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Abstract

This paper explores the optimal role of the tax system in alleviating labour-market imperfections, raising revenue, and correcting the income distribution. For this purpose, the standard search model of the labour market is extended by introducing non-linear vacancy costs due to scarce entrepreneurial talent and by allowing for arbitrage between being a worker and being an entrepreneur. We study how these extensions a®ect the following three major implications of the standard model: (1) only the ad valorum component of the wage tax should be employed to raise revenue; (2) the optimal tax system should not distort labour-market tightness; (3) the tax system cannot redistribute from workers to entrepreneurs.

1 Introduction

The traditional incidence and welfare analysis of taxation assumes perfect markets. This holds true also for the analysis of taxes on labour income, even though the labour market tends to feature several imperfections giving rise to involuntary unemployment. In recent years, however, widespread unemployment in Europe has lead various researchers to reconsider the implications of taxes on labour income within the context of imperfect labour markets (see S¿rensen (1997) and van der Ploeg (1998)). Lockwood and Manning (1993), Bovenberg and van der Ploeg (1994), Holmlund and Kolm (1995), Koskela and Vilmunen (1996) and Kolm (1997) explore wage taxation in union bargaining models, Hoel (1990), Pisauro (1991), Fuest and Huber (1997), Stiglitz (1999), and Kleven and S¿rensen (1999) in e±ciency wage models, and Pissarides (1983, 1985 and 1990), Millard and Mortensen (1996) and Shi and Wen (1999) in search models. Pissarides (1998) and S¿rensen (1999) investigate wage taxation in all these three types of models.

This paper contributes to the literature on taxation in imperfect labour markets by exploring how the features of labour demand and labour supply impact the optimal role of the tax system in correcting labour-market imperfections, raising revenue, and correcting the income distribution. More speci¯cally, we can identify six separate contributions. First, whereas the literature has largely focused on tax reform, we explore the optimal design of the tax system. To

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illustrate, the literature on taxation in imperfect labour markets has demonstrated that a more progressive tax system tends to cut unemployment (see, e.g., Koskela and Vilmunen (1997) and Pissarides (1998)).[1] Whereas recognizing that a more progressive tax system may impose costs, this literature has rarely explored the optimal trade o® between the costs and the bene¯ts of a more progressive tax system.[2] As another illustration, the literature has demonstrated that ad valorum and speci¯c taxes yield di®erent allocative e®ects if competition is imperfect (see Delipalla and Keen (1992)). This is in contrast to competitive models in which ad valorum and speci¯c taxes exert the same impact on prices and the allocation. In the context of imperfect labour markets, we show not only that the government can a®ect the distribution of the tax burden over the demand and supply side of the market by varying the ad valorum component of the wage tax but also how the government can employ the tax system to optimally distribute the tax burden over both sides of the market.[3]

Second, in investigating optimal taxation in imperfect labour markets, this paper both simpli¯es and extends the workhorse of modern labour economics { the search model developed by Mortensen and Pissarides (see, e.g., Pissarides (1990) and Mortensen and Pissarides (1999)). The extensions of the model a®ect the labour-supply and labour-demand elasticities, which are important determinants of the optimal tax system. In particular, we allow for less than in¯nitely elastic labour demand on account of non-linear vacancy costs. Hence, in the extended model, not only labour supply (i.e. search by workers) but also labour demand (i.e. search by entrepreneurs) is less than in¯nitely elastic. As a direct consequence, not only workers but also entrepreneurs earn a positive ex-ante surplus. The positive surplus enjoyed by entrepreneurs can be viewed as a reward for supplying entrepreneurship. Incorporating less than in¯nitely elastic labour demand is important for studying the ultimate incidence of taxation over the demand and supply sides of the labour market and thus the optimal distribution of the tax burden over the two sides of the labour market. Indeed, we show that whether wage taxes (i.e. taxes on labour supply) are preferred over pro¯t taxes (i.e. taxes on labour demand) depends crucially on the features of search cost functions determining the elasticities of labour supply and labour demand. These functions determine also the optimal progressiveness of a linear wage tax.

As another extension, we allow agents to arbitrage between both sides of the labour market in the long-run version of the extended model. Whereas non-linear vacancy cost renders labour demand less elastic, the arbitrage possibility tends to make behaviour more sensitive to tax incentives. Indeed, the additional behavioural margin opens up another channel through which taxes a®ect e±ciency and distribution. We thus show how the optimal tax system depends not only on the nature of search costs at both sides of the market but also on the possibilities for arbitrage between these two sides.

We simplify the standard dynamic model by formulating a one-shot, static

[1] For empirical evidence con¯rming this result, see Lockwood and Manning (1993), Tyrviainen (1995) and Graa°and and Huizinga (1999).

[2] For an exception, see S¿rensen (1999) who numerically explores optimal wage taxation in imperfect labour markets.

[3] For the optimal design of commodity taxation in the presence of imperfect competition, see Myles (1989) and Reinhorn (1999).

version of the model. While it facilitates the interpretation of the results considerably, the simplified model still contains the main determinants of the optimal system. Most importantly, it retains the major market failure of the standard search model: search activities, which amount to specific investments in a labour-market relationship, are non-contractible and may thus be held up.

As a third contribution to the literature, we explore how the tax system, by acting as a commitment device, can avoid hold up of search activities.[4] By efficiently allocating property rights, the tax system in effect acts as a substitute for complete contracts in protecting the appropriate incentives for search activities. In this way, the tax system internalizes both positive and negative search externalities.

Fourth, the paper analyses how the role of the tax system in alleviating labour-market imperfections interacts with the revenue raising task of the tax system. In this context, the paper is related to the literature on the interaction between, on the one hand, distortionary taxation aimed at raising revenues and, on the other hand, pollution taxes targeted at internalizing environmental externalities (see Goulder (1995)). We find that the revenue raising and externality correcting tasks of the tax system are independent. One reason is that the externalities directly affect the production side of the economy. Another reason is constant returns to scale in matching. This implies that internalizing search externalities does not yield any net tax revenues because positive and negative externalities balance exactly; the optimal tax system internalizing these externalities merely redistributes resources from the side of the labour market that imposes negative externalities (i.e. the side that is holding up the other side) to the side of the market that yields positive externalities (i.e. the side that is being held up). In the literature on environmental externalities, in contrast, negative pollution externalities dominate so that the internalization of these externalities through the tax system typically raises net revenues.[5]

A fifth contribution of this paper is that it explores the role of the tax system in correcting the income distribution. By incorporating less than infinitely elastic labour demand, we allow the tax system to redistribute resources between the demand and supply sides of the labour market. In doing so, we decompose the optimal tax system in three terms corresponding to the threefold task of the tax system, namely to correct non-tax distortions (i.e. the missing markets for search activity), to finance government spending, and to correct the income distribution.

As a final contribution to the literature, we study the conditions under which the optimal tax system distorts labour-market tightness. The Diamond-Mirrlees (1971) production efficiency result suggests that labour-market tightness should not be distorted. We show that this latter result holds in the standard model with infinitely elastic labour demand due to linear vacancy costs and in a model with symmetric search cost functions at both sides of the market. If these conditions are violated, however, tightness will typically be distorted in order to redistribute resources either within the private sector (i.e. between workers

[4] For other institutions that alleviate hold up of specific investments in search, see Moen (1997).

[5] For an exception, see Bovenberg and van der Ploeg (1998) who model the interaction between pollution externalities and labour-market imperfections due to a rigid real after-tax wage. Whereas correcting the labour-market imperfection calls for a wage subsidy, the pollution externality demands a pollution tax. The overall effect on tax revenues is ambigious.

and entrepreneurs) or from the private to the public sector.

The rest of this paper is structured as follows. Section 2 formulates a one-shot, static version of the standard search model. Linear vacancy costs imply that labour demand is in¯nitely elastic so that entrepreneurs do not reap any (ex-ante) surplus. Workers, in contrast, enjoy positive ex-ante rents because labour supply is less than in¯nitely elastic. The assumption of in¯nitely elastic labour demand and less than in¯nitely elastic labour supply yields three major implications. First, only the ad valorum component of the wage tax is employed to raise government revenues. Second, in raising public revenues, the optimal tax system does not distort labour-market tightness. Finally, the tax system cannot redistribute from workers to entrepreneurs.

Section 3 extends this model by allowing for non-linear vacancy costs and by keeping the number of entrepreneurs ¯xed. As a direct consequence, labour demand becomes less than in¯nitely elastic and entrepreneurs reap a positive surplus. Section 3 ¯nds that the three major implications of a model with linear vacancy costs are not robust to the introduction of non-linear vacancy costs. In particular, depending on the labour-demand elasticity, the speci¯c component of the wage tax system takes over part or all of the revenue raising role of the ad valorum component. Moreover, labour-market tightness may be distorted for the purposes of either raising government revenues (i.e. redistributing resources from the private to the public sector) or redistributing resources between workers and entrepreneurs.

Section 4 maintains non-linear vacancy costs (and thus less than in¯nitely elastic labour demand) but allows agents to arbitrage between the state of being a worker and that of being an entrepreneur. The model in section 4 can be viewed as an intermediary case between the models of sections 2 and 3. On the one hand, non-linear vacancy costs render labour demand less elastic than in section 2. On the other hand, the arbitrage possibility between the two sides of the labour market tends to make behaviour more sensitive to tax incentives than in section 3. Indeed, compared to the model in section 3, this model implicitly considers a longer time horizon during which the number of agents at both sides of the market can respond to incentives. The implications of the model are also in between those of sections 2 and 3. Just as in the case with linear vacancy costs and free entry of entrepreneurs, long-term arbitrage between the two sides of the market eliminates the role of the tax system in redistributing resources from workers to entrepreneurs. With less than in¯nitely elastic labour demand, however, the optimal tax system may still want to distort labour-market tightness for the purpose of raising public revenues. Moreover, with non-linear vacancy costs, the government adopts not only the ad valorum component but also the speci¯c component of the wage tax to raise revenues. The concluding section 5 summarizes the main results of the paper. The Appendix derives the results presented in the main text.

2 Standard model with linear vacancy costs

2.1 model

This section describes the static search model. This model simpli¯es the dynamic model developed by Mortensen and Pissarides (see Pissarides (1990) and

Mortensen and Pissarides (1999)) but still captures the major market failure in the dynamic search model: the parties who bargain about the wage do not internalize the impact of the negotiated wage on search activities. Hence, the resulting equilibrium is efficient only if the model meets the so-called Hosios condition (see Hosios (1990)).

The sequencing of decisions in the static game is as follows. In the first stage of the one-shot game, tax policy is set. In the second stage, workers and firms (or entrepreneurs or employers), which are unmatched, search for a partner on the labour market. At the supply side of the labour market, workers $i \in [0; 1]$ select their search intensities $0 \leq s_i \leq 1$ at a cost $\gamma(s_i)$, with $\gamma(0) = 0$; $\gamma'(0) = 0$; $\gamma'(.) \geq 0$; $\gamma''(.) > 0$; and $\lim_{s_i \uparrow 1} \gamma'(s_i) \to +\infty$: At the demand side, entrepreneurs simultaneously decide how many vacancies v to create. For each vacancy posted, entrepreneurs bear a fixed cost c:

In the third stage of the game, workers and entrepreneurs are matched; the number of matches equals $m(s; v)$ where $s = \int_0^1 s_i di$: The matching function $m(.; .)$ is increasing in its two arguments. Moreover, it exhibits constant returns in both arguments together but decreasing returns in each of the arguments separately. Since a Cobb Douglas matching function fits the data rather well[6], we assume that the matching function is of the Cobb Douglas form.

After they have been matched, workers and entrepreneurs bargain about the (after-tax) wage rate w in the fourth stage of the game. Workers and entrepreneurs who do not find a match receive a payoff of zero. Finally, output is produced, taxes are collected and tax revenues g are spent on a public good.

The crucial element in the sequencing of decisions is that wages are negotiated after search efforts at both side sides of the labour market have been sunk. The quasi rents from the search activities are thus distributed on the basis of ex-post bargaining power rather than the marginal effectiveness of search in generating matches. Accordingly, if the marginal productivity of search activities exceeds the ex-post bargaining power, specific investments in the match are held up. This hold-up problem arises because the party with excessive bargaining power cannot credibly commit to reward his partner according to her contribution to concluding the match. Indeed, parties can bargain only after they have met. Since contracts can thus only be signed after the contracting parties have sunk their search activities, the market for search is missing. The missing market for specific investments in the match is the key non-tax distortion in the model.

The model is solved backwards. Accordingly, before determining search intensities s and labour-market tightness $\theta \equiv \frac{v}{s}$; we solve for (after-tax) wages.

2.1.1 production

The production technology in the final stage of the game is as follows: each matched firm-worker combination produces y units of output. Output is the numeraire. Output net of search costs, W; is given by

$$W = m(s; v)y - \gamma(s) - cv \tag{1}$$

where $m(s; v)y$ represents total output and $\gamma(s) + cv$ stands for total search costs.

[6]See, e.g., Blanchard and Diamond (1989) and Broersma and Van Ours (1999).

As usual in the labour market literature[7], the public good is financed by a linear tax on wages:[8]

$$g = m(s; v)(\tau w + \tau_a) \tag{2}$$

Here, τ represents the proportional (or ad valorum) tax on wages. The other component of the linear wage tax, τ_a; is a fixed (or specific) tax on the match. This tax depends only on the existence of a match and is not conditioned on how the quasi-rents from the match are shared between firms and workers. Wage taxation is progressive (i.e. the average tax burden rises with the wage) if the marginal tax on wages τ exceeds the average tax burden on wages $\frac{g}{m(s;v)w}$: This implies that the specific tax τ_a is negative.

2.1.2 wage setting

As is common in search models of the labour market, wages are determined by Nash bargaining between risk-neutral agents after a match has been found. The threat points for both parties are zero.[9] The bargaining is about the (after-tax) quasi rent (or surplus) from the match, $y - \tau w - \tau_a$: The after-tax wage w that maximizes the Nash Bargaining function $w^{\beta}(y - (1+\tau)w - \tau_a)^{1-\beta}$ is given by

$$w = \frac{\beta(y - \tau_a)}{1+\tau} \tag{3}$$

This is the value of the match for the worker. The value of a match for the entrepreneur, π; amounts to

$$\pi \equiv y - (1+\tau)w - \tau_a = (1-\beta)(y - \tau_a) \tag{4}$$

The burden of the fixed tax component τ_a is shared between the worker (i.e. the supply side of the labour market) and the firm (i.e. the demand side of the labour market) in proportion to their respective bargaining powers β and $(1-\beta)$; after-tax wages w decline and before-tax wages (i.e. wage costs) $w(1+\tau) + \tau_a$ rise with τ_a. The proportional tax rate τ, in contrast, reduces only the worker's value of a match (3); before-tax wages $w(1+\tau) + \tau_a$ and the firm's value of the match (4) are not affected by τ. The proportional tax rate thus bears on the supply side rather than the demand side of the labour market. Intuitively, by taxing the quasi rents that accrue to workers (i.e. the after-tax wage w); the proportional tax not only reduces the (after-tax) surplus from the match but also raises the effective bargaining strength of employers.[10] In the presence of a higher proportional tax, employers bargain more aggressively because a given increase in the after-tax wage w results in a larger increase in wage costs

[7] See for instance Pissarides (1990), Pissarides (1998) and Mortensen and Pissarides (1999).

[8] The government thus cannot resort to lump sum taxes to finance its spending. Uniform lump-sum taxes are excluded because the individuals who do not find a match do not have any income to finance these taxes. The government cannot impose differential lump-sum taxes either because it does not know which agents will find a match when it sets tax policy in the first stage of the game.

[9] If the bargaining partners cannot agree, the match is dissolved and the parties collect the same pay off (of zero) as the workers and entrepreneurs who are not able to find a match. In equilibrium, a match is never dissolved.

[10] The effective bargaining strength of employers is given by $\frac{(1-\beta)}{\beta/(1+\tau)+(1-\beta)}$:

$w(1+\tau)+\tau_a$. Put differently, workers bargain less hard because, at a higher proportional tax, a given rise in wage costs produces a smaller rise in after-tax wages.[11]

2.1.3 search intensity and vacancies

The wage agreed upon in ex post bargaining (i.e. after the match has been concluded) affects the incentives facing workers and firms to search for a partner in the preceding stage of the game. In selecting their search intensity, workers trade off additional search costs against the higher probability of finding a job. With a constant-returns-to-scale matching function, the probability that a worker with search intensity s_i is matched with a firm can be written as a function of labour-market tightness only: $\frac{s_i}{s}m(s;v) = s_i m(\theta)$ where $m(\theta) \equiv m(1;\theta)$. The risk-neutral worker selects search intensity s_i so as to maximize the expected surplus from search

$$\max_{s_i \geq 0}\{s_i m(\theta)w - \gamma(s_i)\}$$

With homogeneous individuals, all households feature the same search intensity

$$\gamma'(s) = m(\theta)w \tag{5}$$

where the left-hand side represents the marginal costs from higher search intensity and the right-hand side the corresponding expected marginal benefit in terms of raising the probability of finding a job. The net expected surplus for the worker, $sm(\theta)w - \gamma(s) = s\gamma'(s) - \gamma(s)$, is positive (if search intensity is non-zero) because of the properties of the strictly convex cost function (i.e. $\gamma(0) = 0$, $\gamma'(0) = 0$, and $\gamma''(.) > 0$).

The expression for optimal search intensity (5) can be interpreted as the implicit labour supply equation. With the aid of (3), labour supply can alternatively be written as

$$\gamma'(s) = m(\theta)\frac{\beta(y-\tau_a)}{1+\tau} \tag{6}$$

Demand for labour is determined by firms. The probability that a firm is matched with a worker equals $\frac{m(s;v)}{v} = \frac{m(\theta)}{\theta}$. With free entry of firms, expected profits from posting an additional vacancy are zero

$$c = \frac{m(\theta)}{\theta}\pi = \frac{m(\theta)}{\theta}(1-\beta)(y-\tau_a) \tag{7}$$

Here the left-hand side represents the costs of posting a vacancy while the right-hand side stands for the firm's expected benefits of doing so. By reducing the probability of filling a vacancy $\frac{m(\theta)}{\theta}$, a tighter labour market decreases the expected benefits from posting a vacancy. Since labour-market tightness θ is the only endogenous variable in (7), the free-entry condition determines tightness is a function of τ_a.

[11] There is thus no so-called 'real wage resistance' to proportional wage taxation. In competitive models of the labour market, all components of the wage tax (i.e. including the ad valorum component τ and the specific component τ_a) exert the same effect on the equilibrium wage. This is no longer the case in non-competitive labour-market models. In these models, a more progressive tax structure tends to moderate wages and reduce equilibrium unemployment (see Pissarides (1998) and Sørensen (1999)).

2.2 optimal taxes without government spending

After it characterizes the social optimum, this section shows how the parameters τ and τ_a of a linear wage tax should be chosen such that the private outcome coincides with the social optimum in the absence of a positive government financing requirement. The final part of this section demonstrates that a linear wage tax is equivalent to a combination of proportional wage and profit taxes. Although not often employed in the labour market literature, the latter parameterization of the tax system is more convenient to work with when deriving optimal taxes in the presence of positive government spending.

2.2.1 the Hosios condition

We measure welfare by net output (1). The social planner chooses search intensity s and labour-market tightness θ to maximize welfare

$$\max_{s,\theta} sm(\theta)y - \gamma(s) - c\theta s$$

It is routine to verify that the following result follows from the first order conditions for s and θ.

Lemma 1 The socially optimal values of θ and s are determined by

$$c = \frac{m(\theta)}{\theta}\eta y \tag{8}$$

$$\gamma'(s) = (1 - \eta)m(\theta)y \tag{9}$$

where $\eta \equiv \frac{m'(\theta)\theta}{m(\theta)}$.

The elasticity η measures the effectiveness of vacancies in generating matches. Our assumption that the matching function is of the Cobb Douglas form implies that η is constant.

Comparing the equations for the socially optimal labour demand (8) and labour supply (9) with, respectively, the free entry condition (7) and privately optimal search intensity (6) in the laissez fair equilibrium (i.e. $\tau = \tau_a = 0$), we find that the private outcome coincides with the social optimum if and only if

$$1 - \beta = \eta \tag{10}$$

This so-called Hosios condition (see Hosios (1990)) states that the bargaining power $1 - \beta$ of employers should be higher, the larger is the effectiveness of employers in generating matches as measured by η.

One can interpret the balance of (ex-post) power between the two sides of the labour market that is implied by the Hosios condition as the efficient distribution of property rights over the fruits from search.[12] In particular, search activities

[12] Alternatively, one can consider search externalities. By posting more vacancies, a firm reduces the probability that other firms find a match but raises the probability that workers find a job. The lower η, the bigger the negative external effects on the same side of the market and the smaller the positive external effects on the other side of the market. The Hosios condition states that a larger share of output should accrue to the side of the market that causes the smallest negative and the largest positive search externalities.

are specific investments in a future relationship (i.e. a match) between a worker and an employer. These specific investments cannot be contracted on because the worker and the firm meet only after they have searched, i.e. after they have sunk their investments in the relationship. The Hosios condition implies that the party that carries out the most important non-contractible investments should be most powerful ex post so that it can claim most of the quasi rents. In this way, property rights act as a substitute for complete contracts in protecting the incentives for specific investments (see Hart (1995)).

The following result shows how the Hosios condition affects s and θ in the private equilibrium.

Lemma 2 Consider the private outcome determined by (6) and (7) with $\tau = \tau_a = 0$. Then

$$\frac{d\theta}{d\beta} < 0$$

$$(1 - \eta) = \arg\max \{sm(\theta)y - \gamma(s) - c\theta s\}$$

$$(1 - \eta) = \arg\max \{sm(\theta)w - \gamma(s)\}$$

$$(1 - \eta) = \arg\max \{s\}$$

The first result says that labour market tightness is decreasing in workers' bargaining power β. Intuitively, a higher ex-post bargaining power of workers (i.e. labour supply) harms the incentives facing employers to demand labour (i.e. post vacancies). The second result follows from lemma 1 and equation (10): the private outcome coincides with the social optimum if $\beta = 1 - \eta$. With the free entry condition ensuring zero net profits in equilibrium, welfare W in (1) coincides with the ex ante welfare of workers $sm(\theta)w - \gamma(s)$. This implies the third result: the Hosios condition maximizes the ex-ante welfare of workers. Accordingly, if workers could commit ex ante (i.e. before they start searching for a job) to a wage setting rule, they would select the one implied by the Hosios condition. Indeed, if the (ex-post) bargaining power of workers β exceeds the effectiveness of workers in generating matches $(1 - \eta)$, it is in the ex-ante interest of workers to reduce their ex-post bargaining power so as to boost labour demand and thus raise the probability of finding a job. Hence, from an ex-ante perspective of workers, the balance of power reflected in the Hosios condition is optimal.

The Hosios condition implies that the matching process is efficient. This maximizes the incentives of workers to participate in this matching process through search. Accordingly, the Hosios condition maximizes the search intensity of workers (i.e. the last result of the lemma above). If workers' bargaining power is too weak (i.e. $\beta < (1 - \eta)$), workers' search is depressed by excessively low wages. If the bargaining power of workers is too strong (i.e. $\beta > (1 - \eta)$), workers are discouraged from looking for a job by a low probability of finding a job due to a lax labour market (as reflected in a low value for tightness θ).

2.2.2 taxation restores efficiency

If the division of bargaining power between the workers and the firm deviates from that required by the Hosios condition, tax policy can be used to restore efficiency. The following result holds for all the models in this paper.

Lemma 3 With $g = 0$, a linear wage tax exists such that the social optimum determined by (8) and (9) coincides with the private outcome (6) and (7). In particular,

$$\tau_a = \frac{1-\beta-\eta}{1-\beta}y \tag{11}$$

$$\tau = -\frac{1-\beta-\eta}{(1-\beta)(1-\eta)} \tag{12}$$

restore the social optimum with a balanced government budget.

Wage taxation is progressive ($\tau_a < 0$ and $\tau > 0$) if and only if workers hold excessive bargaining power (i.e. $\beta > (1-\eta)$).[13] Intuitively, the positive marginal tax rate reduces the effective bargaining power of workers $\frac{\beta/(1+\tau)}{\beta/(1+\tau)+(1-\beta)}$ to its social optimal value given by the Hosios condition $(1-\eta)$. By refunding the tax revenues from the proportional tax as an effective subsidy to matches ($\tau_a < 0$), the government ensures that employers face sufficient incentives to post the socially optimally amount of vacancies. Indeed, if the ex-post bargaining power of workers is too high, workers in effect levy an implicit tax on the specific investments of employers (i.e. the posting of vacancies) by expropriating part of the marginal social benefits of these investments. In other words, employers are held up by workers. The government in effect undoes the implicit 'hold-up' tax levied by workers on employers through explicit taxes and subsidies; it levies an explicit tax on workers to finance an explicit subsidy to employers.

Tax policy, which is set before search activities are determined, allows workers to commit not to expropriate entrepreneurs. In this way, tax policy effectively creates the market for search that is missing in the laissez fair equilibrium. Before they meet each other after they match, tax policy and tax enforcement in effect allows workers and entrepreneurs to conclude a contract stipulating that their search activities will be rewarded according to the marginal contribution to the match. Indeed, if workers would vote on the tax rate in the first stage of the game (i.e. when they are still unmatched), they would vote for the optimal social contract (i.e. the optimal allocation of property rights) implicit in the optimal tax structure.

The intuition for the balanced government budget is as follows. Since the matching function features constant returns to scale, the output of the matches is exhausted exactly in providing the proper marginal incentives to the providers of inputs. Hence, tax policy only redistributes resources from the side of the market with excessive bargaining power in the laissez fair equilibrium to the other side of the market without generating any net revenues to the government. In other words, through its explicit tax policy, the government undoes the implicit tax levied by the excessively strong side of the market on the excessively weak side. The explicit tax on the activity generating negative search externalities (this activity in effect benefits from an implicit subsidy) is just sufficient to finance an explicit subsidy on the activity that yields positive search

[13] As in most non-competitive models of the labour market, a more progressive tax system will tend to reduce involuntary unemployment by raising the number of matches. However, a more progressive tax system is not neccesarily efficient as vacancy costs are inefficiently high and search intensity of workers is inefficiently low. This shows how a more progressive tax system, while it may reduce involuntary unemployment, can be inefficient.

externalities (this activity in effect suffers from an implicit tax); the positive search externalities and negative search externalities balance exactly.

With decreasing returns in matching, matching would produce a rent. In that case, the government could employ its tax policy to transfer this surplus to the government budget without distorting search decisions. With increasing returns in matching, in contrast, the taxes required to produce efficient matching would yield a deficit for the government: the output from the match would not be sufficient to reward the searching parties according to their marginal contributions in producing the match.

2.2.3 profit and wage taxation

We have parameterized the tax system by the two parameters characterizing a linear wage tax τ and τ_a. For the interpretation of the results in the rest of this paper, however, it turns out to be more convenient to write the tax system as a combination of proportional wage and profit taxes. We can do so without any loss of generality because a linear wage tax is equivalent to an appropriate mix of a proportional wage tax and a proportional profit tax (for which search costs are not deductible). The proportional profit tax can be interpreted also as a tax on entrepreneurial income.

$$g = m(s; v)\left[\tau_w w + \tau_\pi \pi\right] \tag{13}$$

where τ_π stands for the proportional tax on (after-tax) profits (or income accruing to labour demand) and τ_w represents the proportional tax on (after-tax) wages (or labour income or income accruing to labour supply). The profit tax τ_π is the analogue of the proportional wage tax for the demand side of the labour market. Just as the proportional tax τ causes workers to bargain less aggressively, the proportional profit tax τ_π induces entrepreneurs to act less aggressively in wage bargaining because this policy instrument taxes away part of the quasi rents captured by entrepreneurs. Hence, just as a higher proportional wage tax τ_w reduces only the quasi rents accruing to workers (i.e. the after-tax wage w) and leaves unaffected the quasi rents accruing to entrepreneurs (i.e. π); the proportional profit tax τ_π reduces only the quasi rents captured by the demand side π and does not impact the quasi rents captured by the supply side w. The following result is routine to verify.

Lemma 4 With Nash bargaining, proportional profit and wage taxes yield the following division of the surplus

$$w = \frac{\beta y}{1 + \tau_w} \tag{14}$$

$$\pi = \frac{(1 - \beta) y}{1 + \tau_\pi} \tag{15}$$

Furthermore, the proportional wage and profit taxes are related to parameters of an equivalent linear wage tax as follows

$$\tau_\pi = \frac{\tau_a / y}{1 - \tau_a / y} \tag{16}$$

$$\tau_w = \frac{\tau + \tau_a / y}{1 - \tau_a / y} \tag{17}$$

Finally, the following values for τ_w and τ_π

$$\tau_\pi = \frac{1-\beta-\eta}{\eta} \tag{18}$$

$$\tau_w = -\frac{1-\beta-\eta}{1-\eta} \tag{19}$$

cause the private outcome to coincide with the social optimum while balancing the government budget.

The proportional wage and profit taxes that restore the social optimum can be interpreted in terms of the optimal taxation of labour demand and labour supply. In particular, the proportional profit tax can be viewed as a direct tax on labour demand (i.e. abstracting from general equilibrium effects on labour-market tightness θ). The proportional wage tax, in contrast, can be interpreted as a direct tax on labour supply. Labour demand should be taxed (i.e. the profit tax τ_π should be positive) if labour demand is excessive compared to labour supply (i.e. θ is too high because $1-\beta > \eta$): If labour supply is too large compared to demand (i.e. θ is too low because $\beta > 1-\eta$); in contrast, labour supply should be taxed (i.e. the proportional wage tax τ should be positive).

The results can be interpreted also in terms of the distortions due to imperfect competition. If entrepreneurs (the demand side) hold excessive bargaining power (i.e. $1-\beta > \eta$ and θ is too high), the market suffers from monopsony power. A subsidy to supply offsets the implicit tax imposed by the entrepreneurs who hold excessive market power. This subsidy is financed by a tax on monopsony profits. Similarly, if workers exercise too much power ex post (i.e. $\beta > 1-\eta$ and θ is too low), the market can be characterized as being monopolized. Tax policy corrects the associated monopoly distortions by levying a tax on the monopoly profits to finance a subsidy on labour demand. In this way, tax policy undoes the implicit taxes imposed by the party with excessive market power.

2.3 optimal taxes with positive government spending

This section allows for a positive government financing requirement (i.e. $g > 0$, see (2)). Welfare (1) can be written in the following way.

Lemma 5

$$\begin{aligned} W &= s m(\theta) w - \gamma(s) + g \\ &= s\gamma'(s) - \gamma(s) + g \end{aligned} \tag{20}$$

With the free entry condition ensuring a zero expected return for entrepreneurs, welfare consists of the ex ante return to workers $sm(\theta)w - \gamma(s)$ and the resources allocated to the government g: The second equality follows from (6).

The social planner chooses the optimal tax rates so as to maximize net welfare (20) subject to the government budget constraint (2). After exploring the resulting optimal tax structure, we discusse the marginal cost of public funds.

2.3.1 optimal tax structure

In exploring the optimal tax structure for financing a positive government's revenue requirement $g > 0$; we first introduce the following notation.

We define the following overall tax rates

$$\hat{\tau}_a \equiv \tau_a + \tilde{\tau}_a \tag{21}$$

$$\hat{\tau} \equiv \tau + \tilde{\tau} \tag{22}$$

$$\hat{\tau}_\pi \equiv \frac{\eta}{1-\beta}\left(\tau_\pi + \tilde{\tau}_\pi\right) \tag{23}$$

$$\hat{\tau}_w \equiv \frac{1-\eta}{\beta}\left(\tau_w + \tilde{\tau}_w\right) \tag{24}$$

where the implicit tax rates are defined as

$$\tilde{\tau}_a \equiv -\frac{1-\beta-\eta}{1-\beta} \tag{25}$$

$$\tilde{\tau} \equiv \frac{1-\beta-\eta}{(1-\beta)(1-\eta)} \tag{26}$$

$$\tilde{\tau}_\pi \equiv -\frac{1-\beta-\eta}{\eta} \tag{27}$$

$$\tilde{\tau}_w \equiv \frac{1-\beta-\eta}{1-\eta} \tag{28}$$

The overall taxes are defined in deviation of their first best values. The idea is that the government first corrects for the search externalities (a balanced budget correction) and then sets the overall taxes (the taxes with a hat) to optimally raise the positive revenue requirement g. As an illustration, the overall specific tax $\hat{\tau}_a$ equals the explicit tax τ_a minus the first-best tax that corrects for hold up $\frac{1-\beta-\eta}{1-\beta}$ (see (11)). The proportional wage and profit taxes $\hat{\tau}_w$ and $\hat{\tau}_\pi$ can be written as

$$\hat{\tau}_w = \frac{(1-\eta)y - w}{w}$$

$$\hat{\tau}_\pi = \frac{\eta y - \pi}{\pi}$$

The overall labour tax revenue from a match $\hat{\tau}_w w$ thus equals the difference between labour's Hosios share of output, $(1-\eta)y$, and the net wage a worker receives. Similarly, the overall profit tax revenue amounts to the difference between the entrepreneur's Hosios share, ηy, and the net profit received by an entrepreneur.

First, we characterize the optimal tax structure in terms of the proportional tax rates $\hat{\tau}_w$ and $\hat{\tau}_\pi$.

Proposition 6 The optimal overall taxes $\hat{\tau}_w$ and $\hat{\tau}_\pi$, which finance government expenditure g, satisfy

$$\hat{\tau}_\pi \pi = 0$$

$$\hat{\tau}_w w = \hat{g}$$

where $\hat{\tau}_w$ is increasing in ε_s and g; and $\hat{g}$ is defined as $\hat{g} \equiv \frac{g}{sm(\theta)}$:

The government employs only the proportional wage tax to finance government spending per match g. The intuition behind this result is the following. The free entry condition and linear vacancy costs imply that the demand for labour is infinitely elastic. Hence, Ramsey considerations suggest that the overall tax on labour demand, $\hat{\tau}_\pi$, should be zero. In particular, due to infinitely elastic labour demand, entrepreneurs are able to shift the entire burden of this tax on the demand side onto the supply side. Thus, whereas a tax on labour supply, τ_w, taxes the supply side directly through a lower after-tax wage w, a tax on labour demand τ_π is also borne by labour supply – albeit indirectly, namely through the general equilibrium effect of a less tight labour market reducing the probability of finding a job. It is more efficient to tax workers directly through τ_w than indirectly through the general equilibrium effect on θ; both ways distort search intensity but the second way also distorts labour-market tightness.

This result is closely related to the celebrated Diamond-Mirrlees (1971) result on the optimality of production efficiency. With constant returns to scale production (or tax instruments to tax away rents due to decreasing returns) and sufficient tax instruments to tax consumers directly, the government should ensure production efficiency. The government finds it optimal to tax consumers directly through consumer taxes rather than indirectly through taxes that violate production efficiency. Similarly, in the current context, the government should not distort labour-market tightness, θ, through the profit tax $\hat{\tau}_\pi$. Indeed, keeping labour-market tightness at its first-best level can be viewed as maintaining efficiency in the production of matches.

Using the relationship between the proportional taxes $\tau_w; \tau_\pi$ and parameters of a linear wage tax $\tau_a; \tau$ (see (16) and (17)), one can easily verify the following result.

Corollary 7 *The optimal taxes $\hat{\tau}$ and $\hat{\tau}_a$, which finance the government expenditure g, satisfy*

$$\hat{\tau}_a = 0 \tag{29}$$

and $\hat{\tau}$ is increasing in g and ε_s.

The fixed component of the tax, τ_a, is thus set at its first-best level (11). Accordingly, only the ad valorum component, τ, is used as an instrument to finance positive government spending. The intuition is the same as above, since τ is a tax on labour supply only, while τ_a taxes labour demand as well.

2.3.2 marginal cost of public funds

The marginal cost of public funds μ represents the shadow value of government revenue in terms of output. Put differently, it is the Lagrange multiplier in the following optimization problem

$$\max_{\tau_w; \tau_\pi} \{ s m(\theta) w - \gamma(s) + g + \mu [s m(\theta)(\tau_w w + \tau_\pi \pi) - g] \} \tag{30}$$

Lemma 8 *The marginal cost of public funds equals*

$$\mu = \frac{1}{(1 - \varepsilon_s \hat{\tau})} \tag{31}$$

where $\tau \equiv \frac{\tau_w w + \tau_\pi \pi}{w}$ denotes the average tax burden expressed in terms of the after-tax wage and $\varepsilon_s \equiv \left(\frac{s\gamma''(s)}{\gamma'(s)}\right)^{-1}$ stands for the elasticity of the search intensity of workers with respect to the rewards to search. Let g^m denote the value of the following maximization problem

$$\max_{\tau_w;\tau_\pi} s m(\theta)(\tau_w w + \tau_\pi \pi)$$
$$\text{subject to (5), (7), (14) and (15)}$$

Then, defining $\hat{g}^m \equiv \frac{g^m}{s m(\theta)}$, we find

$$\hat{g}^m = \frac{1 - \eta}{1 + \varepsilon_s} y$$

and the corresponding average tax burden τ^m equals

$$\tau^m = \frac{1}{\varepsilon_s}$$

The marginal cost of public funds thus rises with both the elasticity ε_s; which can be interpreted as the labour-supply elasticity, and the average tax burden τ: Since labour demand is infinitely elastic, only the labour-supply elasticity ε_s features in the expression for the marginal costs of public funds. In a model with inelastic labour supply ($\varepsilon_s = 0$), the marginal cost of public funds equals unity. In that case, the government can tax away the ex-ante surplus of workers, $(1 - \eta)y$, without distorting incentives.

The marginal cost of public funds equals unity also if the average tax burden is zero (i.e. if the government financing requirement g is zero). This is a direct consequence of constant returns to scale in matching, which implies that output of the match is exhausted exactly in providing the proper marginal incentives for search at both sides of the market. Taxing away some of the output of the match necessarily implies that the incentives for finding a match are distorted. These distortions in search raise the marginal cost of public funds above unity.

The marginal cost of public funds becomes infinite if the average tax burden τ equals the reciprocal of the elasticity of labour supply, $1/\varepsilon_s$: By imposing a mark up according to the inverse elasticity rule, the government acts as a monopolist who maximizes its revenues. The tax burden $\tau^m = 1/\varepsilon_s$ thus finances the maximum revenue requirement g^m that can be financed.[14] The larger is the elasticity of labour supply, the smaller becomes the maximum revenue requirement that can be financed.

The marginal cost of public funds does not affect the explicit taxes correcting for the search externalities (see (25), (26), (27), and (28)). This is different from optimal pollution taxes offsetting negative consumption externalities due to pollution. These taxes decline with the marginal cost of public funds (see Bovenberg and De Mooij (1994)). If pollution externalities harm the production side of the economy, in contrast, the marginal cost of public funds does not

[14] The appendix shows three (equivalent) ways to calculate the maximal revenue requirement g^m. First, at the maximal revenue requirement, μ tends to infinity. Second, one can solve for the maximal revenue requirement while calculating the optimal tax incidence. Third, at the maximal revenue requirement, the determinant of the matrix of the system of linearized equations determining the private outcome becomes zero.

affect the optimal pollution taxes correcting for these negative externalities (see Bovenberg and Van der Ploeg (1998)). Also in our model, the (search) externalities affect the production side of the economy (i.e. the matching process). Accordingly, the marginal cost of public funds does not enter the expressions for the optimal externality correcting taxes.

3 Non-linear vacancy costs

The previous section established that only the supply side of the labour market should be taxed to finance government spending so that additional government spending leaves labour-market tightness unaffected. This result depends crucially on two important assumptions, namely first, the free entry condition for entrepreneurs (7) and, second, the linear character of vacancy costs. These two assumptions imply that the net surplus of entrepreneurs is zero in equilibrium. Hence, the resources required to finance government spending can come only out of the surplus collected at the supply side by workers.

In order to explore the robustness of the results derived in the previous section, this section modifies these two assumptions by allowing for non-linear vacancy costs and a fixed number of entrepreneurs.[15] The linear vacancy costs assumed in the previous section imply that labour demand is infinitely elastic with respect to wage costs. Empirical evidence, however, suggests that labour demand is less elastic (see Hamermesh (1993: chapter 3)). A labour-demand elasticity $\varepsilon_v \equiv \left(\frac{c''(v)v}{c'(v)}\right)^{-1}$ that is less than infinite is thus more relevant from an empirical point of view. With less than infinitely elastic labour demand, entrepreneurs are no longer able to fully shift the burden of taxes on labour demand. Hence, the government may want to tax the demand side of the labour market to redistribute resources to the public sector or to workers. This section formalizes these insights.

3.1 model

The model presented in section 2 is adapted in two ways. First, there are a fixed number of entrepreneurs $j \in [0;1]$. Second, each entrepreneur selects her search intensity $0 \le v_j \le 1$ at a cost $c(v_j)$ where the function $c(:)$ is increasing and concave ($c(0) = 0; c'(0) = 0; c'(:) \ge 0; c''(:) > 0$; and $\lim_{v_j \uparrow 1} c'(v_j) \to +\infty$). Hence, just as the intensity s with which workers search for a job, the intensity v with which entrepreneurs look for an employee is subject to increasing marginal costs. The rising nature of marginal search costs $c(v_j)$ can be motivated by the entrepreneur inelastically providing a production factor (like supervision or entrepreneurship) in the search process. We adopt the interpretation of this fixed factor as being entrepreneurship and come back to this interpretation in section 4.

[15] The case with non-linear vacancy costs and free entry of firms is equivalent to the model in the previous section. To see this, note that in this case the number of vacancies per firm ψ is fixed by the condition $c'(\psi) = \frac{c(\psi)}{\psi}$, that is average cost per vacancy should be minimized. Pissarides (1990: 76) finds the same condition in a dynamic version of the model. The total number of vacancies posted v equals ψ times the number of firms that enter. Free entry then implies that $c(\psi) = \frac{m(\theta)}{\theta}\pi$ which is the same condition as in section 2.

A risk-neutral entrepreneur j selects vacancy intensity v_j so as to maximize the expected surplus

$$\max_{v_j \geq 0} \left\{ v_j \frac{m(\theta)}{\theta}\pi - c(v_j) \right\} \tag{32}$$

where labour market tightness is defined as $\theta \equiv \frac{v}{s}$. With homogeneous entrepreneurs, all entrepreneurs feature the same search intensity determined by the following first-order condition

$$\begin{aligned} c'(v) &= \frac{m(\theta)}{\theta}\pi \\ &= \frac{m(\theta)}{\theta}(1-\beta)(y-\tau_a) \\ &= \frac{m(\theta)}{\theta}(1-\beta)\frac{y}{1+\tau_\pi} \end{aligned} \tag{33}$$

Substituting (33) into (32), we find that the surplus reaped by entrepreneurs is $vc'(v) - c(v)$. The rising marginal vacancy costs (i.e. $c''(v) > 0$ and $c'(0) = c(0) = 0$) imply that this surplus is positive at non-zero vacancy intensity $v > 0$. This positive surplus can be viewed as a reward for entrepreneurship.

If the social planner attaches equal weight to the surplus of workers and that of entrepreneurs, she chooses the search intensities s and v to maximize net output

$$\max_{s,v} m(s,v)y - \gamma(s) - c(v) \tag{34}$$

As above, the economy still suffers from a missing market for the search intensities s and v. In order to reward these specific investments according to their contributions to net output, the government needs to subsidize the side of the labour market that is being held up and tax the side that is holding up. With constant-returns-to-scale matching, the optimal rewards exhaust output so that the optimal tax policy does not yield any net revenues to the government. This can be summarized as follows.

Lemma 9 If $g = 0$ and if the social planner attaches equal weight to the surpluses of workers and entrepreneurs, then the linear wage tax implied by τ_a and τ in equations (11) and (12) or equivalently the proportional profit and wage taxes τ_π and τ_w in (18) and (19) ensure that the private outcome coincides with the social optimum while maintaining a balanced government budget.

3.2 optimal taxes with positive government spending and distributional considerations

The main difference with the model with linear vacancy costs is that the optimal tax policy no longer maximizes the search intensity of workers, s. With a positive ex-ante surplus for entrepreneurs, net output can be written as the sum of the surpluses accruing to workers, entrepreneurs and the government. This gives us the opportunity to introduce distributional considerations for the social planner. In particular, we assume that the social planner wants to maximize

$$W_\alpha = \alpha[sm(\theta)w - \gamma(s)] + [sm(\theta)\pi - c(v)] + g \tag{35}$$

where α denotes the weight the planner attaches to the ex ante welfare of workers.[16]

Lemma 10 *Welfare W_α can be written as*

$$W_\alpha = \alpha[s\gamma'(s) - \gamma(s)] + [vc'(v) - c(v)] + g$$

Given exogenous government spending g; optimal tax policy no longer optimizes the surplus for workers (and hence search intensity s) but instead trades off a maximal surplus for workers $s\gamma'(s) - \gamma(s)$ against a maximal surplus for entrepreneurs $c'(v)v - c(v)$ (and hence a maximal search intensity v):

The next result characterizes the optimal tax policy if the government features a positive revenue requirement ($g > 0$ in (2)) and cares about the distribution between workers and entrepreneurs.

Theorem 11 *A social planner designing a tax system that maximizes W_α while collecting sufficient tax revenues to finance the revenue requirement g chooses a tax system with the following characteristics*

$$\tilde{\tau}_w w = \zeta \tilde{g} + \xi \tag{36}$$

$$\tilde{\tau}_\pi \pi = (1 - \zeta)\tilde{g} - \xi \tag{37}$$

where

$$\zeta \equiv \frac{\alpha w \varepsilon_v}{\alpha w \varepsilon_v + \pi \varepsilon_s}$$

$$\xi \equiv \frac{(1 - \alpha) w \pi}{\varepsilon_s \pi + \alpha \varepsilon_v w}$$

and

$$\tilde{g} \equiv \frac{g}{sm(\theta)}$$

In the presence of an optimal tax system, the marginal cost of public funds amounts to

$$\mu\left[1 - \frac{\tilde{g}}{\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}}\right] = \frac{\frac{\alpha w}{\varepsilon_s \varepsilon_v} + (1 - \eta)\left(\frac{\alpha w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)}{\frac{w}{\varepsilon_s \varepsilon_v} + (1 - \eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right) + \frac{\xi}{\varepsilon_v}}$$

The expressions for the optimal proportional wage and profit taxes (36) and (37) show that the optimal tax rates can be decomposed in three terms corresponding to the threefold task of the tax system, namely to correct non-tax distortions (i.e. to internalize the search externalities by introducing the missing markets for search activity as reflected in the implicit taxes τ_w and τ_p), to finance government spending (i.e. to redistribute from the private to the public sector as reflected in the parameter ζ), and to correct the income distribution (i.e. to redistribute within the private sector between workers and entrepreneurs as reflected in the parameter ξ). We explore these three roles of the tax system in turn.

[16] We assume that the government financing requirement g is exogenously given. Alternatively, we could endogenously determine g by attaching a welfare weight to g in expression (35).

3.2.1 optimal correction of search externalities

The non-tax distortions, which are represented by the implicit taxes τ_p and τ_w (see equations (27) and (28)) enter the formulas additively. They only affect the definitions of the overall tax wedges $\hat{\tau}_\pi$ and $\hat{\tau}_w$ while leaving the formulas for the optimal relationship between these overall tax wedges unaffected (see the notation introduced in (23) and (24)).

Whereas the optimal tax rates thus feature the so-called additivity property (see Sandmo (1976)), the dichotomy between the revenue-raising task of the tax system and its role in correcting search externalities is even stronger than in the case of other non-tax distortions. First, the externality correcting role of the tax system does not depend on the revenue-raising task of the tax system. The reason is that, in the presence of production externalities due to hold up, the externality correcting parts τ_w and τ_p do not depend on the government financing requirement.[17] Second, the revenue-raising task does not depend on the search externalities because correcting these distortions does not impact the government revenue requirement; the net tax revenues from the associated externality correcting taxes are zero. Hence, the missing market for search activities does not affect the optimal values for $\hat{\tau}_\pi$ and $\hat{\tau}_w$.

3.2.2 optimal revenue raising

In order to investigate the revenue raising role of the tax system, we explore the specific case in which the government does not exhibit any distributional preferences.

Corollary 12 If the government does not feature any distributional preferences (i.e. $\alpha = 1$); the optimal tax system is characterized by the following restriction

$$\frac{\hat{\tau}_w}{\hat{\tau}_\pi} = \frac{\varepsilon_v}{\varepsilon_s} \tag{38}$$

The optimal taxes in terms of the government financing requirement are given by

$$\hat{\tau}_w w = \gamma \bar{g}$$

$$\hat{\tau}_\pi \pi = (1 - \gamma)\bar{g}$$

$$\gamma \equiv \frac{w\varepsilon_v}{w\varepsilon_v + \pi\varepsilon_s}$$

The marginal cost of public funds is given by

$$\mu = \frac{1}{\left(1 - \frac{\bar{g}}{(w/\varepsilon_s) + (\pi/\varepsilon_v)}\right)} = \frac{1}{(1 - \varepsilon\bar{\tau})} \tag{39}$$

and

$$\bar{g}^m = \left\{\frac{1 - \eta}{1 + \varepsilon_s} + \frac{\eta}{1 + \varepsilon_v}\right\} y$$

[17] This is correct only if η does not depend on labour-market tightness (as is the case with a Cobb Douglas matching function). The marginal cost of public funds enters the formulas for taxes correcting for consumption externalities (see Bovenberg and de Mooij (1994)).

where $\varepsilon \equiv \left\{\frac{w}{w+\pi}\frac{1}{\varepsilon_s} + \frac{\pi}{w+\pi}\frac{1}{\varepsilon_v}\right\}^{-1}$ denotes the weighted harmonic average of the demand and supply elasticities, $\tau \equiv \frac{\tau_w w + \tau_\pi \pi}{w+\pi}$ the average tax burden expressed in terms of after-tax incomes, and $\hat{g}^m$ government expenditure per match that maximizes overall government expenditure $g = sm(\theta)\hat{g}$.

Optimal tax rates Equation (38) is the Ramsey rule. The relative overall tax rates on labour demand and labour supply are inversely related to their elasticities. In particular, if the demand elasticity ε_v is large compared to the supply elasticity ε_s; the overall tax rate on labour supply τ_w should be relatively large. At the optimum, a marginal redistribution of tax money from the demand side of the labour market to the supply side (which would raise labour supply depending on the supply elasticity ε_s and reduce labour demand depending on the demand elasticity ε_v) should leave unaffected overall tax revenues (i.e. including the implicit tax revenues), thus $\tau_w \varepsilon_s - \tau_\pi \varepsilon_v = 0$:

The roles of the two taxes in raising government revenues is implicit in the parameter γ which stands for the share of government spending financed by taxes on workers. This share γ rises with both the relative elasticity of labour demand (i.e. $\varepsilon_v/\varepsilon_s$) and the relative size of the tax base of a tax on workers (i.e. w/π):

We can write the optimal tax system also in terms of a linear wage tax by using lemma 4 and (38). This yields

Corollary 13 If the government does not feature any distributional preferences ($\alpha = 1$); the optimal tax system is characterized by the following restriction

$$\varepsilon_s \tau = \tau_a (\varepsilon_v - \varepsilon_s) \tag{40}$$

Expression (40) reveals that the result in corollary 7, that only the ad valorum tax is employed to finance government spending, depends crucially on the assumption of linear vacancy costs; only if labour demand is infinitely elastic ($\varepsilon_v \to +\infty$); is the marginal tax rate τ used to finance government spending ($\tau_a = 0$): In fact, this result of exclusive reliance on τ rather than τ_a is exactly reversed if demand and supply elasticities coincide ($\varepsilon_s = \varepsilon_v$). In that case, only the specific tax component is used for financing spending ($\tau = 0$): Intuitively, the specific tax amounts to an equal tax on both labour demand and supply. Such a tax is optimal if supply and demand elasticities are equal. The ad valorum tax τ; which acts like a tax on labour supply only, is optimal if labour supply is infinitely inelastic compared to demand ($\varepsilon_s/\varepsilon_v \to 0$):

marginal cost of public funds Just as in the case with linear vacancy costs (see (31)), the marginal cost of public funds depends on both the government financing requirement (i.e. the average tax burden) and the sensitivity of private behaviour as reflected in the magnitude of elasticities. In contrast to the case with linear vacancy costs, however, the labour-demand elasticity enters explicitly. In particular, a finite demand elasticity helps to contain the marginal cost of public funds. The marginal cost of public funds is increasing in the demand and supply elasticities. Intuitively, larger elasticities imply that labour-market behaviour is more sensitive to distorted price signals. Hence, taxes exert larger distortionary effects on this behaviour.

The marginal cost of public funds does not exceed unity if either demand or supply is inelastic (i.e. $\varepsilon_s = 0$ or $\varepsilon_v = 0$): In that case, the government can in effect levy a lump-sum tax, which does not affect behaviour.[18] The marginal cost of public funds is unity also for the first dollar of revenue raised. The reason is that the externality correcting taxes do not raise any government revenue with constant returns to scale in matching.

In order to explore the maximum government spending that can be financed, we note that $\varepsilon\hat{\tau} = \hat{\tau}_w\varepsilon_s = \hat{\tau}_\pi\varepsilon_v$ by using the Ramsey rule (38): The marginal cost of public funds thus becomes infinite if the government acts like a monopolist (by adopting the inverse elasticity rules $\hat{\tau}_w = \varepsilon_s^{-1}$ and $\hat{\tau}_\pi = \varepsilon_v^{-1}$) by extracting the maximal surplus from the private sector. At the maximal size of government, the tax burden (in terms of output y) is thus given by $\frac{g^m}{y} = \frac{1-\eta}{1+\varepsilon_s} + \frac{\eta}{1+\varepsilon_v}$. The maximal government size is zero if both labour supply and labour demand are infinitely elastic. If both demand and supply are perfectly inelastic, in contrast, the entire output can be taxed away.

labour-market tightness With linear vacancy costs, the government revenue requirement does not affect labour-market tightness θ. The next result shows that with non linear vacancy costs raising a positive revenue requirement can affect labour-market tightness.

Lemma 14 *If $\alpha = 1$ and assuming that ε_s and ε_v are constant then*

$$\operatorname{sign}\left(\frac{d\theta}{dg}\right) = \operatorname{sign}\left(\frac{g}{\varepsilon_v\varepsilon_s}\left(\frac{\hat{\tau}_\pi}{\hat{\tau}_w} - 1\right)\right)$$

Increasing a positive initial revenue requirement thus strengthens the side of the market that is subject to the highest overall tax rate, that is the inelastic side of the market. To illustrate, if labour supply is relatively inelastic (i.e. $\varepsilon_s < \varepsilon_v$ and hence $\frac{\hat{\tau}_\pi}{\hat{\tau}_w} < 1$), the labour-market becomes less tight (i.e. supply increases compared to demand) if the government needs to raise more revenue. Intuitively, with inelastic labour supply, the government relies relatively heavily on wage taxation τ_w to finance its spending. Thus, wages are a more important tax base than profits. In order to protect wages as a tax base, the government raises wage income at the expense of labour-market efficiency. Indeed, as a stakeholder in wage income, the government shares an interest with workers in monopolizing the labour market. Hence, to protect its interest in revenue raising, the government endows workers with a larger effective bargaining power than is implicit in the Hosios condition. In this way, labour-market tightness is distorted for the purpose of raising public revenue, i.e. for redistributing resources from the private to the public sector. The side of the market that provides the largest revenue base to the government has excessive bargaining power and thus expands compared to the other side.[19]

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[18] These lump-sum taxes, however, are limited by the sizes of the tax base. That is, ηy in the case of profit taxation on inelastic demand ($\varepsilon_v = 0$) and $(1-\eta)y$ in the case of wage taxation on inelastic labour supply ($\varepsilon_s = 0$):

[19] Another way to see this, is the following. At very small revenue requirements (at which $\hat{\tau}_w$ and $\hat{\tau}_\pi$ are close to zero so that $w = (1-\eta)y$ and $\pi = \eta y$), the wage tax share equals $\zeta = \frac{(1-\eta)\varepsilon_s^{-1}}{(1-\eta)\varepsilon_s^{-1} + \eta\varepsilon_v^{-1}}$: At the highest government financing requirement that can be financed (at which $\hat{\tau}_w = \varepsilon_s^{-1}$ and $\hat{\tau}_\pi = \varepsilon_v^{-1}$); this share amounts to $\zeta =$

Labour-market tightness is not distorted in a number of cases. First, the first dollar of revenue raised does not imply a first-order effect on tightness. The reason is that the effect on the tax base is then only second order. Hence, no first-order distortions in the labour-market are needed to protect the tax base. Labour-market tightness is also not distorted if labour demand and supply are equally elastic. If supply and demand respond symmetrically, there is no reason to distort the power balance between workers and entrepreneurs. Finally, in case either demand or supply are infinitely elastic, tightness does not depend on the government financing requirement. In that case, the government does not have any market power to protect the inelastic side because the other side is infinitely elastic. Hence, the tax burden is always born by the inelastic side of the market; the elastic side can fully escape it.[20]

3.2.3 optimal redistribution

optimal taxation The redistributional role of the tax system is implicit in the term ξ in (36) and (37). The impact of the elasticities on this term is closely related to the corresponding impact on the marginal cost of public funds μ in (39); whereas the marginal cost of public funds involves redistribution from the private to the public sector, the term ξ is associated with redistribution within the private sector. If both labour supply and demand are infinitely elastic, redistribution becomes prohibitively expensive and the redistribution term ξ becomes zero while the marginal cost of public funds (39) becomes infinite. More generally, the absolute size of the redistribution term (the marginal cost of public funds) is inversely (positively) related to the supply and demand elasticities ε_s and ε_v: Higher elasticities indicating that behaviour is more sensitive to tax distortions implies that redistribution (either within or away from the private sector) becomes more costly. Hence, efficiency considerations prevent substantial redistribution from the demand to the supply side (or from the private sector to the government).

The distributional term depends also on the parameter α; which represents the distributional preference for workers. If the government cares only about workers (i.e. $\alpha \to +\infty$); the optimal profit tax from (37) is given by the inverse elasticity rule $\hat{\tau}_\pi = \varepsilon_v^{-1}$: This tax system maximizes the ex-ante surplus of workers and hence the search activity at the supply side of the labour market.

labour-market tightness If labour demand is less than infinitely elastic ($\varepsilon_v < +\infty$); a tax policy that maximizes the surplus for workers violates the Hosios condition (even if the government financing requirement g is zero). Indeed, workers would vote for a positive overall tax on labour demand $\hat{\tau}_\pi > 0$. By levying a positive tax on the specific investments of entrepreneurs, workers extract some of the rents enjoyed by entrepreneurs (i.e. the reward to entrepreneurship). At the optimum, workers trade off the additional distortions from a higher tax

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$\frac{(1-\eta)(\varepsilon_s^{-1}/(1+\varepsilon_s^{-1}))}{(1-\eta)(\varepsilon_s^{-1}/(1+\varepsilon_s^{-1}))+\eta(\varepsilon_v^{-1}/(1+\varepsilon_v^{-1}))}$: Accordingly, at a low government financing requirement, differences in demand and supply elasticities exert a larger impact on the relative shares financed by the two sides (if the elasticities η; ε_s;and ε_v are constant). Again we find that the desire to tax the least elastic side most heavily to reduce distortions in private behaviour is offset by the desire to protect the tax base.

[20] Hence, if the demand elasticity exceeds the supply elasticity, the labour market is monopolized as long as the demand elasticity is less than infinite.

on entrepreneurs (which rise with the initial tax τ_π and take the form of a less tight labour market and more unemployment) against the rents captured from the demand side (which rise with the reciprocal of the labour demand elasticity ε_v^{-1} and accrue in the form of higher after-tax wages).

In the equilibrium that maximizes the surplus for workers, the labour market is monopolized. Hence, from a pure efficiency point of view (that is, considering $\alpha = 1$), the labour market is not tight enough. At the margin, the efficiency costs exactly balance the distributional benefits. More generally, if the government cares more about workers than about entrepreneurs (i.e. $\alpha > 1$ but not necessarily infinite), the labour market suffers from monopoly power at a zero government financing requirement (and a less than infinite labour demand elasticity ε_v).[21] In contrast, if the government attaches a higher weight to the surplus of entrepreneurs than that of workers (and if the labour supply elasticity ε_s is less than infinite) then the labour market is monopsonized (i.e. $\tau_w > 0$ so that the labour market is too tight from an efficiency point of view). Accordingly, even without a revenue requirement, labour-market tightness is distorted – not for redistributing resources from the private to the public sector but for redistribution within the private sector. Monopoly and monopsony distortions are the price for redistribution. Unlike the case with linear vacancy costs, voters no longer agree on the optimal tax policy. Compared to workers, entrepreneurs vote for a higher tax on workers and a lower tax on entrepreneurs to protect the rewards for the fixed factor supplied by them.

4 Endogenous worker-entrepreneur choice

In the previous section, the government could freely redistribute resources between workers and entrepreneurs without inducing agents to move between the demand and supply sides of the labour market. This seems a strong assumption – especially in the long run. In this section, therefore, we allow agents to arbitrage between being an entrepreneur and being a worker in a new first stage of the game. At the same time, as in the previous section, we allow for less than infinitely elastic labour demand.

Compared to the standard model outlined in section 2, agents can freely enter and exit not only the demand side of the labour market but also the supply side. Moreover, in entering the labour market as an entrepreneur and demand labour, the agents face the alternative of becoming a worker and supply labour. In contrast to section 2, therefore, the ex-ante value of being an entrepreneur is endogenously determined rather than being exogenously fixed at zero. As an other difference with section 2, non-linear vacancy costs imply that labour demand is finitely elastic with respect to wage costs.

The model discussed in this section can therefore be viewed as an intermediate case between the model of section 2, in which entrepreneurs can freely enter the economy and labour demand is infinitely elastic, and that of section 3, in which the number of entrepreneurs is fixed and labour demand is finitely elastic. Compared to section 3, the additional margin on which agents can respond to

[21] At an infinite demand elasticity (as in the standard model), distributional preferences do not affect the optimal tax system. Intuitively, with infinitely elastic demand, the supply side cannot capture any rents from the demand side.

incentives (the worker-entrepreneur decision) puts an additional constraint on tax policy.

4.1 model

The population of agents is modelled as the unit interval $[0;1]$: The ratio of entrepreneurs to workers is denoted by λ: Accordingly, the proportion of the population that chooses to become entrepreneur is given by $\lambda=(1+\lambda)$ and the share of the population that becomes worker by $1=(1+\lambda)$: Labour market tightness is now defined as

$$\theta \equiv \frac{v}{s}\lambda \tag{41}$$

Labour-market tightness is thus affected not only by the search intensities at both sides of the market, v and s, but also by the proportion of the population that chooses to become entrepreneur, λ.

4.1.1 decentralized equilibrium

Solving the model backwards, we note that the production stage and the wage bargaining process are the same as in the models discussed previously. The first-order condition for the optimal search intensity at the supply side is still given by (6) and that for optimal search intensity at the demand side by (33).

A new stage is introduced in which agents decide whether to become a worker (and search for a job) or to become an entrepreneur (and post vacancies).[22] Accordingly, in contrast to the model of the previous section, the number of agents at both sides of the labour market is endogenous. The following expression describes the first stage of the game in which agents choose whether to enter the labour market on the demand side as an entrepreneur or on the supply side as a worker:

$$sm(\theta)w - \gamma(s) = v\frac{m(\theta)}{\theta}\pi - c(v) \tag{42}$$

Arbitrage ensures that the ex-ante value of being a worker (the left-hand side of (42)) equals the ex-ante value of being an entrepreneur (the right-hand side of (42)). Or equivalently, using equations (5) and (33)

$$s\gamma'(s) - \gamma(s) = vc'(v) - c(v) \tag{43}$$

The following proposition characterizes the private outcome.

Proposition 15 In the private outcome, search s is determined by (6), vacancy intensity v by (33) and the distribution λ of the population by

$$\lambda = \frac{\pi\left(1 - \left(\frac{c'(v)v}{c(v)}\right)^{-1}\right)}{w\left(1 - \left(\frac{\gamma'(s)s}{\gamma(s)}\right)^{-1}\right)} \tag{44}$$

[22] In line with the traditional assumptions in the optimal tax literature, the government is Stackelberg leader in the game with the private sector. Hence, tax policy is set first. If tax policy is determined by voting, voting thus occurs before the private sector implements its economic decisions.

The wage w (profit π) is the ex post pay off or quasi rent on the supply (demand) side of the labour market. The expression $\left(1-\frac{\gamma'(s)s}{\gamma(s)}\right)^{-1}$ $\left(\left(1-\frac{c'(v)v}{c(v)}\right)^{-1}\right)$ determines to which extent this quasi rent is sustained as an ex ante surplus or rent by subtracting the costs associated with the specific investments in search activity.

The endogenous choice to become a worker or an entrepreneur can be viewed as a decision to acquire a particular ability, namely to either search for jobs or search for workers. These abilities are rewarded in equilibrium because both entrepreneurs and workers collect a positive surplus on account of the convex character of search costs. Indeed, these abilities can be viewed as the production factors that give rise to increasing marginal search costs. The following notation formalizes the role of these production factors.

Let f denote an agent's (exogenous) ability to look for agents at the other side of the market. Then the production function of search intensity at the demand (supply) side of the labour market is written as $g(c; f)$ $(h(\gamma; f))$, which is assumed to be homogenous of degree one. The substitution elasticity between ability f and effort c is defined as

$$\sigma_{cf} \equiv \left[\frac{d\left(\frac{\partial g(c;f)/\partial c}{\partial g(c;f)/\partial f}\right)}{d\left(\frac{c}{f}\right)}\left(\frac{\frac{c}{f}}{\frac{\partial g(c;f)/\partial c}{\partial g(c;f)/\partial f}}\right)\right]^{-1}$$

and the elasticity of vacancies with respect to f (or the production share of ability f) is defined as

$$\alpha_{vf} \equiv \frac{\partial g(c; f)}{\partial f}\frac{f}{g(c; f)}$$

Similarly, on the supply side of the labour market these elasticities are defined as

$$\sigma_{\gamma f} \equiv \left[\frac{d\left(\frac{\partial h(\gamma;f)/\partial \gamma}{\partial h(\gamma;f)/\partial f}\right)}{d\left(\frac{\gamma}{f}\right)}\left(\frac{\frac{\gamma}{f}}{\frac{\partial h(\gamma;f)/\partial \gamma}{\partial h(\gamma;f)/\partial f}}\right)\right]^{-1}$$

$$\alpha_{sf} \equiv \frac{\partial h(\gamma; f)}{\partial f}\frac{f}{h(\gamma; f)}$$

The following relationships between the elasticities are used below.

Lemma 16

$$\frac{c(v)}{vc'(v)} = 1-\alpha_{vf}$$

$$\varepsilon_v = \frac{1-\alpha_{vf}}{\alpha_{vf}}\sigma_{cf}$$

$$\frac{\gamma(s)}{s\gamma'(s)} = 1-\alpha_{sf}$$

$$\varepsilon_s = \frac{1-\alpha_{sf}}{\alpha_{sf}}\sigma_{\gamma f}$$

Whereas section 3 assumes that the distribution of abilities over both sides of the market is exogenously given, this section endogenously determines this distribution. Hence, whereas the model of section 3 involved two fixed factors (namely the abilities of the fixed number of agents at both sides of the market), the model of this section involves only a single fixed factor, namely the aggregate supply of abilities in the economy. The overall supply of abilities continues to be determined exogenously by the number of agents in the economy because each agent inelastically acquires her ability to search.

The quasi rents π (w) can be sustained as rents if the production share of ability f, which we will call the rent share, α_{vf} (α_{sf}) is high. The results in Lemma 16 allow us to write the distribution of the population (44) as

$$\lambda = \frac{\pi\alpha_{vf}}{w\alpha_{sf}} \tag{45}$$

A large part of the population is attracted by the side of the market with the largest rents. These rents are determined by two factors: first, the quasi rents (π and w) and the rent shares (α_{vf} and α_{sf}); which represent the shares of the quasi rents that are sustained as rents. General equilibrium is established as a lower probability of finding a match offsets the higher quasi rents. Hence, at the side of the market with relatively small quasi rents a larger part of the ex-ante return is paid out in the form of a high probability of finding a match. At the other side of the market, in contrast, the return accrues mainly through a high return conditional on finding a match.

4.1.2 command equilibrium

The social planner selects s; v; and λ such that net output is maximized

$$\max_{s;v;\lambda} \frac{s}{1+\lambda} m\left(\frac{v\lambda}{s}\right) y - \frac{1}{1+\lambda}\gamma(s) - \frac{\lambda}{1+\lambda} c(v) \tag{46}$$

where we have used (41) to eliminate θ:

Lemma 17 If $g = 0$ the socially optimal split equals

$$\lambda = \frac{\eta\alpha_{vf}}{(1-\eta)\alpha_{sf}} \tag{47}$$

Expression (47) shows that the efficient split between entrepreneurs and workers λ depends on two determinants. The first determinant is the relative effectiveness of the demand and supply sides in generating matches. In particular, if the demand side is more important in generating matches than the supply side (η is large), a larger part of the population should become entrepreneur rather than worker. The second determinant of the optimal population split is the ratio of rent shares α_{vf}/α_{sf}. If vacancy costs are linear ($\alpha_{vf} = 0$), for example, ability is not productive at the demand side of the market. Hence, all agents should go to the supply side where they help to reduce the costs of supplying labour (as long as $\alpha_{sf} > 0$). If it is more costly to raise search intensity (per person) at the demand side than at the supply side (i.e. $c'(v)$ and α_{vf} are large compared to $\gamma'(s)$ and α_{sf}); in contrast, most of the scarce abilities should go to the demand side. Indeed, a large rent share α_{vf} indicates that it is difficult to substitute a higher search intensity per entrepreneur for the number of entrepreneurs in boosting labour demand. Entrepreneurs thus provide an especially important service in raising the efficiency of the matching process. As a direct consequence, a large part of the population should become entrepreneur.

4.2 optimal taxes with positive government spending

The social planner optimizes welfare with respect to the following government budget constraint

$$g = \frac{sm(\theta)}{1+\lambda}\left[\tau_w w + \tau_\pi \pi\right] = \frac{sm(\theta)}{1+\lambda}\left[y - w - \pi\right] \tag{48}$$

Lemma 18 Welfare can be written as

$$W = s\gamma^{0}(s) - \gamma(s) + g \tag{49}$$

As in the standard model discussed in section 2, we find that for given government expenditure g the social planner chooses his instruments so as to maximize search intensity at the supply side of the market. Arbitrage (43) ensures that, in contrast to the model described in section 3, there is no trade off between maximizing the pay off from search at the supply and demand sides. Indeed, the interests of workers and entrepreneurs coincide. Accordingly, the population agrees on optimal tax policy and distributional considerations are not relevant in setting optimal policy.

Theorem 19 A social planner choosing a tax system that maximizes W while collecting enough tax revenues to finance g chooses a tax system with the following feature

$$\frac{\hat{\tau}_w}{\hat{\tau}_\pi} = \frac{((1-\eta)y - w)/w}{(\eta y - \pi)/\pi} = \frac{\alpha_{sf}}{\alpha_{vf}} \tag{50}$$

The marginal cost of public funds equals

$$\mu = \frac{1}{1 - \left[\left(\frac{\lambda}{1+\lambda}\right)\varepsilon_v \hat{\tau}_\pi + \left(\frac{1}{1+\lambda}\right)\varepsilon_s \hat{\tau}_w\right]} \tag{51}$$

An increase in the government revenue requirement while maintaining an optimal tax system yields the following comparative statics

$$\operatorname{sign}\left(\frac{d\theta}{dg}\right) = \operatorname{sign}\left(g\left(\frac{1}{(1-\alpha_{vf})\sigma_{cf}} - \frac{1}{(1-\alpha_{sf})\sigma_{\gamma f}}\right)\right) \tag{52}$$

$$\operatorname{sign}\left(\frac{d\lambda}{dg}\right) = -\operatorname{sign}\left(\frac{d\theta}{dg}\right) \tag{53}$$

We interpret in turn the optimal tax structure (50), the marginal cost of public funds (51), and the comparative statics for tightness (52).

4.2.1 Optimal tax structure

In order to interpret the optimal tax rates, we use the following result, which follows immediately from the theorem by multiplying (50) by $\frac{w}{\pi}$ and using (45).

Corollary 20

$$\lambda = \frac{\hat{\tau}_{\pi}\pi}{\hat{\tau}_w w} \tag{54}$$

$$\hat{\tau}_w w = \frac{1}{1+\lambda}\hat{g} = \frac{\alpha_{sf} w}{\alpha_{sf} w + \alpha_{vf}\pi}\hat{g} \tag{55}$$

$$\hat{\tau}_{\pi}\pi = \frac{\lambda}{1+\lambda}\hat{g} = \frac{\alpha_{vf}\pi}{\alpha_{sf} w + \alpha_{vf}\pi}\hat{g} \tag{56}$$

where $\hat{g} \equiv g(1+\lambda)/sm(\theta) = \hat{\tau}_w w + \hat{\tau}_p \pi$: Expression (54) shows that the ratio of overall tax revenue (i.e. the sum of explicit and implicit revenues) collected from the demand side and overall tax revenue collected from the supply side (i.e. the right-hand side of (54)) should equal the ratio of the number of agents at the demand side to the number of agents at the supply side (i.e. the left-hand side of (54)). Hence, tax revenues collected from each side of the market are proportional to the number of agents active at those sides. Tax revenue collected per person is thus the same. By taxing both sides of the market equally, the tax system does not distort the entrepreneurial-worker choice. Put differently, with arbitrage ensuring that all agents in general equilibrium share equally in the tax burden, it is most efficient to tax the agents directly by collecting most tax revenue from the side of the market with the larger number of agents rather than indirectly through tax shifting and the associated behavioural, distortionary effects on the decision to become a worker or an entrepreneur.

With a zero government financing requirement, the optimal overall tax rates are zero ($\hat{\tau}_{\pi} = \hat{\tau}_w = 0$ so that $\pi = \eta y$ and $w = (1-\eta)y$). Substituting this result into (45), we find that the private split λ coincides with the socially optimal split in (47), $\lambda = \frac{\eta\alpha_v}{(1-\eta)\alpha_s}$. Accordingly, the laissez fair outcome ($\tau_{\pi} = 0$ and $\tau_w = 0$) coincides with the social optimum if the Hosios condition is satisfied (which implies that $\hat{\tau}_{\pi} = \tau_{\pi}$ and $\hat{\tau}_w = \tau_w$). Intuitively, the only missing market in the model is that for search activity. The investments in abilities are not specific to the match and thus are priced appropriately if the Hosios condition is met. This result is reminiscent of a result in Mortensen and Pissarides (1999) where the Hosios condition ensures both optimal search intensities by both sides of the market and an optimal reservation productivity at which to dissolve a match.

If Hosios is not satisfied, the missing market for search distorts not only search intensity (i.e. the intensive margin) but also the split between workers and entrepreneurs (i.e. the extensive margin). Just as in the previous sections, the government can employ tax policy to ensure that the private outcome is efficient. Ex ante, all agents agree on the tax policy implied by (18) and (19), which yields a balanced budget.

Just as in the previous sections, we can express the optimal tax system with positive government spending in terms of a linear wage tax by employing (3) and (4) to eliminate w and π from the second equality in (50).

Corollary 21

$$\alpha_{vf}\hat{\tau} = \tau_a(\alpha_{sf} - \alpha_{vf}) \tag{57}$$

The result from section 2 (in which there are no rents at the demand side, because $\alpha_{vf} = 0$ so that $\tau_a = 0$) that only the ad valorum tax is employed to finance government spending is reversed if the rent shares α_{vf} and α_{sf} coincide. In that case, only the specific tax component is used for financing spending (i.e. $\hat{\tau} = 0$).

The optimal tax structure thus has a similar form as in section 3 (compare (50) with (38) and (57) with (40)). However, whereas without endogenous worker-entrepreneur choice (section 3), the second derivatives of the search costs ($\gamma''(s)$ and $c''(v)$ which are implicit in the elasticities ε_s and ε_v) are relevant, only the first derivatives of these costs ($\gamma'(s)$ and $c'(v)$ which are implicit in the shares α_{sf} and α_{vf}) determine the optimal tax structure with an endogenous worker–entrepreneur choice. Intuitively, with arbitrage between the two sides of the market, the tax structure cannot separately affect the rewards to search intensity at both sides of the market. The search intensities at both sides of the market s and v are affected only by the overall tax burden. The tax structure thus does not impact labour-market tightness through its effect on the relative search intensity s/v. As a direct consequence, the tax structure affects labour-market tightness only through the worker-entrepreneur choice, which is affected by the rent shares rather than the elasticities.

4.2.2 marginal cost of public funds

The elasticities ε_s and ε_v determine the distortionary impact of the overall tax burden on search at both sides of the market. In case the search cost functions at both sides of the markets are identical (i.e. $\alpha_{sf} = \alpha_{vf}$ and $\varepsilon \equiv \varepsilon_s = \varepsilon_v$); the marginal costs of public funds can be written as

$$\mu = \frac{1}{(1 - \varepsilon\bar{\tau})} \tag{58}$$

where $\bar{\tau} \equiv \frac{\tau_w w + \tau_\pi \pi}{w+\pi}$. This expression coincides with the corresponding expression (39) for the case without an endogenous worker-entrepreneur choice.[23]

In order to further increase our intuition for expression (58), we consider the symmetric case where $\sigma \equiv \sigma_{\gamma f} = \sigma_{cf}$ and $\alpha_f \equiv \alpha_{sf} = \alpha_{vf}$. Substituting the expression for ε_v in Lemma 16 into (58), we can write μ as

$$\mu = \frac{1}{(1 - [(\frac{1-\alpha_f}{\alpha_f})\sigma]\bar{\tau})}$$

Just as in section 3, the marginal costs of public funds thus rises with both the average tax burden $\bar{\tau}$ and the substitution elasticity in search σ while it falls with the rent share α_f.

In a model with asymmetric demand and supply, however, raising government spending tends to be more costly in the model of section 4 in which revenue raising can distort the worker-entrepreneur decision. To illustrate, in contrast to

[23] The reason is that in a symmetric model arbitrage does not add anything because the split between workers and entrepreneurs is constant (see below).

the model of section 3, a unitary rent share at only one side of the market (either $\alpha_{sf} = 1$ so that $\varepsilon_s = 0$ or $\alpha_{vf} = 1$ so that $\varepsilon_v = 0$) is not sufficient for a unitary marginal cost of public funds. Intuitively, the aggregate supply of talents in the economy is fixed rather than the supply of talents at each side of the market. Without an endogenous worker-entrepreneur choice, a tax on the side of the market with a unitary rent share is a lump-sum tax (since abilities at this side are fixed). With an endogenous worker-entrepreneur choice, in contrast, this tax is no longer a lump-sum tax because it distorts the worker-entrepreneur choice (as long as the rent share at the other side of the market is positive so that abilities are also allocated to that side of the market). Indeed, the government is less able to exploit the inelastic side of the market (the side of the market with a large rent share) in a model with endogenous worker-entrepreneur choice than without such a choice.

4.2.3 labour-market tightness

To obtain more intuition for the comparative static results of more public spending on labour-market tightness (52) and (53), we consider two special cases.

Cobb Douglas production function In case of a Cobb Douglas production function for search intensities (i.e. $\sigma_{\gamma f} = \sigma_{cf} = 1$); the rent shares α_{sf} and α_{vf} are constant. Then it follows immediately from (52) that in this case the impact of a higher revenue requirement on labour-market tightness is proportional to the term $g(\alpha_{vf} - \alpha_{sf})$. Increasing a positive initial revenue requirement thus strengthens the side of the market on which the highest overall tax rate is levied. The intuition is that the government protects its tax base; the government distorts labour-market tightness for the purpose of raising revenues.

To illustrate, if most rents are at the supply side ($\alpha_{sf} > \alpha_{vf}$); the government relies mainly on wage taxation. As a direct consequence, the government has a direct interest in raising wage income compared to profit income in order to strengthen the tax base. In order to induce the bargaining partners to increase wage income at the expense of profit income, the government increases the profit tax. This discourages agents from becoming entrepreneurs, thereby raising labour supply compared to labour demand and making the labour market less tight.

In case of a Cobb Douglas production function, we can substitute the expressions for ε_s and ε_v in Lemma 16 with $\sigma_{\gamma f} = \sigma_{cf} = 1$ into the optimal tax structure without arbitrage between the demand and the supply side (38) to arrive at

$$\frac{\hat{\tau}_w}{\hat{\tau}_\pi} = \frac{\alpha_{sf}/(1 - \alpha_{sf})}{\alpha_{vf}/(1 - \alpha_{vf})} \tag{59}$$

Comparing this expression with the optimal rule with arbitrage (50), we observe that differences in rent shares have a larger impact on the optimal tax structure in the case without arbitrage. The reason is that without arbitrage, tax policy can exploit the fixed supply of abilities at each side of the market; agents cannot escape the heavy tax burden by moving to the other side of the labour market. With arbitrage, in contrast, taxing the inelastic side of the market relatively heavily induces agents to move to the other side of the market. To illustrate, if the rent share at the supply side α_{sf} approaches unity, the government finds it optimal to only rely on wage taxation in a model without arbitrage (see (59) with $\alpha_{sf} = 1$). With arbitrage, in contrast, the wage tax is no longer non distortionary because it affects the distribution of the population between the supply and demand sides of the labour market. Hence, the optimal tax structure trades off the distortions from the wage tax against the distortions from the profit tax.

Symmetric case The second special case we consider is the symmetric case. If both sides of the market feature the same cost function, a change in the overall tax burden leaves the worker-entrepreneur-split and labour-market tightness unaffected. A case that yields similar results is the case in which a large substitution elasticity $\sigma_{.f}$ compensates for a large rent share $\alpha_{.f}$ such that $\sigma_{\circ f}(1 - \alpha_{sf}) = \sigma_{cf}(1 - \alpha_{vf})$ so that $\varepsilon_s \alpha_{sf} = \varepsilon_v \alpha_{vf}$. It follows from (52) and (53) that under these conditions, a larger tax burden does not impact labour-market tightness [24] nor the worker-entrepreneur choice (and thus neither the relative search intensities v/s). With the worker-entrepreneurship split not being affected, the model in section 3 (in which the split is exogenously fixed) coincides with the model in section 4 (in which the fixed split is an endogenous outcome).[25] Indeed, in this case, the expressions for the optimal tax structure ((38) and (50)) and the marginal costs of public funds ((39) and (51)) coincide in both models.

In this case the share of government spending financed through wage taxation, $\frac{\alpha_{sf} w}{\alpha_{sf} w + \alpha_{vf} \pi}$, remains constant as the overall tax burden increases. Intuitively, the tax base does not erode as a consequence of a higher tax burden because a higher tax burden expands the before-tax rent share (with a substitution elasticity larger than unity, a lower activity level increases the rent share). Hence, the government does not have to distort labour-market tightness to protect the tax base.

5 Conclusions

Table 1 summarizes the results of this paper in case of a Cobb-Douglas formulation of search intensities. In a model with linear vacancy costs, all ex-ante rents accrue to the supply side of the labour market. Hence, after correcting for the Hosios condition, the government optimally taxes only the supply side to finance her expenditure. Furthermore, since it cannot redistribute rents between the two sides of the market, the optimal tax system does not distort labour-market tightness.

These results are not robust to the introduction of convex vacancy costs. If both the number of workers and the number of entrepreneurs are fixed, the optimal tax structure features a Ramsey rule: taxes on labour demand should be high compared to taxes on labour supply if labour demand is inelastic relative to labour supply. If labour demand and labour supply elasticities differ, the

[24] Under the restriction $\varepsilon_{.} \alpha_{.f} = 1$, the elasticity $\varepsilon_{.}$ is not constant. This violates the assumption we used in section 3 to determine the impact of a higher tax burden on labour-market tightness.

[25] Whereas the restriction $\varepsilon_{.} \alpha_{.f} = 1$ and/or symmetry ensure that the model in section 4 boils down to that in section 3, the case with zero rents at the demand side (i.e. $\alpha_{vf} = 0$) implies that the model in section 4 coincides with that in section 2.

optimal tax system distorts labour-market tightness in order to protect the tax base by redistributing rents to the inelastic side of the market.

If agents can arbitrage between the demand and supply sides of the labour-market, the government optimally sets taxes proportional to rent shares. Taxes on labour demand should be high compared to taxes on labour supply if an agent's ¯xed ability is relatively more important in generating search intensity at the demand side than at the supply side of the market. Compared to the case with a ¯xed worker-entrepreneur split, optimal tax rates are less sensitive to di®erences in rent shares because agents can escape taxes not only by reducing their search intensity but also by migrating to the other side of the market. As a direct consequence, behaviour (and hence labour-market tightness) is more elastic with respect to di®erential tax rates on both sides of the market. To illustrate, if search intensity at the supply side is completely inelastic, the government collects all tax revenue from the supply side if agents cannot arbitrage. If arbitrage is feasible, however, it taxes also the demand side in order to prevent too many workers from becoming entrepreneurs.

With long-term arbitrage, the optimal tax system may want to distort labour-market tightness to protect the tax base and thus redistribute resources from the private to the public sector but not for the purpose of redistributing resources within the private sector. Indeed, just as in the case in which one side of the market is perfectly elastic (as in section 2), redistribution is impossible if agents can arbitrage between both sides of the market. Distribution considerations play a role only if search cost functions at both sides of the market are convex and if agents cannot arbitrage between being a worker and being an entrepreneur. If these conditions are met, labour-market tightness is distorted even without a positive government ¯nancing requirement. In particular, the labour market is monopolized if the government favors workers and monopsonized if entrepreneurs are favored.

6 References

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7 Proof of the results

Proof of lemma 2

We linearize the following equations characterizing the decentralized equilibrium

$$\gamma'(s) = m(\theta)\left(y - \pi - \frac{g}{sm(\theta)}\right) \tag{60}$$

$$\frac{c\theta}{m(\theta)} = \pi \tag{61}$$

To derive the first equation, we have substituted the government budget constraint $sm(\theta)y = sm(\theta)[w + \pi] + g$ to eliminate the after-tax wage w from (5) The second equation follows from (4) and (7).

Loglinearization yields

$$\begin{pmatrix} \hat{s} \\ \hat{\theta} \end{pmatrix} = \frac{1}{\frac{c\theta(1-\eta)}{m(\theta)}\left[s\gamma''(s) - \frac{g}{s}\right]} \begin{pmatrix} \frac{c\theta(1-\eta)}{m(\theta)} & m(\theta)\eta(y-\pi) \\ 0 & s\gamma''(s) - \frac{g}{s} \end{pmatrix} \begin{pmatrix} -m(\theta)\pi\hat{\pi} - \frac{g}{s}\hat{g} \\ \pi\hat{\pi} \end{pmatrix} \tag{62}$$

Consider the case with $g = 0$. Then $\frac{d\theta}{d\pi} > 0$ or equivalently (since $\pi = (1-\beta)y$)

$\frac{d\theta}{d} < 0$. Furthermore (the second equality follows from (61)),

$$
\begin{aligned}
\frac{\partial s}{\partial \pi} &= \frac{\pi m(\theta)}{\frac{c\theta(1-\eta)}{m(\theta)} s\gamma''(s)} \left[-\frac{c\theta}{m(\theta)}(1-\eta) + \eta(y-\pi) \right] \\
&= \frac{\pi m(\theta)}{\frac{c\theta(1-\eta)}{m(\theta)} s\gamma''(s)} \left[-\pi + \eta y \right] \\
&= \begin{cases} \geq 0 & \text{if } \pi \leq \eta y \\ < 0 & \text{if } \pi > \eta y \end{cases} \qquad (63)
\end{aligned}
$$

Hence s is maximized if $\pi = \eta y$ or equivalently (since $\pi = (1-\beta)y)$ $\beta = 1-\eta$. Since $\gamma'(s)s - \gamma(s)$ is increasing in s, $\beta = 1-\eta$ maximizes also $s\gamma'(s) - \gamma(s)$. The following equalities show that $\beta = 1-\eta$ maximizes in addition welfare W and workers' ex ante welfare $sm(\theta)w - \gamma(s)$.

$$
\begin{aligned}
W &= sm(\theta)y - \gamma(s) - c\theta s = sm(\theta)[w+\pi] - \gamma(s) - c\theta s \\
&= sm(\theta)w - \gamma(s) \\
&= s\gamma'(s) - \gamma(s)
\end{aligned}
$$

The second equality follows from the fact that (with $g = 0$) output is distributed between workers and firms (i.e. $y = w + \pi$); the third equality from (61) and the last equality from (5).QED

Proof of lemma 3

The optimal values for τ and τ_a follow from a simple comparison of the private outcome (i.e. (6) and (7)) and the social optimum (i.e. (8) and (9)). Substituting (3) into (2) and using the expressions for the optimal tax rates (11) and (12) to eliminate the tax rates from the resulting equation, we find that the optimal tax rates yield zero public revenues. QED

Proof of lemma 5

Substituting the government budget constraint, $sm(\theta)y = sm(\theta)[w+\pi] + g$ into the expression for W in (1), we find

$$
\begin{aligned}
W &= sm(\theta)[w+\pi] + g - \gamma(s) - c\theta s \\
&= s\gamma'(s) - \gamma(s) + g
\end{aligned}
$$

The second equality follows from (5).QED

Proof of proposition 6

Using (5) to eliminate the after-tax wage w from (20), we find

$$
W = s\gamma'(s) - \gamma(s) + g \qquad (64)
$$

Since g is exogenously given and $s\gamma'(s) - \gamma(s)$ is rising in s, maximizing welfare is equivalent to maximizing search s. Thus equation (63) implies that $\pi = \eta y$ or equivalently $\hat{\tau}_\pi = 0$. The government budget constraint, $\hat{g} = \hat{\tau}_w w + \hat{\tau}_\pi \pi$ then implies

$$
\hat{\tau}_w w = \hat{g}
$$

QED

Proof of lemma 8

The private outcome is determined (5) and (7) where w and π are determined by τ_w and τ_π as in (14) and (15). We will determine the optimal incidence w and π directly. The corresponding tax rates then follow from (14) and (15).

Linearizing (5) and (7) with respect to s, θ and w yields

$$\begin{aligned} \frac{ds}{dw} &= \frac{s}{w}\varepsilon_s \\ \frac{d\theta}{dw} &= 0 \end{aligned}$$

Then the first order condition for maximizing welfare (using Lemma 5)

$$s\gamma'(s) - \gamma(s) + g + \lambda\left[sm(\theta)(y - w - \pi) - g\right]$$

with respect to w yields

$$s\gamma''(s)\frac{ds}{dw} + \lambda\left(m(\theta)(y - w - \pi)\frac{ds}{dw} - sm(\theta)\right) = 0$$

This can be written as

$$1 = \lambda\left(1 - \varepsilon_s\frac{\hat{g}}{w}\right)$$

Noting that $\hat{\tau} = \frac{\tau_w w + \tau_\pi \pi}{w} = \frac{y - w - \pi}{w} = \frac{\hat{g}}{w}$, this yields equation (31).

The expressions for $g^m, \hat{g}^m$ and $\hat{\tau}^m$ can be found in three equivalent ways:

(i) Solve

$$\begin{aligned} &\max_{w,\pi} sm(\theta)(y - w - \pi) = sm(\theta)\hat{g} \\ &\text{subject to (60) and (61)} \end{aligned}$$

where $\hat{g} \equiv g/sm(\theta)$: Again we determine the optimal incidence π and w rather than optimizing with respect to τ_π and τ_w.

Log-linearizing (60) and (61) with respect to s, θ, π and $\hat{g} \equiv \frac{g}{sm(\theta)}$, we arrive at

$$\begin{aligned} \frac{ds}{d\pi} &= \frac{-c(1-\eta) + m'(\theta)(y - \pi - \hat{g})}{\gamma''(s)(1-\eta)\frac{c}{m(\theta)}} \\ \frac{ds}{d\hat{g}} &= -\frac{m(\theta)}{\gamma''(s)} \\ \frac{d\theta}{d\pi} &= \frac{1}{(1-\eta)\frac{c}{m(\theta)}} \\ \frac{d\theta}{d\hat{g}} &= 0 \end{aligned}$$

Substituting these expressions into the first order condition for $\hat{g}$

$$m(\theta)\left(\frac{ds}{d\hat{g}}\hat{g} + s\right) = 0$$

we establish

$$-\frac{\hat{g}m(\theta)}{\gamma''(s)} + s = 0$$

This can be written as (with the aid of $\varepsilon_s \equiv \left(\frac{s\gamma''(s)}{\gamma'(s)}\right)^{-1}$;(5), and $w = y - \pi - \hat{g}$)

$$(1 + \varepsilon_s)\hat{g} = y - \pi \tag{65}$$

The first-order condition for the optimization problem with respect to π amounts to

$$\hat{g}\left[m(\theta)\frac{ds}{d\pi} + sm'(\theta)\frac{d\theta}{d\pi}\right] = 0$$

Using the comparative static results and (65), we obtain

$$m(\theta)\left[-c(1-\eta) + m'(\theta)\varepsilon_s\hat{g}\right] + m'(\theta)s\gamma''(s) = 0$$

Using $\frac{c\theta}{m(\theta)} = \pi = y - (1+\varepsilon_s)\hat{g}$ to eliminate c; $\varepsilon_s \equiv \left(\frac{s\gamma''(s)}{\gamma'(s)}\right)^{-1}$;(5), and $w = y - \pi - \hat{g}$, we arrive at

$$\hat{g}^m = \frac{1-\eta}{1+\varepsilon_s}y$$

Hence

$$\pi = y - (1+\varepsilon_s)\hat{g} = \eta y \tag{66}$$

Consequently,

$$\begin{aligned} w &= y - \pi - \hat{g}^m \\ &= (1-\eta)y - \frac{1-\eta}{1+\varepsilon_s}y \\ &= (1-\eta)y\frac{\varepsilon_s}{1+\varepsilon_s} \end{aligned}$$

The corresponding average tax burden is

$$\tau^m = \frac{\tau_w + \tau_a}{w} = \frac{\hat{g}^m}{w} = \frac{1}{\varepsilon_s}$$

(ii) $\tau^m = \frac{1}{\varepsilon_s}$ can be derived directly from the expression (31) for μ, since

$$\lim_{\tau \to \frac{1}{\varepsilon_s}} \mu \to +\infty$$

(iii) Consider the determinant corresponding to the matrix in (62), that is $\frac{c\theta}{m(\theta)}(1-\eta)\left[s\gamma''(s) - \frac{g}{s}\right]$. Then we find that this determinant equals zero (and hence the matrix cannot be inverted) if

$$\gamma'(s)\frac{1}{\varepsilon_s} - \frac{g}{s} = 0$$

which, using (60) and $\hat{g} = \frac{g}{sm(\theta)}$, can be written as

$$m(\theta)(y - \pi - \hat{g})\frac{1}{\varepsilon_s} - m(\theta)\hat{g} = 0$$

With optimal incidence (66), we get

$$\hat{g}^m = \frac{1-\eta}{1+\varepsilon_s}y$$

QED

Proof of lemma 10

Output accrues to workers, entrepreneurs and the government

$$sm(\theta)y = sm(\theta)(w+\pi) + g \tag{67}$$

We can thus write welfare (34) as

$$sm(\theta)\left[w+\pi\right] + g - \gamma(s) - c(v)$$

or equivalently

$$\left[sm(\theta)w - \gamma(s)\right] + \left[sm(\theta)\pi - c(v)\right] + g$$

With distributional considerations (parametrized by α), we obtain

$$\begin{aligned} W_\alpha &= \alpha\left[sm(\theta)w - \gamma(s)\right] + \left[sm(\theta)\pi - c(v)\right] + g \\ &= \alpha\left[s\gamma'(s) - \gamma(s)\right] + \left[vc'(v) - c(v)\right] + g \end{aligned}$$

The second equality follows from the first-order conditions for search intensities at both sides of the market (i.e. (6) and (33)).QED

Proof of theorem 11

We loglinearize the following two equations characterizing the decentralized equilibrium with respect to s, v and π,

$$\begin{aligned} \gamma'(s) &= m\left(\frac{v}{s}\right)(y-\pi) - \frac{g}{s} \\ c'(v) &= \frac{m\left(\frac{v}{s}\right)}{v/s}\pi \end{aligned}$$

The first equation is found by eliminating w from (5) by using (67). The second equation follows from (33)

Loglinearization of these two equations yields

$$\begin{pmatrix} \frac{ds}{s} \\ \frac{dv}{v} \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} \pi\left(\frac{1}{\varepsilon_v} + 1 - \eta\right) & \eta(y-\pi) \\ (1-\eta)\pi & \frac{w}{\varepsilon_s} + \eta(y-\pi) - \hat{g} \end{pmatrix}\begin{pmatrix} -d\pi \\ d\pi \end{pmatrix}$$

where $\hat{g} \equiv \frac{g}{sm(\theta)}$ and Δ is defined as

$$\Delta \equiv \pi\left(\frac{1}{\varepsilon_v} + 1 - \eta\right)\left(\frac{w}{\varepsilon_s} + \eta(y-\pi) - \hat{g}\right) - (1-\eta)\eta\pi(y-\pi)$$

Hence we find

$$\begin{aligned} \frac{1}{s}\frac{ds}{d\pi} &= \frac{-\frac{\pi}{\varepsilon_v} - \pi + \eta y}{\Delta} \\ \frac{1}{v}\frac{dv}{d\pi} &= \frac{-\pi + \frac{w}{\varepsilon_s} + \eta y - \hat{g}}{\Delta} = \frac{\frac{w}{\varepsilon_s} + w - (1-\eta)y}{\Delta} \end{aligned}$$

Maximizing welfare

$$\alpha[s\gamma'(s) - \gamma(s)] + vc'(v) - c(v) + g$$

with respect to profits π, we arrive at the following first-order condition

$$\alpha s\gamma''(s)\frac{ds}{d\pi} + vc''(v)\frac{dv}{d\pi} = 0$$

Substituting the comparative static results and using the definitions of ε_s and ε_v and (5) and (33), we can rewrite this first-order condition as

$$\frac{\alpha}{\varepsilon_s}\frac{\eta y - \pi\frac{\varepsilon_v+1}{\varepsilon_v}}{\pi} + \frac{1}{\varepsilon_v}\frac{-(1-\eta)y + w\frac{1+\varepsilon_s}{\varepsilon_s}}{w} = 0$$

or equivalently

$$\frac{\eta y - \pi}{\pi}\varepsilon_v - 1 = \frac{\varepsilon_s}{\alpha}\frac{(1-\eta)y - w}{w} - \frac{1}{\alpha}$$

Using the definitions of the overall tax rates $\tau_\pi \equiv \frac{\eta y - \pi}{\pi}$ and $\tau_w \equiv \frac{(1-\eta)y - w}{w}$, we can write this equation as

$$-\alpha\varepsilon_v\tau_\pi + \varepsilon_s\tau_w = 1 - \alpha$$

With the government budget constraint

$$\tau_\pi\pi + \tau_w w = \hat{g}$$

we have two linear equations in τ_π and τ_w. Solving this system, we find

$$\begin{pmatrix}\tau_\pi\\ \tau_w\end{pmatrix} = \frac{1}{\varepsilon_s\pi + \alpha\varepsilon_v w}\begin{pmatrix}\varepsilon_s & -w\\ \alpha\varepsilon_v & \pi\end{pmatrix}\begin{pmatrix}\hat{g}\\ 1-\alpha\end{pmatrix}$$

The solution yields the expressions for ζ and ξ in the theorem.

In order to determine μ, we loglinearize the first-order conditions for s and v from (5) and (33) with respect to w, π, s and v.

$$\gamma'(s) = m\left(\frac{v}{s}\right)w \tag{68}$$

$$c'(v) = \frac{m(v/s)}{v/s}\pi \tag{69}$$

This yields

$$\begin{pmatrix}\hat{s}\\ \hat{v}\end{pmatrix} = \frac{1}{\Delta^*}\begin{pmatrix}\frac{1}{\varepsilon_v} + 1 - \eta & \eta\\ 1-\eta & \frac{1}{\varepsilon_s} + \eta\end{pmatrix}\begin{pmatrix}\hat{w}\\ \hat{\pi}\end{pmatrix}$$

where $\Delta^* \equiv \varepsilon_v^{-1}\varepsilon_s^{-1} + \eta\varepsilon_v^{-1} + (1-\eta)\varepsilon_s^{-1} > 0$; $\hat{x} = \frac{dx}{x}$ for $x = s, v, w, \pi$. It follows that

$$\Delta^*\hat{s} = (\varepsilon_v^{-1} + 1 - \eta)\hat{w} + \eta\hat{\pi} \tag{70}$$

$$\Delta^*\tilde{v} = (1 - \eta)\tilde{w} + (\varepsilon_s^{-1} + \eta)\tilde{\pi} \tag{71}$$

so that

$$\Delta^*\tilde{\theta} = \Delta^*(\tilde{v} - \tilde{s}) = -\varepsilon_v^{-1}\tilde{w} + \varepsilon_s^{-1}\tilde{\pi} \tag{72}$$

The marginal cost of public funds μ is the Lagrange multiplier of the government budget constraint in the following optimization program

$$\max_{w;\pi} \alpha\,[s\gamma'(s) - \gamma(s)] + [vc'(v) - c(v)] + g + \mu\,[sm(\theta)\,(y - w - \pi) - g]$$

The first order condition for this optimization program with respect to w can be written as

$$\alpha s\frac{\gamma'(s)}{w\varepsilon_s} + \mu m(\theta)\cdot\frac{s}{w}\Big]\,\frac{\tilde{s}}{\tilde{w}} + v\frac{c'(v)}{w\varepsilon_v}\frac{\tilde{v}}{\tilde{w}} + \mu\eta g\frac{1}{w}\frac{\tilde{\theta}}{\tilde{w}} = \mu sm(\theta)$$

We substitute the comparative static results (70), (71), and (72), multiply the resulting equation by $(\Delta^* w)/(sm(\theta))$ and use the definition of Δ^*. Furthermore, we employ (5) and (33) to eliminate $\gamma'(s)$ and $c'(v)$; respectively. This yields

$$\begin{aligned} &\alpha\frac{w}{\varepsilon_s\varepsilon_v} + (1 - \eta)\left(\alpha\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right) \\ &= \mu\left(\frac{w}{\varepsilon_s\varepsilon_v} + (1 - \eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right) - \frac{(1 - \eta)\pi}{\varepsilon_v} + \frac{\eta w}{\varepsilon_v} - (1 - \eta)\cdot\left(1 + \frac{1}{\varepsilon_v}\right)\right), \end{aligned}$$

This can also be written as

$$\frac{\frac{\alpha w}{\varepsilon_s\varepsilon_v} + (1 - \eta)\Big(\frac{\alpha w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\Big)}{\frac{w}{\varepsilon_s\varepsilon_v} + (1 - \eta)\Big(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\Big)} = \mu\left[1 - \underbrace{\frac{(1 - \eta)\cdot\Big(1 + \frac{1}{\varepsilon_v}\Big) + \frac{(1-\eta)\pi}{\varepsilon_v} - \frac{\eta w}{\varepsilon_v}}{\frac{w}{\varepsilon_s\varepsilon_v} + (1 - \eta)\Big(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\Big)}}_{(*)}\right] \tag{73}$$

To arrive at the expression for μ in the theorem, we rewrite the expression labelled (*). To do so, we employ the expression for $\hat{\tau}_w w = (1 - \eta)y - w$ derived in the first part of the theorem:

$$\begin{aligned} (1 - \eta)y - w &= \frac{w\varepsilon_v}{w\varepsilon_v + \pi\varepsilon_s}\cdot + \left[\frac{\alpha w\varepsilon_v}{\alpha w\varepsilon_v + \pi\varepsilon_s} - \frac{w\varepsilon_v}{w\varepsilon_v + \pi\varepsilon_s}\right]\cdot + \frac{(1 - \alpha)\,w\pi}{\varepsilon_s\pi + \alpha w\varepsilon_v} \\ &= \frac{w\varepsilon_v}{w\varepsilon_v + \pi\varepsilon_s}\cdot + \left[\frac{(\alpha - 1)\,w\varepsilon_v\pi\varepsilon_s}{(\alpha w\varepsilon_v + \pi\varepsilon_s)\,(w\varepsilon_v + \pi\varepsilon_s)}\right]\cdot + \frac{(1 - \alpha)\,w\pi}{\varepsilon_s\pi + \alpha w\varepsilon_v} \\ &= \frac{w\varepsilon_v}{w\varepsilon_v + \pi\varepsilon_s}\cdot + \frac{(1 - \alpha)\,w\pi}{\varepsilon_s\pi + \alpha w\varepsilon_v}\left(1 - \cdot\frac{1}{\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}}\right) \end{aligned}$$

Substitution in expression ($\star$) yields

$$
\begin{aligned}
(\star) &= \frac{(1-\eta)g + \frac{1}{\varepsilon_v}\left((1-\eta)y - w\right)}{\frac{w}{\varepsilon_s\varepsilon_v} + (1-\eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)} \\
&= \frac{(1-\eta)g + \frac{1}{\varepsilon_v}\frac{w\varepsilon_v}{w\varepsilon_v+\pi\varepsilon_s}g + \frac{1}{\varepsilon_v}\left(\frac{(1-\alpha)w\pi}{\varepsilon_s\pi+\alpha w\varepsilon_v}\right)\left[1 - g\frac{1}{\frac{w}{\varepsilon_s}+\frac{\pi}{\varepsilon_v}}\right]}{\frac{w}{\varepsilon_s\varepsilon_v} + (1-\eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)} \\
&= g\left\{\frac{(1-\eta) + \frac{1}{\varepsilon_v}\left(\frac{w\varepsilon_v}{w\varepsilon_v+\pi\varepsilon_s}\right)}{\frac{w}{\varepsilon_s\varepsilon_v} + (1-\eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)}\right\} + \frac{\frac{1}{\varepsilon_v}\frac{(1-\alpha)w\pi}{\varepsilon_s\pi+\alpha w\varepsilon_v}\left[1 - g\left(\frac{1}{\frac{w}{\varepsilon_s}+\frac{\pi}{\varepsilon_v}}\right)\right]}{\frac{w}{\varepsilon_s\varepsilon_v} + (1-\eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)} \\
&= \frac{g}{\frac{w}{\varepsilon_s}+\frac{\pi}{\varepsilon_v}} + \left(\frac{\frac{1}{\varepsilon_v}\frac{(1-\alpha)w\pi}{\varepsilon_s\pi+\alpha w\varepsilon_v}}{\frac{w}{\varepsilon_s\varepsilon_v} + (1-\eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)}\right)\left[1 - g\frac{1}{\frac{w}{\varepsilon_s}+\frac{\pi}{\varepsilon_v}}\right]
\end{aligned}
$$

Substituting this expression for ($\star$) back into (73), we arrive at

$$
\frac{\frac{\alpha w}{\varepsilon_s\varepsilon_v} + (1-\eta)\left(\frac{\alpha w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)}{\frac{w}{\varepsilon_s\varepsilon_v} + (1-\eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)} = \mu\left[1 - g\frac{1}{\frac{w}{\varepsilon_s}+\frac{\pi}{\varepsilon_v}}\right]\left\{1 + \frac{\frac{1}{\varepsilon_v}\left(\frac{(1-\alpha)w\pi}{\varepsilon_s\pi+\alpha w\varepsilon_v}\right)}{\frac{w}{\varepsilon_s\varepsilon_v} + (1-\eta)\left(\frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v}\right)}\right\}
$$

which yields the expression for μ in the theorem.

QED

Proof of Corollary 12

It follows immediately from (36) and (37) that (with $\alpha = 1$)

$$
\begin{aligned}
\hat{\tau}_w &= \frac{\varepsilon_v}{w\varepsilon_v + \pi\varepsilon_s}g \\
\hat{\tau}_\pi &= \frac{\varepsilon_s}{w\varepsilon_v + \pi\varepsilon_s}g
\end{aligned}
$$

This immediately yields the Ramsey rule $\frac{\hat{\tau}_w}{\hat{\tau}_\pi} = \frac{\varepsilon_v}{\varepsilon_s}$.

The expression for μ follows immediately from substituting $\alpha = 1$ in the expression for μ in the theorem. The marginal cost of public funds becomes infinite if

$$
\begin{aligned}
g^m &= \frac{w}{\varepsilon_s} + \frac{\pi}{\varepsilon_v} \\
&= \frac{\hat{\tau}_w w}{\hat{\tau}_w \varepsilon_s} + \frac{\hat{\tau}_\pi \pi}{\hat{\tau}_w \varepsilon_s}
\end{aligned}
$$

where the second equality follows from the Ramsey rule. Substituting the government budget constraint, $g = \hat{\tau}_w w + \hat{\tau}_\pi \pi$; we find that $\hat{\tau}_w = \frac{1}{\varepsilon_s}$ (and thus from the Ramsey rule that $\hat{\tau}_\pi = \frac{1}{\varepsilon_v}$). Substituting these expressions for the overall tax rates back in the government budget constraint and using (from the definitions of the overall tax rates) $w = (1-\eta)y/(1+\hat{\tau}_w)$ and $\pi = \eta y/(1+\hat{\tau}_\pi)$; we establish

$$
\begin{aligned}
\frac{g^m}{y} &= \frac{\hat{\tau}_w}{1+\hat{\tau}_w}(1-\eta) + \frac{\hat{\tau}_\pi}{1+\hat{\tau}_\pi}\eta \\
&= \frac{1-\eta}{\varepsilon_s + 1} + \frac{\eta}{\varepsilon_v + 1}
\end{aligned}
$$

QED

Proof of lemma 14

Log-linearizing the Ramsey equation [(38) in the form of $\frac{\eta y - \pi}{\pi}\varepsilon_v = \varepsilon_s \frac{(1-\eta)y - w}{w}$] under the assumption that the demand and supply elasticities ε_v and ε_s are constant, we find

$$\varepsilon_s \frac{(1-\eta)y}{w}\hat{w} = \varepsilon_v \frac{\eta y}{\pi}\hat{\pi} \tag{74}$$

where $\hat{x} = \frac{dx}{x}$ for $x = w, \pi$. Using (74) to eliminate $\hat{\pi}$ from (72), we establish

$$\begin{aligned}
\hat{\theta} &= -\frac{\hat{w}}{\varepsilon_v} + \frac{1}{\varepsilon_s}\left(\frac{\varepsilon_s}{\varepsilon_v}\frac{(1-\eta)y}{\eta y}\frac{\pi}{w}\right)\hat{w} \qquad (75)\\
&= \frac{\hat{w}}{\varepsilon_v}\frac{1}{(1+\hat{\tau}_\pi)}\left(\frac{\hat{\tau}_\pi}{\hat{\tau}_w} - 1\right)\hat{\tau}_w \\
&= -\frac{\hat{w}}{\varepsilon_v}\frac{1}{(1+\hat{\tau}_\pi)}\left(\frac{\hat{\tau}_\pi}{\hat{\tau}_w} - 1\right)\frac{\frac{1}{\varepsilon_s}}{\frac{w}{\varepsilon_s}+\frac{\pi}{\varepsilon_v}}\hat{g}
\end{aligned}$$

The second equality follows by eliminating π and w from $\pi = \eta y/(1+\hat{\tau}_\pi)$ and $w = (1-\eta)/(1+\hat{\tau}_w)$: By employing the expression for $\hat{\tau}_w$ in terms of $\hat{g}$ from Corollary 12, we arrive at the last equality.

Since $\frac{dw}{dg}$ 0, we find $\text{sign}\left(\frac{d\theta}{dg}\right) = \text{sign}\left(\frac{\hat{g}}{\varepsilon_s\varepsilon_v}\left(\frac{\hat{\tau}_\pi}{\hat{\tau}_w} - 1\right)\right)$. QED

Proof of proposition 15

The proof follows from the following equivalences

$$\begin{aligned}
\left[s\gamma'(s) - \gamma(s)\right] &= \left[vc'(v) - c(v)\right] \\
s\gamma'(s)\left[1 - \left(\frac{\gamma'(s)s}{\gamma(s)}\right)^{-1}\right] &= vc'(v)\left[1 - \left(\frac{vc'(v)}{c(v)}\right)^{-1}\right] \\
sm(\theta)w\left[1 - \left(\frac{\gamma'(s)s}{\gamma(s)}\right)^{-1}\right] &= v\frac{m(\theta)}{\theta}\pi\left[1 - \left(\frac{vc'(v)}{c(v)}\right)^{-1}\right]
\end{aligned}$$

The last step follows from the first order conditions for s and v (i.e. (5) and (33), respectively). The last equation can be written as

$$\frac{s}{v}\theta = \frac{\pi\left(1 - \left(\frac{vc'(v)}{c(v)}\right)^{-1}\right)}{w\left(1 - \left(\frac{\gamma'(s)s}{\gamma(s)}\right)^{-1}\right)}$$

and the result follows since $\lambda = \frac{s}{v}\theta$ by the definition of labour-market tightness (41).QED

Proof of lemma 16

We first write the share parameter α_{vf} in terms of the function describing search costs $c(v)$: Since the function $g(.;.)$ is homogenous of degree 1, we can write

$$v = f g\left(\frac{c}{f}\right)$$

where the function $\tilde g$ is defined as $\tilde g(\cdot) \equiv \frac{1}{f} g(c; f)$. Consequently

$$\frac{dv}{dc} = \tilde g'\left(\frac{c}{f}\right)$$

and

$$\begin{aligned}
\frac{c(v)}{vc'(v)} &= \frac{c}{v\frac{dc}{dv}} = \frac{c}{\frac{f\tilde g\left(\frac{c}{f}\right)}{\tilde g'\left(\frac{c}{f}\right)}} = \frac{\frac{c}{f}\tilde g'\left(\frac{c}{f}\right)}{\tilde g\left(\frac{c}{f}\right)} \qquad (76)\\
&= \frac{\tilde g\left(\frac{c}{f}\right) - \left[\tilde g\left(\frac{c}{f}\right) - \frac{c}{f}\tilde g'\left(\frac{c}{f}\right)\right]}{\tilde g\left(\frac{c}{f}\right)}\\
&= 1 - \alpha_{vf}
\end{aligned}$$

The last equality follows from the definition of $\alpha_{vf} \equiv \frac{\partial g(c;f)}{\partial f}\frac{f}{g(c;f)}$; $\frac{\partial g(c;f)}{\partial f} = \tilde g\left(\frac{c}{f}\right) - \frac{c}{f}\tilde g'\left(\frac{c}{f}\right)$; and $\left[\tilde g\left(\frac{c}{f}\right)\right]^{-1} = \frac{f}{g(c;f)}$.

We now derive an expression for the labour-demand elasticity ε_v in terms of the substitution elasticity σ_{cf} and the share parameter α_{vf}. Ignoring f temporarily for notational convenience, we note that $\tilde g(c)$ is the inverse of the function $c(v)$. Hence we find

$$\begin{aligned}
\frac{dv}{dc} &= \tilde g'(c) = (c'(v))^{-1}\\
\frac{d^2v}{dc^2} &= \tilde g''(c) = -(c'(v))^{-2}c''(v)\frac{dv}{dc}\\
&= -(c'(v))^{-3}c''(v)
\end{aligned}$$

Thus

$$\begin{aligned}
\frac{c\tilde g''(c)}{\tilde g'(c)} &= -\frac{c\,(c'(v))^{-3}c''(v)}{(c'(v))^{-1}} \qquad (77)\\
&= -\frac{vc''(v)}{c'(v)}\frac{c}{c'(v)v}\\
&= -\frac{1-\alpha_{vf}}{\varepsilon_v}
\end{aligned}$$

The last equality follows from the definition of $\varepsilon_v \equiv \left(\frac{vc''(v)}{c'(v)}\right)^{-1}$ and (76). We substitute (77) into the definition of σ_{cf} :

$$
\begin{aligned}
-\frac{1}{\sigma_{cf}} &\equiv \frac{d\left(\frac{\frac{\partial g}{\partial c}}{\frac{\partial g}{\partial f}}\right)}{d\left(\frac{c}{f}\right)}\left(\frac{\frac{c}{f}}{\frac{\frac{\partial g}{\partial c}}{\frac{\partial g}{\partial f}}}\right) \\
&= \frac{d\left(\frac{g'\left(\frac{c}{f}\right)}{g\left(\frac{c}{f}\right)-\frac{c}{f}g'\left(\frac{c}{f}\right)}\right)}{d\left(\frac{c}{f}\right)}\left(\frac{\frac{c}{f}}{\frac{g'\left(\frac{c}{f}\right)}{g\left(\frac{c}{f}\right)-\frac{c}{f}g'\left(\frac{c}{f}\right)}}\right) \\
&= \frac{\frac{c}{f}g''\left(\frac{c}{f}\right)}{g'\left(\frac{c}{f}\right)}\frac{1}{\alpha_{vf}} \\
&= -\frac{1-\alpha_{vf}}{\varepsilon_v}\frac{1}{\alpha_{vf}}
\end{aligned}
$$

Thus we find $\varepsilon_v = \frac{1-\alpha_{vf}}{\alpha_{vf}}\sigma_{cf}$.QED

Proof of lemma 17

The first order conditions for λ; s and v for the maximization problem (46) can be written as

$$
\begin{aligned}
\frac{-s}{(1+\lambda)^2}m(\theta)y + \frac{s}{1+\lambda}m'(\theta)\frac{v}{s}y + \frac{1}{(1+\lambda)^2}\gamma(s) - \frac{1}{(1+\lambda)^2}c(v) &= 0 \\
m(\theta)y(1-\eta) - \gamma'(s) &= 0 \\
m'(\theta)y - c'(v) &= 0
\end{aligned}
$$

Substituting the last two equations to eliminate $m(\theta)$ and $m'(\theta)$, employing arbitrage (43) and using the results of Lemma 16 we arrive at

$$
\frac{v}{s}\frac{1}{\theta}\eta\alpha_{vf} = (1-\eta)\alpha_{sf}
$$

or equivalently

$$
\lambda = \frac{\eta\alpha_{vf}}{(1-\eta)\alpha_{sf}}
$$

QED

Proof of lemma 18

Welfare W (see (46)) can be written as

$$
\begin{aligned}
W &= \frac{s}{1+\lambda}m(\theta)y - \frac{1}{1+\lambda}\gamma(s) - \frac{\lambda}{1+\lambda}c(v) \\
&= \frac{s}{1+\lambda}m(\theta)(w+\pi) + g - \frac{1}{1+\lambda}\gamma(s) - \frac{\lambda}{1+\lambda}c(v) \\
&= \frac{1}{1+\lambda}\left[s\gamma'(s) - \gamma(s) + s\theta c'(v) - \lambda c(v)\right] + g \\
&= \frac{1}{1+\lambda}\left[s\gamma'(s) - \gamma(s) + \lambda(vc'(v) - c(v))\right] + g \\
&= s\gamma'(s) - \gamma(s) + g
\end{aligned}
$$

The second equality follows from the fact that total output is distributed between workers, entrepreneurs and the government: $\frac{s}{1+\lambda}m(\theta)y = \frac{s}{1+\lambda}m(\theta)[w+\pi]+g$; the third equality from the first-order conditions for search intensities at both sides of the market (see (5) and (33)); the fourth equality from the definition (41) of labour-market tightness $\theta \equiv \frac{v}{s}$; and the last equality from arbitrage (43).QED

Proof of theorem 19

We write the equilibrium in terms of the tax parameters τ_w and τ_π. Search at the supply and demand sides of the labour market are derived from, respectively, (6) and (33)

$$\gamma'(s) = m(\theta)\frac{\beta y}{1+\tau_w}$$

$$c'(v) = \frac{m(\theta)}{\theta}\frac{(1-\beta)y}{1+\tau_\pi}$$

Log-linearization of these two equations together with arbitrage (43) yields

$$\begin{pmatrix} \frac{1}{\varepsilon_s} & 0 & -\eta \\ 0 & \frac{1}{\varepsilon_v} & 1-\eta \\ \frac{1}{\alpha_{sf}\varepsilon_s} & \frac{-1}{\alpha_{vf}\varepsilon_v} & 0 \end{pmatrix}\begin{pmatrix} \hat{s} \\ \hat{v} \\ \hat{\theta} \end{pmatrix} = \begin{pmatrix} -\hat{\tau}_w \\ -\hat{\tau}_\pi \\ 0 \end{pmatrix}$$

where $\hat{\tau}_w \equiv d\tau_w/(1+\tau_w)$ and $\hat{\tau}_\pi \equiv d\tau_\pi/(1+\tau_\pi)$: The determinant of the matrix at the left hand side is given by

$$\Delta = \frac{1}{\varepsilon_s\varepsilon_v}\left(\frac{1-\eta}{\alpha_{vf}} + \frac{\eta}{\alpha_{sf}}\right)$$

Using Cramer's rule, we find

$$\hat{s} = -\frac{\hat{\tau}_w(1-\eta) + \hat{\tau}_\pi\eta}{\varepsilon_v\alpha_{vf}\Delta}$$

or equivalently,

$$\hat{s} = -\varepsilon_s\alpha_{sf}\left(\frac{(1-\eta)\hat{\tau}_w + \eta\hat{\tau}_\pi}{(1-\eta)\alpha_{sf} + \eta\alpha_{vf}}\right) \tag{78}$$

Similarly, using Cramer's rule, we find

$$\hat{v} = -\varepsilon_v\alpha_{vf}\left(\frac{(1-\eta)\hat{\tau}_w + \eta\hat{\tau}_\pi}{(1-\eta)\alpha_{sf} + \eta\alpha_{vf}}\right) \tag{79}$$

$$\hat{\theta} = \frac{\alpha_{vf}\hat{\tau}_w - \alpha_{sf}\hat{\tau}_\pi}{(1-\eta)\alpha_{sf} + \eta\alpha_{vf}} \tag{80}$$

The social planner chooses τ_w and τ_π to maximize welfare (49) subject to the government budget constraint (48):

$$\max_{\tau_w,\tau_\pi}[s\gamma'(s) - \gamma(s)] + g + \left[\frac{sm(\theta)}{1+(\theta s/v)}\left[\left(\frac{\tau_w}{1+\tau_w}\right)\beta y + \left(\frac{\tau_\pi}{1+\tau_\pi}\right)(1-\beta)y\right] - g\right]$$

where μ represents the marginal costs of public funds. Here we have used the definition of tightness (41) to eliminate λ and (14) and (15) to eliminate, respectively, w and π.

The first-order condition for optimizing welfare W with respect to the proportional wage tax τ_w amounts to (where we have used (14) to eliminate $(1+\tau_w)$)

$$sW_s(\tilde{s}/\tilde{\tau}_w) + vW_v(\tilde{v}/\tilde{\tau}_w) + \theta W_\theta(\tilde{\theta}/\tilde{\tau}_w) + \mu s m(\theta)(1+\lambda)^{-1} w = 0 \tag{81}$$

where W_s and W_v denote the first-order welfare effects of changes in search intensities of, respectively, workers and entrepreneurs

$$sW_s = \frac{sm(\theta)w}{\varepsilon_s} + \frac{\mu sm(\theta)}{1+\lambda}\hat{g}\frac{1}{1+\lambda} \tag{82}$$

$$vW_v = \frac{\mu sm(\theta)}{1+\lambda}\hat{g}\frac{\lambda}{1+\lambda} \tag{83}$$

W_θ stands for the first-order welfare effect of changes in labour-market tightness

$$\theta W_\theta = \frac{\mu sm(\theta)}{1+\lambda}\hat{g}\frac{\eta - \lambda(1-\eta)}{1+\lambda} \tag{84}$$

Substitution of (78)-(80) and (82)-(84) into (81) yields (where we have used (45) to eliminate λ)

$$w\alpha_s + \pi\alpha_v = \mu\left\{\begin{array}{l} w\alpha_s + \pi\alpha_v - \hat{g}\left[\left(\frac{\lambda}{1+\lambda}\right)\varepsilon_v\alpha_v + \left(\frac{1}{1+\lambda}\right)\varepsilon_s\alpha_s\right] \\ + \frac{\alpha_v}{1-\eta}\left[\hat{g}\frac{\eta-\lambda(1-\eta)}{1+\lambda} + w\eta - \pi(1-\eta)\right] \end{array}\right\} \tag{85}$$

We can write the first-order condition for optimizing welfare W with respect to the wage tax τ_π in the same way. This yields

$$w\alpha_s + \pi\alpha_v = \mu\left\{\begin{array}{l} w\alpha_s + \pi\alpha_v - \hat{g}\left[\left(\frac{\lambda}{1+\lambda}\right)\varepsilon_v\alpha_v + \left(\frac{1}{1+\lambda}\right)\varepsilon_s\alpha_s\right] \\ - \frac{\alpha_s}{\eta}\left[\hat{g}\frac{\eta-\lambda(1-\eta)}{1+\lambda} + w\eta - \pi(1-\eta)\right] \end{array}\right\} \tag{86}$$

Using (85) and (86) to eliminate μ; we find that the last term at the right-hand sides of (85) and (86) should be zero. Using $\hat{g} = \hat{\tau}_w w + \hat{\tau}_\pi\pi$; $w = (1-\eta)y - \hat{\tau}_w w$ and $\pi = \eta y - \hat{\tau}_\pi\pi$ we can write this term as

$$-\hat{\tau}_w w\left(\frac{\lambda}{1+\lambda}\right) + \hat{\tau}_\pi\pi\left(\frac{1}{1+\lambda}\right) = 0 \tag{87}$$

By using (45) to eliminate λ from (87), we find the Ramsey rule (50). Substituting (87) into (85) (or (86)), we arrive at (51).

To find $\frac{d\theta}{dg}$ we log-linearize the following first order conditions for s and v

$$\gamma'(s) = m(\theta)\left[(1-\eta)y - \frac{g}{sm(\theta)}\right]$$

$$c'(v) = \frac{m(\theta)}{\theta}\left[\eta y - \frac{g\theta}{vm(\theta)}\right]$$

These two equations are derived from (5) and (33) where we have used $w = (1-\eta)y - \hat{\tau}_w w = (1-\eta)y - \frac{g}{sm(\theta)}$ to eliminate w from (5) and $\pi = \eta y - \hat{\tau}_\pi \pi = \eta y - \frac{g\theta}{vm(\theta)}$ (because $\hat{\tau}_\pi \pi = \lambda \hat{\tau}_w w = \lambda \frac{g}{sm(\theta)}$ from (48) and (54)). Loglinearization of these two equations and the arbitrage condition (43) with respect to $s; v; \theta$ and g yields

$$\begin{pmatrix} \gamma''(s) - \frac{g}{s^2} & 0 & -\frac{m(\theta)}{\theta}\eta(1-\eta)y \\ 0 & c''(v) - \frac{g}{v^2} & \frac{m(\theta)}{\theta^2}\eta(1-\eta)y \\ s\gamma''(s) & -vc''(v) & 0 \end{pmatrix} \begin{pmatrix} \frac{ds}{dg} \\ \frac{dv}{dg} \\ \frac{d\theta}{dg} \end{pmatrix} = \begin{pmatrix} -\frac{1}{s} \\ -\frac{1}{v} \\ 0 \end{pmatrix}$$

The determinant of the matrix on the left hand side can be written as

$$\det = (1-\eta)\eta y \frac{m(\theta)}{\theta^2}\left[\gamma''(s)vc''(v)(1+\lambda) - g\left(\frac{vc''(v)}{s^2} + \lambda\frac{\gamma''(s)}{v}\right)\right]$$

Using Cramer's rule we determine $\frac{d\theta}{dg}$ as

$$\begin{aligned} \frac{d\theta}{dg} &= \frac{1}{\det}\begin{vmatrix} \gamma''(s) - \frac{g}{s^2} & 0 & -\frac{1}{s} \\ 0 & c''(v) - \frac{g}{v^2} & -\frac{1}{v} \\ s\gamma''(s) & -vc''(v) & 0 \end{vmatrix} \\ &= \frac{1}{\det}\frac{g}{v^2 s^2} s\gamma'(s)\alpha_{sf}\left[\frac{1}{(1-\alpha_{vf})\sigma_{cf}} - \frac{1}{(1-\alpha_{sf})\sigma_{\gamma f}}\right]. \end{aligned}$$

It follows that

$$\operatorname{sign}\left(\frac{d\theta}{dg}\right) = \operatorname{sign}\left(g\left(\frac{1}{(1-\alpha_{vf})\sigma_{cf}} - \frac{1}{(1-\alpha_{sf})\sigma_{\gamma f}}\right)\right)$$

To find $\frac{d\lambda}{dg}$, we note that the private split (45) can be written as

$$\lambda = \frac{\eta}{1-\eta}\frac{\alpha_{vf}(1+\hat{\tau}_w)}{\alpha_{sf}(1+\hat{\tau}_\pi)} \tag{88}$$

where we have used that $\hat{\tau}_w w = (1-\eta)y - w$ and hence $w = \frac{(1-\eta)y}{1+\hat{\tau}_w}$. Similarly $\pi = \frac{\eta y}{1+\hat{\tau}_\pi}$. Using this expression to eliminate α_{vf}/α_{sf} from the expression for the optimal tax structure (50), we find

$$\frac{\hat{\tau}_w}{1+\hat{\tau}_w}\frac{1+\hat{\tau}_\pi}{\hat{\tau}_\pi} = \frac{\eta}{1-\eta}\lambda^{-1}$$

Log-linearizing this equation and using the (55) and (56) to eliminate $\hat{\tau}_w$ and $\hat{\tau}_\pi$; we arrive at

$$\tilde{\tau}_w\alpha_{vf} - \tilde{\tau}_\pi\alpha_{sf} = -\tilde{\lambda}\frac{\hat{g}\alpha_{sf}\alpha_{vf}}{w\alpha_{sf} + \pi\alpha_{vf}} \tag{89}$$

where $\tilde{\tau}_i \equiv d\tau_i/(1+\tau_i) = d\hat{\tau}_i/(1+\hat{\tau}_i)$ $i = 1, 2$. Substitution of this expression in (80) yields $\operatorname{sign}\left(\frac{d\lambda}{dg}\right) = -\operatorname{sign}\left(\frac{d\theta}{dg}\right)$. QED

8 Table

	λ exogenous	λ endogenous
$\alpha_{vf} = 0$ case	$\frac{\partial\tau}{\partial w} = 0$ $\frac{d\theta}{dg} = 0$	same as λ exogenous case
$\alpha_{vf} > 0$ case	$\frac{\partial\tau}{\partial w} = \frac{\varepsilon_s}{\varepsilon_v} = \frac{\frac{\alpha_{vf}}{1-\alpha_{vf}}}{\frac{\alpha_{sf}}{1-\alpha_{sf}}}$ $\mathrm{sign}\left(\frac{d\theta}{dg}\right) = \mathrm{sign}\left(\frac{g}{\varepsilon_v\varepsilon_s}\left(\frac{\partial\tau}{\partial w} - 1\right)\right)$	$\frac{\partial\tau}{\partial w} = \frac{\alpha_{vf}}{\alpha_{sf}}$ $\mathrm{sign}\left(\frac{d\theta}{dg}\right) = \mathrm{sign}\left(g\left(\alpha_{vf} - \alpha_{sf}\right)\right)$

Table 1: summary of optimal taxes
for the Cobb Douglas formulation of search intensities ($\sigma_{\circ f} = \sigma_{cf} = 1$)