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'BE NICE, UNLESS IT PAYS TO FIGHT': A NEW THEORY OF PRICE DETERMINATION WITH IMPLICATIONS FOR COMPETITION POLICY

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'Be nice, unless it pays to fight': a new theory of price determination with implications for competition policy

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Abstract

This paper introduces a simple extensive form pricing game. The Bertrand outcome is a Nash equilibrium outcome in this game, but it is not necessarily subgame perfect. The subgame perfect equilibrium outcome features the following comparative static properties. The more similar firms are, the higher the equilibrium price. Further, a new firm that enters the industry or an existing firm that becomes more efficient can raise the equilibrium price. The subgame perfect equilibrium is used to formalize price leadership, joint dominance and efficiency offence.

Keywords: Bertrand paradox, price leadership, mergers, joint dominance, efficiency offence

J.E.L. codes: D43, L11, L41

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When a move by one seller evidently forces the other to make a counter move, he is very stupidly refusing to look further than his nose if he proceeds on the assumption that it will not.

Chamberlin (1969: 46).

1 Introduction

This paper introduces an intuitive pricing game where firms undercut each other to gain market share. The Bertrand outcome is a Nash equilibrium in this game but it is not necessarily subgame perfect. The interpretation of the subgame perfect equilibrium is that firms behave nicely towards each other, unless it pays to fight. To illustrate, in a duopoly context, it only pays a firm to fight its opponent if the opponent is substantially weaker (less efficient) than the firm itself. If the two firms have similar cost levels, it is more profitable to be "nice", that is to charge a high price. Hence the more similar firms are, the higher the equilibrium price. This effect of cost distribution on conduct is not due to (explicit) collusion. It is the outcome of a pricing game where firms act independently, but understand Chamberlin's observation above.

Although this outcome does not seem unnatural, the subgame perfect equilibrium has surprising comparative statics results. A rise in the number of firms in the market or an increase in efficiency for some firms in the market can raise the equilibrium price by reducing the incentive to behave aggressively. This subgame perfect equilibrium outcome formalizes notions like price leadership, joint dominance, efficiency offence and gives a solution to the Bertrand paradox.

The main idea underlying the analysis can be summarized as follows. The cost distribution in an industry is a major determinant of how competitive or aggressive firms' conduct will be in the industry. This is not the case with the two most used formalizations of competition in economics: Cournot and Bertrand competition. To illustrate, consider the following simple example. There are two industries, denoted I and II, which have the same demand curve for a homogenous good: $X(p) = 1 - p$, where $X(p)$ denotes total demand at price p. In each industry there are three firms. Consumers buy from the cheapest firm(s) only and if more than one firm charges this lowest price p they share the market equally (i.e. consumers randomize to choose a supplier since all firms are identical from their perspective). The industries differ in their cost structure. In industry I the constant marginal costs of firms $1, 2$ and 3 are respectively: $c_1^I = 0, c_2^I = 0.35, c_3^I = 0.4$. In industry II the cost distribution is: $c_1^{II} = 0, c_2^{II} = 0.1, c_3^{II} = 0.4$. In words, in industry II firm 2 is closer to firm 1 in terms of efficiency as compared to industry I. Both Cournot and Bertrand competition predict that the price in industry II is lower than in industry I. This is not always a compelling prediction. As shown in section 3.1, the subgame perfect equilibrium of the pricing game introduced in this paper predicts that the outcome in industry I is more competitive than in industry II. The intuition is the following. In industry I, it pays firm 1 to price very aggressively and keep firms 2 and 3 out of the market. That is, firm 1 chooses the limitprice $p^I = c_2^I = 0.35$ and is the only seller in the market. However, in industry II it is very costly for firm 1 to keep both firm 2 and 3 out of the market because firm 2 is so close in efficiency to 1. So, because $c_2^{II} - c_1^{II}$ is small, it is more profitable for 1 (and 2) to keep only firm 3 out of the market and share the market. Thus, $p^{II} = c_3^{II} = 0.4$. Put differently, in industry II there is a balance of power[1] between firms 1 and 2. Because these firms have similar efficiency levels it is very costly for them to fight each other. In industry I, however, firm 1 is a lot more efficient than firm 2 and hence it pays to fight instead of being nice and sharing the market with firm 2.

Hence the cost distribution, or more precisely the cost gaps between firms, determines how aggressive firms play and how competitive the outcome is in the industry. So a reduction in some firm's cost level (like a reduction in firm 2's cost level from c_2^I to c_2^{II} in the example above) or entry by a new firm can close a cost gap, thereby making the nice outcome more profitable than the aggressive option.

This idea that the cost distribution affects how aggressive firms play, yields six new insights. First, it gives a formalization of the idea of price leadership. The price leader(s) in the framework here is (are) the most efficient firm(s) in

[1] Clearly, this is the same intuition for why the USA and USSR have avoided direct military conflict in the 20th century. A real war (instead of a cold one) with a foe of comparable strength would have been too costly. Also, both countries have picked fights with opponents that they considered rather weak.

the industry. The idea is that if any firm has an incentive to reduce the price at the current price p, it is always profitable for the most efficient firm(s) to reduce the price as well. In this sense, the most efficient firms act as price leaders.

Second, the theory sheds light on the efficiency defense in the case of mergers. The efficiency defense was formalized by Williamson (1968)[2]. He modelled a merger as having two opposite effects on the equilibrium price. On the one hand, a merger reduces the number of firms in the industry and hence tends to raise the price. On the other hand, the merger may through economies of scale and scope raise the efficiency of the merged firms and hence reduce the price. If the efficiency gain is big enough, it may outweigh the former effect on the number of firms and hence the merger reduces price and raises consumer welfare. The efficiency defense, in this view, boils down to showing that the efficiency gains are big enough for the merger to raise welfare and hence it should not be opposed by competition authorities. However, the analysis here suggests that if the efficiency gain closes the gap between the current price and the cost level of price leader, it may well lead to a rise in the equilibrium price. Moreover, a rise in the equilibrium price is more likely if the efficiency gain is big. This can be viewed as a form of an efficiency offence (see Röller et. al (2000)).

Third, the efficiency offence result where big efficiency gains through merger cause an increase in the equilibrium price can also be seen as a formalization of the joint dominance doctrine. The idea of joint dominance is the following. Consider an industry with three firms where one is far bigger than the other two. Now the two small firms merge and become big as well. The risk of joint dominance is that the remaining two big firms start to collude. One way to formalize this is to say that it is easier to collude with two than with three firms. But numbers is not necessarily the main issue here. A more interesting argument here is that the merger creates an industry in which firms are similar and this similarity fascilitates collusion (see, for instance, Scherer and Ross (1990: chapter 7) and Tirole (1988: chapter 6)). The explanation for similarity fascilitating collusion usually depends either on some form of contractual incompleteness

[2] More recent papers using a framework similar to that created by Williamson (1968) include Salant et al (1983), Perry and Porter (1985), Farell and Shapiro (1990), McAfee et al (1992) and Gowrisankaran (1999).

(e.g. side payments between firms are not allowed, and such side payments to sustain collusion are more important with asymmetric firms) or on a bargaining problem created through the asymmetry (e.g. with symmetric firms splitting the surplus equally is a focal point which is absent with asymmetric firms). In other words, asymmetry between firms makes it harder to agree on and sustain a collusive outcome. The theory presented here also predicts that in industries with similar firms the equilibrium price is higher than when firms differ considerably in their efficiency levels. However, this does not happen due to collusive or cooperative behavior by firms. The point is that with similar firms it is very costly to fight in order to force opponents out of the market. With similar firms there is a balance of power and hence firms are 'nice' to each other. In this way the theory formalizes the notion that a merger can change conduct; an aspect that cannot be captured by Bertrand or Cournot competition.

Fourth, as the example above suggests (and as will be proved below), the entrance of a new firm into the market can raise the equilibrium price. This can happen if the new firm closes the cost gap between the active firms in the market and the next efficient firm. Thus it becomes less attractive to be aggressive and choose a low price. In other words, this type of entry causes a switch in conduct from playing aggressively to being nice which raises the equilibrium price. This is not the first paper to consider this non-standard effect of entry on price. An early paper here is Rosenthal (1980) who considers firms selling on different and separate markets but being restricted to charge the same price on all markets. He shows that entry on one of these markets can raise the expected equilibrium price. Another part of the literature has derived conditions (on demand and cost schedules) under which Cournot competition yields the prediction that entry raises the equilibrium price. This literature has been summarized and extended by Amir and Lambson (2000) using lattice theory. However, they do not show that entry can raise the price cost margins of firms (which is important from a competition perspective).[3] I work with constant marginal cost levels implying that a rise in the equilibrium price raises price cost margins for firms. Thirdly, there is a literature analyzing the effect of

[3] In the example given by Amir and Lambson (2000) where entry raises the equilibrium price, it reduces the price cost margin due to the assumption of increasing returns to scale.

entry on price in markets of imperfect information. Stiglitz (1989) surveys this literature and gives the following example. Assume consumers know the price distribution in a market but they have to incur a fixed cost to find out which price a particular firm charges. Consider the incentive of a firm to reduce its price. If there are only 3 firms in the market, reducing your price substantially attracts a large number of new customers because their expected search cost before they find you is low. If there are a 100 firms in the market, consumers' expected search cost to find a particular low cost firm is high and hence their incentive to look for this firm is low. Consequently, a firm's incentive to reduce its price are weak and thus prices are higher with 100 firms than with 3 firms: entry can raise the equilibrium price. Finally, there is a recent auction literature showing that a rise in the number of bidders can lower the price. I discuss this literature in section 3.3.

Fifth, the subgame perfect equilibrium in the pricing game solves the Bertrand paradox. This paradox starts from the observation that Bertrand competition in a duopoly with two firms that produce a homogenous good with the same constant marginal costs yields a price equal to marginal costs. This is called a paradox because it seems counterintuitive that two firms are sufficient to get the perfect competition outcome of price equal to marginal costs. Surely, two firms competing in prices must be able to get to an outcome with a strictly positive price cost margin. This is exactly the prediction of the subgame perfect equilibrium of the pricing game here. If there are no potential entrants, the pricing game comes to an end at the monopoly price and the firms share the market equally. The intuition for this result is discussed below using the concept of conjectural variation. The result that a dynamic extensive form pricing game removes the Bertrand paradox has already been noted by Maskin and Tirole (1988). The main difference between their paper and mine is that here firms do not sell until the pricing game is finished and the equilibrium price reached. In Maskin and Tirole (1988) firms sell to consumers at each stage of the pricing game. This allows them to analyze equilibrium price dynamics, deriving conditions under which Edgeworth cycles are an equilibrium outcome. Since my focus is not on price dynamics but on the effect of industry structure (number of firms and their efficiency levels) on the equilibrium price, I use a simpler pricing game.

Finally, the results in this paper can also be seen as contributing to the literature on conjectural variations and, in particular, to the literature on consistent conjectures. This is not surprising since this literature tries to give a dynamic flavour to an otherwise static model whereas below these dynamics are made explicit. The major advantage of the conjectural variation literature is that it gives a simple intuition of why Bertrand competition yields a more competitive outcome than Cournot competition. Because this intuition will be used below, it is worth stating it explicitly. With Bertrand competition firms conjecture that their opponents keep their price constant (the Nash assumption on the strategic variable price). So when a firm considers expanding its output level a bit, it expects its opponents to reduce their output in response (otherwise they cannot keep their prices constant). In other words, a firm expects its opponents to be soft: a rise in output is met by a reduction in opponents' output. This makes firms aggressive and hence the outcome is very competitive. In contrast, under Cournot competition firms conjecture that their opponents keep their output level constant (the Nash assumption on the strategic variable quantity). This implies that any increase in output is met by a relatively big fall in price. This more aggressive (expected) response by opponents makes firms less aggressive and hence the outcome is less competitive (prices are higher) than under Bertrand competition. The major disadvantage of the conjectural variation literature is that anything goes: every conjecture yields a different equilibrium outcome. Instead of assuming that firms choose price or output as strategic variables, Grant and Quiggin (1994) argue that it is at least as reasonable to view firms as choosing mark-ups or revenues. However, these latter strategies yield a different set of predictions. How does one choose the right conjecture?

The literature on consistent conjectures tries to answer this question: the right conjecture is the one that is consistent. To illustrate, the Bertrand conjecture is not consistent in the following sense. Starting at the monopoly price, firms undercut each other assuming that their opponents keep their price constant but clearly they don't. A seminal paper on consistent conjectures is Bresnahan (1981). However, instead of solving the indetermancy, as Klemperer and Meyer (1988) show, it turns out that every outcome can be a consistent conjecture equilibrium.[4] One way to get back to a unique equilibrium outcome is to add exogenous uncertainty to the firms' problem, as is shown by Klemperer and Meyer (1989) in the context of supply functions. In a sense, the uncertainty in Klemperer and Meyer (1989) plays a similar role as subgame perfection in the pricing game below: both force reactions to be optimal reactions in the (sub)game where the reaction has to be played. This removes threats and promises by players that are not credible.

Also the subgame perfection requirement makes players' believes (conjectures) about other players' behavior correct (consistent). This leads to an outcome that is less competitive than Bertrand competition for the following reason. Firms understand that when they undercut the current price, the optimal response of all other firms (with marginal costs below the new price) is to follow that price cut. This response is more aggressive than the Bertrand conjecture (firms stick to their higher price) and hence firms behave less aggressively. Consequently the outcome is less competitive (higher price) than under Bertrand competition.

Although throughout the paper I emphasize the idea of consistent conjectures to understand the subgame perfect equilibrium outcome, one can also view this outcome in the light of fast responses by firms to new (price) information. I come back to this in section 5.

This paper is organized as follows. Section 2 introduces the pricing game and its subgame perfect equilibrium. Section 3 gives three examples to highlight the main results in a simple setting. Section 4 analyses the subgame perfect equilibrium outcome and shows the implications for competition policy. Section 5 discusses some extensions of the model and section 6 concludes the paper. Proofs of all results can be found in the appendix.

[4] More precisely, the Bresnahan (1981) paper has a problem with existence of the consistent conjecture equilibrium, as pointed out by Robson (1983). In response to this, Bresnahan (1983) generalized the consistent conjecture equilibrium concept. Klemperer and Meyer (1988)'s indetermancy result uses this generalized concept.

2 The model

This section introduces the industry characteristics and describes an extensive form pricing game that determines the industry price. It turns out that the Bertrand outcome is a Nash equilibrium in this game, however it is not necessarily subgame perfect. I characterize the subgame perfect equilibrium and show that it is (generically) unique. Finally, I show that subgame perfection in this game removes the Bertrand paradox.

2.1 Industry conditions

Consider an industry where firms produce a homogenous good with constant marginal cost levels. Firm i produces with constant marginal costs $c_i \geq 0$. Let $X(p)$ denote industry demand which is continuously decreasing in p. The number of firms that can be active in the industry is denoted by $N \in \aleph$. Throughout this paper, I make the following assumptions on the industry cost and demand structures.

Assumption $c_0 \leq c_1 \leq ... \leq c_N \leq p^m = \arg\max_p \{X(p)(p - c_0)\}$. *Further,* $X(p)(p - c_0) > X(p')(p' - c_0)$ *for each* p' *and* p *satisfying* $p' < p < p^m$.

In words, firms are indexed such that higher indices indicate (weakly) higher marginal cost levels. Because firm 0 is never willing to produce at a price above its monopoly price p^m, firms with marginal cost levels exceeding firm 0's monopoly price are excluded, without loss of generality. Finally, I assume that firm 0's profits as monopolist are increasing in $p \in [0, p^m]$.

2.2 Extensive form pricing game

This section describes the extensive form game that leads to the equilibrium price. I assume that all information in this game is publicly known. That is, the demand function and firms' marginal cost levels are common knowledge. None of the results below depend on asymmetric information.

The game starts at a high price, say $p^0 = p^m$, and N firms. A round s in the game is described by the number of players still in the game n^s, their marginal cost levels and the current price p^s. Each round consists of two stages. In the first stage, all remaining bidders simultaneously and independently make bids to undercut the current price p^s (players that do not wish to reduce the price, simply bid p^s again). Let p^{s+1} denote the lowest price bid in the first stage of round s. If no firm strictly undercut the price p^s (i.e. $p^{s+1} = p^s$) the game ends. In this case, the final price equals $p = p^s$ and the number of firms equals $n = n^s$. If $p^{s+1} < p^s$ then firms can leave the game in the second stage of this round s. Firms that do not leave the game, enter round $s + 1$ with the new current price p^{s+1} and number of firms n^{s+1}.

Pay offs in this game are as follows. Any firm that has left the game gets a pay off of 0. A firm i that has not left the game gets a pay off equal to $\frac{X(p)}{n}(p - c_i)$. That is, each of the n firms that are still in the game when the bidding ends, is required to sell at the equilibrium price p and produce $\frac{X(p)}{n}$ units of output. Note that, in contrast to Maskin and Tirole (1988), firms do not produce until the pricing game ends. I come back to this in section 5.

The industry story is as follows. Firms can enter and leave the industry without costs, therefore the pay off of a firm that leaves the bidding game is 0. Further, all firms that are willing to sell at a price p share the market equally. This is not unreasonable, because firms produce homogenous goods and consumers faced with n producers offering an identical product at the same price presumably pick a seller at random. I come back to this in section 5.

The first lemma characterizes a Nash equilibrium of this pricing game.

Lemma 1 *The Bertrand outcome $p = c_1$ is a Nash equilibrium outcome in this game.*

This result implies that profits are zero if $c_1 = c_0$. The result is not surprising since the pricing game described above is a formalization of the stories that are told to give the intuition for the Bertrand outcome (see, for instance, Scherer and Ross (1990: chapter 6) and Tirole (1988: chapter 5)). But, as the next section shows, the Betrand outcome is not necessarily subgame perfect. The subgame perfection captures Fisher's (1898: 126) critisism of the Cournot and Bertrand equilibrium concepts that "no business man assumes either that his rival's output or price will remain constant any more than a chess player assumes that his opponent will not interfere with his effort to capture a knight. On the contrary, his whole thought is to forecast what move the rival will make in response to one of his own".

2.3 Subgame perfect equilibrium

The following notation is used in characterizing the subgame perfect equilibrium outcome of the pricing game.

Notation $p_i^* \equiv \left\{ c_j | \frac{X(c_j)}{j} (c_j - c_i) \geq \frac{X(c_k)}{k} (c_k - c_i) \, for \; each \; k \in \{0, 1, ..., N\} \right\}$

In words, p_i^* denotes the price level that maximizes firm i's pay offs given that only firms with marginal costs below p_i^* will remain active in the production stage (as shown below, this is what happens in equilibrium). In particular, if $p_i^* = c_j$, firms $j, j+1, ..., N$ leave the pricing game and there are j firms left to produce and sell (i.e. firms $0, 1, ..., j-1$ are left). Theorem 3 below shows that these prices form part of firms' subgame perfect strategies and the subgame perfect equilibrium price equals p_0^*. Note that a price level strictly inbetween two firm's cost levels, $p \in \langle c_j, c_{j+1} \rangle$ for some j and $j+1$, can never be optimal for firm i. In such a case, a price equal to c_{j+1} yields higher profits, because it is a higher price but does not lead to more firms producing and selling in the industry. In principle, there could be two or more different values of c_j that satisfy the inequalities in the definition for p_i^*. However, thinking of the cost levels c_j as being drawn from some atomless distribution with support $[\underline{c}, \bar{c}]$ this is a probability zero event. Hence, generically speaking each p_i^* is uniquely defined.

Lemma 2 *(Price leadership) The firm with the lowest marginal cost level, denoted by* 0*, is price leader in the sense that it prefers the lowest price:*

$$p_0^* \leq p_1^* \leq p_2^* \leq ... \leq p_N^*$$

This lemma gives a formalization of price leadership that does not rely on first mover advantages. Firm 0 correctly predicts that firms with marginal cost levels above the equilibrium price p will leave the market. Taking this into account, it chooses the price that is most profitable. Firm 0 is price leader in the sense that no other firm is willing to undercut its profit maximizing price.

The intuition is that a low price raises the amount of output a firm produces (because a price reduction raises demand and reduces the number of firms left in the market), and this is most profitable to a low cost firm.

Theorem 3 *For the pricing game above there exists a subgame perfect equilibrium in pure strategies. The subgame perfect equilibrium outcome is (generically) unique. The equilibrium price equals*

$$p = p_0^*$$

And only firms with $c_j < p_0^*$ *produce output.*
Strategy profiles leading to this equilibrium outcome[5] *prescribe for firm* i *in any round* s *of the pricing game to do the following:*[6]

$$\begin{array}{lll} \textit{if} & p^s > p_i^* & \textit{then bid } p_i^* \\ \textit{if} & p^s \leq p_i^* & \textit{then do not undercut } p^s \\ \textit{if} & p^s > c_i & \textit{then stay in the game} \\ \textit{if} & p^s \leq c_i & \textit{then leave the game} \end{array}$$

In other words, the subgame perfect equilibrium price is determined as follows: $p = c_i$ where i is determined by

$$\frac{X(c_i)}{i}(c_i - c_0) \geq \frac{X(c_j)}{j}(c_j - c_0) \text{ for each } j \in \{1, 2, ..., N\} \tag{1}$$

and the firms that produce (a strictly positive quantity) are firms $0, 1, ..., i-1$. Strategy profiles that bring this equilibrium price about are the following. Each firm i bids its optimal price p_i^* if the current price is above that price. Further, a firm leaves if the price is equal to or below its marginal cost level and stays in the pricing game as long as the price is above its marginal cost level.

[5] Note that, although the subgame perfect equilibrium price is unique, the strategy profiles leading to this equilibrium are clearly not unique. For instance, because there is no cost of delay, a strategy profile for player i saying 'if $p^s > p_i^*$ then undercut the current price by one cent' leads to the same pay offs to everyone as the strategies presented here.

[6] Strictly speaking this is not complete. In principle, also subgames should be considered where firm i is still active but where firms $j < i$ have already left. This does not affect i's exit strategy but does affect i's bid. To illustrate, consider the subgame where only firm 0 has left. Player i's optimal bid now equals $\left\{c_j | \frac{X(c_j)}{j-1}(c_j - c_i) \geq \frac{X(c_k)}{k-1}(c_k - c_i) \text{ for each } k \in \{1, ..., N\}\right\}$. Clearly, taking these possibilities explicitly into account makes the analysis notationally heavy, while no additional insights are gained.

The following corollary shows that the Bertrand paradox cannot happen in this model.

Corollary 4 *If* $c_0 = c_1 = ... = c_j > c_{j+1}$ *then* $p > c_0$.

One way to understand why the subgame perfect equilibrium outcome is less aggressive than the Bertrand outcome is to use the idea of conjectural variation. In the Bertrand outcome a firm expects it opponents to keep their price constant. That is, when the firm underbids the current price it expects its opponents to leave the pricing game and hence a small price reduction gives this firm the whole market. Because it expects it opponents to be soft (to leave the pricing game when it reduces the price) each firm behaves very aggressively and hence the outcome is aggressive. In the subgame perfect equilibrium, a firm understands that each price reduction will be followed by all firms with marginal costs below the price. Hence a firm expects a rather assertive response from its opponents. This reduces the incentive to be aggressive and hence the outcome is less competitive.

3 Three examples

This section presents three simple illustrations of the analysis. First, I analyze the example mentioned in the introduction. Then I analyze the new theory with a continuum of firm types to show how similar analytically the new theory is as compared to the standard theory of firm pricing decisions. Finally, I use auction theory to further illustrate the industrial organization issue I am analyzing here. This makes clear why I do not get the standard second price auction equilibrium outcome which corresponds to the Bertrand outcome.

3.1 Cournot vs Bertrand

This section analyzes more carefully the example with two industries given in the introduction. Recall that in both industries the demand function is given by $X(p) = 1 - p$, where p is the industry price (the lowest price charged by any firm) and X equals total demand (for the homogenous good) at that price. If n firms are charging this price p then consumers choose their seller randomly in such a way that each firm sells $\frac{X(p)}{n}$ units of output. In industry I the (constant marginal) cost distribution of the three firms is $c_1^I = 0, c_2^I = 0.35, c_3^I = 0.4$ and in industry II it is $c_1^{II} = 0, c_2^{II} = 0.1, c_3^{II} = 0.4$.

First, I derive the subgame perfect equilibrium outcomes of the pricing game in these two industries. Then I compare these to the Cournot and Bertrand equilibrium outcomes.

Using equation (1), the subgame perfect equilibrium price is determined as follows. In industry I, firm 1 considers three possible prices: 0.35 and be the only firm in the market, 0.4 and share the market with firm 2 and thirdly a price equal to the monopoly price 0.5 and share the market with firms 2 and 3. The following inequalities show that $p^I = 0.35$ is the equilibrium price. Note that firm 1's profits equal $\frac{X(p)}{n}(p - c_1) = \frac{1-p}{n}p$.

$$\frac{1-0.35}{1}0.35 > \frac{1-0.4}{2}0.4 > \frac{1-0.5}{3}0.5$$

In industry II, however, we have the following inequalities.

$$\frac{1-0.4}{2}0.4 > \frac{1-0.1}{1}0.1 > \frac{1-0.5}{3}0.5$$

Hence the subgame perfect equilibrium price in industry II equals $p^{II} = 0.4$ which is higher than the equilibrium price in industry I. The intuition is that it is too costly for firm 1 to keep firm 2 out of the market in industry II, while this is not very costly in industry I. Hence firm 1 is more aggressive in industry I and the equilibrium price is lower than in industry II.

Can we capture this intuition using the equilibrium concepts of Cournot or Bertrand competition? The answer is yes, but in a way which is not appealing from a conceptual point of view. To see this, consider table 1 which summarizes some characteristics of the equilibria in industries I and II where π_1 denotes the profits of firm 1, p the equilibrium price and n the number of active firms in equilibrium.

Industry		π_1	p	n
I	Cournot	0.19	0.44	3
	Bertrand	0.23	0.35	1
II	Cournot	0.13	0.37	2
	Bertrand	0.09	0.10	1

Table 1: equilibrium configurations for industries I and II with Cournot and Bertrand competition

Note that with Bertrand competition there is only one firm in the market because the most efficient firm (1) chooses a price equal to the cost level of the next efficient firm ($p = c_2$). With Cournot competition more than one firm is active in both industries. Next, note that both Cournot and Bertrand competition predict that the outcome in industry II is more competitive than in industry I ($p^{II} < p^I$). One way to capture the idea above that the outcome in industry I can be more competitive than in II is to consider firm 1's profits. In industry I, firm 1's profits are higher with Bertrand than with Cournot competition, while in industry II firm 1's profits are higher with Cournot competition. Hence, *if firm* 1 *could choose* then it would prefer to play Bertrand competition in industry I ($p^I = 0.35$) and play Cournot competition in II ($p^{II} = 0.37$). However, there is no convincing way in which firm 1 can choose to play Cournot in industry II. How does 1 convince firm 2 that the right conjecture is that 1 keeps its output level constant? Part of the problem is here that both the Bertrand and the Cournot conjecture are incorrect (or inconsistent in the terminology of Bresnahan (1981)) making it hard to select one or the other. However, the subgame perfect equilibrium concept imposes correct believes about opponents' actions and hence is a natural way to formalize the prediction that the outcome in I will be more competitive than in II.

3.2 Continuous types

This section shows that although the analysis is done here with a discrete number of firms (and hence sets of inequalities are used instead of the usual first order conditions) the mathematics is essentially similar to standard monopoly pricing problems. This is done by assuming a continuous distribution of firms.

In particular, assume that firms' costs are distributed on $[c_*, c^*]$ with distribution function $F(.)$. Then using a version of equation (1) defined for continuous (instead of discrete) cost distributions, the firms with $c = c_*$ choose p to solve

$$\max_p \frac{X(p)}{F(p)}(p - c_*)$$

By lowering the price, firms reduce the margin $(p - c_*)$, raise demand $X(p)$ and reduce the number of firms that can profitably produce at that price. The last effect is captured by $F(p)$: the proportion of firms that have marginal cost levels below p and hence stay in the market at that price p. Defining $q(p) \equiv \frac{X(p)}{F(p)}$ as the production of a firm with costlevel c_*, shows that with continuous cost distributions the mathematics of the subgame perfect equilibrium outcome is not more complex than the mathematics of the monopoly problem: the outcome is determined as $\max_p q(p)(p - c_*)$.

3.3 PEA auction

Following Klemperer's (2000) suggestion, I use auction theory to illustrate the I.O. results introduced above. In particular, I modify the English auction in such a way that in equilibrium it is not necessarily the case that the buyer with the highest valuation gets the object at a price equal to the next highest valuation. This modified English auction is denoted a Passive English Auctioneer (PEA) auction. The rules of the game are as follows. The auction starts at a price equal to zero. The buyers indicate whether they are willing to buy at the current price, say by leaving the auction room if they are not willing to buy anymore at that price (or a higher one). In addition to this, any buyer in the auction room can raise the price. If no buyer further raises the price and if there are $n \geq 1$ buyers left, the auctioneer randomly allocates the good to one of the remaining buyers and each buyer has a probability of $\frac{1}{n}$th of winning the good and paying the current price. A buyer who does not get the good pays nothing. In other words, if the number of remaining buyers n equals 2 or more the excess demand is not resolved through increasing the price but through a rationing scheme. This is clearly not an optimal auction, as an active auctioneer could raise the revenue of the auction through raising the price until only one buyer were left. That is why the auctioneer here is called passive.

In an I.O. context this passive auctioneer assumption often makes sense. If a number of firms sell the same product at the same price, consumers are indifferent between the sellers and buy randomly from any firm. Moreover, in this context, there is no auctioneer who can actively change the price to resolve excess supply.

There is a Nash equilibrium of the PEA auction where the buyer with the highest valuation gets the good at a price equal to the second highest valuation. In terms of the conjectural variations literature, this equilibrium comes about if each buyer believes that when he raises the price in the auction, the other buyers keep their bid constant (the Bertrand-Nash assumption). Then the buyer with the highest valuation will raise the price until all other buyers have left the auction. However, the problem is that this belief is not part of a subgame perfect equilibrium, because other buyers will stay in the auction until the price equals or exceeds their valuation.

I want to stress two properties of the subgame perfect equilibrium of the PEA auction.[7] First, that the most efficient player does not necessarily bid up the price until all other buyers leave the auction. Second, that adding another buyer (which has in fact a very high valuation) can reduce the equilibrium price. Below these results are related to recent papers in the auction literature.

Example *Consider a private value auction where the valuations of buyers* $1, 2$ *and* 3 *for the auctioned good equal resp.* $q_1 = 2\frac{1}{2}, q_2 = 2$ *and* $q_3 = 1$*. Using the auction equivalent of equation (1), the subgame perfect equilibrium price equals the price that maximizes bidder* 1*'s pay off. Buyer* 1 *has three relevant choices for the price:* $p = 0$ *and a probability of* $\frac{1}{3}$ *of getting the good,* $p = 1$ *and a probability of* $\frac{1}{2}$ *of winning and* $p = 2$ *in which case buyer* 1 *gets the good for certain. It is straightforward to verify the following inequalities*

$$\frac{2\frac{1}{2} - 0}{3} > \frac{2\frac{1}{2} - 1}{2} > \frac{2\frac{1}{2} - 2}{1}$$

In words, it is most profitable for player 1 *to choose a price equal to* 0 *and 'share' the good with buyers* 2 *and* 3*. Hence the Nash equilibrium where player*

[7] Note that to make these points there is no need to introduce asymmetric information into this auction example. Introducing asymmetric information about other buyers' valuations complicates the analysis, but does not affect these results. I come back to this below.

1 gets the good for certain at a price equal to the second highest valuation is not subgame perfect. Now consider what happens when player 2 is removed from the market. In a standard second price auction, removing the buyer with the second highest valuation will reduce the revenue of the auction. Here we get the opposite result. Player 1 has now two relevant choices for the price: $p = 0$ *and a probability of* $\frac{1}{2}$ *of getting the good and* $p = 1$ *in which case buyer 1 gets the good for certain. The following inequality implies that player 1 chooses* $p = 1$*:*

$$\frac{2\frac{1}{2} - 1}{1} > \frac{2\frac{1}{2} - 0}{2}$$

Hence, as player 2 leaves the market, player 1 becomes more aggressive and the price goes up. The intuition that 1 becomes more aggressive is that the distance between 1 and the next best player 3 is rather big. If this distance is big, it pays off to be aggressive. If this distance is small, being aggressive is rather costly and players behave in a more friendly way.

The result that player 1 prefers to share the market at a low price can be discussed in the light of a paper by Gilbert and Klemperer (2000) where conditions are derived under which rationing is optimal from the auctioneer's point of view. Note that the sharing result is indeed a form of rationing: there is excess demand (three buyers, one product) at the current price ($p = 0$) and yet the price does not go up; the good is allocated randomly. However, above I have argued that this is optimal from the point of view of the buyers, not the seller. The argument given by Gilbert and Klemperer is the following. Suppose that potential buyers need to incur a sunk cost before they enter the auction (e.g. they have to spend resources to see what the auctioned good is worth to them). Then to ensure that the good will actually be sold, the auctioneer may want to attract weak buyers as well (i.e. buyers whose valuations are drawn from distributions that are dominated by the distribution of the strong buyers). One way to do that is for the auctioneer to commit to rationing, because that raises the expected surplus for weak buyers as compared to a market clearing price.

The result that the equilibrium price can fall as another buyer is added has also been noted in Bulow and Klemperer (1999). The mechanism through which this happens is, however, a completely different one. Bulow and Klemperer consider (almost) common value auctions. In such an auction the winner may suffer from the winner's curse: the fact that he wins the auction makes it likely that he has overestimated the value of the good. As the number of bidders goes up, it becomes more likely that the winner has overestimated the value of the good: the curse gets worse. Rational players take this effect into account and shade their bids more strongly as the number of players goes up. Hence, a higher number of bidders can lead to lower expected revenues in the auction.

In the example above, there is a private values auction and hence the winner's curse plays no role. Also, with the winner's curse the problem is that players fear that the entrant has a low signal for (bad news about) the value of the auctioned good. In the example above, however, the price reduction is caused by adding a high value player (bidder 2). This bidder with a high valuation makes it less profitable for player 1 to bid aggressively.

Note that in the PEA auction, although firm 1 is worse off with player 2 added, firm 3 gains when firm 2 enters the market. Below, I come back to this asymmetric reaction of pay offs in response to entry and exit.

Finally, the result in the PEA auction that it can be optimal for bidders to "share" the good at a low price can also be obtained in the context of a simultaneous ascending auction.[8] Consider the case with two bidders and and two goods. Bidder 1 values each of these goods at $q_1 = 2\frac{1}{2}$ and bidder two values each at $q_2 = 2$. That is, if bidder i gets one (both) good(s) at a price equal to p (prices equal to p and p') his pay off is $q_i - p$ $(2q_i - p - p')$. It is routine to verify that in a simultaneous ascending auction the equilibrium price is 0 in this case and each bidder gets one good. If the valuation of bidder 2 is reduced to $q_2 = 1$, the equilirbium price rises to one and bidder 1 obtains both goods.

4 Analysis: implications for competition policy

This section derives a number of properties of the subgame perfect equilibrium. First, I derive a result that limits the relevant comparative statics that need to be considered and a result on the effect on profits of entry and efficiency

[8] I thank Paul Klemperer for bringing this simultaneous ascending auction example to my attention.

gains. Then I analyze the effects on the equilibrium price of entry, efficiency gains by firms, mergers and the introduction of a single market. Since the comparative statics results for the subgame perfect equilibrium differ starkly from the results for the Bertrand-Nash equilibrium, one may ask the question which set of predictions is most convincing for a given market. I come back to this question in the next section (with a formal result in lemma 17).

Consider an industry where the equilibrium price is p. What would the equilibrium price have been if there was an additional firm with cost level $c_e > p$? And what would the equilibrium price have been if an existing firm j's costs $c_j > p$ were lower but still above p?

Lemma 5 *A rise in the number of participating firms N through entry with cost level of the entrant $c_e > p$ or a fall in cost level of an existing firm j from c_j to $c'_j \in \langle p, c_j \rangle$ has no effect on the equilibrium price p.*

Both the entry of an additional firm and the fall in costs described in the lemma, make a price rise above the current equilibrium price p less attractive. Further, nothing changes for potential price choices below the current price p. Hence the equilibrium price will not change in response to such changes.

Although below I derive results that the equilibrium price can rise with the number of firms in the industry and that it can fall with firms' cost levels, it is always the case that the price leader is (weakly) better off with less opponents and with less efficient opponents.

Lemma 6 *The profits of the price leader (firm 0) are non-increasing in the number of other firms N and is non-decreasing in the cost levels of the other firms in the industry.*
However, the profits of firms in the industry with $c_i > c_0$ can rise with the number of firms and fall with other firms' cost levels.

The latter part of the lemma was already illustrated by the auction example in section 3.3. The weak bidder gains by adding another bidder to the auction. The intuition is that entry or efficiency gains by firms with $c_i > c_0$ can make it less profitable for the price leader to price aggressively. If firm 0 decides to raise its price in response to such changes, some firms can now profitably produce that were priced out of the market before the change.

Hence, unlike Bertrand or Cournot competition, entry or efficiency gains by opponents do not affect all firms' profits in the same direction. When discussing mergers, this property gives an explanation of why some firms in an industry oppose a merger while other (non-merging) firms favour the merger.

4.1 Effect of entry on equilibrium price

This subsection presents results on the effect of entry on the equilibrium price. Conventional wisdom says that the price goes down as more firms enter the industry. Results are derived that confirm this intuition for the subgame perfect equilibrium price, but also conditions are derived under which entry leads to a rise in the equilibrium price. The idea of the results is to compare the subgame perfect equilibrium with and without the additional firm.[9]

Lemma 7 *Assume* $\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$ *for each* $j \neq i$ *and a new firm enters with* $c_e \in \langle c_{k-1}, c_k \rangle$ *with* $k < i$. *Then the new equilibrium price* p' *satisfies*

$$p' \notin \{c_k, ..., c_{i-1}\}$$

This result says that if the current equilibrium price is $p = c_i$ and a firm enters with cost $c_e < p$, then the new subgame perfect equilibrium price does not lie strictly between c_e and c_i. In other words, firm 0 then finds it optimal to either price at c_i or higher or at c_e or lower. The intuition is that the price leader wants either to price the entrant out of the market and lower the price substantially. Or the new firm closes a cost gap which makes aggressive behavior by the priceleader no longer profitable. As a result the price goes up.

The next corollory of this result is used in the proof that a rise in the number of firms can raise the equilibrium price.

[9] If one wants to interpret the results as dynamic adjustment from an equilibrium without the entrant to an equilibrium with the entrant, one needs to assume that after entry the pricing game starts again at a very high price. Sometimes it is more reasonable to assume that the pricing game starts at the current price and that firms may raise the price. In that case, a transition to an equilibrium with a higher price may not materialize, because the subgame perfect equilibrium of the pricing game may generate a kinked demand curve type of result. The analysis here has this property in common with the dynamic game analysed by Maskin and Tirole (1988). The issue is left for further research.

Corollary 8 *Assume* $\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$ *for each* $j \neq i$*. If a new firm enters with* $c_e \in \langle c_0, c_1 \rangle$*, then the new equilibrium price* p' *satisfies*

$$p' \in \{c_e\} \cup \{c_i, ..., c_N\}$$

If a new firm enters with $c_e = c_0$*, then the equilibrium price satisfies*

$$p' \in \{c_i, ..., c_N\}$$

This corollary says that due to entry with a cost level very close to c_0 ($c_e \in \langle c_0, c_1 \rangle$) the price either falls dramatically to c_e or it does not fall at all. Clearly, as c_e comes closer and closer to c_0, the option of pricing the entrant out of the market becomes less and less attractive for the priceleader and the price will tend to rise.

This formalizes the intuition that in an industry with similar firms, one should expect a high price. In fact, adding a firm with cost level equal to c_0 never lowers the subgame perfect equilibrium price. Moreover, one can show that by adding enough new firms with cost level c_0, the subgame perfect equilibrium will be the monopoly price p^m. Although having more symmetric firms makes the collusion outcome more likely, the pricing game does not add (explicit) collusive considerations to get the result: every firm acts independently. In the words of Chamberlin (1969: 48), the subgame perfect equilibrium outcome has the property that "since the result of a [price] cut by any one [firm] is inevitably to decrease his own profits, no one will cut, and although the sellers are entirely independent, the equilibrium result is the same as though there were a monopolistic agreement between them".

The next lemma derives results on the effects of exit.

Lemma 9 *Assume* $\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$ *for each* $j \neq i$*.*
(i) If a firm $l \in \{1, 2, ..., i-1\}$ *decides not to enter the pricing game, then the equilibrium price does not rise and may fall.*
(ii) If a firm $l \in \{i+1, ..., N\}$ *decides not to enter the pricing game, then the equilibrium price does not fall and may rise.*

The intuition for the first result is that because a firm that is currently producing leaves the market, a gap is created which makes it (weakly) more profitable for the price leader to become more aggressive. Because by lowering the price a bit the number of firms with which the market is shared can be reduced substantially, the equilibrium price tends to fall. If, on the other hand, a firm with costs above the equilibrium price leaves, a gap may be created which allows the price leader to raise the price substantially without inviting too many firms to start production as well. Hence, if such a firm decides not to enter the pricing game, the equilibrium price may rise.

The following result gives necessary and sufficient conditions for entry to lead to a rise in the price level.

Lemma 10 *Consider an industry with n active firms, that is $p = c_n$. Assume that a firm e enters with cost level $c_e \in \langle c_{l-1}, c_l \rangle$ with $c_l < c_n = p$. Then the price rises to a level $p' = c_k > c_n$ if and only if*

$$(c_k - c_0)\frac{X(c_k)}{k+1} > \begin{cases} (c_j - c_0)\frac{X(c_j)}{j+1} & \text{for} \quad c_j \geq c_l \\ (c_j - c_0)\frac{X(c_j)}{j} & \text{for} \quad c_j \leq c_{l-1} \\ (c_e - c_0)\frac{X(c_e)}{l} & \end{cases} \tag{2}$$

These inequalities are straightforward to check in a particular example. In section 3.3 above, I have already shown that it is possible to construct examples in which entry indeed causes a rise in the equilibrium price level (note that this was done in an auction context and therefore the point was that entry can lower the equilibrium price). One may wonder how easy it is to satisfy the inequalities above. Is this only possible for cases where the number of active firms n is small, or does this hold more generally? The next result shows that indeed such cases can be constructed for arbitrary n.

Proposition 11 *For any $n > 1$ there exists an industry cost distribution such that entry by a firm e causes a rise in the price level.*

In the appendix this result is proved by constructing, for arbitrary n, a cost distribution that satisfies the inequalities in lemma 10 above. So the result that entry can raise the equilibrium price is rather robust and does not depend on small numbers.

The following result derives conditions for the more conventional case where entry leads to a fall in the equilibrium price.

Lemma 12 *Assume that* $\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$ *for each* $j \neq i$.
(i) There exists $\varepsilon > 0$ *such that* $c_e \in \langle c_i - \varepsilon, c_i \rangle$ *implies that* $p' = c_e < c_i = p$.
(ii) For every $c_e \in \langle c_0, c_i \rangle$ *there exists a number* $E \in \aleph_+$ *such that if* E *or more firms enter with cost level* c_e *the equilibrium price falls to or below* c_e.

The first result says that the priceleader prefers to keep an entrant out of the market with a cost level c_e just below the equilibrium price. Hence the price falls in response to such potential entry. The idea of the second result is that by adding enough firms at a cost level strictly above c_0, it becomes profitable for firm 0 to keep these firms out of the industry by choosing a price equal to c_e.

4.2 Effect of cost reductions on price

Usually we think that a reduction in a firm's cost level never leads to an increase in the equilibrium price in the market. Indeed this can never happen in a Cournot or Bertrand model. However, in the model here, we can have that a fall in production costs for a firm leads to a rise in the equilibrium price.

Proposition 13 *If there exists* $j > i$ *such that*

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0) > \frac{X(c_k)}{k}(c_k - c_0) \text{ for each } k \neq i, j$$

then any reduction in c_i *to* $c_i' \in [c_0, c_{i-1}]$ *leads to a rise in the equilibrium price from* $p = c_i$ *to* $p' = c_j$.

In this context we can also get the result that a reduction in costs can lower the equilibrium price.

Lemma 14 *Assume that* $\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$ *for each* $j \neq i$.
(i) There exists $\varepsilon > 0$ *such that a fall in cost* c_i *to* $c_i' \in \langle c_i - \varepsilon, c_i \rangle$ *leads to a reduction in the equilibrium price.*
(ii) If an active firm's cost level $c_k \in \langle c_0, c_i \rangle$ *is reduced below* c_0 *then the equilibrium price does not rise.*

The intuition for the first result is the following. If it is optimal for the price leader to keep firm i out of the market, then a small efficiency gain by i does not change that. Hence, the price leader wants to keep firm i out of the market and hence reduces the price. The second result says that if a firm gains so much in efficiency that it becomes the new price leader, the equilibrium price does not rise. The intuition is that lower costs make higher output and hence lower prices more attractive. Therefore the new price leader will never want to raise the price.

Taking proposition 13 and lemma 14 together we get the following picture. Small cost reductions (from c_i to $c_i - \varepsilon$) lead to a fall in the equilibrium price. Bigger cost reductions (from c_i to $c_i' \in [c_0, c_{i-1}]$) can raise the equilibrium price and major cost reductions (below the costs c_0 of the price leader) tend to reduce the price again. In other words, the relation between industry structure (in terms of firms' efficiency levels) and equilbrium price is rather subtle and there is no reason to expect a simple monotone relation.

If a firm enters with $c_e < c_0$, then the effect on equilibrium price is ambiguous. To see this, entry of a firm with $c_e < c_0$ can be seen as the 'sum' of two comparative statics exercises. First, there is entry by a new firm with cost level $c_e \in \langle c_0, c_i \rangle$. Lemma 9 implies that in this case the price does not fall (and may rise). Second, an existing firm's cost level $c_e \in \langle c_0, c_i \rangle$ is reduced below c_0. Lemma 14 implies that then the price does not rise (and may fall). The overall effect of entry with $c_e < c_0$ on the equilibrium price depends on which of these two effects dominates.

The analysis of cost reductions above can be applied to the introduction of a single market in Europe. As the next example shows, with the subgame perfect equilibrium of the pricing game it is not obvious that a reduction in trade barriers leads to a fall in prices.

Example *Consider two countries A and B, where in each country there is one firm (a and b resp.) producing a homogenous good with zero marginal costs ($c_a = c_b = 0$). In both countries, the demand relation is of the form $x(p) = 1 - p$. Hence the monopoly price in each country is $p^m = 0.5$. Let p_i^j denote the price firm $j = a, b$ charges in country $i = A, B$. I do not require a firm to charge the same price in both countries. Let t denote the import tariff that firm a (b) pays to sell its product in country B (A). Then for $t \geq p^m$*

each firm sells only in its own country and charges the monopoly price. For $t \in [\frac{1}{2} - \sqrt{\frac{1}{8}}, p^m\rangle$[10]*, the price each firms charges equals t and each firm only sells in its own country. For $t \in [0, \frac{1}{2} - \sqrt{\frac{1}{8}}\rangle$ each firm sells in both countries at the monopoly price p^m. Hence, the equilibrium price as a function of t is nonmonotone, as shown in figure 1. Note the difference between the theory here and a theory based on collusive behaviour. With full collusion, for any $t > 0$ firms will choose to sell on their home market only and not export. In contrast, the subgame perfect equilibrium requires firms a and b to share both market A and B equally.*

-Figure 1 around here-

Hence over a range of values of import tariffs, prices fall as tariffs are reduced. However, if firms are symmetric and tariffs fall below a certain treshold, pricing aggressively to keep other firms out of the market is no longer profitable. In contrast, it becomes optimal to accomodate, charge the monopoly price and share the markets with foreign firms.

4.3 Effects of mergers: joint dominance

Finally, I consider the implications of the theory here for merger analysis. In particular, I want to focus on two effects. First, the effect that a merger with big efficiency gains can create an industry where firms are similar enough to raise the equilibrium price. This formalizes the notion of joint dominance. Second, when reading the newspapers on mergers, the situation seems more subtle than the Williamson framework allows. For instance, there are cases where some firms oppose a proposed merger, while other (non-merging) firms favour it. In the Williamson framework, if a merger (due to efficiency gains) reduces one firm's profits, it reduces the profits of all other (non-merging) firms. As an illustration of this issue, consider the Schneider/Legrand merger as described in The Economist (2001). Siemens, one of the rivals of the merging firms, argued that the European Commission should oppose the merger because it would lead

[10] The value for $t = \frac{1}{2} - \sqrt{\frac{1}{8}}$ solves the equation $(1-t)\,t = \frac{p^m(1-p^m)}{2}$. That is, firms are indifferent between limit pricing at t or sharing the market equally at the monopoly price.

to higher (!) prices.[11] The following merger example features each of these effects: joint dominance, higher equilibrium price, a firm that gains and a firm that loses due to the merger.

Example *Consider an industry with demand of the form* $X(p) = 4 - p$ *and four firms with constant marginal cost levels of* $c_0 = 0, c_1 = c_2 = c_3 = 0.3$. *Then the subgame perfect equilibrium price equals* $p = 0.3$. *Now suppose firms* 1 *and* 2 *merge so that they can improve their efficiency through some complementarities. If they merge, their marginal cost level becomes* 0, $c_{1\&2} = 0$. *I assume that the merged entity* 1&2 *keeps the two separate outlets it had before.*[12] *One can readily verify that after the merger the equilibrium price equals the monopoly price for firms with marginal cost levels equal to* 0: $p^m = 2$. *Hence the merger leads to joint dominance: the equilibrium price rises due to the merger (although the merger leads to a big efficiency gain). Next, notice that firm* 0*'s profits are reduced due to the merger. Indeed firm* 0 *will oppose the merger on the ground that it raises the equilibrium price! Finally, note that firm* 3*'s profits are raised due to the merger. Before the merger, firm* 3 *could not produce while it produces a strictly positive output level after the merger.*

As the example illustrates, the theory of price competition introduced in this paper gives a richer framework to analyze merger issues that cannot be addressed in, say, a Cournot model.

[11] One explanation for this is that Siemens opposed the merger because it expected a price reduction (say, because of efficiency gains for the merged firms). However, arguing to the European Commision that a merger should be opposed because it reduces prices does not seem to be convincing. So to convince the Commission, Siemens claimed it expected a price increase. The example I give, using the subgame perfect equilibrium of the pricing game, does not rely on Siemens misrepresenting its views to mislead the Commission.

[12] An issue in modelling mergers when marginal costs are constant is the effect of the merger on the market share of the merged firm (see, for instance, Salant et al. (1983), Perry and Porter (1985), McAfee et al. (1992)). In the example here, the question is whether a merged firm with $n-1$ active opponents, has a market share of $\frac{1}{n}$ or $\frac{2}{n+1}$? For my purposes, however, this is not an issue as the merger is profitable under either assumption.

5 Discussion and extensions

This section discusses the following five topics: entry deterence, asymmetric information, rationing rules, sales at (non)equilibrium prices and tacit collusion.

Entry deterence An issue in entry deterence is that the incumbent firm wants to convince entrants that competition will be fierce after they enter. In terms of table 1 in section 3.1, the incumbent firm would like entrants (that are equally or less efficient than the incumbent) to think that the industry is characterized by Bertrand competition. But how can the incumbent convince the entrant that it should have Bertrand conjectures?[13] The theory proposed here answers this question. If the incumbent is far more efficient than the entrant then it will indeed be optimal (and hence the threat credible) to price very aggressively to keep the entrant out of the market. If, on the other hand, the entrant has an efficiency level similar to the incumbent, it will be optimal to accomodate the entrant and hence threats to limit price are not credible. If the entrant does not know the cost level of the incumbent, the incumbent may want to signal that it is very efficient. In this case, there is an element added to the analysis of entry deterence under asymmetric information as in Milgrom and Roberts (1982). Now there are two reasons why the incumbent wants to convince the entrant that its marginal costs are low. First, to signal that it will be a tough opponent (for given aggressiveness of interaction) as in Milgrom and Roberts. Second, to signal that competition will indeed be fierce after entry, because the cost gap between the incumbent and entrant is big.

Asymmetric information In the analysis above it is assumed that firms marginal cost levels are common knowledge. What will change if firm's cost levels are private information? The main change is to firms' bidding strategies.

[13] Although the Bertrand conjecture is always wrong in equilibrium, there is an additional problem in the case of entry. If the entrant expects the incumbent to keep prices constant in response to its own price changes (Bertrand conjecture) it does not expect the incumbent to keep its price constant in response to its entry decision. The latter conjecture would, in fact, make entry very attractive. In a sense this is counterintuitive: the entrant understands that the incumbent changes its price in response to the entrant's entry decision but still believes that the incumbent does not change its price in response to price decisions by the entrant.

Now firms will reduce prices step by step to see which of its opponents will leave the market after a certain price reduction. So in contrast to theorem 3 firm i will not bid p_i^* in the first round. To see this consider the following duopoly with demand of the form $X(p) = 4 - p$ and where firm 0 has marginal costs equal to 0, $c_0 = 0$. Firm 0 does not know the costs of its opponent, firm 1. Firm 0 has the following information on 1's marginal cost level

$$c_1 = \begin{cases} 0 & \text{with probability} \quad y \\ 1 & \text{with probability} \quad \frac{1}{4} \\ 1.5 & \text{with probability} \quad \frac{3}{4} - y \end{cases}$$

with $y \in [0, \frac{3}{4}]$. To determine firm 0's bidding strategy, I use backward induction. First, note that reducing the price to 0 can never be optimal for firm 0, hence the choice is between a price equal to 1, 1.5 or the monopoly price 2. Given that firm 0 ends up at a price equal to 1.5 and its opponent has not left the pricing game, should it reduce the price to 1? If it does so, then with (conditional) probability $\frac{1}{4y+1}$ firm 1 leaves the game and with probability $\frac{4y}{4y+1}$ firm 1 stays in the game. Hence the expected pay off of such a price reduction from $p = 1.5$ to $p = 1$ equals

$$\frac{1}{4y+1}3 + \frac{4y}{4y+1}\frac{3}{2} = \frac{3+6y}{1+4y}$$

On the other hand, keeping the price at $p = 1.5$ with the opponent active yields a pay off equal to $1\frac{7}{8}$. Hence for all $y < \frac{3}{4}$ it is optimal to reduce the price from $p = 1.5$ to $p = 1$ if firm 1 has not left the pricing game at a price $p = 1.5$. If firm 1 does leave at a price equal to $p = 1.5$, it is clearly optimal for firm 0 to stay put.

Knowing this, under which conditions is it optimal for firm 0 to reduce the price from $p = 2$ to $p = 1.5$? Expected pay off from reducing the price from 2 to 1.5 equals

$$\left(\frac{3}{4} - y\right)\frac{15}{4} + \left(y + \frac{1}{4}\right)\frac{3+6y}{1+4y} = \frac{57}{16} - \frac{9}{4}y$$

Staying put at a price equal to 2 yields a pay off equal to 2. Hence, if $y > \frac{25}{36}$ it is not optimal to undercut the monopoly price for firm 0 while for $y < \frac{25}{36}$ it is optimal for firm 0 to reduce the price to $p = 1.5$.

In other words, the more likely it is that firm 1 is very efficient (i.e. y is high) the higher the equilibrium price. If it is unlikely that firm 1 is very efficient (i.e.

y close to 0) the equilibrium price is lower. This is line with the results found above under symmetric information. The additional result is that for $y < \frac{25}{36}$, firm 0 first bids $p = 1.5$. If firm 1 stays in the market, firm 0 bids $p = 1$ in the next round. So unlike the result in theorem 3, the equilibrium is not necessarily reached immediately after the first round but the price develops over time.

Rationing rules Above I have assumed that when n sellers are willing to sell at the lowest price, each seller serves $\frac{1}{n}$th of the market. Although this assumption is reasonable in an I.O. context and more or less standard, one can ask whether the results derived above critically depend on this assumption.

So if there are n firms willing to sell at the lowest price, one can define a more general rationing rule as follows. Firm i gets a market share β_i where the β_i's satisfy

$$\begin{aligned} \beta_i &\geq 0 \text{ for all } i \in \{0, 1, ..., n-1\} \\ \sum_{i=0}^{n-1} \beta_i &= 1 \end{aligned} \tag{3}$$

For instance, Cournot competition in a homogenous good market where firms differ in efficiency yields an outcome where all firms sell at the same price but with different market shares. The question above can now be phrased as follows: Can we have an equilibrium with $p > c_1$ for all rationing rules satisfying (3)? Clearly, the answer is "no". Consider the following two extreme cases (i) $\beta_0 = 0$ and (ii) $\beta_0 = 1$. In the first case, the most efficient firm will never want to share the market with another firm and the subgame perfect equilibrium price equals the Bertrand price ($p = c_1$). The second case is more interesting. Here the other firms will never want to share the market. There is always a firm $i \geq 1$ that undercuts any price $p > c_1$. In other words, although it seems advantegous for firm 0 to have a lot of market power ex post, ex ante firm 0 would like to commit to a lower market share. In particular, if firm 0's profit maximizing price in lemma 2 satisfies $p_0^* = c_i > c_1$ then firm i is better off with a rationing rule $\beta_j = \frac{1}{i}$ for all $j \in \{0, 1, ..., i-1\}$ than with $\beta_0 = 1$.

Although there are rationing rules that destroy any subgame perfect equilibrium price above the Bertrand price, I now show that the results above do not depend on the equal sharing rationing rule. In particular, I show that for every subgame equilbrium with equal market shares where the price is above the Bertrand price, there exists a subgame equilibrium where firms have different market shares at the same market price. Second, I derive conditions on the rationing rule (3) such that the price leadership lemma continues to hold.

I focus in the remainder on rationing rules which imply higher market shares for more efficient firms, that is

$$\beta_0 \geq \beta_1 \geq ... \geq \beta_n$$

which seems a reasonable assumption. The following result formalizes the idea that the results derived in this paper do not hinge on the equal sharing assumption.

Lemma 15 *Assume that* $c_j > c_{j-1}$ *for all* $j \geq 1$.[14] *If it is the case that*

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_k)}{k}(c_k - c_0) \text{ for all } k \neq i$$

then there exist functions $\beta_j(n)$ *satisfying*

$$\begin{aligned} \beta_0(n) &> \beta_1(n) > ... > \beta_{n-1}(n) > 0 \qquad (4) \\ \beta_j(n) &= 0 \text{ for all } j \geq n \\ \sum_{j=0}^{n-1} \beta_j(n) &= 1 \end{aligned}$$

such that $p = c_i$ *is the equilibrium price under this rationing rule.*

Note that a rationing rule $\beta_j(n)$ has two arguments: the position of the firm (j) and the number of firms in the market (n). The idea of the proof is to construct rationing rules $\beta_j(n)$ close enough to the equal sharing rule that all inequalities continue to hold for the equilibrium $p = c_i$. Since the equilibrium is defined with strict inequalities, this is always possible.

Finally, the next result derives sufficient conditions on $\beta_j(n)$ for the price leadership lemma to hold.

Lemma 16 *The price leadership lemma 2 continues to hold with a general rationing rule* $\beta_j(n)$ *if* $\beta_j(n)$ *is decreasing in* j *and in* n *and the following condition is satisfied*

$$\frac{\beta_j(n)}{\beta_i(n)} > \frac{\beta_j(n')}{\beta_i(n')} \qquad (5)$$

[14] This assumption on costlevels is mainly made for notational convenience as presumably one would like $c_i = c_j$ $(i \neq j)$ to imply $\beta_i = \beta_j$.

for all $n' > n > i > j$.

The assumptions on the rationing rule are that more efficient firms have a larger market share, that market shares are smaller if there are more firms in the market and equation (5) implies that as competition intensifies through falling prices the market share of firm j rises compared to a less efficient firm i. For every rationing rule this assumption holds for the marginal firm ($n' - 1$ in the lemma), equation (5) extends this assumption to all firms in the market. The intuition is that as the market becomes more competitive (and the price falls) firm j gains market share at the expense of less efficient rivals. Therefore, the firm that stands to gain the most from a price reduction is firm 0 and the price leadership lemma continues to hold.

Sales at (non)equilibrium prices An important assumption in the model above is that no sales are made before the pricing stage has come to an end and the equilibrium price is reached. This implies that firms can initially charge high prices without the risk of loosing sales or customers if they are undercut by opponents. Firms simply follow the lowest price and continue in the next round (or leave) without loss of profits. This is different in the duopoly model of Maskin and Tirole (1988) where firms choose prices in alternating periods. Choosing a high price today may be costly in the next period if your opponent undercuts the price and increases his market share at your expense.

The question addressed in this extension is: does the possibility of loosing sales when a firm bids high prices in early rounds of the pricing stage destroy the subgame perfect equilibrium with prices above the Bertrand equilibrium price? Before formalizing this question in the current framework, note that Maskin and Tirole (1988) also find equilibria with prices above the Bertrand price. In other words, in their fully dynamic model there are equilibria with a flavour similar to the subgame perfect equilibrium above.

I extend the pricing stage described in section 2 as follows. In each round the remaining bidders bid prices, as above. After these bids have been made, there is a probability q that the pricing game ends immediately. That is, before other firms can indicate whether they want to follow the lowest price bid. In other words, demand is served before the other firms can react to the lowest price.

The firms that actually bid the lowest price in this round serve demand at that price sharing the market equally. With a probability $1-q$ the round continues as above. That is, firms can indicate whether they are willing to charge the lowest price or leave the pricing game. The pricing stage then goes into the next round where the firms can underbid the current lowest price etc. In a given round, the pricing game can thus end exogenously with probability q or endogenously (as above) when no firm undercuts the current lowest price. The difference with the analysis above is that it becomes more profitable to undercut the current lowest price. If you are the only firm to undercut then with probability q you gain the whole market and the game ends.

Note that with $q = 1$ the game is the Bertrand pricing game and the unique equilibrium is $p_{q=1} = c_1$. With $q = 0$ the game is as in section 2 and the equilibrium price is

$$p_{q=0} = p_0^* = \left\{ c_j | \frac{X(c_j)}{j}(c_j - c_0) \geq \frac{X(c_k)}{k}(c_k - c_0) \, \textit{for each } k \neq j \right\}$$

Another way to view the model here is that there is only one customer with a demand function $X(p)$. This customer arrives at the shops with a probability q and then chooses the shop that at that moment charges the lowest price. In this sense, q can also be viewed as the speed with which firms can react to price changes by opponents. Low values of q imply that firms can react quickly (relative to consumers). I come back to this below.

To analyze this game I need an additional (technical) assumption. To ensure that optimal reactions exist (in particular, to make sure that slightly undercutting your opponent is well defined) assume that price bids are integers, $p \in \aleph$. To avoid rounding issues, I also assume that marginal cost levels are integers, $c_i \in \aleph$ for all $i = 0, 1, 2, ...$[15]

The following result shows that the equilibrium analyzed above is not a knife edge phenomenon which only occurs for $q = 0$.

[15] Note that the assumption that $c_i \in \aleph$ rules out the argument above that having two cost levels c_j and $c_{j'}$ (with $c_j \neq c_{j'}$) such that $\frac{X(c_j)}{j}(c_j - c_0) = \frac{X(c_{j'})}{j'}(c_{j'} - c_0)$ is a probability zero event. Hence, in the lemma it is taken into account that the set $\left\{ c_j | \frac{X(c_j)}{j}(c_j - c_0) \geq \frac{X(c_k)}{k}(c_k - c_0) \, \textit{for each } k \neq j \right\}$ can have more than one element.

Lemma 17 *There exists* $\bar{q} > 0$ *such that for all* $q \in [0, \bar{q}\rangle$ *it is the case that*

$$\tilde{p} \equiv \min \left\{ c_j | \frac{X(c_j)}{j}(c_j - c_0) \geq \frac{X(c_k)}{k}(c_k - c_0) \text{ for each } k \neq j \right\}$$

is an equilibrium price of the game described above.

This shows that the subgame perfect equilibrium analyzed in this paper is robust to the introduction of a probability $q > 0$ that there are sales at nonequilibrium prices, in the sense that the pricing game in section 2 has not come to an end yet. If q is small enough, this incentive to undercut does not destroy the subgame perfect equilibrium. If q is big (e.g. $q = 1$) the incentive to undercut is too big for any price $p > c_1$ to survive as an equilibrium.

If there are two prices (or more) c_j and $c_{j'}$ (with $c_j > c_{j'}$) that lead to the same maximal profits for the price leader, then the equilibrium price is the lowest of these prices. Indeed, at $p = c_j$, firm 0 can undercut (e.g. to $c_{j'}$) thereby gaining the whole market with probability q and losing no profits with probability $1 - q$. Hence the higher prices (like c_j) do not survive the introduction of the probability q.

The reader may wonder why the extension of sales at nonequilibrium prices is not done within the Maskin and Tirole (1988) framework (henceforth MT). There are two reasons for this. First, MT analyze the case of a symmetric duopoly and it is not clear whether meaningful analytical results can be derived with three or more firms that differ in efficiency. Second, and more important, there is a conceptual difficulty when extending MT to more than two firms. In MT firms change prices in an alternating way (firm 1 changing its price in even periods, firm 2 in odd periods). This makes sense with two firms, because the main reason why a firm may want to update its price is that the other firm made a price change. However, when there are five firms it is not convincing to argue that each firm can only adjust its price once every five periods. If one of the five firms undercuts the others in a certain period, it is reasonable to expect that more than one firm will want to react by adjusting its price. But if one allows more than one firm to adjust its price in a period, the unique equilibrium is the Bertrand outcome.

My model avoids this problem by introducing the idea that undercutting your opponents does not necessarily give you the whole market because no customer may arrive before your opponents react. If q is low, it is likely that your opponents can react to your price cut before customers arrive and hence undercutting is not profitable.

Tacit collusion The result found above that the equilibrium price tends to be higher the more symmetric firms are, has a collusive flavour. Although I have stressed that this result is not due to (explicit) collusion (because firms act independently), there is a link here with the literature on implicit collusion using supergames (see, for instance, Green and Porter (1984), Rotemberg and Saloner (1986) and Abreu (1986)). This literature derives conditions under which firms can coordinate on a collusive outcome due to the (infinite) repetition of the stage game.

The main differences between this supergame approach and the one proposed in this paper are the following. First, a disadvantage of the supergame approach is that there is a multiplicity of equilibria, while the game proposed above yields a unique subgame perfect equilibrium. Second, the supergame approach predicts that collusion is harder to sustain if there are more firms. In the game above it is not so much the number of firms that may hinder a collusive outcome as the differences between firms. Third, the game described above is essentially a one-shot game (although the pricing stage has a number of rounds). This contrasts sharply with the supergame approach where repetition of the game is important.

Another aspect that is stressed in the literature on collusion is how quickly firms can react to changing strategies by opponents (that is, how quickly do they learn that an opponent deviates and how fast can they react to that information). The faster they can react to deviating behavior, the more collusive outcome they can sustain. This suggests an alternative intuition for the observation that the subgame perfect equilibrium outcome is less competitive than the Bertrand outcome. In the standard Bertrand game, firms have no information about other firms' prices and they cannot react at all if an opponent charges a different price from what they expected. This is an extreme form of price commitment. The pricing game in this paper makes the opposite extreme assumption: firms can react immediately to price changes by opponents. More precisely, firms can react to such price changes before consumers do. In reality, this is usually a matter of degree. In some industries it is the case that choosing a price which turns out to be too high is very costly. The price is published in a catalogue, cannot be easily changed and a lot of sales are lost before the price is adjusted (i.e. q is high in the extension of the model in lemma 17). In other industries, it is straightforward for firms to get information on other firms' prices and they can lower their price in reaction to a price cut before losing much sales (i.e. q is low in lemma 17).

As an illustration of this, Bailey (1998) finds that prices for books, CDs and software are higher on the internet than for traditional retailers. The point is that for traditional retailers it is harder to figure out out what prices other retailers are charging and it is more costly to reprice the books in store. For an internet firm, in contrast, it is easy and cheap to check opponents' prices (one mouseclick) and prices can be changed very quickly and cheaply (another mouseclick) in response. Hence the theory of this paper would indeed predict that prices are higher for internet firms as the pricing game above (with its subgame perfect equilibrium) is a more reasonable description of their situation than the standard Bertrand pricing game. And this is true to a lesser extent for the traditional retail stores.

6 Conclusion

This paper proposes a simple and intuitive pricing game formalizing the idea of firms undercutting each other to gain market share. Although the Bertrand outcome is a Nash equilibrium in this game, it is not necessarily subgame perfect. The subgame perfect equilibrium outcome formalizes the quote from Chamberlin at the beginning of the paper. The outcome is determined by the cost distribution in the industry. If firms are fairly symmetric, there is a balance of power and therefore it is very costly for firms to price aggressively. Consequently, firms behave nicely toward each other and the outcome is a high price. If, on the other hand, one firm (or a group of firms) is far more efficient than its opponents, there is no balance of power and it pays to be aggressive to price the other firms out of the market. Consequently, the equilibrium price is low.

Because of this property, it is easy to see why entry or efficiency gains by existing firms can raise the equilibrium price: cost gaps are reduced and balance of power is established. With this theory it is straightforward to formalize a number of issues in competition policy that are harder to capture with traditional industrial organization models, like Cournot or Bertrand competition. Examples here are joint dominance, efficiency offence and the idea that the single European market may raise prices instead of reducing them.

I have argued that the subgame perfect equilibrium is most convincing as a predictor of market outcomes in markets where firms can react quickly to price reductions by opponents. In markets where firms can react to new price information only with a considerable time lag, the Bertrand equilibrium seems a more convincing prediction of the market outcome than the subgame perfect equilirbium.

Although the theory is formulated here in an I.O. and auction context, the underlying mechanism of balance of power is more widely applicable. In all situations where aggressive play is not immediatele rewarded and hence leaves time for opponents to react before pay offs are realized, it is not obvious that more players or better players leads to a more aggressive outcome. Coming back to the cold war example in the introduction, it is indeed the case that before a superpower can reap the benefits of starting a nuclear war its opponent can react.

References

Abreu, D., (1986), Extremal equilibria of oligopolistic supergames, *Journal of Economic Theory*, 39: 191-225.

Amir, R. and V. Lambson, (2000), On the effects of entry in Cournot markets, *Review of Economic Studies*, Vol. 67 (2): 235-254.

Bailey, J. (1998), Electronic commerce: prices and consumer issues for three products: books, compact discs and software, OECD.

Bresnahan, T., (1981), Duopoly models with consistent conjectures, *American Economic Review*, Vol. 71 (5), 934-945.

Bresnahan, T., (1983), Existence of consistent conjectures: Reply, *American Economic Review*, Vol. 73 (3), 457-458.

Bulow, J. and P. Klemperer, (1999), Prices and the winner's curse, mimeo

Nuffield College, Oxford: www.nuff.ox.ac.uk/economics/people/klemperer.htm.

Chamberlin, E., (1969), *The theory of monopolistic competition: A re-orientation of the theory of value*, Cambridge, Mass.: Harvard University Press.

Farrell, J. and C. Shapiro, (1990), 'Horizontal mergers: an equilibrium analysis', *American Economic Review*, Vol. 80: 107-126.

Fisher, I., (1898), Cournot and Mathematical Economics, *Quarterly Journal of Economics*, Vol. XII (2): 119-138.

Fudenberg, D. and J. Tirole, (1991), *Game Theory*, MIT Press.

Gilbert, R.J. and P. Klemperer, (2000), An equilibrium theory of rationing, *RAND Journal of Economics*, Vol. 31 (1): 1-21.

Gowrisankaran, G., (1999), 'A dynamic model of endogenous horizontal mergers', *RAND Journal of Economics*, Vol. 30 (1): 56-83.

Grant, S. and J. Quiggin, (1994), Nash equilibrium with mark-up-pricing oligopolists, *Economic Letters*, Vol. 45: 245-251.

Green, E. and R. Porter, (1984), Noncooperative collusion under imperfect information, *Econometrica* Vol. 52: 87-100.

Klemperer, P., (2000), 'Why every economist should learn some auction theory', Invited Symposium paper for the World Congress of the Econometric Society, Seattle: www.nuff.ox.ac.uk/economics/people/klemperer.htm.

Klemperer, P. and M. Meyer, (1988), Consistent conjectures equilibria: a reformulation showing non-uniqueness, *Economic Letters*, Vol. 27: 111-115.

Klemperer, P. and M. Meyer, (1989), Supply function equilibria in oligopoly under uncertainty, *Econometrica*, Vol. 57 (6): 1243-1277.

Maskin, E. and J. Tirole, (1988), 'A theory of dynamic oligopoly, II: price competition, kinked demand curves, and Edgeworth cycles, *Econometrica*, Vol. 56 (3): 571-599.

McAfee, R.P., J.J. Simons and M.A. Williams, (1992), 'Horizontal mergers in spatially differentiated noncooperative markets', *Journal of Industrial Economics*, Vol. XL: 349-358.

Perry, M.K. and R.H. Porter, (1985), Oligopoly and the incentive for horizontal merger, *American Economic Review*, Vol. 75 (1): 219-227.

Robson, A., (1983), Existence of consistent conjectures: Comment, *American Economic Review*, Vol. 73 (3), 454-456.

Röller, L., J. Stennek and F. Verboven, (2000), Efficiency gains from mergers, Discussion Paper FS IV 00-09, WZB.

Rosenthal, R., (1980), A model in which an increase in the number of sellers leads to a higher price, *Econometrica*, Vol. 48 (6): 1575-1579.

Rotemberg, J. and G. Saloner, (1986), A supergame-theoretic model of business cycles and price wars during booms, *American Economic Review* Vol 76: 390-407.

Salant, S.W., S. Switzer and R.J. Reynolds, (1983), 'Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium', *Quarterly Journal of Economics*, Vol. XCVIII: 185-199.

Scherer, F. and D. Ross, (1990), *Industrial market structure and economic performance*, Houghton Mi- in Company.

Stiglitz, J., (1989), Imperfect information in the product market, In. R. Schmalensee and R. Willig, ed, *Handbook of Industrial Organization*, Volume I, Elsevier Science Publishers.

The Economist, (2001), 'A shocking denouement', October 13th-19th.

Tirole, J., (1988), *The theory of industrial organization*, MIT Press.

Williamson, O.E., (1968), 'Economies as an anti-trust defense: the welfare trade-offs', *American Economic Review*, Vol. 58: 18-36.

7 Appendix: proof of results

This appendix contains the proofs of the results in the main text.

Proof of lemma 1

It is routine to verify that the following strategy profiles form a Nash equilibrium and that they yield the Bertrand outcome. Any player i with cost level $c_i > c_0$ stays in the pricing game at any price p strictly above c_i and leaves the pricing game if $p \leq c_i$. Further, such a player i undercuts any price p strictly above c_i to (say) $\frac{p+c_i}{2}$. Any player j with $c_j = c_0$ stays in the pricing game at any price $p \geq c_j$ and leaves the pricing game at $p < c_j$. Further, such a player j bids the price $p = c_1$. Q.E.D.

Proof of lemma 2

This is proved by contradiction. Suppose that there exists $i > j$ (i.e. $c_i \geq c_j$)

such that $p_i^* = c_{i'} < c_{j'} = p_j^*$ (i.e. $i' < j'$).

$$\begin{aligned} 0 &> \frac{X(c_{i'})}{i'}(c_{i'} - c_j) - \frac{X(c_{j'})}{j'}(c_{j'} - c_j) \\ &= \frac{X(c_{i'})}{i'}c_{i'} - \frac{X(c_{j'})}{j'}c_{j'} - c_j\left(\frac{X(c_{i'})}{i'} - \frac{X(c_{j'})}{j'}\right) \\ &\overset{(*)}{\geq} \frac{X(c_{i'})}{i'}c_{i'} - \frac{X(c_{j'})}{j'}c_{j'} - c_i\left(\frac{X(c_{i'})}{i'} - \frac{X(c_{j'})}{j'}\right) \\ &= \frac{X(c_{i'})}{i'}(c_{i'} - c_i) - \frac{X(c_{j'})}{j'}(c_{j'} - c_i) \end{aligned}$$

The first inequality follows from the fact that $p_j^* = c_{j'}$. The second inequality (labelled $(*)$) follows from the observations that $c_i \geq c_j$ and that $c_{i'} < c_{j'}$ implies that $\frac{X(c_{i'})}{i'} - \frac{X(c_{j'})}{j'} > 0$. It follows that

$$\frac{X(c_{j'})}{j'}(c_{j'} - c_i) > \frac{X(c_{i'})}{i'}(c_{i'} - c_i)$$

contradicting the initial assumption that $p_i^* = c_{i'} < c_{j'}$. Q.E.D.

Proof of theorem 3

First, note that every subgame perfect equilibrium implies that each player i leaves the pricing game when the price falls below his marginal cost level c_i and stays in the pricing game when the price is above c_i. Any threat or promise to do otherwise is not credible (i.e. is a dominated strategy). To make sure equilibria are well defined, assume that a player i with $c_i > c_0$ does not produce when $p = c_i$ while a player i with $c_i = c_0$ does produce when $p = c_i$.

Second, note that the assumption that $(p - c_0)X(p) > (p' - c_0)X(p')$ for all prices p, p' satisfying $p' < p < p^m$ implies that for each i it is the case that

$$(p - c_i)X(p) > (p' - c_i)X(p')$$

for all prices p, p' satisfying $p' < p < p^m$. Suppose not, i.e. suppose that for some player i it is the case that

$$(p - c_i)X(p) < (p' - c_i)X(p')$$

for some pair of prices p, p' with $p' < p$. This implies

$$(p - c_0)X(p) + (c_0 - c_i)X(p) < (p' - c_0)X(p') + (c_0 - c_i)X(p')$$

or equivalently

$$(p - c_0)X(p) - (p' - c_0)X(p') < (c_i - c_0)[X(p) - X(p')] \leq 0$$

where the last inequality on the right follows from the observations that $c_i \geq c_0$ and $X(p) < X(p')$. However, this contradicts the starting assumption that $(p - c_0) X(p) > (p' - c_0) X(p')$. Hence, this proves (by contradiction) that for each i it is the case that

$$(p - c_i) X(p) > (p' - c_i) X(p')$$

for all prices p, p' satisfying $p' < p < p^m$. Hence choosing a price $p \in \langle c_j, c_{j+1} \rangle$ for some $j \geq 0$ cannot be optimal for any player i . Profits for i are always higher by choosing $p = c_{j+1}$ since this higher price raises $(p - c_i) X(p)$ and the market is shared with the same j players as with a price $p \in \langle c_j, c_{j+1} \rangle$. Hence, one only needs to consider prices $p = c_j$ $(j = 1, 2, ..., N)$.

Now turn to the claim that the (generically) unique subgame perfect equilibrium price equals p_0^*. Given the dominant exit strategy of players, it is optimal for player 0 to undercut any price $p > p_0^*$. This follows directly from the definition of p_0^*. Hence any subgame perfect equilibrium price satisfies $p \leq p_0^*$. So I need to prove that a price $p < p_0^*$ cannot be subgame perfect.

Consider a subgame (round) s where the current price satisfies $p^s \geq p_0^*$. Given the specified exit strategy, is it in any player's interest to bid $p^{s+1} < p_0^*$? By definition of p_0^* it is not in player 0's interest to bid strictly below p_0^*. Further, the following inequalities imply that if firm 0's profits are higher at a price p than at a lower price $p' < p$ then all firms profits are higher at p than at p':

$$\begin{aligned}
0 &> \frac{X(p')}{n(p')}(p' - c_0) - \frac{X(p)}{n(p)}(p - c_0) \\
&= \frac{X(p')}{n(p')}p' - \frac{X(p)}{n(p)}p - c_0\left(\frac{X(p')}{n(p')} - \frac{X(p)}{n(p)}\right) \\
&\overset{(*)}{\geq} \frac{X(p')}{n(p')}p' - \frac{X(p)}{n(p)}p - c_i\left(\frac{X(p')}{n(p')} - \frac{X(p)}{n(p)}\right) \\
&= \frac{X(p')}{n(p')}(p' - c_i) - \frac{X(p)}{n(p)}(p - c_i)
\end{aligned}$$

where $n(p)$ denotes the number of active firms at price p. The inequality labelled $(*)$ follows from the fact that $c_i \geq c_0$ and $\frac{X(p')}{n(p')} \geq \frac{X(p)}{n(p)}$ (at a lower price there is more demand X and less firms n that are active in the market). So given that player 0 prefers p_0^* above any price strictly below p_0^*, it is the case that no other player has an incentive to bid a price strictly below p_0^*. Thus the (generically) unique subgame perfect equilibrium outcome is $p = p_0^*$ and all firms with $c_i < p_0^*$

are active. Q.E.D.

Proof of corollary 4

Since $\frac{X(c_{j+1})}{j+1}(c_{j+1} - c_0) > 0$, equation (1) implies that $p = c_0$ cannot be a subgame perfect equilibrium price. Q.E.D.

Proof of lemma 5

If $c_e > p$ and assume that $p = c_i$ for some i before entry, that is

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$$

for all $j \neq i$. Then for any $c_k < c_e$ this inequality implies that c_i is also optimal after entry. For any $c_k > c_e$ the inequality above implies

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_k)}{k}(c_k - c_0) > \frac{X(c_k)}{k+1}(c_k - c_0)$$

and hence c_i yields higher profits to firm 0 then $c_k > c_e$. Finally, a price $p = c_e$ was feasible before entry as well but then dominated by $p = c_i$.

Any cost reduction from $c_j > p$ to $c_j' \in \langle p, c_j \rangle$ will not change the subgame perfect equilibrium price because any price p' firm 0 might switch to was available before the cost reduction as well but then it was dominated by p. Moreover, any $p' > c_j'$ is weakly less profitable after the cost reduction because the market has to be shared with an additional firm. Q.E.D.

Proof lemma 6

Effect on firm 0's profits follows from a revealed preference argument. Any price firm 0 chooses after entry (or cost reduction by opponent), it could have chosen before but it did not. Moreover, any p is weakly less profitable after entry (or cost reduction) than before.

Lemma 10 (proposition 13) derives conditions under which entry (cost reduction) leads to a higher price. Clearly any firm with a marginal cost level strictly between the equilibrium prices before and after entry (cost reduction) gains in profits. Q.E.D.

Proof lemma 7

Because

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_l)}{l}(c_l - c_0)$$

for all $l \in \{k, k+1, ..., i-1\}$ it follows that

$$\frac{X(c_i)}{i+1}(c_i - c_0) > \frac{X(c_l)}{l+1}(c_l - c_0)$$

for all $l \in \{k, k+1, ..., i-1\}$, as the following argument shows. Suppose this is not the case, that is suppose that

$$\frac{X(c_i)}{i+1}(c_i - c_0) \leq \frac{X(c_l)}{l+1}(c_l - c_0)$$

for some $l \in \{k, k+1, ..., i-1\}$. This can be rewritten as

$$\frac{X(c_i)}{i}(c_i - c_0) - \frac{X(c_l)}{l}(c_l - c_0) \leq \frac{X(c_l)(c_l - c_0) - X(c_i)(c_i - c_0)}{il}$$

The right hand side of this inequality is strictly negative by the assumption that $X(p)(p - c_0)$ is increasing in $p < p^m$ and $c_l < c_i$. Hence it follows that

$$\frac{X(c_i)}{i}(c_i - c_0) < \frac{X(c_l)}{l}(c_l - c_0)$$

which contradicts that $p = c_i$ is the equilibrium price before entry. Q.E.D.

Proof of Lemma 9

(i) This is proved by contradiction. Suppose the price rises to $c_k > c_i$. Then it must be the case that

$$\frac{X(c_k)}{k-1}(c_k - c_0) \geq \frac{X(c_i)}{i-1}(c_i - c_0)$$

This can be rewritten as

$$\frac{X(c_i)(c_i - c_0) - X(c_k)(c_k - c_0)}{ki} \geq \frac{X(c_i)}{i}(c_i - c_0) - \frac{X(c_k)}{k}(c_k - c_0)$$

The left hand side of this inequality is strictly negative by the assumption that $X(p)(p - c_0)$ is increasing in $p < p^m$ and $c_k > c_i$. This contradicts that $p = c_i$ is the equilibrium price before firm l decides not to enter the pricing game.

(ii) Because a firm with $c_l > c_i$ decides not to enter the pricing game, the optimality of $p = c_i$ implies that also after firm l has left, it is the case that

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$$

for each $j < i$. Hence the subgame perfect equilibrium price cannot fall. The price may rise because there can be a firm $k > l$ such that

$$\frac{X(c_k)}{k-1}(c_k - c_0) \geq \frac{X(c_i)}{i}(c_i - c_0)$$

Q.E.D.

Proof of lemma 10

Equation (2) gives necessary and sufficient conditions for $p = c_k$ to be the subgame perfect equilibrium price after entry. To see this, note that for firms j with $c_j \geq c_l$ pricing at $p = c_j$ implies that there are $j + 1$ firms in the market. For prices $c_j \leq c_{l-1}$ there are j firms in the market (as before entry). Pricing at $p = c_e$ implies that there will be l firms in the market (namely firms $0, 1, ..., l-1$). Q.E.D.

Proof of proposition 11

To simplify notation, choose $c_0 = 0$. Monopoly price p^m is defined as $p^m = \arg\max_p \{pX(p)\}$. Choose c_n such that the following inequality holds

$$\frac{c_n X(c_n)}{n+1} < \frac{p^m X(p^m)}{n+2} \tag{6}$$

I construct the industry cost distribution in such a way that in the initial equilibrium (before entry) $p = c_n$ and hence we have n active firms in the industry. After entry, the price rises to $p' = c_{n+1} > c_n$. In order to get this price rise, $c_{n+1} < p^m$ must be chosen such that the following two inequalities hold

$$\begin{array}{ccc} \frac{c_n X(c_n)}{n} & > & \frac{c_{n+1} X(c_{n+1})}{n+1} \\ \frac{c_n X(c_n)}{n+1} & < & \frac{c_{n+1} X(c_{n+1})}{n+2} \end{array}$$

Because of equation (6) and the assumption that $\frac{d[pX(p)]}{dp} > 0$ for all $p < p^m$, such a value of c_{n+1} can be found, as shown in figure 2. Next, assign firms $1, ..., n-1$ all the same cost level c_1 where c_1 is chosen to satisfy the following inequality

$$\frac{c_n X(c_n)}{n} > \frac{c_1 X(c_1)}{1}$$

This makes sure that before entry the equilibrium price equals $p = c_n$. Finally, assign entering firm e cost level $c_e = c_0 = 0$. Then entry by firm e causes the equilibrium price to rise from $p = c_n$ to $p = c_{n+1}$, as can be seen as follows. First, note that $p = c_e$ cannot be an equilibrium. Next, note that the inequality $\frac{c_n X(c_n)}{n+1} < \frac{c_{n+1} X(c_{n+1})}{n+2}$ implies that the priceleaders prefer $p' = c_{n+1}$ to $p = c_n$. Finally, note that $p' = c_{n+1}$ is also preferred to $p' = c_1$ because $\frac{c_n X(c_n)}{n} > \frac{c_1 X(c_1)}{1}$ for $n > 1$ implies that $\frac{c_n X(c_n)}{n+1} > \frac{c_1 X(c_1)}{2}$ and hence $\frac{c_{n+1} X(c_{n+1})}{n+2} > \frac{c_1 X(c_1)}{2}$. Q.E.D.

-Figure 2 around here-

Proof of Lemma 12

(i) By continuity of the expression $X(p)(p - c_0)$, it follows from

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$$

for each $j \neq i$ that

$$\frac{X(c_i - \varepsilon)}{i}(c_i - \varepsilon - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$$

for ε small enough.

(ii) Let j denote smallest value such that $c_{j-1} < c_e \leq c_j$. Then the number of firms with costs strictly below c_e is j. Now note that there exists $E \in \aleph_+$ such that

$$\frac{X(c_e)}{j}(c_e - c_0) > \frac{X(c_k)}{k+E}(c_k - c_0)$$

for each $k \geq j$. Hence the subgame perfect equilibrium price falls to or below c_e. Q.E.D.

Proof proposition 13

The assumed inequalities

$$\frac{X(c_j)}{j}(c_j - c_0) > \frac{X(c_k)}{k}(c_k - c_0) \text{ for each } k \neq i, j$$

imply that $p = c_j$ is preferred above any price c_k with $k \neq i, j$. Can a price equal to c'_i yield higher profits to firm 0 than $p = c_j$? The inequality above together with the observation that

$$\frac{X(c_g)}{g}(c_g - c_0) \geq \frac{X(c'_i)}{g}(c'_i - c_0) \text{ for } c'_i \in \langle c_{g-1}, c_g] \text{ and } g < i$$

implies that

$$\frac{X(c_j)}{j}(c_j - c_0) > \frac{X(c'_i)}{g}(c'_i - c_0)$$

and thus $p = c_j$ gives a higher profit to the price leader than $p = c'_i$. Q.E.D.

Proof of Lemma 14

(i) By continuity of the expression $X(p)(p - c_0)$, it follows from

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$$

for each $j \neq i$ that

$$\frac{X(c_i - \varepsilon)}{i}(c_i - \varepsilon - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$$

for ε small enough.

(ii) Assume that before the cost reduction $p = c_i$, that is

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_j)}{j}(c_j - c_0)$$

for each $j \neq i$. This implies that

$$\frac{X(c_i)}{i}(c_i - c_0') > \frac{X(c_j)}{j}(c_j - c_0') + \left(\frac{X(c_j)}{j} - \frac{X(c_i)}{i}\right)(c_0' - c_0)$$

It follows for $j > i$ that $\frac{X(c_j)}{j} < \frac{X(c_i)}{i}$ and $c_0' < c_0$ imply that

$$\frac{X(c_i)}{i}(c_i - c_0') > \frac{X(c_j)}{j}(c_j - c_0')$$

for all $j > i$. Hence the equilibrium price does not rise. Q.E.D.

Proof of lemma 15

It is routine to verify that for ε close enough to 0 the following rationing rule

$$\beta_j^{\varepsilon}(n) = \begin{cases} \frac{1}{n} + \varepsilon\left(\frac{n-1}{2} - j\right) & \text{if} \quad j \leq n-1 \\ 0 & \text{if} \quad j \geq n \end{cases}$$

satisfies $\beta_j^{\varepsilon}(n) \geq 0$ for all $j \in \{0, 1, ..., n-1\}$, $\sum_{j=0}^{n-1} \beta_j^{\varepsilon}(n) = 1$ and does not violate the equilibrium inequalities at $p = c_i$. Q.E.D.

Proof of lemma 16

This is proved by contradiction following the notation and the argument in the proof of lemma 2. Suppose that there exists $i > j$ (i.e. $c_i \geq c_j$) such that $p_i^* = c_{i'} < c_{j'} = p_j^*$ (i.e. $i' < j'$).

$$\begin{aligned}
0 &> \beta_j(i')X(c_{i'})(c_{i'} - c_j) - \beta_j(j')X(c_{j'})(c_{j'} - c_j) \\
&= \beta_j(i')X(c_{i'})c_{i'} - \beta_j(j')X(c_{j'})c_{j'} - c_j\left(\beta_j(i')X(c_{i'}) - \beta_j(j')X(c_{j'})\right) \\
&\overset{(*)}{\geq} \beta_j(i')X(c_{i'})c_{i'} - \beta_j(j')X(c_{j'})c_{j'} - c_i\left(\beta_j(i')X(c_{i'}) - \beta_j(j')X(c_{j'})\right) \\
&= \beta_j(i')\left[X(c_{i'})(c_{i'} - c_i) - \frac{\beta_j(j')}{\beta_j(i')}X(c_{j'})(c_{j'} - c_i)\right] \\
&\overset{(**)}{\geq} \beta_j(i')\left[X(c_{i'})(c_{i'} - c_i) - \frac{\beta_i(j')}{\beta_i(i')}X(c_{j'})(c_{j'} - c_i)\right]
\end{aligned}$$

The first inequality follows from the fact that $p_j^* = c_{j'}$. The second inequality (labelled $(*)$) follows from the observations that $c_i \geq c_j$ and that $c_{i'} < c_{j'}$ implies that $\beta_j(i')X(c_{i'}) - \beta_j(j')X(c_{j'}) > 0$. The third inequality (labelled $(**)$) follows from equation (5). It follows that

$$\beta_i(j')X(c_{j'})(c_{j'} - c_i) > \beta_i(i')X(c_{i'})(c_{i'} - c_i)$$

contradicting the initial assumption that $p_i^* = c_{i'} < c_{j'}$. Q.E.D.

Proof of lemma 17

To prove this result, I use the one stage deviation principle (see, for instance, Fudenberg and Tirole (1991)). Note that the assumption made in the text that price bids are integers, $p \in \aleph$, implies that the pricing game becomes a finite horizon game because it can have at most $\bar{p} - c_1$ rounds, where $\bar{p}$ denotes the price at which the pricing game starts. This is due to the fact that either the current lowest price is undercut by (at least) 1 or if no bidder undercuts the current price the pricing stage ends.

In order to show that the price $\tilde{p}$ defined as

$$\tilde{p} \equiv \min \left\{ c_j | \frac{X(c_j)}{j}(c_j - c_0) \geq \frac{X(c_k)}{k}(c_k - c_0) \, for\ each\ k \neq j \right\}$$

is an equilibrium price of the game with q close enough to 0, I prove that no firm has an incentive to deviate from this equilibrium by undercutting this price $\tilde{p}$.

For notational convenience, let $n(p)$ denote the number of firms with marginal costs strictly below p. Then the pay off to firm i (with $c_i < \tilde{p}$) if it does not deviate from the equilibrium equals $\frac{X(\tilde{p})}{n(\tilde{p})}(\tilde{p} - c_i)$. Now consider player i deviating from the equilibrium by undercutting to a price $p_i' < \tilde{p}$. Its expected pay off can then be determined as follows. First, there is a probability q that the game ends and firm i gets a pay off equal to $X(p_i')(p_i' - c_i)$. Second, with probability $(1-q)$ a new subgame is entered where the current lowest price equals p_i'. The best pay off that firm i can hope to get in this subgame equals

$$\max_{p_i'' \leq p_i'} \left\{ \frac{X(p_i'')}{n(p_i'')}(p_i'' - c_i) \right\}$$

Clearly this pay off is an upperbound on the pay off that player i can expect in this subgame as other players may have an incentive to undercut p_i''.

It follows immediately from the price leadership lemma 2 that

$$\Delta_i \equiv \frac{X(\tilde{p})}{n(\tilde{p})}(\tilde{p} - c_i) - \max_{p_i'' \leq p_i'} \left\{ \frac{X(p_i'')}{n(p_i'')}(p_i'' - c_i) \right\} > 0$$

A sufficient condition for player i not to deviate from the equilibrium price $\tilde{p}$ can now be written as

$$\frac{X(\tilde{p})}{n(\tilde{p})}(\tilde{p} - c_i) > q \left[\max_{p_i' \leq \tilde{p}-1} \{X(p_i')(p_i' - c_i)\} \right] + (1-q) \left[\frac{X(\tilde{p})}{n(\tilde{p})}(\tilde{p} - c_i) - \Delta_i \right]$$

This condition is satisfied if

$$q < \bar{q}_i$$

where $\bar{q}_i$ is defined as

$$\bar{q}_i \equiv \frac{\Delta_i}{\Delta_i + \max_{p'_i \leq \tilde{p}-1} \left\{ X\left(p'_i\right)\left(p'_i - c_i\right)\right\} - \frac{X(\tilde{p})}{n(\tilde{p})}\left(\tilde{p} - c_i\right)}$$

Define $\bar{q}$ as $\bar{q} \equiv \min\{\bar{q}_0, \bar{q}_1, ...\}$ and the result follows.

If there are two (or more) values of j such that

$$\frac{X\left(c_j\right)}{j}\left(c_j - c_0\right) = \frac{X\left(c_{j'}\right)}{j'}\left(c_{j'} - c_0\right) \geq \frac{X\left(c_k\right)}{k}\left(c_k - c_0\right) \textit{ for each } k$$

for $j > j'$ (and thus $c_j > c_{j'}$), then firm 0 will undercut a price equal to c_j until the smallest element in the set $\left\{c_j | \frac{X(c_j)}{j}\left(c_j - c_0\right) \geq \frac{X(c_k)}{k}\left(c_k - c_0\right) \textit{for each } k \neq j\right\}$ is reached. This follows from the fact that

$$\Delta_0 = \frac{X\left(c_j\right)}{j}\left(c_j - c_0\right) - \frac{X\left(c_{j'}\right)}{j'}\left(c_{j'} - c_0\right) = 0$$

in this case and hence firm 0 has nothing to lose by undercutting the price $p = c_j$. Q.E.D.

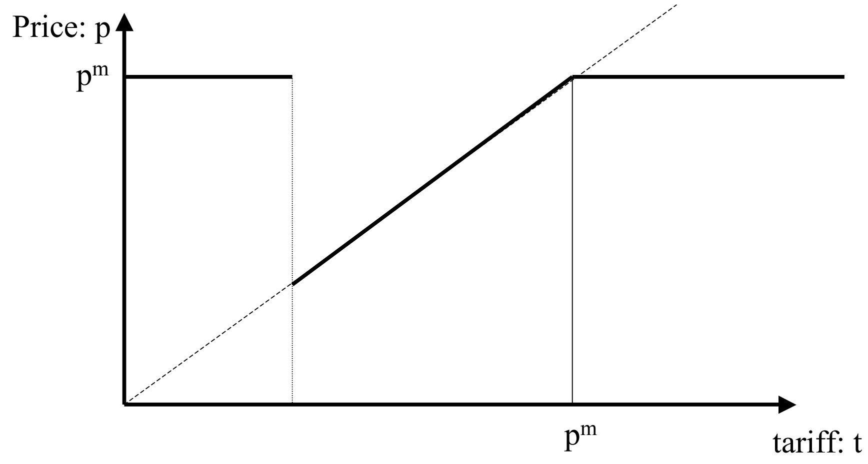

Figure 1: Single European market: equilibrium price as a function of the tariff t

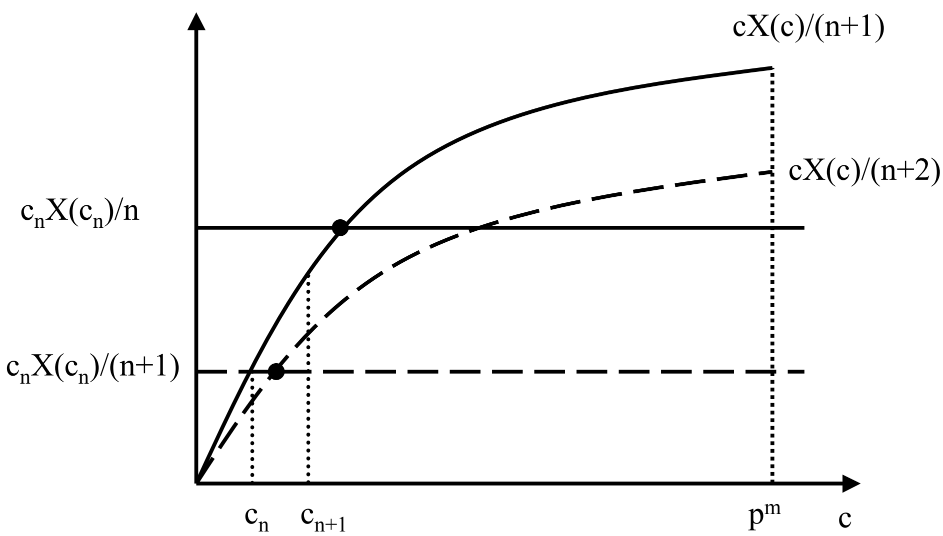

Figure 2: finding a value for c_{n+1} satisfying $c_{n+1}X(c_{n+1})/(n+1) < c_nX(c_n)/n$ and

$c_{n+1}X(c_{n+1})/(n+2) > c_nX(c_n)/(n+1)$