-	-
U_{-1}	-	-	-	-	-
U_0	+	-	-/+*	-	-
long run					
L^Y, g	-	+	-	+	-
κ	0	+	0	+	+
$p = w$	-	+	-	+	-
c	-	+	-	+	-
d	+	-	+	-	-
U	-	+	-	+	-

* Depending (among other things) on the size of the PAYG-scheme.

5 Extensions and modifications

In this section we explore the effects of privatisation and population ageing when the model presented above is extended or modified in several ways. First, we consider the consequences of ageing if this is due to a lower rate of population growth. Second, the assumption that only old individuals demand services is modified, and finally, a pension system that promises the retired a fixed amount of services is taken into consideration.

5.1 Population ageing as a lower fertility rate

Suppose the economy is inhabited by N_t young people at time t, and population grows at rate n, such that $N_{t+1} = (1 + n_t)N_t$. Population ageing takes the form of a lower value of n, and equilibrium on the labour market implies that $N_t = L_t^Y + L_t^D$. We now assume that the productivity growth rate depends on the fraction of young people employed in the productive sector: $g_t = g(l_t^Y)$, with $l_t^Y \equiv \frac{L_t^Y}{N_t}$.[30] Furthermore, we take ε constant and equal to unity, and the social security tax τ does not change either. The PAYG-benefit that an old individual at time $t+1$ receives now equals $(1+n_{t+1})\tau w_{t+1}$. Assumption III is now $g' < \frac{(1+\gamma)\left((1+g(l_t^Y))\right)^2(1+n)}{r\gamma(1-\tau)}$ $\forall t$.

5.1.1 The case of a small open economy

Equilibrium on both the labour and services market is given by

$$l_t^Y = 1 - \frac{\gamma(1-\tau)r}{(1+n_t)\left(1+g(l_t^Y)\right)} - \gamma\tau.$$

Comparative statics of this equation gives $\frac{dl^Y}{dn} = \frac{\gamma(1-\tau)r(1+g)}{(1+\gamma)(1+n)(1+g)^2 - \gamma(1-\tau)rg'} > 0$, so again, ageing reduces the (relative) number of people working in the commodity sector and thereby economic growth, as is the case when lifespan increases. Notice that $\frac{d^2l^Y}{dnd\tau} < 0$, so the negative effect of a lower fertility rate on economic growth is curbed by a PAYG-scheme.

However, the short run effects are different. If the fertility rate decreases unexpectedly at time $t=0$, the economy will again be in its new steady state as from $t=1$ on. In this case, equation (14) can be written as

$$l_0^Y = 1 - \frac{r\gamma(1-\tau)}{(1+\gamma)(1+n_0)\left(1+g(l_0^Y)\right)} + \frac{(1+n)\tau(1+g)}{(1+\gamma)\left(1+g(l_0^Y)\right)} - \tau,$$

where g (n) denotes the growth (fertility) rate at the time before the shock occurred (that people took into account when making their savings decision at time $t=0$). Differentiating this equation we arrive at

$$\frac{dl_0^Y}{dn_0} = \frac{rs(1+\gamma)(1+g)}{(1+\gamma)(1+n)(1+g)^2 - r\gamma(1-\tau)g' + (1+n)(1+g)\tau g'}.$$

The denominator is negative when assumption III holds. Ageing implies a relatively higher number of elderly individuals. If the country is a net lender ($s > 0$), then relatively more people will spend their income on services, and l_0^Y decreases, so productivity growth will already decrease in the short run. Consequently, the price of the services that these old individuals buy decreases, so the direct negative effect of a higher dependency ratio on the PAYG-benefit will be alleviated by an lower inflation rate, i.e., an increase in the real interest rate. The reverse holds if the economy is

[30] This assumption is necessary to calculate an equilibrium.

a net borrower ($s < 0$), which is the case if the social security tax is rather high. Then, elderly individuals who borrowed an amount of $N_{t-1}s_{t-1}$ from foreigners when they were young, have to pay off this same amount from a lower transfer they receive from their children, who have decreased in number: $N_{t-1}(1+n_t)\tau w_t$. So, there is less money left for the purchase of services, and l_0^Y increases.

5.1.2 The case of a closed economy

In a closed economy, the long-run effects of a lower rate of fertility are the same as those of a longer lifespan. In both cases, the dependency ratio increases and people have a stronger incentive to save, causing the capital-labour ratio to increase and the (relative) number of employees in the commodity sector to increase (decrease) if capital and labour are complements (substitutes) in that sector. The same can be said for the short run in case fertility (unexpectedly) decreases. This can be seen by differentiating the equilibrium condition for the commodity market, $l_t^Y = (1-\tau)\left(1-\frac{\kappa_t r_t}{f(\kappa_t)}\right)$ at time $t = 0$, which gives

$$\frac{dl_0^Y}{dn_0} = \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)}\frac{d\kappa_0}{dn_0}.$$

Furthermore, equilibrium on the capital market is now given by $\frac{s_t}{A_t\left(1+g(l_{t+1}^Y)\right)} = \kappa_{t+1}l_{t+1}^Y(1+n_{t+1})$; taking savings as given, differentiating this equation results in

$$\frac{d\kappa_0}{dn_0} = -\kappa\left(\frac{g'}{1+g}+\frac{1}{l^Y}\right)\frac{dl_0^Y}{dn_0} - \frac{\kappa}{1+n}.$$

Combining these two expressions and rewriting gives

$$\frac{dl_0^Y}{dn}\left[1+\frac{(1-\tau)(\sigma-1)\kappa^2 f''(\kappa)}{f(\kappa)}\left(\frac{g'}{1+g}+\frac{1}{l^Y}\right)\right] = -\frac{(1-\tau)(\sigma-1)\kappa^2 f''(\kappa)}{f(\kappa)(1+n)}.$$

The RHS of this expression is positive (negative) if $\sigma > (<)1$, whereas the LHS can numerically be shown to be always positive. Hence, $\frac{dl_0^Y}{dn} > 0$ if $\sigma > 1$, and the opposite holds if $\sigma < 1$. From this it immediately follows that $\frac{d\kappa_0}{dn} < 0$. In case of a Cobb-Douglas production function ($\sigma = 1$), $\frac{dl_0^Y}{dn} = 0$ and $\frac{d\kappa_0}{dn} = -\frac{\kappa}{1+n} < 0$.

So contrary to increasing longevity, a lower population growth rate does affect productivity growth in the short run. This is so because a lower fertility rate immediately decreases the number of young individuals, whereas the capital stock is fixed in the short run. Hence, the capital is becoming relatively abundant in the commodity sector. This attracts labour to that sector if both production factors are complements, whereas l^Y decreases in case of substitutability.

The short-run effects of population ageing are summarised in Table 2.

Table 2

		Decreasing fertility			Increasing longevity	
variable	SOE	CLE (σ<1)	CLE (σ>1)	SOE	CLE (σ<1)	CLE (σ>1)
short run						
L_0^Y, g_0	-	+	-	0	0	0
k_0	0	+	+	0	0	0
$p_0=w_0$	-	+	+	0	0	0
c_0	-	+	+	-	-	-
d_0	+	-	-	-	-	-
U_{-1}	+	-	-	-	-	-
U_0	-	-	-	-/+*	-	-

* Depending (among other things) on the size of the PAYG-scheme.

5.2 Allowing for service demand by the young

In this section we investigate the effects of ageing for a small open economy in case individuals also demand services when they are young. The effects of privatisating social security and the effects of ageing in a closed economy are qualitatively not different from the effects when services demand by the young is absent.

Expected lifetime utility of a representative agent is now given by the following function,

$$E_t U(c_t, d_t^y, d_{t+1}^o) = \alpha \log c_t + \beta \log d_t^y + \gamma \varepsilon_{t+1} \log d_{t+1}^o, \tag{17}$$

where d_t^y (d_{t+1}^o) is the number of services enjoyed by the agent when young (old). The budget constraints are

$$\begin{aligned} c_t + p_t d_t^y &= (1-\tau) w_t - s_t, \\ p_{t+1} d_{t+1}^o &= \frac{r s_t}{\varepsilon_{t+1}} + \frac{(1+n_{t+1}) \tau w_{t+1}}{\varepsilon_{t+1}}. \end{aligned}$$

This leads to the following demand for services,

$$d_t^y = \frac{\beta(1-\tau)}{1+\beta+\gamma\varepsilon_{t+1}} + \frac{\beta(1+n_{t+1})\tau w_{t+1}}{w_t(1+\beta+\gamma\varepsilon_{t+1})},$$
$$d_{t+1}^o = \frac{\gamma(1-\tau)w_t r}{(1+\beta+\gamma\varepsilon_{t+1})w_{t+1}} + \frac{\gamma(1+n_{t+1})\tau}{1+\beta+\gamma\varepsilon_{t+1}}.$$

Equilibrium on the services and labour market is given by

$$L_t^Y = N_t - N_t d_t^y - \varepsilon_t N_{t-1} d_t^o. \tag{18}$$

Note that the current state of the economy, as summarised by (18), is not related to its past. Instead, the solution of (18) is a terminal-value problem (see Appendix for details). The effects of a change in the average life span can be traced by linearising (18) around the initial steady state. In case the small open economy is unexpectedly hit by a shock, the forward-looking variable L^Y (or l^Y) will jump to a new value at the time of the shock, and jumps to its new steady-state value after one period, which can easily be seen from the fact that equation (18) only contains one parameter (ε_{t-1}) and no variable (L^Y) that refers to the past (see Appendix for details). Furthermore, Assumption III now states that $g' < \frac{(1+\beta+\gamma\varepsilon)\left(1+g(l_t^Y)\right)^2(1+n)}{\gamma\varepsilon(1-\tau)r}$ $\forall t$.

5.2.1 Increasing longevity

The long run effects of a longer average life span can be traced by linearising equation (18), taking $n = 0$ and $N = 1$, which results in

$$\frac{dL^Y}{d\varepsilon} = \frac{\gamma(1+g)\left(\beta(1+g) - r(1+\beta)\right)\left(\tau(1+g) + r(1-\tau)\right)}{(1+\beta+\gamma\varepsilon)\left(r(1+\beta+\gamma\varepsilon)(1+g)^2 + \beta\tau(1+g)^2 g' - r^2\gamma(1-\tau)\varepsilon g'\right)},$$

which is clearly negative if Assumptions III and IV hold. Naturally, the fact that young individuals demand services too does not change the result that ageing leads to lower long-run productivity growth in a small open economy.

In the short run, however, the effects are different from the case that $\beta = 0$. This can be seen by looking at the equilibrium condition for the services and labour market at time $t = 0$ (when the shock unexpectedly occurs),

$$L_0^Y = 1 - \frac{\beta(1-\tau)}{1+\beta+\gamma\varepsilon_1} - \frac{\beta\tau\left(1+g(L_1^Y)\right)}{r(1+\beta+\gamma\varepsilon_1)} - \frac{r(1-\tau)\gamma\varepsilon}{(1+\beta+\gamma\varepsilon_1)\left(1+g(L_0^Y)\right)}$$
$$- \frac{(1-\beta)\tau(1+g)}{(1+\beta+\gamma\varepsilon_1)\left(1+g(L_0^Y)\right)} - \tau,$$

where ε (g) refers to the initial steady state value, i.e., the one that individuals born at time $t=-1$ took into account when they made their savings decision. Differentiating this expression gives us

$$\frac{dL_0^Y}{d\varepsilon}=\frac{(1+g)^2\left(\beta\gamma r(1-\tau)+\beta\gamma(1+g)\tau-(1+\beta+\gamma\varepsilon)\beta\tau g'\frac{dL^Y}{d\varepsilon}\right)}{r(1+\beta+\gamma\varepsilon)\left((1+\beta+\gamma\varepsilon)(1+g)^2+r(1-\tau)\gamma\varepsilon g'-(1+\beta)\tau(1+g)g'\right)}.$$

Knowing that $\frac{dL^Y}{d\varepsilon}<0$, it immediately follows that $\frac{dL_0^Y}{d\varepsilon}>0$ if $s>0$. That is, increasing longevity will stimulate growth in the short run if young individuals also demand services. This is so because these young individuals expect to spend a longer time in retirement and therefore decide to increase their savings. In order to do this, they will ask less services when young. Another reason is that future services will be cheaper, because L^Y decreases at time $t=1$, so current services are substituted for future services. The group of elderly individuals as a whole at time $t=0$ will buy as many services as before, because each old individual receives a proportionately lower return on his savings and social security benefit. So fewer people will be employed in the services sector, and more in the commodity sector, which stimulates productivity growth.

5.2.2 Decreasing population growth

The long run effects of a lower fertility rate can be traced by linearising equation (18), taking $\varepsilon=1$, which results in

$$\frac{dl^Y}{dn}=\frac{(1+g)\left(\gamma(1-\tau)r^2-\beta\tau(1+g)^2(1+n)^2\right)}{(1+n)\left((1+\beta+\gamma\varepsilon)r(1+g)^2(1+n)+\beta(1+n)^2(1+g)^2\tau g'-\gamma(1-\tau)r^2g'\right)},$$

which is positive under Assumptions III and IV, and a social security tax that is not too high, i.e., $\tau<\frac{\gamma}{\beta+\gamma}$. So now too, population ageing will lower the long-run rate of economic growth.

In the short run, the effects are different, both from the case in which $\beta=0$ and from the effect of increasing longevity. Differentiating (18) at time $t=0$, taking the savings of the old as given, we arrive at

$$\frac{dl_0^Y}{dn}=\frac{(1+g)\left(-\beta\tau(1+g)^2(1+n)^2-\beta(1+n)^3(1+g)\tau g'\frac{dl^Y}{dn}+r^2(1-\tau)\gamma-(1+\beta)\tau(1+g)^2(1+n)\right)}{r(1+n)((1+\beta+\gamma\varepsilon)(1+g)^2(1+n)+r(1-\tau)\gamma g'-(1+\beta)\tau g')}. \quad (19)$$

Due to assumption III, the denominator is positive. If $\tau=0$, then $\frac{dl_0^Y}{dn}>0$, so ageing leads to a lower productivity growth in the short run. If $\tau=1$, then $\frac{dl_0^Y}{dn}<0$ and ageing implies a higher productivity growth in the short run. This, together with the fact that (19) is a continuous and decreasing function of τ, implies that there is a unique $\tau^* \in (0,1)$ for which $\frac{dl_0^Y}{dn} = 0$. Consequently, for all $\tau < \tau^*$ it holds that $\frac{dl_0^Y}{dn} > 0$, so unexpected ageing leads to a lower value of l_0^Y, and the reverse holds if $\tau > \tau^*$.

Population ageing has four effects on the division of labour over the two sectors at time $t = 0$. First, the relative number of old individuals who spend their savings on services increases, so l^Y decreases. If the social security tax is high, individual savings are relatively low and the pension benefit decreases considerably, so this effect will be moderate. Second, young agents at that time foresee a lower pension benefit when old, and thus decide to save more, i.e., consume fewer services, which affects l^Y positively. This is especially so if the social security tax is quite high, because then the drop in the pension benefit due to ageing will be rather sizeable. Therefore, the second effect will dominate the first effect if the PAYG-scheme is relatively generous. Economic growth will then increase in the short run due to ageing, whereas the reverse holds if τ is relatively low. Third, the economy is in its new steady state as from time $t = 1$ on, with a lower rate of economic growth, so current services become more expensive relative to future services. Hence, the social security benefit the young expect to receive when old decreases in terms of current services, which decreases the current demand for services by the young, driving l^Y up, again especially if the PAYG-scheme is rather extensive. Finally, the change in the current division of labour affects the productivity growth, and thereby the current wage and price of services. If l^Y initially increases, services become more expensive, so the elderly's real interest rate is lower, urging them to demand fewer services, pushing l^Y down (the opposite holds if l_0^Y decreases). Naturally, this second-order effect does not dominate the other effects. The second and third effects disappear if young individuals do not demand any services (i.e., if $\beta = 0$).

Figure 5 shows the consequences of population ageing on the lifetime utility of successive generations, for different values of the social security tax ($\tau = 0$ and $\tau = 0.2$ respectively).[31]

[31] Simulations are based on the same production function as applied in Figure 1. Furthermore, $\beta = 1$, $\gamma\varepsilon = \frac{2}{3}$, $r = 3.25$, $\rho = 2$ in case of endogenous growth and n is decreased from 0.2 to 0.

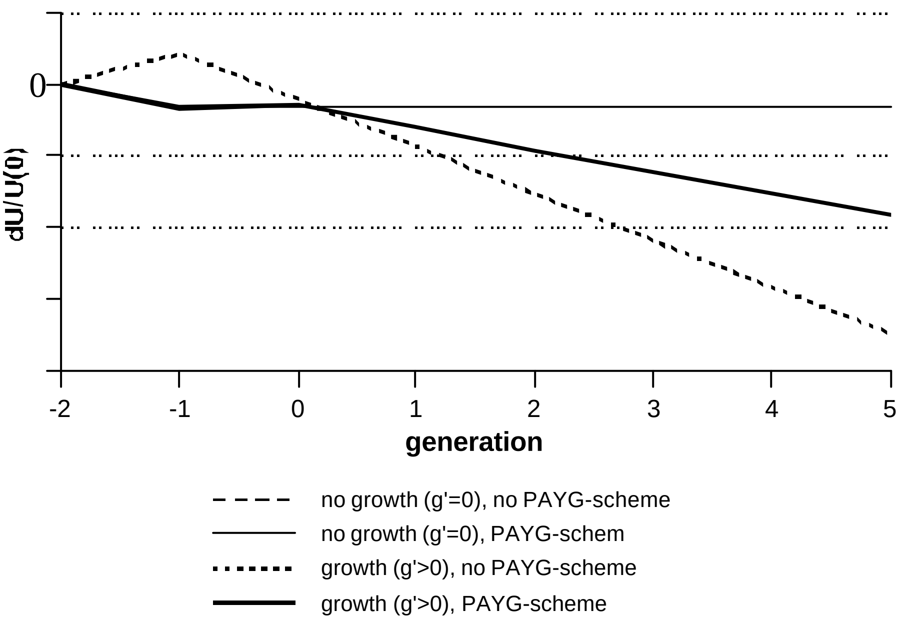

Figure 5 *Decreasing fertility in a small open economy when* $\beta > 0$

If there is no PAYG-scheme, relatively more people will be employed in the services sector at the time of the shock. This implies a lower productivity growth, so wages decline and the elderly pay a lower price for their services, allowing them to enjoy more services which increases their utility. However, the wage that the young at time $t = 0$ receive has become lower, and thereby their lifetime income. The consumption of commodities therefore decreases, whereas the number of services enjoyed when young stays the same. Because of a lower future price of services, the number of services enjoyed when old increases. Overall, utility decreases due to ageing. The same holds for all subsequent generations.

On the other hand, if the social security scheme is relatively extensive, more people will be employed in the commodity sector at the time of the shock. This increases their productivity, and thus increases the wages, so the then living elderly can enjoy fewer services. Apart from that, their utility also decreases because of the lower social security benefit they receive due to ageing. As for the young generation at that time, although they earn a higher wage, the price of current services rises just as much; furthermore, the PAYG-benefit they expect to receive when old decreases in terms of current services, both due to a lower population growth and a lower future wage growth. So they decide to save more, i.e., consume fewer services and commodities when young. When they are old, they have a higher capital income at their disposal, and face lower prices of services. This allows them to enjoy more services when retired, even though the public pension benefit has decreased.

This also holds for all future generations. They suffer utility losses because the growth rate is lower, and thereby their lifetime income. However, they are less worse off when the PAYG-scheme is generous because in that case, the reduction of l^Y due to ageing is moderate.

5.3 Fixed-benefit PAYG-pensions

In the previous sections, the social security tax that young individuals pay was kept constant, or, in the case of privatisation, reduced to a lower constant level. So elderly individuals receive a pension benefit that decreases if the dependency ratio rises. So the retired bear the full risk of adverse demographic changes. The alternative would be a social security scheme that ensures each pensioner a fixed amount of services. In that case, the PAYG-tax is flexible and responds to changes in the demographic structure and factor rewards. This subsection investigates the effects of population ageing if such a pension scheme is in place.

Suppose that the government ensures every retiree η services. The social security tax is then equal to $\tau_t = \eta\varepsilon_t$. Differentiating the equilibrium condition for the services and labour market in a *small open economy* gives for the long run

$$\frac{dL^Y}{d\varepsilon} = \frac{dL^Y}{d\varepsilon}[\bar{\tau}] + \frac{(1+g)\gamma\tau(r-1-g)}{(1+\gamma\varepsilon)(1+g)^2 - \gamma\varepsilon r(1-\tau)g'},$$

where $\frac{dL^Y}{d\varepsilon}[\bar{\tau}] < 0$ is the change in the labour share of the commodity sector if the social security tax is kept constant, as given by (15). One can clearly see that if the economy is dynamically efficient, the decline of L^Y, and thereby of economic growth in the long run, is smaller in case of a fixed-benefit pension scheme.[32] Increasing longevity implies that the social security tax has to rise in order to finance the higher amount of public pensions paid out to retirees. In a dynamically efficient economy, this means a decline in the lifetime income of individuals, so it decreases the demand for commodities and services and thereby offsets part of the increased demand for services that is directly caused by a higher dependency ratio.

[32] Assuming that for both pension systems, the initial steady state is the same.

In a *closed economy*, long-run equilibrium on the commodity market is given by $L_t^Y = (1-\eta\varepsilon_t)\left(1 - \frac{\kappa_t f'(\kappa_t)}{f(\kappa_t)}\right)$, so

$$dL_{t+1}^Y = \delta_1 d\kappa_{t+1} + \delta_2 d\varepsilon_{t+1},$$

with $\delta_1 \equiv \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)}$ and $\delta_2 \equiv -\eta\left(1 - \frac{\kappa f'(\kappa)}{f(\kappa)}\right) < 0$. The labour and services market clear if $L_{t+1}^Y = 1 - \frac{\gamma\varepsilon_{t+1} r_{t+1} w_t (1-\eta\varepsilon_{t+1})}{(1+\gamma\varepsilon_{t+1}) w_{t+1}} - \frac{\gamma\eta\varepsilon_{t+1}^2}{1+\gamma\varepsilon_{t+1}}$, which results in

$$dL_{t+1}^Y = \lambda_1 d\kappa_t + \lambda_2 d\kappa_{t+1} + \lambda_3' d\varepsilon_{t+1},$$

where $\lambda_3' = \lambda_3 + \frac{(1+g)\gamma\tau(r-1-g)}{(1+\gamma\varepsilon)(1+g)^2 - \gamma\varepsilon r(1-\tau)g'}$, and $\lambda_{1,2,3}$ are given in the Appendix. Combining these two differential equations we arrive at

$$\frac{d\kappa}{d\varepsilon} = \frac{\delta_2 - \lambda_3'}{\lambda_1 + \lambda_2 - \delta_1}.$$

Knowing that $\delta_2 = 0$ in case pension benefit are not defined, and that $\lambda_3 > \lambda_3' > 0$, we can conclude that the capital-labour ratio rises to a smaller extent due to ageing if the social security scheme pays a fixed number of services to each retired individual. The change in the employment share of the commodity sector is

$$\frac{dL^Y}{d\varepsilon} = \delta_1 \frac{d\kappa}{d\varepsilon} + \delta_2.$$

Knowing that $\frac{d\kappa}{d\varepsilon}$ is smaller in case of a fixed-benefit scheme and $\delta_2 < 0$, it is clear that $\frac{dL^Y}{d\varepsilon}$ will be smaller if $\delta_1 \geq 0$, i.e., if $\sigma \leq 1$ (capital and labour being complements in the production of commodities), so the effects of population ageing on long-run growth are more moderate. But if $\sigma > 1$ (substitutability), such a conclusion can not be drawn since it is not clear whether $\left|\frac{dL^Y}{d\varepsilon}\right|$ is smaller or bigger than without fixing the PAYG-benefit.

6 Conclusions

Elderly people typically have two sources of income at their disposal: the assets they accumulated during their working life and the transfers they receive from the current young generation. It is well known that the second source will increasingly be under pressure with a rising dependency ratio that is due to population ageing. This does not mean, however, that the first income source is completely safe and should therefore be entirely relied upon. If many people save and thus invest, capital will become relatively abundant and the interest rate decreases, which implies a lower return on savings. This is especially so if the demand for labour increases due to a longer life span. Furthermore, elderly people are particularly vulnerable to high inflation rates, as this excavates the purchasing power of the capital they accumulated and depend so heavily upon. A factor that is of great importance in this respect is the fact that elderly people seem to spend relatively much on services that typically use a lot of labour. This makes them vulnerable for wage increases, which are due to productivity improvements caused by increasing capital accumulation and technological progress in the productive sector. Reducing the unfunded part of social security, a policy measure that is often presented as the solution to an ageing population, will only aggravate this and thus quickly lead to a situation of dynamic inefficiency.

Appendix

Steady-state equilibrium

Equation (8) implicitly gives the steady-state equilibria for $L_t^Y = L_{t+1}^Y$. In order for a steady-state equilibrium to exist, the following assumption with respect to the growth function is made.

Assumption I *The function g(x) is such that* $\exists x \in (0,1) \mid 1 - \frac{r\gamma\varepsilon(1-\tau)}{(1+\gamma\varepsilon)(1+g(x))} + \frac{\gamma\varepsilon\tau}{1+\gamma\varepsilon} = x.$

Appendix to Subsection 3.2: Privatisation in a closed economy

Comparative dynamics

Linearising the equilibrium condition for the commodity market at time $t+1$, equation (10), yields

$$dL_{t+1}^Y = \delta_1 d\kappa_{t+1} + \delta_2 d\tau_{t+1}, \tag{A.1}$$

with $\delta_1 \equiv \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)}$ and $\delta_2 \equiv \frac{\kappa r - f(\kappa)}{f(\kappa)}$. Linearising (9), which describes equilibrium on both the labour and services market, at time $t+1$ around the initial steady state gives

$$dL_{t+1}^Y = \lambda_1 d\kappa_t + \lambda_2 d\kappa_{t+1} + \lambda_3 d\tau_t + \lambda_4 d\tau_{t+1}, \tag{A.2}$$

where $\lambda_1 \equiv \frac{(1+g)\gamma(1-\tau)r\kappa f''(\kappa)}{((1+\gamma)(1+g)^2 - \gamma(1-\tau)rg')(f(\kappa)-\kappa r)}$, $\lambda_2 \equiv -\lambda_1 \frac{f(\kappa)}{\kappa r}$, $\lambda_3 \equiv \frac{(1+g)\gamma r}{(1+\gamma)(1+g)^2 - \gamma(1-\tau)rg'}$ and $\lambda_4 \equiv -\lambda_3 \frac{1+g}{r}$.
Combining these two equations yields the following first-order difference equation,

$$dL_{t+1}^Y = \frac{\lambda_1}{\delta_1 - \lambda_2} dL_t^Y + \frac{\lambda_1\lambda_4 - \delta_2\lambda_2}{\delta_1 - \lambda_2} d\tau_{t+1} + \frac{\delta_1\lambda_3 - \delta_2\lambda_1}{\delta_1 - \lambda_2} d\tau_t. \tag{A.3}$$

This first-order difference equation is stable if $\left|\frac{\lambda_1}{\delta_1 - \lambda_2}\right| < 1$, i.e.,

$$\left| \left(\frac{((1+\gamma)(1+g)^2 - \gamma(1-\tau)rg')(\sigma-1)(f(\kappa)-\kappa r)}{(1+g)\gamma r f(\kappa)} + \frac{f(\kappa)}{\kappa r)} \right)^{-1} \right| < 1.$$

Knowing that $f(\kappa) > \kappa r$ and taking Assumption III into account, it can easily be seen that this condition always holds if $\sigma \geq 1$. Furthermore, simulations with a linear growth function ($g_t = \bar{g} + \rho L_t^Y$) show that the stability condition also holds for all values of σ below unity for which a steady-state equilibrium exists.

Using the equilibrium condition for the capital market, it can be shown that $\delta_1 - \lambda_2 < 0$. This implies that the stability condition boils down to $\delta_1 - \lambda_1 - \lambda_2 < 0$.

Comparative statics

Setting $dL^Y_{t+1} = dL^Y_t = dL^Y$ and $d\tau_{t+1} = d\tau_t = d\tau$ in (A.3) gives

$$\frac{dL^Y}{d\tau} = \frac{\delta_1\lambda_3 - \delta_2\lambda_1 + \delta_1\lambda_4 - \delta_2\lambda_2}{\delta_1 - \lambda_1 - \lambda_2}. \tag{A.4}$$

The denominator of this equation is always negative. Furthermore, simulations with a linear growth function ($g_t = \bar{g} + \rho L^Y_t$) show that the nominator of (A.4) is always positive, so $\frac{dL^Y}{d\tau} < 0$.

Furthermore, equation (A.2) becomes in steady state $\left(\lambda_1\left(1 - \frac{f(\kappa)}{\kappa r}\right)\right)\frac{d\kappa}{d\tau} = \frac{dL^Y}{d\tau} - \lambda_3\left(1 - \frac{1+g}{r}\right)$. Knowing that $\lambda_1, \frac{dL^Y}{d\tau} < 0$, $\lambda_3 > 0$ and $1+g < r$, it follows that $\frac{d\kappa}{d\tau} < 0$.

Short-run effect expected privatisation

If the reduction in τ is announced at time $t = 0$ to take place at $t = 1$, the change in the capital-labour ratio can be found by differentiating (11), taking savings as given, which gives

$$\frac{d\kappa_0}{d\tau} = -\kappa\left(\frac{g'}{1+g} + \frac{1}{L^Y}\right)\frac{dL^Y_0}{d\tau}. \tag{A.5}$$

Furthermore, (A.1) boils down to

$$\frac{dL^Y_0}{d\tau} = \delta_1\frac{d\kappa_0}{d\tau}. \tag{A.6}$$

This implies that $\frac{d\kappa_0}{d\tau} = \frac{dL^Y_0}{d\tau} = 0$. As in a small open economy, young individuals decide to save more, but because they do not demand services, this merely implies a lower consumption demand for commodities and an equally higher investment demand for commodities, leaving the labour division unchanged.

Appendix to Subsection 4.2: Ageing in a closed economy

Linearising the equilibrium condition for the commodity market at time $t+1$, equation (10), yields

$$dL^Y_{t+1} = \delta d\kappa_{t+1}, \tag{A.7}$$

with $\delta \equiv \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)}$, and linearising (9) for time $t+1$, which describes equilibrium on both the labour and services market, gives

$$dL^Y_{t+1} = \lambda_1 d\kappa_t + \lambda_2 d\kappa_{t+1} + \lambda_3 d\varepsilon_{t+1}, \tag{A.8}$$

where $\lambda_1 \equiv \frac{(1+g)\gamma\varepsilon(1-\tau)r\kappa f''(\kappa)}{((1+\gamma\varepsilon)(1+g)^2-\gamma\varepsilon(1-\tau)rg')(f(\kappa)-\kappa r)}$, $\lambda_2 \equiv -\lambda_1 \frac{f(\kappa)}{\kappa r}$, $\lambda_3 \equiv -\frac{\gamma(1+g)(r(1-\tau)+\tau(1+g))}{((1+\gamma\varepsilon)(1+g)^2-\gamma\varepsilon(1-\tau)rg')(1+\gamma\varepsilon)}$. Combining these two equations yields the following first-order difference equation,

$$dL_{t+1}^Y = \frac{\lambda_1}{\delta - \lambda_2} dL_t^Y + \frac{\delta\lambda_3}{\delta - \lambda_2} d\varepsilon_{t+1}. \tag{A.9}$$

As was shown above, this first-order difference equation is stable.

Comparative statics

Setting $dL_{t+1}^Y = dL_t^Y$ and $d\varepsilon_{t+1} = d\varepsilon$ in (A.9) gives

$$\frac{dL^Y}{d\varepsilon} = \frac{\delta\lambda_3}{\delta - \lambda_1 - \lambda_2}. \tag{A.10}$$

Knowing that $\lambda_3 < 0$ and $\delta - \lambda_1 - \lambda_2 < 0$ (stability), it follows that $\frac{dL^Y}{d\varepsilon} \gtrless 0$ if $\sigma \lesseqgtr 1$. Furthermore, equation (A.7) together with (A.10) shows that $\frac{d\kappa}{d\varepsilon} > 0$ if $\sigma \neq 1$. If $\sigma = 1$, then $\delta = 0$ so $\frac{dL^Y}{d\varepsilon} = 0$ and $\frac{d\kappa}{d\varepsilon} = -\frac{r(1-\tau)+\tau(1+g)}{(1+\gamma\varepsilon)(f(\kappa)-\kappa r)\varepsilon f''(\kappa)(1-\tau)} > 0$.

Short-run effect expected ageing

Replacing $\frac{d\kappa_0}{d\tau}$ by $\frac{d\kappa_0}{d\varepsilon}$ and $\frac{dL_0^Y}{d\tau}$ by $\frac{dL_0^Y}{d\varepsilon}$ in (A.5) and (A.6) gives the same results as with expected privatisation.

Appendix to Subsection 5.2

Linearising (18) around the initial steady state,. we get the following expression,

$$\begin{aligned} dl_t^Y &= \frac{\gamma\varepsilon(1-\tau)rg'}{(1+\beta+\gamma\varepsilon)(1+g)^2(1+n)} dl_t^Y - \frac{\beta(1+n)\tau g'}{1+\beta+\gamma\varepsilon} dl_{t+1}^Y \\ &\quad - \frac{(1+\beta)d^o}{(1+n)(1+\beta+\gamma\varepsilon)} d\varepsilon_t + \frac{\gamma d^y}{1+\beta+\gamma\varepsilon} d\varepsilon_{t+1}. \end{aligned}$$

In case all shocks are foreseen as of time $t-1$, we can write this expression as follows,

$$dl_{t+1}^Y = \Phi dl_t^Y + \frac{\Delta_t}{\beta(1+n)\tau g'}, \tag{A.11}$$

with $\Phi \equiv \frac{\gamma\varepsilon(1-\tau)rg' - (1+\beta+\gamma\varepsilon)(1+g)^2(1+n)}{\beta(1+g)^2(1+n)^2\tau g'} < 0$ (due to Assumption III) and $\Delta_t \equiv \gamma d^y d\varepsilon_{t+1} - \frac{(1+\beta)d^o}{1+n} d\varepsilon_t$. Because l^Y is a forward-looking variable and there is no backward-looking variable in this economy, the solution of (A.11) results from iterating this equation forward. The state of the economy at time t can thus be written as a function of its future state at time $t+s$ and all shocks between $t-1$ and $t+s$,

$$dl_t^Y = \Phi^s dl_{t+1}^Y - (\beta(1+n)\tau g')^{-1} \sum_{j=0}^{s-1} \Phi^j \Delta_{t+j}.$$

Letting $s \to \infty$, the change in l_t^Y is written as a function of the change in its steady-state value and all future shocks. For the steady-state equilibrium to be well-defined, we therefore need to assume that $|\Theta| < 1$. With assumption III, the assumption that $\beta(1+n)^2(1+g)^2\tau g' > (1+\beta+\gamma\varepsilon)(1+g)^2(1+n) - \gamma\varepsilon(1-\tau)rg'$ assures that the steady-state equilibrium is asymptotically stable. Then, the change in l_t^Y can merely be written as a function of the sequence of exogenous future shocks,

$$dl_t^Y = -(\beta(1+n)\tau g')^{-1} \sum_{j=0}^{s-1} \Phi^j \Delta_{t+j}.$$

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