

Hypothetical Intertemporal Consumption Choices

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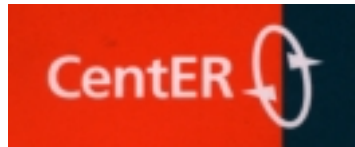
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**HYPOTHETICAL INTERTEMPORAL
CONSUMPTION CHOICES**

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Discussion paper

Hypothetical intertemporal consumption choices*

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Abstract

The paper extends and replicates part of the analysis by Barsky, Juster, Kimball, and Shapiro (1997), which exploits hypothetical choices among different consumption streams to infer intertemporal substitution elasticities and rates of time preference. We use a new and much larger dataset than Barsky et al. Furthermore, we estimate structural models of intertemporal choice, while parameterizing the parameters of interest as a function of relevant individual characteristics. We also consider "behavioral" extensions, like habit formation. Models with habit formation appear to be superior to models with intertemporally additive preferences.

JEL codes: C5, C9, D9

1 Introduction

Increasingly, recent findings cast doubt on the conventional intertemporally additive utility formulation of dynamic choice. The inspiration for alternative models often comes from other disciplines than economics. This is not surprising as the time dimension has been actively investigated in the past in several areas, such as personality theory and social psychology, and of course in individual decision theory.

Generally, in both psychology and economics most of the empirical findings have led to the conclusion that value and time exhibit a negative relation, so that the value of a certain good is discounted as a function of the time when it will be received (Horowitz, 1988). The most usual discounting function for representing time preferences has an exponential shape. An alternative, more concave curve has been proposed (Ainslie, 1975) as the result of observations of animal behavior. More

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recently, Laibson (1997) and Laibson, Repetto and Tobacman (1998), among others, have modelled the apparent empirical regularity that short-run discount rates are much higher than long-run ones by means of a hyperbolic function. People are more sensitive to a given time delay if it occurs earlier rather than later. As a consequence, behavior is dynamically inconsistent (Strotz, 1956; Ainslie, 1975; Elster, 1977) and discount rates decrease as a function of the time delay over which they are estimated. Ample evidence has accumulated for the fact that the conventional intertemporally additive utility formulation with exponential discounting provides an inadequate model of dynamic choice. As a result of this, alternative formulations have been developed. Two important strands are, what might be called, changing-tastes models and self-control models.

Changing tastes models account for the fact that preferences change over time so that, as time passes, a consumer may revise his consumption plans. The main point is how such an agent might behave in order not to become dynamically inconsistent. Early work in this line of research was Allais' (1947), who considered the welfare implications of a consumer with exogenously changing tastes. Strotz's (1956) paper discusses the consistency problem for a particular type of consumer planning his savings over a finite time-interval. He showed that inconsistency arises if and only if the individual discounts the utility of future consumption with a non-exponential discount function. A revisited version of Allais' model can be found in Pollak (1968). Blackorby et al. (1973) examined the general problem of a consumer with changing tastes making choices from budget sets. Their main result is that naive intertemporal optimization is consistent only if intertemporal preferences are structured so that the future is functionally separable from the present. Two illustrative models describing the behavior of an economic agent through time have been studied by Peleg and Yaari (1973). In one model, the agent is a producer-consumer with investment in one period affecting output in the next period. In the other model, the agent is a consumer operating successively in a sequence of competitive markets, with saving in one period affecting income in the next period. Hammond (1976) considered dynamic choice within an entirely general framework. When dynamic choice is inconsistent, two alternative procedures are examined: naive and sophisticated choice. In the first case, the agent ignores the fact that his tastes are changing; the resulting choices will be coherent only in very special circumstances. In the second case, the agent anticipates his future choices and chooses the best path from amongst those he is actually prepared to follow through to their end. If sophisticated choice is well defined, then it is consistent.

Closely connected to time consistency is the concept of self-control. This has been investigated by means of models where the agent is assumed to be both a farsighted planner and a myopic doer. The individual is seen as an organization, so that the problem of self-control is basically the same as the agency conflict between the owner and the manager of a firm. It is therefore not surprising that most of the work done in this field of research is based on a game-theoretic analysis. Some of the applications

of the principal-agent model have been investigated by Thaler and Schefrin (1981), particularly in the study of individual saving behavior. Both rules and incentives have been extensively taken into account. The work by Laibson (1997) squarely falls into this category.

Some authors have found that socio-economic conditions seem to play a relatively modest role in the explanation of dynamic choices, like the tendency to delay reward or punishment or a tendency to procrastinate (Strauss, 1962; Yates, 1972). In his study on intertemporal choice, health behavior and health status, Fuchs (1982) finds a positive, though not particularly strong, relation between time preference and schooling.

Two different empirical approaches have been developed in the literature in order to estimate individual preference parameters: the revealed preference approach and, what we shall call, the experimental approach. The first method consists of making a set of assumptions on the true individuals' preferences, observing the actual behavior and inferring the preference parameters. The experimental method consists of posing direct choices to respondents, which may involve real or hypothetical payoffs. The advantage of experiments is that one does not have to model the full environment in which agents operate (particularly constraints and uncertainty), but rather that one can fully specify scenarios oneself. On the other hand, of course, one may wonder how seriously respondents are trying to give honest answers, in particular to questions that have no consequences for themselves (hypothetical payoffs). We will briefly review some empirical literature using the two different approaches.

Exploiting an Euler equation approach, Lawrance (1991) estimates subjective rates of time preference for different permanent income classes, by using US data from the Panel Study of Income Dynamics (PSID). In her wealth-varying RTP model, she assumes a constant intertemporal elasticity of substitution (IES) as in the standard economic models with isoelastic utility functions. She finds that there exist wide differences in intertemporal preferences across households in a given age group. Particularly, estimated time preference rates exhibit a strong negative correlation with measures of presample labor income and levels of education.

Atkeson and Ogaki (1996) estimate a wealth-varying IES model using panel data on the consumption of Indian households. Ogaki and Atkeson (1997) use the same Indian panel data to estimate a model where both the RTP and the IES may change systematically between rich and poor households. Their main result is that the RTP is constant across wealth levels, while the IES is larger for the rich than for the poor, implying a more volatile consumption growth for the former than for the latter. This finding is basically in line with that of Mankiw and Zeldes (1991), that consumption growth is more volatile for stockholders than nonstockholders in the PSID. Given the inverse relation between the intertemporal elasticity of substitution and the coefficient of relative risk aversion, a model with wealth-varying IES would predict that the wealthy should hold a disproportionate share of aggregate risk and have more volatile consumption than the poor.

A detailed study by Hall (1988) on data for the twentieth-century United States shows little evidence for a large, positive IES. The main conclusion of the paper is that this parameter may be close to zero and probably not above 0.2. Earlier findings of substantially positive elasticities are reversed when appropriate estimation methods are used.

Hausman (1979) used individual household data on the purchase and utilization of room air conditioners to estimate the intertemporal discount rates used by consumers in order to evaluate the trade-off between present and future costs. These discount rates are intertemporal marginal rates of substitution that do not separate heterogeneous tastes from the influence of differences in taxes, capital market imperfections and income uncertainty. He found an estimated average annual discount rate of 26.4% (clearly above any relevant interest rate) and an inverse relation with income. Loewenstein and Thaler (1989) cite a number of other studies that find even larger discount rates. Explanations that have been offered include information barriers and liquidity constraints. Further explanations have been provided by Kooreman (1995), whose main conclusion is that if risk-neutral consumers anticipate a random lifetime of a durable, the assumption of a deterministic lifetime results in an upward bias, as large as 35%, of estimated discount rates.

Time preference has been extensively measured through survey techniques. In general, respondents face a hypothetical situation involving different amounts of money at different points in time and are asked to express a preference: this approach implicitly reveals a rate of time discount. Implicit discount rates were found to be negatively correlated with future time orientation and positively correlated with big spending (Thomas and Ward, 1979). Kurz et al. (1973) asked a sample of participants in the Seattle and Denver Income Maintenance Experiments a series of hypothetical questions such as: “What size bonus would you demand today rather than collect a bonus of \$100 in 1 year?”. They found a mean rate of time preference between 0.36 and 0.76, and between 0.40 and 1.22 for whites and for blacks, respectively.

Donkers et van Soest (1997) and Donkers et al. (1999) used data from the VSB panel survey¹ of Dutch households in order to elicit information about subjective measures of household preferences, financial decisions and risk attitudes. Their main results are as follows: the rate of time preference is negatively correlated with age and uncorrelated with other background variables or family income; the subjective interest rate is positively related to the decision to hold risky assets; the rate of risk aversion is positively related to the value of the house and negatively related to the decision to invest in risky assets.

During the Sixties, several experiments (Metzner and Mischel, 1962; Grusec and Mischel, 1967; Grusec, Masters and Mischel, 1969) on rewards and punishments run with adults and children revealed that the greater the period of delay, the lower was the probability that subjects would choose deferred rather than immediate rewards. Moreover, adults generally preferred immediate rather than deferred punishments.

¹Now called the CentER Savings Survey (CSS).

Similarly, in a study of the role of time preference in the intergenerational transmission of income inequality, Maital and Maital (1978) pointed out that the ability to defer gratification is part of the process of socialization and that after adolescence the propensity to delay gratification is quite stable. Later on, it has been found (Loewenstein, 1988) that the amount required to compensate for delaying the receipt of a real reward by a given interval was from two to four times greater than the amount subjects were willing to sacrifice to speed up consumption by the same interval.

Our study falls in the experimental strand of literature in that we follow Barsky, Juster, Kimball, and Shapiro (1997) (BJKS from now on) in using observed hypothetical choices between different consumption patterns to estimate both the rate of time preference and the elasticity of intertemporal substitution. As a theoretical framework we take the conventional intertemporally additive utility model as a starting point. We then extend the model by allowing for habit formation. By using hypothetical questions, the issue of self-control does not arise. The hypothetical nature of the questions allows a respondent to our questions to be a planner rather than a doer. Also, uncertainty is not part of the framework in which questions are asked. So, when we speak of relative risk aversion this refers to nothing else than the curvature of the intratemporal utility function. The goal of the paper is to investigate the structure of intertemporal preferences over consumption streams in as clean an environment as possible. Thus we abstract from uncertainty and present respondents in a survey with straightforward choices that do not involve complicated utility maximization tasks.

The remainder of the paper has the following structure: Section 2 describes the dataset we used for the experiment and presents the most relevant summary statistics. The basic model (with intertemporally additive utility) is explained in Section 3. In the same section we also present estimation results for this model. Habit formation is introduced in Section 4, where two model specifications are described, as well as the estimation outcomes for these two models. One of the distinguishing features of the conventional intertemporally additive utility model is the inverse relationship between the IES and the relative risk aversion parameter (in our context without uncertainty merely interpreted as a measure of the concavity of the utility function). Introduction of habit formation breaks this relation. Section 4 also includes a derivation of the IES in the model with habit formation. Section 5 concludes.

2 The dataset

In the empirical analysis we use data from the *CentERpanel*. The CentERpanel comprises some 2000 households in the Netherlands. The members of those households answer a questionnaire at their home computers every week. These computers may either be their own computer or a PC provided by CentERdata, the agency running

the panel². The CentERpanel is representative of the Dutch population. In the weekends of August 7-10 and August 14-17 of 1998 a questionnaire was fielded with a large number of questions on hypothetical choices. The questionnaire was repeated in the weekends of November 20-23 and November 27-30 of 1998 for those panel members who had not responded yet. The questions we use present respondents with five different hypothetical consumption paths and then ask them to choose one of them. A typical question reads as follows:

Now imagine that you (and your partner) decide to take financial advice and set up an expenditure plan, starting now and ending when you are ENDAGE years old. The financial advisor tells you that, in your situation, there are a number of options. These options will be presented on the screen below.

Please indicate by selecting a number, which expenditure pattern you would prefer. When making this choice, please consider your family situation to remain unchanged.

Please indicate your preferred expenditure pattern by selecting a number.

	<i>EXPENDITURE PATTERNS</i>				
	<i>(expenditures in guilders per month)</i>				
<i>AGE</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
<i>AGE1</i>	<i>PAT111</i>	<i>PAT112</i>	<i>CONS</i>	<i>PAT114</i>	<i>PAT115</i>
<i>AGE2</i>	<i>PAT121</i>	<i>PAT122</i>	<i>CONS</i>	<i>PAT124</i>	<i>PAT125</i>
<i>AGE3</i>	<i>PAT131</i>	<i>PAT132</i>	<i>CONS</i>	<i>PAT134</i>	<i>PAT135</i>
<i>AGE4</i>	<i>PAT141</i>	<i>PAT142</i>	<i>CONS</i>	<i>PAT144</i>	<i>PAT145</i>
<i>AGE5</i>	<i>PAT151</i>	<i>PAT152</i>	<i>CONS</i>	<i>PAT154</i>	<i>PAT155</i>
<i>AGE6</i>	<i>PAT161</i>	<i>PAT162</i>	<i>CONS</i>	<i>PAT164</i>	<i>PAT165</i>
<i>AGE7</i>	<i>PAT171</i>	<i>PAT172</i>	<i>CONS</i>	<i>PAT174</i>	<i>PAT175</i>
<i>AGE8</i>	<i>PAT181</i>	<i>PAT182</i>	<i>CONS</i>	<i>PAT184</i>	<i>PAT185</i>
<i>AGE9</i>	<i>PAT191</i>	<i>PAT192</i>	<i>CONS</i>	<i>PAT194</i>	<i>PAT195</i>

Select expenditure pattern 1, 2, 3, 4 or 5.

- After this question, three more, similar, questions are asked. The variables *ENDAGE*, *AGE1-AGE9*, *PAT111-PAT195* vary depending on the respondent's characteristics:
- *ENDAGE* depends on the respondent's age: if the respondent is younger than 56, *ENDAGE* is equal to 65; if the respondent is between 56 and 65, *ENDAGE*

²The description refers to the time of the survey. Nowadays, CentERdata does not provide a PC any longer but a set-top box.

is equal to 75; if the respondent is between 66 and 75, *ENDAGE* is 85. The questions are not posed to respondents older than 76.

- The variables *AGE1-AGE9* divide up the interval between the respondent's own age and *ENDAGE* in (almost) equal parts. *AGE1* is equal to the respondent's own age and *AGE9* is equal to *ENDAGE*.

2.1 The data generating process

The format of the questions cited above is very similar to that used by BJKS. The main difference is that in contrast to BJKS the computerized nature of the CentER-panel allows us to present each respondent with a series of choices that is related to the respondent's own circumstances. Not only is the age range a function of the respondent's own age, also the amounts are chosen in such a way that they are not too far from the respondent's own current consumption level.

The various consumption patterns have been generated as follows. Let y be the income of the household. Let ϵ be a uniformly distributed random variable on $[.8, 1.2]$. For each respondent we draw one value from the distribution of ϵ and compute $C = .9y\epsilon$. C is the flat consumption path that is given to the respondent as one of the possible choices (the middle column in the question cited above). The remaining consumption paths are derived from C as follows.

Draw four random variables: α_1 from $U[-.05, -.025]$, α_2 from $U[-.025, 0]$, α_3 from $U[0, .025]$, and α_4 from $U[.025, .05]$. These random variables are growth rates. Let the current age of a respondent be l (i.e. *AGE1* in the question) and let the final year of his or her consumption horizon be L (*ENDAGE* in the question). Furthermore let c_l^i be the consumption level at age l in path $i = 1, \dots, 4$ (the construction of c_l^i is described below). Then the consumption level in path i at any given age τ between l and L is given by $c_\tau^i = c_l^i \cdot (1 + \alpha_i)^{\tau-l}$. In other words, a consumption pattern is completely characterized by its initial value c_l^i and the growth rate α_i . It remains to describe the way the initial values c_l^i are generated.

The following somewhat artificial procedure has been adopted. Let r_i be a random *interest rate* drawn from a uniform distribution on $[-15, 15]$. Given the interest rate r_i and the growth rate α_i we choose c_l^i in such a way that the present discounted value of the consumption stream $\{c_\tau^i = c_l^i \cdot (1 + \alpha_i)^{\tau-l}\}_{\tau=l}^L$ is equal to the present discounted value of the constant consumption pattern C . In other words we impose:

$$c_l^i + c_l^i \frac{(1 + \alpha_i)}{1 + \frac{r_i}{100}} + \dots + c_l^i \frac{(1 + \alpha_i)^{\tau-l}}{(1 + \frac{r_i}{100})^{\tau-l}} + \dots c_l^i \frac{(1 + \alpha_i)^{L-l}}{(1 + \frac{r_i}{100})^{L-l}} \quad (1)$$

$$= C \left[1 + \frac{1}{1 + \frac{r_i}{100}} + \dots + \frac{1}{(1 + \frac{r_i}{100})^{\tau-l}} + \dots \frac{1}{(1 + \frac{r_i}{100})^{L-l}} \right] \quad (2)$$

Define $A_i \equiv 1 + \alpha_i$ and $R_i \equiv \frac{1}{1 + \frac{r_i}{100}}$. Then the equality can be written as

$$c_l^i [1 + A_i R_i + \dots + (A_i R_i)^{\tau-l} + \dots + (A_i R_i)^{L-l}] = C [1 + R_i + \dots + R_i^{\tau-l} + \dots R_i^{L-l}] \quad (3)$$

Solving for c_l^i yields

$$c_l^i = C \frac{1 - A_i R_i}{1 - R_i} \frac{1 - R_i^{L-l+1}}{1 - (A_i R_i)^{L-l+1}} \quad (4)$$

Although for our analysis the exact way in which the consumption paths have been generated is of no great consequence, the procedure followed ensures that respondents are faced with choices that do not deviate wildly from their own consumption level.

2.2 Descriptive statistics

In total, the questions have been posed to 1711 respondents. After deleting observations with missing values on some relevant variables we are left with a sample of 1557 observations. We constructed three levels of education: the “low level of education” consists of primary school, low-level high school, junior high school, junior vocational training, special low-level education and apprentice system; the “middle level of education” consists of senior high school and senior vocational training; the “high level of education” consists of vocational colleges and university education. There are 885 males and 672 females and their ages range from 22 to 75. More than 80% of the sample respondents have a partner, whereas the three levels of education we constructed are homogeneously represented (Table 1). The age distribution of the respondents is depicted in Figure 1.

Table 2 presents the frequency with which certain columns are chosen. One observes that the middle columns are chosen more frequently than the extreme patterns 1 and 5. Observe also that as a result of the way in which consumption patterns have been generated the first two consumption patterns presented are always upward sloping, whereas consumption patterns 4 and 5 are always downward sloping. One should probably suspect that not all respondents are equally conscientious in carrying out the task of selecting the optimal consumption path. One way in which this may show up is in “routine selection”. For instance, respondents may always pick the first consumption path or always the middle one, etc. It turns out that about half (48.9%) of the respondents pick the same column in all four questions. See Table 3. This may reflect a genuine preference for the column they pick, but it may also reflect an arbitrary choice. Clearly, with hindsight the order of the columns in the questions should have been randomized.

Table 1: Distribution of sample respondents across age classes, gender, education and marital status

Age classes	Gender		Level of Education			Marital status	
	<i>Fem.</i>	<i>Males</i>	<i>Low</i>	<i>Middle</i>	<i>High</i>	<i>Single</i>	<i>Married</i>
<i>22-32 yrs</i>	87	75	33	79	50	51	111
<i>33-43</i>	194	226	121	164	135	72	348
<i>44-55</i>	206	269	185	132	158	79	396
<i>56-66</i>	126	197	126	96	101	57	266
<i>67-75</i>	59	118	70	45	62	38	139
<i>Total</i>	<i>672</i>	<i>885</i>	<i>535</i>	<i>516</i>	<i>506</i>	<i>297</i>	<i>1260</i>

Table 2: Frequency of consumption patterns choice

Cons. pattern	Question 1		Question 2		Question 3		Question 4	
	Freq.	%	Freq.	%	Freq.	%	Freq.	%
1	179	11.50	165	10.60	161	10.34	149	9.57
2	395	25.37	408	26.20	385	24.73	407	26.14
3	476	30.57	473	30.38	516	33.14	509	32.69
4	331	21.26	341	21.90	320	20.55	315	20.23
5	176	11.30	170	10.92	175	11.24	177	11.37
Total	1557	100	1557	100	1557	100	1557	100

Table 3: Persistence of consumption patterns choice

Cons. pattern	% choosing only one pattern across four questions
1	4.4
2	11.4
3	18.5
4	9.1
5	5.5
Total	48.9

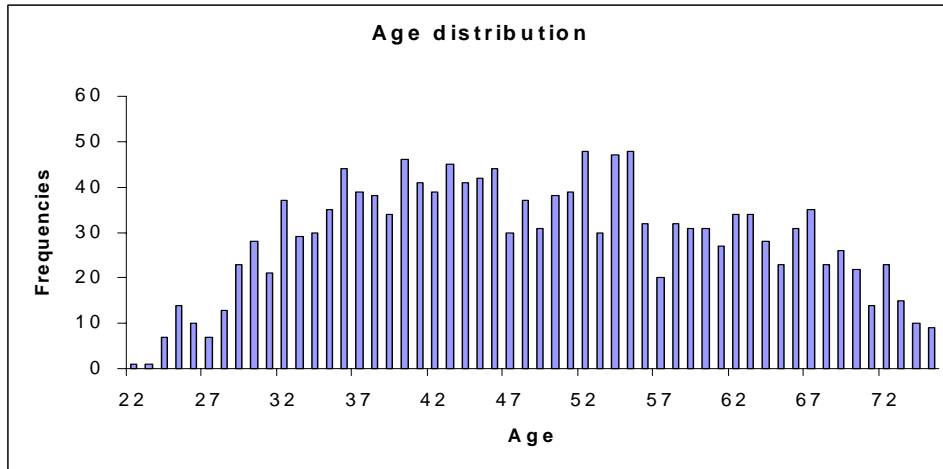


Figure 1: Age distribution of respondents

3 The basic model

As a starting point we model the choice of consumption path as the result of maximizing an intertemporally additive utility function. That is, a respondent prefers

a sequence of consumption levels, (c_l, \dots, c_L) to an alternative sequence, c'_l, \dots, c'_L , if and only if

$$\bigtimes_{t=l} \beta_t u(c_t) > \bigtimes_{t=l} \beta_t u(c'_t), \quad (5)$$

where $u(c)$ is a concave utility function and β_t is the weight given to utility in period t . Given that AGE1 is equal to the respondent's own age, we can interpret period $l \equiv$ AGE1 as being the present, whereas L is the period in which the respondent either turns 65, or 75, or 85. Thus, depending on a respondent's age, the time period over which the consumption path is defined will vary. For instance, if a respondent is 40, the time period will cover 25 years: 40-65. If a respondent is 75 years of age, the time period will cover only 10 years: 75-85. The wording of the question does not specify the consumption level in years in between the ages specified. We will interpret the consumption levels in the questions as representative of a smooth path from AGE1 to AGE9. Given the data generating process, the utility of a consumption path can be written as a function of the parameters of the data generating process, as will be shown below. In order to arrive at an estimable model, we make a number of additional assumptions. To begin with we assume exponential discounting; that is, the utility of a consumption path is written as³:

$$\bar{u} \equiv \bigtimes_{t=l} (1 + \delta)^{-t+1} u(c_t), \quad (6)$$

To allow for random variation in choices (for instance due to non-observable variation in preferences), we add an i.i.d. extreme value distributed error term to the utility function:

$$u_p^* = \bar{u}_p + \varepsilon_p \quad p = 1, \dots, 5 \quad (7)$$

where u_p^* represents the level of utility associated with consumption path p . Consumption pattern p is chosen (which we denote as $d_p = 1$) whenever it yields a level of utility greater than the one associated with all other paths. Formally, this means that

$$d_p = \begin{cases} \frac{1}{2} & \text{if } u_p^* > u_q^* \quad \forall q \neq p \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

According to this model we then have the familiar logit form for the probability that consumption path p is chosen:

$$\Pr(d_p = 1) = \frac{\exp(\bar{u}_p)}{\sum_{j=1}^5 \exp(\bar{u}_j)} \quad (9)$$

³Although we will later generalize this model to allow for habit formation, we will maintain the assumption of exponential discounting throughout. The motivation for this is that in the hypothetical and long term planning context that we study here, issues of self-control are much less likely to arise. In that sense the results presented do indeed refer to dynamic choices over a long horizon and not to short term intertemporal trade offs.

To close the model, we specify the form of the instantaneous utility function u and we exploit the specifics of the data generating process. For the instantaneous utility function we adopt the CRRA-specification:

$$u(c) \equiv (1 - \rho)^{-1} c^{1-\rho} \quad (10)$$

The parameter ρ is the coefficient of relative risk aversion and $\frac{1}{\rho}$ is the IES. As mentioned above, we can write the utility function (6) as a function of the underlying parameters of the data generating process. Recall that the consumption at age τ is equal to $c_\tau^i = c_l^i A_i^{(\tau-l)}$, where $A_i \equiv 1 + \alpha_i$. Inserting this into (6) and using the CRRA specification (10) we obtain for the intertemporal utility function:

$$\begin{aligned} \bar{u} &= \prod_{\tau=l}^{\mathcal{X}} (1 + \delta)^{l-\tau} \frac{1}{1 - \rho} (c_l^i A_i^{\tau-l})^{1-\rho} = \frac{(c_l^i)^{1-\rho}}{1 - \rho} \prod_{\tau=l}^{\mathcal{X}} (1 + \delta)^{l-\tau} (A_i^{\tau-l})^{1-\rho} \\ &= \frac{(c_l^i)^{1-\rho}}{1 - \rho} \prod_{\tau=l}^{\mathcal{X}} \Phi_i^{\tau-l} = \frac{(c_l^i)^{1-\rho}}{1 - \rho} \frac{1 - \Phi_i^{L-l+1}}{1 - \Phi_i} \end{aligned} \quad (11)$$

where $\Phi_i \equiv \frac{A_i^{1-\rho}}{1+\delta}$.

Each respondent is asked to choose four times among five consumption patterns. Thus we observe four choices per respondent. Estimation of the underlying parameters (δ, ρ) by maximum likelihood is straightforward. It is worth mentioning that our methodology differs from the one adopted by BJKS, as these authors estimate the rate of time preference and the intertemporal elasticity of substitution by means of the Euler equation deriving from a standard formulation of the consumer's maximization problem. We instead do not assume individuals to solve this problem.

3.1 Empirical results for the basic model

Table 4 presents the estimation results for the basic model. To allow for a preference for a certain column, we add dummies to the utility of each consumption path corresponding to the particular column corresponding to that consumption path. The choice of the first column serves as a reference category. Table 4 shows that these dummies are quite significant, with the middle column being the favorite and the two extreme columns (the first one and the fifth one) the least preferred ones. The estimated value of δ is in contrast with the findings of BJKS, who find a preference for upward sloping consumption paths. The estimated value of ρ ($\exp(.283) = 1.33$) is smaller than usually found in the literature. The implied IES $\frac{1}{\rho} = 0.75$ is larger than usually found. For instance, BJKS (who use the individual data to derive bounds on individual parameters) report an average upper bound on the IES equal to 0.36. Hall (1988) using a revealed preference representative agent approach estimates the IES to be around 0.1.

Table 4: Preference parameters for consumption paths (basic model)

<i>Parameters</i>	<i>Coeff.</i>	<i>s.e.</i>	<i>t-value</i>
delta	.175	.067	2.59
ln(rho)	.283	.049	5.80
Dummy for column 2	.934	.078	12.05
Dummy for column 3	1.86	.291	6.39
Dummy for column 4	.786	.097	8.06
Dummy for column 5	.216	.130	1.66
Log likelihood	-9468.0		

3.2 Parameterization of δ and ρ

We allow for variation in preferences, by making both δ and ρ a function of observable characteristics. The notion that preference parameters may vary with background characteristics has a long history. For instance, the idea of a connection between culture and patience has already been pointed out by Fisher (1930)⁴.

The estimation results with parametrization of δ and ρ are given in Table 5. The parameters of age and age squared are jointly significant at the 1% level for both ρ and δ . Figures 2 and 3 provide a picture of the implied age functions for the rate of time preference and the IES (or equivalently the coefficient or relative risk aversion), respectively⁵. Both graphs are non-monotonic in age. For δ , the graph first increases and then starts falling after the age of 35. Thus, roughly speaking people become increasingly patient beyond the age of 35. For ρ the graph first falls and starts rising beyond age 40. The graph for ρ implies that risk aversion is lowest at middle ages. The education dummies are jointly significant at the 5% level for both preference parameters. An increase in education appears to reduce risk aversion and increase impatience. The effects of household income and gender are both significant at the 5% level. Women are more patient than men and are more risk averse. Patience rises with income, as also found by Lawrance (1991). The latter result seems to give support to the idea that poverty “... *increases the want for immediate income even more than it increases the want for future income*” (Fisher, 1930, p.72). Risk aversion appears to increase slightly with income.

⁴“*In the case of primitive races, children, and other uneducated groups in society, the future is seldom considered in its true proportions*” (Fisher, 1930 p.81)

⁵For both functions, the origin of the vertical scale has no meaning, as we can arbitrarily change the intercept in the estimated index function and the origin of the age function.

Table 5: Preference parameters for consumption paths

<i>Parameter</i>	<i>Coeff.</i>	<i>s.e.</i>	t-value
<i>log(rho)</i>			
age	.0050	.002	2.65
age squared	.00024	.0002	1.51
gender	.216	.088	2.47
middle education	.056	.047	1.19
higher education	-.076	.060	1.27
log(household inc.)	.019	.006	3.47
constant	.048	.097	.50
<i>delta</i>			
age	-.0097	.002	4.32
age squared	-.00034	.0004	.95
gender	-.239	.110	2.18
middle education	-.111	.075	1.48
higher education	.362	.247	1.46
log(household inc.)	-.013	.010	1.30
constant	.477	.166	2.87
Dummy for column 2	.934	.077	12.2
Dummy for column 3	1.57	.170	9.18
Dummy for column 4	.781	.093	8.42
Dummy for column 5	.202	.117	8.42
No. of obs.		1557	
Log likelihood	-9415.0		

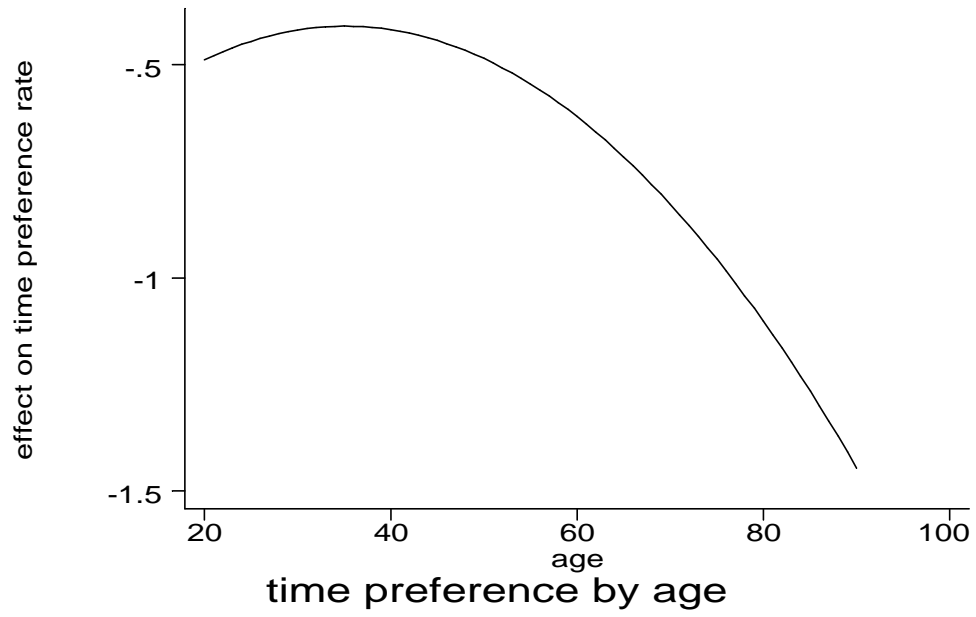


Figure 2: Time preference as a function of age

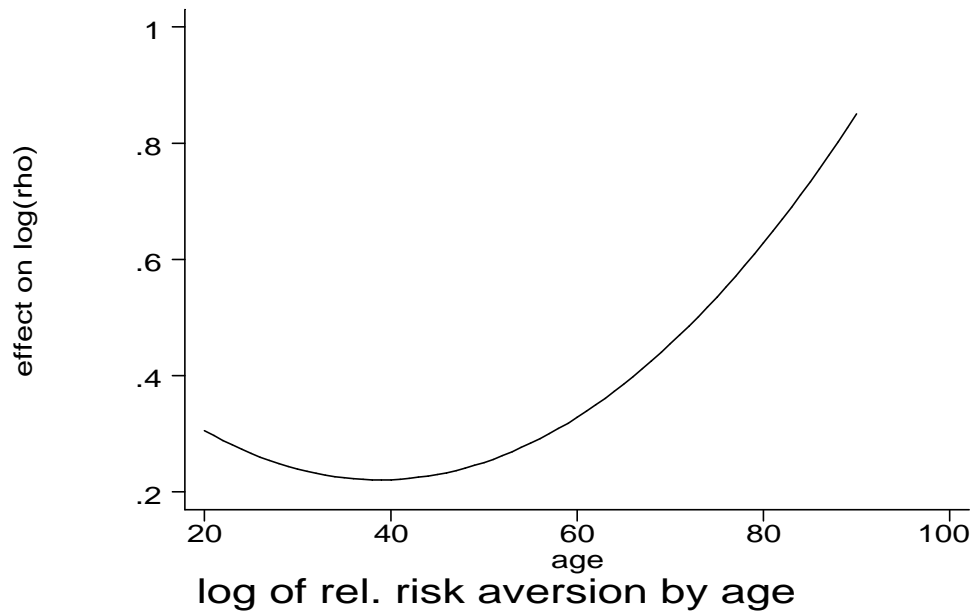


Figure 3: Log-riskaversion as a function of age

4 Habit formation

We now relax the assumption of intertemporal additivity of the utility function, so that the marginal rate of substitution between any two periods is no longer independent of the level of consumption in any two other periods. Thus, we consider life time utility functions of the following form:

$$\bar{u} = \sum_{t=l}^{\infty} (1 + \delta)^{l-t} v(c_t, z_t) \quad (12)$$

where, as before, L is the horizon, δ the discount rate, t denotes the time period, v is the intratemporal utility function, c_t is consumption in period t , and z_t now reflects the stock of habits⁶. To complete the model we have to describe the evolution of the stock of habits. We consider two alternative cases: First order Markov (AR) habits and Moving Average (MA) habits.

4.1 First order Markov habits

This is the simplest case. We specify z_t as a function of c_{t-1} . An obvious specification for the intratemporal utility function would then be $v(c_t - \theta z_t)$. Of course we may wish to consider transformations of c_t , e.g. its logarithm. Let $h(\cdot)$ be a monotonically increasing function, then a fairly general specification for the intratemporal utility function would be $v(h(c_t) - \theta h(z_t))$.

Choosing the logarithm for the transformation function h , the intertemporal utility function is then:

$$\bar{u} = \sum_{\tau=l}^{\infty} (1 + \delta)^{l-\tau} \frac{1}{1 - \rho} \left(\frac{c_\tau^i}{(z_\tau^i)^\theta} \right)^{1-\rho} \quad (13)$$

where $z_\tau^i \equiv c_{\tau-1}^i$ for $\tau = l, \dots, L$, and $z_l^i \equiv c_0^i$. The variable c_0^i (consumption at age $l - 1$) will be defined below. We can write for $\tau > l$:

$$\frac{c_\tau^i}{(z_\tau^i)^\theta} = \frac{c_l^i A_i^{(\tau-l)}}{(c_l^i)^\theta A_i^{(\tau-l-1)\theta}} = (c_l^i)^{1-\theta} A_i^{(\tau-l)(1-\theta)+\theta} = (c_l^i)^{1-\theta} A_i^\theta A_i^{(\tau-l)(1-\theta)} \quad (14)$$

Hence,

$$\begin{aligned} \bar{u} &= \frac{1}{1 - \rho} \left(\frac{c_l^i}{(c_0^i)^\theta} \right)^{1-\rho} + \frac{1}{1 - \rho} (c_l^i)^{(1-\theta)(1-\rho)} A_i^{\theta(1-\rho)} \sum_{\tau=l+1}^{\infty} (1 + \delta)^{l-\tau} \frac{1}{1 - \rho} A_i^{(1-\theta)(1-\rho)(\tau-l)} \\ &= \frac{1}{1 - \rho} \left(\frac{c_l^i}{(c_0^i)^\theta} \right)^{1-\rho} + \frac{1}{1 - \rho} (c_l^i)^{(1-\theta)(1-\rho)} A_i^{\theta(1-\rho)} \frac{\Psi_i - \Psi_i^{L-l+1}}{1 - \Psi_i} \end{aligned} \quad (15)$$

⁶Notice that in this context we only deal with rational habit formation (cf., e.g. Spinnewijn, 1981) as the respondents are assumed to consider the whole consumption path.

with $\Psi_i \equiv \frac{A_i^{(1-\theta)(1-\rho)}}{1+\delta}$

4.2 Moving Average habits

In this case we let habits evolve according to $h(z_t) = \beta h(z_{t-1}) + (1 - \beta)h(c_{t-1})$, with $0 < \beta < 1$. In contrast to the previous case, habits now have an infinite memory. We can rewrite the specification for habits as $(1 - \beta L)h(z_t) = (1 - \beta)h(c_{t-1})$, where L is the lag operator. We can then rewrite the expression for z_t as:

$$h(z_t) = \frac{1 - \beta}{1 - \beta L} h(c_{t-1}) \quad (16)$$

In a more cumbersome notation this can also be written as:

$$h(z_t) = \sum_{\tau=-\infty}^{\infty} \beta^{t-\tau} h(c_\tau) = \beta^t h(z_0) + (1 - \beta) \sum_{\tau=1}^{\infty} \beta^{t-\tau-1} h(c_\tau) \quad (17)$$

where

$$h(z_0) = \sum_{\tau=-\infty}^{\infty} \beta^{-\tau} h(c_{\tau-1}) \quad (18)$$

The only difference with the AR-case lies in the definition of the habit variable z_τ^i . We now define:

$$\ln z_t^i = \beta^{t-l} \ln z_0 + (1 - \beta) \sum_{\tau=l}^{\infty} \beta^{t-1-\tau} \ln c_\tau^i, \quad t > l \quad (19)$$

For age l we define $\ln z_l \equiv \ln z_0^i$. Furthermore, $\ln c_\tau^i = (\tau - l) \ln A_i + \ln c_l^i$, for $\tau > l$. We can use this to rewrite (19):

$$\begin{aligned} \ln z_t^i &= \beta^{t-l} \ln z_0^i + (1 - \beta) \left[\sum_{\tau=l}^{\infty} \beta^{t-1-\tau} (\tau - l) \ln A_i + \sum_{\tau=l}^{\infty} \beta^{t-1-\tau} \ln c_l^i \right], \quad t > l \\ &= \ln z_0, \quad t = l \end{aligned} \quad (20)$$

Now consider

$$\begin{aligned}
\ln c_t^i - \theta \ln z_t^i &= (t-l) \ln A_i + \ln c_l^i - \theta(\beta^{t-l} \ln z_0^i + & (21) \\
&\quad \prod_{\tau=l}^{t-1} \beta^{t-1-\tau}(\tau-l) + \ln c_l^i \prod_{\tau=l}^{t-1} \beta^{t-1-\tau}) \\
&= (t-l) \ln A_i + \ln c_l^i - \theta(\beta^{t-l} \ln z_0^i + \\
&\quad (1-\beta)[\ln A_i \frac{\beta^{t-l} - 1 + (1-\beta)(t-l)}{(1-\beta)^2} \\
&\quad + \ln c_l^i \frac{1 - \beta^{t-l}}{1-\beta}]) \\
&= (t-l)(1-\theta) \ln A_i + (1-\theta) \ln c_l^i + \theta \frac{\ln A_i}{1-\beta} \\
&\quad - \theta \beta^{t-l} [\ln z_0^i + \frac{\ln A_i}{1-\beta} - \ln c_l^i], \quad t > l \\
&= \ln c_l^i - \theta \ln z_0^i \quad t = l
\end{aligned}$$

Next we consider the utility function:

$$\begin{aligned}
\bar{u} &= \prod_{\tau=l}^t (1+\delta)^{t-\tau} \frac{1}{1-\rho} \left(\frac{c_\tau^i}{(z_\tau^i)^\theta} \right)^{1-\rho} & (22) \\
&= \prod_{\tau=l}^t (1+\delta)^{t-\tau} \frac{1}{1-\rho} \exp[(1-\rho)(\ln c_\tau^i - \theta \ln z_\tau^i)] \equiv \frac{1}{1-\rho} \prod_{\tau=l}^t \exp[\Omega_{\tau-l}]
\end{aligned}$$

with $\Omega_{\tau-l}$ defined as follows:

$$\Omega_{\tau-l} \equiv (l-\tau) \ln(1+\delta) + (1-\rho)(\ln c_\tau^i - \theta \ln z_\tau^i) \quad (23)$$

Upon using (21) this can be written as

$$\Omega_{\tau-l} = (\tau-l)[(1-\rho)(1-\theta) \ln A_i - \ln(1+\delta)] \quad (24)$$

$$\begin{aligned}
&\quad -\beta^{\tau-l}(1-\rho)\theta[\ln z_0^i + \frac{\ln A_i}{1-\beta} - \ln c_l^i] \\
&\quad + (1-\rho)[(1-\theta) \ln c_l^i + \theta \frac{\ln A_i}{1-\beta}], \quad \tau > l \\
&= (1-\rho)[\ln c_l^i - \theta \ln z_0^i], \quad \tau = l & (25)
\end{aligned}$$

Let us define

$$\Pi_i \equiv (1-\rho)(1-\theta) \ln A_i - \ln(1+\delta) \quad (26)$$

$$\Lambda_i \equiv (1 - \rho)\theta[\ln z_0^i + \frac{\ln A_i}{1 - \beta} - \ln c_i^i] \quad (27)$$

$$\Xi_i \equiv (1 - \rho)[(1 - \theta) \ln c_i^i + \theta \frac{\ln A_i}{1 - \beta}] \quad (28)$$

Then we can write (22) as

$$\begin{aligned} \bar{u} &= \frac{1}{1 - \rho} \frac{\mu}{z_0^\theta} c_i^i \uparrow_{1-\rho} + \frac{1}{1 - \rho} \frac{\times}{\tau=l+1} \exp[\Pi_i(\tau - l) - \Lambda_i \beta^{\tau-l} + \Xi_i] \quad (29) \\ &= \frac{1}{1 - \rho} \frac{\mu}{z_0^\theta} c_i^i \uparrow_{1-\rho} + \frac{\exp(\Xi_i)}{1 - \rho} \frac{\times}{\tau=l+1} \exp[\Pi_i(\tau - l) - \Lambda_i \beta^{\tau-l}] \end{aligned}$$

4.3 The intertemporal elasticity of substitution for the model with habit formation.

The introduction of habit formation breaks the tight link between the coefficient of relative risk aversion and the intertemporal elasticity of substitution⁷. Below we derive the IES for the model with moving average habit formation. This also covers the AR-case, as the AR-case is a special case of MA, obtained by setting $\beta = 1$. Our derivation closely follows Carroll (2000), which provides a derivation of the Euler equation with a slightly different specification of habit formation. For the purpose of the derivation we simplify the notation somewhat by omitting the superscript i . For a start, recall the specification of the utility function and the specification of the habit formation equation:

$$u(c, z) = \frac{1}{1 - \rho} \left(\frac{c}{z^\theta}\right)^{1-\rho} \quad (30)$$

$$\ln z_t = \beta \ln z_{t-1} + (1 - \beta) \ln c_{t-1} = \ln z_{t-1} + (1 - \beta)[\ln c_{t-1} - \ln z_{t-1}] \quad (31)$$

Some useful derivatives of the utility function are:

$$u^c = \left(\frac{c}{z^\theta}\right)^{-\rho} z^{-\theta} \quad (32)$$

$$u^z = -\theta \left(\frac{c}{z^\theta}\right)^{-\rho} \frac{c}{z^{\theta+1}} \quad (33)$$

The Bellman equation for the problem of maximizing the additive intertemporal utility function is:

$$v_t(x_t, z_t) = \max_{c_t} u(c_t, z_t) + \frac{1}{1 + \delta} v_{t+1}(x_{t+1}, z_{t+1}) \quad (34)$$

⁷Of course, habit formation is not the only way to break the link between risk aversion and intertemporal substitution, cf. e.g., Epstein and Zin (1989, 1991).

subject to

$$x_{t+1} = R(x_t - c_t) + y_{t+1} \quad (35)$$

$$\ln z_{t+1} = \ln z_t + (1 - \beta)[\ln c_t - \ln z_t] \quad (36)$$

where R is one plus the interest rate, x_t is cash on hand, y_{t+1} is income in period $t + 1$. The first order condition for a maximum is:

$$u_t^c + \frac{1}{1 + \delta} [(1 - \beta)v_{t+1}^z \frac{z_{t+1}}{c_t} - Rv_{t+1}^x] = 0 \quad \implies \quad (37)$$

$$u_t^c = \frac{1}{1 + \delta} [Rv_{t+1}^x - (1 - \beta)v_{t+1}^z \frac{z_{t+1}}{c_t}] \quad (38)$$

Next, exploit the envelope theorem to obtain

$$v_t^x = \frac{\partial v_t}{\partial x_t} + \frac{\overset{=0}{\partial v_t}}{\partial c_t} \frac{\partial c_t}{\partial x_t} = \frac{\partial v_t}{\partial x_t} = \frac{R}{1 + \delta} v_{t+1}^x \quad (39)$$

Combining this with (38) yields

$$v_t^x = u_t^c + \frac{1 - \beta}{1 + \delta} \frac{z_{t+1}}{c_t} v_{t+1}^z \quad (40)$$

Similarly,

$$v_t^z = \frac{\partial v_t}{\partial z_t} + \frac{\overset{=0}{\partial v_t}}{\partial c_t} \frac{\partial c_t}{\partial z_t} = \frac{\partial v_t}{\partial z_t} = u_t^z + \frac{1}{1 + \delta} v_{t+1}^z \frac{\partial z_{t+1}}{\partial z_t} = u_t^z + \frac{\beta}{1 + \delta} v_{t+1}^z \frac{z_{t+1}}{z_t} \quad (41)$$

This implies

$$v_{t+1}^z = u_{t+1}^z + \frac{\beta}{1 + \delta} v_{t+2}^z \frac{z_{t+2}}{z_{t+1}} \quad (42)$$

Substituting the expression for v_{t+1}^z in (40) yields

$$u_t^c = v_t^x - \frac{1 - \beta}{1 + \delta} \frac{z_{t+1}}{c_t} u_{t+1}^z - \frac{\beta(1 - \beta)}{(1 + \delta)^2} \frac{z_{t+2}}{c_t} v_{t+2}^z \quad (43)$$

(40) implies

$$v_{t+1}^x = u_{t+1}^c + \frac{1 - \beta}{1 + \delta} \frac{z_{t+2}}{c_{t+1}} v_{t+2}^z \quad \implies \quad (44)$$

$$\frac{1 - \beta}{1 + \delta} z_{t+2} v_{t+2}^z = c_{t+1} v_{t+1}^x - c_{t+1} u_{t+1}^c \quad (45)$$

Substituting this in (43) yields

$$\begin{aligned}
u_t^c &= v_t^x - \frac{1-\beta}{1+\delta} \frac{z_{t+1}}{c_t} u_{t+1}^z - \frac{\beta}{1+\delta} \frac{c_{t+1}}{c_t} [v_{t+1}^x - u_{t+1}^c] \\
&= v_t^x - \frac{\beta}{1+\delta} \frac{c_{t+1}}{c_t} v_{t+1}^x - \frac{1}{1+\delta} [(1-\beta) \frac{z_{t+1}}{c_t} u_{t+1}^z - \beta \frac{c_{t+1}}{c_t} u_{t+1}^c] \\
&= v_t^x - \frac{\beta}{R} \frac{c_{t+1}}{c_t} v_t^x - \frac{1}{1+\delta} [(1-\beta) \frac{z_{t+1}}{c_t} u_{t+1}^z - \beta \frac{c_{t+1}}{c_t} u_{t+1}^c] \\
&= [1 - \frac{\beta}{R} \frac{c_{t+1}}{c_t}] v_t^x - \frac{1}{1+\delta} [(1-\beta) \frac{z_{t+1}}{c_t} u_{t+1}^z - \beta \frac{c_{t+1}}{c_t} u_{t+1}^c] \tag{46}
\end{aligned}$$

(46) implies

$$u_{t+1}^c = [1 - \frac{\beta}{R} \frac{c_{t+2}}{c_{t+1}}] v_{t+1}^x - \frac{1}{1+\delta} [(1-\beta) \frac{z_{t+2}}{c_{t+1}} u_{t+2}^z - \beta \frac{c_{t+2}}{c_{t+1}} u_{t+2}^c] \tag{47}$$

So that

$$v_{t+1}^x = [1 - \frac{\beta}{R} \frac{c_{t+2}}{c_{t+1}}]^{-1} \frac{1}{1+\delta} [(1-\beta) \frac{z_{t+2}}{c_{t+1}} u_{t+2}^z - \beta \frac{c_{t+2}}{c_{t+1}} u_{t+2}^c] + u_{t+1}^c \tag{48}$$

Define

$$B_t \equiv \frac{1 - \frac{\beta}{R} \frac{c_{t+1}}{c_t}}{1 - \frac{\beta}{R} \frac{c_{t+2}}{c_{t+1}}} \tag{49}$$

Combining (48) with (46) and (39) yields

$$\begin{aligned}
u_t^c &= [1 - \frac{\beta}{R} \frac{c_{t+1}}{c_t}] \frac{R}{1+\delta} v_{t+1}^x - \frac{1}{1+\delta} [(1-\beta) \frac{z_{t+1}}{c_t} u_{t+1}^z - \beta \frac{c_{t+1}}{c_t} u_{t+1}^c] \\
&= B_t \frac{R}{1+\delta} \frac{1}{1+\delta} [(1-\beta) \frac{z_{t+2}}{c_{t+1}} u_{t+2}^z - \beta \frac{c_{t+2}}{c_{t+1}} u_{t+2}^c] + u_{t+1}^c \\
&\quad - \frac{1}{1+\delta} [(1-\beta) \frac{z_{t+1}}{c_t} u_{t+1}^z - \beta \frac{c_{t+1}}{c_t} u_{t+1}^c] \tag{50}
\end{aligned}$$

which is the Euler equation for consumption.

Using (32) and (33) this can be written as

$$\begin{aligned}
\frac{c_{t+1}}{c_t} \frac{\mu}{z_t^\rho} \Pi_{1-\rho} &= -B_t \frac{R}{1+\delta} \left(\frac{1}{1+\delta} \frac{\mu}{z_{t+2}^\rho} \Pi_{1-\rho} [\theta(1-\beta) + \beta] - \frac{\mu}{z_{t+1}^\rho} \Pi_{1-\rho} \right) \\
&\quad + \frac{1}{1+\delta} \frac{c_{t+1}}{c_t} \frac{\mu}{z_{t+1}^\rho} \Pi_{1-\rho} \{\theta(1-\beta) + \beta\} \tag{51}
\end{aligned}$$

In principle equation (51) can be used to derive the IES for any given consumption history. Such a derivation leads to rather messy formulas. It is therefore probably of more interest to find the IES for a steady state consumption path.

Let us assume therefore that consumption grows at a constant rate σ , i.e.

$$\ln c_{t+1} - \ln c_t = \ln \sigma \quad (52)$$

and similarly for the stock of habits. Equation (31) then implies that

$$\ln \sigma = (1 - \beta)(\ln c_t - \ln z_t) \quad (53)$$

so that

$$\frac{c_t}{z_t} = \sigma^{\frac{1}{1-\beta}} \quad \text{and} \quad \frac{c_t}{z_t^\theta} = c_t^{1-\theta} \sigma^{\frac{\theta}{1-\beta}} \quad (54)$$

The last expression has a straightforward interpretation. For example, if $\theta = 1$, utility only depends on the growth rate of consumption, but not on consumption itself. If θ approaches 0, utility only depends on the level of consumption. If θ is not equal to 0, the parameter β determines how strongly consumption growth affects utility. If β tends to 1 (i.e., when the habit formation process has a long memory and last period's consumption has a small effect on the stock of habits), utility at a given rate of growth is higher than in the case where β tends to 0. The reason for this is simply that at a given rate of growth the ratio between current consumption and the stock of habits is bigger with a long memory than with a short memory.

The assumption of a steady state consumption growth simplifies (51) considerably. The variable B_t reduces to 1. Furthermore, define the parameter $\chi \equiv \sigma^{\frac{1}{1-\beta}}$, so that

$$\frac{c_t}{z_t} = c_t^{1-\theta} \chi^\theta \quad (55)$$

Then we can write (51) as

$$\begin{aligned} \sigma [c_t^{1-\theta} \chi^\theta]^{1-\rho} &= -\frac{R}{1+\delta} \frac{1}{1+\delta} \sigma^{2(1-\theta)} c_t^{1-\theta} \chi^\theta \sigma^{\theta(1-\rho)} [\theta(1-\beta) + \beta] - \sigma^{1-\theta} c_t^{1-\theta} \chi^\theta \sigma^{\theta(1-\rho)} \\ &\quad + \frac{1}{1+\delta} \sigma \sigma^{1-\theta} c_t^{1-\theta} \chi^\theta \sigma^{\theta(1-\rho)} [\theta(1-\beta) + \beta] \end{aligned} \quad (56)$$

Or

$$\sigma = -\frac{R}{1+\delta} \frac{1}{1+\delta} \sigma^{2(1-\theta)(1-\rho)} [\theta(1-\beta) + \beta] - \sigma^{(1-\theta)(1-\rho)} + \frac{\sigma}{1+\delta} \sigma^{(1-\theta)(1-\rho)} [\theta(1-\beta) + \beta] \quad (57)$$

Define $\kappa \equiv \theta(1-\beta) + \beta$ and

$$\eta \equiv \frac{\sigma^{(1-\theta)(1-\rho)}}{1 + \delta} \quad (58)$$

Then (57) can be written as:

$$\sigma = -R \kappa \eta^2 - \eta + \sigma \kappa \eta \quad \implies \quad (59)$$

$$R\kappa\eta^2 - [R + \sigma\kappa]\eta + \sigma = 0 \quad (60)$$

(60) is a quadratic equation in η . Solving for η yields the following two solutions:

$$\eta_1 = \frac{1}{\kappa} \quad (61)$$

$$\eta_2 = \frac{\sigma}{R} \quad (62)$$

To check which solution corresponds to a utility maximum, we consider the special case $\theta = 0$. It turns out that for (62) we retrieve the usual Euler equation for the case without habit formation. (58) and (62) imply

$$\frac{\sigma^{(1-\theta)(1-\rho)}}{1 + \delta} = \frac{\sigma}{R} \quad \implies \quad (63)$$

$$\begin{aligned} \ln \sigma &= \frac{1}{\rho + \theta - \rho\theta} [\ln R - \ln(1 + \delta)] \\ &\approx \frac{1}{\rho + \theta - \rho\theta} (r - \delta) \end{aligned} \quad (64)$$

where $r = R - 1$, i.e. the interest rate. Thus the steady state IES with habit formation is equal to $\frac{1}{\rho + \theta - \rho\theta}$ ⁸.

4.4 Empirical results for the models with habit formation

We estimate AR and MA specifications, both with and without parameterization of the preference parameters. The estimation results for the AR and MA specifications without parameterization are reported in Tables 6 and 7, respectively. For all cases, the number of observations is 1557, as in the previous sections. In the specification of the stock of habits we face the problem that we do not know z_0 (or c_0 in the case of first order Markov habits). Somewhat arbitrarily we have set both z_0 and c_0 equal to .9 times household income.

The log-likelihood values for the two specifications is virtually identical (-9440.8). The estimates of ρ and δ are quite similar in both specifications. The parameter β in

⁸Carrol (2000) specifies (36) in linear form rather than in log-linear form. Although his Euler equations are different from ours, the implied IES in steady state turns out to be identical.

the MA specification is positive but not significant. The parameter θ capturing habit formation is estimated close to one. (Rational) habits appear to play an important role in the choice of consumption paths in the sense that consumption in a given period is almost completely evaluated relative to the stock of habits. The most striking difference between the results presented in Tables 6 and 7 and the results presented in Table 4 (the specification without habit formation) lies in the value of ρ . The estimates in Tables 6 and 7 imply much more curvature than the estimate from Table 4. The steady state IES implied by Table 6 is .89, whereas the steady state IES implied by Table 7 is equal to .88. This is slightly higher than the IES implied by the estimate of ρ in Table 4 (.75).

Table 6: AR-specification

<i>Parameters</i>	<i>Coeff.</i>	<i>s.e.</i>	<i>t-value</i>
delta	.121	.029	4.16
ln(rho)	1.19	.062	19.06
theta	.945	.014	69.36
Dummy for column 2	.940	.075	12.52
Dummy for column 3	2.167	.259	8.36
Dummy for column 4	.846	.093	9.13
Dummy for column 5	.432	.127	3.39
log-likelihood	-9440.82		

Table 7: MA-specification

<i>Parameters</i>	<i>Coeff.</i>	<i>s.e.</i>	<i>t-value</i>
delta	.117	.030	3.89
ln(rho)	1.21	.063	19.16
theta	.942	.016	60.23
beta	.475	.701	0.68
Dummy for column 2	.941	.075	12.53
Dummy for column 3	2.151	.269	7.98
Dummy for column 4	.846	.093	9.14
Dummy for column 5	.430	.127	3.39
Log likelihood	-9440.80		

In order to analyse whether and how the parameters are related to background characteristics of the individuals, we once again parameterize the models. Results are reported in Tables 8 and 9. To avoid an unwieldy number of parameters to be looked at, we have adopted a general-to-specific strategy and restricted parameters to zero that were (very) insignificant in a saturated specification.

Unlike the previous case, the MA-specification now exhibits a substantially larger log-likelihood value than the AR-case (-9318 versus -9375). As β tends to zero, the MA-specification model converges to the one with the AR-specification. Thus, the AR-specification is a special case of the MA-specification and the differences in log-likelihood indicate rejection of the AR-specification. Considering the demographic variables, age influences δ , θ and β , whereas it turned out to be fully insignificant for ρ .

The relation between age and δ is almost linear, as shown in Figure 4: the rate of time preference increases monotonically with age. The habit formation parameter θ increases until the age of 62 (Figure 5) and then declines. Gender has a significant effect, suggesting that for females habit formation is less important than for males.

Figure 6 presents a picture of the age function for β . The graph is non-monotonic in age: it first decreases until the age of 53 and then starts increasing. This implies that young and, to a lesser extent, old individuals have a longer memory for the past levels of consumption than middle-age respondents. Also gender has a significant influence on β , as females exhibit a shorter memory of past consumption levels than males. Both income and education are insignificant.

Table 8: AR-specification and parameterization

<i>Parameters</i>	<i>Coeff.</i>	<i>s.e.</i>	<i>t-value</i>
<i>delta</i>			
age	.005	.002	2.12
agesqu	-.00002	.00005	0.65
constant	.111	.042	2.63
<i>ln(rho)</i>			
log(household income)	.011	.005	2.10
middle education	-.021	.034	0.63
high education	.002	.045	0.04
constant	1.386	.069	20.18
<i>theta</i>			
age	.002	.001	4.54
agesqu	-.00006	.00001	4.71
gender	-.006	.003	1.97
constant	.986	.006	154.67
Dummy for column 2	.977	.075	13.03
Dummy for column 3	3.044	.439	6.93
Dummy for column 4	.964	.095	10.16
Dummy for column 5	.747	.129	5.78
Log likelihood	-9375.7		

Table 9: MA-specification and parameterization

<i>Parameters</i>	<i>Coeff.</i>	<i>s.e.</i>	<i>t-value</i>
<i>delta</i>			
age	.004	.001	3.01
agesqu	-.00004	.00005	0.45
constant	.106	.039	0.01
<i>ln(rho)</i>			
log(household income)	.009	.006	1.34
middle education	-.044	.041	1.09
high education	-.085	.056	1.53
constant	1.425	.095	15.01
<i>theta</i>			
age	.002	.0003	5.92
agesqu	-.00006	.00001	3.61
gender	-.008	.003	2.25
constant	.985	.010	102.95
<i>beta</i>			
age	-.012	.004	3.29
agesqu	.001	.0003	2.63
gender	-.093	.030	3.07
log(household income)	-.058	.033	1.74
middle education	-.143	.251	0.57
high education	-.675	.435	1.55
constant	.003	.350	0.01
Dummy for column 2	.991	.074	13.40
Dummy for column 3	2.841	.398	7.13
Dummy for column 4	1.009	.096	10.47
Dummy for column 5	.826	.137	6.04
Log likelihood	-9318.19		

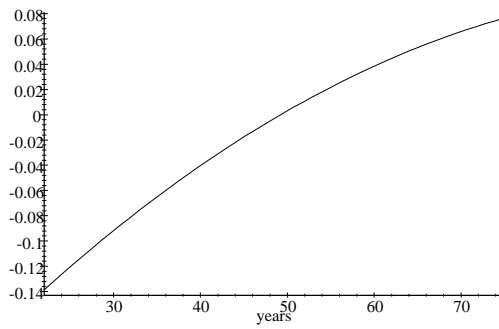


Figure 4: Age function for time preference rate δ

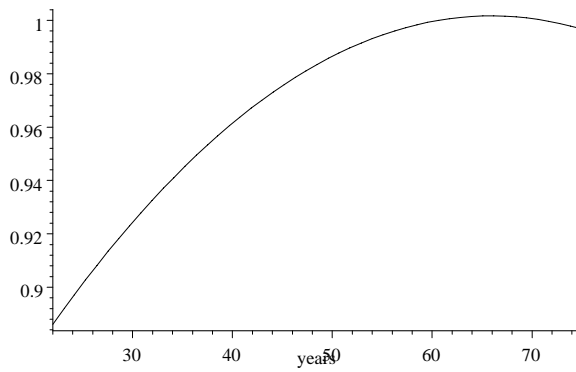


Figure 5: Age function for the parameter θ

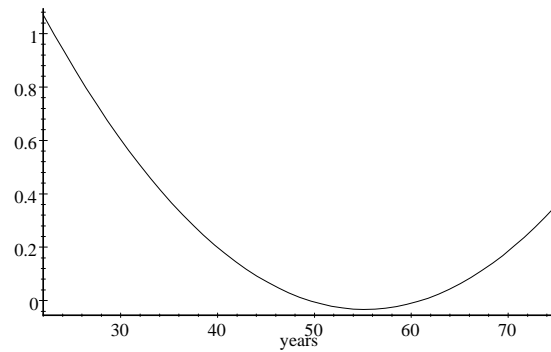


Figure 6: Age function for the parameter β

5 Concluding remarks

Our analysis is based on information from direct questions about hypothetical intertemporal consumption choices. In comparison with revealed preference approaches, the use of direct questioning to elicit dynamic preferences over consumption has the

advantage of simplicity and the avoidance of strong assumptions on the constraints faced by an individual. The results obtained in this paper appear to be plausible. The rate of time preference in the preferred specification (MA habits) is equal to .12, which seems more reasonable than values found in many other empirical studies. The estimate for the constant of relative risk aversion (3.4) is in line with what has been found elsewhere. The main finding is the rejection of intertemporal additivity. Habit formation appears to be quite strong and breaks the simple link between relative risk aversion and IES inherent in simpler models.

The set-up of the questionnaire can be further improved, however. The method effects ("routine selection") point at the need to randomize the order in which the consumption paths are presented to a respondent. In particular the fact that the constant consumption path is always in the middle is unfortunate. Also, the way the consumption paths have been presented by giving consumption levels at specified ages potentially leads to ambiguity. A better way to present consumption paths may be to show full graphs to respondents, or to assign all respondents the same horizon of ten years (say). We could then also systematically vary the horizon and investigate the effect of this on the elicited choices.

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