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Suijs, J.P.M.

Publication date:
2001

Citation for published version (APA):
Suijs, J. P. M. (2001). *Voluntary Disclosure and Risk Sharing*. (CentER Discussion Paper; Vol. 2001-90). Accounting.

No. 2001-90

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By Jeroen Suijs

November 2001

ISSN 0924-7815

Discussion paper

Voluntary disclosure and risk sharing

JEROEN SUIJS *

November 9, 2001

Abstract

This paper analyzes the disclosure strategy of firms that face uncertainty regarding the investor's response to a voluntary disclosure of the firm's private information. This paper distinguishes itself from the existing disclosure literature in that firms do not use voluntary disclosures to separate themselves from the less profitable firms. Here, voluntary disclosures are used to redistribute risk. It is shown that in a partial disclosure equilibrium, a firm discloses relatively bad news and withholds relatively good news. The reason for nondisclosure is that a firm is not willing to risk a negative response by the investor. However, if private information is relatively bad, nondisclosure imposes such a high risk on the investor, that he invests most of his capital in investment opportunities other than the firm. In that case, the firm is better off by disclosing its private information as this reduces the risk of the investor and increases the expected investment in the firm.

Keywords: voluntary disclosure, risk sharing.
JEL codes: D820, M410.

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*CentER Accounting Research Group, Tilburg University, PO Box 90153, 5000 LE, Tilburg, The Netherlands, phone: +31 13 466 2441, fax: +31 13 4663066, e-mail: j.p.m.suijs@kub.nl. The research of this author has been made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences. This paper was written while visiting the Kellogg Graduate School of Management at Nothwestern University. I wish to thank the faculty members of the Accounting Information and Management department for their hospitality. The comments and suggestions made by Ronald Dye, Frank Gigler, Michal Matejka, Sri Sridhar and workshop participants at Northwestern University and Tilburg University are gratefully acknowledged.

1 Introduction

Information asymmetries between firms and investors hamper the allocation of capital across the most productive investment opportunities. Conditional on disclosure being truthful and costless, Grossman (1981) and Milgrom (1981) show that in equilibrium, firms resolve these information asymmetries by fully disclosing their private information. This result is in strong contrast with empirical observations of firms not engaging in such full disclosure behavior and has lead subsequent research to finding theoretical explanations for nondisclosure. To this purpose, Verrecchia (1983), Dye (1986), Wagenhofer (1990), and Feltham and Xie (1992), amongst others, introduce (non)proprietary costs of disclosure, while Dye (1985) introduces uncertainty about the existence of private information. In this paper, the incentive to nondisclose arises because firms have imperfect information regarding investors' expectations. Due to this information asymmetry, disclosure brings about a certain amount of risk as firms do not perfectly know ex ante, how investors will respond to a disclosure. In this respect, the paper is similar to Kanodia and Lee (1998), where disclosure also puts a certain amount of risk on the disclosing firm. In Kanodia and Lee (1998), however, this risk serves as a cost to improve the quality of the disclosed signal, meaning that disclosure plays a similar role as retained equity does in Leland and Pyle (1977). Then, in equilibrium, firms with better private information make higher quality disclosures. This paper distinguishes itself from the existing literature in that firms do not use voluntary disclosures to separate themselves from firms with worse private information. Here, voluntary disclosures serve to redistribute the risk that firms and investors bear as a result of the existing information asymmetry.

The model considers a single investor and two firms. The investor has a limited amount of capital available that he can invest in any of the two firms or in some outside investment project with known return. A firm's return can either be high or low and is private information of the firm. To model the uncertainty regarding the investor's response, I assume that a firm knows the absolute value of its return, but not whether this return is the high or the low return. Since the investor wants to invest in the most profitable investment opportunity, disclosing that one is a low type yields a negative response, as the investor prefers investing in the other firm to investing in the disclosing firm. Similarly, disclosing that one is a high type yields a positive response, as the investor prefers investing in the disclosing firm to investing in the other firm. As a result, a firm is willing to disclose its return only if, conditional on its private information, it assigns a high probability to being the most profitable firm, i.e. having the high return.

The main results of the paper are the following. First, a full disclosure equilibrium always exists and is supported by skeptical beliefs of the investor. Second, a full nondisclosure equilibrium exists if the firms' posterior beliefs of being the most profitable firm are sufficiently low. In that case, firms prefer nondisclosure because the risk of getting an unfavorable response by the investor is too high. Third, a partial disclosure equilibrium may exist in which firms disclose low returns and withhold high returns. The explanation for this is the following. Let the cost of disclosure be the change in the firm's risk between nondisclosure and disclosure, and let the benefit of disclosure be the respective change in expected investment in the firm. Then the incentive to disclose relatively low profits arises because nondisclosure would impose too much risk on the investor, so that risk diversification arguments induce the investor to invest part of his capital in the outside option. Since this implies that the firm's expected investment under nondisclosure is relatively low, the benefit of disclosure is relatively high. In addition, nondisclosure imposes risk on the firm; because the firm has imperfect knowledge about the investor's expectations, it does not know how much the investor will invest in the outside option and how much he will invest in the firm. This uncertainty makes the cost of disclosure relatively low. Conversely, if profits are relatively high, nondisclosure imposes only little risk on the investor so that no investments in the outside option are made. Since in that case the firm's expected investment under nondisclosure is relatively high, the benefit of disclosure is relatively low. Furthermore, since zero investment in the outside option implies that nondisclosure imposes no risk on the firm, the cost of disclosure is relatively high.

In a partial disclosure equilibrium, voluntary disclosures are used to redistribute the risk that arises from the information asymmetry between the investor and the firms. Nondisclosure allocates most of the risk to the investor, while disclosure transfers (part of) this risk to the firms. The results imply that one should observe more voluntary disclosures in industries with higher risk. Since investors are unfavorably disposed towards investing in risky industries, firms need to use voluntary disclosures to reduce the risk of the investors, for otherwise they will invest their capital in other industries. In low risk industries, on the other hand, firms are aware that investors will invest in their industry anyway. Hence, nondisclosure should occur more frequently as firms are not willing to risk a negative response by the investors.

In a partial disclosure equilibrium, the equilibrium allocation of risk need not be socially efficient. It is shown that Pareto improvements can be obtained by allowing for transfer payments. For instance, in a nondisclosure equilibrium, all three parties may be better off if the investor compensates the firms for disclosing their information. Disclosure also need not be the socially efficient outcome. Only if the risk borne by the investor is relatively large compared to the risk borne by the firms, is disclosure socially efficient and are mandatory disclosure regulations desirable.

The remainder of this paper is organized as follows. Section 2 provides a formal description of the model. Section 3 presents the equilibrium results and discusses the multiplicity of equilibria and the efficiency of risk sharing. Section 4 discusses the results and Section 5 concludes. All proofs have been relegated to the appendix.

2 The model

The model consists of three decision makers, two firms (i.e. their managers) and a single investor. A state of the world is described by a triple (ϕ_1, ϕ_2, ψ), where ϕ_i denotes the private information of firm $i \in \{1, 2\}$ and ψ denotes the private information of the investor. ϕ_i represents the return of a constant return to scales investment project of firm i and is net of the manager's compensation payments. ψ represents the investor's expectations regarding the industry that the two firms operate in. Let v denote some publicly unobservable industry characteristic that determines the return of each firm and the beliefs of the investor. The value of v is randomly distributed according to the density function f with support V. The domain V is unbounded from above by assumption. A firm's investment project can be one of two types, a high profitable project or a low profitable project. This means that, conditional on v, the return ϕ_i equals $v + \Delta$ with probability $p \in (0, 1)$ and it equals $v - \Delta$ with probability $1 - p$, where $\Delta > 0$ is common knowledge. Notice that since the firm's managers do not know the value of v, they do not know whether they are a high type or a low type. For mathematical convenience, I assume for the investor's beliefs that $\psi = v$.[1] So, conditional on v, there are four possible states of the world, namely $(v+\Delta, v+\Delta, v)$, $(v+\Delta, v-\Delta, v)$, $(v-\Delta, v+\Delta, v)$, $(v-\Delta, v-\Delta, v)$ that occur with probabilities $p^2, p(1-p), p(1-p)$, and $(1-p)^2$, respectively.

To undertake the project, each firm needs to acquire capital from the investor. The investor has an endowment of capital $c > 0$ and, without loss of generality, I assume that $c = 1$. The investor can invest this capital in any of the two firms, or he can invest in some outside option generating a return on investment ϕ_0.[2] By investing in firm i, the investor

[1] This assumption has no significant effect on the main results of this paper. Modeling ψ as a noisy signal of v only increases the number of possible states of the world without any significant effect on the structure of the firms' and investor's decision problems. In either way of modeling, the investor's beliefs concerning a firm's type differ from the firm's beliefs of what the investor believes. It is this difference that drives the results.

[2] One can think of the outside option as investing in the risk free asset. To avoid confusion, however, I will not refer to ϕ_0 as being the risk free asset, since the firms' investment projects are modeled as risk free investments as well. In this model, risk only arises from information asymmetry. This simple structure reduces the complexity of the analysis without significantly affecting the results.

receives the firm's ownership rights so that each dollar invested earns the investor the return ϕ_i. A firm is called profitable if $\phi_i > \phi_0$ and nonprofitable if $\phi_i \leq \phi_0$. Prior to the investor's investment decision, each firm can voluntarily disclose the return ϕ_i of its investment project to the investor. I assume that all disclosures are truthful and that the disclosure decision is made simultaneously by both firms. The investor is not able to credibly disclose any information on ψ to the firms.[3]

The investor's objective is to maximize the expected utility from the investments' returns. The utility function is denoted by U_I and it is assumed that $U_I' > 0$ and $U_I'' \leq 0$. Since each project features constant returns to scale, the size of each project and the size of each managers' compensation are determined by the amount of capital that each manager acquires. Managers are assumed to be risk averse and maximize the expected utility of their compensation, i.e. the amount of capital that they acquire. For each $i \in \{1, 2\}$, the utility function of firm i's manager is denoted by U_i. Furthermore, it is assumed that $U_i(0) = 0$, $U_i' > 0$, and $U_i'' \leq 0$.

3 Equilibrium analysis

The equilibrium concept that I will use to analyze the disclosure behavior of the firms is that of a Bayesian equilibrium. In brief, a Bayesian equilibrium consists of a set of disclosure/investment strategies and beliefs such that the strategies are optimal with respect to the beliefs and the beliefs are rational with respect to the strategies. A more formal description follows below.

Let me start with a description of the beliefs and strategies of the firms. Given its private information ϕ_i, firm i knows that there are four possible states of the world, namely $(\phi_i, \phi_i, \phi_i - \Delta)$, $(\phi_i, \phi_i - 2\Delta, \phi_i - \Delta)$, $(\phi_i, \phi_i + 2\Delta, \phi_i + \Delta)$, and $(\phi_i, \phi_i, \phi_i + \Delta)$. The beliefs of firm i then specify the probability that firm i assigns to the occurence of each state. Notice that firm i has the high profit investment project if $v = \phi_i - \Delta$. Using Bayes' rule, the posterior probability that ϕ_i is the high profit project equals

$$p(\phi_i) = \frac{f(\phi_i - \Delta)p}{f(\phi_i - \Delta)p + f(\phi_i + \Delta)(1 - p)} \tag{1}$$

for all ϕ_i. Notice that $p(\phi_i) > p$ if and only if $f(\phi_i - \Delta) > f(\phi_i + \Delta)$, that is firm i updates its beliefs about being a high type upwards if $v = \phi_i - \Delta$ is more likely to occur than $v = \phi_i + \Delta$.

[3] Notice that the investor has a strong incentive to understate ψ, for revealing a low ψ induces more disclosure by firms as they are more likely to believe to be a high type.

The beliefs β_i of firm i are given by

$$\begin{aligned}
&\beta_i(\phi_i, \phi_i, \phi_i - \Delta) = p(\phi_i)p, \\
&\beta_i(\phi_i, \phi_i - 2\Delta, \phi_i - \Delta) = p(\phi_i)(1-p), \\
&\beta_i(\phi_i, \phi_i + 2\Delta, \phi_i + \Delta) = (1 - p(\phi_i))p, \\
&\beta_i(\phi_i, \phi_i, \phi_i + \Delta) = (1 - p(\phi_i))(1-p),
\end{aligned} \tag{2}$$

for each ϕ_i and each $i \in \{1, 2\}$.

Let d_i denote the disclosure strategy of firm $i \in \{1, 2\}$. For each return on investment ϕ_i, let $d_i(\phi_i) \in \{\phi_i, \tilde{\phi}_i\}$ with the interpretation that $d_i(\phi_i) = \phi_i$ corresponds to disclosure of ϕ_i and $d_i(\phi_i) = \tilde{\phi}_i$ corresponds to nondisclosure, where

$$\tilde{\phi}_i = \begin{cases} v + \Delta, & \text{with probability } p, \\ v - \Delta, & \text{with probability } 1 - p. \end{cases} \tag{3}$$

This notation allows for a convenient representation of the investor's private information. Prior any disclosures, the investor's private information is described by $(\tilde{\phi}_1, \tilde{\phi}_2, v)$; posterior any disclosures, it is described by $(d_1(\phi_1), d_2(\phi_2), v)$.

The investment decision of the investor depends on v, the disclosure decision of the firms, and the investor's posterior beliefs concerning the profitability of a nondisclosing firm. The posterior beliefs β_I assign to each possible state (ϕ_1, ϕ_2, v) of $(d_1(\phi_1), d_2(\phi_2), v)$ its likelihood of occurence $\beta_I(\phi_1, \phi_2, v)$. To illustrate, suppose $d_i(\phi_i) = \phi_i$ if and only if $\phi_i \leq \phi^*$ for all $i \in \{1, 2\}$, that is only relatively low profitability is disclosed. If $v + \Delta < \phi^*$, then both firms always disclose their private information so that $(d_1(\phi_1), d_2(\phi_2), v) = (\phi_1, \phi_2, v)$ and $\beta_I(\phi_1, \phi_2, v) = 1$. If $v - \Delta \leq \phi^* < v + \Delta$, then the investor knows that a nondisclosing firm is a high type, for otherwise it would have disclosed its private information. Since the investor can perfectly infer the state of the world from the disclosure strategies, it again holds that $\beta_I(\phi_1, \phi_2, v) = 1$. Finally, if $v - \Delta > \phi^*$, then firms always withhold their private information so that $(d_1(\phi_1), d_2(\phi_2), v) = (\tilde{\phi}_1, \tilde{\phi}_2, v)$ and $\beta_I(v + \Delta, v + \Delta, v) = p^2$, $\beta_I(v + \Delta, v - \Delta, v) = \beta_I(v - \Delta, v + \Delta, v) = p(1-p)$, and $\beta_I(v - \Delta, v - \Delta, v) = (1-p)^2$.

Let $x(d_1(\phi_1), d_2(\phi_2), v)$ describe the investment decision conditional on the investor's posterior information $(d_1(\phi_1), d_2(\phi_2), v)$. The investment decision $x(d_1(\phi_1), d_2(\phi_2), v)$ is a vector $(x_0(d_1(\phi_1), d_2(\phi_2), v), x_1(d_1(\phi_1), d_2(\phi_2), v), x_2(d_1(\phi_1), d_2(\phi_2), v))$ that describes how much the investor invests in the outside option, firm 1, and firm 2, respectively. Recall that the investor is constrained by his endowment $c = 1$ and that the investor does not receive any utility from not investing. Furthermore, since I exclude short selling, feasible investment decisions are characterized by $x_0 + x_1 + x_2 = 1$ and $x_i \geq 0$, $i \in \{0, 1, 2\}$.

The disclosure strategies d_1^*, d_2^*, investment decision x^*, and beliefs $\beta_1^*, \beta_2^*, \beta_I^*$ constitute a Bayesian equilibrium if

(a) the disclosure strategy d_i^* of firm $i \in \{1, 2\}$ is an optimal disclosure strategy, i.e.

$$d_i^*(\phi_i) = \begin{array}{rl} \arg\max & E\left(U_i(x_i^*(d_i(\phi_i), d_j(\phi_j), v)|\beta_i^*\right) \\ \text{s.t.:} & d_i(\phi_i) \in \{\phi_i, \tilde{\phi}_i\} \end{array} \tag{4}$$

for all ϕ_i,

(b) the investment decision x^* is optimal,

$$x^*(d_1^*(\phi_1), d_2^*(\phi_2), v) = \begin{array}{rl} \arg\max & E\left(U_I(x_0\phi_0 + x_1 d_1^*(\phi_1) + x_2 d_2^*(\phi_2)|\beta_I^*\right) \\ \text{s.t.:} & x_0 + x_1 + x_2 = 1, \\ & x_0, x_1, x_2 \geq 0, \end{array} \tag{5}$$

for all $v \in V$.

(c) the beliefs β_1^*, β_2^*, and β_I^* are rational with respect to the disclosure strategies d_1^*, d_2^* and investment strategy x^*.

Condition (a) requires that the disclosure strategy maximizes the firm's expected utility given the other firm's disclosure strategy, the investor's investment decision and the investor's beliefs. Notice that the expectation is taken with respect to the posterior beliefs β_i^* of firm i. Condition (b) requires that the investment decision maximizes the expected utility of the investor given his beliefs β_I, which are determined by v, d_1^*, and d_2^*. Condition (c) requires that the posterior beliefs follow Bayes' rule whenever possible.

An equilibrium is called a full disclosure equilibrium if both firms always reveal their private information to the investor, i.e. $d_i(\phi_i) = \phi_i$ for all ϕ_i and each $i \in \{1, 2\}$. Conversely, an equilibrium is called a full nondisclosure equilibrium if both firms always withhold their private information from the investor, i.e. $d_i(\phi_i) = \tilde{\phi}_i$ for all ϕ_i and each $i \in \{1, 2\}$. An equilibrium is called a partial disclosure equilibrium in all other cases.

Proposition 1 A full disclosure equilibrium always exists.

A full disclosure equilibrium is supported by skeptical beliefs of the investor. This means that if the investor observes nondisclosure by a firm, he beliefs that this firm has the low type investment project. Hence, disclosure (weakly) dominates nondisclosure. Notice that in order to sustain a full disclosure equilibrium, the investor must be able to commit to skeptical beliefs *ex ante*. To see this, suppose that firm j discloses its private information and that the investor observes an off-equilibrium nondisclosure by firm i.

If $\phi_j = v - \Delta > \phi_0$, the investor knows that the disclosing firm j is a low type. In that case, the investor's private information is given by $(\tilde{\phi}_i, \phi_j, v)$ which consists of the two possible states $(v-\Delta, v-\Delta, v)$ and $(v+\Delta, v-\Delta, v)$. If the investor can commit to skeptical beliefs, he beliefs that $(v-\Delta, v-\Delta, v)$ is the true state of the world so that his investment decision will be $x_i^*(\tilde{\phi}_i, v-\Delta, v) = x_i^*(v-\Delta, v-\Delta, v) = \frac{1}{2}$ and $x_j^*(\tilde{\phi}_i, v-\Delta, v) = x_j^*(v-\Delta, v-\Delta, v) = \frac{1}{2}$.[4] If, however, the investor cannot commit to skeptical beliefs, the latter investment decision is not optimal. Since the state of the world is either $(v-\Delta, v-\Delta, v)$ or $(v+\Delta, v-\Delta, v)$, investing in the nondisclosing firm i is optimal, i.e. $x_i^*(\tilde{\phi}_i, \phi_j, v) = 1$ and $x_j^*(\tilde{\phi}_i, \phi_j, v) = 0$.

If $\phi_j = v + \Delta > \phi_0$, the investor knows that the disclosing firm j is a high type. The investor's private information is given by $(\tilde{\phi}_i, \phi_j, v)$ which consists of the two possible states $(v-\Delta, v+\Delta, v)$ and $(v+\Delta, v+\Delta, v)$. In this case, it is does not matter whether the investor can commit to skeptical beliefs or not; the optimal investment strategy allocates all of the investor's capital to the high type firm j, i.e. $x_i^*(\tilde{\phi}_i, v+\Delta, v) = 0$ and $x_j^*(\tilde{\phi}_i, v+\Delta, v) = 1$.

So, conditional on firm j disclosing its private information, the optimal disclosure decision of firm i is determined as follows. Since disclosure yields an expected utility of

$$\begin{aligned}
&p(\phi_i)\left[pU_i(x_i^*(\phi_i, \phi_i, \phi_i - \Delta)) + (1-p)U_i(x_i^*(\phi_i, \phi_i - 2\Delta, \phi_i - \Delta))\right] \\
&+(1-p(\phi_i))\left[pU_i(x_i^*(\phi_i, \phi_i + 2\Delta, \phi_i + \Delta)) + (1-p)U_i(x_i^*(, \phi_i, \phi_i, \phi_i + \Delta))\right] \\
&= p(\phi_i)\left[pU_i(\tfrac{1}{2}) + (1-p)U_i(1)\right] + (1-p(\phi_i))\left[pU_i(0) + (1-p)U_i(\tfrac{1}{2})\right]
\end{aligned} \tag{6}$$

and nondisclosure yields

$$\begin{aligned}
&p(\phi_i)\left[pU_i(x_i^*(\tilde{\phi}_i, \phi_i, \phi_i - \Delta)) + (1-p)U_i(x_i^*(\tilde{\phi}_i, \phi_i - 2\Delta, \phi_i - \Delta))\right] + \\
&(1-p(\phi_i))\left[pU_i(x_i^*(\tilde{\phi}_i, \phi_i + 2\Delta, \phi_i + \Delta)) + (1-p)U_i(x_i^*(\tilde{\phi}_i, \phi_i, \phi_i + \Delta))\right] \\
&= p(\phi_i)\left[pU_i(0) + (1-p)U_i(1)\right] + (1-p(\phi_i))\left[pU_i(0) + (1-p)U_i(1)\right],
\end{aligned} \tag{7}$$

nondisclosure is preferred to disclosure if (7) exceeds (6), that is if

$$p(\phi_i) \leq \frac{(1-p)\left(U_i(1) - U_i\left(\frac{1}{2}\right)\right)}{(1-p)\left(U_i(1) - U_i\left(\frac{1}{2}\right)\right) + pU_i\left(\frac{1}{2}\right)}.$$

Hence, if the investor cannot commit to skeptical beliefs, full disclosure of private information need not occur.

The structure of the information asymmetry is such that each party has imperfect information about the type of the investment projects. Although firms know the return of their own

[4] Throughout the paper it is assumed that if the investor has identical posterior information about both firms, he invests the same amount in each firm, i.e. $x_1^*(\phi_1, \phi_2, v) = x_2^*(\phi_1, \phi_2, v)$ if $\phi_1 = \phi_2$. Notice that for $(\phi_1, \phi_2, v) \in \{(v+\Delta, v+\Delta, v), (v-\Delta, v-\Delta, v)\}$ this investment decision is not the unique optimal investment decision. Uniqueness, however, can be easily obtained by extending the model to allow for risky projects with return $\phi_i + \tilde{\varepsilon}$, where firm i only knows the expected return ϕ_i of its investment project.

investment project, they do not know whether their investment project is the high or the low type project. This lack of information puts a certain amount of risk on the disclosure decision of the firms and the investment decision of the investor. In this model, disclosure strategies can serve as instruments to allocate this risk.

The investor bears most of the risk if both firms withhold their private information. To diversify this risk, the investor will invest part of his capital in the outside option. More specifically, if neither firm discloses, the investor's private information equals $(\tilde{\phi}_1, \tilde{\phi}_2, v)$. Hence, if $v + \Delta \leq \phi_0$,[5] investing in the outside option dominates investing in a firm, so that $x_0^*(\tilde{\phi}_1, \tilde{\phi}_2, v) = 1$ and $x_i^*(\tilde{\phi}_1, \tilde{\phi}_2, v) = 0$ for $i \in \{1, 2\}$. Similarly, if $v - \Delta > \phi_0$, investing in a firm dominates investing in the outside option so that $x_0^*(\tilde{\phi}_1, \tilde{\phi}_2, v) = 0$. Since the firms are symmetric, optimal risk diversification yields that $x_i^*(\tilde{\phi}_1, \tilde{\phi}_2, v) = \frac{1}{2}$. If $v - \Delta \leq \phi_0 < v + \Delta$, investing in a firm yields a loss of $\phi_0 - (v - \Delta)$ if this firm turns out to be a low type. To diversify this risk, the investor will invest in the outside option as well as in both firms, i.e. $x_i^*(\tilde{\phi}_1, \tilde{\phi}_2, v) > 0$ and $x_1^*(\tilde{\phi}_1, \tilde{\phi}_2, v) = x_2^*(\tilde{\phi}_1, \tilde{\phi}_2, v) < \frac{1}{2}$. More specifically, define

$$\underline{v} = \sup\{v \in V | x_0^*(\tilde{\phi}_1, \tilde{\phi}_2, v) = 1\},$$
$$\bar{v} = \inf\{v \in V | x_0^*(\tilde{\phi}_1, \tilde{\phi}_2, v) = 0\}.$$

Since $\phi_0 - \Delta \leq \underline{v} \leq \bar{v} \leq \phi_0 + \Delta$, the investor invests all of his capital in the outside option if $v < \underline{v}$ and he invests all of his capital in the firms if $v > \bar{v}$. If $\underline{v} \leq v \leq \bar{v}$, the size of the position $x_0^*(\tilde{\phi}_1, \tilde{\phi}_2, v)$ depends on the loss $\phi_0 - (v - \Delta)$, the probability p, and the risk aversion of the investor. Notice that since the loss decreases in v, $x_0^*(\tilde{\phi}_1, \tilde{\phi}_2, v)$ also decreases in v. Furthermore, notice that the disclosure decision is irrelevant if $\phi_i \leq \underline{v} - \Delta$. In that case, $v \leq \underline{v}$ implies that the investor will not invest in the firms whatever their disclosure decisions.

Nondisclosure also imposes risk on the firms if the returns are relatively low. Given its private information ϕ_i, firm i knows that with probability $p(\phi_i)$ it is a high type, i.e. $v = \phi_i - \Delta$, and with probability $1 - p(\phi_i)$ it is a low type, i.e. $v = \phi_i + \Delta$. If $v = \phi_i - \Delta$, the investor invests $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta)$ in firm i, while if $v = \phi_i + \Delta$, he invests $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta)$. Notice that since $\phi_i - \Delta < \phi_i + \Delta$, it holds true that $0 \leq x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta) \leq x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) \leq \frac{1}{2}$. Furthermore, notice that if $\underline{v} - \Delta < \phi_i < \bar{v} + \Delta$, then the firm's payoff is risky as $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta) < x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta)$. So, if both firms withhold their private information, the firm's uncertainty regarding the investor's private information implies, that the firm is uncertain about the amount of capital that the investor will invest in the outside option and the firms. This

[5]Throughout the paper I assume that if $\phi_i = \phi_0$, the investor does not invest in firm i. Notice that such investment decision arises endogenously if the investor is risk averse, the outside option is risk free, and the investment project yields a risky return $\phi_i + \tilde{\varepsilon}$ instead of ϕ_i (cf. footnote 4).

uncertainty puts risk on nondisclosing firms. Observe that this risk is maximal if $\bar{v} - \Delta \leq \phi_i \leq \underline{v} + \Delta$, as this implies that $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta) = 0$ and $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) = \frac{1}{2}$, and that this risk is nonexistent if $\phi_i \geq \bar{v} + \Delta$, as this implies that $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta) = x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) = \frac{1}{2}$.

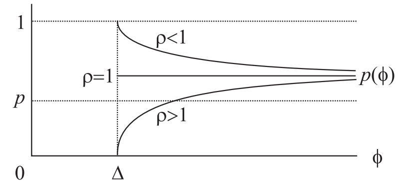
The risk that arises from the uncertainty regarding the firms' types is (partly) transfered from the investor to the firms if either one or both firms disclose their private information. Let me start with discussing the case that only one of the two firms discloses its private information, firm i say. First, observe that firm i has no incentive to disclose a nonprofitable return $\phi_i \leq \phi_0$. Hence, assume that $\phi_i > \phi_0$. Since the investor learns firm i's type, he no longer bears the risk associated with firm i's type. This risk is now borne by the disclosing firm i *and* the nondisclosing firm j. To see this, consider the payoff of firm i if firm i discloses its private information $\phi_i > \phi_0$ and firm j does not disclose. Then from firm i's perspective, firm i is a high type with probability $p(\phi_i)$ which yields the investment $x_i^*(\phi_i, \tilde{\phi}_j, \phi_i - \Delta)$, and firm i is a low type with probability $1 - p(\phi_i)$ which yields $x_i^*(\phi_i, \tilde{\phi}_j, \phi_i + \Delta)$. Notice that $x_i^*(\phi_i, \tilde{\phi}_j, \phi_i - \Delta) \geq x_i^*(\phi_i, \tilde{\phi}_j, \phi_i + \Delta)$, i.e. the investor will invest more in firm i if it is a high type than if it is a low type. The disclosure of firm i also affects the payoff of firm j. From firm j's perspective, firm i is a high type with probability p which yields firm j the investment $x_j^*(\phi_i, \tilde{\phi}_j, \phi_i - \Delta)$, and firm i is a low type with probability $1 - p$ which yields firm j the investment $x_j^*(\phi_i, \tilde{\phi}_j, \phi_i + \Delta)$. Notice that $x_j^*(\phi_i, \tilde{\phi}_j, \phi_i + \Delta) \geq x_j^*(\phi_i, \tilde{\phi}_j, \phi_i - \Delta)$, i.e. the investor will invest more in firm j if firm i is the low type than if firm i is the high type.

Summarizing, if both firms do not disclose, then firm i receives with probability $p(\phi_i)$ the low outcome $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta)$ and with probability $1 - p(\phi_i)$ the high outcome $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta)$. If only firm i discloses, then firm i receives with probability $p(\phi_i)$ the high outcome $x_i^*(\phi_i, \tilde{\phi}_j, \phi_i - \Delta)$ and with probability $1 - p(\phi_i)$ the low outcome $x_i^*(\phi_i, \tilde{\phi}_j, \phi_i + \Delta)$. Clearly, given that firm j does not disclose, firm i prefers disclosure to nondisclosure if the posterior probability $p(\phi_i)$ of being a high type is sufficiently high. One can show that a similar result holds if both firms disclose their private information. So, the posterior probability $p(\phi_i)$ plays a primary role in the firm's disclosure decision. Essential in this regard is whether or not $p(\phi_i)$ converges to one as ϕ_i becomes infinitely large. Convergence, namely, induces full disclosure by the usual unravelling argument. Firms with high returns will disclose their information as these firms know almost certainly that they are a high type. Then, conditional on this behavior, the investor perceives nondisclosure as a bad signal so that the firms with lower returns will also disclose their information. Table 1 shows for various classes of probability distributions the posterior probability $p(\phi_i)$ as a function of ϕ_i. Observe that convergence occurs only for normal distributions. Divergence from one occurs for Gamma, Pareto, and

Gamma distribution:

$$f(v) = \frac{\lambda^\beta}{\Gamma(\beta)} v^{\beta-1} e^{-\lambda v}, v > 0$$

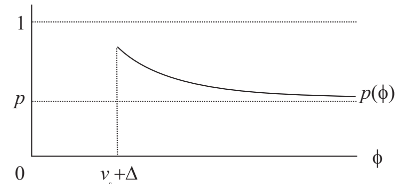
$$p(\phi) = \begin{cases} 0, & \phi \leq \Delta, \\ \frac{p}{p + \left(\frac{\phi+\Delta}{\phi-\Delta}\right)^{\beta-1} e^{-2\lambda\Delta}(1-p)}, & \phi > \Delta. \end{cases}$$



Pareto distribution:

$$f(v) = \frac{\lambda {v_0}^\beta}{v^{\beta+1}}, v > v_0 > 0$$

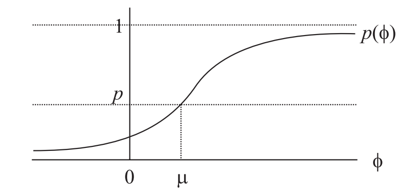
$$p(\phi) = \begin{cases} 0, & \phi \leq v_0 + \Delta, \\ \frac{p}{p + \left(\frac{\phi-\Delta}{\phi+\Delta}\right)^{\beta+1}(1-p)}, & \phi > v_0 + \Delta. \end{cases}$$



Normal distribution:

$$f(v) = \frac{1}{\sigma\sqrt{2\pi}} e^{-0.5\left(\frac{v-\mu}{\sigma}\right)^2}, v \in \mathbb{R}$$

$$p(\phi) = \frac{p}{p + e^{-\frac{\Delta(\phi-\mu}{\sigma^2}}(1-p)}, \phi \in \mathbb{R}.$$



Cauchy distribution:

$$f(v) = \frac{\delta}{\pi} \frac{1}{\delta^2 + (v-\alpha)^2}, v \in \mathbb{R}$$

$$p(\phi) = \frac{p}{p + \frac{\delta^2 + (\phi-\Delta-\alpha)^2}{\delta^2 + (\phi+\Delta-\alpha)^2}(1-p)}, \phi \in \mathbb{R}.$$

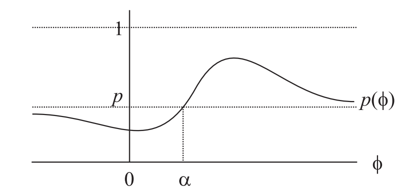


Table 1: The posterior probability $p(\phi)$ for various classes of probability distributions.

Cauchy distributions because of the relatively fat tails that these distributions exhibit.

Proposition 2 A full nondisclosure equilibrium exists if the firms' posterior beliefs of being a high type are sufficiently low, that is if $p(\phi_i) \leq \bar{p}_i(\phi_i)$ for all ϕ_i, where

$$\bar{p}_i(\phi_i) = \begin{cases} 1, & \text{if } \phi_i \leq \phi_0, \\ \frac{U_i\left(\frac{1}{2}\right)}{U_i(1) + U_i\left(\frac{1}{2}\right) - U_i(x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta))}, & \text{if } \phi_0 < \phi_i, \end{cases} \tag{8}$$

for each $i \in \{1, 2\}$.

Given the firm's incentives to disclose, a full nondisclosure equilibrium only exists if the posterior probability $p(\phi_i)$ of being a high type remains sufficiently low. $\bar{p}_i(\phi_i)$ denotes the probability for which firm i is indifferent between nondisclosure and disclosure given the equilibrium nondisclosure strategy of firm j. Observe that nondisclosure is always optimal if $\phi_i \leq \phi_0$. Since disclosing $\phi_i \leq \phi_0$ yields zero investment in firm i, firm i cannot be worse off by concealing ϕ_i. Furthermore, $\bar{p}_i(\phi_i) < \frac{U_i\left(\frac{1}{2}\right)}{U_i(1)}$ if $\phi_0 < \phi_i \leq \bar{v} + \Delta$ and $\bar{p}_i(\phi_i) = \frac{U_i\left(\frac{1}{2}\right)}{U_i(1)}$ if $\phi_i > \bar{v} + \Delta$. The explanation that $\bar{p}_i(\phi_i)$ is lower for low values of ϕ_i is the following. If $\phi_0 < \phi_i \leq \bar{v} + \Delta$ then $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta) < x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) = \frac{1}{2}$, so that nondisclosure also imposes some risk on the firm. This risk arises from the fact that a firm does not know how much the investor will invest in the outside option, and thus, how much the investor will invest in the firms. Since this risk is nonexistent if $\phi_i > \bar{v} + \Delta$, the cost of disclosure, i.e. the change in the firm's risk between disclosure and nondisclosure, is relatively low for $\phi_0 < \phi_i \leq \bar{v} + \Delta$. So, to sustain nondisclosure as the optimal disclosure decision, the benefits of disclosure, i.e. the probability of being a high type, must be relatively low too. Hence, $\bar{p}_i(\phi_i)$ is relatively low for $\phi_0 < \phi_i \leq \bar{v} + \Delta$.

Finally, observe that since $\bar{p}_i(\phi_i) \geq \frac{U_i\left(\frac{1}{2}\right)}{U_i(1)+U_i\left(\frac{1}{2}\right)} \geq \frac{1}{3}$, a full nondisclosure equilibrium can easily be constructed by appropriately choosing p and the parameters of the distribution of v if v is Gamma, Pareto, or Cauchy distributed (cf. Table 1).

Proposition 3 A partial disclosure equilibrium with disclosure strategies of the form

$$d_i^*(\phi_i) = \begin{cases} \tilde{\phi}_i, & \text{if } \phi_* < \phi_i \leq \phi_0, \\ \phi_i, & \text{if } \phi_0 < \phi_i \leq \phi^*, \\ \tilde{\phi}_i, & \text{if } \phi^* < \phi_i, \end{cases} \tag{9}$$

for each $i \in \{1, 2\}$, exists if for some $\phi^* > \phi_0$ it holds that

(3.a) $\phi_* = \max(\phi^* - 2\Delta, \underline{v} - \Delta)$,

(3.b) $p(\phi_i) \geq \underline{p}_i(\phi_i)$ for $\max(\phi^* - 2\Delta, \phi_0) \leq \phi_i < \phi^*$ and $i \in \{1, 2\}$, where

$$\underline{p}_i(\phi_i) = \begin{cases} \frac{pU_i(\frac{1}{2})+(1-p)\left(U_i(1)-U_i(\frac{1}{2})\right)}{2\left(pU_i(\frac{1}{2})+(1-p)\left(U_i(1)-U_i(\frac{1}{2})\right)\right)+(1-p)U_i(\frac{1}{2})}, & \text{if } \phi_0 < \phi_i \leq \phi_0 + 2\Delta, \\ \frac{1}{2}, & \text{if } \phi_0 + 2\Delta < \phi_i, \end{cases} \tag{10}$$

(3.c) $p(\phi_i) \leq \bar{p}_i(\phi_i)$ for all ϕ_i satisfying $\phi^* < \phi_i \leq \max(\phi^*, \phi_0 + 2\Delta)$ or $\phi_i > \phi^* + 2\Delta$, and $i \in \{1, 2\}$.

In a partial disclosure equilibrium, the disclosure decision of a firm with private information $\phi_i \leq \phi_*$ is irrevelant. Reason for this is that in equilibrium, the investor always learns that $\phi_i \leq \phi_0$ and will therefore not invest in this firm.

The partial disclosure equilibrium characterized in Proposition 3 features one nondisclosure interval if $\phi_* \geq \phi_0$, namely the interval (ϕ^*, ∞), and it features two nondisclosure intervals if $\phi_* < \phi_0$, namely the intervals $(\phi_*, \phi_0]$ and (ϕ^*, ∞). If only one nondisclosure interval arises in equilibrium, the investor always learns whether a firm is profitable or not. If two nondisclosure intervals arise in equilibrium, the non-profitable firms in the interval $(\phi_*, \phi_0]$

nondisclose as this enables them to pool with the profitable firms in the interval $(\phi^*, \phi_0 + 2\Delta]$. That pooling only occurs if $\phi_* < \phi_0$ is straightforward. Suppose firm i with $\phi_i < \phi_0$ does not disclose. Then with probabiltiy $p(\phi_i)$, firm i is a high type, that is $v = \phi_i - \Delta$. Since in that case the investor knows that firms are not profitable, firm i receives zero investment. With probability $1 - p(\phi_i)$, however, firm i is a low type, that is $v = \phi_i + \Delta$. Since in that case $v + \Delta = \phi_i + 2\Delta \geq \phi^*$, a high type firm also does not disclose. Then if the investor observes nondisclosure, his posterior beliefs equal his prior beliefs so that he invests $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta)$ in firm i. From $\phi_i + \Delta > \underline{v}$ it follows that $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) > 0$. Hence, a nonprofitable firm $\phi_i \in (\phi_*, \phi_0]$ strictly prefers nondisclosure to disclosure. If $\phi_* \geq \phi_0$, pooling is not beneficial/possible for the nonprofitable firms either because $\underline{v} - \Delta \geq \phi_0$ implies that $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) = 0$, or because $\phi^* - 2\Delta \geq \phi_0$ implies that the investor can infer that a nondisclosing firm is a low type.

Condition (3.b) requires that for $\max(\phi^* - 2\Delta, \phi_0) < \phi_i \leq \phi^*$ the posterior probability of being a high type is sufficiently high, so that disclosure of ϕ_i is optimal for firm i. Since for $\phi_0 < \phi_i \leq \min(\phi^* - 2\Delta, \phi_0)$ disclosure is driven by skeptical beliefs of the investor, no conditions on $p(\phi_i)$ are required. Notice that skeptical beliefs are not rational if $\max(\phi^* - 2\Delta, \phi_0) < \phi_i \leq \phi^*$. If $v = \phi_i + \Delta$, rationality implies that a nondisclosing firm is a high type. Furthermore, notice that $\underline{p}_i(\phi_i) < \frac{1}{2}$ if $\phi_0 < \phi_i \leq \phi_0 + 2\Delta$. The reason that $\underline{p}_i(\phi_i)$ is lower for $\phi_0 < \phi_i \leq \phi_0 + 2\Delta$ is again driven by the fact that, if $\phi_0 < \phi_i \leq \phi_0 + 2\Delta$, then nondisclosure also imposes some risk on the firms. Since this risk reduces the cost of disclosure, disclosure is optimal for lower values of $p(\phi_i)$. In particular, this implies that in a partial disclosure equilibrium, disclosure can be optimal even for firms that expect to be of a low type, i.e. firms that satisfy $\underline{p}_i(\phi_i) \leq p(\phi_i) < \frac{1}{2}$.

Condition (3.c) requires that $p(\phi_i)$ is sufficiently low for $\phi^* < \phi_i \leq \max(\phi^*, \phi_0 + 2\Delta)$ or $\phi_i > \phi^* + 2\Delta$, so that nondisclosure is optimal for firm i. From Proposition 2 it follows that this holds true if $p(\phi_i) \leq \bar{p}_i(\phi_i)$. Notice that since $\bar{p}_i(\phi_i) = \frac{U_i(\frac{1}{2})}{U_i(1)} \geq \frac{1}{2}$ for $\phi_i > \phi^* + 2\Delta$, nondisclosure can be optimal even for firms that expect to be of a high type, i.e. firms that satisfy $\bar{p}_i(\phi_i) \geq p(\phi_i) > \frac{1}{2}$. Furthermore, notice that for $\max(\phi^*, \phi_0 + 2\Delta) < \phi_i \leq \phi^* + 2\Delta$, nondisclosure is always optimal whatever the value of $p(\phi_i)$. The explanation for this is the following. If $\max(\phi^*, \phi_0 + 2\Delta) < \phi_i \leq \phi^* + 2\Delta$, then firm i knows that $v \in \{\phi_i - \Delta, \phi_i + \Delta\}$. First, consider the case $v = \phi_i - \Delta$. Since $v - \Delta \leq \phi^*$, the investor knows that a nondisclosing firm is a high type. Hence, firm i does not benefit from disclosing its private information. Next, consider the case $v = \phi_i + \Delta$, so that firm i is a low type. Since disclosing that one is a low type yields no benefits, nondisclosure is the optimal decision for firm i.

The partial disclosure equilibria characterized by Proposition 3, have the common feature that firms do not disclose relatively high returns. The following proposition shows that a reverse partial disclosure strategy in which firms only disclose high returns cannot not be supported in equilibrium.

Proposition 4 A partial disclosure equilibrium in which firms only disclose high returns, i.e. for some $\phi^* > \phi_0$ it holds that $d_i^*(\phi_i) = \phi_i$ if and only if $\phi_i \geq \phi^*$, $i \in \{1,2\}$, does not exist.

Reason for the nonexistence of such disclosure equilibria is that with such disclosure strategies, the investor perceives nondisclosure as a bad signal. Hence, upon observing nondisclosure, the investor believes that the nondisclosing firm is a low type, so that this firm is (weakly) better off by disclosing its private information. As a result, the firm ends up disclosing the low returns as well and a full disclosure equilibrium emerges.

3.1 Multiplicity of equilibria

The equilibrium analysis shows that various disclosure strategies can be supported in equilibrium. Besides the two extreme equilibria of full disclosure and full nondisclosure, multiple partial disclosure equilibria can exist. In fact, the following example shows that the number of partial disclosure equilibria can be infinite.

Example 1 Consider a risk neutral investor and risk averse firms with utility function $U_i(y) = \sqrt[3]{y}$, $y \geq 0$, for $i \in \{1,2\}$. Take $\phi_0 = 1$, $\Delta = 0.5$, and $p = 0.4$. Let v be exponentially distributed with expectation 1.2 so that $p(\phi_i) = 0.6888$ for all $\phi_i \geq \Delta$. Since the investor is risk neutral, it holds that

$$\underline{p}_i(\phi_i) = \begin{cases} 0.2259, & \text{if } \phi_0 < \phi_i \leq \phi_0 + 2\Delta, \\ 0.5, & \text{if } \phi_i > \phi_0 + 2\Delta. \end{cases}$$

Risk neutrality implies that $\underline{v} = \bar{v} = \phi_0 + (1-2p)\Delta$. Hence,

$$\bar{p}_i(\phi_i) = \begin{cases} 0.4142, & \text{if } \phi_0 \leq \bar{v} + \Delta, \\ 0.7071, & \text{if } \phi_i > \bar{v} + \Delta. \end{cases}$$

for each $i \in \{1,2\}$. Figure 1 depicts $p(\phi_i)$, $\bar{p}_i(\phi_i)$, and $\underline{p}_i(\phi_i)$. Notice that in this particular situation, any $\phi^* \geq \bar{v} + \Delta$ can be supported by a partial disclosure equilibrium. Since $p(\phi_i) > 0.5 \geq \underline{p}_i(\phi_i)$, condition (3.b) is satisfied, and since $p(\phi_i) < 0.7071 = \bar{p}_i(\phi_i)$ for $\phi_i \geq \bar{v} + \Delta$, condition (3.c) is satisfied.

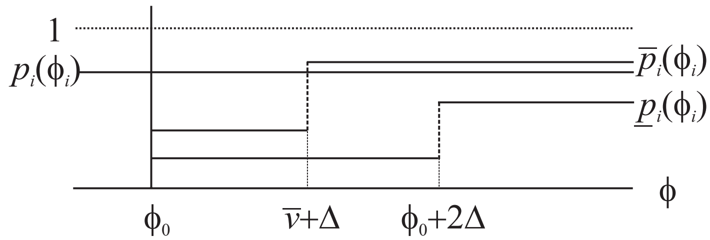


Figure 1: Partial disclosure equilibrium.

With the existence of multiple disclosure equilibria comes the problem of equilibrium selection. Since none of the three parties involved can enforce a particular equilibrium, there is no unambiguous answer to the question which equilibrium will emerge. Obviously, the investor always prefers the disclosure equilibrium payoffs to the nondisclosure equilibrium payoffs.[6] For the firms, however, this only holds if the posterior beliefs of being a high type are sufficiently high. More specifically,

Proposition 5 Let $\phi_i \geq \phi_0$. Then firm $i \in \{1, 2\}$ prefers the disclosure equilibrium payoffs to the nondisclosure equilibrium payoffs if $p(\phi_i) \geq \pi_i(\phi_i)$ where

$$\pi_i(\phi_i) = \frac{pU_i\left(\frac{1}{2}\right)}{pU_i\left(\frac{1}{2}\right) + (1-p)\left(U_i(1) - U_i\left(\frac{1}{2}\right)\right) + U_i\left(\frac{1}{2}\right) - U_i(x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta))}, \quad (11)$$

for all $\phi_i \geq \phi_0$.

Notice that since $\pi_i(\phi_i)$ is increasing in ϕ_i, disclosure is more likely to be preferred for low values of ϕ_i than for high values. Again, the reason for this is that for low values of ϕ_i, nondisclosure also puts some risk on the firms. Since this implies that the cost of disclosure is lower for low values of ϕ_i, the disclosure equilibrium yields a higher payoff for lower values of $p(\phi_i)$.

Example 1 (continued) Applying Proposition 5 yields that $\pi_i(\phi_i) = 0.3466$ if $\phi_0 \leq \phi_i \leq \bar{v}+\Delta$ and $\pi_i(\phi_i) = 0.7218$ if $\phi_i \geq \bar{v} + \Delta$. Since $p(\phi_i) = 0.6888$ for all $\phi_i > \Delta$, each firm prefers

[6]The disclosure equilibrium payoffs for a particular state of the world are the payoffs that the subjects receive if, in equilibrium, both firms disclose their private information in this particular state of the world. The equilibrium nondisclosure payoffs are defined similarly.

the nondisclosure equilibrium payoffs to the disclosure equilibrium payoffs if $\phi_i > \bar{v} + \Delta$ and they prefer the disclosure equilibrium payoffs if $\phi_0 \leq \phi_i < \bar{v} + \Delta$. Hence, from the firms' perspective, the partial disclosure equilibrium with $\phi^* = v + \Delta$ dominates the other partial disclosure equilibria, the full disclosure equilibrium and the full nondisclosure equilibrium.

3.2 Risk sharing efficiency

In a partial disclosure equilibrium, firms use their disclosure strategies to redistribute the risk that arises from the information asymmetry between the investor and the firms. In equilibrium, however, efficient risk sharing is not guaranteed. The following example shows that, in equilibrium, agents can obtain a Pareto improvement by using transfer payments.

Example 2 Let the investor and firms be constant absolute risk averse expected utility maximizers, i.e. $U_i(y) = 1 - e^{-\alpha_i y}$, $i \in \{I, 1, 2\}$. Take $\alpha_I = \alpha_1 = \alpha_2 = 1$ so that all three parties are equally risk averse. Let v be exponentially distributed with expectation 1 and let $\phi_0 = 1$. Notice that since all decision makers are constant absolute risk averse, the value of v does not affect their valuation of risk. Finally, let $p = \frac{1}{2}$ so that both firm types are equaly likely.

Take $\Delta = 0.1$. Given that v is exponentially distributed, it holds that $p(\phi_i) = 0.5498$ for all $\phi_i > \Delta$. Since $p(\phi_i) = 0.5987 < 0.6225 = \bar{p}_i(\phi_i)$ for all $\phi_i \geq \Delta$, both a full disclosure and a full nondisclosure equilibrium exist. Suppose the full disclosure equilibrium arises. Then the expected utility for firm $i \in \{1, 2\}$ equals 0.3705 while the investor's expected utility equals 0.0451. If $\phi_i > \phi_0 + 2\Delta$ for each firm $i \in \{1, 2\}$, nondisclosure by both firms yields a certain investment of $x_i = 0.5$ for each firm $i \in \{1, 2\}$. Since the utility of nondisclosure by both firms equals 0.3935 for each firm $i \in \{1, 2\}$, firms prefer the nondisclosure equilibrium payoffs. A Pareto improvement is realized if each firm pays the investor a compensation of 0.03, i.e. 6% of the invested capital, for not disclosing their private information. In that case, the utility of firm $i \in \{1, 2\}$ equals 0.3750, which is higher than the expected utility 0.3705 they receive under full disclosure. In addition, the expected utility of the investor increases from 0.0451 to 0.0559.

Next, take $\Delta = 0.2$. Since $p(\phi_i) = 0.5987 < 0.6225 = \bar{p}_i(\phi_i)$ for all $\phi_i \geq \Delta$, again both a full disclosure and a full nondisclosure equilibrium exist. Suppose the full nondisclosure equilibrium arises. If $\phi_i > \phi_0 + 2\Delta$ for each firm $i \in \{1, 2\}$, nondisclosure yields a certain investment of $x_i = 0.5$. Hence, the expected utility for each firm equals 0.3935. The expected utility of the investor equals -0.0100. If both firms would disclose their private information, the investor would receive an expected utility of 0.0806. In this case, a Pareto improvement arises if the investor pays each firm a compensation of 0.03 to reveal their information. Then the investor's expected utility increases from -0.01 to 0.0238, and the firms' expected utility increases from 0.3935 to 0.4041.

Example 2 shows that all three parties may benefit from a collective agreement in which either the firms compensate the investor for concealing their private information, or the investor compensates the firms for revealing their private information. Notice that such Pareto improvements are not driven by differences in risk aversion. The main reason for the existence of such Pareto improvements, is the fact that the risk that nondisclosure puts on the investor differs from the risk that disclosure puts on the firms. To see this, observe that the dispersion in the investor's payoff under nondisclosure is 2Δ, i.e. the difference between the high profit project $v + \Delta$ and the low profit project $v - \Delta$, while the dispersion of the firm's payoff of disclosure is at most 1, i.e. the firm receives the maximum investment 1 if it discloses that it is of a high type and the firm receives zero investment if it discloses that it is of a low type. So, if Δ is relatively small, nondisclosure puts relatively little risk on the investor. Then a cooperative agreement in which the firms pay the investor to conceal their information might be beneficial as the investor requires only a small compensation for the risk that he bears under nondisclosure. Similarly, if Δ is relatively high, nondisclosure puts a relatively high risk on the investor. Then a cooperative agreement in which the investor pays the firms to disclose their information may be beneficial as the investor is willing to pay a relatively large amount of money for the firms' private information.

4 Discussion

The primary determinant of a firm's disclosure decision is the firm's beliefs of having good information. If the firm believes it has good information, a voluntary disclosure of this information is likely to trigger a favorable reaction by the investor. Hence, the more positive a firm is about the value of its own information, the more likely it will disclose this information. These beliefs, however, are sensitive to the distribution of the investor's private information. The relationship between the profitability of the firm's investment project and his beliefs about the investor's valuation of this profitability need not be monotonically increasing. In fact, it is well possible that higher profits reduce the firm's beliefs of having good information. Consequently, firms may prefer to conceal good information.

A secondary determinant of the firm's disclosure decision is the disclosure decision of the other firm. This relationship is best explained on the basis of Example 1 and Figure 1.

Disclosure of information is optimal for firm i if its beliefs of having good information, i.e. of being a high type, are sufficiently high. In particular, given that firm j discloses, disclosure is optimal if $p(\phi_i) \geq \underline{p}_i(\phi_i)$, and, given that firm j does not disclose, disclosure is optimal if $p(\phi_i) \geq \bar{p}_i(\phi_i)$. Since $\bar{p}_i(\phi_i) > p(\phi_i) > \underline{p}_i(\phi_i)$ for all $\phi_i \geq \bar{v} + \Delta$, the disclosure decision of firm i depends on the disclosure decision of firm j. If firm i believes that firm j will disclose its private information, it is optimal for firm i to disclose as well. Conversely, if firm i believes that firm j will not disclose, it is optimal for firm i to nondisclose as well. Notice, however, that this type of herding behavior does not always occur. If, for instance, $\phi_0 \leq \phi_i < \bar{v} + \Delta$ in Example 1, firm i prefers disclosure to nondisclosure independent of the disclosure decision of firm j.

In this model, the interdependency between the firm's disclosure decisions arises from the fact that firms compete for the investor's limited amount of capital. A disclosure by one firm does not only affect the payoff of this disclosing firm, but also the payoff of the other firm. Disclosure dynamics within an industry of firms are more extensively examined in Dye and Sridhar (1995). There, however, the source of the interdependency is correlation in the distribution of the firms' private information.

In a partial disclosure equilibrium, firms disclose relatively low profits and conceal relatively high profits. The explanation for this is the following. Let the cost of disclosure be the change in the firm's risk between nondisclosure and disclosure, and let the benefit of disclosure be the respective change in expected investment in the firm. Then the incentive to disclose relatively low profits arises because nondisclosure would impose too much risk on the investor, so that risk diversification arguments induce the investor to invest part of his capital in the outside option. Since this implies that the firm's expected investment under nondisclosure is relatively low, the benefit of disclosure is relatively high. In addition, nondisclosure imposes risk on the firm. Since the firm does not have perfect knowledge about the investor's private information, it does not know how much the investor will invest in the outside option and how much he will invest in the firm. This uncertainty makes the cost of disclosure relatively low. Conversely, if profits are relatively high, nondisclosure imposes only little risk on the investor so that no investments in the outside option are made. Since in that case the firm's expected investment under nondisclosure is relatively high, the benefit of disclosure is relatively low. Furthermore, since zero investment in the outside option implies that nondisclosure imposes no risk on the firm, the cost of disclosure is relatively high.

In the model, one may think of the investor as representing a capital market and the firms as representing a specific industry. In this context, one can interpret v as an indicator of the industry's risk. A low value of v implies that an industry has high risk as investing in a nondisclosing firm can result in opportunity losses if the disclosing firm turns out to be a low type. A high value of v, on the other hand, implies that an industry has low risk as even the low type firms in the industry yield higher returns than the outside option. Applying the partial disclosure equilibrium results of Proposition 3 then implies, that one should observe more disclosures in a high risk industry than in a low risk industry. Firms in a high risk industry know that they have to disclose additional information so as to convince investors of their quality. If they do not disclose, investors will put their capital in other investment opportunities. In particular, this may induce disclosure by firms that expect to be of a low type, i.e. bad news disclosures may occur. Firms in a low risk industry, on the other hand, know that investors will invest in their industry independent of their disclosure decision. Since they are not willing to risk a negative response by investors, they do not disclose their private information. In particular, firms that expect to be a high type may not disclose, i.e. good news is concealed.

5 Conclusion

This paper analyzed the equilibrium disclosure strategy of firms that face uncertainty regarding the market's response to a voluntary disclosure of their private information. It is shown that this uncertainty provides an incentive for the firm to withhold its private information as the firm does not want to risk an unfavorable response by the market. Under certain conditions, this uncertainty can give rise to a full nondisclosure equilibrium.

In a partial disclosure equilibrium, firms use voluntary disclosures to redistribute the risk between the investor and the firms. In this respect, the paper differs form the existing literature, where firms generally use voluntary disclosures to separate themselves from less profitable firms. In a partial disclosure equilibrium, firms disclose relatively low returns because if returns are low, nondisclosure imposes too much risk on the investor, which adversely affects the investments in the firms. If returns are relatively high, the risk that nondisclosure imposes on the investor is small and does not significantly affect the investments in the firms. Hence, disclosure does not occur as firms are not willing to take the risk of an unfavorable response by the investor.

A Appendix

PROOF OF PROPOSITION 1: The expected utility of firm $i \in \{1,2\}$ under full disclosure by both firms equals

$$p(\phi_i)\left[pU_i(x_i^*(\phi_i,\phi_i,\phi_i-\Delta)) + (1-p)U_i(x_i^*(\phi_i,\phi_i-2\Delta,\phi_i-\Delta))\right] + \\ (1-p(\phi_i))\left[pU_i(x_i^*(\phi_i,\phi_i+2\Delta,\phi_i+\Delta)) + (1-p)U_i(x_i^*(\phi_i,\phi_i,\phi_i+\Delta))\right] \tag{12}$$

for each ϕ_i. If firm i nondiscloses, its expected utility equals

$$p(\phi_i)\left[pU_i(x_i^*(\tilde{\phi}_i,\phi_i,\phi_i-\Delta)) + (1-p)U_i(x_i^*(\tilde{\phi}_i,\phi_i-2\Delta,\phi_i-\Delta))\right] + \\ (1-p(\phi_i))\left[pU_i(x_i^*(\tilde{\phi}_i,\phi_i+2\Delta,\phi_i+\Delta)) + (1-p)U_i(x_i^*(\tilde{\phi}_i,\phi_i,\phi_i+\Delta))\right]. \tag{13}$$

Skeptical beliefs imply that

$$x_i^*(\tilde{\phi}_i,\phi_i,\phi_i-\Delta) = x_i^*(\phi_i-2\Delta,\phi_i,\phi_i-\Delta) \leq x_i^*(\phi_i,\phi_i,\phi_i-\Delta)$$
$$x_i^*(\tilde{\phi}_i,\phi_i-2\Delta,\phi_i-\Delta) = x_i^*(\phi_i-2\Delta,\phi_i-2\Delta,\phi_i-\Delta) \leq x_i^*(\phi_i,\phi_i-2\Delta,\phi_i-\Delta)$$
$$x_i^*(\tilde{\phi}_i,\phi_i+2\Delta,\phi_i+\Delta) = x_i^*(\phi_i,\phi_i+2\Delta,\phi_i+\Delta)$$
$$x_i^*(\tilde{\phi}_i,\phi_i,\phi_i+\Delta) = x_i^*(\phi_i,\phi_i,\phi_i+\Delta).$$

Hence, (12) is weakly greater than (13) so that for each ϕ_i, firm i (weakly) prefers disclosure of ϕ_i to nondisclosure. □

PROOF OF PROPOSITION 2: The expected utility of firm $i \in \{1,2\}$ under full nondisclosure by both firms equals

$$p(\phi_i)U_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta)) + (1-p(\phi_i))U_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i+\Delta)), \tag{14}$$

for all ϕ_i. If firm i discloses ϕ_i, the expected utility equals

$$p(\phi_i)U_i(x_i^*(\phi_i,\tilde{\phi}_j,\phi_i-\Delta)) + (1-p(\phi_i))U_i(x_i^*(\phi_i,\tilde{\phi}_j,\phi_i+\Delta)). \tag{15}$$

Then nondisclosure is (weakly) preferred to disclosure if (14) is (weakly) greater than (15) for all ϕ_i and all $i \in \{1,2\}$.

Take $\phi_i < \phi_0$. If $v = \phi_i - \Delta$ then $x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta) = 0$ as investing in the outside option dominates investing in the firms. From $\phi_i < \phi_0$ it follows that $x_i^*(\phi_i,\tilde{\phi}_j,\phi_i-\Delta) = x_i^*(\phi_i,\tilde{\phi}_j,\phi_i+\Delta) = 0$. Since $x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i+\Delta) \geq 0$, (14) exceeds (15) if $p(\phi_i) \leq 1$.

Take $\phi_0 \leq \phi_i < \bar{v}+\Delta$. Since $\bar{v} \leq \phi_0+\Delta$ implies $\phi_i+\Delta \geq \bar{v}$, it holds that $x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i+\Delta) = \frac{1}{2}$. From $\phi_i > \phi_0$ it follows that $x_i^*(\phi_i,\tilde{\phi}_j,\phi_i-\Delta) = 1$ and $x_i^*(\phi_i,\tilde{\phi}_j,\phi_i+\Delta) = 0$, so that (14) exceeds (15) if

$$p(\phi_i) \leq \frac{U_i(\frac{1}{2})}{U_i(1) + U_i(\frac{1}{2}) - U_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta))}.$$

$\square$

PROOF OF PROPOSITION 3: Let the disclosure strategy d_i^* of firm $i \in \{1, 2\}$ be as formulated in (9). I start with proving the equilibrium result for $\phi_0 < \phi^* < \phi_0 + 2\Delta$, that is there are two nondisclosure intervals $(\phi_*, \phi_0]$ and (ϕ^*, ∞), where $\phi_* = \max(\phi^* - 2\Delta, \underline{v} - \Delta)$.

First, let me specify the posterior beliefs of the investor. Notice that if $v \leq \underline{v}$, the posterior beliefs are irrelevant as the investor does not invest in any of the two firms. If $v > \underline{v}$, the posterior beliefs are taken as follows:

- if $\underline{v} < v \leq \phi_* + \Delta$, posterior beliefs are skeptical;
- if $\phi_* + \Delta < v \leq \phi_0 + \Delta$, the posterior beliefs are equal to the prior beliefs as $\phi^* - 2\Delta < v - \Delta \leq \phi_0$ and $v + \Delta > \phi^*$ imply that both the low and the high type firm do not disclose under d_i^*;
- if $\phi_0 + \Delta < v \leq \phi^* + \Delta$, posterior beliefs are optimistic. Since $v - \Delta \leq \phi^*$ and $v + \Delta > \phi^*$ only the high type firm does not disclose under d_i^*;
- if $\phi^* + \Delta < v$, posterior beliefs are equal to the prior beliefs. Since $v - \Delta > \phi^*$, both the low and the high type firm do not disclose under d_i^*.

Next, I show that given the posterior beliefs of the investor and the disclosure strategy d_j^* of firm j, the disclosure strategy d_i^* is optimal if the conditions of Proposition 3 are satisfied.

Consider $\phi_i \leq \phi_*$. Since $v \in \{\phi_i - \Delta, \phi_i + \Delta\}$ implies that $v \leq \underline{v}$, the investor invests all of his capital in the outside option. Hence, the disclosure decision of the firm does not matter, i.e. $d_i^*(\phi_i) \in \{\phi_i, \tilde{\phi}_i\}$.

Consider $\phi_* < \phi_i \leq \phi_0$. Since $\phi_i \leq \phi_0$, disclosure of ϕ_i yields firm i zero investment. Nondisclosure of ϕ_i yields with probability $p(\phi_i)$ the investment $x_i^*(\tilde{\phi}_i, d_j^*(\phi_j), \phi_i - \Delta)$ and with probability $1 - p(\phi_i)$ the investment $x_i^*(\tilde{\phi}_i, d_j^*(\phi_j), \phi_i + \Delta)$. If $v = \phi_i - \Delta$, then $v \leq \phi_0 - \Delta \leq \underline{v}$ and $\phi_i \leq \phi_0$ imply that $x_i^*(\tilde{\phi}_i, d_j^*(\phi_j), \phi_i - \Delta) = 0$. If $v = \phi_i + \Delta$, it follows from $\phi_* + \Delta < v \leq \phi_0 + \Delta$ that the posterior beliefs of the investor equal the prior beliefs. Furthermore, $\phi_* < v - \Delta \leq \phi_0$ and $\phi_* + 2\Delta < v + \Delta \leq \phi_0 + 2\Delta$ imply that, whatever its type, firm j does not disclose, i.e. $d_j^*(\phi_j) = \tilde{\phi}_j$. Hence, $x_i^*(\tilde{\phi}_i, d_j^*(\phi_j), \phi_i + \Delta) = x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta)$. In particular, $\phi_i + \Delta = v > \underline{v}$ implies that $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) > 0$. Since nondisclosure yields firm i with probability $p(\phi_i)$ zero investment and with probability $1 - p(\phi_i)$ the investment $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) > 0$, firm i strictly prefers nondisclosure to disclosure, i.e. $d_i^*(\phi_i) = \tilde{\phi}_i$ for $\phi_* < \phi_i \leq \phi_0$.

Consider $\phi_0 < \phi_i \leq \phi^*$. Then disclosure of ϕ_i yields firm i the expected utility

$$
\begin{aligned}
&p(\phi_i)\left[pU_i(x_i^*(\phi_i, d_j^*(\phi_i), \phi_i - \Delta)) + (1-p)U_i(x_i^*(\phi_i, d_j^*(\phi_i - 2\Delta), \phi_i - \Delta))\right] + \\
&(1-p(\phi_i))\left[pU_i(x_i^*(\phi_i, d_j^*(\phi_i + 2\Delta), \phi_i + \Delta)) + (1-p)U_i(x_i^*(\phi_i, d_j^*(\phi_i), \phi_i + \Delta))\right].
\end{aligned} \tag{16}
$$

From the definition of d_j^* it follows that $d_j^*(\phi_i) = \phi_i$, $d_j^*(\phi_i + 2\Delta) = \tilde{\phi}_j$, and $d_j^*(\phi_i - 2\Delta) \in \{\phi_i - 2\Delta, \tilde{\phi}_j\}$. Hence, $x_i^*(\phi_i, d_j^*(\phi_i), \phi_i - \Delta) = x_i^*(\phi_i, \phi_i, \phi_i - \Delta) = \frac{1}{2}$ and $x_i^*(\phi_i, d_j^*(\phi_i), \phi_i + \Delta) = x_i^*(\phi_i, \phi_i, \phi_i + \Delta) = \frac{1}{2}$. If $v = \phi_i - \Delta$, then $v \leq \phi^* - \Delta$ implies that the investor's posterior beliefs are skeptical so that $x_i^*(\phi_i, d_j^*(\phi_i - 2\Delta), \phi_i - \Delta) = x_i^*(\phi_i, \phi_i - 2\Delta, \phi_i - \Delta) = 1$. If $v = \phi_i + \Delta$, then $\phi_0 + \Delta < v \leq \phi^* + \Delta$ implies that the posterior beliefs are optimistic so that $x_i^*(\phi_i, d_j^*(\phi_i + 2\Delta), \phi_i + \Delta) = x_j^*(\phi_i, \phi_i + 2\Delta, \phi_i + \Delta) = 0$. Substituting the above values in (16) yields

$$
p(\phi_i)\left[pU_i\left(\tfrac{1}{2}\right) + (1-p)U_i(1)\right] + (1-p(\phi_i))(1-p)U_i\left(\tfrac{1}{2}\right). \tag{17}
$$

Nondisclosure yields firm i the expected utility

$$
\begin{aligned}
&p(\phi_i)\left[pU_i(x_i^*(\tilde{\phi}_i, d_j^*(\phi_i), \phi_i - \Delta)) + (1-p)U_i(x_i^*(\tilde{\phi}_i, d_j^*(\phi_i - 2\Delta), \phi_i - \Delta))\right] + \\
&(1-p(\phi_i))\left[pU_i(x_i^*(\tilde{\phi}_i, d_j^*(\phi_i + 2\Delta), \phi_i + \Delta)) + (1-p)U_i(x_i^*(\tilde{\phi}_i, d_j^*(\phi_i), \phi_i + \Delta))\right].
\end{aligned} \tag{18}
$$

From the definition of d_j^* it follows that $d_j^*(\phi_i) = \phi_i$, $d_j^*(\phi_i + 2\Delta) = \tilde{\phi}_j$, and $d_j^*(\phi_i - 2\Delta) \in \{\phi_i - 2\Delta, \tilde{\phi}_j\}$. If $v = \phi_i - \Delta$, the investor has skeptical beliefs so that $x_i^*(\tilde{\phi}_i, d_j^*(\phi_i), \phi_i - \Delta) = x_i^*(\tilde{\phi}_i, \phi_i, \phi_i - \Delta) = x_i^*(\phi_i - 2\Delta, \phi_i, \phi_i - \Delta) = 0$. Using $\phi_i - 2\Delta < \phi_0$ yields $x_i^*(\tilde{\phi}_i, d_j^*(\phi_i - 2\Delta), \phi_i - \Delta) = x_i^*(\phi_i - 2\Delta, \phi_i - 2\Delta, \phi_i - \Delta) = 0$. If $v = \phi_i + \Delta$, the investor has optimistic beliefs so that $x_i^*(\tilde{\phi}_i, d_j^*(\phi_i + 2\Delta), \phi_i + \Delta) = x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) = x_i^*(\phi_i + 2\Delta, \phi_i + 2\Delta, \phi_i + \Delta) = \frac{1}{2}$ and $x_i^*(\tilde{\phi}_i, d_j^*(\phi_i), \phi_i + \Delta) = x_i^*(\tilde{\phi}_i, \phi_i, \phi_i + \Delta) = x_i^*(\phi_i + \Delta, \phi_i, \phi_i + \Delta) = 1$. Substituting the above values in (18) yields

$$
(1-p(\phi_i))\left[pU_i(\tfrac{1}{2}) + (1-p)U_i(1)\right]. \tag{19}
$$

So, disclosure is (weakly) preferred to nondisclosure for all ϕ_i satisfying $\phi_0 \leq \phi_i < \phi^*$, if (17) exceeds (19), i.e. if

$$
p(\phi_i) \geq \frac{pU_i(\frac{1}{2}) + (1-p)\left(U_i(1) - U_i(\frac{1}{2})\right)}{2\left(pU_i(\frac{1}{2}) + (1-p)\left(U_i(1) - U_i(\frac{1}{2})\right)\right) + (1-p)U_i(\frac{1}{2})} \tag{20}
$$

for all $\phi_0 < \phi_i \leq \phi^*$.

Consider $\phi^* < \phi_i \leq \phi_0 + 2\Delta$. Since $d_j^*(\phi_j) = \tilde{\phi}_j$, nondisclosure of ϕ_i yields firm i the expected utility

$$p(\phi_i)\left[pU_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta))+(1-p)U_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta))\right]+ \\ (1-p(\phi_i))\left[pU_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i+\Delta))+(1-p)U_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i+\Delta))\right], \tag{21}$$

while disclosure yields

$$p(\phi_i)\left[pU_i(x_i^*(\phi_i,\tilde{\phi}_j,\phi_i-\Delta))+(1-p)U_i(x_i^*(\phi_i,\tilde{\phi}_j,\phi_i-\Delta))\right]+ \\ (1-p(\phi_i))\left[pU_i(x_i^*(\phi_i,\tilde{\phi}_j,\phi_i+\Delta))+(1-p)U_i(x_i^*(\phi_i,\tilde{\phi}_i,\phi_i+\Delta))\right]. \tag{22}$$

If $v=\phi_i-\Delta$, posterior beliefs equal prior beliefs since $\phi^*-\Delta<v\leq\phi_0+\Delta$. Similarly, if $v=\phi_i+\Delta$, posterior beliefs equal prior beliefs since $v>\phi^*+\Delta$. The fact that posterior beliefs equal prior beliefs implies that nondisclosure is optimal if $p(\phi_i)\leq\bar{p}_i(\phi_i)$ (cf. Proposition 2).

Consider $\phi_0+2\Delta<\phi_i\leq\phi^*+2\Delta$. Since $d_j^*(\phi_j)=\phi_j$ only if $\phi_j=\phi_i-2\Delta$, the expected utility of nondisclosure and disclosure of ϕ_i are given by

$$p(\phi_i)\left[pU_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta))+(1-p)U_i(x_i^*(\tilde{\phi}_i,\phi_i-2\Delta,\phi_i-\Delta))\right]+ \\ (1-p(\phi_i))\left[pU_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i+\Delta))+(1-p)U_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i+\Delta))\right], \tag{23}$$

and

$$p(\phi_i)\left[pU_i(x_i^*(\phi_i,\tilde{\phi}_j,\phi_i-\Delta))+(1-p)U_i(x_i^*(\phi_i,\phi_i-2\Delta,\phi_i-\Delta))\right]+ \\ (1-p(\phi_i))\left[pU_i(x_i^*(\phi_i,\tilde{\phi}_j,\phi_i+\Delta))+(1-p)U_i(x_i^*(\phi_i,\tilde{\phi}_j,\phi_i+\Delta))\right], \tag{24}$$

respectively. If $v=\phi_i-\Delta$, posterior beliefs of the investor are optimistic since $\phi_0+\Delta<v\leq\phi^*+\Delta$. If $v=\phi_i+\Delta$, posterior beliefs equal prior beliefs since $v>\phi^*+\Delta$. Applying the beliefs of the investor yields $x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta)=x_i^*(\phi_i,\phi_i,\phi_i-\Delta)$, $x_i^*(\tilde{\phi}_i,\phi_i-2\Delta,\phi_i-\Delta)=x_i^*(\phi_i,\phi_i-2\Delta,\phi_i-\Delta)$, and $x_i^*(\phi_i,\tilde{\phi}_j,\phi_i-\Delta)=x_i^*(\phi_i,\phi_i,\phi_i-\Delta)$. Since $x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i+\Delta)=\frac{1}{2}\geq 0=x_i^*(\phi_i,\tilde{\phi}_j,\phi_i+\Delta)$, it follows that the expected utility from nondisclosure in (23) is (weakly) greater than the expected utility from disclosure in (24). Hence, $d_i^*(\phi_i)=\tilde{\phi}_i$ for all $\phi^*\leq\phi_i<\phi^*+2\Delta$.

Consider $\phi_i>\phi^*+2\Delta$. Then $v\in\{\phi_i-\Delta,\phi_i+\Delta\}$ implies that $v>\phi^*+\Delta$ so that the investor's posterior beliefs are equal to his prior beliefs. Hence, nondisclosure is preferred to disclosure if $p(\phi_i)\leq\bar{p}(\phi_i)$ for all $\phi_i\geq\phi^*+2\Delta$ (see Proposition 2).

The proof for $\phi^*\geq\phi_0+2\Delta$ is similar to the proof above with the following exceptions. First, since $\phi^*\geq\phi_0+2\Delta$, the nondisclosure interval $[\phi_*,\phi_0)$ no longer exists. Second, posterior beliefs are skeptical if $v\leq\phi^*-\Delta$, posterior beliefs are optimistic if $\phi^*-\Delta<v\leq\phi^*+\Delta$, and posterior beliefs equal prior beliefs if $v>\phi^*+\Delta$. Third, disclosure is optimal for $\max(\phi^*-2\Delta,\phi_0+2\Delta)<\phi_i\leq\phi^*$ if $p(\phi_i)\geq\frac{1}{2}$. The reason that this condition differs from (20) is that $\phi_i-2\Delta>\phi_0$ implies that $x_i^*(\phi_i-2\Delta,\phi_i-2\Delta,\phi_i-\Delta)=\frac{1}{2}$ instead of $x_i^*(\phi_i-2\Delta,\phi_i-2\Delta,\phi_i-\Delta)=0$. This changes the expected utility of nondisclosure to

$$p(\phi_i)(1-p)U_i(\tfrac{1}{2}) + (1-p(\phi_i))\left[pU_i(\tfrac{1}{2}) + (1-p)U_i(1)\right], \tag{25}$$

so that disclosure is (weakly) preferred to nondisclosure if

$$p(\phi_i) \geq \frac{1}{2} \tag{26}$$

for all $\phi_0 + 2\Delta < \phi_i \leq \phi^*$. □

PROOF OF PROPOSITION 4: Take $\phi_i > \phi_0$ such that $\phi_i < \phi^* < \phi_i + 2\Delta$. Substituting $d_j^*(\phi_i) = \phi_j$, $d_j^*(\phi_i - 2\Delta) = \tilde{\phi}_j$, and $d_j^*(\phi_i + 2\Delta) = \phi_i + 2\Delta$ in expressions (16) and (18), respectively, yields an expected utility of disclosure of

$$\begin{aligned} &p(\phi_i)\left[pU_i(x_i^*(\phi_i, \tilde{\phi}_j, \phi_i - \Delta)) + (1-p)U_i(x_i^*(\phi_i, \tilde{\phi}_j, \phi_i - \Delta))\right] + \\ &(1-p(\phi_i))\left[pU_i(x_i^*(\phi_i, \phi_i + 2\Delta, \phi_i + \Delta)) + (1-p)U_i(x_i^*(\phi_i, \tilde{\phi}_j, \phi_i + \Delta))\right] \end{aligned} \tag{27}$$

and an expected utility of nondisclosure of

$$\begin{aligned} &p(\phi_i)\left[pU_i(x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta)) + (1-p)U_i(x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta))\right] + \\ &(1-p(\phi_i))\left[pU_i(x_i^*(\tilde{\phi}_i, \phi_i + 2\Delta, \phi_i + \Delta)) + (1-p)U_i(x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta))\right]. \end{aligned} \tag{28}$$

If $v = \phi_i - \Delta$, then $v + \Delta < \phi^*$ implies that posterior beliefs coincide with prior beliefs, so that $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta) < x_i^*(\phi_i, \tilde{\phi}_j, \phi_i - \Delta)$. If $v = \phi_i + \Delta$, then $v - \Delta < \phi^* < v + \Delta$ implies that posterior beliefs are skeptical, so that $x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta) = x_i^*(\phi_i, \phi_i, \phi_i + \Delta)$ and $x_i^*(\tilde{\phi}_i, \phi_i + 2\Delta, \phi_i + \Delta) = x_i^*(\phi_i, \phi_i + 2\Delta, \phi_i + \Delta)$. Hence, disclosure is strictly preferred to nondisclosure, which contradicts that in equilibrium, $d_i^*(\phi_i) = \tilde{\phi}_i$. □

PROOF OF PROPOSITION 5: If both firms disclose their private information, the expected utility of firm $i \in \{1, 2\}$ equals

$$\begin{aligned} &p(\phi_i)\left[pU_i(x_i^*(\phi_i, \phi_i, \phi_i - \Delta)) + (1-p)U_i(x_i^*(\phi_i, \phi_i - 2\Delta, \phi_i - \Delta))\right] + \\ &(1-p(\phi_i))\left[pU_i(x_i^*(\phi_i, \phi_i + 2\Delta, \phi_i + \Delta)) + (1-p)U_i(x_i^*(\phi_i, \phi_i, \phi_i + \Delta))\right]. \end{aligned} \tag{29}$$

If both firms do not disclose, the expected utility equals

$$p(\phi_i)U_i(x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i - \Delta)) + (1-p(\phi_i))U_i(x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta)). \tag{30}$$

If $\phi_i < \phi_0$, the expected utility of disclosure equals zero while the expected utility of nondisclosure equals $(1-p(\phi_i))U_i(x_i^*(\tilde{\phi}_i, \tilde{\phi}_j, \phi_i + \Delta))$. Hence, nondisclosure is always (weakly) preferred.

If $\phi_0 \leq \phi_i$, the expected utility of disclosure equals

$$p(\phi_i)\left[pU_i\left(\tfrac{1}{2}\right)+(1-p)U_i(1)\right]+(1-p(\phi_i))(1-p)U_i\left(\tfrac{1}{2}\right) \tag{31}$$

and the expected utility of nondisclosure equals

$$p(\phi_i)U_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta))+(1-p(\phi_i))U_i\left(\tfrac{1}{2}\right). \tag{32}$$

Hence, disclosure is preferred to nondisclosure if (31) exceeds (32), i.e. if

$$p(\phi_i)\geq\frac{pU_i\left(\frac{1}{2}\right)}{pU_i\left(\frac{1}{2}\right)+(1-p)\left(U_i(1)-U_i\left(\frac{1}{2}\right)\right)+U_i\left(\frac{1}{2}\right)-U_i(x_i^*(\tilde{\phi}_i,\tilde{\phi}_j,\phi_i-\Delta))}. \qquad \square$$

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