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**ON THE SEQUENCING OF PROJECTS,  
REPUTATION BUILDING, AND RELATIONSHIP  
FINANCE**

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# On the Sequencing of Projects, Reputation Building, and Relationship Finance

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## Abstract

We study the decision entrepreneurs face in financing multiple and independent projects. If strategic defaults are assessed likely to occur, for example if judicial efficiency is low, entrepreneurs delay projects to seek sequential financing from a relationship lender. Such commitment-type borrowing allows the entrepreneur to build a private reputation for repayment and consequently reduces the cost of financing. However, if the ex-ante risk of strategic default in the economy is low, the benefits of building a private reputation are outweighed by the holdup rents extractable by the incumbent financier. In this environment, entrepreneurs choose to finance all projects at once from single or multiple, arm's-length lenders.

JEL code: G21

Keywords: project sequencing, reputation building, relationship financing, contract enforcement, judicial efficiency

# 1 Introduction

Previous theory teaches us that close and repeated financing, such as that provided by banks, is most important to so-called “informationally-opaque” borrowers. According to theory, these borrowers - typically small, young firms with no public track record - value this relationship-based lending because they are unable to credibly communicate their repayment ability to a wider set of “arm’s-length” lenders. In reality, many informationally-transparent firms also rely on some form of relationship-based financing. For instance, Houston and James (2000) report that relatively large, publicly-traded firms obtain an average of 67% of their debt from banks and only 16% from public issues. Ongena and Smith (2000a) show that large, established publicly-traded firms actually maintain longer bank relationships than small, young publicly-traded firms. In this paper, we provide a rationale for why all types of firms may at times prefer relationship-based financing. In particular, we study the decision firms face in financing multiple independent investment projects.

In our model, an entrepreneur determines the sequencing for investment in two projects according to the availability and cost of funds. The entrepreneur can either try to finance both projects up front or sequence the financing and investment over two periods. The entrepreneur also decides whether to finance the projects using one or multiple lenders. A lender bases its financing decision on the perceived likelihood that an entrepreneur will strategically default on a loan, and its ability to extract holdup rents. We assume that some entrepreneurs are “good”, in that they never default on a loan, while others are “bad” in the sense that they will always default when it pays to do so. Lenders cannot observe an entrepreneur’s default type, but know the unconditional likelihood of facing a bad borrower. We show that firms operating in an environment where strategic defaults are likely choose to have their projects sequentially financed by the same lender. We term this behavior “relationship financing”. The intuition for this result is straightforward. The relationship lender observes individual loan repayments in the first period, which increases the ex-ante likelihood of repayment in both periods. The resulting decrease in the interest rate charged to the entrepreneur seeking to finance the sequenced projects more than offsets holdup rents accruing to the incumbent lender.

Our model illustrates that even in cases of severe holdup, relationship financing may still be preferable to arm’s-length financing if the assessed

likelihood of repayment is sufficiently low. However, relationship financing is not always optimal. If the ex-ante risk of strategic default in the economy is low, then the benefits of building a reputation are outweighed by the holdup rents extractable by the relationship lender. In this environment, firms choose to finance both projects upfront either from a single lender or from multiple lenders. We term this, as well as the opportunity to sequence projects using multiple lenders, “arm’s-length financing”.

The main contributions of our paper are two-fold. First, we demonstrate that relationship financing can arise endogenously, even when firms have equal access to arm’s-length financing and banks are able to extract holdup rents. In our model, all entrepreneurs start with the opportunity for financing their projects with arm’s length securities and then choose whether or not to invest in a relationship. Second, we link capital budgeting concepts like project timing to financing method. Our model provides a rationale for why entrepreneurs may optimally choose to delay financing a project even when there is no uncertainty about project payoffs or discount rates (Dixit and Pindyk, 1994; Berk, 1999), nor a need to monitor progress through stages of financing (Gompers, 1995). Firms sequence projects when reputational gains from paying off early projects reduce future lending costs. Because a reputation for repayment can only be gained by borrowing from the same lender, when firms choose to sequence projects, they do so through relationship financing.

Our theoretical framework also has several more specific applications. For instance, our model embodies characteristics of a revolving line of credit. Lines of credit are capped, forcing firms to repay their drawn credit before financing new projects. A pattern of drawdowns and repayments enables a firm to build a reputation for repayment with its bank. Given this interpretation, our model implies that firms should opt for lines of credit financing with a low credit limit over a large term loan when operating in an environment where strategic default is likely. On the other hand, large term loans should be preferred in settings where strategic default is unlikely. By assigning more meaning to our strategic default parameter, we can also gain insight into cross-sectional differences in financing behavior. For instance, if a country’s legal system can reduce the incentive for firms to strategically default, then our model also suggests that relationship financing will be more prevalent in countries with weaker contract enforcement and less efficient judicial systems. We expand these interpretations later in the paper.

The rest of the paper is organized as follows. We discuss associations

with the related literature in Section 2. Section 3 introduces the model. In Section 4, we explore the characteristics of arm's-length financing, while in Section 5 we focus on relationship financing. Section 6 describes the possible equilibria and Section 7 introduces extensions to the model. In Section 8, we explore different applications of the model, and Section 9 concludes.

## 2 Related Literature

### 2.1 Bank Relationships

Our paper is most closely related to the literature exploring the value of bank relationships. These papers are predicated on the idea that banks, as "inside" lenders, can observe and monitor borrowers in a way that allows them to finance firms that are otherwise unable to obtain valuable financing. Banks, it is argued, enjoy scale and scope economies in financing such informationally-opaque borrowers, and can therefore improve borrower welfare.<sup>1</sup> However, the ability for a bank to privately observe proprietary information and maintain a close relationship with customers may create a holdup problem, in the sense that the bank can use its information monopoly to extract above cost rents from its good borrowers.<sup>2</sup>

We differ from this extant literature along several dimensions. First, although information asymmetries exist between lenders and borrowers in our model, the asymmetry itself does not influence the choice between relationship and arm's length financing. Therefore, our model moves away from relating the value of bank financing to whether or not a firm is informationally opaque. Second, rather than assume that firms require repeated financing through time, we allow firms the choice between repeated lending and one-shot financing and derive conditions under which repeated financing with one bank is optimal. Third, by assuming that entrepreneurs have multiple projects to finance, we are able to relate the timing and sequencing of projects to financing choice. This provides a novel approach to thinking about some common capital budgeting issues.

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<sup>1</sup>For instance, see Campbell (1979), Diamond (1984, 1991), Fama (1985), Rajan (1992), Ramakrishnan and Thakor (1984), Sharpe (1990), and von Thadden (1995). For formal reviews of this literature, see Boot (2000) and Ongena and Smith (2000b).

<sup>2</sup>The holdup problem is explored by Fischer (1990), Greenbaum, Kanatas, and Venezia (1989), Rajan (1992), Sharpe (1990), and von Thadden (1998).

## 2.2 Reputation

Because we focus on the influence of borrower reputation on financing choice, our paper also shares similarities with Diamond (1991). In Diamond's model, borrowers always borrow repeatedly, taking into account the impact of current actions on their future reputation, which is publicly observable. In his model, only high-rated borrowers with a good reputation receive arm's-length financing. In contrast, in our stylized framework an entrepreneur chooses whether or not to borrow repeatedly depending on the assessed likelihood of repayment to build a privately-observed track record (the repayment history is private information and cannot be reported credibly by an incumbent bank). Hence our setup highlights the trade-off between building a privately-observable reputation and an ex post monopoly relationship between the entrepreneur and the bank (rather than a firm-specific track record known to the entire banking sector and perfect competition) to explain the firm's choice between single-shot arm's-length financing and relationship financing.

Like our paper, Boot and Thakor (1994) also model repeated borrowing. They show that even without learning or risk aversion, bank-borrower relationships are welfare enhancing and benefit the borrower. Borrowers in their model commit to a long-term contract that requires paying an above-market borrowing rate and committing collateral until a good project outcome is realized, then paying an infinite stream of below market rates with no collateral requirements after the realization. Hence in their model durable relationships permit long-term contracting and efficient intertemporal taxation and subsidization to reduce the use of costly collateral. In contrast, in our model durable relationships enhance efficiency by enabling financing of sequenced projects in cases where financing of all projects at once can not take place.

## 2.3 Relationship and Arm's-Length Lending

Our modeling is further related to the literature exploring the contrast between relationship and arm's length lending. Rajan (1992) for example argues that relationship lending is beneficial because a relationship bank's threat to withdraw funding induces firm managers to accept positive net present value projects. In Boot and Thakor (2000) banks determine the allocation of their lending capacity across relationship and arm's-length or "transaction" lending. Relationship lending is assumed to result in a higher probability of non-zero payoffs for the borrowers' projects and is further assumed to be

increasing in the degree of the banks' sector specialization. Boot and Thakor show that an increase in interbank competition increases relationship lending but decreases the banks' sector specialization, while increased capital market competition reduces relationship lending but increases the banks' sector specialization.

Das and Nanda (1999) also analyze bank specialization by analyzing the trade-off between offering commercial and investment banking services. In their model, commercial banking activities entail long term relationships with ex-post payoffs for the bank resulting in low levels of bank specialization. Investment banking activities go hand in hand with short term relationships and ex-ante payoffs causing 'too high' levels of specialization. In contrast to both Boot/Thakor and Das/Nanda, we focus on the entrepreneur's financing decision and show that financing relationships may arise endogenously, result in a higher probability of repayment, and hence assuage asymmetric information problems.

## 2.4 Project Timing

Because we link project timing to financing method, our setup is further related to work focusing on the option value of waiting to invest and optimal contracting under uncertainty. For example, entrepreneurs may optimally choose to delay financing a project when there is uncertainty about investment returns or discount rates (Dixit and Pindyk, 1994; Berk, 1999). Staged financing of a single entrepreneurial venture may further be optimal if there is a need to monitor its progress (Sahlman, 1990; Gompers, 1995).

While Admati and Pfleiderer (1994) derive the optimality of a fixed-fraction contract under uncertainty in a multistage setting, Bergemann and Hege (1998) show that a time-varying share contract is optimal in a more general, dynamic agency model. The optimality of staging itself and the optimal number of stages is derived in Neher (1999). In his model, staging investment in a single project mitigates the commitment problem of the entrepreneur not to renegotiate down the investor's claim once the investment is sunk.

Complementing these papers, our model considers multiple projects and provides an additional rationale for why entrepreneurs may optimally choose to delay financing a project. In our model entrepreneurs sequence projects when reputational gains from paying off early projects reduce future lending costs.

### 3 The Model

An entrepreneur has access to two independent projects A and B. Both projects require an initial investment  $k$  and yield certain payoffs  $\frac{1}{4}_A$  and  $\frac{1}{4}_B$ . Both payoffs exceed the initial investment  $k$ .<sup>3</sup> The entrepreneur can either realize both projects simultaneously (henceforth, 'joint projects') or delay one project and in effect pursue the projects sequentially (henceforth, 'sequential projects'). In the latter case the entrepreneur can start with either project A or project B: Without loss of generality, we assume that  $\frac{1}{4}_A > \frac{1}{4}_B$ : To ease notation, let  $\Phi = 2k = (\frac{1}{4}_A + \frac{1}{4}_B)$  and  $\Phi_j = k = \frac{1}{4}_j$ ,  $j = A; B$ , be the inverse profitability measures for the joint and sequential projects respectively. Notice that by definition  $\Phi_A < \Phi < \Phi_B$ :

We assume that the entrepreneur has no initial wealth, hence she has to seek outside financing from one or two lenders. Each project is nondivisible, so the entrepreneur must borrow the entire amount for a project from one lender. She also has no mechanism for storing excess cash, so that all financing must be done just prior to investment. In other words, if the entrepreneur chooses to sequence the projects, she must also sequence her financing. Moreover, if the projects are sequenced, the entrepreneur consumes all surplus from the project payoff at the end of period 1, such that she finances the entire period 2 project from outside sources.<sup>4</sup> As part of the financing decision, the entrepreneur must also choose whether to borrow from one lender or two. If the borrower chooses to finance sequential projects from one lender, we label it "relationship financing" since the lender learns from the entrepreneur's first period behavior. We label as "arm's-length financing" the funding of the two projects in one shot in the first period (through either one or two lenders) and sequential funding by two different lenders.

In the beginning of a relationship, the entrepreneur has full bargaining power. However, we assume that the lender acquires full bargaining power in the second period of relationship financing. The entrepreneur retains full bargaining power if she switches lenders in the second period. Switching lender does not entail any direct costs, but the new lender does not know the repayment history of the entrepreneur, and the incumbent lender is not able to report it credibly to the new lender.

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<sup>3</sup>In a somewhat related setup, Aerni and Egli (2000) start with different investment sizes to study progressive lending in microfinance programs.

<sup>4</sup>This assumption is not restrictive. We will discuss the issue of retained earnings in the concluding section.

Lenders are uncertain whether or not repayment,  $r_t$ ; with  $t = 1; 2$ , will occur. With an initial probability  $p_0 \in (0; 1]$ , the entrepreneur is good, and will always repay any amount up to the project payoff. With probability  $1 - p_0$ ; the entrepreneur is bad and has the option to strategically default. She will exercise the option if this is optimal for her to do so. We assume that the lenders cannot recuperate any positive payment when a bad entrepreneur decides to default. Moreover a bad entrepreneur cannot precommit to a positive level of repayment.

The entrepreneur knows her own type, whereas the lenders do not. Initially, the lenders only know the prior probability  $p_0$ : In case the projects are sequenced, the lender that finances the first project also knows whether the entrepreneur pays  $r_1$ . Denote  $\pi \in [0; 1]$  to be the (endogenously determined) probability that a bad entrepreneur pays  $r_1$ : Given  $\pi$ ; the lender can deduce the total probability  $q = p_0 + \pi(1 - p_0)$  of receiving the payment  $r_1$ : Given that  $r_1$  is paid, the lender updates its prior belief,  $p_0$ ; that the entrepreneur is good using Bayes' rule, i.e.  $p_1 = p_0/q$ :

Both the entrepreneur and the lenders maximize their expected income, with  $\beta$  as discount factor of the entrepreneur. In order to facilitate the formal exposition, we assume that  $\beta > \frac{p_0}{C_B}$ : This assumption requires either that project B be quite profitable or that the entrepreneur does not discount the future by very much.

This completes the description of the game setup. We proceed as follows. We investigate the cases of arm's length and relationship financing separately. We fully describe the equilibrium outcome for each of the two cases. Next we analyze the entrepreneur's choice to sequence projects and to switch lenders after period 1. The lenders anticipating the entrepreneur's choice may structure contracts accordingly.

## 4 Arm's Length Financing

If the entrepreneur seeks to finance joint projects simultaneously, she proposes a contract specifying the investment amount  $2k$  and the repayment level  $r$ : Obviously, a bad entrepreneur never repays  $r$  as she is always better off repudiating. The good entrepreneur pays  $\min\{r; \frac{1}{4}A + \frac{1}{4}B\}$  by assumption. The risk of repudiation influences negotiations at the beginning of the game. A lender anticipates a breach of contract with probability  $1 - p_0$ : Hence a lender is only willing to sign a contract under which its expected repayment under

the best of circumstances  $p_0 r$  at least covers investment  $2k$ ; i.e.  $r \geq 2k=p_0$ . On the other hand, it is known that any repayment  $r$  exceeding  $\frac{1}{4}A + \frac{1}{4}B$  is impossible since the entrepreneur has no initial wealth, i.e.  $r \leq \frac{1}{4}A + \frac{1}{4}B$ . The two constraints are compatible if and only if  $p_0 \geq 2k=(\frac{1}{4}A + \frac{1}{4}B) = \Phi$ : If this is the case, the good entrepreneur offers a repayment  $r = 2k=p_0$  which makes the lender indifferent between signing and rejecting and maximizes the entrepreneur's income  $\frac{1}{4}A + \frac{1}{4}B$  given  $r$ . A bad entrepreneur is forced to imitate the behavior of a good entrepreneur. As a bad entrepreneur always wants to default, a rational lender never signs a contract once it is clear the entrepreneur is bad. Aware of this intention, a bad entrepreneur will conceal her intentions in contract negotiations by mimicking the behavior of a good entrepreneur.

Denote  $\alpha \in [0; 1]$  be the probability with which the lender accepts the proposed repayment  $r$ .<sup>5</sup> Hence given the arguments above, in equilibrium we have:

$$\alpha = \begin{cases} 0 & \text{if } r < 2k=p_0; \\ \in [0; 1] & \text{if } r = 2k=p_0 \text{ and } p_0 = \Phi; \\ 1 & \text{if } r \geq 2k=p_0 \text{ and } p_0 > \Phi; \end{cases} \quad (1)$$

The lender rejects if  $r < 2k=p_0$ . On the other hand, the lender accepts if  $r \geq 2k=p_0$ . For  $r = 2k=p_0$  and  $p_0 > \Phi$ , the lender accepts with certainty. There is no equilibrium profile under which it rejects with positive probability, because the entrepreneur would then propose a repayment  $r$  slightly above  $2k=p_0$ , so that no best response for the lender exists. For  $r = 2k=p_0$  and  $p_0 = \Phi$ , any  $\alpha \in [0; 1]$  represents a best response for the lender, since the only acceptable repayment leading to a nonnegative income for the entrepreneur is  $r = 2k=p_0$ .

We assumed that the entrepreneur has full bargaining power in both periods under arm's-length financing and that the 'repayment history' of the entrepreneur is only known by the first lender. It is important to note that, in the second period, the outside lenders cannot learn from the fact that the entrepreneur is seeking financing from them. To put it differently, the outside lenders are not exposed to a Winner's Curse problem. The reason for this

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<sup>5</sup>For a sequential equilibrium to exist in the two-period case, it may be necessary that a second-period contract is randomly signed. In general, it is possible for the entrepreneur to randomize in equilibrium between proposing a contract promising zero expected income and proposing a contract leading to a certain rejection. Alternatively, when indifferent between accepting and rejecting, a bank may randomize in equilibrium. We can assume that in these cases the entrepreneur proposes a contract with certainty.

is that lenders are able to compute a good entrepreneur's optimal choice of financing scheme. A bad entrepreneur pursuing a different strategy than a good entrepreneur is immediately revealed. Therefore, a bad entrepreneur only chooses to switch after period 1 when it is also in the interest of a good entrepreneur to do so. Since the outside lenders do not know the repayment history of the entrepreneur, they expect to be faced with a good entrepreneur with probability  $p_0$ . Therefore, the two periods are structurally identical, and we can directly apply the analysis derived above. The results are summarized in Proposition 1.

**Proposition 1 Arm's-Length Financing:**

- (i) Joint projects: If  $p_0 \geq \Phi$ ; the entrepreneur, either good or bad, proposes a repayment  $r^* = 2k=p_0$  in exchange for an investment  $2k$  in equilibrium, and the lender accepts. The bad entrepreneur defaults on  $r$  with certainty. If  $p_0 < \Phi$ ; no contract is signed.
- (ii) Sequential projects: If  $p_0 \geq \Phi_j$ ;  $j = A; B$ ; the entrepreneur, either good or bad, proposes repayment  $r^* = k=p_0$  in exchange for investment  $k$ , and the lender accepts. A bad entrepreneur defaults with certainty. If  $p_0 < \Phi_j$ ; no contract is signed.

The entrepreneur, is able to finance the sequential projects if and only if  $p_0 \geq \Phi_B$ : For  $\Phi_A < p_0 < \Phi_B$ ; she can only finance the more profitable project A: In that case, due to discounting, she chooses to realize project A in the first period. Also due to discounting and the fact that the repayment is independent from the project choice, she realizes project A before project B: The profits of a good entrepreneur are:

- (i) Joint projects:

$$V_i^{ALF} = \begin{cases} \frac{1}{2}(\frac{1}{4}A + \frac{1}{4}B) - 2k=p_0 & \text{if } p_0 \geq \Phi \\ 0 & \text{if } p_0 < \Phi \end{cases}$$

- (ii) Sequential projects:

$$V_i^{ALF} = \begin{cases} \frac{1}{2}(\frac{1}{4}A) - k=p_0 + \frac{1}{2}(\frac{1}{4}B) - k=p_0 & \text{if } p_0 \geq \Phi_B \\ \frac{1}{4}A - k=p_0 & \text{if } \Phi_B > p_0 \geq \Phi_A \\ 0 & \text{if } p_0 < \Phi_A \end{cases}$$

Comparing projects, it becomes clear that the entrepreneur always chooses to realize both projects jointly if the lender is willing to finance such engagement. As  $\Phi < \Phi_B$ , the entrepreneur always undertakes joint projects if the lender is willing to fund. This result is summarized in Corollary 2.

**Corollary 2** Under arm's-length financing, joint projects are realized whenever  $p_0 \geq \Phi$ ; For  $\Phi > p_0 \geq \Phi_A$ ; only project A is realized in the first period, and there is no additional financing provided in the second period. For  $p_0 < \Phi_A$ ; no financing takes place at all.

## 5 Relationship Financing

The main difference between arm's-length and relationship financing is that the repayment behavior of the bad entrepreneur at the end of period 1 will play an important role. As the ensuing analysis will show, there are four types of possible equilibria. We define a 'reputational equilibrium' as a sequential equilibrium in which the bad entrepreneur pays  $r_1$  with probability  $\gamma \in (0; 1)$ . In contrast, we define an equilibrium in which the bad entrepreneur always defaults ( $\gamma = 0$ ) as a 'separating equilibrium', and an equilibrium in which the bad entrepreneur never defaults ( $\gamma = 1$ ) as a 'pooling equilibrium'. Equilibria in which no project is financed are labelled 'no investment equilibria'.

We solve the model by backwards induction. Applying Proposition 1 to the second period gives us a complete description of the equilibrium in the second period.

**Corollary 3** Suppose no project has been carried out in the first period. If  $p_0 \geq \Phi_A$ ; the lender proposes repayment  $r_2^B = \frac{1}{2}k_A$  in exchange for investment  $k$ ; and the entrepreneur accepts. The bad entrepreneur defaults on  $r_2^B$  with certainty. If  $p_0 < \Phi_A$ ; no second-period contract is signed.

**Corollary 4** Suppose project  $i$  has been carried out in the first period. If  $r_1$  has been repaid and  $p_1 \geq \Phi_j$ , the lender proposes with probability  $\alpha^B$  a contract with repayment  $r_2^B = \frac{1}{2}k_j$  and investment  $k$ ; where  $\alpha^B$  is:

$$\alpha^B = \begin{cases} \gamma \in [0; 1] & \text{if } r_2 = k=p_1 \text{ and } p_1 = \Phi_j; \\ = 1 & \text{if } r_2 \geq k=p_1 \text{ and } p_1 > \Phi_j; \end{cases}$$

The bad entrepreneur defaults on  $r_2^B$  with certainty. If repayment  $r_1$  has not been paid or  $p_1 < \Phi_j$ ; no second-period contract is signed.

We now turn to the end of period 1. Suppose project  $i$  has been financed and realized, and repayment  $r_1$  is due. Anticipating the outcome of the second period (Corollaries 3 and 4), a bad entrepreneur knows that she collects the payoff of project  $j$  with present value  $\frac{1}{2}V_j$  in case she pays  $r_1$  with probability  $\bar{p}$  such that  $p_1 \geq \Phi_j$ . Obviously, she is better off defaulting when the costs  $r_1$  of 'building up a reputation' exceed the potential gain  $\frac{1}{2}V_j$  of having the reputation, i.e. in equilibrium,  $\bar{p} = 0$ ,  $r_1 > \frac{1}{2}V_j$ :

In contrast, for  $r_1 \leq \frac{1}{2}V_j$ ; a bad entrepreneur will choose  $\bar{p}$  as high as possible in order to maximize the probability to collect the reputational rent  $\frac{1}{2}V_j - r_1$ : For  $p_0 \geq \Phi_j$ , she can choose  $\bar{p} = 1$  without risking to lose the second-period contract. For  $p_0 < \Phi_j$ , she can maximally choose:

$$\bar{p} = \bar{p} = \frac{p_0 - \Phi_j}{1 - p_0} < 1; \quad (2)$$

implying  $p_1 = \Phi_j$ , and according to Corollary 4,  $\bar{p} = \bar{p}$  successfully induces a second-period contract with probability  $\bar{p} \in [0; 1]$ . For  $\bar{p} = \bar{p}$  to be an equilibrium, the bad entrepreneur must be indifferent between  $\bar{p}$  and any other  $\bar{p}$  increasing her reputational rent based on initial beliefs  $\bar{p}$ .<sup>6</sup> Hence in equilibrium, the expected reputational rent  $\bar{p}(\frac{1}{2}V_j - r_1)$  must be equal to zero implying  $\bar{p} = r_1 / (\frac{1}{2}V_j)$ . The results are summarized in

**Lemma 5** Suppose project  $i$  has been financed. When repayment  $r_1$  is due and  $r_1 \leq \frac{1}{2}V_j$ ; a bad entrepreneur repays with probability  $\bar{p} = \min\{\bar{p}; 1\}$  where  $\bar{p}$  is given by (2). For  $\bar{p} = \bar{p} < 1$ , a second-period contract for project  $j$  is induced with probability  $\bar{p} = r_1 / (\frac{1}{2}V_j)$ . If  $r_1 > \frac{1}{2}V_j$ ; a bad entrepreneur defaults, i.e.  $\bar{p} = 0$ :

The probability  $q^1 = p_0 + (1 - p_0)\bar{p}$  of a repayment in the first period is:

$$q^1 = \begin{cases} 1 & \text{if } r_1 \leq \frac{1}{2}V_j \text{ and } p_0 \geq \Phi_j; \\ p_0 = \Phi_j & \text{if } r_1 \leq \frac{1}{2}V_j \text{ and } p_0 < \Phi_j; \\ p_0 & \text{if } r_1 > \frac{1}{2}V_j; \end{cases} \quad (3)$$

and the updated equilibrium beliefs of the lenders about the proportion of good entrepreneurs in case of repayment  $r_1$  has occurred are:

$$p_1^1 = \frac{p_0}{q^1} = \begin{cases} p_0 & \text{if } r_1 \leq \frac{1}{2}V_j \text{ and } p_0 \geq \Phi_j; \\ \Phi_j & \text{if } r_1 \leq \frac{1}{2}V_j \text{ and } p_0 < \Phi_j; \\ 1 & \text{if } r_1 > \frac{1}{2}V_j; \end{cases} \quad (4)$$

<sup>6</sup> Given small non-transferable private benefits of running projects she will choose  $\bar{p} = \bar{p}$ .

Note that  $p_1^a$  is never less than  $\Phi_j$ : Let us now turn to the contracting problem at the beginning of period 1. Anticipating  $q^a$ , the lender expects a repayment of  $q^a r_1$ : To cover its investment  $k$ ; it only accepts a contracted repayment equal to:

$$r_1 \geq k/q^a \quad (5)$$

On the other hand, it also knows that any repayment promise  $r_1$  exceeding  $\frac{1}{2} \Phi_j$  is impossible as entrepreneurs have no initial wealth, hence:

$$r_1 \leq \frac{1}{2} \Phi_j \quad (6)$$

We now derive conditions for a reputational equilibrium, i.e. an equilibrium in which  $q^a \in (0; 1)$ : According to Lemma 5, a positive repayment probability less than 1 implies  $q^a = \bar{q}$  and is only possible if the reputational rent is nonnegative, hence:

$$r_1 \geq \frac{1}{2} \Phi_j \quad (7)$$

and if the choice of  $q^a$  matters,  $p_0 < \Phi_j$ . Taking (3) into account, inequalities (5), (6) and (7) are compatible if and only if:

$$\min\{\frac{1}{2} \Phi_j; \frac{1}{2} \Phi_j\} \geq r_1 \geq \frac{k}{p_0} \Phi_j \quad (8)$$

implying:

$$p_0 \geq \max\left(\Phi_A \Phi_B; \frac{\Phi_j^2}{\frac{1}{2}}\right)$$

Combined with  $p_0 < \Phi_j$ , the former condition implies  $k < \frac{1}{2} \Phi_j$ :

Suppose all conditions stated so far are fulfilled. Then, if the good entrepreneur chooses a contract promising a repayment  $r_1$  satisfying (8), she will choose  $r_1$  as low as possible in order to maximize her income. Hence she proposes:

$$r_1^a = \frac{k}{p_0} \Phi_j$$

A bad entrepreneur is forced to mimic the good type as any other proposal would reveal her true type. In the Appendix, it is shown that proposing a contract promising  $r_1^a$  in exchange for investment  $k$  actually maximizes the good entrepreneur's income. Hence we arrive at a Lemma detailing the Reputational Equilibrium.

**Lemma 6 Reputational Equilibrium:** For  $\Phi_j > p_0$ ,  $\max\{\Phi_A, \Phi_B\} < \frac{1}{2}g$ , which implies  $k < \frac{1}{2}\Phi_j$ , there exists a unique reputational equilibrium in which the entrepreneur, whether good or bad, proposes a contract promising repayment  $r_1^a = k = p_0$  in exchange for investment  $k$  in the first period, and the lender accepts. At the end of period 1, the bad entrepreneur repays with probability  $\alpha = \frac{1}{2} \in (0, 1)$ :

We now derive conditions under which a pooling equilibrium exists ( $\alpha = 1$ ): The way we do this is analogous to the derivation of the reputational equilibrium. According to Lemma 5, a repayment probability  $\alpha = 1$  is only possible if the reputational rent is nonnegative, i.e.  $r_1 \leq \frac{1}{2}\Phi_j$ , and if the choice of  $\alpha$  does not matter, i.e.  $p_0 \leq \Phi_j$ . Recalling inequalities (5), (6), (7), and taking (3) into account, we arrive at  $\min\{\frac{1}{4}i; \frac{1}{2}\Phi_j\} \geq r_1 \geq k$ , which is only possible if  $k \leq \frac{1}{2}\Phi_j$ .

If  $k \leq \frac{1}{2}\Phi_j$  and  $p_0 \leq \Phi_j$  are satisfied, and if the good entrepreneur chooses a contract promising a repayment  $r_1$  within the interval  $[k; \min\{\frac{1}{4}i; \frac{1}{2}\Phi_j\}]$ ; she will choose  $r_1$  as low as possible in order to maximize her income. Hence she proposes  $r_1^a = k$ : Again, a bad entrepreneur is forced to mimic the good type to prevent detection. In the Appendix, we show that proposing a contract promising  $r_1^a$  actually maximizes the good entrepreneur's income.

**Lemma 7 Pooling Equilibrium:** Suppose  $k \leq \frac{1}{2}\Phi_j$  and  $p_0 \leq \Phi_j$ : Then there exists a unique pooling equilibrium in which the entrepreneur, whether good or bad, proposes a contract promising repayment  $r_1^a = k$  in exchange for investment  $k$  in the first period, and the lender accepts. At the end of period 1, the bad entrepreneur repays with certainty.

Next we derive conditions for a separating equilibrium to exist ( $\alpha = 0$ ): According to Lemma 5, a repayment probability  $\alpha = 0$  is only possible if the reputational rent is negative ( $r_1 > \frac{1}{2}\Phi_j$ ): Recalling inequalities (5), (6), and taking (3) into account,  $r_1$  must also satisfy  $\frac{1}{4}i \geq r_1 \geq k = p_0$ ; implying  $p_0 \leq \Phi_j$ : To analyze this configuration, we consider the following two cases:  $k > \frac{1}{2}\Phi_j$  and  $k \leq \frac{1}{2}\Phi_j$ .

Suppose  $p_0 \leq \Phi_j$  and  $k > \frac{1}{2}\Phi_j$ : According to Lemma 5 and inequality (5), a bad entrepreneur will never repay  $r_1$  since  $r_1 \geq k = p_0 > \frac{1}{2}\Phi_j$ : Hence  $r_1$  must be at least  $k = p_0$ : If the good entrepreneur is to choose a contract promising repayment  $r_1 \geq k = p_0$ ; she will choose  $r_1$  as low as possible in order to maximize her income. She proposes  $r_1^a = k = p_0$ ; and the bad entrepreneur is forced to mimic the good type.

Now suppose  $p_0 > \Phi_i$  and  $k < \frac{1}{2} \frac{1}{4} j$ : From Lemmata 6 and 7 we know that the good entrepreneur prefers to propose a repayment  $r_1^a = k \max\{\Phi_j = p_0; 1\}g$  as long as  $p_0 > \max\{\Phi_A \Phi_B; \Phi_j^2 = \frac{1}{2}g\}$ : Hence a separating equilibrium only exists if  $\max\{\Phi_A \Phi_B; \Phi_j^2 = \frac{1}{2}g\} > p_0 > \Phi_i$ : This is possible for  $\Phi_j^2 = \frac{1}{2}g > p_0 > \Phi_i$ : If that is the case, the good entrepreneur will propose  $r_1^a = k = p_0$  in order to maximize her income, and the bad entrepreneur mimics this behavior.

Taking the Appendix into account, where we show that proposing  $r_1^a = k = p_0$  actually maximizes the good entrepreneur's income in both cases, we end up with a Lemma describing the Separating Equilibrium.

**Lemma 8 Separating Equilibrium:** Suppose (i)  $k > \frac{1}{2} \frac{1}{4} j$  and  $p_0 > \Phi_j$ ; or (ii)  $k < \frac{1}{2} \frac{1}{4} j$  and  $\Phi_j^2 = \frac{1}{2}g > p_0 > \Phi_i$ : Then there exists a unique separating equilibrium in which the entrepreneur, whether good or bad, proposes a contract promising repayment  $r_1^a = k = p_0$  in exchange for investment  $k$  in the ...rst period, and the lender accepts. At the end of period 1, the bad entrepreneur defaults with certainty.

To complete the analysis, we need to state the conditions under which a equilibrium with no investment exists. This is done by summarizing the logical counter-arguments of Lemmata 6, 7, and 8.

**Lemma 9 No Investment Equilibrium:** Suppose either  $k > \frac{1}{2} \frac{1}{4} j$  and  $p_0 < \Phi_i$ ; or  $k < \frac{1}{2} \frac{1}{4} j$  and  $p_0 < \min\{\max\{\Phi_A \Phi_B; \Phi_j^2 = \frac{1}{2}g\}; \Phi_i g\}$ : Then no contract is signed in the ...rst period.

Figure 1 illustrates the results as well as the influence of the choice of the project sequencing. Because of the assumptions  $\frac{1}{4} A > \frac{1}{4} B$  and  $\frac{1}{2} > \frac{p_0}{\Phi_B}$ ; there are only three cases we have to consider.<sup>7</sup> Let us ...rst look at the project sequence fA; Bg: Figure 1 (i) depicts the case where the ...rst period project A is very profitable compared to project B: For low values of  $p_0$  ( $\Phi_A - p_0 < \Phi_B = \frac{1}{2}$ ); a Separating Equilibrium exists. Here, the second period is of no interest for the bad entrepreneur since the ...rst-period repayment is too high ( $r_1^a = k \Phi_B = p_0$ ). If  $\Phi_B > p_0 > \Phi_B = \frac{1}{2}$ ; this is no longer the case, and a Reputational Equilibrium exists. In case of  $p_0 > \Phi_B$ ; there is a Pooling Equilibrium.

Decreasing  $\frac{1}{4} A$  while holding  $\frac{1}{4} B$  ...x has the effect that  $\Phi_A$  gets larger than  $\Phi_B = \frac{1}{2}$ : Therefore, there is no more room for a Separating Equilibrium. This

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<sup>7</sup>Relaxing the assumption on the discount factor does not affect the results qualitatively.

is shown in Figures 1 (ii) and (iii). The difference between the two figures has no influence for sequence fA;Bg:

Looking at project sequence fB;Ag; one notices the absence of a Separating Equilibrium. Due to the assumption  $\frac{1}{4}_A > \frac{1}{4}_B$ ; the second period project A is attractive enough for the bad entrepreneur wishing to get financed. For the same reason, equilibria exist for lower ranges of  $p_0$  than for the project sequence fA;Bg: The next Lemma 10 summarizes these results.

**Lemma 10** For  $\max(\Phi_A, \Phi_B; \Phi_A^2 = \frac{1}{2})$   $p_0 < \min(\Phi_A; \Phi_B^2 = \frac{1}{2})$ ; there exists a Reputational Equilibrium for project sequence fB;Ag; and a No Investment Equilibrium for project sequence fA;Bg:

From Lemma 10 follows that the good entrepreneur will choose project sequence fB;Ag; whenever  $p_0$  is too low to allow for financing using project sequence fA;Bg: For higher values of  $p_0$ ; we have to compare the profits for the good entrepreneur in order to know which project sequence is chosen in equilibrium. For project sequence fi;jg; the profits are given by

$$\begin{aligned} \pi_i^{RF}(RE; fi;jg) &= \frac{1}{4}_i \frac{k\Phi_j}{p_0} \\ \pi_i^{RF}(PE; fi;jg) &= \frac{1}{4}_i \frac{k}{p_0} \\ \pi_i^{RF}(SE; fi;jg) &= \frac{1}{4}_i \frac{k}{p_0} \end{aligned} \quad (9)$$

Comparing these payoffs allows us to formulate Lemma 11.

**Lemma 11** For  $\frac{1}{4}_A > \frac{1}{2}\frac{1}{4}_B^2 = k$  and  $k = (\frac{1}{4}_A \wedge \frac{1}{4}_B + k) < \Phi_B^2 = \frac{1}{2}$ ; the good entrepreneur chooses project sequence fB;Ag if  $p_0 < k = (\frac{1}{4}_A \wedge \frac{1}{4}_B + k)$  and project sequence fA;Bg otherwise. For  $\frac{1}{4}_B < \frac{1}{4}_A$   $\frac{1}{2}\frac{1}{4}_B^2 = k$  or  $k = (\frac{1}{4}_A \wedge \frac{1}{4}_B + k) > \Phi_B^2 = \frac{1}{2}$ ; the good entrepreneur chooses project sequence fB;Ag if  $p_0 < \Phi_B^2 = \frac{1}{2}$  and project sequence fA;Bg otherwise.

We relegate the proof of Lemma 11 to the Appendix. Combining Lemmata 6 to 11 allows us to fully describe the equilibrium for relationship financing.

### Proposition 12 Relationship Financing

- <sup>2</sup>  $\frac{1}{4}A > \frac{1}{2}\frac{1}{4}B^2=k$  : There is a No Investment Equilibrium for  $p_0 < \Phi_A\Phi_B$ ; there is a Reputational Equilibrium with project sequence fB; Ag for  $\Phi_A\Phi_B$   $p_0 < \Phi_A$ ; there is a Pooling Equilibrium with project sequence fB; Ag for  $\Phi_A$   $p_0 < k=(\frac{1}{4}A \text{ ; } \frac{1}{4}B + k)$ ; there is a Separating Equilibrium with project sequence fA; Bg for  $k=(\frac{1}{4}A \text{ ; } \frac{1}{4}B + k)$   $p_0 < \Phi_B^2=\frac{1}{2}$ ; there is a Reputational Equilibrium with project sequence fA; Bg for  $\Phi_B^2=\frac{1}{2} < p_0 < \Phi_B$ ; and there is a Pooling Equilibrium with project sequence fA; Bg for  $p_0 \geq \Phi_B$ :
- <sup>2</sup>  $\frac{1}{2}\frac{1}{4}B^2=k \leq \frac{1}{4}A > \frac{1}{4}B=\frac{1}{2}$  : There is a No Investment Equilibrium for  $p_0 < \Phi_A\Phi_B$ ; there is a Reputational Equilibrium with project sequence fB; Ag for  $\Phi_A\Phi_B$   $p_0 < \Phi_B^2=\frac{1}{2}$ ; there is a Reputational Equilibrium with project sequence fA; Bg for  $\Phi_B^2=\frac{1}{2}$   $p_0 < \Phi_B$ ; and there is a Pooling Equilibrium with project sequence fA; Bg for  $p_0 \geq \Phi_B$ :
- <sup>2</sup>  $\frac{1}{4}B=\frac{1}{2} \leq \frac{1}{4}A > \frac{1}{4}B$  : There is a No Investment Equilibrium for  $p_0 < \Phi_A^2=\frac{1}{2}$ ; there is Reputational Equilibrium with project sequence fB; Ag for  $\Phi_A^2=\frac{1}{2}$   $p_0 < \Phi_B^2=\frac{1}{2}$ ; there is a Reputational Equilibrium with project sequence fA; Bg for  $\Phi_B^2=\frac{1}{2}$   $p_0 < \Phi_B$ ; and there is a Pooling Equilibrium with project sequence fA; Bg for  $p_0 \geq \Phi_B$ :

Hence Proposition 12 establishes a No Investment Equilibrium for values of  $p_0$  equal to or somewhat larger than 0, a Separating and/or Reputational Equilibrium for intermediate values of  $p_0$ , and a Pooling Equilibrium for values of  $p_0$  close to or equal to 1. Border values of  $p_0$  depend on project returns and ordering. We provide a graphical representation of this Proposition in Figure 1.

## 6 Choice of Financing Method

Arm's-length financing occurs when either (i) both projects are realized at once, or (ii) the entrepreneur sequences projects and switches to another lender at the end of the first period. Relationship financing is characterized by sequencing projects but no switching of lender. We assume that under arm's-length financing the entrepreneur retains full bargaining power in the second period, while under relationship financing the lender obtains full bargaining power in the second period. Clearly, the entrepreneur switches lender

whenever the second period's profit is positive, and an outside lender is willing to finance the second period project. Applying Proposition 1 (ii) shows that outside lenders are willing to finance project  $i$  whenever  $p_0 \geq \Phi_i$  and that in this case the entrepreneur's profit is nonnegative.

Suppose that project A has been realized in period one. The entrepreneur gets project B financed from an outside lender whenever  $p_0 \geq \Phi_B$ : We know from Corollary 2, that the entrepreneur does not sequence projects whenever  $p_0 \geq \Phi$ : Therefore, since  $\Phi_B > \Phi$ ; for  $p_0 \geq \Phi_B$ ; undertaking both projects jointly strictly dominates sequencing fA;Bg. Similarly suppose now that project B has been realized in period one. The entrepreneur switches whenever  $p_0 \geq \Phi_A$ : Then, financing project B in the first period only takes place if  $p_0 \geq \Phi_B$ : If  $p_0 \geq \Phi_B$ , however, it follows from Corollary 2 that the entrepreneur does not sequence the projects. Summarizing, we find that the entrepreneur never switches lender, since, whenever it would be profitable for her to do so, it is even more profitable not to sequence the projects in the first place.

This limits the analysis of the entrepreneur's choice between financing joint and sequential projects. In case the entrepreneur sequences projects, she also has to decide project sequence. From Proposition 1 (i) follows that joint projects can be financed if  $p_0 \geq \Phi$ : Therefore, for  $p_0 < \Phi$ , Proposition 12 applies. As already shown, relationship financing is not feasible for  $p_0 \geq \Phi_B$ ; since then the entrepreneur would switch the lender after period one. For  $\Phi > p_0 < \Phi_B$ ; we have to compare the entrepreneur's profits. From  $\Phi > k = (\frac{1}{4}A + \frac{1}{4}B + k)$  and Lemma 11 follows that the good entrepreneur chooses fA;Bg if she chooses relationship financing. From Lemma 7 we know that for relationship financing we only need to look at the profits for a Separating and a Reputational Equilibrium. Therefore, we have to compare the profits of the entrepreneur for a Separating and a Reputational Equilibrium with project sequence fA;Bg under relationship financing and the profit under arm's length financing with joint projects. It turns out that the entrepreneur always chooses relationship financing.

Lemma 13 For  $\Phi > p_0 < \Phi_B$ ; the entrepreneur sequences projects.

Proof.

1.  $\pi^{RF}(SE; fA; Bg) > \pi^{ALF}$ : Suppose not:  $\frac{1}{4}A + \frac{1}{4}B + k = p_0 \geq \Phi_B \geq 1$ ; a contradiction.

2.  $\pi^{RF}(RE; fA; Bg) > \pi^{ALF}$ : Suppose not:  $\frac{1}{4}A + \frac{1}{4}B + k = p_0 < \Phi_B < 2$ ; a contradiction.

From Lemma 6 follows that a reputational equilibrium only exists if  $p_0 < \Phi_B$ : Combining the two inequalities leads to  $\Phi_B > 1$ ; a contradiction. ■

Proposition 14 summarizes the main result.

**Proposition 14** For  $p_0 > \Phi_B$ ; arm's length financing emerges and the projects are jointly financed in period 1. For  $\max(\Phi_A \Phi_B; \Phi_A^2 = 1/2) > p_0 > \Phi_B$ ; relationship financing emerges. For  $p_0 < \max(\Phi_A \Phi_B; \Phi_A^2 = 1/2)$ ; no financing takes place.

This we find an interesting and straightforward result. Although the entrepreneur would be able to finance the projects jointly, for intermediate values of  $p_0$ ; she chooses to sequence and to accept that the lender will collect the entire second period surplus. The reason is that under joint project financing, the repayment  $r = 2k = p_0$  rises quicker with a decreasing  $p_0$  than for sequential project financing and a Reputational Equilibrium where  $r_1 + r_2 = k\Phi_B = p_0 + 1/4B$ ; or a Separating Equilibrium where  $r_1 + r_2 = k = p_0 + 1/4B$ . Since relationship financing only exists for relatively low values of  $p_0$ , this effect is strong and compensates more than adequately for the loss of the entire second period payoff.

## 7 Extensions

Our stylized model shows that if the lender assesses repayment to be unlikely, an entrepreneur will defer a project and borrow repeatedly from the same lender in order to build a reputation for repayment. Such relationship financing is chosen, even in the presence of holdup. If the likelihood of repayment becomes really low, the entrepreneur may even reverse project order, exacerbating holdup costs.

To illustrate the main ingredient of the model further, assume an entrepreneur has access to exactly one project in each period and hence cannot decide to delay a project. In that case the entrepreneur picks ordering fA; Bg. If  $p_0 > \Phi_B > \Phi_A$ , she switches lender and the bad entrepreneur can default both in the first and second period. If  $p_0 < \Phi_B$ , the entrepreneur does not swap lender and the bad entrepreneur repays with probability  $1 - \alpha < 1$ . Alignment fB; Ag may not be chosen. If  $p_0 > \Phi_A$ , new lenders always provide funds implying that switching is optimal. Consequently, lenders in the first period only fund project B if  $p_0 > \Phi_B$ . Now, introduce immediate

access to both projects including the option to delay. If  $p_0 \leq C_B$ , good entrepreneurs tackle both projects such that bad entrepreneurs never get the switching option. In general and even in the case of mixed project alignment (but retaining free project access), showing up in the second period as a new customer exposes the entrepreneur as being bad. Hence sequencing projects and relationship financing go hand-in-hand in our model.

This main result is robust to various model alterations and extensions. Consider for example the case of endogenous project split up. Let  $\frac{1}{4}A + \frac{1}{4}B = C$ ; a constant, and assume that the entrepreneur mixes project size by determining the proportion  $\theta$  of  $C$  to be allocated to project A and the proportion  $(1 - \theta)$  allocated to project B. We have to consider only the representative project sequence  $fB; Ag$ . For this project sequence;  $p_0 \leq \max\{C_A, C_B\}$ ;  $C_A^2 = \frac{1}{2}g$  is a necessary condition for a Reputational Equilibrium to exist. In case of endogenous project split the latter condition changes to  $p_0 \leq \max\{k^2 = (\theta(1 - \theta)C^2); k^2 = (\frac{1}{2}\theta^2 C^2)\}$ : Obviously,  $k^2 = (\theta(1 - \theta)C^2)$  is minimized for  $\theta = \frac{1}{2}$ ; and increases in  $\theta$  for  $\theta > \frac{1}{2}$ : However for  $\frac{1}{4}A$  close to  $\frac{1}{4}B$ ;  $p_0 \leq k^2 = (\frac{1}{2}\theta^2 C^2)$  is the relevant condition (see Figure 1 (iii)), and  $k^2 = (\frac{1}{2}\theta^2 C^2)$  decreases in  $\theta$ : Hence by increasing  $\theta$ , that is by placing more weight on the second-period project, the entrepreneur is able to broaden the range of  $p_0$  for which financing is feasible. The maximum she can attain is to set  $\theta = \theta^* = \frac{1}{1 + \frac{1}{2}}$ . Increasing  $\theta$  beyond  $\theta^*$  makes  $k^2 = (\theta(1 - \theta)C^2)$  the relevant condition, which, as mentioned before, increases in  $\theta$ . To conclude, endogenous project split up and an increase in total project payoffs ( $C$ ) widens the reach of both arm's-length and relationship financing versus the No Investment outcome.

However, arms'-length financing may become less prevalent, if the size of the investment in the joint projects exceeds the credit limit set for the entrepreneur by each lender and, in addition, if the entrepreneur incurs a mixed cost when approaching a second lender. Credit limits could be the result of small bank size and corresponding lack of diversification, and may be self-imposed or a result of regulation.

On the other hand, introducing a mixed cost to sequencing projects may make relationship financing less attractive. For example, sequencing product development and generating a payoff on a partly finished product to repay a loan may at best be suboptimal. Revolving finance of retail inventories may be easier to accomplish. A reduction in the discount rate (i.e., an increase in the discount factor  $\frac{1}{2}$ ) similarly makes project deferral less costly and increases the reach of relationship financing versus the No Investment

area. Increased bank fragility, i.e. the expectation that the relationship bank may not be around in later periods, may make arm's-length financing more attractive.

The main intuition of the model also remains broadly intact in generalizations to multiple projects and/or multiple periods. Enabling the entrepreneur and the financiers to write long-term contracts similarly does not alter the results. We so far implicitly assumed that the entrepreneur is not able to credibly commit to stay with the incumbent financier. The threat of switching in the second period disappears if she is able to commit. Hence it is possible that the entrepreneur prefers relationship financing for  $p_0 \geq \Phi_B$ : In order to check for this possibility, we compare profits for a Pooling Equilibrium for  $f_A; B_g$  with the profits for arm's-length financing of the joint projects. As  $\pi^{ALF} > \pi^{RF}(PE; f_A; B_g)$ ,<sup>8</sup> we can conclude that the writing of long-term contracts does not alter our results.

On the other hand, introducing second period financing by retained earnings impedes the existence of a Reputational Equilibrium because the value of built-up reputation decreases. However, if the initial proportion of good debtors is too low to establish a Reputational Equilibrium, the good debtor could offer a contract to the second-period lender at the beginning of the first period. The contract should specify the investment and a condition that the project will proceed only in case the first-period lender is repaid. The introduction of such contract re-establishes the Reputational Equilibrium because the bad entrepreneur is once again forced to imitate the good debtor by proposing a similar contract.

## 8 Applications

### 8.1 Loan Commitments

Because of the similarity between relationship lending in our model and a bank line of credit, our paper is closely related to the literature analyzing the optimality of loan commitment lending.<sup>9</sup> According to this literature, loan commitments are mechanisms designed to optimally balance reputational and

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<sup>8</sup>Suppose not, then  $\frac{1}{4}_A + \frac{1}{4}_B \leq 2k = p_0 \frac{1}{4}_A \leq k$  ( $\frac{1}{4}_B > k$ ): Combined with  $p_0 < 1$  leads to  $\frac{1}{4}_B < k$ ; a contradiction.

<sup>9</sup>See Boot, Thakor and Udell (1987, 1991), Houston and Venkataraman (1994), Morgan (1994), and Shockley and Thakor (1997).

...nancial capital, to forecast future loan demand, to lower regulatory taxes, or to exploit cost advantages in providing liquidity. Commitments can further mitigate investment distortions and suboptimal liquidation problems, enable borrowers to signal unobservable characteristics, and function as insurance contracts to risk-averse borrowers.

Complementing this literature, our model aims to demonstrate why it may be optimal to have repeated borrowing instead of one-shot ...nancing. We do so by formally showing that a ...rm may opt for project delay to allow the bank to learn from observing both drawdowns and repayments. The ensuing but voluntary exposure to the bank's scrutiny may render better contract terms for the entrepreneur, even in the presence of anticipated holdup.

## 8.2 Contract Enforcement

Recent empirical work documents a strong positive correspondence between judicial e¢ciency, development of ...nancial intermediation, and ultimately economic growth. For example, Levine (1999) and Levine, Loayza, and Beck (2000) show that cross-country di¢ferences in creditor rights, the quality of contract enforcement, and accounting standards help explain cross-country di¢ferences in ...nancial intermediary development.<sup>10</sup> The component of ...nancial development determined by the legal and regulatory environment in turn helps account for cross-country di¢ferences in economic growth. In particular, these studies document a strong positive association between proxies for the quality of contract enforcement in a country and the overall size of the ...nancial intermediary sector. Our model illustrates this positive association.

In our setup only bad entrepreneurs have the option not to repay. This proportion of bad entrepreneurs may in reality directly stem from the quality of the available contract enforcement mechanism. Stringent contract enforcement leaves few entrepreneurs with the strategic option to default. Lax enforcement, on the other hand, creates opportunities for many entrepreneurs never to repay.<sup>11</sup> For example, an entrepreneur may know the local judge

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<sup>10</sup>For example, La Porta, de Silanes, Shleifer, and Vishny (1997, 1998, 2000).

<sup>11</sup>Contract enforcement is not only framed by the legal environment but may also be the outcome of political processes determining ...nancial development (Rajan and Zingales, 2000). Such conjecture is implicit in Levine (1999) and Levine et al. (2000), where contract enforcement quality is alternatively gauged by (1) the risk that a government will - and therefore can - modify (i.e. repudiate, postpone, or reduce) a contract after it has been signed, (2) the law and order tradition in the country (measured by the International

or in general have enough legal skills and resources to elude, delay, and ultimately derail any weak attempts at judicial enforcement. Lenders may not know ex-ante whether or not an entrepreneur has access to such skills and resources.

### 8.3 Judicial Efficiency

A more general interpretation of the likelihood of repayment as a function of judicial efficiency may also be fruitful. Relationship banking may thrive in countries where the judicial system is weaker in apprehending and removing 'swindlers' from the credit market. Swindlers don't simply 'work' a legal system to avoid repayment. Swindlers are crooks and thieves, they never repay, they disappear, are unable, or simply refuse to repay even under legal duress and in jail.<sup>12</sup> Banks may assess pools of entrepreneurs in countries with weaker judicial systems to contain a high percentage of such risky, i.e. non-repaying, borrowers. Residing in such a country, entrepreneurs may do better delaying projects and seeking relationship-type financing.

Our stylized model not only links contract enforcement and judicial efficiency with decisions about project sequencing, but ultimately also with the development of the financial intermediary sector and the level of investment. Indeed, when the judicial system is efficient, entrepreneurs will immediately undertake all accessible projects and engage in arm's-length borrowing. An inefficient judicial system on the other hand impels entrepreneurs to delay projects to build a reputation for repayment. As a result, judicial inefficiency may hamper current investment and reduces contemporaneous demand for funding as entrepreneurs choose for relationship financing.<sup>13</sup> Hence we identify project delay to build a private reputation for repayment as an endoge-

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Country Risk Guide), or (3) an average of the latter two measures.

<sup>12</sup>In Bolton and Scharfstein (1990) all entrepreneurs are bad but the financier can promise a second-period loan in case the entrepreneur repays at the end of first period. The second-period losses are then outweighed by the first-period profits. In our model, we have both good and bad entrepreneurs and contracts in both periods have to be profitable for the financier.

<sup>13</sup>Our framework may complement recent static models by, for example, Fabbri (2000) and Iacovoni and Zazzaro (2000). Fabbri assumes that weak contract enforcement increases the cost of repossessing collateral in case of default, while Iacovoni and Zazzaro postulate that legal inefficiencies increase the banks' screening and monitoring costs. Both papers arrive, like ours, at demonstrating a positive link between the quality of contract enforcement and investment.

nously arising, 'insidious' cost of judicial inefficiency.

## 9 Conclusion

To conclude, the model suggests that repeated funding of sequential projects may arise as the dominant form of finance when many entrepreneurs are expected not to repay. This may be the case when the quality of contract enforcement or judicial efficiency is poor. In that case, many entrepreneurs can get away with and may decide not to repay and building a reputation for repayment constitutes a costly substitute for contract enforcement. To build a reputation, good entrepreneurs will delay projects to seek repeated finance from the same bank. Hence a low ex-ante likelihood of repayment may go hand-in-hand with project delay, reduced current investment, and possibly a smaller relationship-oriented financial sector. On the other hand, a larger arm's-length oriented financial sector will facilitate investment if the ex-ante likelihood of repayment is high.

While our stylized framework links judicial efficiency and the prevailing type of financing, it remains silent on the precise linkage between judicial efficiency and the number of financing relationships. For example, Detragiache, Garella, and Guiso (2000) and Ongena and Smith (2000c) document a negative correspondence between different proxies for judicial efficiency and the occurrence of multiple bank-firm relationships in samples containing Italian and large European firms respectively. Their results may suggest that in regions where judicial efficiency is poor, relationship financing forces project delay, in effect reducing per period funding and worsening holdup. Multiple bank arrangements may then arise to increase per period access to funding and to abate holdup. On the other hand, in regions where judicial efficiency is high, firms can immediately finance all currently accessible projects possibly using a single lender. Such arm's-length financing is further untainted by holdup, even when firms would borrow repeatedly from the same bank. However, we leave investigating these conjectures for future research.

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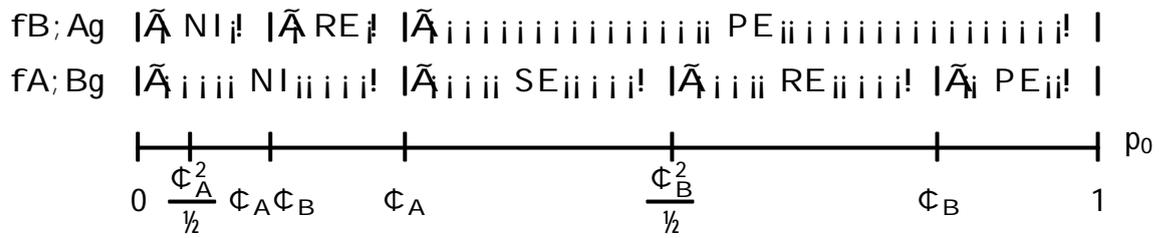
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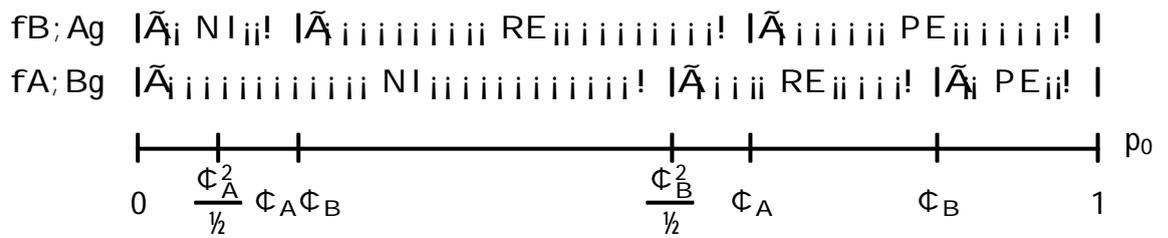
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Figure 1

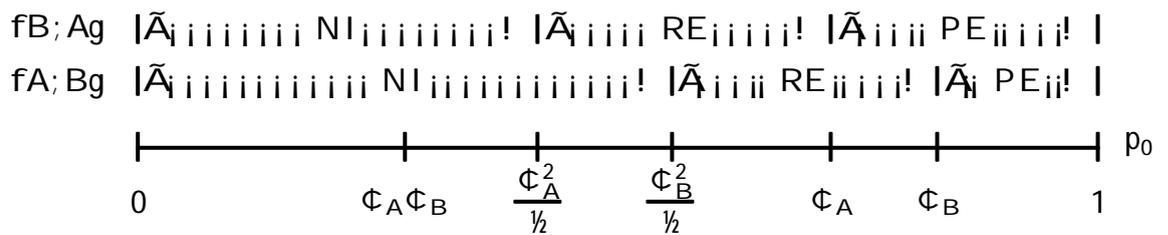
(i)  $\frac{1}{4}A > \frac{1}{2}\frac{1}{4}B^2 = k$



(ii)  $\frac{1}{2}\frac{1}{4}B^2 = k$ ,  $\frac{1}{4}A > \frac{1}{4}B = \frac{1}{2}$



(iii)  $\frac{1}{4}B = \frac{1}{2}$ ,  $\frac{1}{4}A > \frac{1}{4}B$



# Appendix

## Proof of Lemma 6

From Proposition 12 follows that for  $p_0 \leq \Phi_A$ ; project sequence  $fA; Bg$  results. It was shown in the text that given  $\Phi_B > p_0 \leq \max\{\Phi_A, \Phi_B\}$ ;  $\Phi_B^2 = \frac{1}{2}g$  (implying  $k < \frac{1}{2}\Phi_B$ ); repayment  $r_1^* = k\Phi_B = p_0$  satisfies the relevant rationality constraints (5), (6) and (7). To complete the proof, we show that promising repayment  $r_1^*$  maximizes the good entrepreneur's income. Recalling Corollary 4, the good entrepreneur's income  $P_g^*$  under  $r_1^*$  over both periods is given by

$$P_g^* = \frac{1}{4}A + k\Phi_B = p_0.$$

Note that given the assumptions made on  $p_0$ ; income  $P_g^*$  is nonnegative.

Step 1: Consider any repayment  $r_1^0$  a lender is not willing to sign. Then, according to Corollary 3, a contract over project A promising repayment  $r_2^* = k = p_0$  is signed in the second period if  $p_0 \leq \Phi_A$ ; For  $p_0 < \Phi_A$ ; no contract is signed. Hence by promising  $r_1^0$  in the first period, the good entrepreneur achieves income  $P_g^0 = \max\{0; \frac{1}{4}A + k = p_0\}$  in the second period. It is straightforward to show that  $P_g^0 \leq P_g^*$ .

Step 2: Any repayment promise  $r_1^0 < k\Phi_B = p_0$  violates the lender's rationality constraint (5) since in that case  $q^0 = p_0 = \Phi_B$ : Hence the lender rejects, and we are back at Step 1.

Step 3: Consider any repayment  $r_1^0$  such that  $k\Phi_B = p_0 < r_1^0 < \frac{1}{2}\Phi_B$ : If the lender accepts, it follows from equation (4) that  $p_1^* = \Phi_B$ ; and according to Corollary 4, a second-period contract with  $r_2 = \frac{1}{4}\Phi_B$  is induced. The good entrepreneur's income is then given by  $P_g^0 = \frac{1}{4}A + r_1^0$ ; which is less than  $P_g^*$ :

Step 4: Consider any repayment  $r_1^0 > \frac{1}{2}\Phi_B$ : If the lender accepts, Lemma 5 implies  $q^* = 0$ : According to (5), this is only rational for the lender if  $r_1^0 \leq k = p_0$ : If that is the case, the good entrepreneur's income is given by  $P_g^0 = \frac{1}{4}A + r_1^0 + \frac{1}{2}(\frac{1}{4}\Phi_B + k)$ : This is at least as big as profit  $P_g^*$  if  $r_1^0 \geq k\Phi_B = p_0 + \frac{1}{2}(\frac{1}{4}\Phi_B + k)$ : But this is only compatible with the lender's constraint  $r_1^0 \leq k = p_0$  if  $p_0 \leq \Phi_B$ ; leading to a contradiction with the assumptions made on  $p_0$ :

Summarizing Step 1 to 4, proposing to repay  $r_1^* = k\Phi_B = p_0$  maximizes the good entrepreneur's income. ■

### Proof of Lemma 7

The proof is analogous to the proof of Lemma 6. We showed in the text that given  $k \geq \frac{1}{2}\frac{1}{4}B$  and  $p_0 \leq \Phi_B$ ; repayment  $r_1^a = k$  satisfies the relevant rationality constraints. To complete the proof, we show that promising repayment  $r_1^a = k$  maximizes the good entrepreneurs's income. Recalling Corollary 4, the good entrepreneur's income  $P_g^a$  under repayment  $r_1^a$  over both periods is given by

$$P_g^a = \frac{1}{4}A \int k + \frac{1}{2}(\frac{1}{4}B \int k=p_0):$$

Note that given the assumptions made on  $p_0$ ; income  $P_g^a$  is nonnegative.

Step 1: Consider any repayment  $r_1^o$  the lenders are not willing to sign. Then, according to Corollary 3, a contract promising repayment  $r_2^a = k=p_0$  is signed in the second period if  $p_0 \leq \Phi_A$ : For  $p_0 < \Phi_A$ ; no contract is signed. Hence by proposing  $r_1^o$  in the first period, the good entrepreneur achieves income  $P_g^o = \max\{0; \frac{1}{4}A \int k=p_0\}$  which is less than  $P_g^a$ :

Step 2: Any repayment promise  $r_1^o < k$  violates the lenders' rationality constraints (5). Hence the lenders reject, and we are back at Step 1.

Step 3: Consider any repayment  $r_1^o$  such that  $k < r_1^o \leq \frac{1}{2}\frac{1}{4}B$ : the lender accepts  $r_1^o$ ; and from equation (4) it follows that  $p_1^o = p_0$ : According to Corollary 4, a second-period contract with  $r_2^a = k=p_0$  is induced. The good entrepreneur's income is then given by  $P_g^o = \frac{1}{4}A \int r_1^o + \frac{1}{2}(\frac{1}{4}B \int k=p_0)$ : Since  $r_1^o > r_1^a = k$ ; this is less than  $P_g^a$ :

Step 4: Consider any repayment  $r_1^o > \frac{1}{2}\frac{1}{4}B$ : If a lender accepts, Lemma 5 implies  $p_1^o = 0$ : According to (5), this is only rational for the lender if  $r_1^o \leq k=p_0$ : If that is the case, the good entrepreneur's income is given by  $P_g^o = \frac{1}{4}A \int r_1^o + \frac{1}{2}(\frac{1}{4}B \int k)$ : This is at least as high as profit  $P_g^a$  if  $r_1^o \leq k(1 + \frac{1}{2}) + \frac{1}{2}k=p_0$ : Since  $r_1^o > k=p_0$ ; this implies  $p_0 > 1$ ; leading to a contradiction.

Steps 1 to 4 show that choosing  $r_1^a = k$  maximizes the income of the good entrepreneur. ■

### Proof of Lemma 8

We showed in the text that given (i)  $k > \frac{1}{2}\frac{1}{4}B$  and  $p_0 \leq \Phi_A$ ; or (ii)  $k \leq \frac{1}{2}\frac{1}{4}B$  and  $\Phi_B^2 = \frac{1}{2} > p_0 \leq \Phi_A$ ; repayment  $r_1^a = k=p_0$  satisfies the relevant rationality constraints. Note that in both cases (i) and (ii),  $k=p_0$  exceeds  $\frac{1}{2}\frac{1}{4}B$  since (i)  $k=p_0 > k > \frac{1}{2}\frac{1}{4}B$ ; and (ii)  $k=p_0 > \frac{1}{2}\frac{1}{4}B = \Phi_B > \frac{1}{2}\frac{1}{4}B$ : This implies that

in both cases only separating equilibria exist. Recalling Corollary 4, the good entrepreneur's income  $P_g^a$  under repayment  $r_1^a$  over both periods is given by

$$P_g^a = \frac{1}{4}A \mid k=p_0 + \frac{1}{2}(\frac{1}{4}B \mid k):$$

It remains to show that proposing  $r_1^a = k=p_0$  dominates the strategy to sign no contract in the first period and, according to Corollary 3, a contract with repayment  $r_2^a = k=p_0$  in the second period. Following the latter strategy, the entrepreneur achieves an income with present value  $\frac{1}{2}(\frac{1}{4}A \mid k=p_0)$ ; which is less than income  $P_g^a$ . ■

### Proof of Lemma 11

- 2  $\mid^{RF}(PE; fA; Bg) > \mid^{RF}(PE; fB; Ag)$  since  $\frac{1}{4}A > \frac{1}{4}B$ :
- 2  $\mid^{RF}(RE; fA; Bg) \succ \mid^{RF}(RE; fB; Ag)$  for  $p_0 \succ \Phi_A \Phi_B$ ; which, for a RE, is always given.
- 2  $\mid^{RF}(RE; fA; Bg) > \mid^{RF}(PE; fB; Ag)$ : Suppose not. This leads to  $p_0 \leq k \Phi_B = (\frac{1}{4}A \mid \frac{1}{4}B + k)$ : For  $\frac{1}{4}A > \frac{1}{2}\frac{1}{4}B^2 = k$ ; a RE with sequence fA; Bg and a PE with sequence fB; Ag only exists if  $p_0 \leq \Phi_B^2 = \frac{1}{2}$ : Combining the inequalities leads us to  $\frac{1}{4}A \leq \frac{1}{2}k = \Phi_B + \frac{1}{4}B \mid k$ : Combining this with  $\frac{1}{4}A > \frac{1}{2}\frac{1}{4}B^2 = k$  leads to  $\frac{1}{2} < \Phi_B$ ; a contradiction. For  $\frac{1}{4}A \leq \frac{1}{2}\frac{1}{4}B^2 = k$ ; a RE with sequence fA; Bg and a PE with sequence fB; Ag only exists if  $p_0 \leq \Phi_A$ : Combining the two inequalities leads to  $\frac{1}{4}A \leq (\frac{1}{4}B \mid k) = (1 \mid \Phi_B)$ : Combining this with  $\frac{1}{4}A > \frac{1}{4}B$  leads to  $1 \mid \Phi_B < 1 \mid \Phi_B$ ; a contradiction.
- 2  $\mid^{RF}(SE; fA; Bg) \succ \mid^{RF}(PE; fB; Ag)$  for  $p_0 \leq k = (\frac{1}{4}A \mid \frac{1}{4}B + k)$ : In addition, a SE with sequence fA; Bg only exists if  $p_0 < \Phi_B^2 = \frac{1}{2}$  (see Lemma 8).
- 2 All other combinations are irrelevant; see Figure 1. ■