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1 INTRODUCTION

A ¯rm's tax loss carryforward is valuable because it shelters some portion of the ¯rm's future income from tax. The ¯nancial accounting system re°ects a tax loss carryforward as a deferred tax asset, perhaps o®set by a valuation allowance. This paper derives the ratio of the market value to book value of a ¯rm's tax loss carryforward.

We show that the market-to-book ratio of the tax loss carryforward re°ects three factors. First, because neither the deferred tax asset nor the valuation allowance is discounted to its present value, the book value tends to exceed the market value. Second, a valuation allowance is not established under generally accepted accounting principles (GAAP) as long as the probability that some of the loss carryover will expire is less than 50 percent. This also causes the book value to tend to exceed the market value for those ¯rms without a valuation allowance, but with some positive probability of having a tax loss carryforward expire. Third, the market value of the tax loss carryforward re°ects the mean level of future tax savings associated with the carryforward, whereas the net book value re°ects the median level of future tax savings. If the distribution of future tax savings is positively skewed, the market value can exceed the book value. Taken in combination, these factors imply that the market-to-book ratio of a ¯rm's tax loss carryforward is less than one if the ¯rm does not have a valuation allowance, whereas the market-to-book ratio of a ¯rm with a valuation allowance could be less than or greater than one.

We also show that the e®ect of the size of the loss and the expiration date of the loss on the market-to-book ratio of the tax loss carryforward depends on whether the ¯rm has a valuation allowance. This suggests that ¯rms with and without a valuation allowance should be analyzed separately rather than being aggregated into a single analysis.

Amir et al. (1997) and Ayers (1998) empirically investigate the relations between market and book values using linear regression models. Both report regression coe±cients on the valuation allowance variable in excess of one in their 1992 regressions, and Ayers reports a coe±cient in excess of one in his 1993 regression as well. These somewhat surprising results are consistent with the theoretical relations we establish in this paper.

Miller and Skinner (1998) and Schrand and Wong (2000) examine the extent to which managers use the deferred tax asset valuation allowance to manage earnings. We do not consider earnings management in this study. Instead, we examine a benchmark case in which both the deferred tax asset and valuation allowance comply with the literal requirements of GAAP. Our study is similar in spirit to Sansing (1998) and Guenther and Sansing (2000) in that we examine the relations between stock price and financial accounting variables in a benchmark case in which stock price equals the present value of the firm's expected future cash flows; therefore, earnings management plays no role in our study.

Section 2 presents the model in the case in which future income is certain. In the certainty case, the market-to-book ratio only reflects time value of money considerations. Section 3 examines the uncertainty case. We examine the difference between the book value of the loss carryforward and the expected future tax savings associated with the carryover by focusing on the special case in which the interest rate is zero. Section 4 extends our analysis to cases following a merger in which the use of the acquired corporation's loss carryforward is limited under Internal Revenue Code (IRC) §382. Section 5 concludes the paper.

2 THE DISCOUNTING EFFECT

In this section, we derive the market value of the firm's tax loss carryforward and the book value of the firm's deferred tax asset and valuation allowance assuming that future cash flows are known with certainty. We then derive the market-to-book ratio of the tax loss carryforward. Because there is no uncertainty regarding the eventual tax savings from the tax loss carryforward, the market–to-book ratios reflect only time value of money considerations.

Valuation

A firm owns assets on date zero that will generate a constant pretax cash flow of y per unit of time in perpetuity. The firm has a net operating loss carryforward (NOL) equal to L that will expire on date w if it is not used. The firm faces a tax rate τ on its taxable income. Taxable income is y per unit of time if there is no NOL, and zero otherwise; in the latter case, the NOL decreases at the rate of y per unit of time until it is either fully used, or until it expires on date w. All after-tax cash flows are distributed to the shareholders as dividends as they are generated. The stock price P is equal to the present value of all future after-tax cash flows, discounted at the interest rate r. In the absence of a loss carryforward ($L = 0$) the firm's stock price P is:

$$P = \int_0^\infty (1 - \tau) y e^{-rt} dt = \frac{y(1 - \tau)}{r}. \tag{1}$$

We distinguish between two different cases. In the first case, $L \leq wy$, which implies that the NOL is fully used before it expires. We refer to a firm that fully uses its NOL as a type-A firm. The stock price of a type-A firm consists of two parts. The first part is the present value of pretax cashflows earned between dates zero and L/y, at which point the NOL is fully used. The second part is the present value of future after-tax cash flows earned after date L/y. This yields:

$$\begin{aligned} P_A &= \int_0^{L/y} ye^{-rt}dt + \int_{L/y}^{\infty} y(1-\tau)e^{-rt}dt \\ &= \frac{y(1-\tau)}{r} + \frac{\tau y(1-e^{-rL/y})}{r}. \end{aligned} \tag{2}$$

The value of the NOL carryforward for a type-A firm, denoted VCF_A, is the difference between equations (1) and (??).

$$VCF_A = P_A - P = \frac{\tau y(1-e^{-rL/y})}{r} \tag{3}$$

In the second case, $L > wy$, which implies that some of the loss L expires on date w. We refer to a firm that loses part of its NOL as a type-B firm. The stock price of a type-B firm also consists of two parts. The first part is the present value of pretax cashflows earned between date zero and w, at which point the NOL expires. The second part is that present value of future after-tax cash flows earned after date w. This yields:

$$\begin{aligned} P_B &= \int_0^{w} ye^{-rt}dt + \int_{w}^{\infty} y(1-\tau)e^{-rt}dt \\ &= \frac{y(1-\tau)}{r} + \frac{\tau y(1-e^{-rw})}{r} \end{aligned} \tag{4}$$

The value of the NOL carryforward for a type-B firm, denoted VCF_B, is the difference between equations (1) and (??).

$$VCF_B = P_B - P = \frac{\tau y(1-e^{-rw})}{r}. \tag{5}$$

Deferred tax asset

We now consider how L is reflected in the firm's financial accounting statements. The firm pays zero tax when the loss is incurred and, assuming the loss cannot be carried back, records a deferred tax asset (DTA) equal to ¿L. If some of the NOL will expire, a valuation allowance is recorded under Statement of Financial Accounting Standards No. 109, Accounting for Income Taxes, (SFAS No. 109), to re°ect the portion of the future tax savings that will not be realized due to the expiration of the carryforward period. If all of the NOL will be used before date w, no allowance is recorded. The valuation allowance, denoted VA, is:

$$VA = \max f0; ¿(L ¡ wy)g: \tag{6}$$

Market-to-book ratios

Next, we derive the market-to-book ratio of the NOL. For a type-A ¯rm, $VA = 0$: We let $¯_A$ denote the ratio of the market value of the loss carryforward, VCF_A; to its book value.

$$¯_A = \frac{VCF_A}{DTA} \tag{7}$$

Substituting ¿L for DTA and using equation (3) yields:

$$¯_A = \frac{y(1 ¡ e^{¡rL=y})}{rL}: \tag{8}$$

Equation (8) shows that the coe±cient $¯_A$ is between zero and one, is increasing in y; and is decreasing in r and L: Under certainty, the future tax savings associated with L is equal to the book value of the deferred tax asset. Therefore, the term $¯_A$ diverges from one only because of time value of money considerations. The factors that cause $¯_A$ to diverge from one are the length of time it takes to realize the tax bene¯ts $\frac{L}{y}$ and the opportunity cost to the ¯rm of delaying the realization of the tax bene¯ts (r): As either the length of time it takes to realize the bene¯ts or the interest rate approaches zero, $¯_A$ approaches one.

Unlike a type-A ¯rm, a type-B ¯rm has both a deferred tax asset and a valuation allowance, so the book value of the deferred tax asset is $DTA ¡ VA$:

$$¯_B = \frac{VCF_B}{DTA ¡ VA} \tag{9}$$

Substituting ¿L for DTA, ¿(L ¡ wy) for VA; and using equation (5) yields:

$$¯_B = \frac{1 ¡ e^{¡rw}}{rw}: \tag{10}$$

As was the case of β_A; β_B is between zero and one because the coefficient only reflects time value of money considerations. In this case, the length of time it takes to use the tax loss L reflects the remaining carryforward period w instead of $\frac{L}{y}$.
We now compare the coefficients β_A and β_B, holding the pretax income y constant for each firm. The rankings of these coefficients are formalized in proposition 1.

Proposition 1 For firms of type A ($yw_A > L_A$) and B ($yw_B < L_B$) with identical pretax cash flows y:

$$\beta_A > \beta_B \quad \text{iff} \quad \frac{L_A}{y} < w_B.$$

The proof appears in the appendix.

Proposition 1 shows that the market-to-book ratio of a firm's NOL depends on the length of time the loss carryforward shelters the firm's income from tax. In the certainty case, the book value of a firm's tax loss carryforward equals the future tax savings associated with that loss. A firm that fully uses its loss carryover does so by date $\frac{L}{y}$, while a firm that loses part of its loss carryforward uses losses until date w. The longer it takes a firm to use the loss carryover, the lower the market-to-book ratio of that loss carryforward.

Example 2 Suppose $y = 2$; $L_A = L_B = 30$; $w_A = 20$; and $w_B = 10$. In this case, $DTA = 30\tau$, firm A has no valuation allowance, and $VA = 10\tau$ for firm B. $\beta_A < \beta_B$ because firm B uses its asset faster than does firm A; in 10 years for B as opposed to 15 years for A. Likewise, suppose $y = 2$; $w = 20$; $L_A = 30$; and $L_B = 50$. Then $\beta_A > \beta_B$ because firm A uses its loss carryforward for 15 years whereas firm B uses its loss carryforward for 20 years.

Therefore, if firms A and B have the same L but some of firm B's loss expires unused because it has a shorter carryforward period w over which the NOL can be used, the market-to-book ratio of firm B is higher than that of firm A. In contrast, if firms A and B have the same w, but some of the NOL of firm B expires because it has a greater loss carryforward, then the market-to-book ratio of firm A is higher. The consequence for an empirical study is that one should be very careful when aggregating firms with and without valuation allowance, since the effect of the expiration of some of the NOL on the market-to-book ratio is ambiguous.

3 THE UNCERTAINTY EFFECT

In this section, we derive the market-to-book ratio of a firm's NOL assuming that the rate of future income y is uncertain. The stochastic rate of income is denoted Y, and a possible outcome is again denoted y. We assume that $Y > 0$ is a random variable with a probability density function $f(\cdot)$ and a cumulative density function $F(\cdot)$. Note that Y is uncertain as of date zero, but is constant in the sense that once Y is realized on date zero, it does not vary over time subsequent to date zero.

Valuation

The stock price on date zero reflects the possibility that $y < \frac{L}{w}$, which implies that some of the NOL carryover will expire on date w, and the possibility that $y \geq \frac{L}{w}$, which implies that all of the NOL carryover will be used. Therefore, the stock price reflects an average of P_B, the price when $y < \frac{L}{w}$, and P_A, the price when $y \geq \frac{L}{w}$.

$$P = \int_0^{L/w} P_B f(y)dy + \int_{L/w}^{\infty} P_A f(y)dy. \tag{11}$$

Substituting in the values of P_A and P_B from equations (??) and (??) into equation (11) and subtracting the stock price when $L = 0$ yields the value of a firm's loss carryforward under uncertainty.

$$VCF = \frac{\tau}{r}\int_0^{L/w} y(1 - e^{-rw})f(y)dy + \frac{\tau}{r}\int_{L/w}^{\infty} y(1 - e^{-rL/y})f(y)dy. \tag{12}$$

Deferred tax asset

We now consider how L is reflected in the firm's financial accounting statements when future income is uncertain. The firm pays zero tax when the loss is incurred and, assuming the loss cannot be carried back, records a deferred tax asset (DTA) equal to τL, less any valuation allowance under SFAS No. 109. Because NOLs can only be carried forward a limited number of years (IRC §172(b)(1)), SFAS No. 109 requires that a valuation allowance must be established under certain circumstances. Paragraph 96 reads as follows:

> "The Board believes that the criterion required for measurement of a deferred tax asset should be one that produces accounting results that come closest to the expected outcome, that is, realization or nonrealization of the deferred tax

asset in future years. For that reason, the Board selected more likely than not as the criterion for measurement of a deferred tax asset. Based on that criterion, (a) recognition of a deferred tax asset that is expected to be realized is required, and (b) recognition of a deferred tax asset that is not expected to be realized is prohibited."

Paragraph 97 reads in part:

> "The Board intends more likely than not to mean a level of likelihood that is more than 50 percent."

Paragraph 98 reads in part:

> "The board acknowledges that future realization of a tax benefit sometimes will be expected for a portion but not all of a deferred tax asset, and that the dividing line between the two portions may be unclear. In those circumstances, application of judgment based on a careful assessment of all available evidence is required to determine the portion of a deferred tax asset for which it is more likely than not a tax benefit will not be realized."

We define the median y^* of the function $F(y)$ to be the solution to:

$$F(y^*) = 1/2. \tag{13}$$

Because a valuation allowance is required if there is a greater than 50 percent probability that some of the loss L will not yield a future tax benefit, a valuation allowance must be established if $L > wy^*$, and cannot be established otherwise. The valuation allowance is:

$$VA = \max\{0, \tau(L - wy^*)\}. \tag{14}$$

Market-to-book ratios

We now examine the market-to-book ratios under uncertainty. As in the preceding section, we consider two types of firms. A type-C firm is one that has not recognized a valuation allowance, so $VA = 0$. A type-D firm has recognized a valuation allowance, so $VA > 0$. First we consider a type-C firm. Because $VA = 0$ for a type-C firm:

$$\beta_C = \frac{VCF}{DTA}. \tag{15}$$

Substituting $DTA = \tau L$ and VCF from equation (12) yields:

$$\beta_C = \frac{\int_0^{L/w} y(1 - e^{-rw}) f(y)dy + \int_{L/w}^{\infty} y(1 - e^{-rL/y}) f(y)dy}{rL}. \tag{16}$$

The market-to-book ratio β_C is analogous to β_A; the difference is that Y is a random variable instead of a constant. However, whereas β_A only reflected time value of money considerations, β_C reflects both time value considerations and the possibility that part of the loss L expires unused. To quantify the effect of an expiring loss on β_C; we derive upper and lower bounds on β_C in absence of time value considerations, i.e., when the interest rate r equals zero.

Proposition 3 $\frac{1}{2} < \lim_{r \to 0} \beta_C < 1.$

The proof appears in the appendix.

The lower and upper bounds of β_C reflect the valuation allowance rules of SFAS 109. The probability of losing a portion of a firm's loss carryover can be as low as zero percent or as high as 50 percent without recognizing a valuation allowance. When L is sufficiently small, the probability that the tax benefit associated with L is fully used is close to one, and so β_C is close to one when L is close to zero. As L increases, β_C falls because the probability that some of the loss L will expire unused grows. This probability can be as high as 50 percent without recognizing a valuation allowance.

Next, we consider the market-to-book ratio for a firm for which $VA = \tau(L - wy^*)$; which we refer to as a type-D firm. Because $VA > 0$ for a type-D firm:

$$\beta_D = \frac{VCF}{DTA - VA}. \tag{17}$$

Substituting $DTA = \tau L$; $VA = \tau(L - wy^*)$ and VCF from (12) yields:

$$\beta_D = \frac{\int_0^{L/w} y(1 - e^{-rw}) f(y)dy + \int_{L/w}^{\infty} y(1 - e^{-rL/y}) f(y)dy}{rwy^*}. \tag{18}$$

As was the case with β_C; β_D reflects both time value of money considerations and the difference between the expected future tax savings and the book value of the tax loss carryforward. To quantify this difference, we again determine the upper and lower bounds of β_D when $r = 0$:

Proposition 4 $\frac{1}{2} < \lim_{r \to 0} \beta_D < \frac{E[Y]}{y^*}.$

The proof appears in the appendix.

As L grows sufficiently large, the probability that some of the loss will expire converges to one. As that happens, the market-to-book ratio β_D converges to $\frac{E[Y]}{y^*}$; which is the ratio of the expected level of future tax benefits ($\tau wE[Y]$) to the amount of future tax benefits that are reflected on the balance sheet ($\tau[DTA - VA] = \tau wy^*$): The VA may overstate the expected unused portion of the loss because VA reflects the median unused loss while the stock price reflects the mean unused loss. If the distribution $f(y)$ is positively skewed, β_D may exceed one. The fact that the coefficient can become greater than one if $f(y)$ has positive skew is illustrated in the following example.

Example 5 Let Y be lognormally distributed with a location parameter μ and dispersion parameter σ^2. Then $E[Y] = e^{\mu+\sigma^2/2}$; $y^* = e^{\mu}$; and $\frac{E[Y]}{y^*} = e^{\sigma^2/2}$. Because an increase in σ^2 increases $E[Y]$ but not y^*, $\frac{E[Y]}{y^*}$ could exceed one by a substantial margin.

Note that, since the ratio $\frac{E[Y]}{y^*}$ can become substantially larger then one, the market-to-book ratio can also exceed one for positive r.

Next, we examine the effects of the parameters L and w on the market-to-book ratios β_C and β_D: As before, we focus on the special case in which $r = 0$ in this section so as to distinguish between the effects of present value discounting from the differences between the book value of the deferred tax asset and the expected future tax savings associated with that asset.
Proposition 6 examines the effect of the loss L on the market-to-book ratios.

Proposition 6 When $r = 0$:
(i) $\lim\limits_{L \to 0} \beta_C = 1$
(ii) $\frac{\partial \beta_C}{\partial L} < 0$
(iii) When $L = wy^*$; $\beta_C = \beta_D$
(iv) $\frac{\partial \beta_D}{\partial L} > 0$
(v) $\lim\limits_{L \to \infty} \beta_D = \frac{E[Y]}{y^*}$

The proof appears in the appendix.

Proposition 6 shows that the relation between L and the market-to-book ratio β is not monotone. When L is close to zero, the probability that it will yield a tax benefit of τL is close to one, so the market-to-book ratio is close to one. As L grows, both the market and book values grow; however, the market value grows more slowly because the probability that some of the loss will expire unused grows with L; this causes the market-to-book ratio to decline when $0 < L < wy^*$: When $L > wy^*$, the market value continues to grow with L, while the book value remains at wy^*; this causes the market-to-book ratio to increase as L increases. As L becomes arbitrarily large, the market-to-book ratio converges to the ratio of the mean future tax savings to the median future tax savings.

Proposition 7 examines the effect of the expiration date w on the market-to-book ratios.

Proposition 7 When $r = 0$:

(i) $\lim_{w \to 0} \beta_D = \frac{E[Y]}{y^*}$

(ii) $\frac{\partial \beta_D}{\partial w} < 0$

(iii) When $w = \frac{L}{y^*}$, $\beta_C = \beta_D$

(iv) $\frac{\partial \beta_C}{\partial w} > 0$

(v) $\lim_{w \to \infty} \beta_C = 1$

The proof appears in the appendix.

As was the case in proposition 6, proposition 7 shows that the market-to-book ratios of type-C and type-D firms respond differently to changes in w: When w is close to zero, the probability that some of the loss carryover will expire unused is close to one, causing the market-to-book ratio to converge to the ratio of the mean future tax savings to the median future tax savings. As w increases, both the market and book values increase; however, the market value grows more slowly, which causes the market-to-book ratio to decline when $0 < w < \frac{L}{y^*}$: When $w > \frac{L}{y^*}$, the market value continues to grow, while the book value remains at τL; this causes the market-to-book ratio to increase as w increases. As w becomes arbitrarily large, the market-to-book ratio converges to one because the probability that the loss will yield a future tax benefit converges to one.

Propositions 6 and 7 suggest that if one wants to examine the cross-sectional variation in market-to-book ratios, firms with and without valuation allowances should be examined separately because β behaves differently as w and L change for firms with and without valuation allowances.

4 EFFECTS OF §382 LIMITATIONS

Section 382 of the Internal Revenue Code limits the use of the tax loss carryforward of a corporation that is acquired in a merger or stock purchase. The annual limitation is the product of the value of the acquired corporation and the long-term tax-exempt interest rate (IRC §382(b)(1)). In this section, we examine the effects of the §382 limitation on the market-to-book ratio of the loss carryforward. As in section 3, we pay particular attention to the special case in which $r = 0$.

We let the parameter π denote the maximum amount of loss carryforward that can be used per unit of time under §382. This implies that the amount of the loss that is used per unit of time equals:

$$Z = \min\{\pi, Y\}. \tag{19}$$

Valuation

First, we consider the market value of the NOL carryforward. There are two cases to consider. First, when $\pi w < L$, some part of the loss will expire unused at date w, because the maximum amount of loss that can be used equals $\min\{\pi, Y\}w \leqslant \pi w < L$. Equation (12) and the fact that the amount of the loss used per unit of time equals π when $y > \pi$ implies that:

$$VCF = \frac{\tau}{r}\int_0^{\pi} y(1 - e^{-rw}) f(y)dy + \frac{\tau}{r}\int_{\pi}^{\infty} \pi(1 - e^{-rw}) f(y)dy. \tag{20}$$

Second, when $\pi w \geqslant L$, the level of income y will determine whether some of the loss will expire unused. When $y < L/w$, part of the loss will expire unused at date w. When $L/w < y < \pi$, all the loss will be used by date L/y. Finally, when $y > \pi$, all the loss will be used, but due to the §382 limitation, this will only happen at date $L/\pi > L/y$ because the amount that can be used per unit of time equals $\pi < y$. Therefore, equations (3) and (5) lead to the following expression for the market value of the NOL carryforward:

$$\begin{aligned} VCF = {} & \tfrac{\tau}{r}\textstyle\int_0^{L/w} y(1 - e^{-rw}) f(y)dy \\ & + \tfrac{\tau}{r}\textstyle\int_{L/w}^{\pi} y(1 - e^{-rL/y}) f(y)dy + \tfrac{\tau}{r}\textstyle\int_{\pi}^{\infty} \pi(1 - e^{-rL/\pi}) f(y)dy. \end{aligned} \tag{21}$$

Valuation Allowance

Because the amount of the loss that can be used per unit of time is the stochastic variable $Z = \min\{\pi, Y\}$, SFAS No. 109 implies that the valuation allowance equals:

$$VA = \max\{0, \tau(L - wz^*)\}, \tag{22}$$

where z^* denotes the median of Z. This in turn implies:

$$z^* = \min\{\pi, y^*\}. \tag{23}$$

Therefore, it follows that

$$\begin{aligned} VA &= \max\{0; \tau(L - wy^*)\} \text{ if } y^* < \pi \\ &= \max\{0; \tau(L - w\pi)\} \text{ if } y^* \geq \pi; \end{aligned}$$

so that the valuation allowance is not affected by §382 as long as either $\pi \geq y^*$ or $w\pi \geq L$: If $\pi < y^*$ and $w\pi < L$; then the §382 limitation changes the book value of the deferred tax asset by increasing the valuation allowance.

Market-to-book ratio

The effect of a §382 limitation on the market-to-book ratio depends on whether the limitation changes the valuation allowance. If it does not, the limitation decreases the market value of the carryforward without decreasing its book value, which causes the market-to-book ratio to decrease.

Proposition 8 If either $\pi \geq y^*$ or $\pi \geq \frac{L}{w}$; the §382 limitation decreases the market-to-book ratio.

The proof appears in the appendix.

Next, we consider the case in which the §382 limitation affects both the market value and the book value of the loss carryforward, which occurs when $\pi < y^*$ and $\pi < \frac{L}{w}$: In that case, the net book value of the loss carryforward is $\tau w\pi$; and the market-to-book ratio (β_E) is:

$$\beta_E = \frac{\int_0^\pi y(1 - e^{-rw}) f(y) dy + \int_\pi^1 \pi(1 - e^{-rw}) f(y) dy}{rw\pi}. \tag{24}$$

When the §382 limitation reduces the net book value of the loss carryforward by increasing the valuation allowance VA; the limitation causes the market-to-book ratio to increase.

Proposition 9 Let $\pi < \frac{L}{w}$; $\pi < y^*$; and $r = 0$: Then the §382 limitation increases the market-to-book ratio.

The proof appears in the appendix.

Propositions 8 and 9 show that the §382 limitation could either increase or decrease a firm's market-to-book ratio. The limitation always decreases the market value of the loss carryforward, but only decreases the net book value of the loss carryforward when the limitation is su±ciently low. Therefore, for su±ciently large values of ¼ (¼ > minfL=w; y¤g), the limitation decreases the market-to-book ratio because it decreases the market value but has no e®ect on the book value. But if the limitation is low enough to a®ect the ¯rm's valuation allowance, then the x382 limitation increases the market-to-book ratio.

5 CONCLUSIONS

This paper examines the ratio of the market value to book value of a ¯rm's tax loss carryforward. We examine three settings: certainty, uncertainty without a x382 limitation, and uncertainty with a x382 limitation. In the last two settings we focus on the special case in which the interest rate is zero so as to distinguish between the e®ects of time value of money considerations and the e®ects of losing a tax bene¯t due to the statutory expiration of a tax loss carryforward.

The certainty case shows that the failure to discount the book value of a loss carryforward to its present value causes the market-to-book ratio to be less than one. Under certainty, the market-to-book ratio depends on the number of years that the loss carryover will shelter a ¯rm's income from tax. The market-to-book ratio of a ¯rm that will lose a tax bene¯t because the loss carryover expires unused could be greater than or less than the ratio of a ¯rm with a loss that will not expire, because expiration causes both the market value and the book value of the loss to decrease. The critical feature is the time period over which the loss is used, not whether some of the loss expires unused.

The uncertainty case shows that the ratio of the future expected tax bene¯t from the loss carryforward (that is, the market value when the interest rate is zero) to the book value of the loss carryforward could be less than or greater than one. When there is more than a 50 percent chance that the loss will yield a future tax bene¯t, the full amount of the loss carryforward is recorded as a deferred tax asset; in that case, the market-to-book ratio is less than one. But when there is more than a 50 percent chance that part of the loss will expire unused, the market-to-book ratio can exceed one. This occurs because the market value re°ects the mean future tax bene¯t, whereas the book value re°ects the median future tax bene¯t. Positive skewness in the distribution of future taxable income can cause the market-to-book ratio to exceed one. The uncertainty case also suggests that the e®ects of the size of the loss and the length of time until the loss expires have di®erent e®ects on the market-to-book ratio for ¯rms with and without a valuation allowance (which is recorded when the probability of a loss expiring exceeds

50 percent.) Our results suggest that when conducting an empirical analysis of ¯rms with tax loss carryforwards, one should segregate ¯rms with and without a valuation allowance, because the relation between the market value and book value of the loss carryforwards are di®erent for the two types of ¯rms.

The presence of a x382 limitation triggered by the acquisition of a corporation with a loss carryforward could either decrease or increase the market-to-book ratio, depending on whether the limitation a®ects both the market value and book value or just the market value. If the limitation does not a®ect the ¯rm's valuation allowance, then the x382 limitation decreases the ¯rm's market-to-book ratio; if the limitation causes the valuation allowance to increase (thus decreasing the net book value of the deferred tax asset), then the limitation increases the ¯rm's market-to-book ratio.

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APPENDIX

Proof of Proposition 1: i) The definitions of type-A and type-B firms imply that $\frac{L_A}{w_A} < y < \frac{L_B}{w_B}$. The ratios β_A and β_B are both of the form $\frac{1-e^{-z}}{z}$, where $z = \frac{rL_A}{y}$ for firm A and $z = rw_B$ for firm B. The expression $\frac{1-e^{-z}}{z}$ is decreasing in z, which implies that $\beta_A > \beta_B$ if and only if $\frac{L_A}{y} < w_B$. $\blacksquare$

Proof of Proposition 3: Applying L'Hopital's rule to equation (16) and evaluating it at $r = 0$ yields:

$$\beta_C = \frac{w}{L}\int_0^{L/w} y f(y)dy + \int_{L/w}^{1} f(y)dy. \tag{A.1}$$

Differentiating β_C with respect to L indicates that when $r = 0$, β_C is decreasing in L. For any type-C firm, $0 \le L \le wy^*$. Once again applying L'Hopital's rule shows that, when $r = 0$, β_C approaches one as L approaches zero, which yields the upper bound of $\beta_C = 1$. Substituting $L = wy^*$ into equation (A.1) yields $\beta_C = \int_0^{y^*}\frac{y}{y^*}f(y)dy + \int_{y^*}^{1} f(y)dy$. The first term could be arbitrarily close to zero; the second term equals $1 - F(y^*) = \frac{1}{2}$, and thus the lower bound of β_C is $\frac{1}{2}$. $\blacksquare$

Proof of Proposition 4: Applying L'Hopital's rule to equation (18) and evaluating it at $r = 0$ yields:

$$\beta_D = \frac{w\int_0^{L/w} y f(y)dy + L\int_{L/w}^{1} f(y)dy.}{wy^*} \tag{A.2}$$

Differentiating β_D with respect to L indicates that when $r = 0$, β_D is increasing in L. For any type-D firm, $L > wy^*$. Substituting $L = wy^*$ into equation (A.2) yields

$$\beta_D = \int_0^{y^*}\frac{y}{y^*}f(y)dy + \int_{y^*}^{1} f(y)dy.$$

The first term could be arbitrarily close to zero; the second term equals $1 - F(y^*) = \frac{1}{2}$, and thus the lower bound of β_D is $\frac{1}{2}$. Applying L'Hopital's rule shows that, when $r = 0$, β_D converges to $\int_0^{1}\frac{y}{y^*}f(y)dy$ as L approaches infinity, so the upper bound of β_D is $\frac{E[y]}{y^*}$. $\blacksquare$

Proof of Proposition 6: When $0 < L \le wy^*$, $VA = 0$ and thus the market-to-book ratio is β_C. When $L > wy^*$, $VA > 0$ and thus the market-to-book ratio is β_D.

(i) Applying L'Hopital's rule to equation (A.1) and evaluating it at $L = 0$ yields $\beta_C = \int_0^{1} f(y)dy = 1$.

(ii) $\frac{\partial \beta_C}{\partial L} = -\frac{w\int_0^{L/w} y f(y)dy}{L^2} < 0.$

(iii) Substituting $L = wy^*$ into equations (A.1) and (A.2) yields $\beta_C = \beta_D = \int_0^{y^*} \frac{y}{y^*} f(y)dy + \int_{y^*}^{\infty} f(y)dy.$

(iv) $\frac{\partial \beta_D}{\partial L} = \frac{\int_{L/w}^{\infty} f(y)dy}{wy^*} > 0.$

(v) Applying L'Hopital's rule to equation (A.2) and evaluating it as $L \to \infty$ yields $\beta_D = \frac{\int_0^{\infty} y f(y)dy}{y^*} = \frac{E[Y]}{y^*}.$ ¤

Proof of Proposition 7: When $0 < w < \frac{L}{y^*}$, $VA > 0$ and thus the market-to-book ratio is β_D. When $w \geq \frac{L}{y^*}$, $VA = 0$ and thus the market-to-book ratio is β_C.

(i) Applying L'Hopital's rule to equation (A.2) and evaluating it at $w = 0$ yields $\beta_D = \int_0^{\infty} \frac{y}{y^*} f(y)dy = \frac{E[Y]}{y^*}.$

(ii) $\frac{\partial \beta_D}{\partial w} = -\frac{L\int_{L/w}^{\infty} f(y)dy}{w^2 y^*} < 0.$

(iii) Substituting $w = \frac{L}{y^*}$ into equations (A.1) and (A.2) yields $\beta_C = \beta_D = \int_0^{y^*} \frac{y}{y^*} f(y)dy + \int_{y^*}^{\infty} f(y)dy.$

(iv) $\frac{\partial \beta_C}{\partial w} = \frac{\int_0^{L/w} y f(y)dy}{L} > 0.$

(v) Applying L'Hopital's rule to equation (A.1) and evaluating it as $w \to \infty$ yields $\beta_C = \int_0^{\infty} f(y)dy = 1.$ ¤

Proof of Proposition 8: There are two cases to consider. If $\pi < \frac{L}{w}$, then the effect of the §382 limitation on VCF is equal to the difference between equations (12) and (20). Differentiating equation (20) with respect to π yields:

$$\frac{\partial VCF}{\partial \pi} = \frac{\tau(1 - e^{-rw})\int_{\pi}^{\infty} f(y)dy}{r} > 0. \tag{A.3}$$

Therefore, we need only show that equation (12) exceeds equation (20) when $\pi = \frac{L}{w}$. Subtracting equation (20) from equation (12) and setting $\pi = \frac{L}{w}$ yields:

$$\int_{L/w}^{\infty} \left[\frac{y(1 - e^{-rL/y})}{rL} - \frac{(1 - e^{-rw})}{rw}\right] f(y)dy > 0. \tag{A.4}$$

If $\pi \geq \frac{L}{w}$; then the effect of the §382 limitation on VCF is equal to the difference between equations (12) and (21). Differentiating equation (21) with respect to π yields:

$$\frac{\partial VCF}{\partial \pi} = \frac{\tau\left(1 - e^{-rL/\pi} - \frac{rLe^{-rL/\pi}}{\pi}\right)\int_{\pi}^{\infty} f(y)dy}{r} > 0. \tag{A.5}$$

Furthermore, equation (21) converges to equation (12) as π approaches infinity, and thus equation (12) exceeds equation (21) for all finite values of π. $\square$

Proof of Proposition 9: There are two cases to consider, $y^* \geq \frac{L}{w}$ and $y^* < \frac{L}{w}$. If $y^* \geq \frac{L}{w}$, then without the §382 limitation, $VA = 0$ and the market-to-book ratio is β_C from equation (A.1). Applying L'hopital's rule to equation (24) shows that when $r = 0$:

$$\beta_E = \int_0^{\pi} \frac{y}{\pi} f(y)dy + \int_{\pi}^{\infty} f(y)dy. \tag{A.6}$$

Differentiating equation (A.6) with respect to π yields:

$$\frac{\partial \beta_E}{\partial \pi} = -\frac{\int_0^{\pi} y f(y)dy}{\pi^2} < 0. \tag{A.7}$$

As π converges to $\frac{L}{w}$, β_E converges to β_C. Therefore, $\beta_E > \beta_C$ whenever $0 < \pi < \frac{L}{w}$. If $y^* < \frac{L}{w}$, then without the §382 limitation, $VA = \tau(L - wy^*)$ and the market-to-book ratio is β_D from equation (A.2). As π converges to y^*, β_E converges to β_D. Because $\frac{\partial \beta_E}{\partial \pi} < 0$, $\beta_E > \beta_D$ whenever $0 < \pi < y^*$. $\square$