GENERAL-EQUILIBRIUM EFFECTS OF PRIVATISATION: THE MISSING PIECE IN SOCIAL SECURITY REFORM

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General-Equilibrium Effects of Privatisation: The Missing Piece in Social Security Reform

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Abstract
This paper analyses the effects of reducing unfunded social security in a closed economy that consists of a service sector and a commodity sector. It is shown that if old agents mainly demand labour intensive services, a modest decrease of the pay-as-you-go pension scheme still raises long-run utility as long as the economy is dynamically efficient. However, entirely privatising the social security system will sooner lead to dynamic inefficiency than in the conventional one-sector model, leading to a different conclusion about the desirability of unfunded pensions.

JEL classification: D91, E60, H55, J14
Keywords: overlapping generations, pensions, privatisation, social security reform

1 Introduction
Many developed countries are and will increasingly be confronted with an ageing of their populations due to lower fertility rates and a longer life span. These upcoming demographic changes will cause the relative number of elderly individuals to increase, and thus urge many to reconsider the design of existing social security arrangements that involve transfers between generations. One of the main concerns is the sustainability of unfunded Pay-As-You-Go (PAYG) pension schemes and health care arrangements that are widely observed. As a matter of fact, contributing a constant

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fraction $\tau$ of the wage to the PAYG-scheme when young yields a pension benefit of $\tau(1+n)(1+g)$ when old, where $n$ is the rate of population growth and $g$ denotes the rate at which wages grow. Hence, as Aaron (1966) pointed out, the internal rate of return from contributing to an unfunded social security scheme equals $(1+n)(1+g)$. But investing the same amount on the capital market yields a rate of return equal to the interest rate $r$. So as long as the economy is dynamically efficient, i.e., if $r > (1+n)(1+g)$, a social security scheme decreases long-run welfare and individuals would be better off if it were reduced. In case of dynamic inefficiency, individuals save too much, in the sense that the capital stock is above the level at which consumption per capita is maximal. This is due to the fact that individuals do not take into account that increasing their own savings implies a lower interest rate for all individuals and thereby reduces the return to the savings of others.\(^1\) Diamond (1965) showed that in that case, government intervention is desirable: by transferring resources from young to old agents (e.g. through government debt or a PAYG-pension scheme), welfare is increased.

Population ageing deteriorates the internal rate of return of unfunded schemes compared to the return that can be obtained from investing in physical capital. The plea for replacing unfunded pension schemes by funded ones (often referred to as privatising social security) is therefore more often heard, because it would raise long-run welfare. Martin Feldstein, for instance, asserted that a complete and fast transition from unfunded to funded schemes by privatising social security was just the missing piece in social security analysis (see Feldstein 1995, 1996). That piece was soon found back: in the 2000 US presidential campaign, social security reform was a key issue. However, an important piece seems to be missing in Feldstein’s analysis.\(^2\)

A shift to a fully-funded system will boost savings\(^3\) and thereby lead to a lower interest rate. The lower interest rate induces people to substitute away from old-age consumption towards young-age consumption. Although this is dictated by individual

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\(^1\)Naturally, this does not hold for a small open economy that faces an exogenously given (world) interest rate. This paper therefore focuses on a closed economy.

\(^2\)One missing piece is the negative effects on the utility of the elderly alive at the time of the transition to a privatised system. Social security reform is therefore not a Pareto-improving policy, as was demonstrated by Verbon (1988) and Breyer (1989). In this paper, we restrict ourselves to the long-run consequences of privatisation.

\(^3\)As was shown by e.g. Feldstein (1974), Kotlikoff and Summers (1981) and Modigliani (1988), transfers from the young to the old, such as unfunded social security, discourage private savings, which Feldstein dubbed the asset-substitution effect.
utility maximization, the result might be that old people will have rather meagre levels of consumption, while the young ones have abundant consumption streams. On the other hand, if people wish to have a fairly constant standard of living over their lifetime, and thus have a low elasticity of substitution between young and old-age consumption, they will decide to save even more than due to the asset-substitution effect alone so as to transfer part of the higher income when young to their old age. But the higher savings intensify the general-equilibrium effects, making individual savings less effective as a device to smooth consumption over the lifetime.

This result will hold even stronger if people do not spend their money on the same items during their entire lifetime and currently living generations face different inflation rates as suggested in a recent issue of *The Economist*:

“One consequence of wide changes in relative prices is that individuals may face widely differing inflation rates, if their spending differs much from the national average. Older people, for instance, are likely to spend more on medical care and domestic services, such as those of gardeners and cleaners, that have risen sharply in price. Younger people are likely to spend more on new products, such as mobile phones or computers, which tend to fall rapidly in price during their early years.” (The Economist “The Price of Age”, Dec. 21st 2000)

It seems indeed plausible to assume that the elderly spend relatively more on labour-intensive services, both because of disability (which raises the demand for e.g. care services and housekeeping assistance) and a lower preference for (high-tech) commodities, because the varieties of this kind of commodities constantly changes due to innovation, and elderly people have more difficulties learning how to apply these new technologies and products. The price index of the elderly’s consumption basket is then particularly sensitive to wage inflation. As noted above, such wage inflation is one of the general-equilibrium effects that accompany a transition to funded pensions. So, if old-age consumption requires relatively much labour, the gap between young-age and old-age consumption resulting from privatisation will be even larger, and closing this gap through higher savings will be harder. More capital accumulation will only intensify the decrease in the real interest rate but not increase old-age consumption possibilities. It seems that this is a missing piece in the debate on social security reform. The question this paper therefore addresses is whether the condition of dynamic efficiency is still decisive with respect to the desirability of a PAYG-public pension scheme. In other words, does it still hold in a multi-sector economy with dif-
ferent generational demand patterns that privatising social security raises (long-run) welfare as long as the interest rate exceeds the rate of economic growth?

It appears that this question can be answered affirmatively, at least for the case of a small reduction of the unfunded part of social security: partially privatising social security in a dynamically-efficient economy increases individuals’ lifetime income, both in terms of commodities and services, and utility. However, it is likely that after having totally privatised social security, the economy will end up in a situation of dynamic inefficiency, something that may not happen in the standard one-sector economy as applied by e.g. Feldstein. In that case, the optimal PAYG-tax is strictly positive and complete privatisation of social security is not the best policy.

The remainder of this paper is organised as follows. In Section 2, we present a two-sector, 2-OLG closed economy with a government that runs a PAYG-public pension scheme. In one production sector, both physical capital and labour are employed to produce commodities that are bought by young individuals for consumption and investment purposes. The other sector deals with the provision of services that do not use capital goods. These services are demanded by old individuals only. In Section 3, we look at how dynamic (in)efficiency is linked to the desirability of PAYG-pensions. In Section 4, the optimal social security tax is calculated and compared to the one that follows from the standard one-sector model. Section 5 concludes.

2 The model

Production of commodities and services

Following Baumol (1967), we distinguish two sectors of production. In the commodity sector (labelled Y), homogeneous goods are produced that either serve as consumption or investment good. The production process involves the employment of both physical capital and labour according to the following Cobb-Douglas production function: \( Y_t = K_t^\alpha (L_t^Y)^{1-\alpha} \), where \( K_t \) stands for the domestic capital stock and \( L_t^Y \) is the number of people employed in the commodity sector.\(^4\) Production per employee is described by \( \kappa_t^\alpha \), where \( \kappa_t \equiv \frac{K_t}{L_t^Y} \) denotes the capital-labour ratio. Firms are fully competitive and maximise profits, so the interest and wage rate are given by \( r_t = \alpha \kappa_t^{\alpha-1} \) and \( w_t = (1 - \alpha) \kappa_t^\alpha \) respectively.

In the service sector (labelled D), labour is the only factor of production.\(^5\) One

\(^{4}\)Allowing for a more general CES production function does not alter the results.

\(^{5}\)These are both personal services (domestic services such as gardeners, cleaners, housekeepers,
unit of labour translates into one service, so the total provision of services equals total labour supply in this sector \( L_t^D \). The price of services in terms of commodities, \( p_t \), is therefore equal to the wage rate in this sector. As labour is homogeneous and and perfectly mobile across sectors, there is only one wage rate, so \( p_t = w_t \).

**Households**

In each period, a young and an old generation of constant size \( N \) are alive. Individuals inelastically supply one unit of labour when young and are retired in the second period of their life. To simplify the analysis, we will assume young individuals to consume only commodities \( c \), while elderly demand services \( d \) only. Lifetime utility of a non-altruistic representative agent born at any time \( t \) is represented by a CIES utility function,

\[
U(c_t, d_{t+1}) = \frac{c_t^{1-\theta}}{1-\theta} + \gamma \frac{d_{t+1}^{1-\theta}}{1-\theta},
\]

where \( \gamma \) is the private discount factor and \( \theta > 0 \), so that the elasticity of substitution is the constant \( 1/\theta \).

The government runs a PAYG social security scheme financed by a wage tax \( \tau \), so the benefit level at time \( t \) equals \( \tau w_t \). Individual consumption possibilities are thus given by the following intertemporal budget constraint,

\[
c_t + \frac{w_{t+1}d_{t+1}}{r_{t+1}} = (1-\tau)w_t + \frac{\tau w_{t+1}}{r_{t+1}}.
\]

Assuming perfect foresight, maximising (1) subject to (2) results in the following first-order condition,

\[
\frac{c_t}{d_{t+1}} = \left( \frac{w_{t+1}}{\gamma r_{t+1}} \right)^\frac{1}{\theta}.
\]

This condition shows that the ratio of young-age consumption over old-age services is determined by the real interest rate \( \frac{r_{t+1}}{w_{t+1}} \) (unless they are perfect substitutes, i.e., butlers, and recreation and cultural services) and social services like nurses and physicians. Note that these are not services like bank services which extensively use high-tech goods like computers.

6 Another interpretation would be that the quality of services increases with the number of employees.

7 Allowing for population growth does not alter the results.

8 Note the similarity with Samuelson (1958). In that model, individuals are not able to save at all because the goods are perishable. In our model, people can save, but when old, they do not derive utility from the goods that they stored when young. Instead, they have to exchange these commodities for services by paying a certain price to the young.
\( \theta \to \infty \). Individual demand and saving functions are given by

\[
\begin{align*}
  c_t &= \frac{w_t(1 - \tau)}{1 + \Gamma_t} + \frac{w_{t+1} \tau}{r_{t+1}(1 + \Gamma_{t+1})}, \\
  d_{t+1} &= \frac{\Gamma_{t+1} w_t(1 - \tau) r_{t+1}}{(1 + \Gamma_{t+1}) w_{t+1}} + \frac{\Gamma_{t+1} \tau}{1 + \Gamma_{t+1}}, \\
  s_t &= \frac{\Gamma_{t+1}(1 - \tau) w_t}{1 + \Gamma_{t+1}} - \frac{w_{t+1} \tau}{r_{t+1}(1 + \Gamma_{t+1})},
\end{align*}
\]

with \( \Gamma_t \equiv \gamma^\frac{1}{\phi} \left( \frac{w_t}{w} \right)^{\frac{\phi}{\phi - 1}} \).

The model presented above comprises four markets, all characterised by fully flexible prices and are therefore simultaneously in equilibrium at each point in time.

**The commodity market**

Demand for commodities only comes from young individuals who spend their entire after-tax wage income on commodities, both as consumption and investment. Accordingly, the equilibrium condition reads \( Y_t = N(1 - \tau) w_t \), which in case of the specified production function boils down to

\[
l^Y_t = (1 - \alpha)(1 - \tau),
\]

with \( l^Y_t \equiv L^Y_t / N \). So as long as the social security tax is constant, the share of young individuals employed in the commodity sector is constant as well. If the tax is increased, young agents have less to spend on commodities and fewer people will be active in the commodity sector.

**The services market**

As described before, one service requires the input of exactly one employee, so the total provision of services is \( L^D_t \), which equals the total demand for services at time \( t \), so \( L^D_t = N d_t \).

**The labour market**

The total labour force at time \( t \) consists of all young individuals, who each inelastically supply one unit of labour in one of the two sectors.\(^9\) Equilibrium on the labour market therefore implies that

\[
L^Y_t + L^D_t = N.
\]

\(^9\)One could also assume that every person works part-time in both sectors.
This, together with equilibrium on the commodities and services market, gives the individual number of services that each person uses when old,

\[ d_t = \alpha + \tau - \alpha \tau. \tag{8} \]

Although the individual demand for old-age consumption will generally depend on the substitution elasticity, the services that will be supplied to a retired person depend, according to equation (8), only on the production elasticity \( \alpha \) and the social security tax \( \tau \).

**The capital market**

Since there is no scope for international lending and borrowing, aggregate savings are entirely invested domestically. If capital fully depreciates after one period,\(^{11}\) this implies that the capital market clears when \( N_s = K_{t+1} \) holds, or

\[ \kappa_{t+1} = \frac{s_t}{(1 - \tau)(1 - \alpha)} = \frac{\kappa_t \Gamma_{t+1}}{1 + \Gamma_{t+1}} - \frac{\kappa_{t+1}}{\alpha(1 - \tau)(1 + \Gamma_{t+1})}. \tag{9} \]

From this, the steady state capital-labour ratio can be calculated. Totally differentiating this expression gives the following difference equation

\[
\frac{\alpha(1 - \tau)(1 + \theta \Gamma) + \tau}{\alpha(1 - \tau)(1 + \Gamma)\theta} d\kappa_{t+1} = \frac{\alpha \Gamma \kappa^{\alpha - 1}}{1 + \Gamma} d\kappa_t - \frac{\kappa}{\alpha(1 - \tau)^2(1 + \Gamma)} d\tau, \tag{10}
\]

so the steady state is stable if \( \frac{\alpha \Gamma \kappa^{\alpha - 1}}{1 + \Gamma} - \frac{\alpha(1 - \tau)(1 + \Gamma) \theta}{\alpha(1 - \tau)(1 + \theta \Gamma)^{\tau}} < 0 \). This is assumed to be the case.

### 3 Privatising social security

Privatisation of social security as proposed by Feldstein is one big jump from an unfunded to a funded scheme. A less extreme approach would be to reduce the PAYG-tax to a permanently lower but still positive level. The option we consider first is a marginal decrease, so if the economy is initially dynamically efficient, it

\(^{10}\)This result is due to the Cobb-Douglas specification of production. For more general specifications, the parameter \( \theta \) will affect the equilibrium quantity of \( d_t \) as well, but the positive relation between \( \tau \) and \( d_t \) remains.

\(^{11}\)Allowing for depreciation would not change the results.
can be assumed to remain so. After that, we analyse a complete privatisation of social security. In contrast to what Feldstein (1996) assumes, the economy may then become dynamically inefficient, especially in the two-sector model we apply.

In a dynamically-efficient economy, a marginal reduction of the social security tax raises lifetime income (*ceteris paribus*). Moreover, the reverse asset-substitution effect induces the young to save more. This leads to the following proposition.

**Proposition 1** In the two-sector model as presented above, a PAYG-system reduces the long-run capital-labour ratio. (Partially) privatising social security therefore increases $\kappa$.

**Proof** From (10) it follows that

$$ \frac{d\kappa}{d\tau} = \frac{\kappa}{\alpha(1 - \tau)^2(1 + \Gamma)} \left( \frac{\alpha\Gamma\kappa^{\alpha-1}}{1 + \Gamma} - \frac{\alpha(1 - \tau)(1 + \Gamma)\theta}{\alpha(1 - \tau)(1 + \theta\Gamma) + \tau} \right) < 0. \ (11) $$

With a lower PAYG-tax, people have a stronger incentive to save,$^{12}$ because a smaller part of their old-age income is provided through the government by their children. These higher national savings imply a higher capital stock. However, as can be seen from (6), less labour is attracted to the services sector, leaving more employees to work with the capital stock. Because the first effect is dominant, the capital labour ratio increases.$^{13}$

In a closed economy, the return to the funds invested at the capital market will be lower, and at the same time, employees can work with a higher number of capital goods, so their productivity and thereby their wage rate increases. In order to prevent labour moving out of the services sector, wages will rise in that sector too, making services more expensive.$^{14}$ An individual is therefore confronted with higher earnings when young, enabling him to purchase more commodities which raises his utility, but when old, he faces both a lower return to his savings and a higher inflation, so the purchasing power when old could decrease.

$^{12}$From the equilibrium condition for the capital market, $s = \kappa(1 - \alpha)(1 - \tau)$, it follows that $\frac{ds}{d\tau} = (1 - \alpha)(1 - \tau)\frac{d\kappa}{d\tau} - \kappa(1 - \alpha) < 0$.

$^{13}$If $\kappa$ would decrease if $L^Y$ increased strongly. But then, the interest rate would increase and the wage would decrease, thus stimulating the demand for services, which contradicts the sharp rise of $L^Y$.

$^{14}$This is the well-known Baumol-effect: services become more expensive because of rising productivity in the capital-intensive sector.
Proposition 2 Marginally reducing the unfunded part of pension schemes leads to a higher lifetime income if the economy is dynamically efficient, both in terms of commodities and (current) services. The consumption level for young individuals rises whereas the number of services enjoyed when old decreases.

Proof Lifetime income measured in commodities is equal to $z \equiv (1 - \tau)w + \frac{\tau w}{r}$, so $\frac{dz}{dr} = w \left( \frac{1 - \tau}{r} \right) + (1 - \alpha)\alpha^\alpha - 1 \left( \frac{1 - \tau}{r} \right) + \kappa^{\alpha - 2} w \tau \tau - 2 \frac{ds}{dr}$, which is negative if $r > 1$. Lifetime income in terms of services is $z/w = 1 + \tau \left( \frac{1}{r} \right)$, so $\frac{dz/w}{dr} = \frac{1 - \tau}{r} + (1 - \alpha)\alpha^{\alpha - 2} \tau \tau - 2 \frac{ds}{dr}$, which is also negative if the economy is dynamically efficient. Equilibrium on the capital market can be written as $\kappa = \kappa^\alpha - \frac{r}{(1 - \alpha)(1 - \tau)}$, so $\frac{dc}{dr} = (1 - \alpha)\left( \frac{\alpha^\alpha - 1}{\alpha(1 - \tau)} \right) \left( (1 - \tau) \frac{ds}{dr} - \kappa \right)$, Inserting $\kappa^{\alpha - 1}$ that follows from (9) thus gives $\frac{dc}{dr} = (1 - \alpha) \left( \frac{\alpha^\alpha - 1}{\alpha(1 - \tau)} \right) \left( (1 - \tau) \frac{ds}{dr} - \kappa \right) < 0$. Furthermore, equation (8) immediately results in $\frac{dd}{dr} = 1 - \alpha > 0$. 

So the Aaron-condition also holds in the two-sector model. The change in consumption when young and old is due to several effects. The first is the positive income effect of a lower social security tax on lifetime income, which raises $c$ and $d$. Second, the interest rate decreases. This causes a negative income effect on $c$ and $d$, a positive substitution effect on $c$ and a negative substitution effect on $d$. Third, the wage rises, which implies a higher labour income and thus has a positive income effect on both $c$ and $d$, but because it is also the price of services, it implies a positive substitution effect on $c$ and a negative substitution effect on $d$. The total effect is a higher consumption level of commodities when young and a lower amount of services when old. Table 1 summarises these effects.

<table>
<thead>
<tr>
<th>$\tau \downarrow$</th>
<th>$r \downarrow$</th>
<th>$w \uparrow$</th>
<th>total</th>
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<tr>
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<td>$c$</td>
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<td>$d$</td>
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Table 1 Effects of (marginally) reducing the social security tax

The question then is whether lifetime utility increases as well if the social security system is privatised. The most obvious way to evaluate this is to compare the implicit return to individuals from contributing to the unfunded social security scheme (keeping the contribution rate constant) with the explicit return that can be obtained from investing the same amount at the capital market, which in our model corresponds to
the interest rate. If the latter exceeds the former, which seems to be the case in most countries, the economy is said to be dynamically efficient and transferring an amount through a PAYG-scheme enables individuals to purchase less commodities when old than what they could have bought if the same amount were invested in a funded scheme. Consequently, reducing the unfunded part of social security raises long-run welfare, and the reverse holds in a dynamically-inefficient economy. However, this reasoning does not take account of general-equilibrium effects and the different spending patterns of young and old generations. In particular, contributing to the unfunded social security scheme via a proportional wage tax yields a ‘return’ that is linked to the wage that the next generation earns, which is also the price of the services an individual buys when old. Hence, a PAYG-scheme yields a fixed return in terms of services. On the other hand, investing at the capital mark through a funded scheme gives a return in commodities, so that the elderly are sensitive to price increases. One can wonder whether the higher consumption level when young compensates for the loss of services when old in terms of lifetime utility. That is, does a reduction of the social security tax raise lifetime utility in the two-sector model?

**Proposition 3** (Marginally) reducing the unfunded part of pension schemes implies a higher lifetime utility.

**Proof** The effect of a marginal change of the social security tax on lifetime utility can be traced by linearising around the initial steady state. Utility changes according to

\[
\frac{dU}{d\tau} = c^{-\theta} \frac{dc}{d\tau} + \gamma d^{-\theta} \frac{dd}{d\tau} = (1 + \Gamma) \frac{dc}{d\tau} - \frac{\alpha}{\theta \kappa} \frac{dc}{d\tau}. \tag{12}
\]

After differentiating (3) we arrive at

\[
\frac{dU}{d\tau} = \left[ \frac{1 - \alpha}{\alpha} \kappa(1 - r) + \frac{dc}{d\tau} \left( \alpha(1 - \alpha)(1 - \tau) \kappa^{\alpha - 1} + \frac{\alpha(1 - \alpha)}{\alpha} - \frac{\alpha}{\kappa} \right) \right].
\]

\[ \frac{dU}{d\tau} \]

Clearly, the first term is negative if the economy is dynamically efficient, i.e., if \( r = \alpha \kappa^{\alpha - 1} > 1 \), and positive in the case of dynamic inefficiency. From (9) it follows that \( \kappa^{\alpha - 1} = \frac{\alpha \Gamma}{1 - r} \left( 1 + \frac{\tau}{\alpha(1 - \tau)(1 + \Gamma)} \right) \), so dynamic efficiency is equivalent to \( \alpha(1 - \tau)(1 + \Gamma) + \tau - (1 - \tau)\Gamma > 0 \). Inserting this and (3) into (12) implies that the last term of (12) is positive (negative) if the economy is dynamically efficient (inefficient). Knowing that \( \frac{dc}{d\tau} < 0 \), it follows that \( \frac{dU}{d\tau} \geq 0 \) if \( r \geq 1 \). □

One can conclude from this that despite the fact that individuals face different prices over their lifetime, the golden rule still holds in the two-sector model: lifetime utility is maximal if the interest rate equals the rate of economic growth. If the social security tax is such that \( \frac{dU}{d\tau} > 0 \), people save too much in the sense that the utility of
current and future generations is increased when the PAYG-scheme is (marginally) extended. Elderly then benefit from a higher social security benefit, and the current young and all future generations will have a higher lifetime income. This is so because individuals do not take into account that their own savings have a downward effect on the (future) interest rate and an upward effect on the (future) wage, which also reflects the price of the services they will enjoy when old. By creating a disincentive to save, the government internalises this negative externality and thereby increases welfare. In the opposite case (dynamic efficiency), a marginal increase of the PAYG-tax reduces the utility of currently young individuals and all future generation (although it still increases the utility of the current elderly), because their lifetime income is reduced when the inefficient unfunded scheme is extended. Therefore, the proposal to decrease the unfunded part of social security and simultaneously increase the funded part would still benefit current young and future generations if \( r > 1 \).

However, this conclusion is based on a small reduction of the PAYG-scheme. Some proposals encompass the entire abolition of the PAYG-scheme, for this would bring about maximum welfare gains. The following section deals with this issue.

### 4 Abolishing PAYG: When does it go wrong?

If the social security system is marginally privatised, lifetime income increases in a dynamically-efficient economy, both in terms of commodities and services. Figure 1 shows consumption when young and services when old as a function of the social security tax, for two different substitution elasticities. From this figure, we can see that a reduction of the social security tax always goes along with lower service provision to the elderly. If people prefer a smooth consumption pattern over their lifetime (i.e., if \( \theta \) is rather high) they will try to carry over a large part of the net-income gain when young to their old-age by saving more. We know from equation (2), however, that market forces will not allow larger old-age consumption: higher savings only cause wage inflation which diminishes retirees’ purchasing power. For low tax rates (\( \tau < 0.1 \)), the attempts to close the gap between consumption of commodities and services may eventually even result in lower young-age consumption. Obviously, complete privatisation of social security will then not be the optimal policy. That is,

15 The underlying parameter values of this picture are \( \alpha = 0.3, \gamma = 0.7 \) and \( \theta = 0.5 \) and 2 respectively. Furthermore, \( L^Y \) is multiplied by 2 in the production function.
the economy will become dynamically inefficient at a positive tax rate.\footnote{As a matter of fact, the economy is only dynamically efficient in Figure 1 for values of $\tau$ exceeding 19\% if $\theta = 2$ and 10\% if $\theta = 0.5$.}

![Figure 1](image_url)

Figure 1 Level of consumption when young and services when old

Of course, since Diamond’s seminal (1965) paper it is well-known that in a one-sector model the market can generate dynamic inefficiency as well. In a two-sector economy, however, the different spending patterns of generations spur the savings incentives of privatisation, thus creating a larger bias towards dynamic inefficiency. This can clearly be seen if one compares the golden-rule tax rate for a one-sector and a two-sector model.

The capital-labour ratio that coincides with the golden rule in our two-sector model equals $c_1 = (1 - c^\alpha) / (1 - \mu)$, which determines the value of $\Gamma$ at $\Gamma^{GR} = \gamma^{1/\theta} \left( \frac{\alpha/(\alpha-1)}{1-\alpha} \right)^{(1-\theta)/\theta}$. From (9) we can then calculate the social security tax that accords with the golden rule for the two-sector model: $\tau_{2s}^{GR} = \Gamma^{GR - \alpha(1+\Gamma^{GR})} / (1-\alpha)(1+\Gamma^{GR})$.

A one-sector model

Suppose individuals consume the same items when young and old. In that case, we can replace $d_{t+1}$ in the utility function by $c_{t+1}^\alpha$, and the intertemporal budget...
constraint becomes
\[ c_t + \frac{\sigma_{t+1}}{r_{t+1}} = (1 - \tau)w_t + \frac{\tau w_{t+1}}{r_{t+1}}. \]

The first-order condition for which utility is maximised is then \( \sigma_{t+1} = (\gamma r_{t+1})^{1/\theta} c_t \), so that individual savings can be calculated to be
\[
 s_t = (1 - \tau)w_t - c_t = \frac{(1 - \tau)w_t \Delta_{t+1}}{1 + \Delta_{t+1}} - \frac{\tau w_{t+1}}{r_{t+1}(1 + \Delta_{t+1})},
\] (13)

with \( \Delta_{t+1} \equiv \gamma^{1/\theta} (r_{t+1})^{(1-\theta)/\theta} \). Equilibrium on the capital market implies \( N s_t = K_{t+1} \), so that \( \kappa_{t+1} = s_t \). Combining this with (13) and again assuming a Cobb-Douglas production function, the resulting steady-state capital-labour ratio for the one-sector economy can be calculated, and the interest rate equals \( r^{1s} = \left( \frac{\alpha(1+\Delta)+\tau}{(1-\alpha)\Delta(1-\tau)} \right) \). Hence, the social security tax for which the golden rule is achieved in the one-sector model is \( \gamma^{GR}_{1s} = \frac{(1-2\eta)\gamma^{1/\theta}}{1+(1-\alpha)\gamma^{1/\theta}} \).

Figure 2 shows these social security taxes that belong to the golden rule as a function of \( \theta \).
Note that in the two-sector model the optimal tax rate is strictly positive for nearly all values of $\theta$. Only if extremely high elasticities of substitution prevail complete privatisation of social security will unambiguously imply long-run welfare gains. The difference with a one-sector model is striking. For a large range of substitution elasticities (i.e., for $0.3 < \theta < 1.25$) privatisation increases welfare in the one-sector economy, but not necessarily in the two-sector economy. If this is the relevant range to consider, then Feldstein’s assertion that the PAYG-system should be abolished in order to raise the utility of future generations only makes sense if the economy looks like a one-sector economy, or, in other words, when the consumption patterns of young and old individuals do not differ too dramatically. If, however, the elderly consume relatively many labour-intensive services, pleas for privatisation are very misleading. In that case, the best policy would be to increase rather than decrease the social security tax. As shown in Figure 2, the optimal tax rate for a substitution elasticity around one is almost 17%, a value that is very much unprecedented in the US. In reality, the curve will lie somewhere in between, but nevertheless, the optimal tax is positive for realistic values of $\theta$.

5 Conclusion

Feldstein’s claim that abolishing PAYG-schemes raises long-run welfare is missing an important point. If old individuals mostly rely on labour-intensive services, then extra savings, generated by privatisation of social security, create general-equilibrium effects that corrode their purchasing power. A vicious circle might arise: individuals save more in order to protect themselves against rising prices of services that are urgently needed in old age. However, these additional savings become futile, as they lead to even higher prices of these services. From that perspective, privatising social security may one day turn into a political dead-end street. The position of the elderly then mimics their position in the Great Depression when the value of their savings dwindled to negligible values, prompting the introduction of social security by president Roosevelt.
References


